Nonlinear Vibration

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Lecture - 32
Cantilever beam with tip mass for combination resonance

Welcome to today class of Non-linear Vibration. In this class, we are going to continue what we have studied in the last class that is of a flexible beam or we are going to study the non-linear dynamics of a flexible beam. Today class we are going to particularly study about different types of beam, elastic beam, viscoelastic beam and magneto elastic beam.

Last class we have derived this equation of motion using both the principle that is Newton's second law and also by using this extended Hamilton principle and we have seen how we can solve for a system with principle parametric resonance condition. And today let us start with this combination parametric resonance condition.

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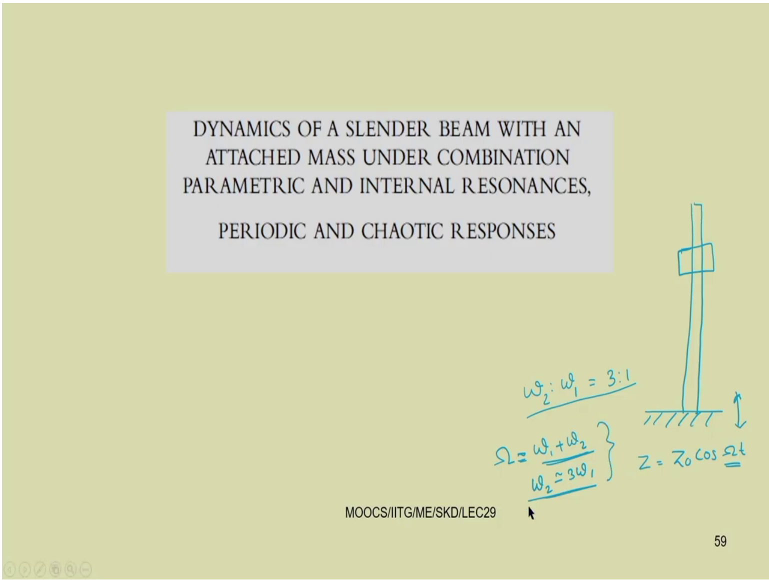

So, we have taken the same beam so, same cantilever beam, base excited cantilever beam with an arbitrary mass position. So, the mass is at arbitrary position. Putting this mass at arbitrary position or by adjusting this mass we can generate or we can have the frequency of the second mode and first mode in such a way that it can have two mode interaction or it may have three mode interactions also.

So, in case of two mode interactions we have continuing that thing. So, here we have taken this omega 2 by omega 1 equal to 3 is to 1. So, this gives rise to combination parametric resonance condition and here particularly we will be interested to see these periodic and chaotic responses in case of combination parametric resonance condition.

So, here the base excitations can be written in this form that is Z equal to Z 0 cos omega t. So, here this omega in case of the combination parametric resonance condition, so omega will be equal to omega 1 plus omega 2. So, as we are considering near to this omega 1 plus omega 2 so, here we can use some detuning parameter also to consider the nearness of this external frequency to this frequency omega 1 plus omega 2.

In addition to that so, we have taken this omega 2 nearly equal to 3 times omega 1. So, here also we will take one detuning parameter to see the nearness of this omega 2 to that of 3 times omega 1. So, by substituting this thing in the governing equation and proceeding as we have already seen in case of the method of multiple scale, we can get a set of first order differential equation.

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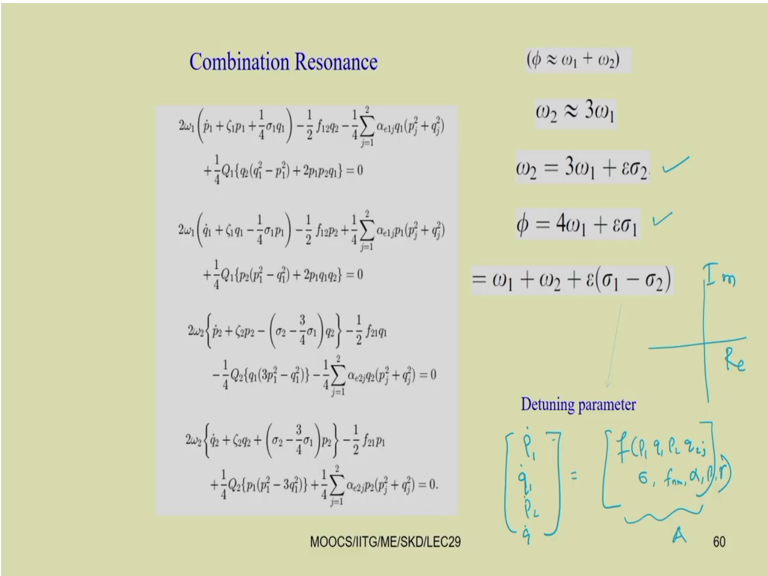

So, here also we have used this transformation that is p equal to a cos gamma and q equal to a sin gamma or p 1 equal to a 1 cos gamma 1, and q 1 equal to a 1 sin gamma 1 and similarly a

2 equal to a 2 cos gamma 2 and q 2 equal to a 2 sin gamma 2 and taking this omega 2 equal to 3 omega 1 plus epsilon sigma 2 and phi that is the external frequency equal to.

So, omega 1 plus omega 2 so, this is nearly equal to so, omega 2 equal to 3 times omega 1. So, that is why phi equal to 4 omega 1 plus epsilon sigma 1. So, here either you can write this phi equal to 4 omega 1 plus epsilon sigma 1 or you can write this is equal to omega 1 plus omega 2 plus epsilon sigma 1 minus sigma 2 so, these are the detuning parameter. We can get this equation.

So, this for this combination resonance, we have say get a set of equation and by putting this p q or p dot q p 1 dot q 1 dot p 2 dot and q 2 dot equal to 0, for steady state we can find the response and also by perturbing these equations. So, writing these equation in this form that is p 1 dot q 1 dot p 2 dot and q 2 dot equal to so, we can write this equation equal to equation so then by perturbing these things so, we can get the Jacobian matrix.

So by perturbing these equations so, let this equation is A. So, it is a function of so, this is a function of p 1, q 1, p 2, q 2 and system parameters like this sigma and other system parameters alpha 1 alpha 2 or beta 1. So, this coefficient of cubic nonlinearity coefficient of this forcing function, it will be a function of all these things forcing f and m and then alpha, beta, and gamma.

Now, perturbing this equation perturbing this part so, we will get the Jacobian matrix and by finding the eigenvalue of the Jacobian matrix then, you can study the stability. In this particular case when you are studying the stability then, for the fixed point response. So, if the eigenvalue of all the Jacobian matrix are in the left hand side of the s plane so, this is the real axis, this is the imaginary axis.

So, if they are in the left hand side of the s plane then the system is stable, otherwise it is unstable and at the bifurcation point. So, we know we have the static bifurcation and dynamic bifurcation. So, we can have the saddle node bifurcation then, we can have the pitchfork type of bifurcation or the transcritical bifurcation and Hopf bifurcations in case of the dynamic bifurcation of the system.

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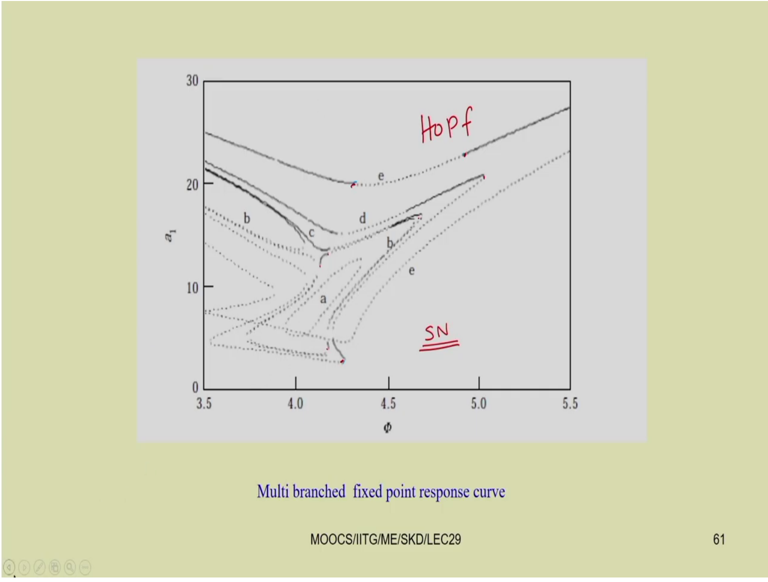

We can first see how we can obtain the multi branched fixed point response curve. So, this these are the multi branched fixed point response curve, we have obtained in case of a combination parametric resonance condition. Here it is plotted a 1 versus phi so many branches you can see, so many branches are present in this case.

Here clearly you can see this bifurcation point, so these two points these are the bifurcation points similarly here also we have bifurcation point. So here also so, we can have bifurcation point perpendicularly this bifurcation point you can recognise easily that these bifurcation points where there is a bend so, or here so where there is a bend and then changing stability.

So they are saddle node bifurcation point SN bifurcation point that is saddle node bifurcation point and these bifurcation points this point and this point. So, these are the Hopf bifurcation point Hopf bifurcation point and this is saddle node bifurcation point. So, we have a combination of Hopf and saddle node bifurcation point in this case for different value of the system parameters then, after getting this response plot.

So, how you are getting this response plot? You can get this response plot by solving the set of equation first order equation by putting equal to 0, where the resulting equations are this algebraic and transcendental equation and so, you have four such equation. So, these four equations you can solve by using this Newton's method to find this frequency response plot.

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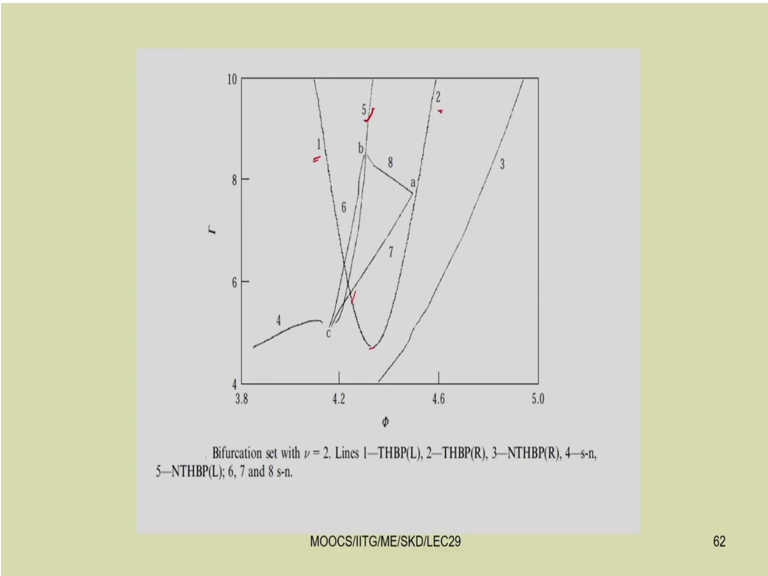

Similarly, so if you are interested so you can plot the bifurcation diagram. So, here the bifurcation diagram is plotted we in the phi and gamma plane. Phi is the frequency of external excitation and gamma is the amplitude of external excitation. So, you can see for example; let us take these 1 and 2 curve, the curve marked with 1 and 2. So, below these things so you can find this point below this thing always the system is stable.

So, this 1 and 2 are trivial Hopf bifurcation point to the left and trivial Hopf bifurcation point to the right. So, these two these lines shows the Hopf bifurcation point observed in the system. So, this line 3 the non trivial Hopf bifurcation point in the right side and the 4 curve, curve 4 shows so, this is curve 4.

So, it shows a non trivial Hopf bifurcation point in the right side, 4 shows this SN bifurcation that is the saddle node bifurcation point. So, 4 is the saddle node bifurcation, 3 is non trivial Hopf bifurcation point right then, 5 is non trivial Hopf bifurcation point left. So, 5 let you see where is the line 5 so, this is the line 5 so it is the non trivial Hopf bifurcation point towards left and 6 is the 7 and 8 6 7 and 8 are the saddle node bifurcation point.

So, many bifurcation points you are getting so by changing the or by plotting all these bifurcation point in the phi and gamma plane so, we can get the bifurcation set. So, this is the bifurcation diagram or bifurcation set and by plotting this bifurcation set you can know the critical value at which the system can be operated from this thing you can find where the system has to be.

So, for example, in between 1 and 2 the system has so as it is Hopf bifurcation point as 1 and 2 are trivial state Hopf bifurcation point; so, clearly so you can have periodic response in between 1 and 2. Similarly, between these 3 and 5 so 5 is also non trivial Hopf bifurcation point so, 5 is the non trivial Hopf bifurcation point and 3 is also the non trivial Hopf bifurcation point.

So, these 2, 3 and 5 are non trivial Hopf bifurcation points. So, this is 3 and this is 5 so in between also you can have this periodic response and so within 2 and 5 so, you can have a set of periodic response; some of them will have their route in the trivial state and some of them will have their route in the non trivial state. So, due to presence of all these periodic response, so, there may be a chance that that period doubling route to chaos or some other different type of route to chaos also can be observed in the system.

This is a very very critical system though the system is very simple that is this is a simple cantilever beam with an attached mass and it is excited at its base with a harmonic force, but the system yield a number of bifurcation points and we can see in addition to the fixed point response we have periodic quasi periodic and chaotic response.

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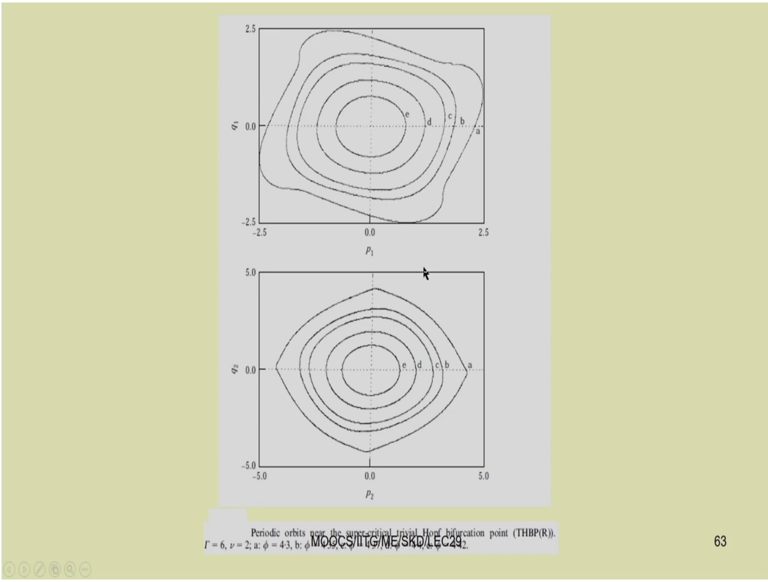

Near this Hopf bifurcation point trivial state trivial state Hopf bifurcation point so, you can see there are so for a different value of phi by changing different value of phi so, we can get different periodic response. So, you just see these are different periodic response, and here you can see at a equal to 4.3, then a is 4.3 so, b is phi equal to 4.35 then 4.37. So, you are going to increase the phi here. So, when you are increasing phi so, you can see the periodic orbit that the orbit reduces.

So, initially you have a very large orbit, which contains the harmonics so, due to the presence of harmonics it is no longer the circular in nature, so it contains harmonics so this bending and so the shapes are there or irregular shapes are there; slowly by increasing this phi. So, you can see as you are going away from the combination resonance condition, the shapes are getting more and more a regular shape and at 4.92 you can see you have a periodic response close periodic response.

So, it is plotted in p 1 q 1 and p 2 q 2 so, this was the first mode and this was the second mode periodic response. So, you can have this periodic orbit and with the increase or with the decrease in the phi value increase in the phi value it will go from e to so, e highest phi value and a is the lowest phi value. So, it will decrease from e to a, so by decreasing these frequency so we can getting so we are getting bigger and bigger orbits.

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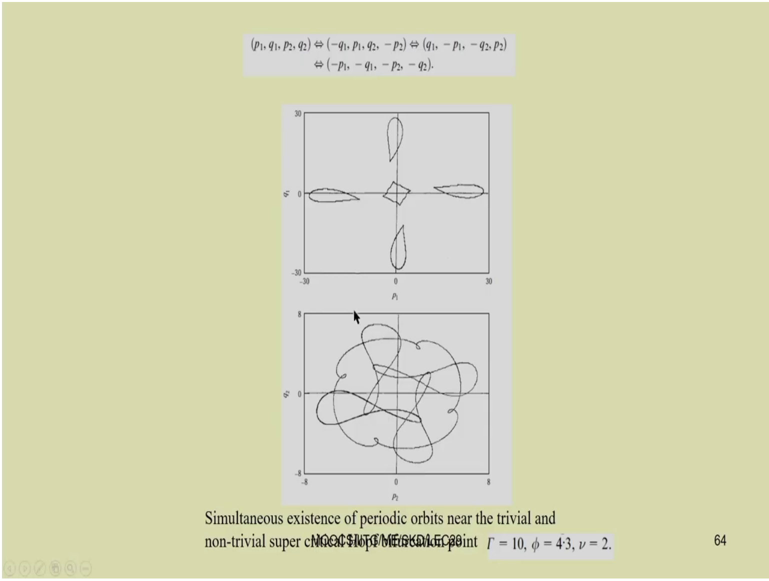

Similarly, so here you can see clearly these simultaneous existence of periodic orbit near the trivial and non trivial super critical Hopf bifurcation points. Already we have seen so, it is in between, in between, we have this trivial state Hopf bifurcation point and non trivial state Hopf bifurcation point that is between 2 and 5.

So, there are symmetrically so, you can see symmetrically four periodic orbits are there and so, there are five orbits periodic orbits simultaneously existing, and by so here you can see the transformation so, from the equation p q equation you can see under this transformation. So, the equations will remain same; that means, p 1, q 1, p 2, q 2 so, if you replace it by minus q 1, p 1, q 2 and minus p 2 or by q 1 minus p 1 minus q 2, p 2 or by minus p 1 minus q 1 minus p 2 minus q 2.

The equation remain unchanged due to the presence of if we have a square term then q 1 square plus q 1 square and minus q 1 square will be same, that is why due to the presence of this type. So, these terms in the in that equation so reduced equation so you can see; so it is invariant under this transformation.

As it is invariant under this transformation so you can have so four different type of response. So, these are the four symmetric responses present in the system. So, later you can see that the symmetry breaking, so by changing the system parameters.

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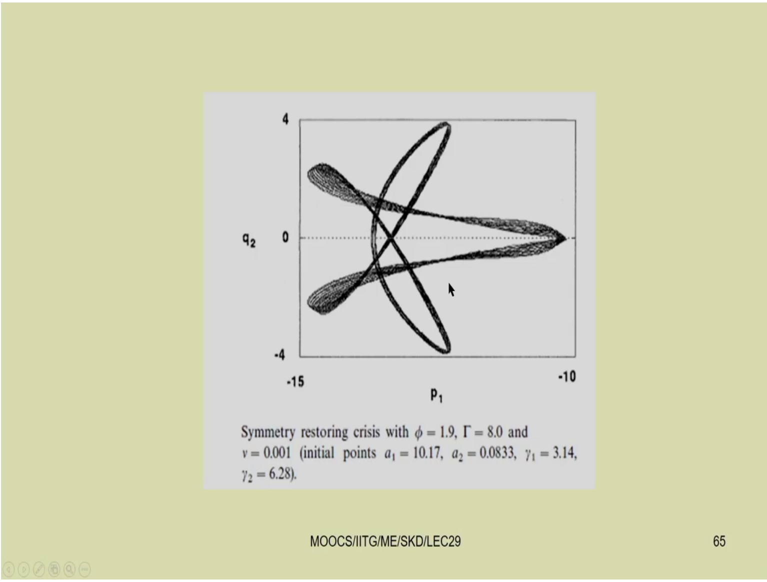

So you can see, so symmetry restoring crisis we can get. So, the symmetry will restored in this type of crisis so, initial point by taking these initial points so we can plot these curves and you can see the symmetry restoring crisis. Already you know a crisis occur when a chaotic orbit come in contact with an unstable fixed point or unstable periodic response.

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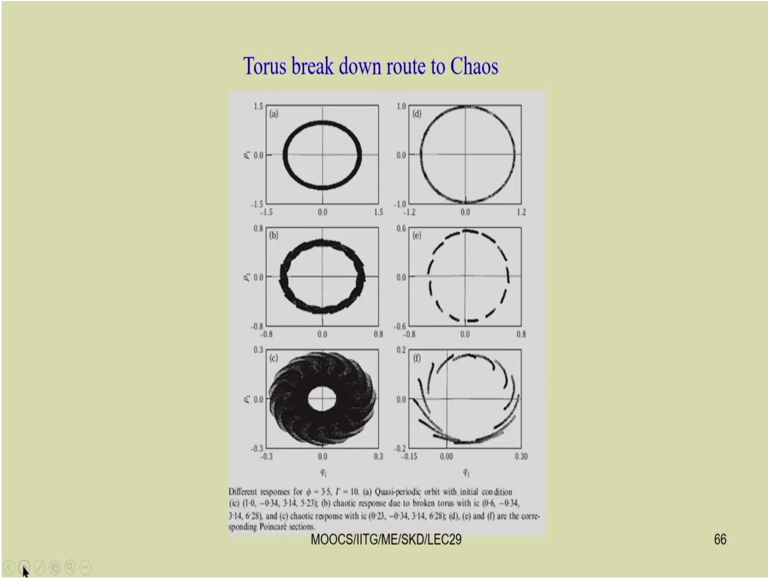

So, here we are getting the symmetry restoring crisis. So, here another thing also you can see so initially we have these torus or periodic orbit initially then, it converted to torus. So, here you can see this torus and then this torus get breakdown so we are getting the torus breakdown route to torus breakdown route to chaos.

So, initially we have a torus then this torus get breakdown and finally, it leads to a chaotic orbit. So, by slightly changing the system parameters, we can get this torus breakdown route to chaos in this combination parametric resonance case.

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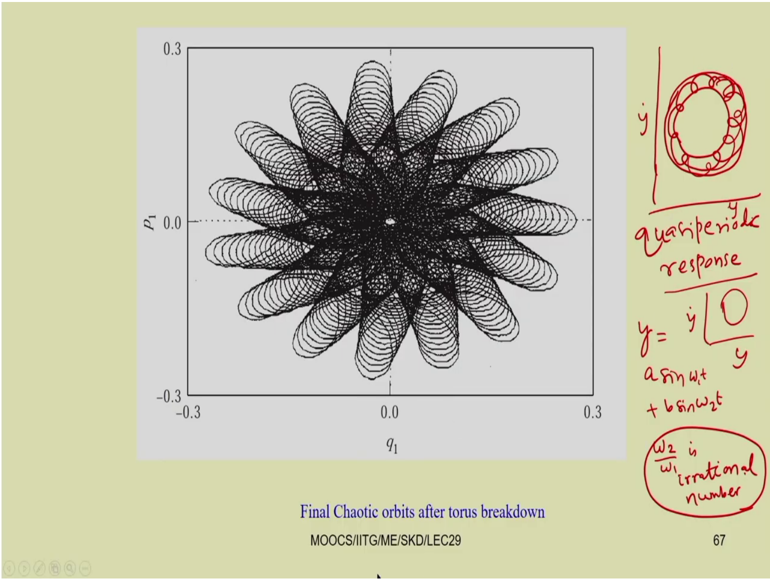

Similarly, you just see finally this is the chaotic final chaotic orbit after the torus breakdown. You can recall that we can get a torus so, if the response is quasi periodic for example, in case of a quasi periodic response, so you can write your y equal to a sin omega 1 t plus b sin omega 2 t here this omega 2 by omega 1 is irrational number, this is irrational number.

So, if it is irrational number, so then we can get a quasi periodic response quasi periodic response. So, these are in case of quasi periodic response, we can get the torus so, in case of torus for example, so you have two loop so if you plot the phase portrait the shape will be like this. So, inside you have two loops and they are connected by this. So, if you draw the phase portrait so the shape will be like this is the phase portrait that is your y versus y dot.

So, now you can draw the Poincare section, so in case of Poincare section so this y versus y dot curve will be a closed loop. So, in case of Poincare sections so you can get a closed loop in case of the quasi periodic response or this torus, so this torus get breakdown by changing the system parameter and finally, you are getting a chaotic orbit. So, this is the final chaotic orbit in case of the torus breakdown.

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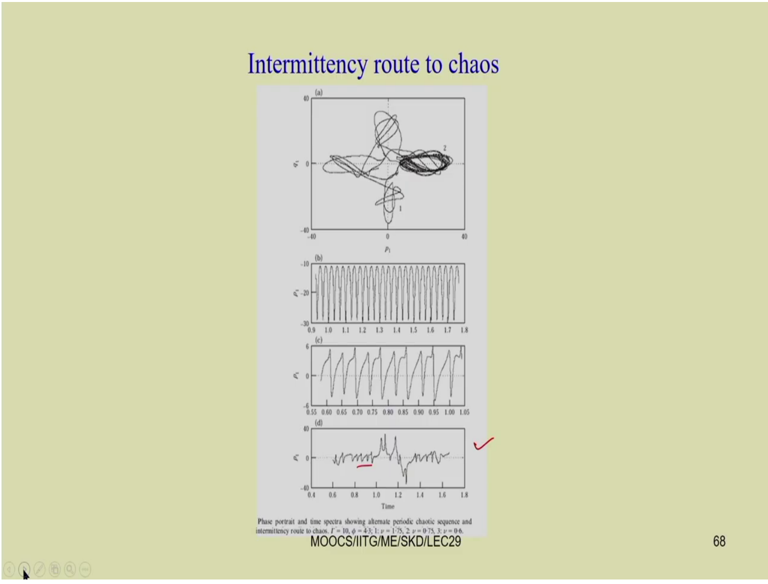

Similarly, we can have the intermittency route to chaos intermittency route to chaos. So, you can see initially we have this periodic response and this periodic response by changing the system parameter. So, you can see, this is converted to a chaotic response where this may revisit this chaotic response may revisit the initial periodic response intermittently.

So, you can have intermittently the periodic response some part of this thing will be periodic and then again it suddenly wonders and it can go to other orbits. So, you can recall that initially we have four symmetrically placed periodic response, a chaotic attractor can be so due to the presence of so many bifurcation point; so here a chaotic attractor or chaos chaotic attractor may also exist which can connect all these attractor all these periodic attractor can form these intermittent route to chaos.

So, this way we can absorb this intermittency route to chaos in this simple system, so you just see initially we have the periodic response.

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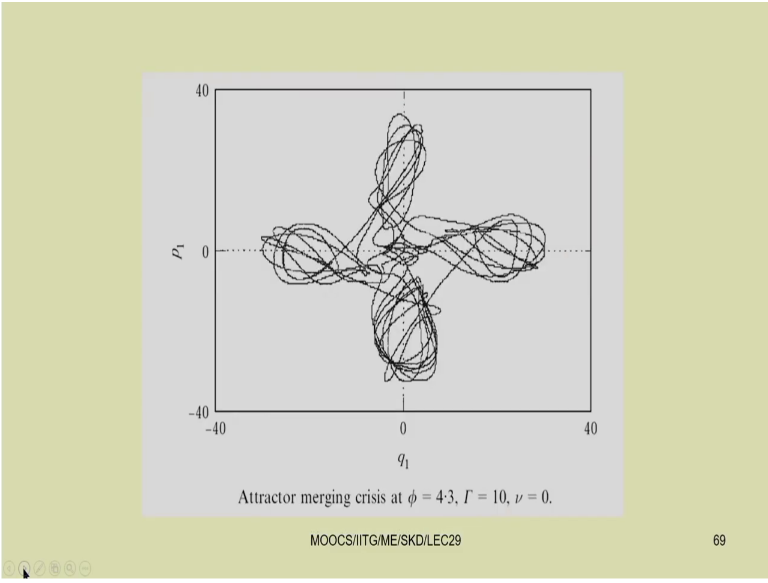

Now it wanders it move from one place to another place and we are getting these intermittency route to chaos in this case.

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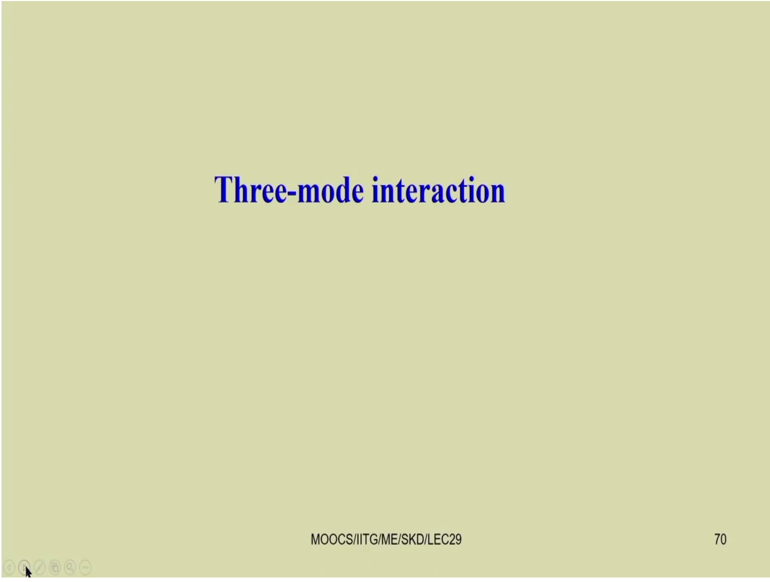

So, this way we have seen many different periodic quasi periodic and chaotic responses and the routes to chaos that is three different routes to chaos we have seen. Till now, so we have seen the period doubling route to chaos then, these torus doubling route to chaos and torus break down route to chaos also and we have seen this crisis that is intermittency route to chaos also we have seen and here also we have seen this symmetry restoring crisis.

So, initially it was asymmetric and this chaotic response also it makes it to be chaotic, but this chaotic response is symmetric also. So, that is why it is symmetry restoring crisis and further we have seen this torus break down route to chaos and intermittency route to chaos. So, let us see another example or another type of system so, where we can take the three mode interaction.

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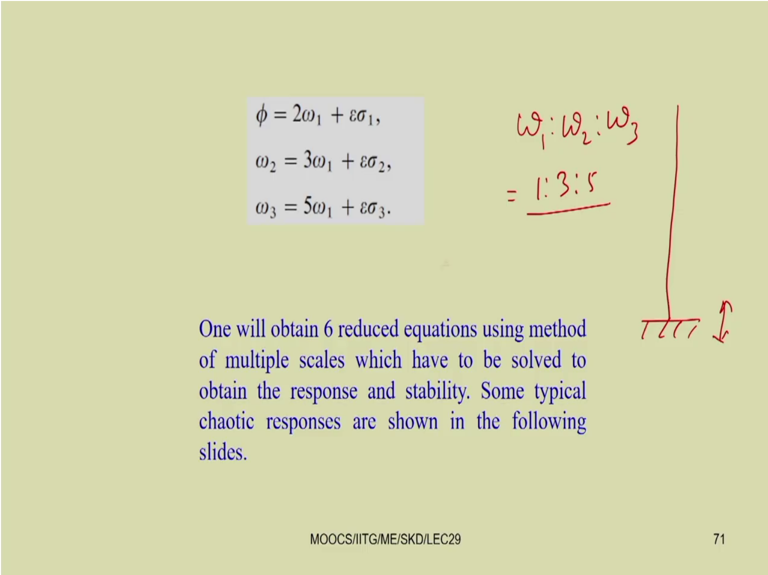

So, we have the same cantilever beam same base excited cantilever beam bought with three mode interaction. So, three mode interaction means, we have taken. So, this phi that is equal to external frequency equal to twice 2 times omega 1 plus epsilon sigma 1 and here we are taking for example, it is omega 1 is to omega 2 is to omega 3 can be taken in this way 1 is to 3 is to 5 1 can take 1 is to 3 is to 5 or 1 is to 3 is to 9, in this case we have taken it is equal to 1 is to 3 is to 5.

That means, the second mode is nearly three times the first mode and the third mode frequency is nearly five times the first mode frequency by using this detuning parameter so, we can write this phi equal to 2 omega 1 plus epsilon sigma 1, omega 2 equal to 3 omega 1 plus epsilon sigma 2 and omega 3 equal to 5 omega 1 plus epsilon sigma 3.

So, here we can get 6 reduced equation previously we have taken only two modes. So, there we got 4 equations, so in case of (Refer Time: 22:31), so they have taken one mode only so, they have the two reduced equation. So, depending on the number of modes you are considering, the number of equations reduced equation will be increased. So, here as we are taking three mode interaction so here we have 6 reduced equations;

So, one will obtain 6 reduced equation using method of multiple scale, which have to be solved to obtain the response and stability. Some typical chaotic responses are shown in the following slides.

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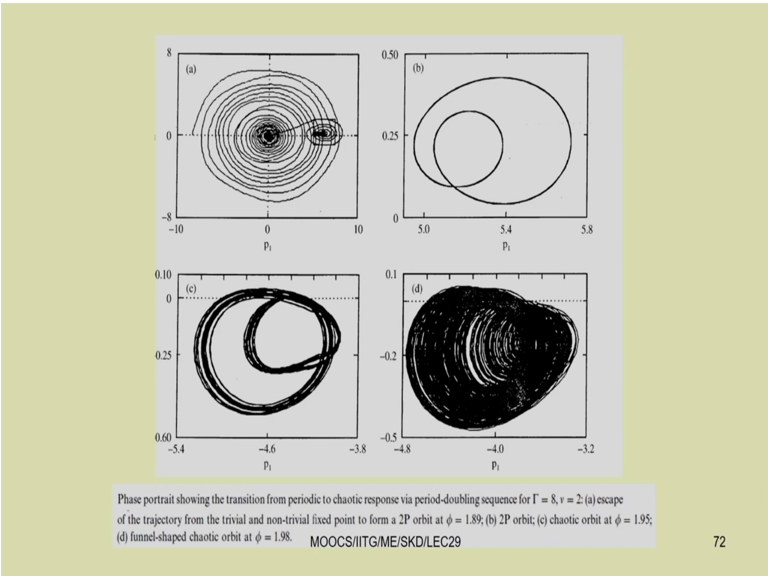

So, here only we are showing some very typical chaotic orbit. So, you can see the transition from the periodic to chaotic response via a period doubling sequence. So, initially we have a periodic response then, it becomes period doubling then period doubling route to chaos.

So, we have seen these things. So, in the first figure you can see the escape of the trajectory from the trivial and non-trivial fixed point response to form a 2 periodic orbit at phi equal to 1.89 initially, so this is the 0 0 line you can see. So, this is the trivial state and we have a periodic response. So, it is escaping.

So, you can see that it is escaping from this one so escape of the trajectory from the trivial and non trivial fixed point. So, it will come out and now it will form a 2 periodic orbit at phi equal to 1.89. So, here this is we have shown the 2 periodic orbit and then, if you change further this system parameter, so it will lead to this period doubling route to chaos.

So, chaotic orbit at phi equal to 1. So this is 9 5 and then funnel shaped chaotic orbit; so you can see a funnel shaped chaotic orbit at phi equal to 1.98. So, here we just see we can we have seen different type of chaotic orbits like, you have seen previously the Lorenz attractor or the Rousselot curve.

So, similarly in the simple physical system so, this is a simple physical system that is a base excited cantilever beam, if we are considering different model interaction; so, we can see different type of chaotic orbits present in the system.

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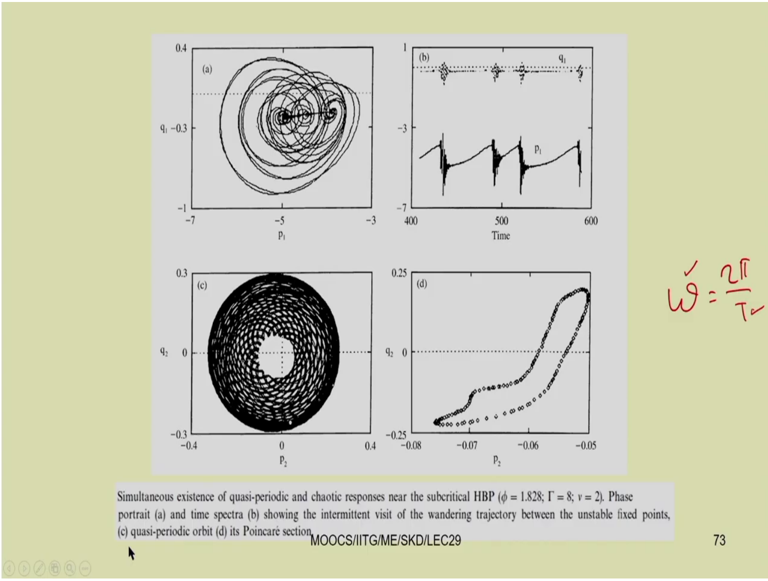

Here again you can see simultaneous existence of quasi periodic and chaotic responses near the subcritical Hopf bifurcation point. So, we have a subcritical Hopf bifurcation point in three mode interactions I have not shown you the fixed point responses, but you can go through the detail paper I will give you the list of papers from which all these things have been taken. So, these are part of my PhD work.

So, here you can see simultaneous existence of quasi-periodic and chaotic responses near the subcritical Hopf bifurcation point phi equal to 1.828; gamma equal to 8 and nu equal to t 2. So, phase portrait a and you just see the time response here so, you have a periodic response slightly periodic then it is wandering and from the time response clearly you can see that the this chaotic orbit is wandering between different fixed point responses ok.

So, wandering trajectory between unstable fixed points, here you can see so this is a quasi-periodic orbit clearly the this is a quasi periodic orbit. So, to know whether it is a quasi periodic orbit or not so, you can draw the Poincare section; so this is the Poincare section plotted for this p 2, q 2 and how to draw the Poincare sections already I have explained.

So, you can take the lowest frequency or lowest not lowest frequency you just take the lowest time period. So, taking the lowest time period lowest time period means, so omega equal to 2 pi by t so 2 pi by t corresponding to lowest time period you have the highest frequency, so, you can get the FFT and you can get what are the frequency present taking the highest frequency.

So, you can find what is the lowest time period? So taking this time period you can sample the time response and from that sample so, you can get all the points on the curve so, by taking all these points if you plot then for a quasi periodic orbit so it will be a closed loop. So, here clearly we have seen a closed loop here.

So, due to the presence of these quasi periodic orbits so, you have seen the Poincare section to be in the shape. So, here what we have seen? So, simultaneous existence of quasi periodic orbit and chaotic response, so at the same value of phi so, you can see there we have the quasi periodic orbit and also the chaotic response.

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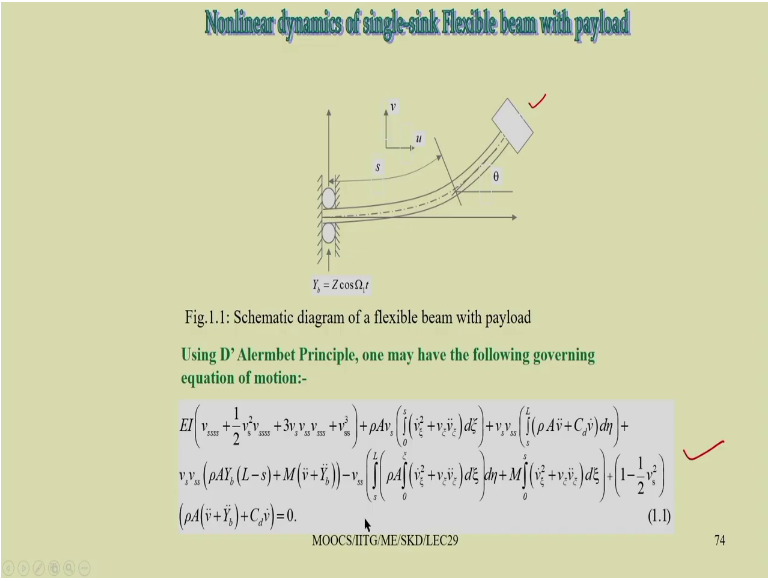

So, both of them are present simultaneously, in these 1 is 2, 3 is to 8 resonance condition. You can see in details all these responses in the references what I will show you after a few minutes. Let us take a simplified model. So, I have shown you a complicated model with internal resonance that is two mode interaction and three mode interaction.

If we take a simple cantilever beam also single link so, which can be used as a single flexible link with a payload, let us take the system and let it is subjected to this base excitation in vertical direction.

So, previous case we have taken a vertical cantilever beam. So, here we are taking a horizontal cantilever beam by changing the orientation also your equation of motion will be different. So, here by using this D’ Alermbet principle, so you can have this is the equation of motion.

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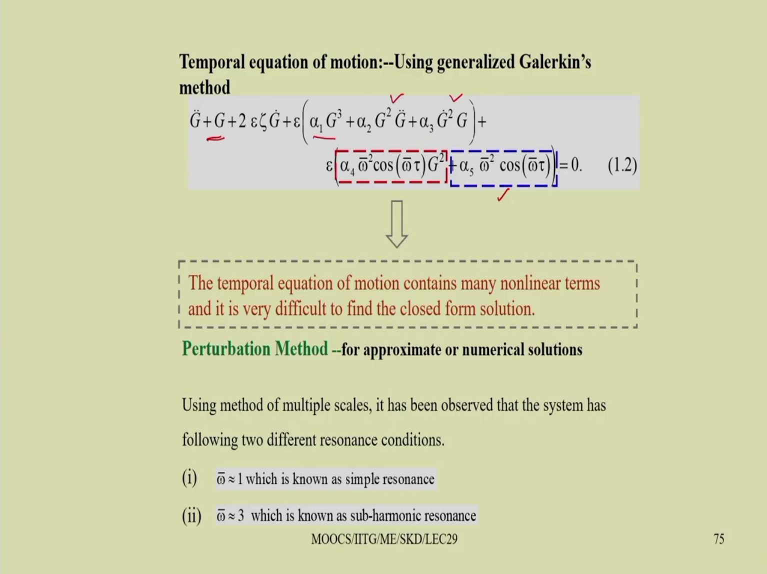

So, this equation motion here now by applying this Galerkin principle. So, you can see that it is reducing to this form that is G double dot plus G so, here previously I have taken q and here it is taken G.

So, this is the work of my PhD student Dr. Barun Patiher so, who is now a faculty member at IIT Jodhpur using this Galerkin method, he has reduced this equation to this form that is G double dot plus G plus 2 epsilon zeta G dot plus epsilon into alpha 1 G cube plus alpha 2 into G square G double dot plus alpha 3 into G dot square G here only single mode approximation is taken.

So, by taking the single mode approximation, we can get these non-linear terms cubic non-linear terms so, this part is the cubic non-linear term and these two are the inertia non-linear terms. So, in addition to that so we have in this case you can see this term last term that is phi alpha into omega square cos omega t so, this is a forcing term.

So, you just see it is not coefficient of this G so, that is why it is this is give rise to an external forcing term and for this part that is epsilon alpha 4 omega square cos omega t into G square. So, into G square as G is the time modulation, whose coefficient is a periodic function.

So, that is why the system is parametrically excited and also it is subjected to external forcing. So, it is a direct and parametrically excited system. As the temporal equation of motion contains many non-linear terms and it is very difficult to find the closed form solution.

So, one can go or use this perturbation method. So, there are several way you can use this perturbation method or you can use this harmonic balance method also. So, I will show you use of harmonic balance method in some other examples also, also you may use this method of normal form or Lindstedt Poincare technique in this case. So, using this method of multiple scale so, we can see we have two different resonance condition.

So, one is the simple resonance condition and second one is the simple resonance condition. When so, this omega bar equal to coefficient of G that is equal to 1 when omega bar equal to 1. So, this is known as the simple resonance condition, and also we can have the sub harmonic resonance condition, when omega bar nearly equal to 3 sub harmonic resonance when omega bar nearly equal to 3.

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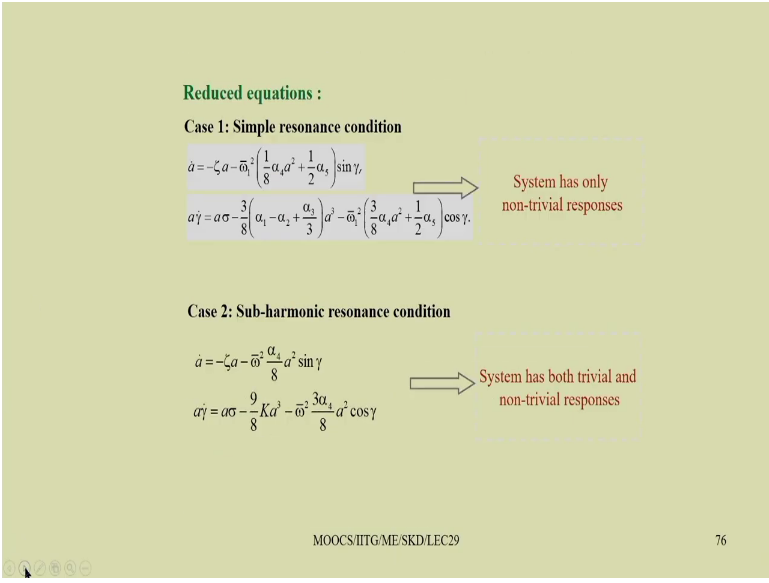

So, in this case in case of simple resonance so we can have; we can have two equation two reduced equation. So, system has only non trivial response so here you can see by substituting this a dot equal to 0. So, you cannot take a common from here because, this last term that is half alpha 5 sin gamma so does not contain a term. So, you cannot take a common to make it equal to 0 so, there is non trivial state and the system has only non-trivial response.

So, by substituting a dash equal to 0 and gamma dash equal to 0 so, you can have a equation. Similarly, for sub harmonic resonance condition the system has both trivial and non-trivial response because, here you can take a common. So, put a dash a dot equal to 0 by taking a common so, this becomes minus zeta minus omega bar square alpha 4 by 8 a sin gamma.

So, either a will be 0 or this part will be equal to 0 so, as a equal to 0 is a solution a trivial state present in this system. So, also in addition to that we have the non trivial state, trivial state means a equal to 0 that is the amplitude becomes 0.

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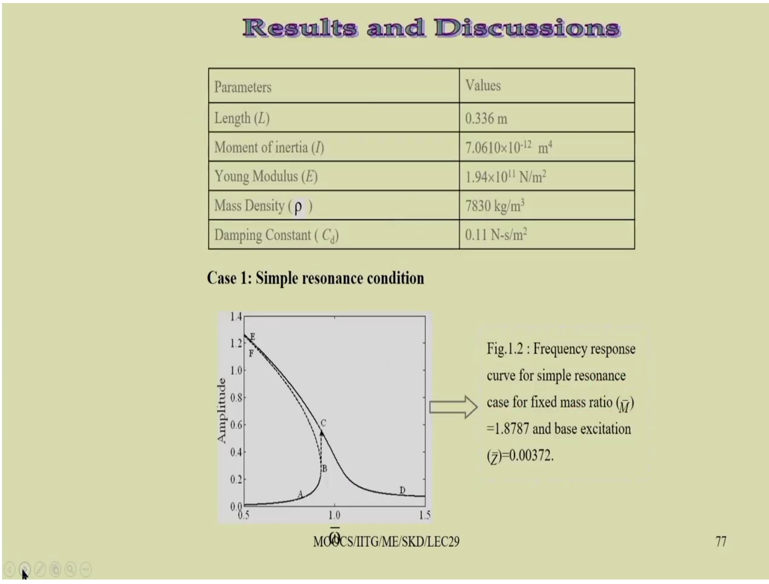

Now, we can plot taking all these system parameters. So, you just see here the length is taken to be 0.336 metre and then moment of inertia I equal to 7.0610 into 10 to the power minus 12 metre 4th. So, you may note that the dimensions are taken very small. So, if you slightly vary the dimensions the natural frequency that will be large variation in natural frequency.

So, they are very sensitive to the variation in the dimensions of the system. So, you have to take proper dimension system to get the required results. So, the Young's modulus is taken to be 1.94 into 10 to the power 11 Newton per metre square, mass density equal to 7830 kg per metre cube and this damping constant C d is taken equal to 0.11 Newton's second per metre square.

So, you can see a typical frequency response plot in case of the simple resonance condition. So, here there is no trivial state. So, trivial state is not there so we have all non all the points and in the non trivial state. So, now by increasing these frequencies so, you can see we have a saddle node bifurcation point here saddle node bifurcation point here, and it will jump from this B to C.

So, you can observe a jump of phenomena in this case and due to this jump the beam may break again so, it will follow then it will follow this A, B, C and further increase it will go to D so, there will be a jump here. But, if you reduce the frequency that is it will go from d, so it will follow this path D, C, E and so, it will jump from this point.

So, as this is a saddle node bifurcation point. So, from here it will jump down to this lower branch. So, you can see this hysteretic effect; that means, while you are taking the frequency upward direction while you are sweeping the frequency it follow A, B, C, D, where it will a jump of phenomena. So, while you are reducing the frequency it will not follow the same path, but it will follow another path that is D, C, E and it will jump down to a point here in the lower branch.

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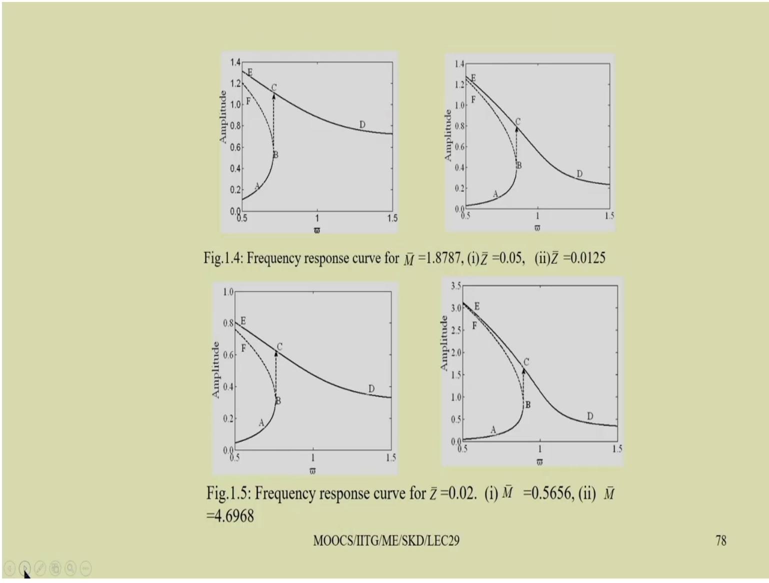

So, by changing the system parameters, so now, for example, you just take this mass ratio different mass ratio and also different. So, you can take different mass ratio and also different amplitude of excitation and one can plot. So, these bifurcation points there will be only change in these bifurcation points.

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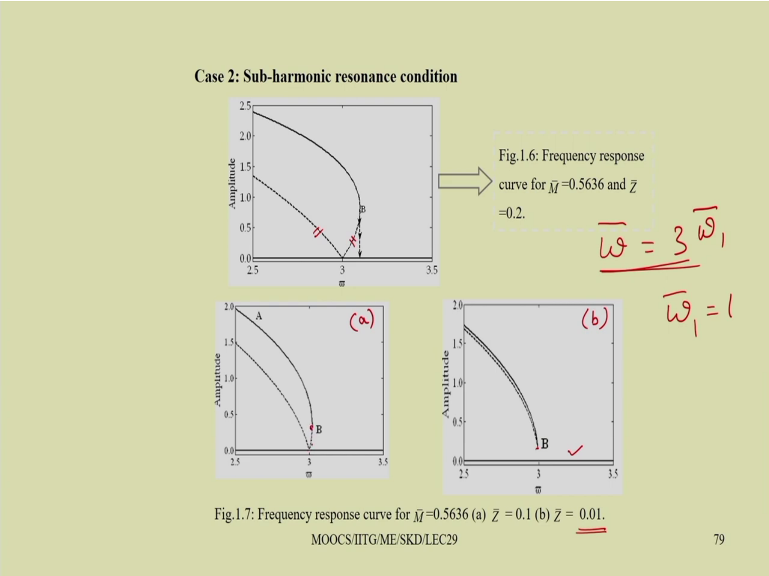

And one can get the required result by properly checking or properly finding the system parameters, in case of the sub critical sub harmonic resonance condition. So, you can see both trivial and non trivial responses are there. Here you can see at 3 so, omega bar equal to here, so you just see exactly at three as we are taking omega two omega here you are considering the single mode approximation and sub harmonic resonance condition.

So, your omega bar you have taken nearly equal to 3. So, as you have taken omega bar nearly equal to 3 that is 3 times the first natural frequency, the first natural frequency we have taken the non dimensional first natural frequency equal to 1. So, that is why it is equal to 3 at 3.

So, the system this unstable branch and this unstable branch meet at this point, and we have this curve and here so, you have a clearly you have a saddle node bifurcation point. And with increase in this Z value so for example, by increasing Z or decreasing Z here we have decreased the Z value a in curve a Z equal to point 1 and in curve b so, Z equal to 0.01 so it is decreased.

So, by decreasing this thing so, this is curve a this is curve b so the previous one frequency response curve M bar equal to 0.563 and Z bar equal to 0.2. So, here Z bar equal to 0.2 in this case Z bar equal to 0.1 and in this case Z bar equal to 0.01 so that means, for very less value of amplitude of external excitation. So, you can have the trivial state.

So, the trivial state and non trivial state do not colossus at this point 3, you have clearly a saddle node bifurcation point here and by increasing this Z bar. So, you can see so this point single saddle node point. So this branch becomes a it takes a different shape. So, here you can see you have a saddle node bifurcation point here and also a saddle node bifurcation point, but this is unstable here.

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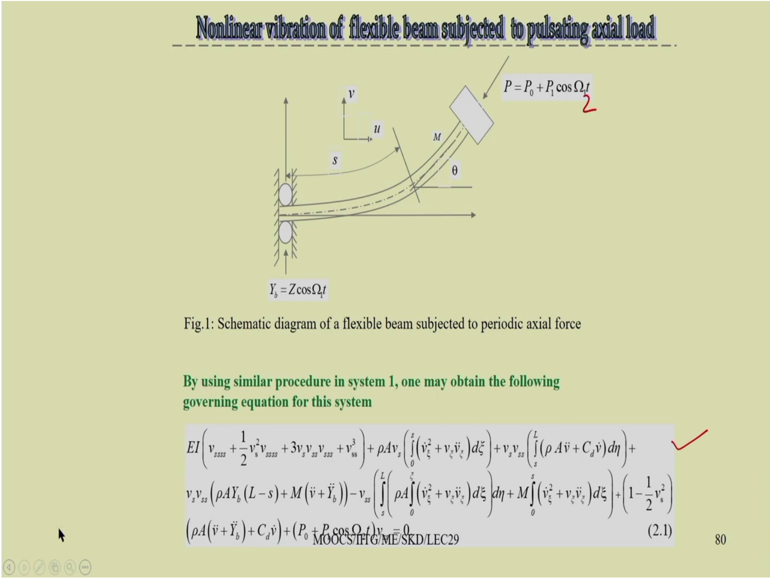

So, this way you can see different responses in case of a simple cantilever beam base excited cantilever beam. So, there is no external load acting in the system, the response we have obtained that is simple resonance condition and sub harmonic resonance condition.

But in the same system, so if we are applying now let us apply a load P equal to P 0 plus P 1 cos omega t. That is one axial load if we are applying to the system so for example, this is in case of a follower force acting on a system. So, in that case, the system response will be modified to this form and by applying this Galerkin method by applying Galerkin method, so it is reduced to this equation.

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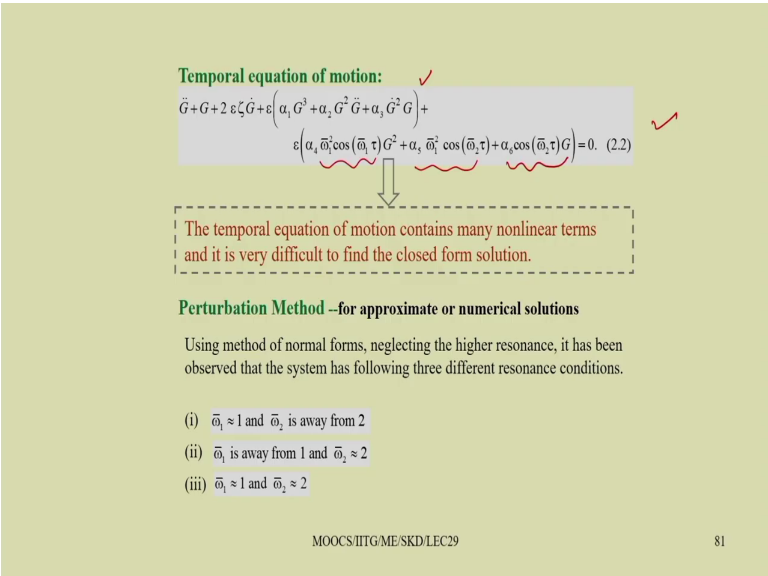

So, here you just see in up to this thing the response become the equation is same. Now, we have this epsilon term that is epsilon alpha 4 omega bar square cos omega tau into G square plus alpha 5 omega 1 square cos omega 2 tau plus alpha 6 cos omega 2 tau into G. So, this term is similar to this Mathieu Hill type of equation, presents in Mathieu Hill type of equation where this periodic term is coefficient of G.

So, here it is coefficient of G square and this term is the simple external excitation or direct excitation. So, this temporal excitation of motion contain many non-linear terms. So, that is why already you have seen. So, we cannot have a closed form solution here. So, here in this case if you go on deriving using this method of multiple scale you can see so, there are several resonance conditions.

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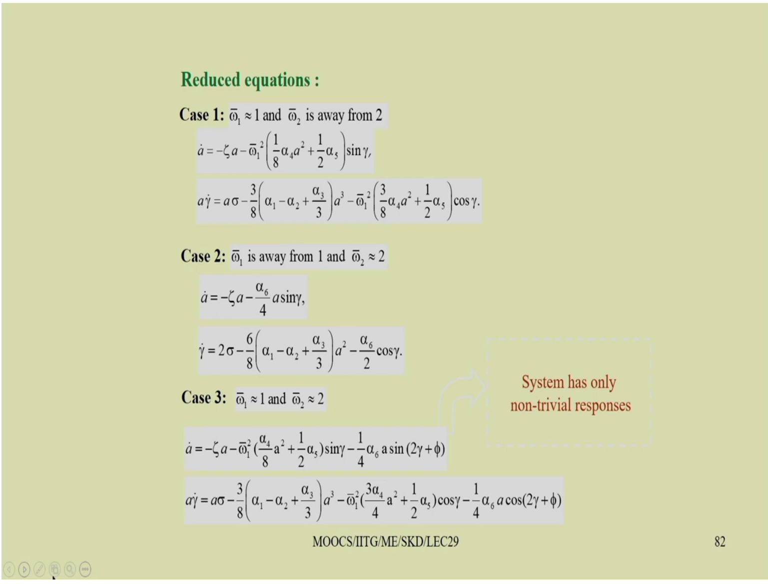

So for example, so we have these 3 resonance condition in this case. So, the first case is omega nearly equal to 1 and omega 2 bar is away from 2. What is omega 2 bar and what is omega 1 bar? So, this thing you can see omega 1 bar so, it is coming from these external excitation omega 1 and here we can put. So, this axial forcing actually we can put this is omega 2 so this omega 2. So, we have to frequency excitation in this case.

So, previous case we have taken only single frequency excitation that is base excited, but now we have two frequency excitation that is base is also excited with a frequency omega 1 and we have applied an axial force at the end with a frequency of omega 2. As we have two frequency excitation in this case. So, we have this omega 1 bar equal to 1 and omega 2 bar is away from 2.

Similarly, omega 1 bar is away from 1 omega 2 bar nearly equal to 2 and we can have simultaneous omega 1 bar equal to 1 and omega 2 bar equal to 2, there is no restriction in omega 1 bar and omega 2 bar independently one can control these two parameters.

So, that is why, in 1st case you can take one can take this omega 1 bar that is base excitation nearly equal to 1, but this omega 2 is away from 2 due to this axial loading that is no resonance condition; 2nd case omega 1 bar is away from 1 and omega 2 bar is taken nearly equal to 2 and, 3rd case we have this omega 1 bar equal to 1 and omega 2 bar nearly equal to 2.

1st case so, we have this these are the reduced equation, in 2nd case we have this reduced equation, and in 3rd case so, these are the reduced equation. So, system has only non trivial response in this 3rd case. So, we will see all these three cases some results there.

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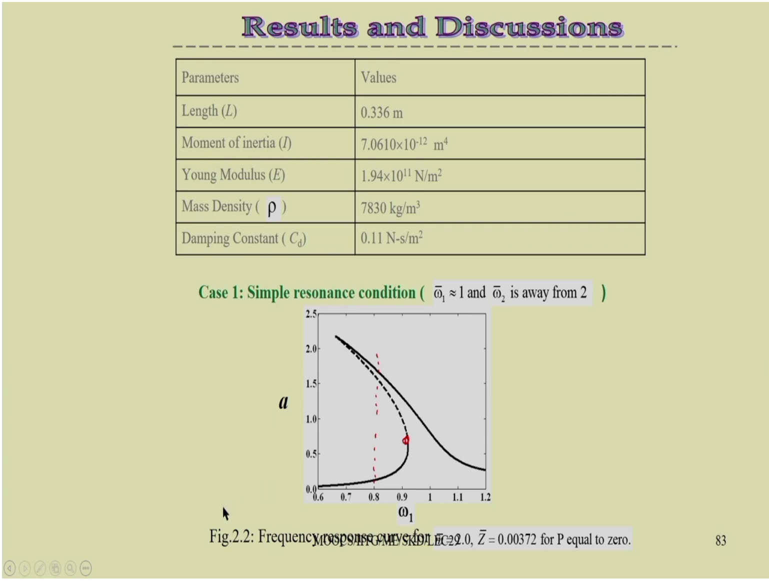

So, taking these system parameters similar system parameter, so we can study these three resonance condition. So, in case of simple resonance condition you just see the response is like this. So, it is having the sub training effect if you can compare this equation to that (Refer Time: 42:16) of equation.

So, we can see. So, it has a sub tuning type of response, and here again we can have the saddle node bifurcation point and you can see at omega 1 around 0.8. So, if you see if you take around 0.8 so, it has two stable and one unstable response so that means, the system has a bistable region along with the presence of a unstable fixed point response. So, to which branch it will go so depends on the initial conditions. So, that is why one has to study the basin of attraction.

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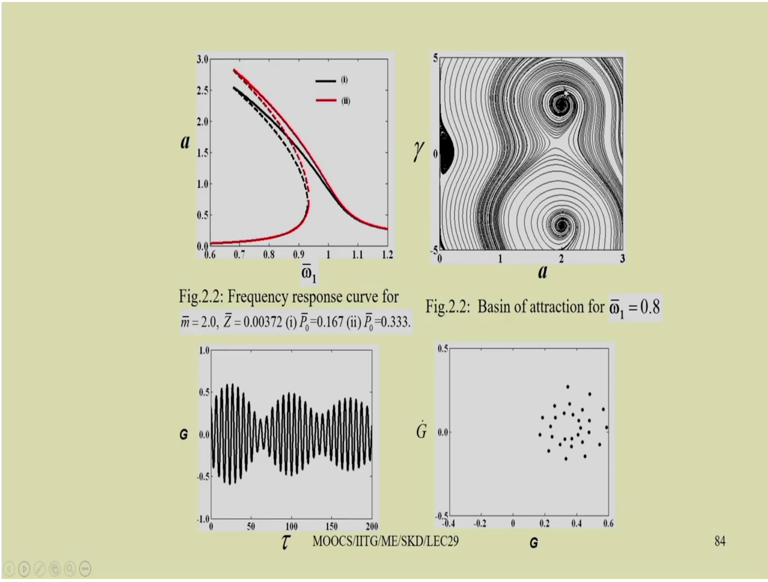

So, here we have plotted the basin of attraction in that case and you can see. So, we can clearly so, these are 2 fixed point response and this is you can find this is the saddle node point this gives rise to the saddle node point and these and these. So, this is the so this point is the replica of this one.

So, that is at 2 and gamma equal to so in a gamma plane we have plotted. So, this is also a fixed point and this is a fixed point. So, this fixed point and this fixed point are the same fixed point, but with different gamma and this is the at 0. So, we have the trivial. So, near 0 also we have a fixed point.

So, this is exactly not 0, but a smaller value. So, another branch is here that is near this branch is near this two. So, that way you can plot it and you can verify the basin of attraction and one can plot the time response and clearly one can absorb that this beating type of phenomena is occurring in this case also and for a different value of P 0, P 0 for example, P 0 equal to 1.67 0.167 and 0.333.

So, P 0 is so this is higher value of P 0 what is P 0? So, you can see we are applying a force. So, this force is P 0 plus P 1 cos omega t. So, this is the; this is the fixed part of the periodic response and this is the time varying part of the periodic response. So, the if the fixed part is increased actually by increasing these fixed parts.

So, we can increase the stiffness of the system and due to that so, you can see the response nature of the response changes. So, we can have the saddle node bifurcation point at different locations.

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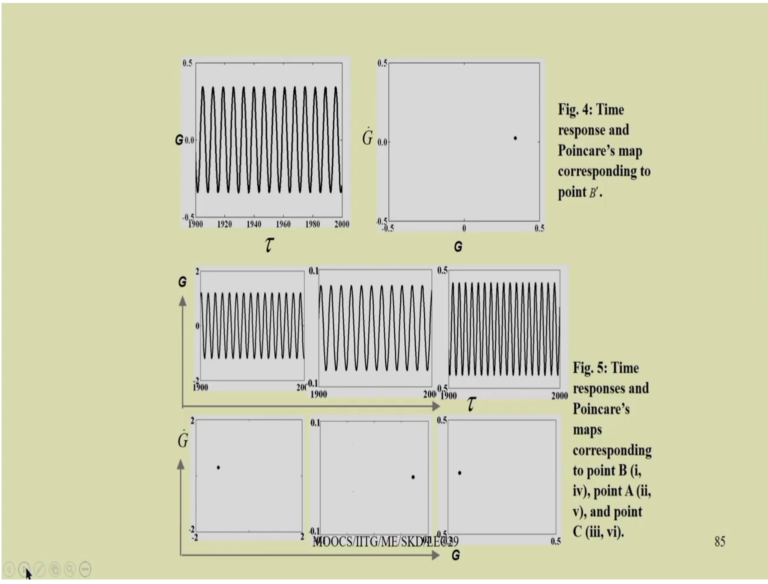

So, time response and Poincare maps initially you have a periodic response. So, in the Poincare section we have only a single point. So, here you can see in Poincare section time response showing a fixed point.

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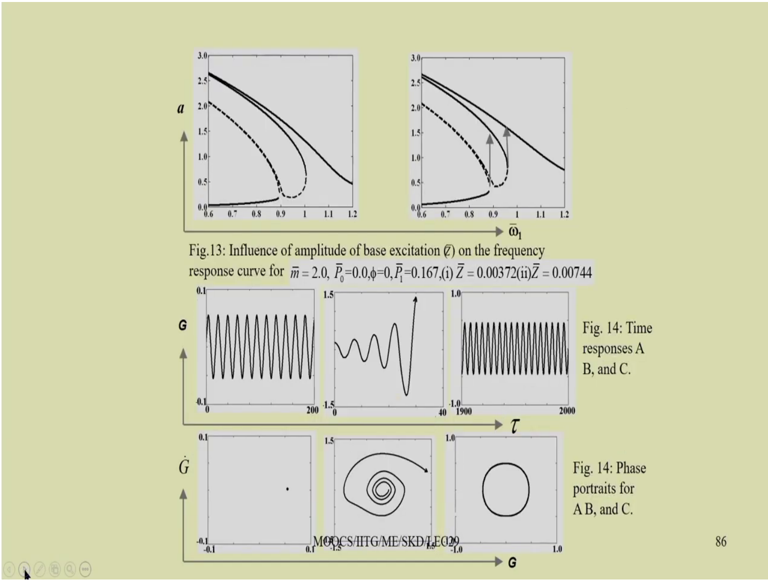

So, this way you can show the fixed point by using a Poincare section and here you can see influence of amplitude of base excitation on the frequency response. So, this math ratio equal to 2 P 0 equal to 0 and phi equal to 0. So, in P 1 bar equal to 0.167 so, in first figure this Z bar equal to 0.00372 and in 2nd case Z bar equal to 0.00744.

So, here these base excitation is increased amplitude of base excitation is increased. So, thereby so you just see the nature is remaining same, but the amplitude is slightly increasing.

So, we have taken three points in these. So, one point it is two points are stable and one is giving rise to unstable response. So, in phase portrait at Poincare section.

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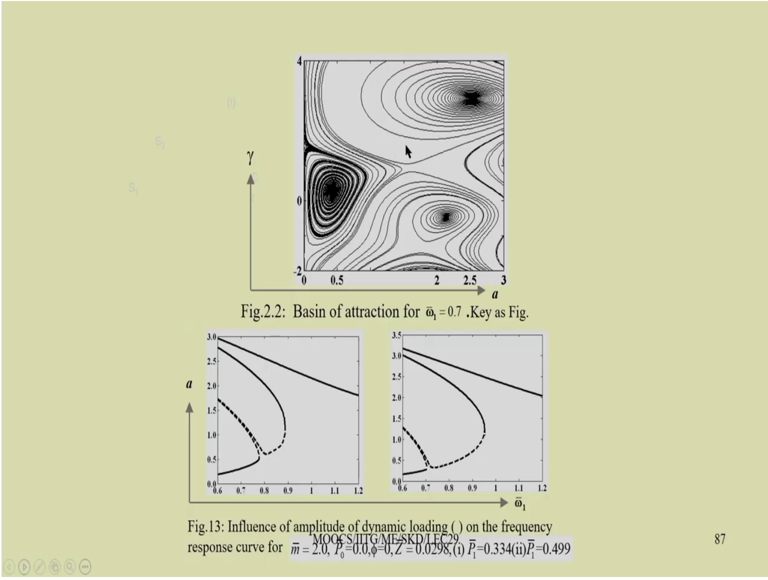

So, you can see this thing, so here clearly you can see the basin of attraction in this type of response. So, depending on the so clearly this is the saddle point. So, you can see different saddle points and these fixed point responses in this case. So, you can have a tri stable region here 1, 2, 3. So, three branches are stable.

So, you can get three stable branch and so, you just see this point and this in the previous case these and the above point where at the same place that is for a equal to 2, but in this case so, this a point are at different points.

So, they are at different points and you can have three values who as we have three stable points along with that we have unstable points also two unstable point. So, two saddle point can be so found here one and here one two saddle points also maybe observed.

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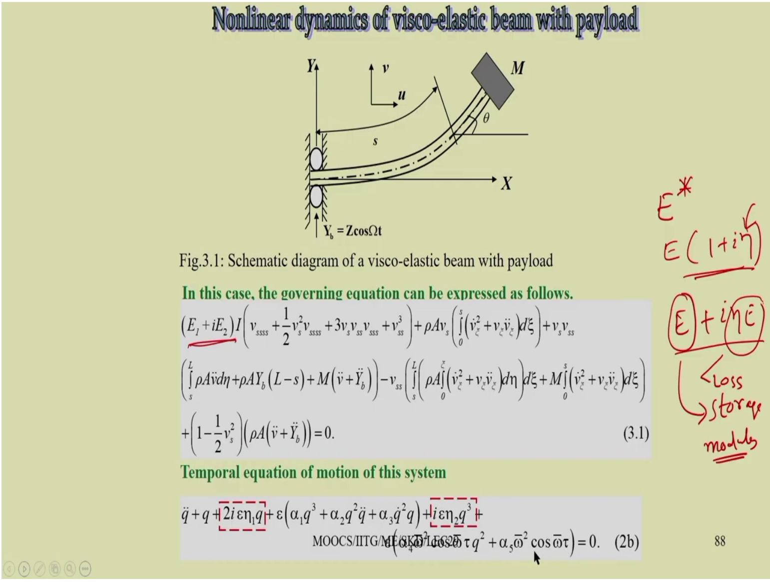

So, previously we have taken a elastic beam. So, if we take a viscoelastic beam. So, in case of viscoelastic beam, so the equation will be modified by replacing this i by i plus i i 1 E 1 plus i E 2. So, this E that is the Young's modulus will be a complex number in case of the viscoelastic material.

So, in that case E can be replaced by E star. So, this E star also can be retained by E into 1 plus i eta so, one can write in this form that is where eta is the loss factor or one can write expanding this thing this will be E plus i eta E also. So, here it is written in terms of E 1 and E 2.

This part this first part is the shear modulus storage modulus and this is the loss modulus. So, this is the loss modulus, this is the storage modulus, and this part is the loss modulus. So, this is the loss modulus and other one is the storage modulus of the system by putting this complex number E 1 plus iE 2 so, you can see the equation E changed.

So, the now the equation becomes q double dot plus q plus 2i epsilon eta 1 q plus epsilon alpha 1 q cube alpha 2 q square q double dot plus alpha 3 q dot square q plus i eta i epsilon eta 2 q cube. So, plus these non-linear term and then this forcing term direct forcing and parametric forcing terms are there.

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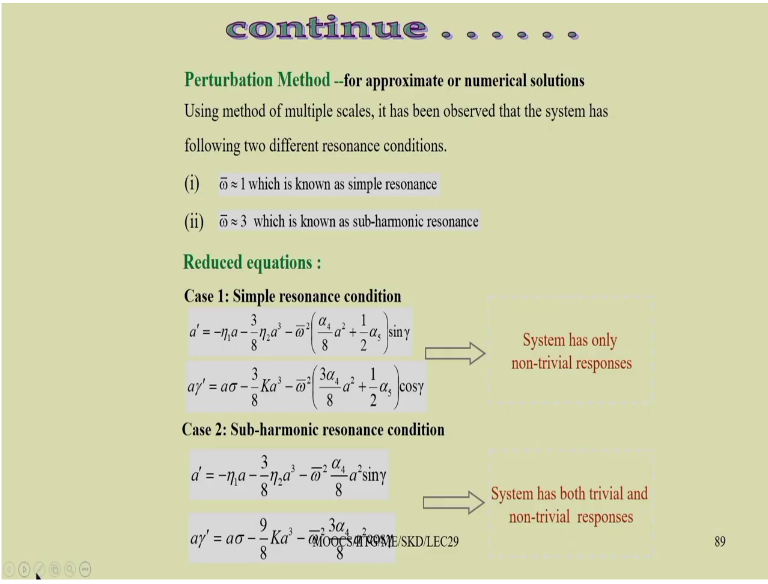

So, we can use this perturbation method for analysis. So, here we can have a simple resonance condition and sub harmonic resonance condition like previous case, but here we have the loss factor instead of loss factor in this case loss factor is written in terms of eta and this a dash equation and a gamma dash equation you can get. So, system has only non trivial response, but in case of sub harmonic resonance we have these both trivial and non trivial response present in the system.

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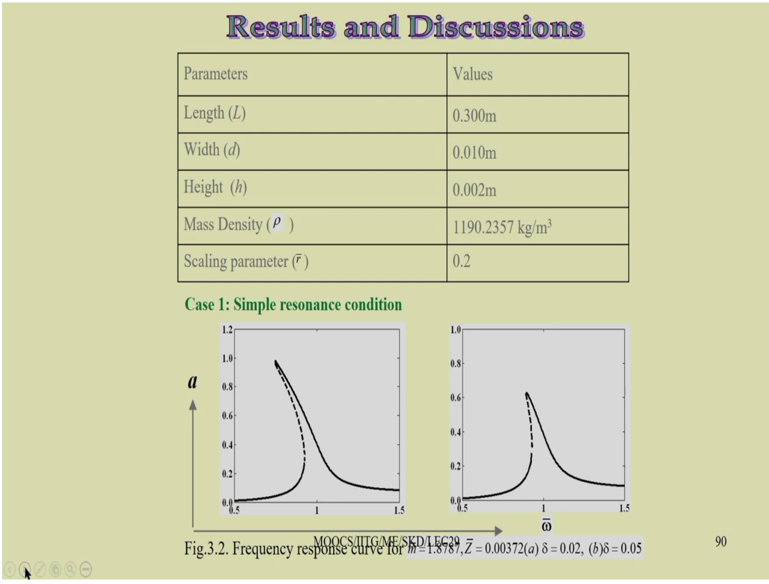

So, taking this system parameters you just see we can take the system parameter here we have taken a material with density 1190.2357 kg per metre cube, scaling parameter r equal to taken 0.2, so you can see we can plot this response. So, due to the presence of this viscoelastic material like this damping property is high so, in this case we can get the maximum response amplitude less than that of the elastically excited elastically elastic beam used in our previous analysis.

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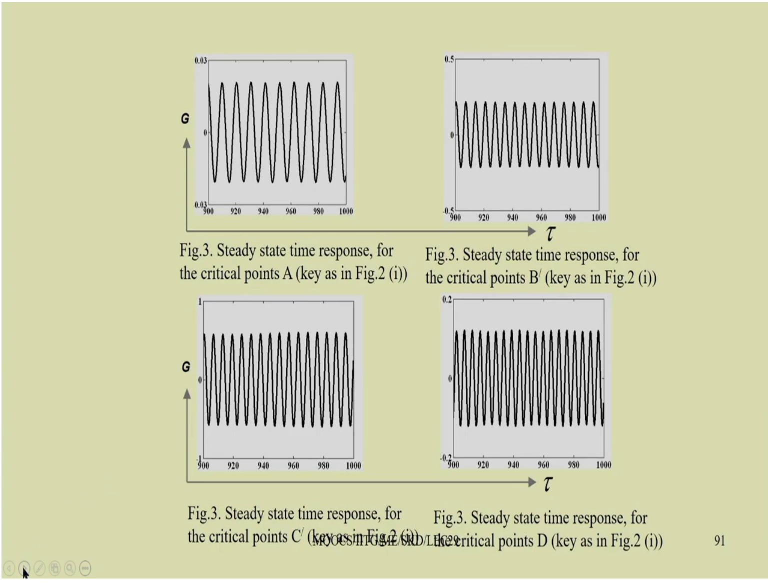

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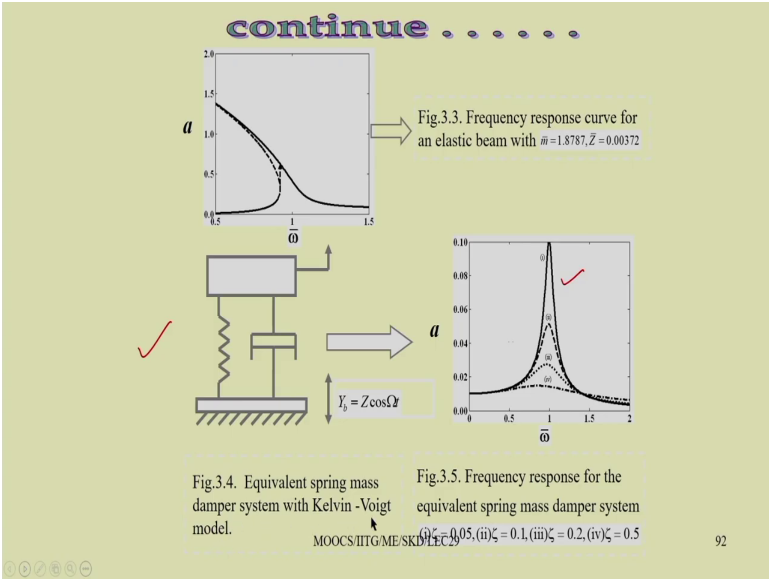

So, one can see this steady state response and the time response one can convert that thing to an equivalent system equivalent Kelvin Voigt model. So, this is the equivalent Kelvin Voigt model in the linear case, if you take the equivalent Kelvin Voigt model. So, this is the linear response curve linear frequency response plot one can obtain. So, frequency response plot for an equivalent spring mass damper system. So, this is the equivalent spring mass damper system with Kelvin Voigt model one can take.

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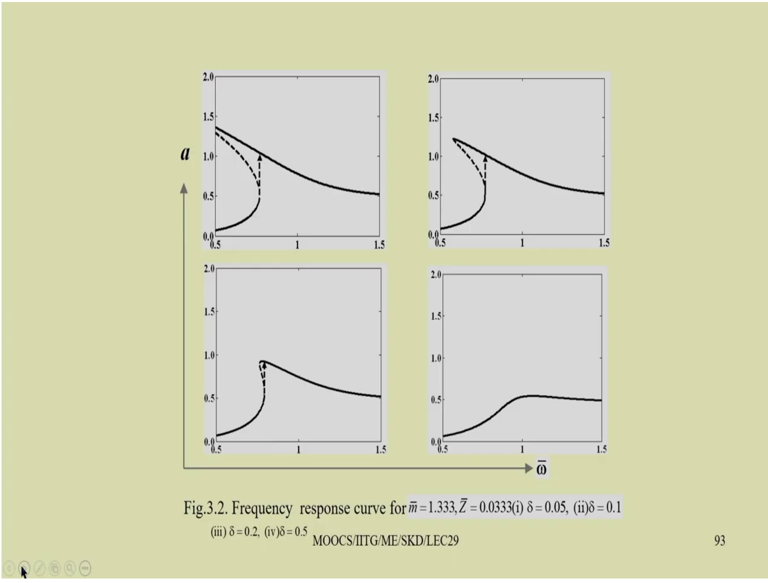

Now, the frequency response plot one can plot this frequency response and can see. So, in the previous case you just see the response is showing similar to that of a non-linear thing, non-linear system a system of a equivalent to a linear system and the response amplitude is found not to be same as that in case of the non-linear system.

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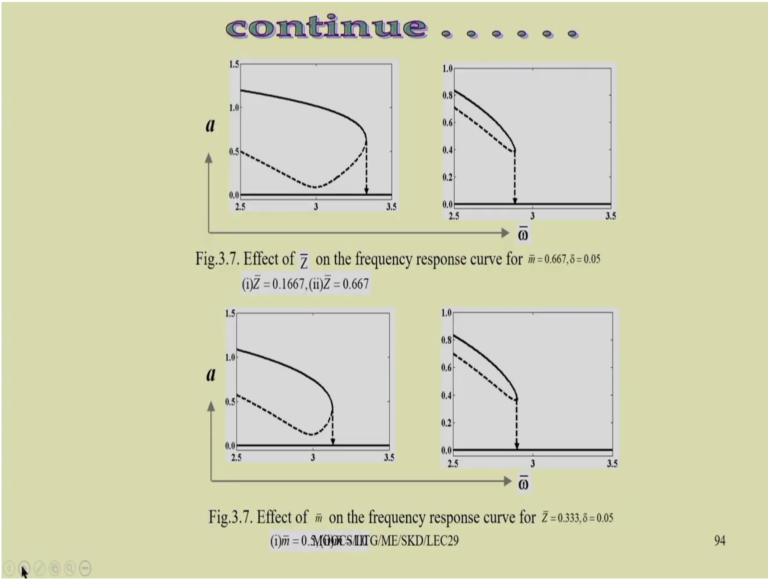

So, this one can avoid this linear analysis. So, one can avoid the linear analysis and perform the non-linear analysis to actually find the response of the system.

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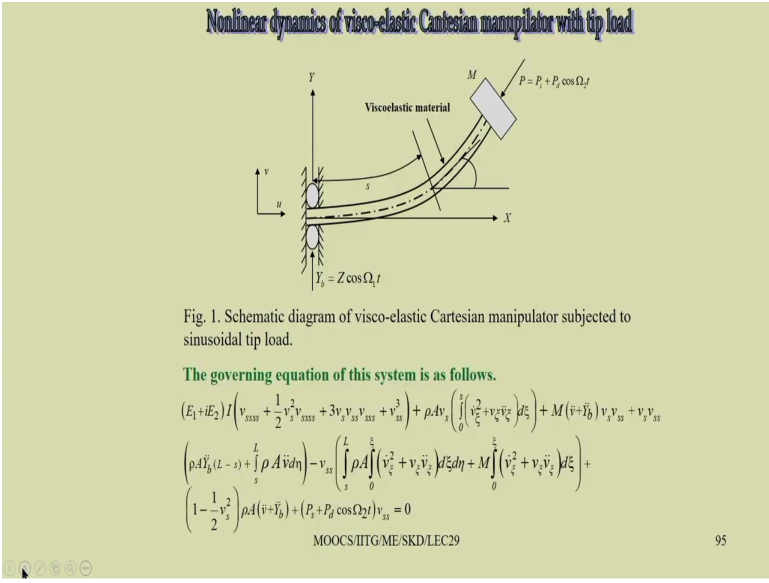

So, if we extend to the second problem that is a viscoelastic beam with tip mass and this axially period periodically applied axial load. So, or a follower load P s plus P d cos omega t here also equation will be same, but E will be replaced by this complex number.

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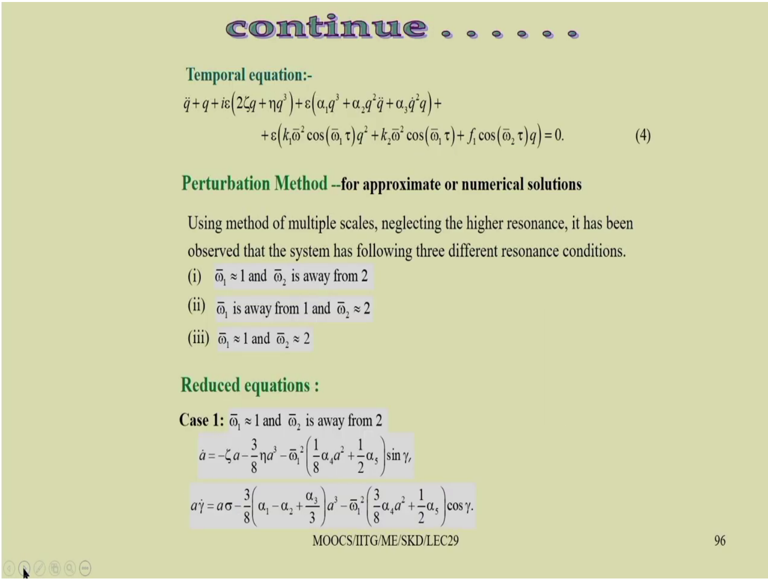

So, due to the presence of complex number so, we can see we have these three different equation, one is omega 1 equal to 1 omega 2 is away from 2, omega bar omega 1 bar is away from 1 and omega 2 bar nearly equal to 2, and this omega 1 bar equal to 1 and omega 2 bar equal to 2. So, this way we can have these three different resonance conditions so, for these three different resonance conditions.

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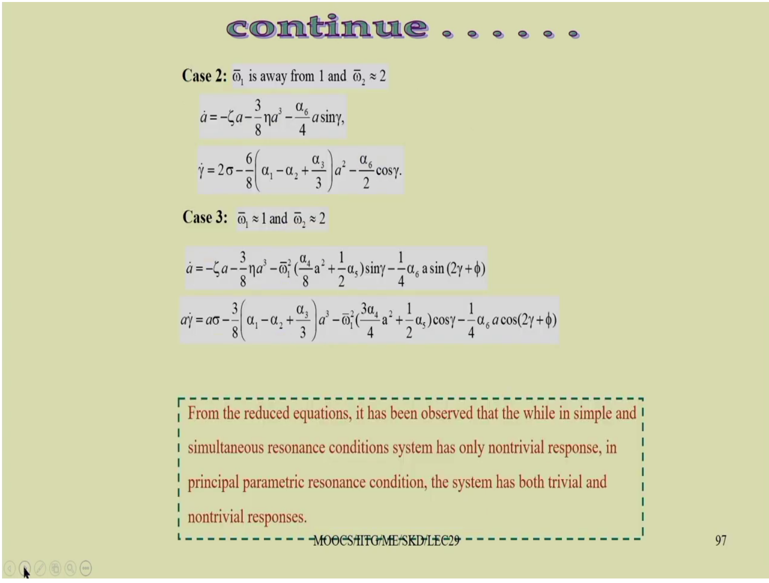

So, we have three sets of reduced equation. So, from the reduced equation, it has been observed that while in simple and simultaneous resonance condition the system has only non trivial response, in principal parametric resonance condition, the system has both trivial and non trivial response.

So, here we are studying the principal parametric resonance condition also because, omega 2 you are taking nearly equal to 2. So, it will leads to principal parametric resonance condition.

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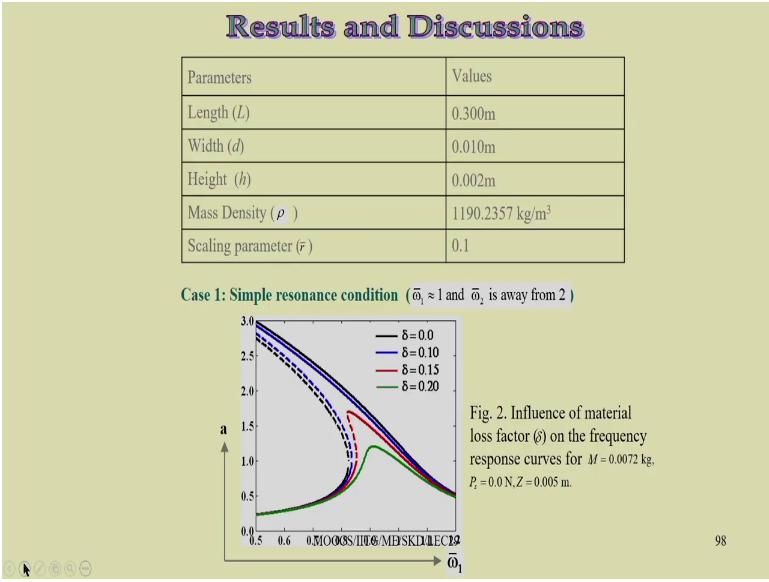

Here so, for different value of delta that is loss factor. So, you can plot the response plot and you can clearly observe by increasing this loss factor the response amplitude of the system decreases drastically it decreases.

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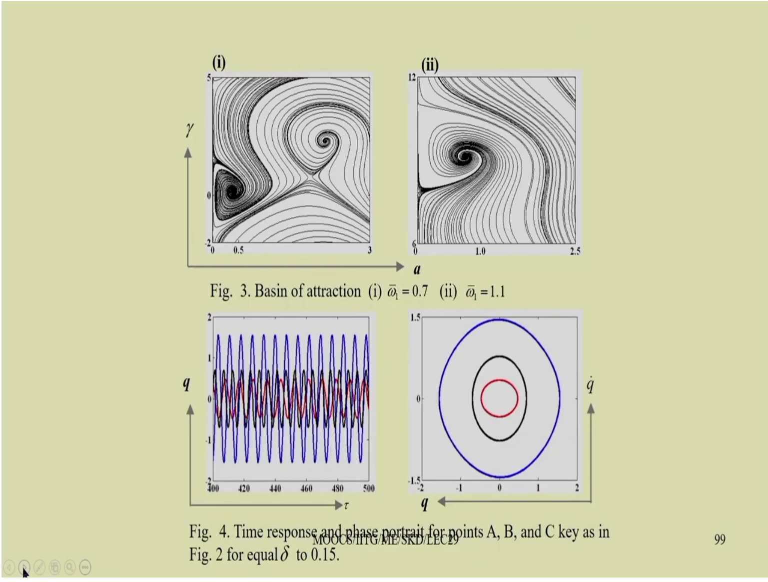

So, by using this viscoelastic beam so, you can reduce the vibration of the system extensively.

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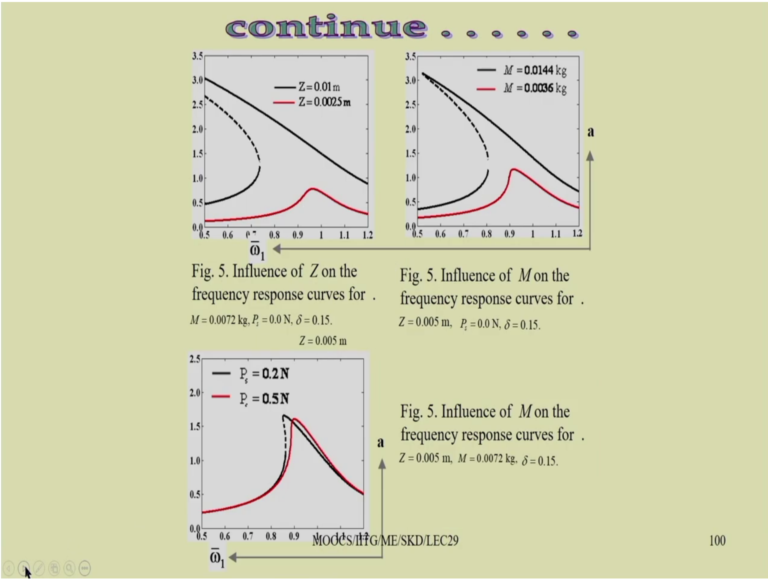

So, these are the basin of attractions also plotted in this case and you can study the influence of P s and Z and N. So, you just see by varying the mass ratio so, you can control the vibration of the system also by changing different response amplitude. So, the response of the system can be controlled in a significant way so here you just see.

So, you have a bi stable region so, but with this Z equal to smaller value if you take smaller value of Z the response changes and it is completely stable response throughout the frequency range.

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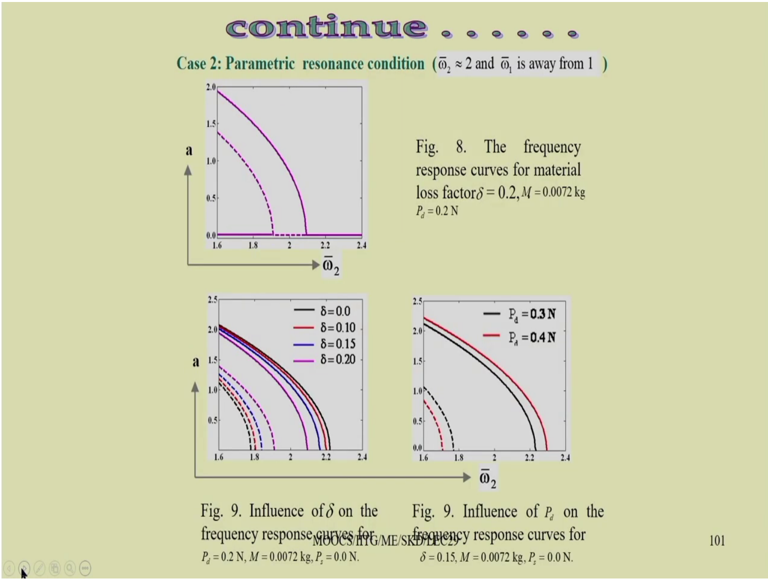

Similarly, here also you can plot the frequency response for different value of P s that static value. And previously we just see you have only this saddle node bifurcation point, but here in addition to saddle node bifurcation point. So, if you are considering principal parametric resonance condition that is omega 2 nearly equal to 2 and omega 1 is away from 1.

So, we have this pitchfork bifurcation points. So, here we have pitchfork bifurcation point and you can study the response of the system for a different pitchfork bifurcation points.

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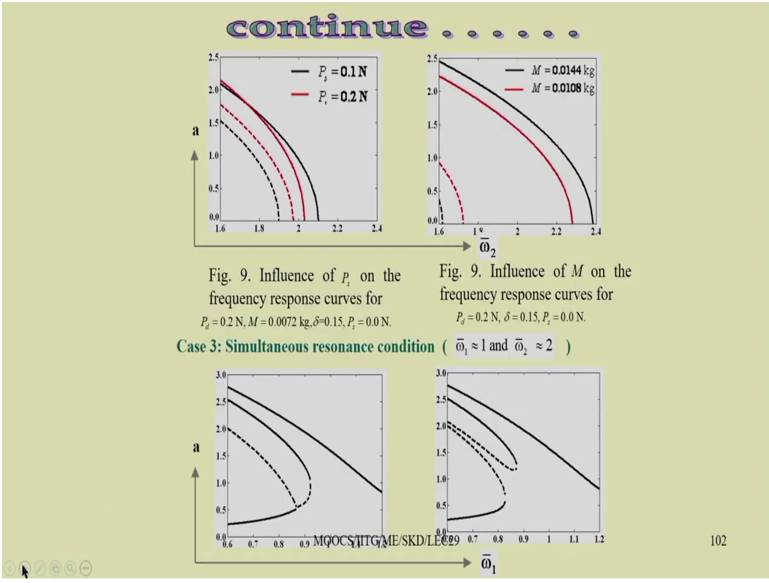

So, in case of simultaneous resonance omega 1 nearly equal to 1 and omega 2 nearly equal to 2 so, we have a combination of these thing, but the trivial state no longer become stable and it is shifted this curve is shifted and we can have this multiple branch.

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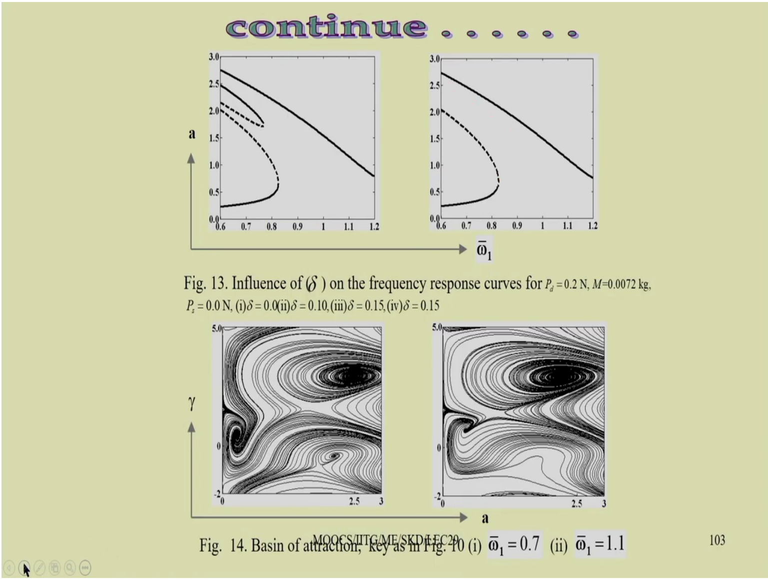

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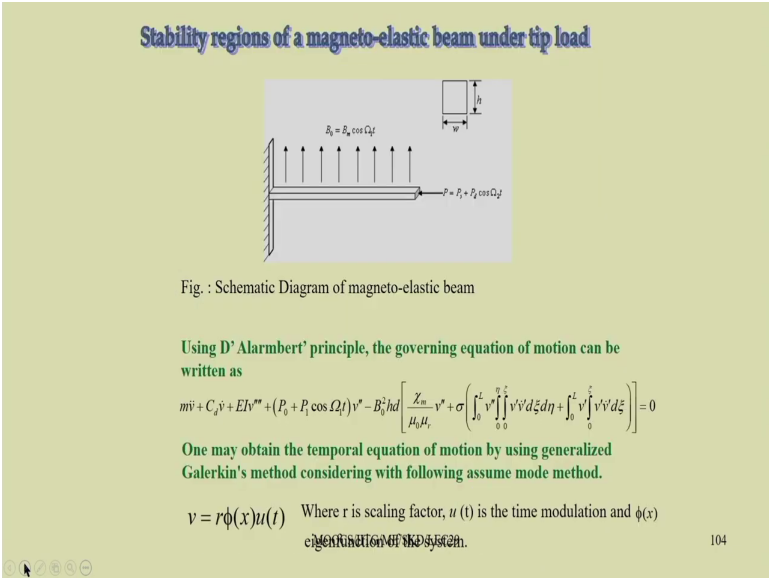

So, due to the presence of multiple branch so, one has to study the one has to study the basin of attraction. So, another problem we may study that is stability region of a magneto elastic beam.

So, instead of taking a elastic or viscoelastic beam. So, if we take a elastic beam, but made up of these magnetic magnetically magneto elastic beam so that means, ferrous material. So, if it is made up of ferrous material then by applying this magnetic field, we can easily control the stiffness of this beam.

So, we can control the stiffness of the beam and in this type of system for example, let us apply let us apply a magnetic field B 0 equal to B m cos omega t and also a load of P equal to

P s plus P d cos omega t; you can see the reduced equation you can see the equation reduced to this form. So, here this term is due to the magnetic field.

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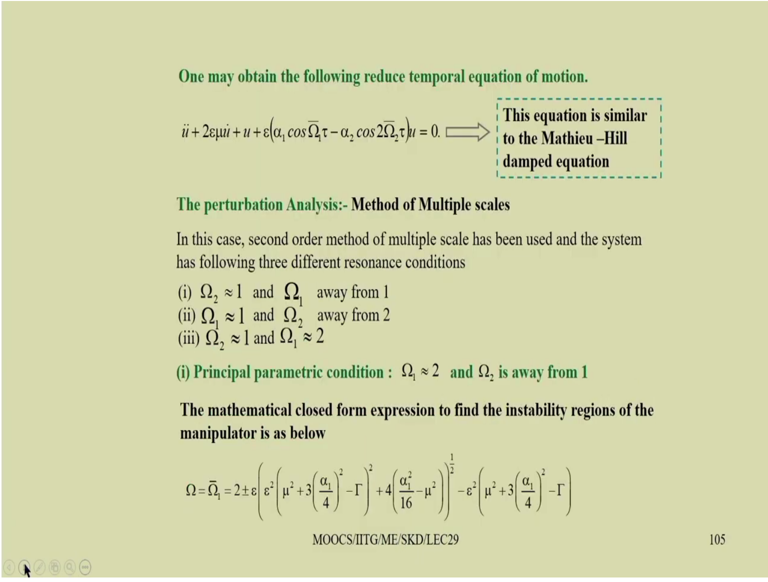

So, taking this Galerkin method single mode Galerkin method so, this equation is reduced to a very simpler form that is to that of a Mathieu- Hill type of equation. That is u double dot plus 2 epsilon mu u dot plus u equal plus epsilon into alpha 1 cos omega bar tau minus alpha 2 cos 2 omega 2 tau into u equal to 0. So, this is simply it is reducing to that of a Mathieu-Hill type of equation.

So, here a different resonance conditions can be considered omega 2 nearly equal to 1 and omega 1 away from 1, omega 1 nearly equal to 1 and omega 2 away from 2, and both omega 2 nearly equal to 1 and omega 1 nearly equal to 2. So, in first case it is the principal parametric. Principal parametric resonance condition so, you can study; so where this omega

1 equal to 2 omega 1 equal to this is away from 1 so; that means, we can take it nearly 2 and omega 2 is away from 1.

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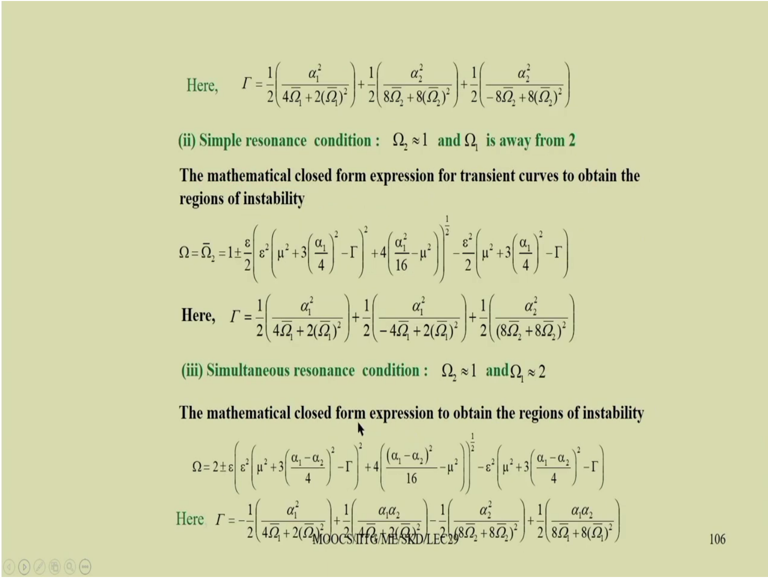

So, we can get this resonance condition. So, here you can get a closed form type of equation, similarly for simple resonance condition. So, you can get a closed form solution so, as the equations are simple.

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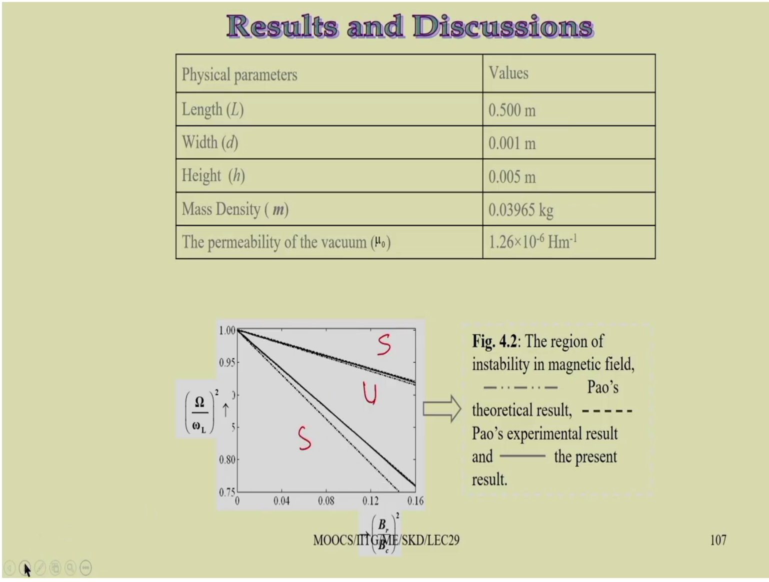

So, you can get a closed form solution and from that thing you can plot this instability region. So, in the omega by omega L square beta B r by B c square B critical square, so this is the curve one can find the instability region. So, outside the curve the system is stable and inside the curve the system is or inside these two curve the system is unstable.

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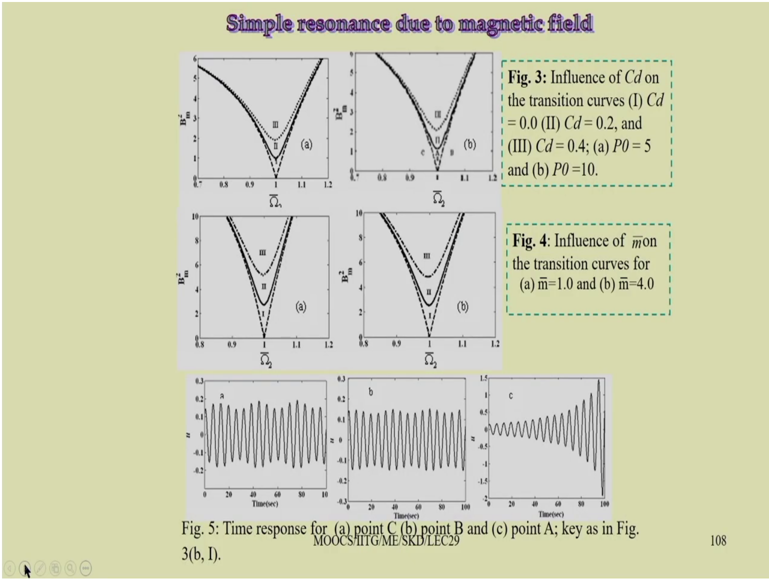

So, this way you can plot the stability diagram. So, you just see the stability diagram so these are the critical value. So, these are the critical value below which the system, whatever value of B omega you apply the system will be stable.

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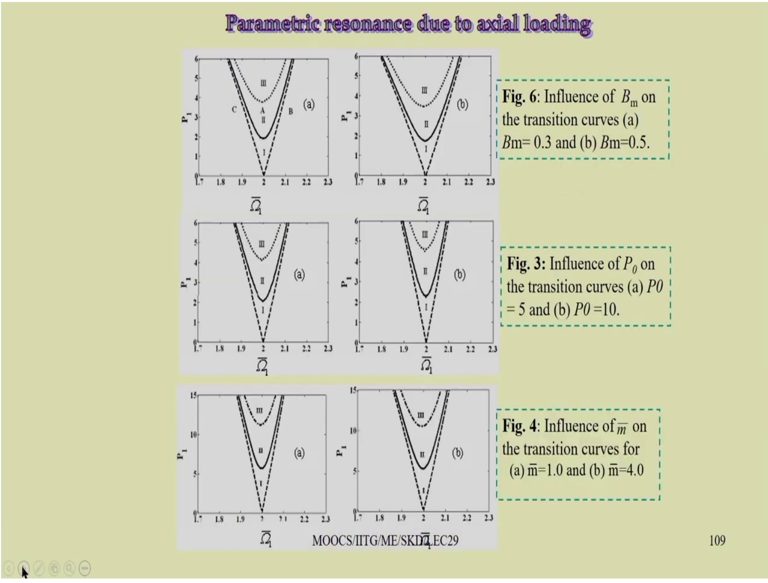

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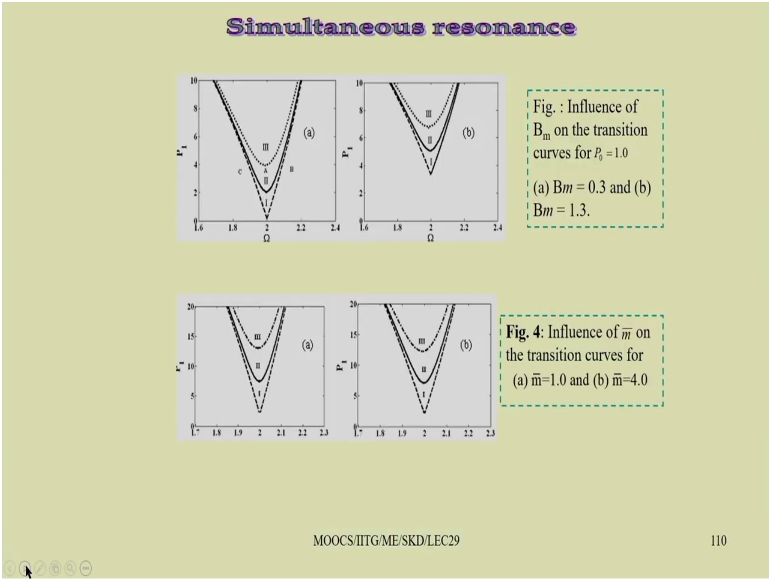

So, one must find all these critical value below which the systems will be stable and apply the concept to control the vibration of the system.

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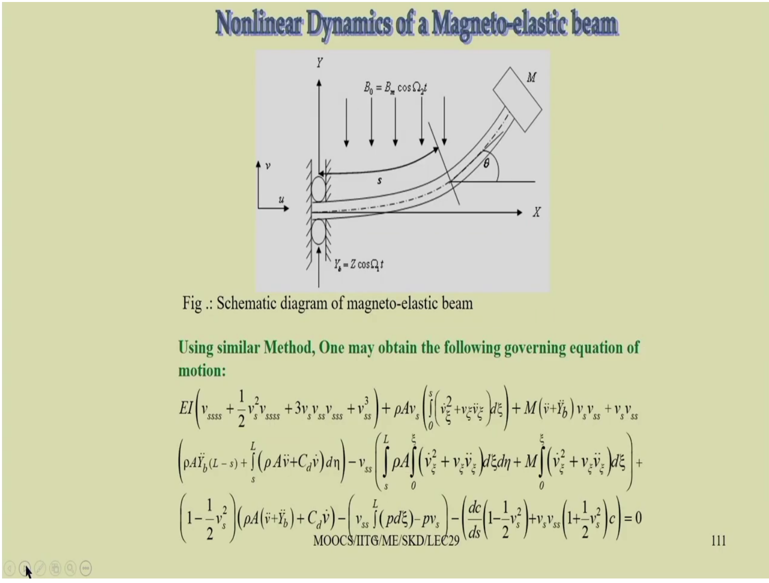

Similarly, for different parameter it has been plotted. So, if you take this same magneto elastic beam, and apply this magnetic field instead of applying axial load. So, with a tip mass you can have this equation motion.

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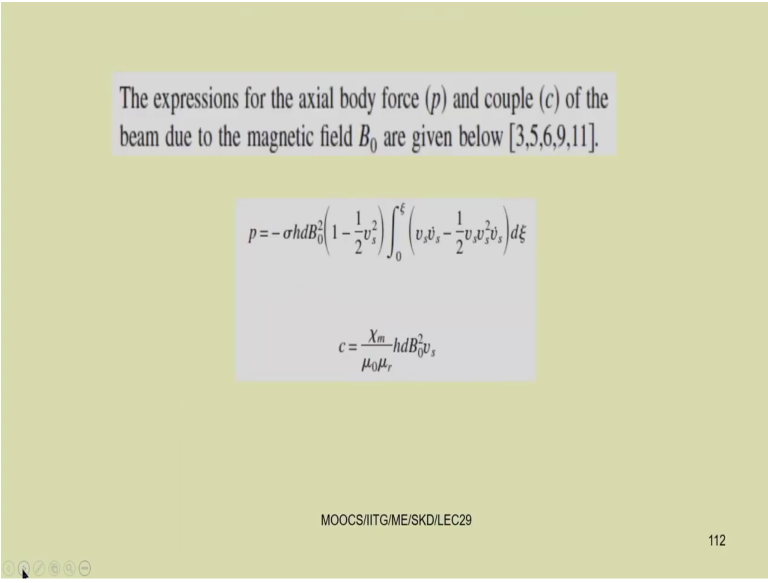

So in this equation motion, you can study you just see same different type of resonance condition in this case you just see this equation.

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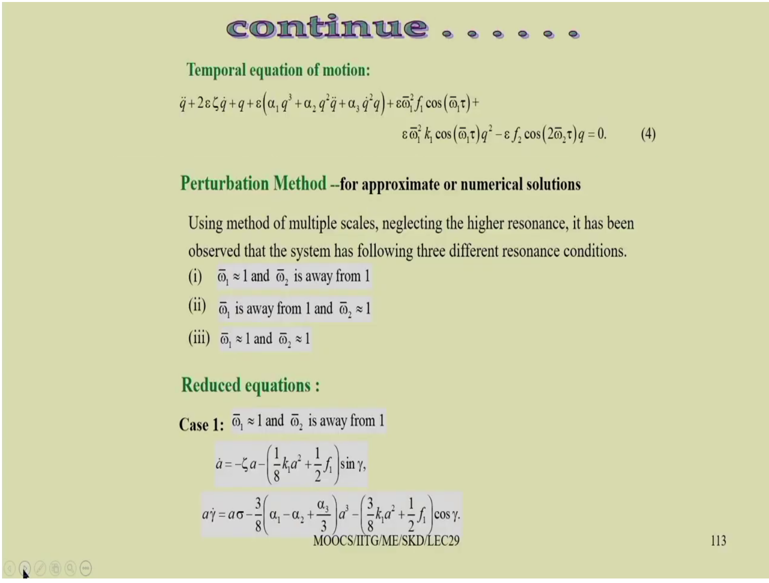

So, here this is the direct excitation, the second term this is the non-linearity with non-linear term is there so, parametric excitation and the third term is a parametric excitation term.

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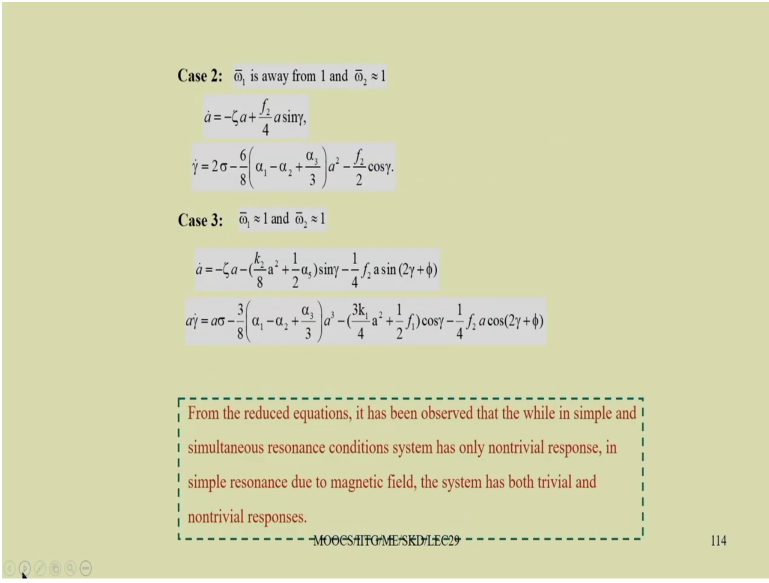

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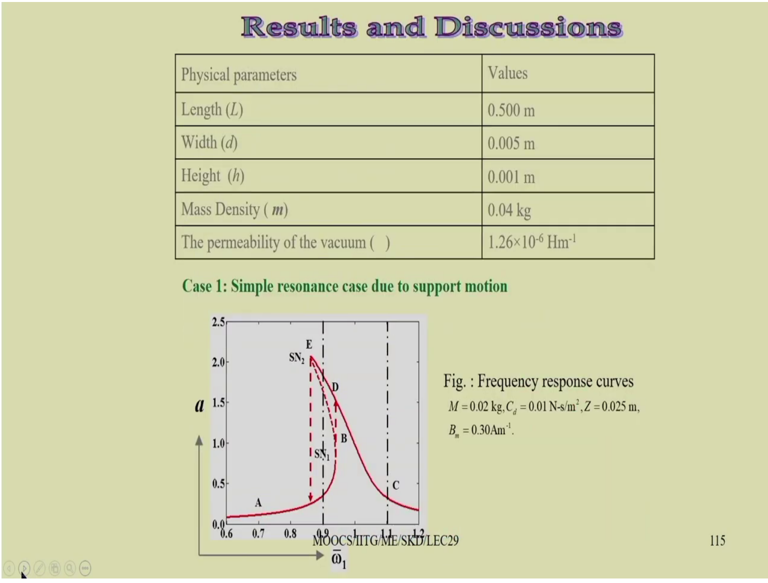

So, we have this these are the reduced equations you can get for different cases, and you can study similarly.

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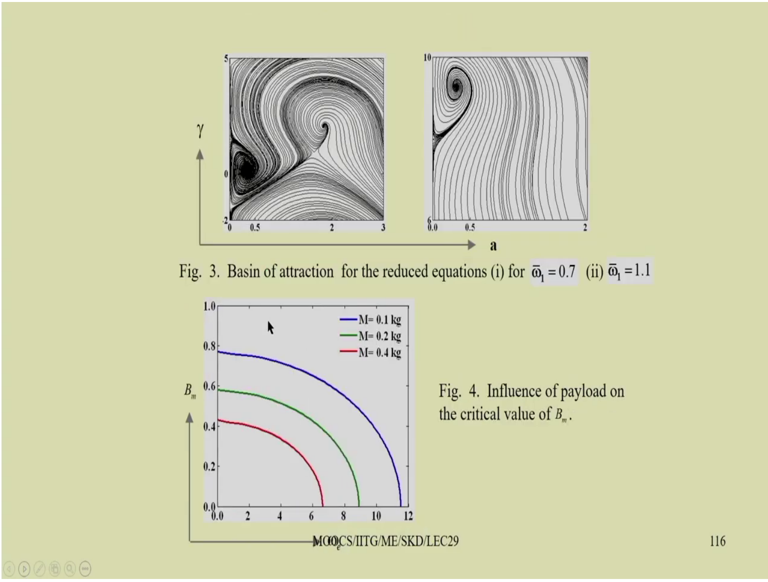

So, you can study the frequency response curve and here you can see how with varying these magnetic field and other parameters. So, we can have the response of the system.

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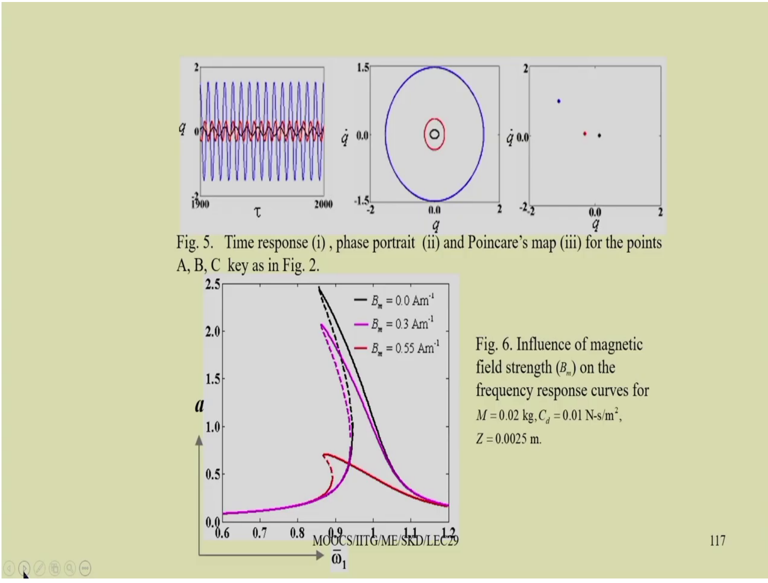

So, clearly you can see you can control the vibration of the system by applying different magnetic field. For example, this magnetic field 0 magnetic field there is no magnetic field this black curve is there then this one the middle one is with B m equal to 0.3 and when you are increasing B m to 0.55 you can see you can control or you can reduce the response amplitude by 5 times.

So, by using this magneto elastic beam and applying suitable magnetic field so, you can control the vibration of the system.

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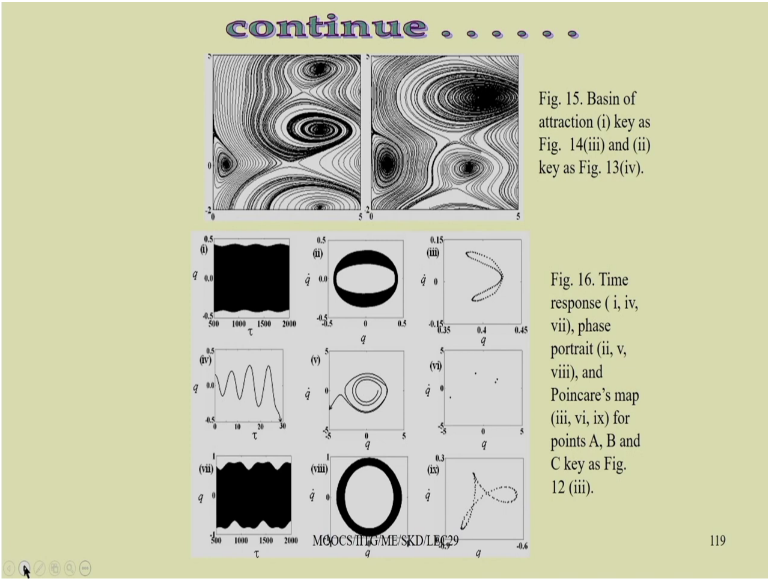

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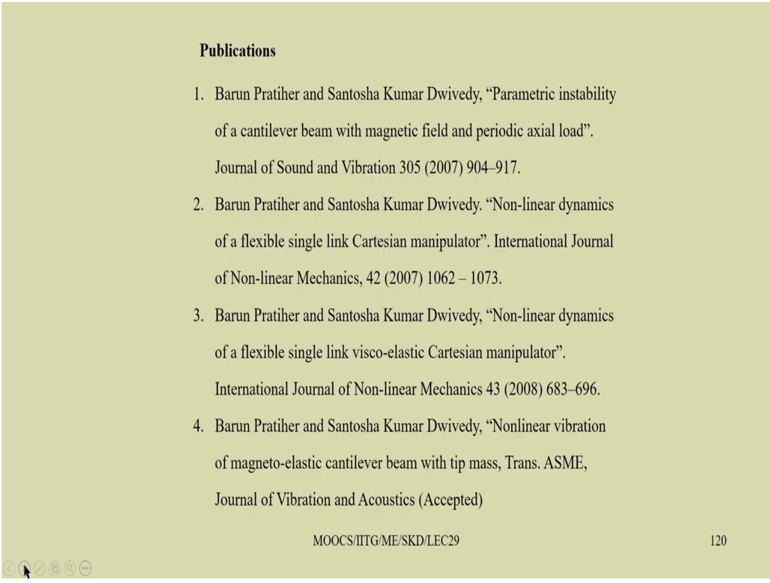

So, these are similarly one can plot this basin of attraction and study the system. So, these are detail literature on which this today's presentation has been made. So, the work so this is the part of the work is my PhD student Dr. Barun Pratiher so Barun Pratiher and Dwivedy, so "parametric instability of a cantilever beam with magnetic field and periodic axial load".

So, this is published in Journal of Sound and Vibration then, Barun Pratiher and SK Dwivedy. So "non-linear dynamics of a flexible single link Cartesian manipulator" so, it is International Journal of Non-linear Mechanics, in 2007. The 3rd paper is "Non-linear dynamics of a flexible single link viscoelastic Cartesian manipulator", it is also published in the same journal International Journal of Non-linear Mechanics.

Then "Non-linear vibration of a magneto-elastic cantilever beam with tip mass, Journal of Vibration and Acoustics this is a semi journal.

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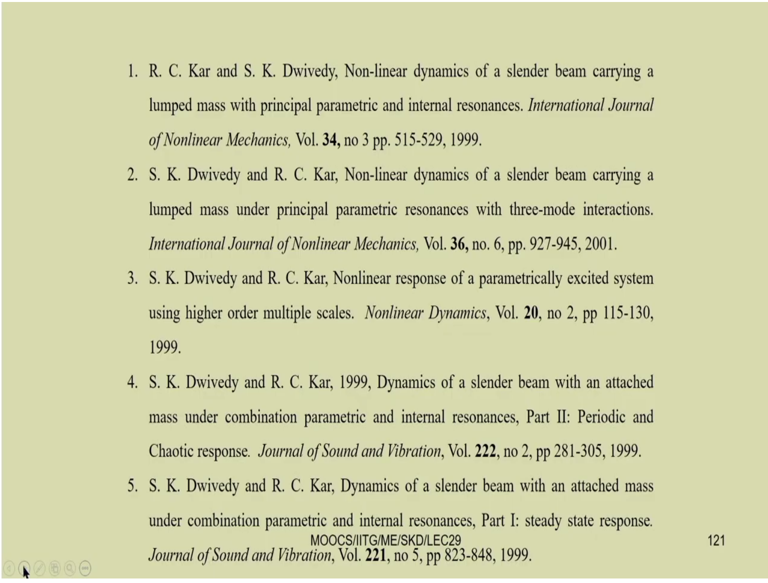

So, there are some other papers also you can refer this work by this R.C. Kar and S.K. Dwivedy, Professor R.C. Kar was my PhD supervisor of IIT, Kharagpur. So, Non-linear Dynamics of a slender beam carrying a lumped mass with principal parametric and internal resonance. So, this was published in Non-linear International Journal of Non-linear Mechanics.

Then, Non-linear Dynamics of a slender beam carrying a lumped mass under principal parametric resonance with three mode interaction. So, this is published in International Journal of Non-linear Mechanics. Then, Non-linear response of a parametrically excited system using higher order method of multiple scales. So, that is published in Non-linear Dynamics.

Then Dynamics of a slender beam with an attached mass under combination parametric and internal resonance, periodic and chaotic response, this is published in Journal of Sound and Vibration. Dynamics of a slender beam with an attached mass under combination parametric and internal resonance, so this is part I: so, steady state response this is also published in Journal of Sound and Vibration.

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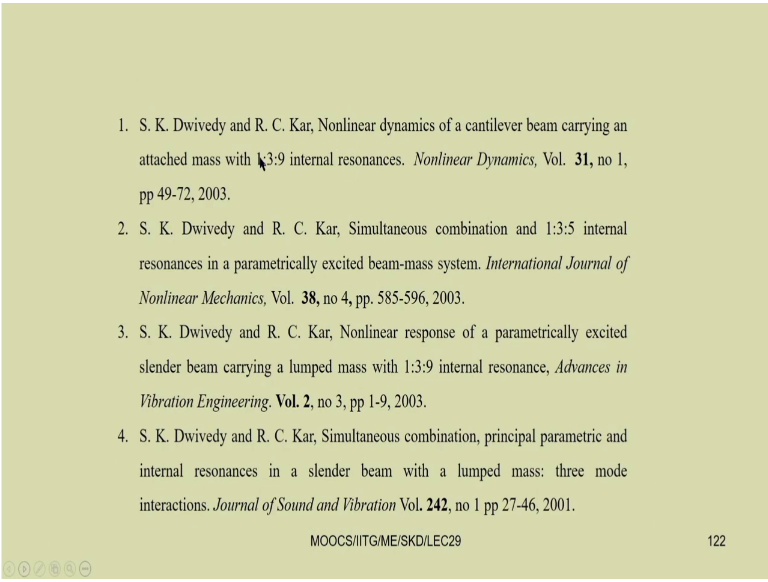

So, some other papers are like these Non-linear dynamics of a cantilever beam carrying an attached mass with 1 is to 3 is to 9 internal resonance, that is published in Non-linear Dynamics. Simultaneous its combination of 1 is to 3 is to 5 internal resonance in a parametrically excited beam mass system. So, this is International Journal of Non-linear

Mechanics. Non-linear response of a parametrically excited slender beam carrying lumped mass with 1 is to 3 is to 9 internal resonance.

So, Advances in Vibration Engineering then, Simultaneous combination, principal parametric and internal resonance of a slender beam with lumped mass: three mode interaction. So, this is published in Journal of Sound and Vibration. So, you can refer all these papers for understanding more and more of these how a slender cantilever beam can be used for generating different type of responses fixed point response, quasi periodic response, quasi periodic response, and chaotic response.

So, next class we will extend this idea to develop an energy harvester by adding this piezoelectric pairs to this cantilever type base excited cantilever type of beam, so.

Thank you very much.