Nonlinear Vibration

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Lecture - 30
Application of Active vibration absorber with combination feedback

Welcome to this course of Nonlinear Vibration. So, last two classes we are studying regarding this vibration absorber, today class also we will continue with the same vibration absorber, so where I will tell you regarding different other different types of vibration absorber.

For example, so we have taken the vibration absorber to 2 degrees of freedom system. So here, either one can use this passive vibration absorber or tuned vibration absorber, similar to in case of a linear system, but when we are considering non-linear system, non-linear primary system, non-linear secondary system and active vibration absorber so we can use some active elements like the smart material.

For example, this piezoelectric material or someone can use this shape memory alloy or other different type of smart materials can be used to control the vibration of the system. So, today class we will see two different types of vibration absorber, particularly we are going to see the traditional and non traditional vibration absorber. So already, you are familiar with the traditional vibration absorber and I will tell you something regarding this non traditional vibration absorber.

So, one applications of the torsional vibration absorbers also I can show you which is used for controlling the tremor of the hand, and also in case of continuous systems how we can reduce or control the vibration so that thing also we will see in today class.

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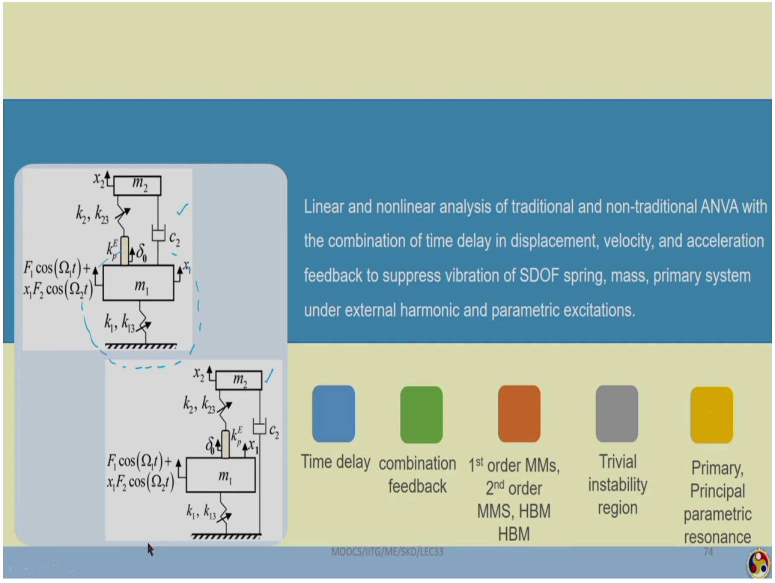

So, this work is part of the PhD thesis of Mr. Sibananda Mohanty, who is working under my guidance. Here, let us see this linear and non-linear analysis of traditional and non traditional active non-linear vibration absorber with combination of time delay in displacement, velocity and acceleration feedback to suppress the vibration of single degree of freedom spring, mass, primary system under external harmonic and parametric excitation.

Here, we are considering all types of vibration absorber, here we are taking the time delay in displacement, velocity and acceleration. Also, the system is subjected to external, harmonic and parametric excitation. So, you can see the system, so this is the primary system. The primary system, there is no damping is considered in the primary system, so for example, the damping may be very less, so I can neglect the damping. Later one can study the use of damping also in the system.

Here, so we have put a simplified system where the piezoelectric stack actuator is connected to a spring and also a damper is there and the system is subjected to both direct and parametric excitation.

So, in direct excitation the term F 1 cos omega 1 t is there and in case of parametric excitation the term is x 1 F 2 cos omega 2 t. Here, one can take two different type of forcing amplitude and frequency F 1 and F 2 are the forcing amplitude and omega 1 and omega 2 are the forcing frequency.

So, the stack actuator has a displacement of delta 0 so which is a function of the voltage applied to the system, and here k 2 is the linear spring constant and k 23 is the non-linear spring constant of the system. The mass m 2 has a displacement of x 2 and mass m 1 has a displacement of x 1.

So, this is the traditional vibration absorber in the primary system, so this is the primary system. So this part is the primary system to which a secondary system is added. Here, so the secondary system is now directly connected to the ground, but the primary system is connected to the peaks based or ground. But in case of non traditional vibration absorber the secondary system itself is connected to the ground.

So here, the damper c 2 is connected to the ground, and the system m 1 that is the primary system is connected to the ground through the non-linear spring, with a linear spring constant k 1 and non-linear spring constant k 13. So, we have two different type of vibration absorber considered in this case. You can easily write down this equation of motion of these two system by either applying this Newton's 2nd law or D Alembert principle or energy based principle like Lagrange principle or Hamilton principle.

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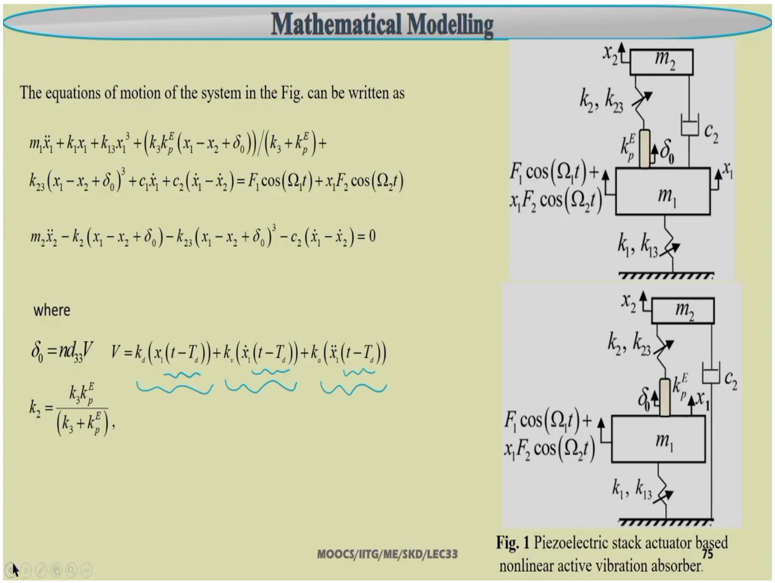

So after deriving this equation of motion, so you can see so how we can give the time delay to the system. So here we are considering a time delays; that means, whenever we are applying this force it will take some time to react, that is why there is a time delay in the response term.

So, the equation of motion of this case can be written as m 1 x 1 double dot, this is the inertia force then plus k 1 x 1 plus k 13 x 1 cube plus k 3 k p E x 1 minus x 2 plus delta 0 divided by k 3 plus k p E. So here we are taking this equivalent spring stiffness and then multiplied that thing by the displacement. So the displacement of the stack actuator which is connected to the mass m 1 can be written in that form.

So then we have this k 23 k 23 x 1 minus x 2 plus delta 0 x 3 plus c 1 x 1 dot plus c 2 x 1 dot minus x 2 dot equal to F 1 cos omega 1 t plus x 1 F 2 cos omega 2 t. And for the secondary system it can be m 2 x 2 double dot minus k 2 x 1 minus x 2 plus delta 0 minus k 23 into x 1 minus x 2 plus delta 0 cube minus c 2 into x 1 dot minus x 2 dot equal to 0, so here this delta 0 is written as nd 33 into voltage V.

So this voltage can be written by using this PDE controller, so V can be written as k d into x 1 minus t minus tau d, so here you just see how the delay term is included in the system. So, V equal to k d into x 1 minus t minus T d plus k v into x 1 dot into t minus T d and ka into x 1 double dot t minus T d. So here you can note that this T d is the time delay term associated with the feedback system, that is the voltage. When we are applying this voltage we are assuming so there is a delay in the system here.

This is the displacement feedback so these are the displacement feedback so this part is the velocity feedback and this part is the acceleration feedback. So you can take a combination of all these three or one of them or two of them at a time and you can do this analysis for completeness purpose.

So, all these three have been shown, but in practical case you may take one or two or any combination of these two or these three so depending on the system or applications so you can take these feedback.

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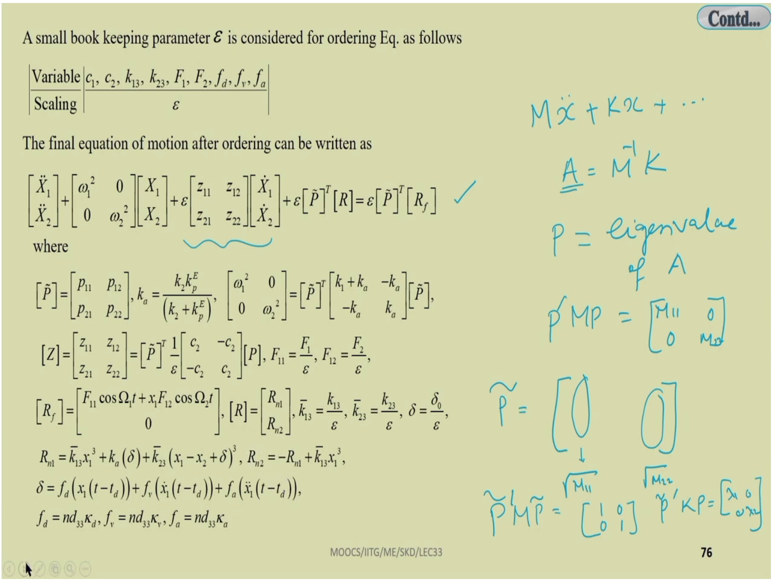

Here, by using a small book keeping parameter and ordering this equation of different order of scaling for example, the c 1, c 2, k 13 k, 23, F 1, F 2, f d, f b and f a are taken as order of epsilon.

The final equation of motion after ordering can be written in this form; this is a are 2 degrees of freedom system form. We can write that is X 1 double dot X 2 double dot plus omega 1 square 0 0 omega 2 square into X 1 X 2 plus epsilon into z 1, z 11, z 12, z 21, z 22 into X 1 dot Z 2 dot plus epsilon P weighted modal matrix transpose into R equal to epsilon P weighted modal matrix transpose into R f.

So, here you can note so originally you have a system so where mass matrix is coupled so you have a coupled mass matrix so your equation previously becomes Mx double dot only the linear part if I am writing then this is Kx plus the other terms will be there damping term and other terms are there so here what you are doing.

So, initially you write A equal to M inverse K, M inverse K and find the eigenvalue of A to get the P matrix. So P matrix that is modal matrix so you can find P matrix equal to eigenvalue find the eigenvalue of eigenvalue and eigenvector so this eigenvalue of matrix A.

So after getting this eigenvalue of matrix a so you can find the generalized mass. So generalized mass can be obtained this way P transpose MP will give the generalized mass matrix so you will get a mass matrix where you can have these M 11. Now, M 11 0 M 22.

So, in the modal matrix, so modal matrix you have in this modal matrix you have two columns, so to get the weighted modal matrix what do you have to do so in the first column so divide this first column by root over M 11 so divide the first column by root over M 11. Similarly, divide the second column by root over M 22, so that will give you this weighted modal matrix.

In this weighted modal matrix the property of this weighted modal matrix if you know so you can verify that P trans weighted modal matrix MP is P weighted modal matrix transpose MP is nothing, but your identity matrix. So, this is 0 1 0 0 1. Similarly, so if you find this P weighted modal matrix transpose KP so it is nothing, but the eigenvalue so that is why it is written omega 1 square 0 and 0 omega 2 square so this will be lambda 1 0 0 lambda 2. So this way by using this weighted modal matrix so you can derive this equation.

So, you just see now when you are multiplying all the terms so of the order of we have kept only up to these first two terms are from the linear then you can use this damping so this correspond to damping. Sometimes you may use this Rayleigh damping also. So in case of Rayleigh damping. So, you can assume so this part damping equal to alpha m plus beta k and taking that then you can multiply that thing by weighted modal matrix and you can find this equation.

So here the basic assumption is that so you can take these X 1 X 2 equal to P weighted modal matrix into X 1 capital X 1 X 2, that is why it will reduce to this form. So after getting these equations so you know so these are the terms is retained so by taking all these terms you just note that so here we have the time delay also considered in this case. Now, the equation is reduced to this form and after getting this equation.

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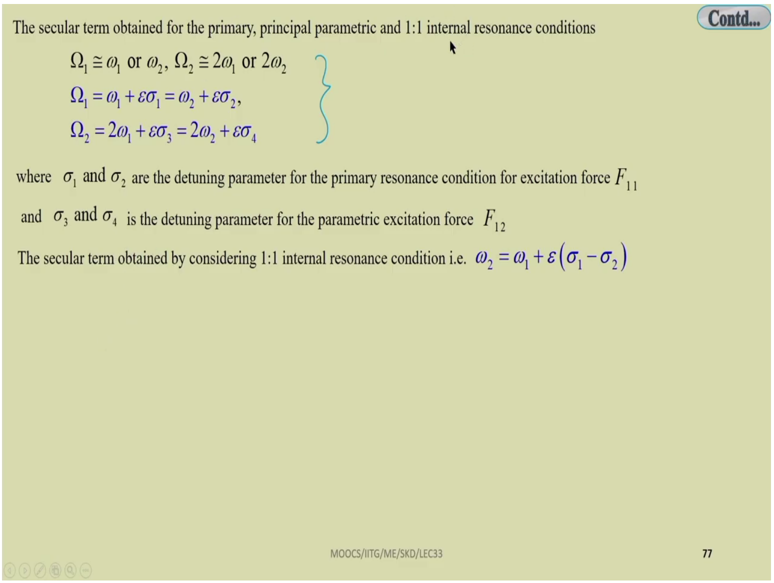

So, you can see, so there are different resonance conditions will be there the resonance conditions. For example, so you can had omega or you can have omega 1 equal to omega 1 that is external frequency capital omega 1; capital omega 1 is for the direct forcing. So, omega 1 equal to nearly equal to omega 1 or omega 2.

Similarly, omega 2 as it is principal parametric if you want to take so it may be nearly equal 2 omega 1 or nearly equal to 2 omega 2 similarly, so if you want to use the detuning parameter to represent that one. So, you can write this omega 1 equal to small omega 1 plus epsilon sigma 1 equal to omega 2 plus epsilon sigma 2, similarly this capital omega 2 equal to 2 omega 1 plus epsilon sigma 3 and it can be written also equal to 2 omega 2 plus epsilon sigma 4.

So here you just see as we are taking 2 frequency that is omega 1 and omega 2. And also we are considering this natural frequency or this modal frequency as omega 1 and omega 2. Here, we are using 4 detuning parameter that is sigma 1, sigma 2, sigma 3, sigma 4 to source the nearness of those terms to that the external excitation.

This way by writing or by taking all these frequency combination, so we can apply this method of multiple scales and study that resonance condition. Here, you can get the secular terms also one can take this 1 is to 1 internal resonance condition.

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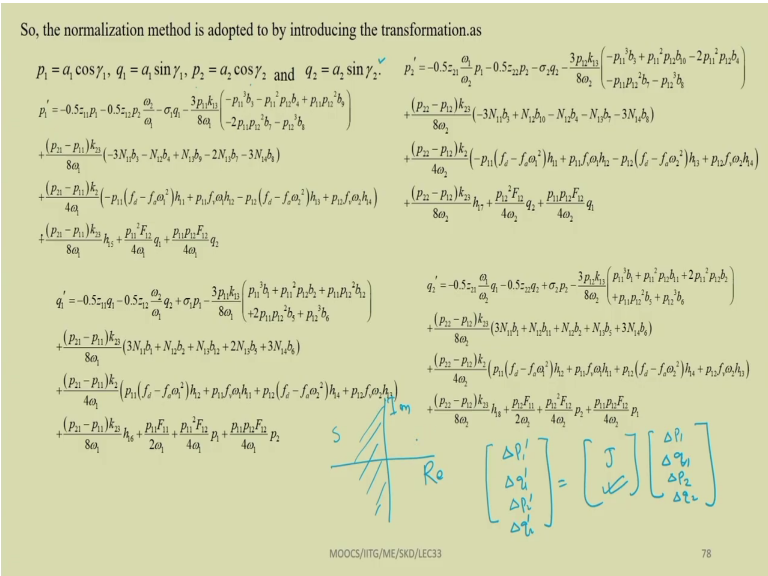

So 1 is to 1 internal resonance condition is nothing but, so here we are considering the second mode is nearly equal to the first mode and by taking that thing so one can find this equation reduced equation in terms of a 1 and gamma 1 so for finding the stability of the trivial state.

So, you can convert that thing into the normalized form that is this p 1 equal to a 1 cos gamma 1, q 1 equal to a 1 sin gamma 1 and p 2 equal to a 2 cos gamma 2 and q 2 equal to a 2 sin gamma 2. Then it will reduce to this form that is p 1 dash p q 1 dash p 2 dash and q 2 dash. Now, by perturbing these p 1 dash p 2 dash and p 1 dash p 2 dash q 1 dash and q 2 dash, so we can obtain this equation that is delta p 1 dash delta q 1 dash delta p 2 dash and delta q 2 dash.

So, this will be equal to the Jacobian matrix J into it will be equal to delta p 1 delta p 2 delta p delta so it will be delta q 1 delta q 1 p 1 q 1, this is p 2 and q 2. So this form you can reduce it to in this form. So then finding the eigenvalue of the Jacobian matrix, so we can find stability of the system.

So, if the eigenvalues are on the left hand side of this plane so this is the real part this is the imaginary part. So, if they are in the left hand side of the S plane so then the system is stable. So, if they are on the right hand side of the S plane then the system is unstable so this way we can study the stability of the system.

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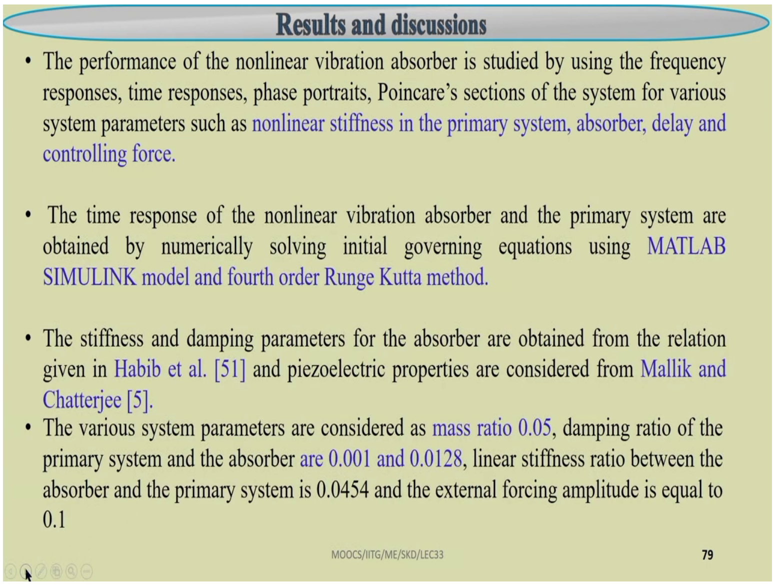

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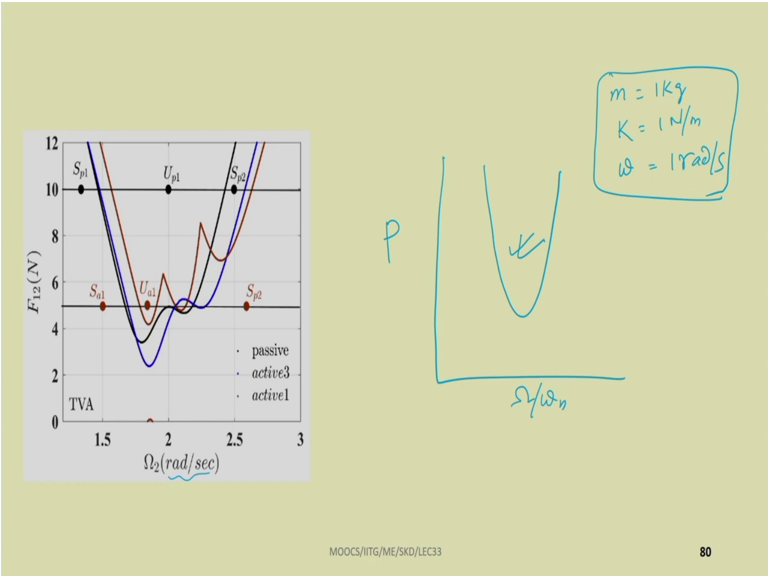

And, so we can study the stability of the system for different system parameters. You can use the Simulink to perform the same analysis also so here, as we have already seen so it can give rise to parametric excitation principal parametric excitation. So principal parametric excitation will occur near the non dimensional frequency. So here it is taken omega 2 nearly equal to 2 in the simulation it may be noted that the mass is taken to be 1 kg and the spring constant k also equal to taken 1 Newton per meter.

So, this omega equal to 1 is we got omega equal to 1 here omega 11 equal to 1, that is why this whether it is in whether this is written in dimensional or non dimensional form it is immaterial because this is taken to be 1 radian per second. Either one can plot this versus omega 2 or omega 2 by omega so that will give rise to non dimensional frequency.

So here one can observe so unlike in case of Mathieu Hill equation, so where one can get only 1 lobe, so here multiple lobes for example, in Mathieu Hill equation pressure this force versus omega by omega N if 1 plot then this is the stability region. So, here in addition to the single lobe so here multiple lobes are there. So these multiple lobes occurred so due to the presence of multiple frequency and also due to the presence of internal resonance condition.

So here we are considering the internal resonance condition that is omega 2, nearly equal to omega 1, so that is why we have several (Refer Time: 18:46) are available in this instability region. Already, we are familiar that outside this instability region the system is stable and so this is that is why this is S a1 and S p2, so they are stable condition and this U that is inside this thing is unstable condition.

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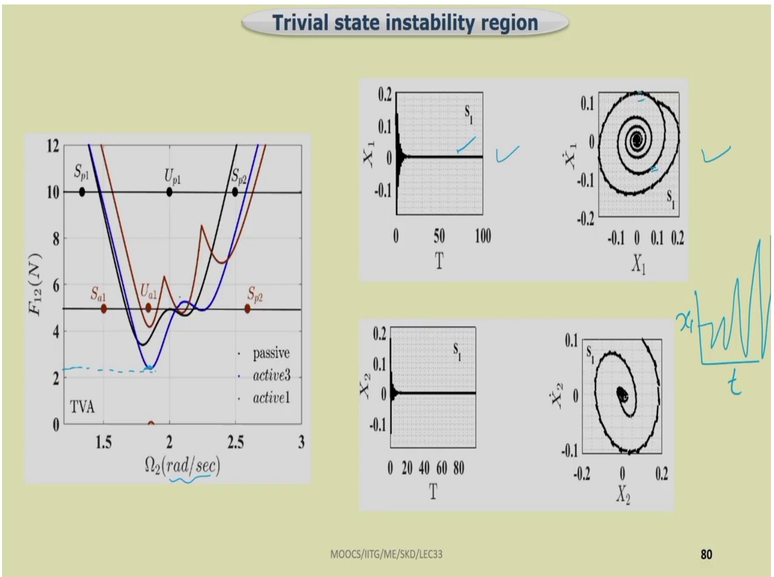

So at a different position, so you can plot the response plot also. So by plotting this response you can see the system to be the you can study the stability of the system. So for example, here it is written so here we have found the response at the stable point so you can see in phase portrait and in so this is the phase portrait plotted and this is the time response X 1 versus T is plotted.

So, clearly you can see so in steady state the response is trivially stable so; that means, so it is not vibrating. And here also, so this is outside region it has been plotted so if you plot it inside region so the response is not shown here, but you can see this is given as assignment so that is why it is not plotted here so you can verify that the response will grow with time so this is time and this is X 1.

So, in this unstable region you can find the response will grow with time. To observe the vibration of the system, so one must have to operate the system at a frequency where it is marked S that is it is in stable region. So here you can note so if you see this point particularly, if you see this point this critical point so in this critical point so you can observe that below this value of this F 12, whatever may be the frequency of the system the system remain stable.

This is the critical value below which the system remain always stable for vibration observation purpose. So, you can operate the system at a frequency or at a forcing F 12 below these critical value. Similarly, so you can see so here, three cases have been shown, one is the passive, second one is active. So two different type of active cases have been considered, by taking two different type of active conditions so you can see the active 1 case has very high F 12 than active 2 and passive case has the least value of F 12.

If you are using this active vibration control, so you can see so in that case very high value of F 12 is required to vibrate the system. It may also be noted that this vibration absorber can also be used for energy harvesting when it is operated in this unstable range. The same system can be used as a vibration absorber or energy harvesting depending on the application of the system and application or the range of frequency in which it is operated.

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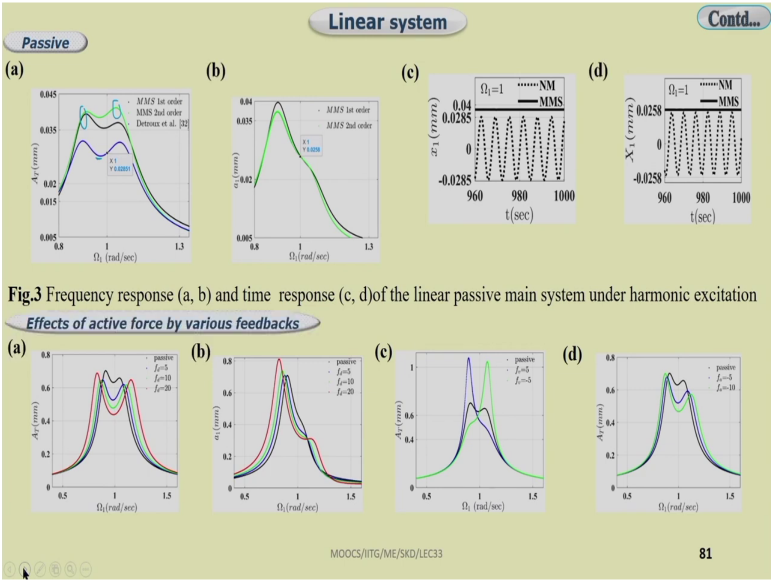

So, here also some comparison have been given these MMS 1st order MMS, 2nd order MMS and also with the one reference value. So you can see as we are using or we have a 2 degrees of freedom system, so here so one can observe 2 peaks so this is one peak and this is the other peak so these correspond to the shifting of the natural frequency or modal frequencies from the omega by omega 1 equal to 1.

It can be observed that it has a lower value, this value is previously in case of the single or primary system so one can observe the highest value or the resonance condition at nearly omega equal to omega 1.

But, when we are adding the secondary system the frequency is shifted to 2 value so that is why we are having 2 peaks, but at omega equal to omega n the response amplitude is minimum different conditions can be taken so for example, f d can be taken so for 3 different value of f d so passive cases have been taken so for different conditions it has been considered.

All those cases you have to verify as part of your assignment in this course. So effect of active force by various feedback and then passive case also you can see.

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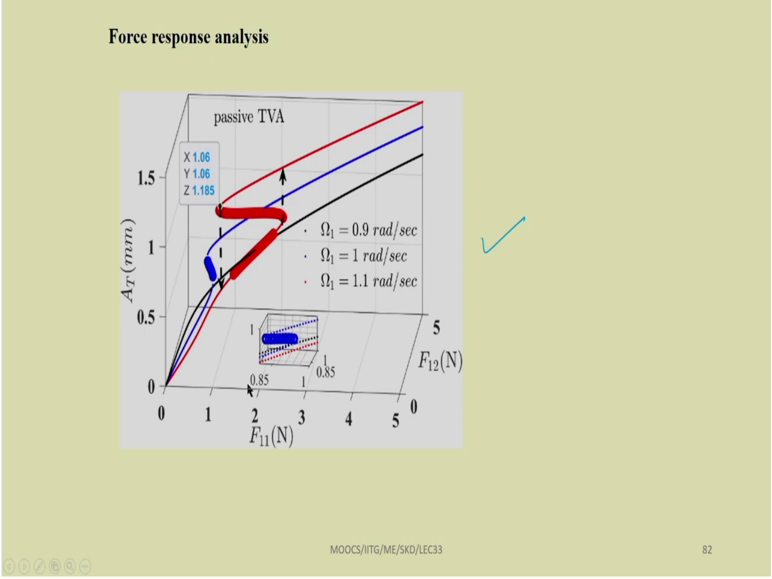

So, you can plot the force response plot also so for example, so you can take this F 11 2, you can vary this forcing F 11, F 12, simultaneously and see what will happen to this response amplitude in a 3D plot you can plot these thing and you can verify the response. So this way you can verify the system.

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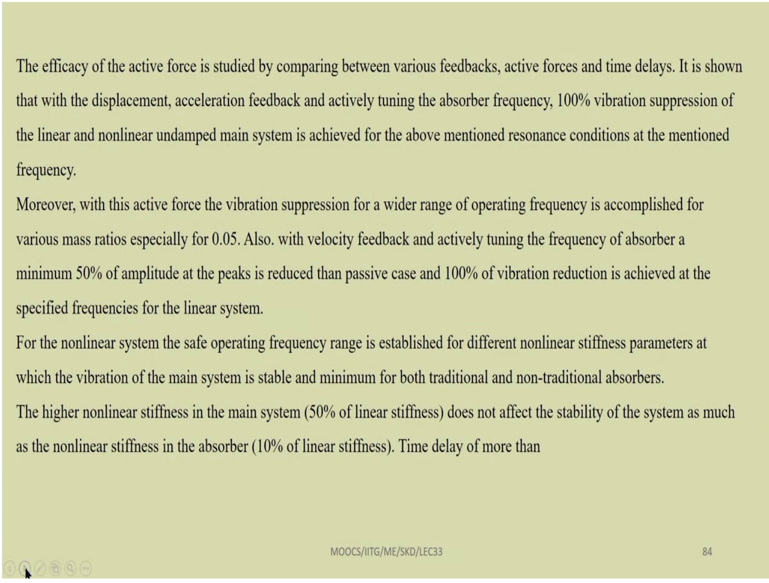

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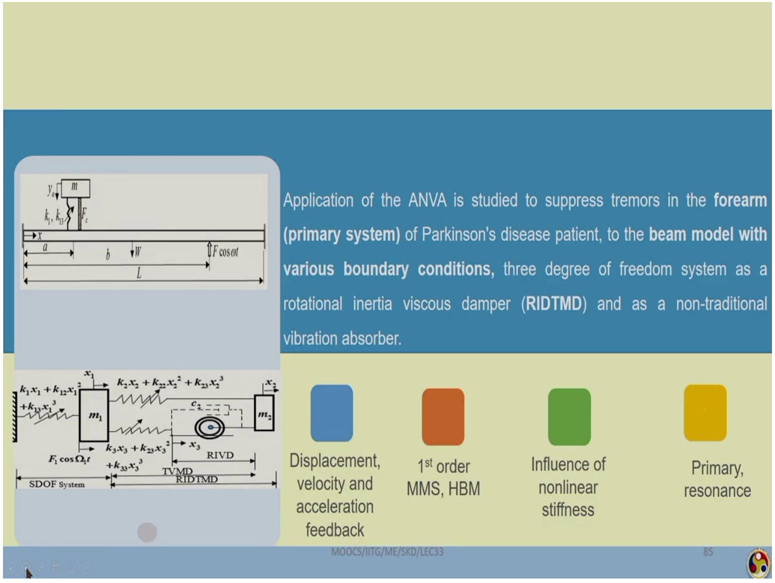

So we have now we have seen what we mean by this traditional and non traditional. So just I have shown you one case, but you can take this as an assignment and solve the problems for both traditional and non traditional active vibration absorber case.

So, let us now use this non-linear, active non-linear vibration absorber to suppress tremor in the forearm primary system for Parkinson's disease patient and also one can consider another system where we can take this as a continuous system that is a beam model with various boundary condition.

So, it can be considered as a 3 degrees of freedom system. So, let us take two cases and consider these two cases.

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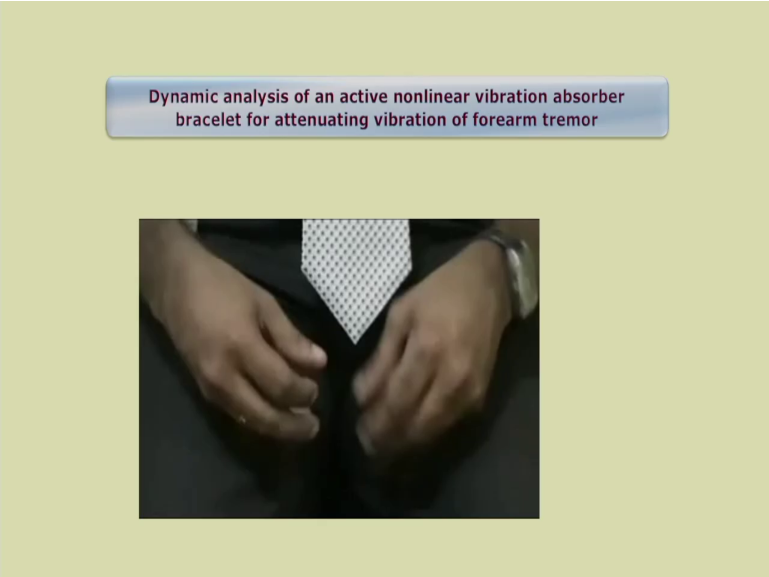

So, here also we can take the velocity, displacement and acceleration feedback. So you can see sometimes, one can absorb the tremor in the hand so now to control the tremor in the hand one can put a band here. As we know, by adding a mass to a vibrating system so we can suppress the vibration by shifting the natural frequency, actually resonance will occur when it is near the natural frequency.

So, when you are adding additional mass and spring to the system so we are shifting the natural frequency thereby we are reducing the vibration of the system.

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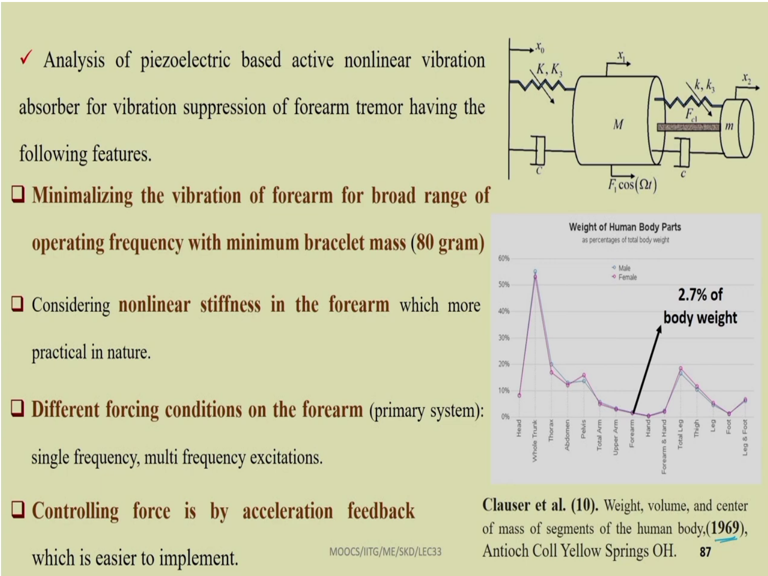

So, to control the vibration of the system one can use a bracelet so in the forearm one can use a bracelet having these effective mass and spring conditions and then it can be studied. So, you just see while designing such a system analysis of piezoelectric based active non-linear vibration absorber can be used for the suppression of the tremor in the hand using the following features.

So, one is minimizing the vibration of forearm for broad range of operating frequency with minimum bracelet mass of 80 gram. So in this case we have taken a minimum bracelet mass of 80 gram. It can be seen what is the different percentage of body mass where it is concentrated.

For examples, we have the head, then whole trunk. Whole trunk is more than 50 percent head is less than 8 percent then this thorax region so we have less than 20 percent, so for the forearm particularly if you see this is 2.7 percent of the body weight. So different part of the body so it is taken from this work of Antioch Coll Yellow Springs oh this is in 1969 work.

So, it is weight, volume, and center of mass of the segment of a body human body soul so one is for the male and other one is for the female body, so here the forearm is shown to be 2.7 percent of the whole body weight. One can consider the non-linear stiffness in the forearm which may be more practical in nature. So different forcing conditions of the forearm primary system so single frequency one can consider single frequency or one can consider multi frequency excitation to study the system.

So, controlling force is by acceleration feedback which is easier to implement because we can have the accelerometer mounted on the system.

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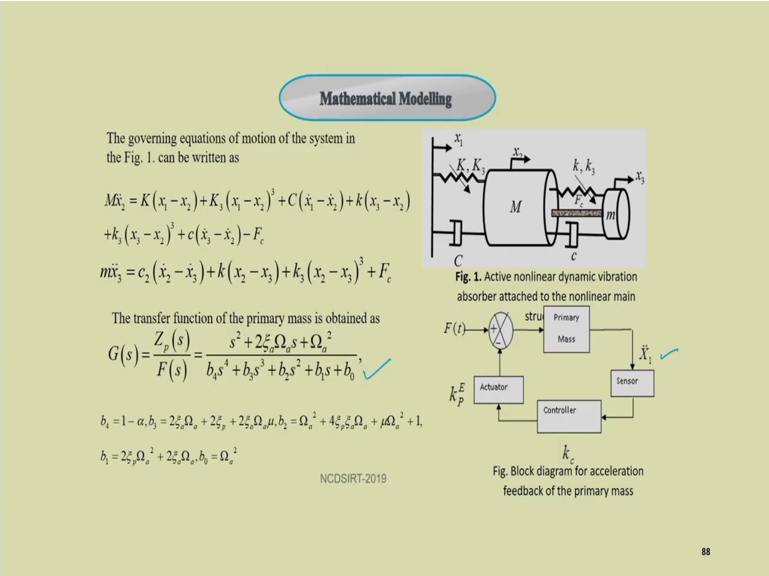

Using the acceleration feedback only acceleration feedback, so now, you can model the system, one can consider this as a linear system or non-linear system. So the equation of motion this is a torsional system.

Here the equation of motion can be written as M 2 capital M that is the primary mass so that is the hand or forearm so here equation can be written M x 2 double dot equal to K x 1 minus x 2 plus K 3 x 1 minus x 2 whole cube plus C into x 1 dot minus x 2 dot plus small k into x 3 minus x 2 plus k 3 into x 3 minus x 2 whole cube plus c into x 3 dot minus x 2 dot minus F c.

So, this F c is due to the piezoelectric stack actuator similarly m x 3 double dot can be written as c 2 into x 2 dot minus x 3 dot plus k into x 2 minus x 3 plus k 3 into x 2 minus x 3 cube plus F c. Considering the linear part so one can write the transfer function. So after writing the transfer function so then from the characteristic equation that is from the bottom denominator part either one can apply this Rutherford's criteria or some other criteria to study the stability of the system.

So, here we have a primary mass, so from the primary mass we are getting the acceleration with sensor. Then, we have a controller so k c x 1 double dot so that is the control force we are giving so we have the actuator and this is given the force to the system.

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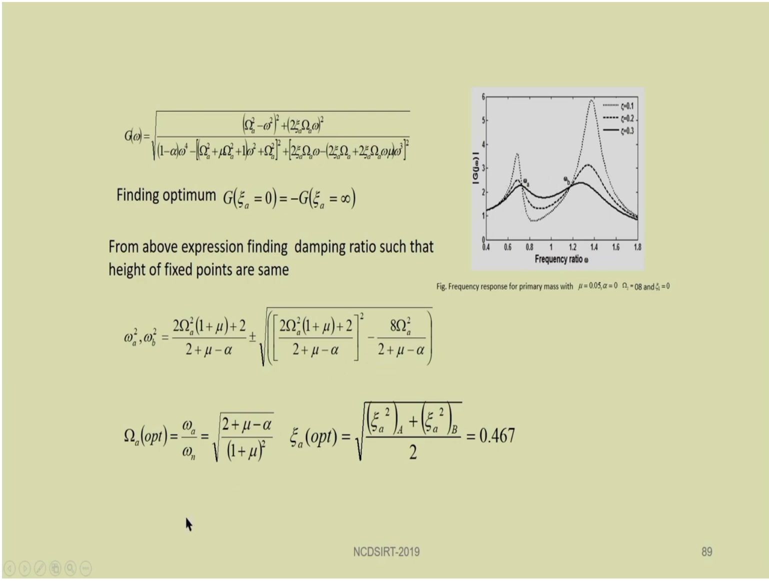

Similarly, here we can by using this transfer function. So we can write down these equations of the transfer from the transfer function we can find this thing G omega so that is the response amplitude so versus this response amplitude versus the frequency ratio. One can plot and find the value of these omega a omega b at which irrespective of damping so it is having the same value of response amplitude.

So, this value one can find this value and one can find the minimum or optimum parameter and using that optimum parameter.

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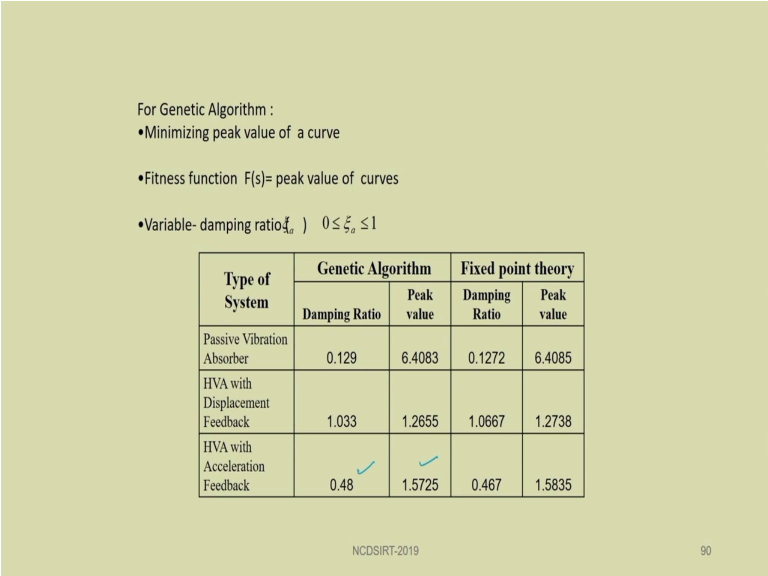

Type of System	Genetic Algorithm		Fixed point theory	
	Damping Ratio	Peak value	Damping Ratio	Peak value
Passive Vibration Absorber	0.129	6.4083	0.1272	6.4085
HVA with Displacement Feedback	1.033	1.2655	1.0667	1.2738
HVA with Acceleration Feedback	0.48	1.5725	0.467	1.5835

So, one can study the system there are many methods one can use; one can use the genetic algorithm also so minimizing peak value of a curve so one can use this fitness function F s peak value of the curve. So, variable damping ratio also can be taken so less than zeta a less than 1 so 0 less than zeta a less than 1. In case of genetic algorithm so some study has been carried out.

So passive vibration absorber so damping ratio was found to be 0.129, so this hybrid vibration absorber with displacement feedback this damping ratio is obtained to be 1.033 and in this hybrid vibration absorber with acceleration feedback it can be seen that the damping ratio is 0.48.

So, peak value is 6.0 in case of vibration absorber, the peak value is 6.4083 and HVA with the displacement feedback d is 1.26, HVA is hybrid vibration absorber 1.265 and in case of acceleration feedback you can see the active value has reduced to 1.5. So using this fixed point theory so this damping ratio is found in case of passive to be 0.1272 and the peak value is 6.0483, it is matching with the previous one.

Then with HVA, so taking that same value 1.0667, so the peak value equal to 1.27 in case of displacement feedback. And in case of acceleration feedback the peak value is observed to be 1. 5835.

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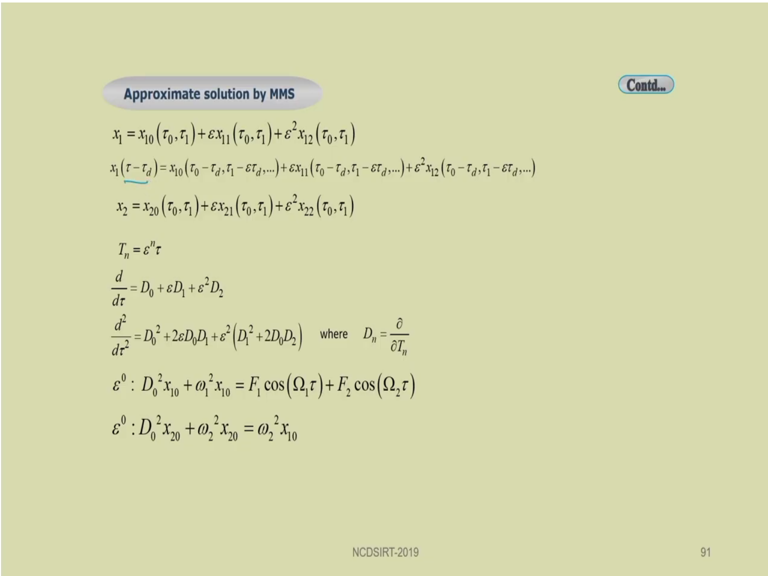

Now, by considering these non-linear systems so one can take use this method on multiple scales and also one can take the delay also in the system tau minus tau d and then by solving these equations one can obtain this reduced equation.

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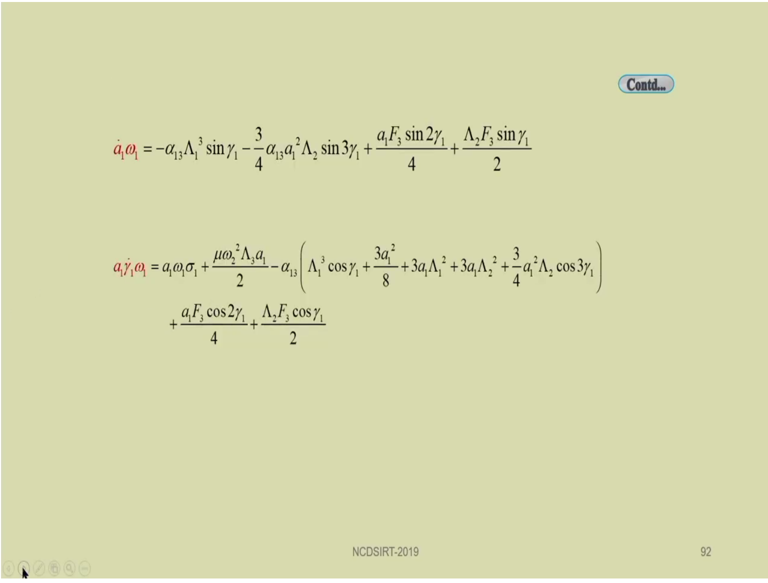

So, by using these reduced equation 4 reduced equation will be obtained and by using these reduced equation.

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Contd...

$$\dot{a}_2\omega_2 = -\frac{F_{c1}}{\mu}\Omega_2^2\Lambda_2\sin(\gamma_2-\Omega_2\tau_d)+z_2\Omega_2\Lambda_2\cos\gamma_2(\Lambda_2-\Lambda_5)-z_2\omega_2\frac{a_2}{2}+\omega_2^2\begin{pmatrix}\mu\omega_2^2\frac{\Lambda_5\sin\gamma_2}{\omega_1^2-\Omega_2^2}-\frac{3\alpha_{13}\sin\gamma_2}{\omega_1^2-\Omega_2^2}\left(\Lambda_2^3+2\Lambda_1^2\Lambda_2+\frac{a_1^2}{2}\Lambda_2\right)\\-\frac{\alpha_{13}a_1^3\sin(\gamma_2-3\gamma_1)}{8\left(\omega_1^2-(3\omega_1)^2\right)}-\frac{2\Omega_2\Lambda_2\sigma_2}{\omega_1^2-\Omega_2^2}\cos\gamma_2\end{pmatrix}$$

$$+\alpha_{23}\begin{pmatrix}\sin\gamma_2\left(\frac{3a_1^2}{2}(\Lambda_2-2\Lambda_2\Lambda_3-\Lambda_5+\Lambda_2\Lambda_3^2+2\Lambda_3\Lambda_5-\Lambda_3^2\Lambda_5)+\frac{3a_2^2}{2}(\Lambda_2-\Lambda_5)\right)+\sin\gamma_2\begin{pmatrix}3\Lambda_2^3+6\Lambda_1^2\Lambda_2-(6\Lambda_1^2\Lambda_5+9\Lambda_2^2\Lambda_5+12\Lambda_1\Lambda_2\Lambda_4)\\+(6\Lambda_2\Lambda_4^2+9\Lambda_2\Lambda_5^2+12\Lambda_1\Lambda_4\Lambda_5)-3(\Lambda_5^3+2\Lambda_4^2\Lambda_5)\end{pmatrix}\\+\frac{a_1^3}{8}\sin(\gamma_2-3\gamma_1)(1-3\Lambda_3+3\Lambda_3^2-\Lambda_3^3)-\frac{3a_2^2}{4}\sin\gamma_2(\Lambda_2-\Lambda_5)-\frac{3a_2}{2}\sin(2\gamma_2)(\Lambda_2^2+\Lambda_5^2)+3a_2\Lambda_2\Lambda_5\sin(2\gamma_2)\end{pmatrix}$$

$$a_2\omega_2\dot{\gamma}_2 = a_2\omega_2(\sigma_2-\sigma)-\frac{F_{c1}}{\mu}\Omega_2^2\Lambda_2\cos(\gamma_2-\Omega_2\tau_d)-z_2\Omega_2\sin\gamma_2(\Lambda_2-\Lambda_5)+\frac{\mu\omega_2^4\Lambda_5\cos\gamma_2}{\omega_1^2-\Omega_2^2}+\frac{\mu\omega_2^4a_2}{2(\omega_1^2-\omega_2^2)}+\omega_2^2\begin{pmatrix}-\frac{3\alpha_{13}\cos\gamma_2}{\omega_1^2-\Omega_2^2}\left(\Lambda_2^3+2\Lambda_1^2\Lambda_2+\frac{a_1^2}{2}\Lambda_2\right)+\\\frac{2\Omega_2\Lambda_2\sigma_2}{\omega_1^2-\Omega_2^2}\sin\gamma_2-\frac{\alpha_{13}a_1^3\cos(\gamma_2-3\gamma_1)}{8\left(\omega_1^2-(3\omega_1)^2\right)}\end{pmatrix}$$

$$+\alpha_{23}\begin{pmatrix}\cos\gamma_2\begin{pmatrix}\frac{3a_1^2}{2}(\Lambda_2-2\Lambda_2\Lambda_3-\Lambda_5+\Lambda_2\Lambda_3^2+2\Lambda_3\Lambda_5-\Lambda_3^2\Lambda_5)+\frac{3a_2^2}{2}(\Lambda_2-\Lambda_5)++3\Lambda_2^3+6\Lambda_1^2\Lambda_2\\-6\Lambda_1^2\Lambda_5-9\Lambda_2^2\Lambda_5-12\Lambda_1\Lambda_2\Lambda_4+6\Lambda_2\Lambda_4^2+9\Lambda_2\Lambda_5^2+12\Lambda_1\Lambda_4\Lambda_5-3(\Lambda_5^3+2\Lambda_4^2\Lambda_5)\end{pmatrix}+\frac{3a_1^2a_2}{4}(2\Lambda_3-\Lambda_3^2-1)-\frac{3}{8}a_2^3\\+\frac{a_1^3}{8}\cos(\gamma_2-3\gamma_1)(1-3\Lambda_3+3\Lambda_3^2-\Lambda_3^3)+\frac{3a_2^2}{4}\cos(\gamma_2)(\Lambda_2-\Lambda_5)-\frac{3a_2}{2}\cos(2\gamma_2)(\Lambda_2^2+\Lambda_5^2)+3a_2\Lambda_2\Lambda_5\cos(2\gamma_2)+3a_2(2\Lambda_1\Lambda_4+2\Lambda_2\Lambda_5-\Lambda_1^2-\Lambda_2^2-\Lambda_4^2-\Lambda_5^2)\end{pmatrix}$$

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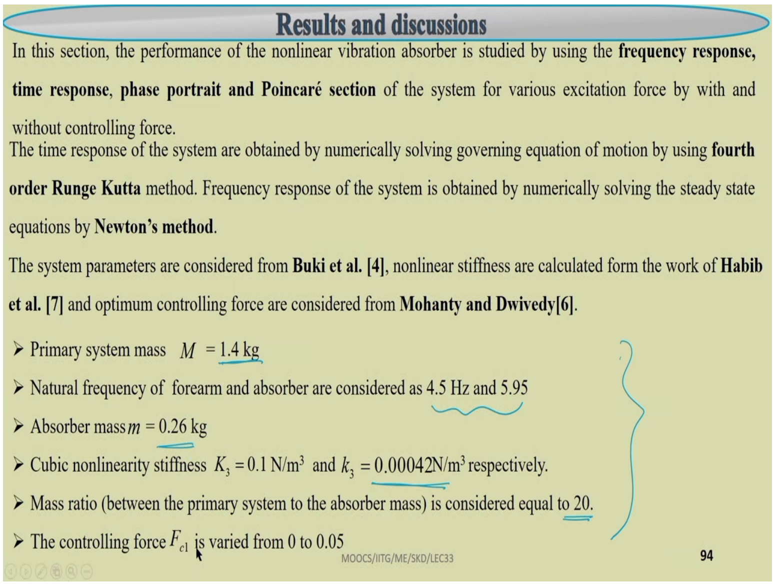

And, plotting the time response phase portrait Poincare section and using these fourth order Runge-Kutta method or by solving this set of equations by using Newton's method, one can study the system. So here, the study in the study mass is taken to be 1.4 kg, so natural frequency of forearm absorber considered to be 4.5 Hertz and 5.95 Hertz.

The forearm natural frequency 4.5 Hertz and the absorber frequency equal to 5.95 Hertz, then the forearm mass the absorber mass forearm mass we have taken 1.4 kg and absorber mass. We have taken only 0.26 kg 0.26 kg that is 20.26 kg is taken. The cubic nonlinearity stiffness K 3 equal to 0.1 Newton per meter cube K 3 equal to K 3 equal to very very small is taken that is non-linear spring constant.

And this mass ratio between the primary system and the absorber is considered to be 20, so that is why this is taken to be 0.26 the controlling force F c 1 is valid from 0 to 0.05.

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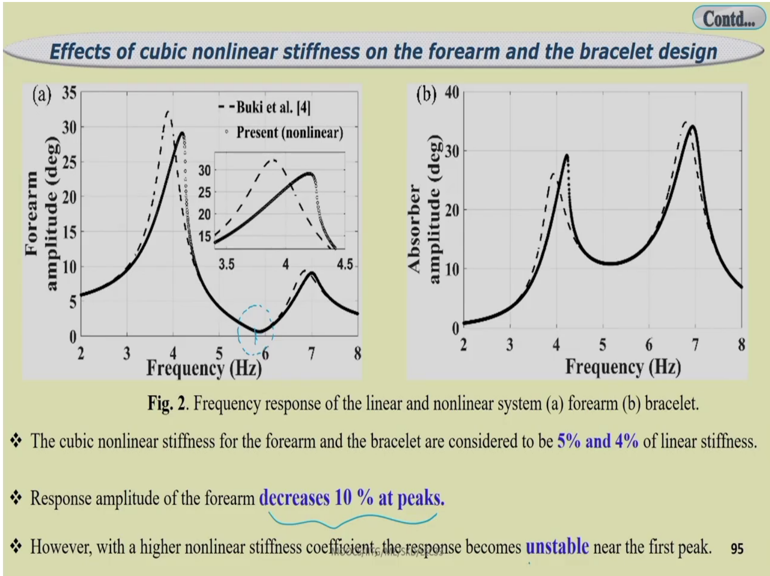

So, in case of passive it is 0 and in other case it is more. So here you just see at omega equal to omega n the response amplitude is very high, that is 25 degrees or tremor of 25 degree up to 25 degree forearm and tremor can be observed this way the forearm will 25 up to 25 degree it can move, but here you can see the response amplitude so by if it is shifted to this region so by putting these absorber so the response amplitude can be now be in this region so which is having this minimum amplitude.

So, the arm now you can observe that arm vibration is reduced in this region, so we have to put the system in such a way that so it will vibrate in this frequency range. But, the absorber amplitude you can see so the absorber amplitude so at these 2 peak region it is around 30, but at this natural frequency near the natural frequency so it is around 10 degree. So the cubic non-linear stiffness of the forearm and the bracelet are considered to be 5 percent and 4 percent of the linear stiffness.

So, the response amplitude of the forearm you can absorb decreases by 10 percent at the peak or the peak the so you just see this Buki et al, so this comparison has been made with Buki et al and the present case and it can be seen that the response amplitude of the forearm decreased by 10 percent; however, with higher non-linear stiffness coefficient, the response becomes unstable the response becomes unstable near the first peak.

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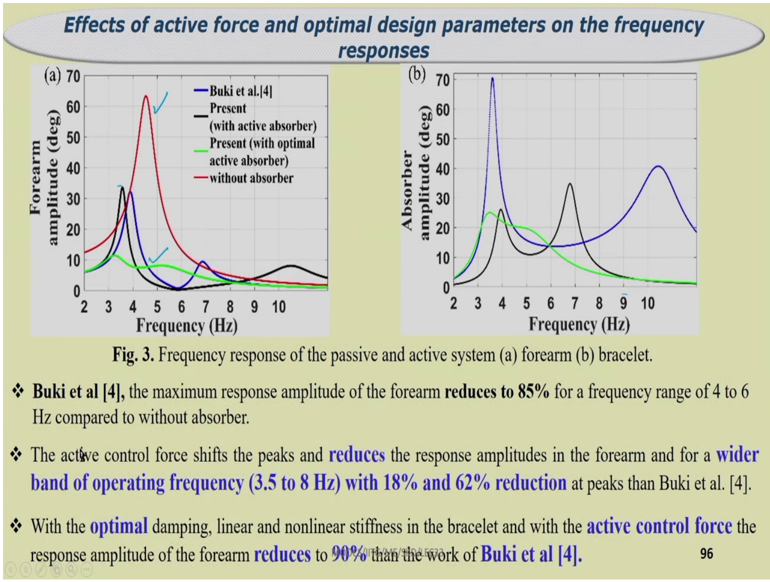

So, here the response become unstable. If we are using this effect of active force and optimal design parameter in the frequency response, so you can observe that for a wide range so it can be observed for that for a wide range we can control the maximum or peak amplitude.

Without absorber this red colour curve shows without absorber, so without absorber the amplitude is around 60 degree.

So now, by putting with active so without absorber so with present case it is so this is present absorber this one, so you can observe that for a wide range the response amplitude has been reduced. So, it is reduced below 10 degree also the vibration, and if you are operating at a frequency very higher value of that one, so it will further decrease to near to the 0 value.

It has been this work has been compared with that of the Buki et al. So in Buki et al, the maximum response amplitude of the forearm reduces to 85 percent of the frequency range of 4 to 6 hours compared to without absorber the active vibration force shifts the peaks and reduce the response amplitude of the forearm and for a wider band of operating frequency that is 3.5 to 8 hours, with 18 percent and 62 percent reduction at the peak.

So, you just see so it has been reduced further from those obtained in the literature. So here, you can see the peak value has been reduced here, so it is significantly reduced in this case. With the optimal damping linear and non-linear stiffness of the bracelet and with the active control force the response amplitude of the forearm reduces to 90 percent than the work of Buki et al. So in this way, you can have a study of the non-linear vibration absorber for a different purpose so you can see the response amplitude also.

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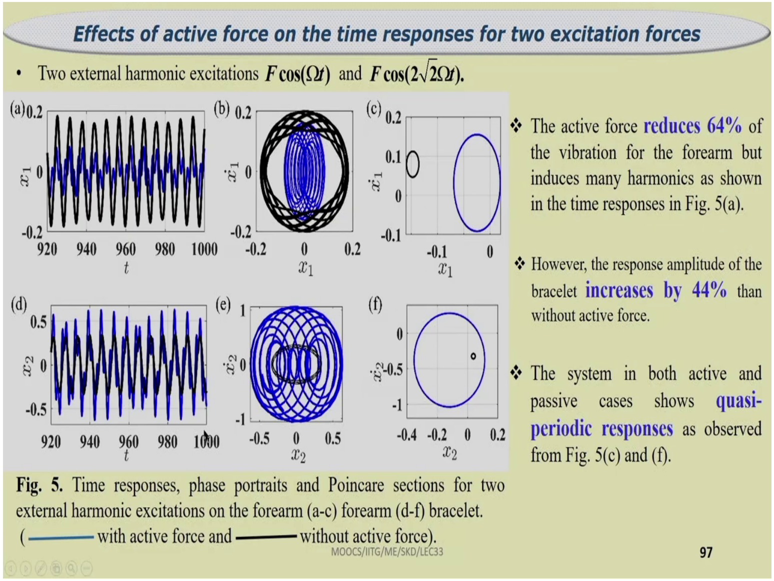

So, response amplitude clearly you can see so it is reducing significantly. So the active force produces 64 percent of the vibration of the forearm, but induces many harmonics as shown in this figure so it reduces many harmonics.

And however, the response amplitude of the bracelet increases by 44 percent than without active force, so the system in both active and passive cases so quasi periodic response as observed in this figure so you can obtain this quasi periodic response so this is the Poincare section zone so on.

So, in case of the quasi periodic response so if you plot the Poincare section so we can clearly observe a closed loop so here so without so one is with the active force and without active force so both the things you have can observe here.

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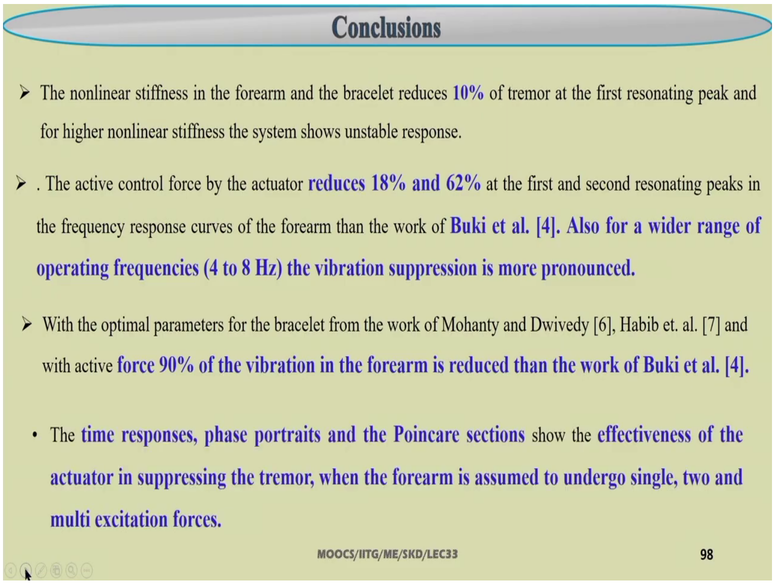

So this way you can reduce, so in this case we have seen why we can use this as a torsional vibration absorber.

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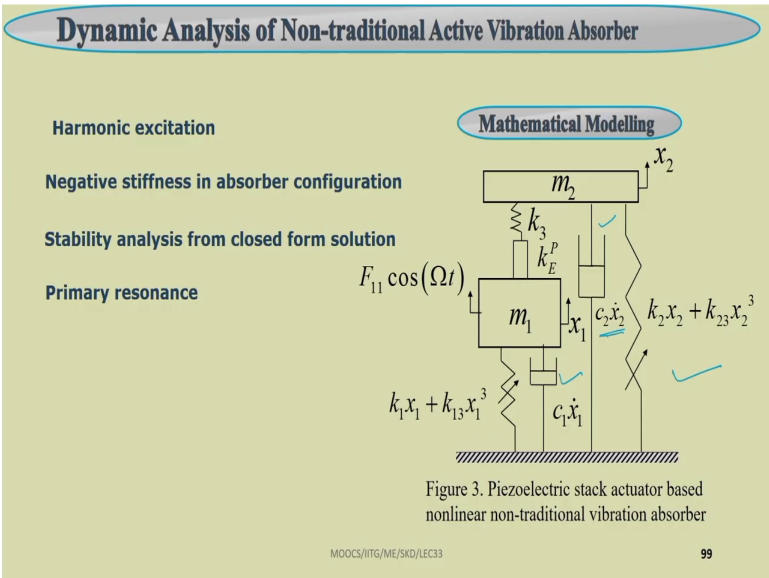

And, we can effectively control the vibration of the forearm significantly by using a vibration absorber. So we have used the system as a 2 degrees of system. So, also we have used a torsional system also. Previously, longitudinally vibrating system, now we have studied a torsional vibration absorber. Similarly, you can study the system with many different kinds of application.

So the system can also be modelled or the system can be modelled in this way also so where you can use this non traditional active vibration absorber using, so previous case we have not taken a damper so one can take a damper here. So in case of the non traditional you just see.

So, previously we have connected only one spring. So now, a non-linear spring is also added, so this is previously we have used a damper here, between the secondary mass and the base what one can use in addition to these damping, so one can use a non-linear spring also.

So the primary system previously we have considered only with this non-linear spring, so one can add this damper also to the primary system. So one can do or one can study several combinations of all these systems and depending on the application so all those things can be effectively checked.

So, here a forcing of F 11 cos omega t is added. So here only harmonic excitation is considered so previous case so we have taken both direct and parametric excitation. So in this case we have not considering the parametric excitation we can perform the stability analysis from closed loop solution.

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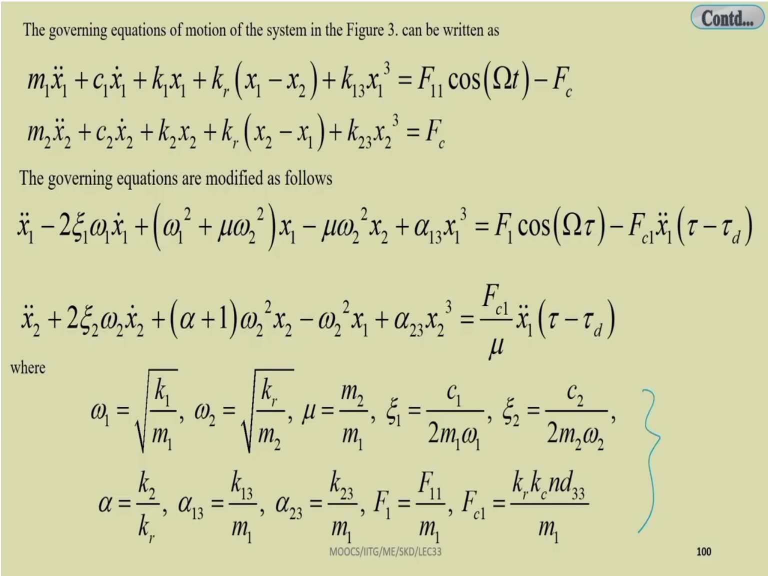

So, primary resonance condition can be considered. Already you know how to write down this equation of motion. So write down this equation of motion and then dividing these mass m 1 and m 2.

So, I am taking these non dimensional time one can take, or taking only this way these parameters, you can reduce this governing equation to this form where this they are coefficient of these acceleration term that is x 1 double dot x 2 double dot r equal to 1 so here omega 1 is considered to be k 1 by m 1 and omega 2 is taken as k r by m 2 and mu is this mass ratio m 2 by m 1 zeta 1 is c 1 by 2 omega 2 m 1 omega 1 zeta 2 equal to c 2 by 2 m 2 omega 2.

So, you can take this parameter for example, alpha equal to k 2 y k r alpha 13 equal to k 13 by m 1 alpha 2 3 equal to k 23 by m 1 F 1 equal to F 11 by m 1 and F c 1 equal to k r k c n nd 33 by m 1.

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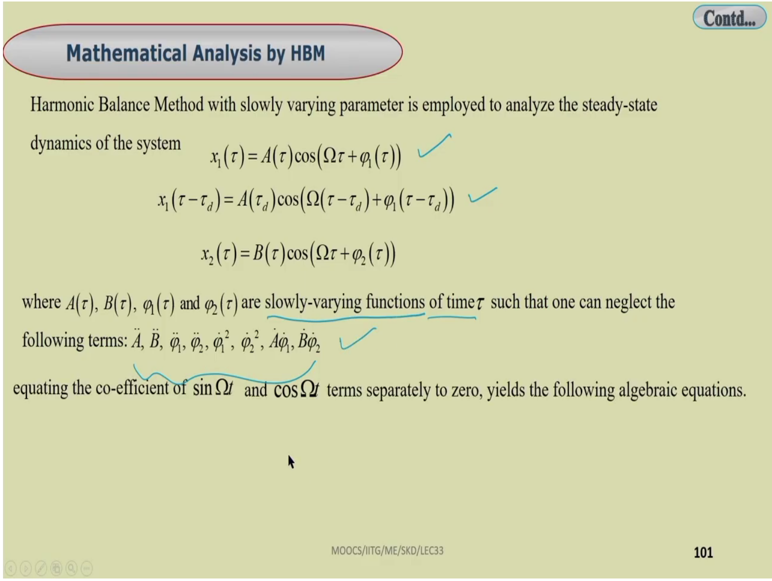

So, taking different system parameter. Previously, we have used this method of multiple scales. So here, I am showing you how you can use this harmonic balance method to solve similar system. So when you are applying this harmonic balance method, so you can take many terms or you can consider this way also.

So, when you are taking a number of terms actually the mathematical complexity will increase, but in that case you must use the symbolic software package to derive this equation motion rather than deriving it manually. So, you can take this x 1 tau equal to A tau cos omega tau plus pi 1 tau. Similarly, x 1 tau minus tau d so you are if you are considering the delay system so then x 1 tau minus tau d equal to A tau d cos omega tau minus tau d plus psi 1 tau minus tau d.

So, similarly x 2 can be considered as B tau cos omega tau plus psi 2 tau. So here you just note unlike in these previous harmonic balance or previous cases where we use to consider these A and B are constant, but here in this case we are considering A and B to be the time varying term. So by considering these A and B to be time varying term. So, in this case you can have a set of equations where you can perform the stability analysis simultaneously.

So, otherwise you have to again for after getting the response, again you have to perturb these equations to find the stability of the system. So here, A tau B tau psi 1 tau and psi 2 tau are considered to be slowly varying function of time tau. So as it is observed to be slowly varying function of tau so then you can neglect this term.

For example, A double dot that is d square A by d tau square. Similarly, B double dot that is d square B by d double d t d tau square all these terms. So, here you have a higher order so you can neglect these term, because you are considering only slowly varying function of time so now, by equating the coefficient of sin omega t and cos omega t separately.

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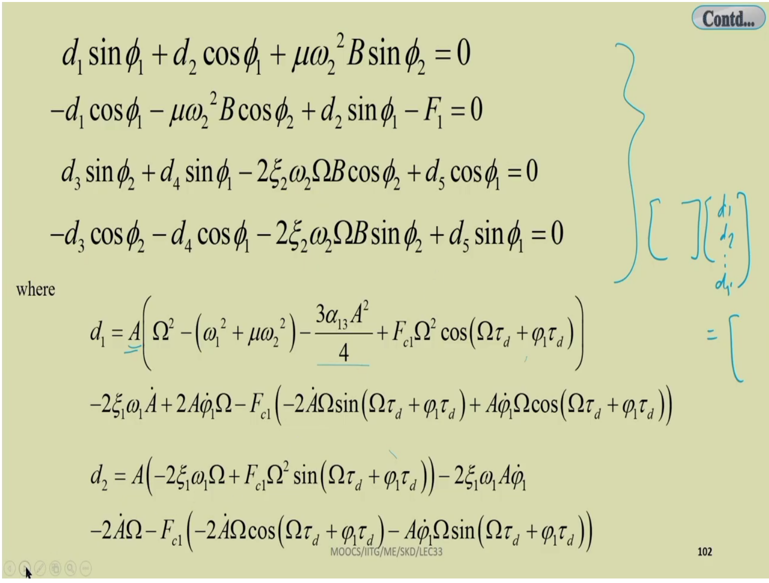

So, you can get these equations so you just see you can get a set of 4 equations so which contain these terms so where these d 1, d 2, d 3 and d 4, d 5 so are these function of the system parameter. So, you can reduce all the equation to this form now you can write this thing in a matrix form where you have this d 1, d 2, d 5 so will be equal to you just see d 1, d 2 ok. So here what are the terms are used you just see.

So, this is constant d 1 is d 1 contains A into the c A A square all these terms are there d 1, d 2, d 3 and it is written d 5, d 1, d 2, d 3, d 5.

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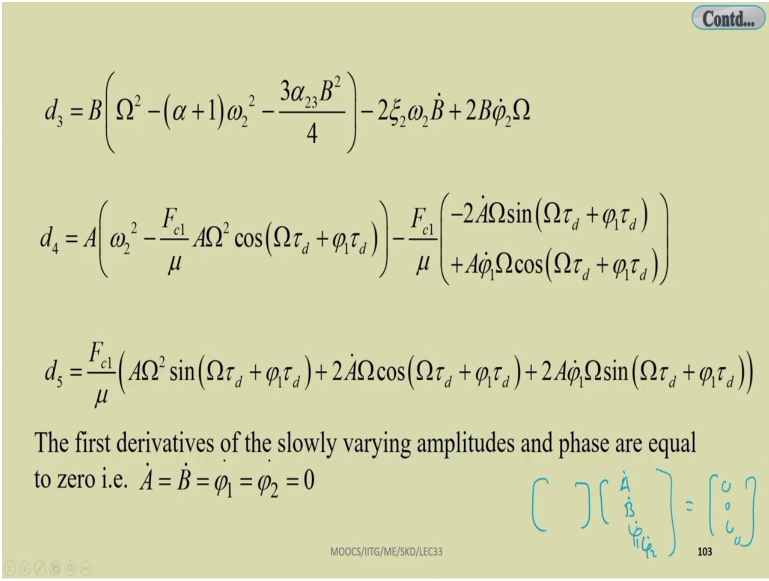

So, you can find this thing d 4, d 5, so these are the expression the first derivative of the slowly varying amplitude and phases equal to A dot B dot, so your system you can write in terms of so these into. So, it will be A dot B dot then psi dot and psi 1 dot psi 2 dot, so this way you can write this equation. Or it will be a function of some parameters also maybe there in the right hand side.

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Contd...

Solving the equations

$$P_1^2 + P_2^2 - P_3^2 - P_4^2 = 0$$

where

$$P_1 = B\left(\left(d_1^2 + d_2^2\right)\left(\mu\omega_2^2 d_5 - d_2 d_3\right) - \mu\omega_2^2 d_1\left(d_1 d_5 - d_2 d_4\right)\right)$$

$$P_2 = B\left(\left(d_1^2 + d_2^2\right)2\xi_2\omega_2\Omega d_2 + \left(d_1 d_5 - d_2 d_4\right)\left(\mu\omega_2^2 d_2\right)\right)$$

$$P_3 = F_1 d_2\left(d_1 d_5 - d_2 d_4\right),\ P_4 = F_1\left(d_5\left(d_1^2 + d_2^2\right) + d_1\left(-d_1 d_5 + d_2 d_4\right)\right)$$

$$A = \sqrt{\frac{B^2\left(d_3^2 + \left(2\xi_2\omega_2\Omega\right)^2\right)}{\left(d_4^2 + d_5^2\right)}},\ \tan\varphi_1 = \frac{(q_3 t_3 - d_3 t_2)(t_4 t_1 - t_1 t_3)}{(q_3 t_4 - d_3 t_1)(t_3 t_1 - t_2 t_4)},\ \tan\varphi_2 = \frac{j_1 j_3 + j_2 j_4}{j_2 j_3 - j_1 j_4}$$

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You just see solving this equation so you can get this, so this is the expression for A.

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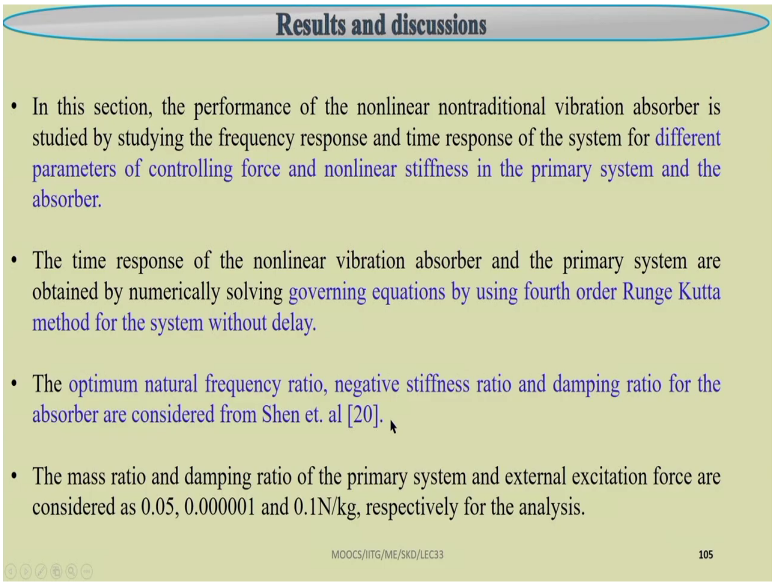

Similarly, the expression so response amplitude expression you got.

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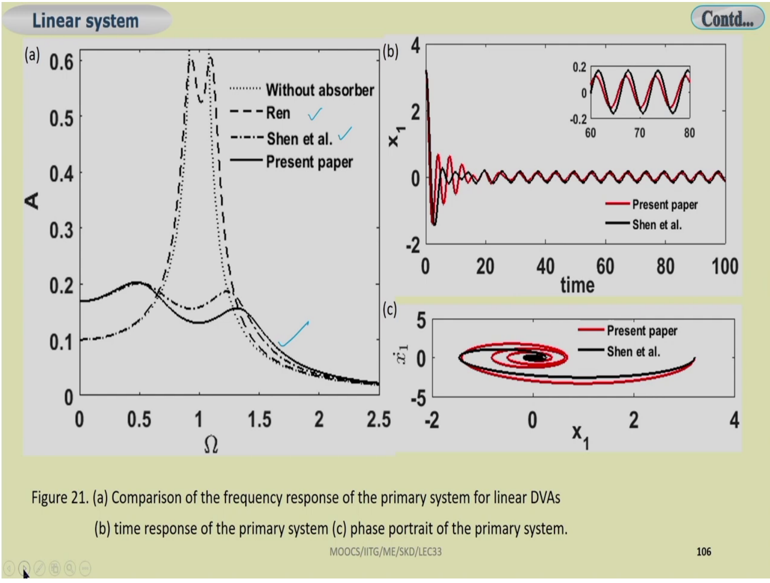

Figure 21. (a) Comparison of the frequency response of the primary system for linear DVAs (b) time response of the primary system (c) phase portrait of the primary system.

So, now by you can see the response so without absorber so it is comparison is given here. So you just see this curve is without absorber so if damping is considered. So you have two equal peaks here. So, now these two are by other two different researcher and in the present case by considering properly the system parameter, so you can observe that it is reduced significantly.

The response amplitude is reduced significantly. You can observe the time response also, present case and Shen refer so you have a less settling time also. So quickly it settles to its steady state that is the 0 oscillation.

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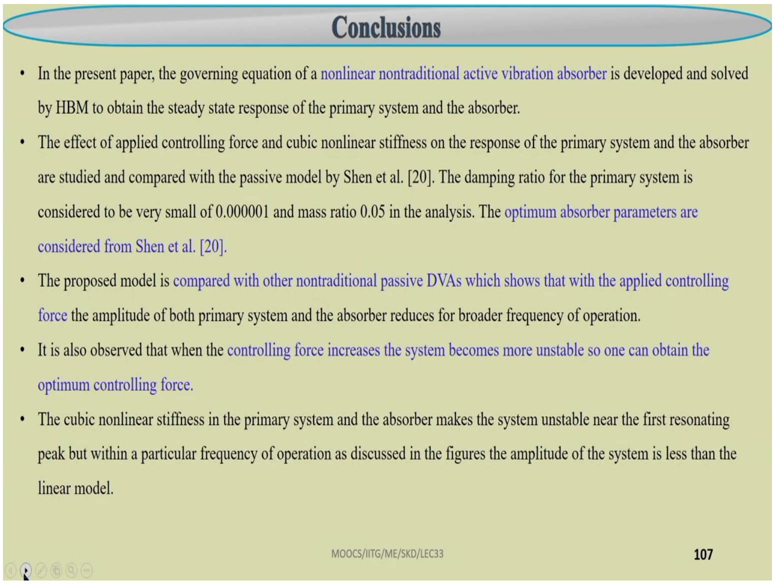

So, properly by using the system parameters so in this way so you can reduce the response amplitude of any system.

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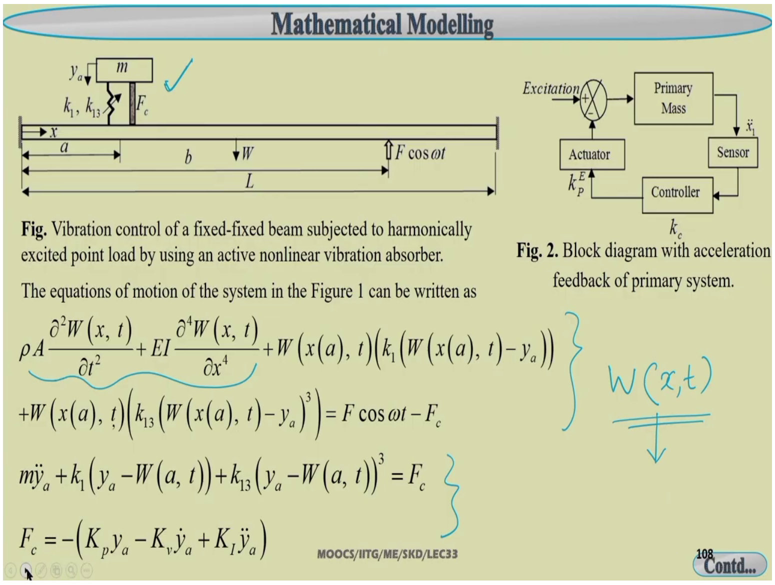

So, if you want to take so another system, I am it can show you so you can take this as a continuous system so you can in this continuous system so you can apply force F cos omega t and here this is the absorber part. So this system you can take as an assignment and you can solve the system and you can find so. In case of the beam so you can take as a different boundary conditions also. So this is a fixed beam is taken.

So, we can take this as a simply supported beam or cantilever beam also. So the first part is nothing but the Euler Bernoulli beam equation. So rho A del square W by del tau square plus E I del 4th by del x 4th, then other terms are there. So here are the forcing F cos omega t is acting this weight of the system W is there then in addition to that so you have a secondary system. So this is the equation of the secondary system and this is the equation of the primary system.

So, the primary part you can see this is W, so where W is a function of both x and time. So you have to reduce this system that is x W x and time to its temporal form first by applying this Galerkin method and then you can solve the system.

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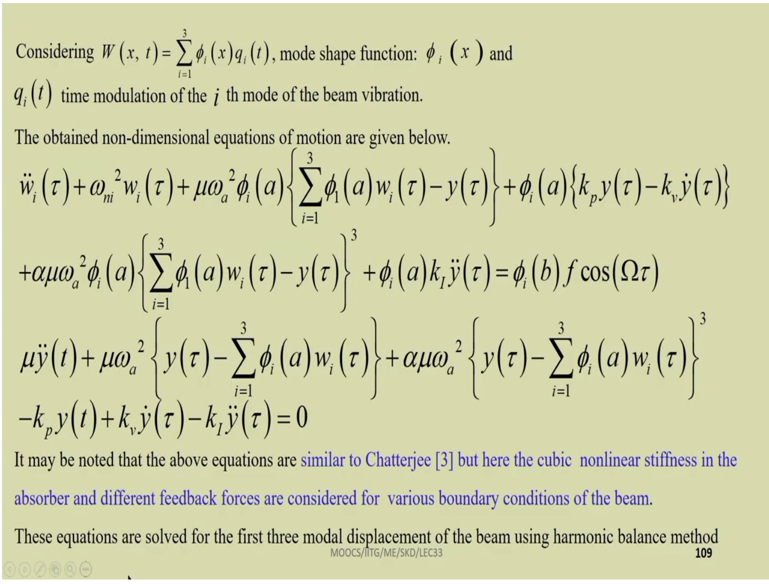

When you are applying this Galerkin method, so for example, W x t is considered to be pi x into q it so depending on the number of modes you are considering your equation of motion will be the coefficient of the equation of motion will be different.

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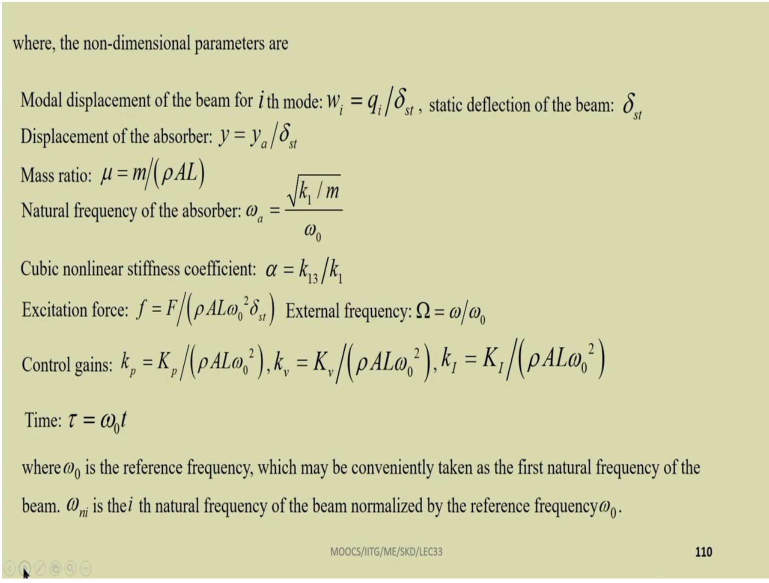

So, you can use this orthogonality principle by using these eigenvectors and then it can be reduced to a simpler form.

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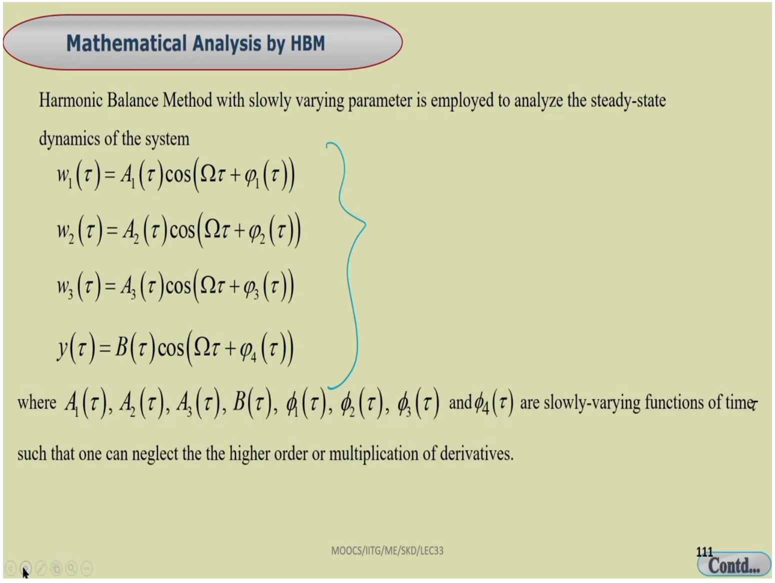

Then you can either use this method of multiple scale or use these harmonic balance method so here harmonic balance method is used.

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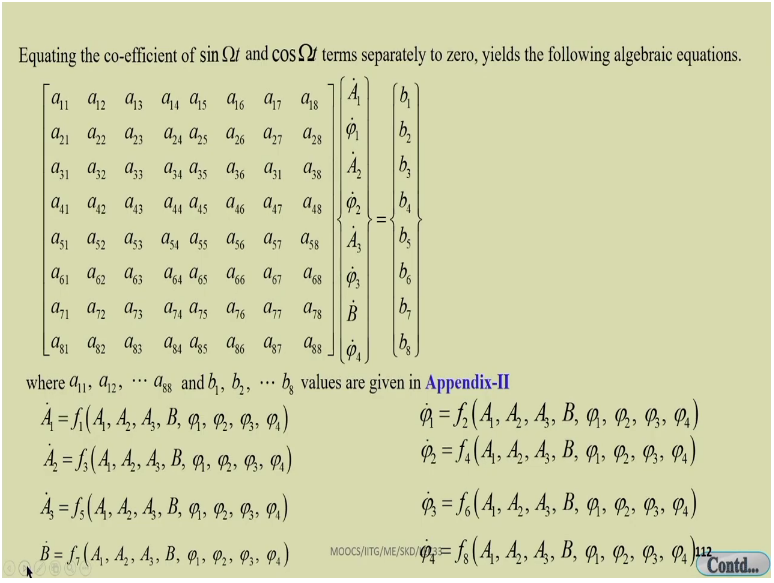

So, you can find the reduced or you can find a set of equations so you just see. So now, we have considered three modes, so by considering three modes we have A 1 dot, A 2 dot, A 3 dot and for the absorber we have this B. So, now you see this as reduced to a 4 degrees of freedom system by taking three mode into account. So, depending on the number of modes you can you are considering so the size of the matrix will be different. Now, you can solve this equation to find the response of the system.

So you can solve this is a first order set of 1st order equation either you can use this Runge-Kutta method to solve these equations to find the response or you can use some other numerical techniques also to find the response so you can see these are function of the system parameters.

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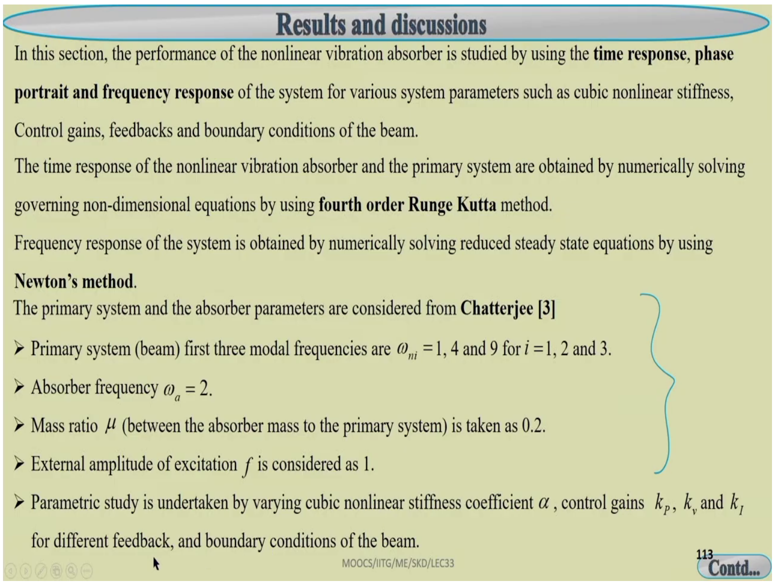

So, this way you can study this paper or this work by taking the primary system, the first 3 modes has been taken. So here the omega 1 is considered to be or taken to be 1 omega 2 is taken to be 4 and omega 3 is considered to be 9 so you just see that is integer relationship between these first 2 second and third modes. The absorber frequency is taken to be omega a equal to 2, the mass ratio between the absorber mass and the primary system is taken to be 0.2, the external amplitude of excitation f is considered to be 1.

Parametric study undertaken by varying different cubic non-linear stiffness coefficient control gain k P k V and k I for different feedback and boundary conditions of the system.

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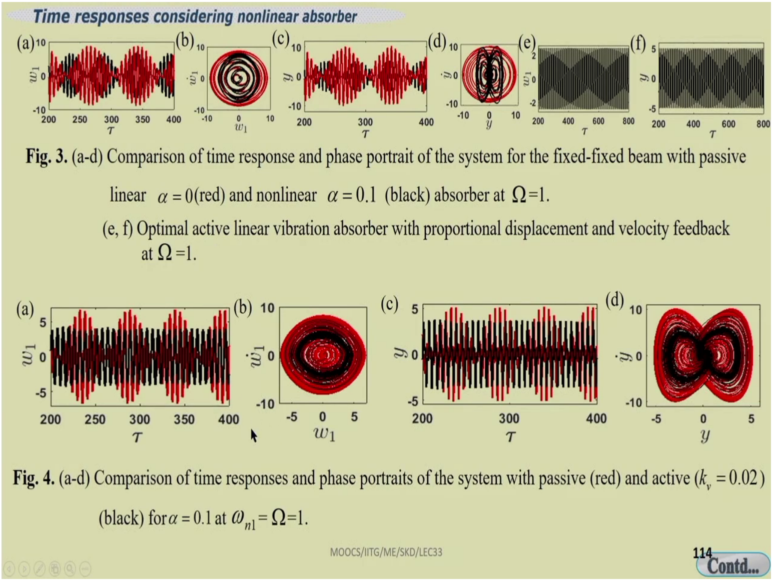

So, that way you can study and I am not going to show you all the results.

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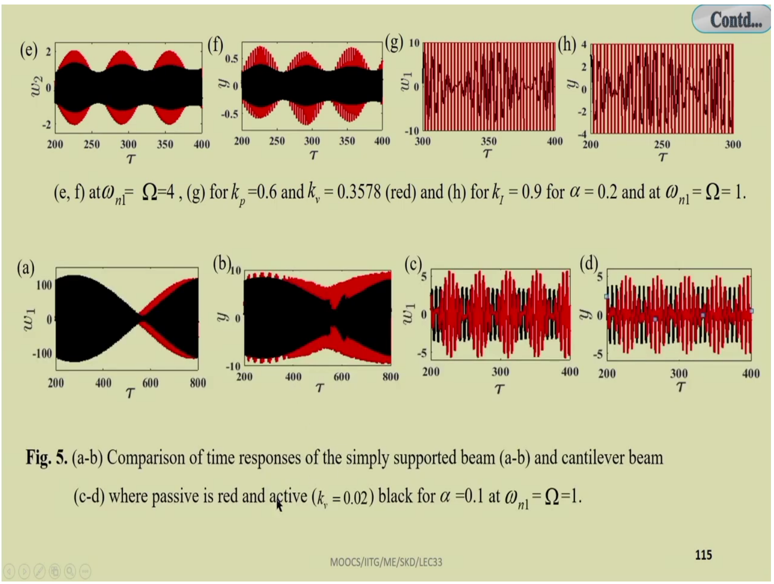

So, you can easily observe that you can by using different controlling parameter.

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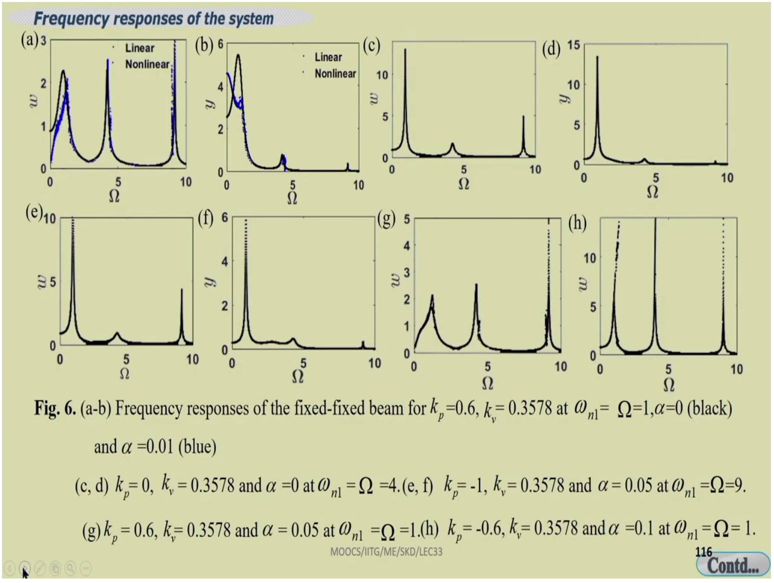

So, you can easily control the response of the system so you can easily find the difference between the linear and non-linear systems.

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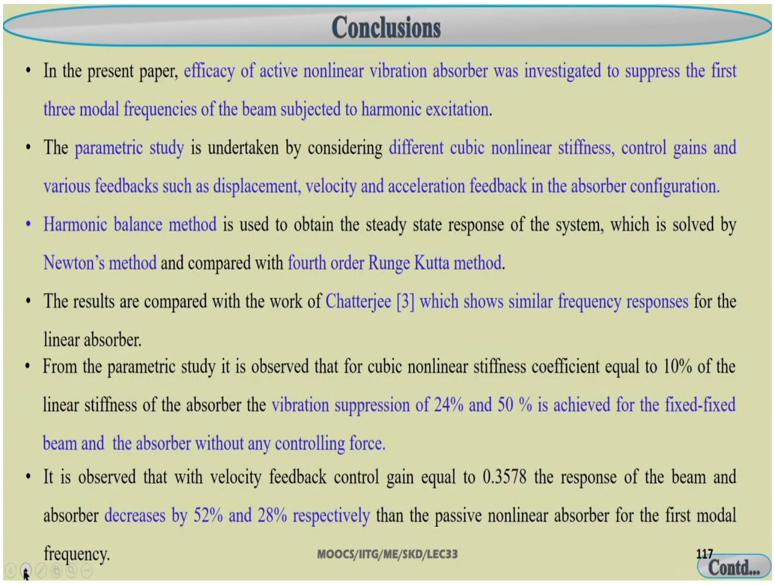

And also, how you can control the response of the system. So, with this so we can conclude this vibration absorber here and we will see some other applications of the non-linear vibration. Particularly, the last three classes we are going to study, how we can control chaos in the system. So we will take some chaotic system and study how to characterize the chaos in the non-linear system.

And, then we will study how we can have different crisis and how we are going to control the chaotic response, and of the system. So whether the chaotic response are helpful or useful or for what applications they are useful and where they are not useful so all those things also we are going to study.

Thank you very much.