Nonlinear Vibration
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Lecture - 03
Equilibrium points: potential function

So, welcome to today class of Non-linear Vibration. So, today is the 3rd class in the introduction session in module 1.

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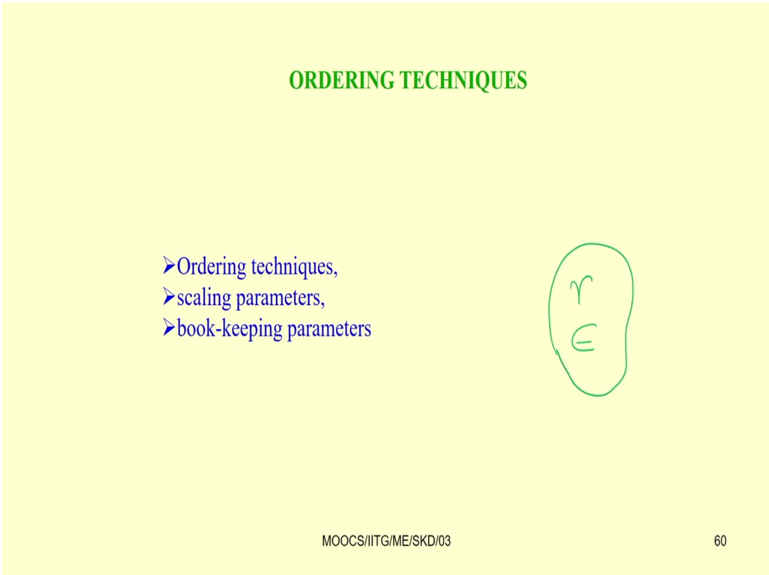

So, last class we have studied about the ordering techniques and also we have reviewed the linear systems and in ordering techniques particularly I told you regarding the scaling parameter. So, we can use this scaling parameter and book keeping parameters; scaling parameter so, r and book keeping parameter epsilon. So, these two terms I have included or introduced to you in the last class.

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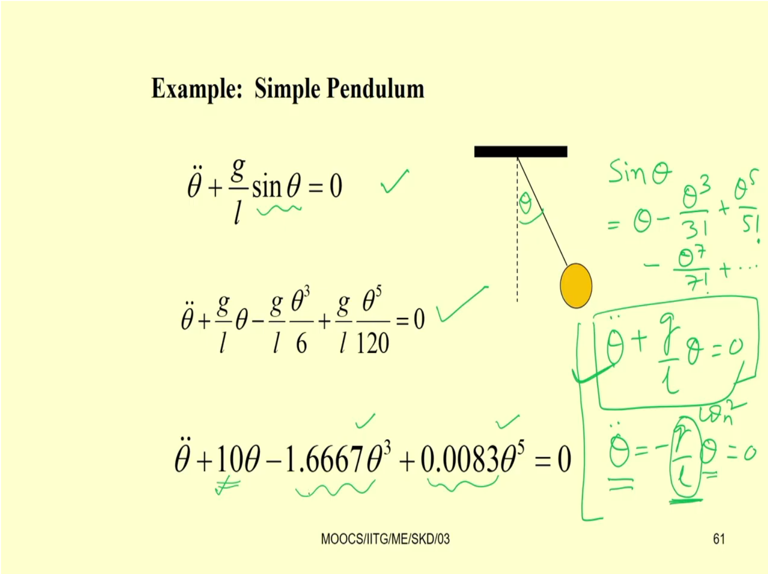

So, let us revise that thing briefly and then we can see what are the commonly used non-linear equations which we are going to study in this course; so, for example, so, already you are familiar with the simple pendulum and that example we have solved in last class again I am repeating that thing, so that you can better understand this thing.

So, in case of the simple pendulum you know how to derive this equation motion. So, you can derive this equation motion by applying either this Newton Euler formula Newton equation, Newton second law by drawing the free body diagram or by using this energy principle; for example, Lagrange principle or Hamilton principle. So, derivations of equation of motion we will see in the 2nd module and that time again we will derive this equation motion.

So, for the time being so, as you know the equation of motion of the simple pendulum can be written; so, taking this angle as theta as theta double dot plus g by l sin theta equal to 0. So, if theta is small then the sin theta you can write the sin theta equal to theta and then this equation will be theta double dot plus theta or g by l theta; g by l theta equal to 0.

So, in this case; in this case this is the equation of motion of a other equation is for a simple harmonic motion because you can write this theta double dot equal to minus g by l theta equal to 0. Here this acceleration term proportional to this displacement theta and takes place in a direction in the opposite to the acceleration that is why the motion is simple harmonic motion and this g by l can be replaced by omega n square.

So, this is the natural square of the natural frequency of the system. So, this part you know very well. So, now, if theta is not small, so, you can expand this sin theta. So, you can expand sin theta equal to theta minus theta cube by factorial 3 plus theta 5th by factorial 5 minus theta 7 by factorial 7 and in this way you go expanding this thing particularly by taking these first three term.

So, you can write this equation equal to. So, this equation equal to theta double dot plus g by l theta minus g by l theta q by 3 6; this 3 factorial is equal to 3 into 2 that is 6 plus g by l into theta to the power of 5 by 5 factorial. So, 5 factorial equal to 5 into 4 into 3 into 2 into 1 that is equal to 120; so, this way you can write this equation.

So, now, by taking for example, this g equal to approximately 10, so, we can and l equal to 1 meter then this equation can be written in this form that is theta double dot plus 10 theta minus 1.6667 theta cube plus 0.0083 theta to the power 5 equal to 0. So, always you will have a tendency to neglect this term and this term when you compare the term with the coefficient of theta.

Coefficient of theta is the linear term; coefficient of linear term similar to your omega square, but I have described it before. So, you will have always a tendency to neglect this 0.0083. So, this 5th order always you can neglect. Similarly, many times we neglect this cubic order nonlinearity also. So, by expanding this sin theta so, now, you can see so, we have a non-linear equation.

So, you have already seen how to distinguish between the linear and non-linear equation. So, the linear and non-linear equation you can separate by applying the super position rule and you can easily see that due to the presence of this theta cube term and theta 5th term no longer this equation is linear.

But, if you have simple this theta double dot plus g by l theta equal to 0 so, easily you can apply the super superposition rule and you can tell this equation to be linear. So, now, we can use a scaling parameter to write down this equation in a form where you will not have a tendency to neglect this term.

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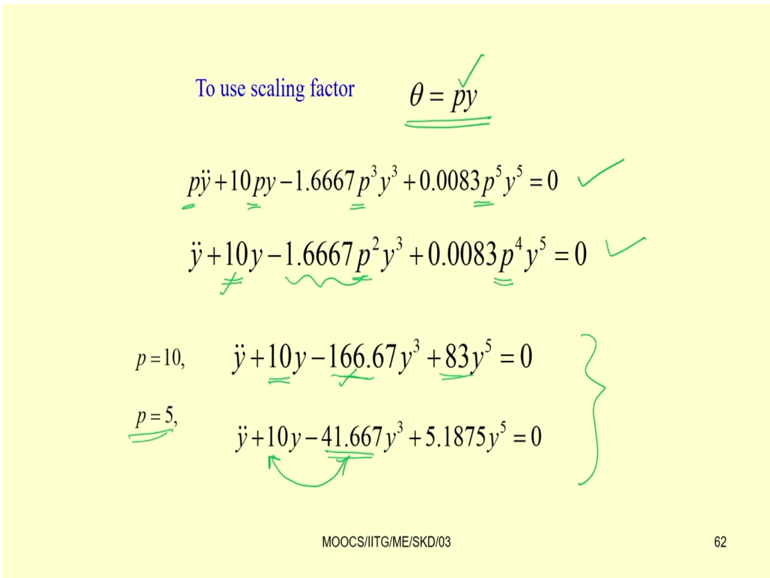

So, let us see. So, let us substitute this theta equal to p y. So, if we are substituting this theta equal to p y so, we can write this equation p y double dot plus 10 p y minus 1.6667 p cube y cube plus 0.0083 p to the power 5 y to the power 5 equal to 0. Similarly, so, now we can substitute or this equation you just see we can take this p common from this equation as p is present in all the term. So, here you have p, here also p. So, in every terms you have p. So, you can take a common p and you can write.

So, this equation as p into y double dot plus 10 y minus 1.6667 p square y cube plus 0.0083 p 4 y 5th equal to 0 as p naught equal to 0 we are taking if p equal to 0 then theta will be equal to so, p naught equal to 0. So, as p naught equal to 0 so, this must be equal to 0, so, this is the governing equation now. So, your y double dot plus 10 y minus 1.6667 p square y cube plus 0.0083 p to the power 4 y to the power 5 equal to 0.

So, now, our objective is to take the p in such way that so, this will be similar to a number which is coefficient of y that is the here in this particular case it will be near to 10. So, if we are taking p equal to 10 you can see this equation reduce to y double dot plus 10 y minus we have taken p equal to 10 so, p square equal to 100.

So, then this becomes 166.67 and this is as we are taking this is equal to p to the power 4 then 10 to the power 4 we have to multiply with these thing. So, this becomes 83, y to the power 5 equal to 0. So, you just see in this particular case you have seen the coefficient of y cube equal to 166 and the coefficient of y equal to 10 only.

So, this is no longer a weak a non-linear term. So, you may not have the urge to neglect this term. So, now, what we can do? So, let us take some other numbers. So, by taking this p equal to 5 you just see now this 166 now reduce to 41.667. So, this equation now becomes y double dot plus 10 y minus 41.667 y cube plus 5.1875 y to the power 5 equal to 0. So, similarly so, here you just see it is more or so, this term is coming closer by taking a number 5.

So, it is coming closer to 10. Similarly, if you can take some other number and check so, you can find some value of y. So, in which these numbers will be closer to the coefficient of the linear term. So, this is the use. So, this way you can use a scaling parameter. So, this is p is here the scaling parameter. So, you can use a scaling parameter to order the equation of motion.

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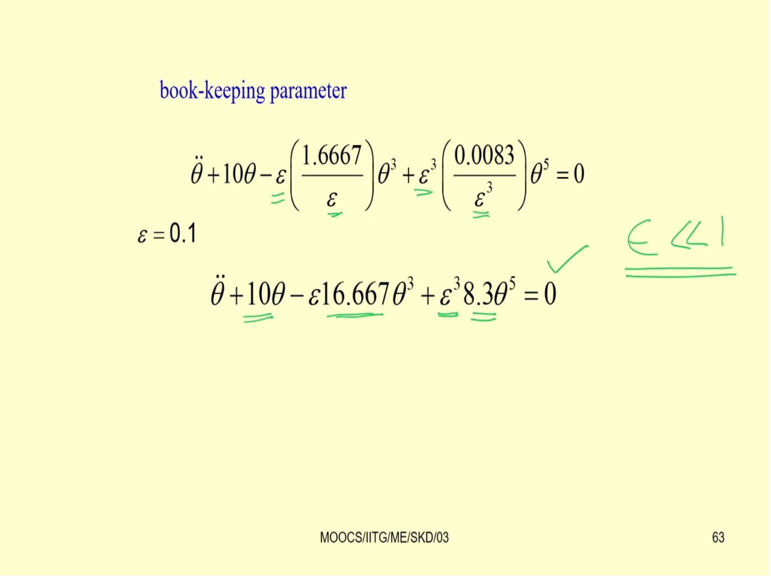

So, now, let us see how we can use a book keeping parameter. So, book keeping parameter epsilon, so, that is a parameter. So, which is very very less than 1 so, which is very very less than 1. So, you can take a parameter which is very very less than 1. So, you just see you have seen that scaling parameter may or may not be less than 1, but this book keeping parameter always less than 1.

So, in this governing equation so, you just multiply and divide by epsilon, so, the number will not change. Similarly, here if you divide and multiply by epsilon cube also this number will not change. So, now, taking this epsilon equal to 0.1, so, you can write down this equation equal to theta double dot plus 10 theta minus 16.667 epsilon theta cube plus 8.3 theta to the power 5 so, where it is multiplied by epsilon cube.

So, now just see this number 16.667 and 8.3, so, they are more closer to 10. So, by using this book keeping parameter epsilon so, you have clearly ordered this equation to a form where the non-linear terms are more or less closer to the that of the coefficient of non-linear terms are more or less closer to that of the linear term.

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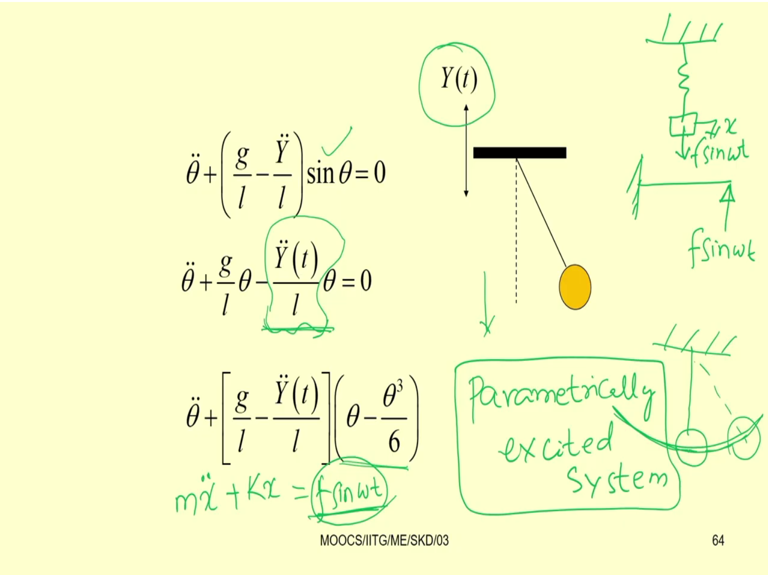

So, let us see few more examples. So, for example, so, we have a simple pendulum. So, simple pendulum attached to a platform and the platform is moving up and down. So, what is this equation of motion? So, previous case you have seen only the simple pendulum with a fixed support. So, in this fix support this is the simple pendulum and it has come to this position and then it will move up and down.

So, this way it will oscillate and you already you have seen this equation, but if the platform itself is moving up and down so, your equation of motion if you derive this equation of motion it will simply modified to this form because this we have to take this acceleration term here. So, this is Y t. So, this Y t, then acceleration will be Y double dot t.

So, as the gravity is always taking place in the downward direction and so, this equation you can write in this form that is theta double dot plus g by l minus Y double dot by l sin theta equal to 0. So, in this case you can see this equation by expanding this equation, you can write this is equal to theta double dot plus g by l theta minus Y double dot t by l. So, Y double dot t by l theta equal to 0.

So, here Y is the motion of the platform. So, this motion can be in the form of a periodic force or it is a function of time. So, when it is a periodic function of time, so, the inclusion of this term you can clearly see that inclusion of this term where the coefficient of theta that is the response is a time varying term; so, this is a time varying term; coefficient of theta is a time varying term.

So, due to the presence of this type of term this equation is known or this coefficient, this parameter time varying parameter as it is coefficient of theta. So, these type of equations are known as parametrically excited, the equation of a parametrically excited system parametrically excited system.

So, later we will see what is the difference between a parametrically excited system and a directly excited system. Directly excited system in this case; so, for example, let us take a system like this. So, this is a cantilever beam it is subjected to a force like this. So, this is the this force is f sin omega t or take the case of the simple pendulum. So, in which, so, this mass is subjected to a force f sin omega t.

So, in this case so, the response of the system is written mx double dot plus Kx equal to f sin omega t. So, if you compare this equation. So, that equation is mx double dot plus Kx equal to f sin omega t. Here the forcing term is not the coefficient of x or response, but in this case the so, in this case the time varying term that is the excitation term is the coefficient of response that is theta.

So, if it is the coefficient of the response or this time varying parameter is the coefficient of the response then that type of systems are known as parametrically excited system and if it is written as a homogenous function without the without writing as a coefficient of response then it is directly excited system. So, later we will study regarding more detail regarding this parametrically excited system and what is the difference of that system with respect to this directly excited system.

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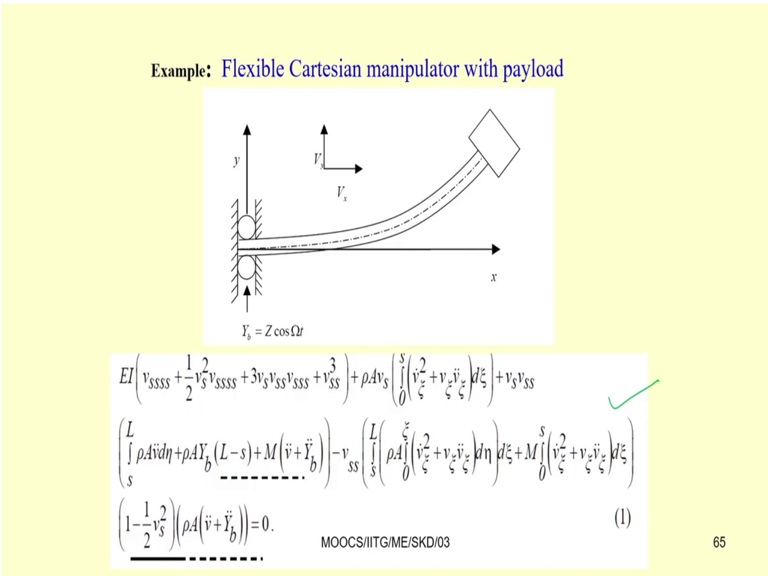

So, few more examples of non-linear equations so, you can see. So, this is a cantilever beam and here the base is excited. The system is similar to that you have seen in this previous case. So, where the platform is moving up and down and platform is excited also. In that case you can see the equation of motion. So, we will see later how the equation of motion will be there.

So, in this case you can see the equation of motion will contain many non-linear terms. In the previous case also when we expand this sin theta so, we can get a non-linear equation here. So, here sin theta can be written as theta minus theta cube by 6. And, the governing equation it will be a non-linear equation.

Similarly here so, for a base excited cantilever beam. So, we can get a equation similar to this. So, these type of equations we will study later in detail. So, how to derive these equations and how to solve these equation so, this is the material course material of this course non-linear vibration.

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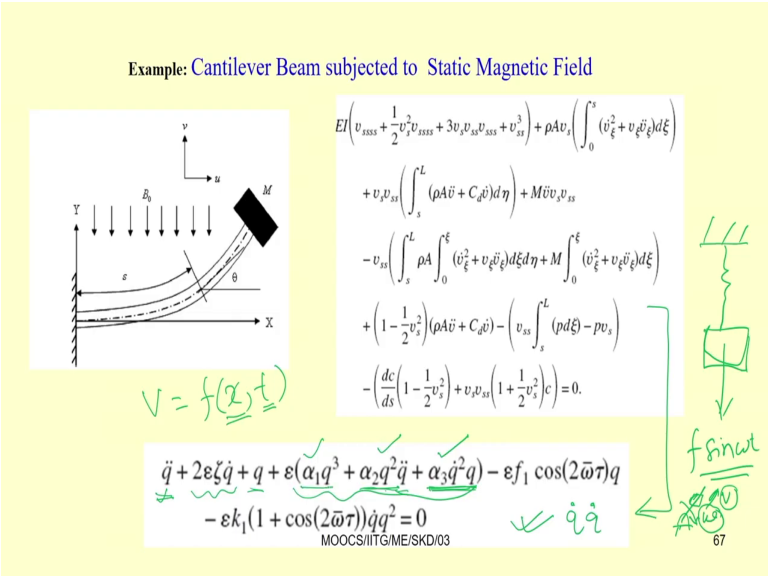

So, let us see some more examples also. So for example, same cantilever beam, so, if it is subject to a magnetic force. So, in this case you just see the magnetic force is a non contact type of force unlike your the force what I have shown for a spring mass system. So, that is; so that is the force is directly attached or force is directly applied to the system here f sin omega t force is directly applied to the system here.

So, this is contact type of force. So, it is in contact to the body, but so, if you are applying a magnetic field. So, it can be a non contact type of force. So, due to this magnetic field so, the beam will be subjected to forcing and movement and then you can write down this equation of motion.

So, here you can see so, this is a continuous system. So, unlike the lump parameter system, in continuous systems the response for example, in this case the response let me write in terms of v. So, this v is a function. So, it will be a function of both distance and time.

So, along this length along the length of the beam so, the response will vary and also the response will vary with time. So, this is a function of both space and time. So, by applying this generalized Galerkin method so, you can reduce this spatio temporal equation to its temporal form. So, later we will see how we can reduce this equation to its temporal form. So, you can see this is in its temporal form.

So, here this equation if you see clearly. So, this equation is written this is q double dot then 2 epsilon zeta q dot. So, this is the damping term, so, this is the inertia term. So, this is due to the stiffness parameter that is k q and these terms are non-linear term. So, you can see this non-linear term that is alpha q cube. So, this is due to this geometric nonlinearity and these two terms you just see q double dot is nothing, but this acceleration.

So, here you have a non-linear terms in acceleration. So, this is this term is known as inertia non-linear term and here also you have a term q dot square. So, q dot multiplication of two velocity term q dot into q dot multiplication of two velocity terms. So, can be giving rise to acceleration.

So, for example, in case of the coriolis components so, you know so, if you have a system. So, let you have a link. So, this link along with in the link we have a slider. So, if the link is moving with angular velocity omega and the slider is moving with v then you know the coriolis component equal to 2v omega coriolis acceleration equal to 2v omega.

So, product of this so, this is velocity angular velocity and this v is also linear velocity here the product of two velocity is giving rise to this Coriolis acceleration. So, that is why the product of two velocity terms sometimes will give you acceleration. So, due to the presence of acceleration so, these term is also known as so, this term is also a inertia and non-linear term.

Inertia is coming mass into acceleration will give you the inertia force. So, due to the presence of this acceleration term q dot square so, this is also inertia force and this term is due to inertia force or this is inertia nonlinearity this is also inertia nonlinearity and this term is for geometric non-linear term.

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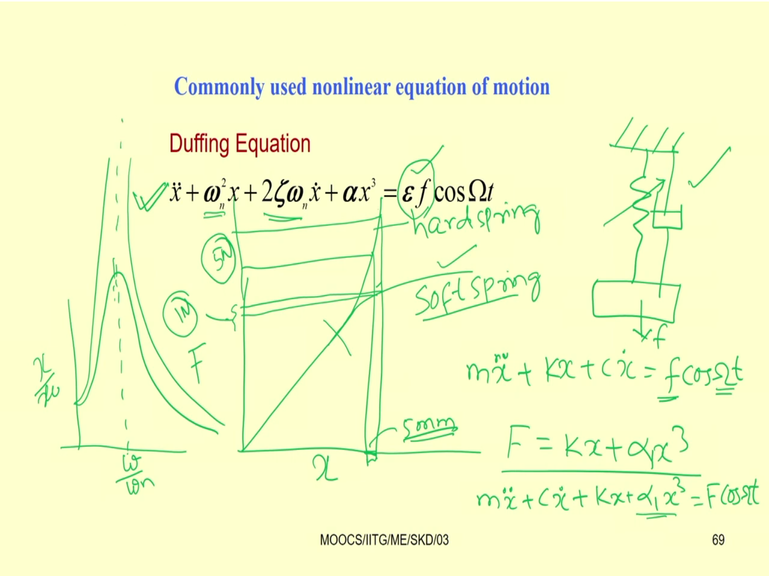

So, let us see some commonly used non-linear equation of motion. For example, let us take the duffing equation. So, in simple spring mass systems we have seen the equation of motion. So, this is a simple spring mass and damper system. So, let us put a damper also. So, in this case equation of motion is mx double dot plus Kx plus cx dot equal to if we are applying a force f then this is equal to f sin omega t or f cos omega t.

So, where f is the amplitude of the force and omega is the frequency of the force. So, if this spring is not linear. So, what do you mean by linear spring? So, if you draw the force verses displacement curve. So, for a linear spring, so, it will be straight. So, if the spring is no longer linear. So, in that case the curve no longer will be like this. So, it may bend this way or it may bend this side.

So, this is the non-linear. So, in this case the characteristic of the spring is a non-linear characteristic it shows non-linear characteristic. So, up to certain length the spring force is proportional to displacement, but after that thing no longer it is proportional to displacement. So, it can be so for example, in this past case. So, let us take this one. So, let us take two points in this.

So, here you can easily observe that by applying more force. So, you have to apply more force to have the same displacement as compared to here. So, here you have to apply less force. So, here you have to apply less force and you have the same displacement. So, in this case it is very soft then this is known as softening soft spring because you have to apply less force.

So, for example, let this displacement equal to 5 mm. So, to get a displacement of 5 mm. So, here you have to apply for example, you may apply a very small amount of force. So, this is the force you have to apply here for example, let me take this is 1 Newton force. So, you have to apply 1 Newton force to have a displacement of 5 mm, but in this case in the second case you may have to apply force of 5 Newton to have 5 mm displacement.

So, here you are applying more force to have 5 mm displacement here you are applying less force to have same displacement. So, that is why it is soft and this is known as this is hard spring. So, you can have a hard spring or you can have a soft spring. So, this is soft spring and this is hard spring. So, if it is soft spring then you can have a response or force displacement response like this and you have a hard spring the force displacement response will be like this.

So, now, let us consider a hard spring. So, in that case the force can be written. So, in that case spring force can written as Kx plus alpha x cube. The spring force is Kx plus alpha x cube and the damping will be there. So, the equation of motion so, or we can write K alpha 1 x cube K alpha 1 x cube. So, the equation of motion will be mx double dot plus cx dot plus

Kx plus alpha 1 x cube equal to. So, I can write this thing equal to for example, let me write this F equal to capital F cos omega t.

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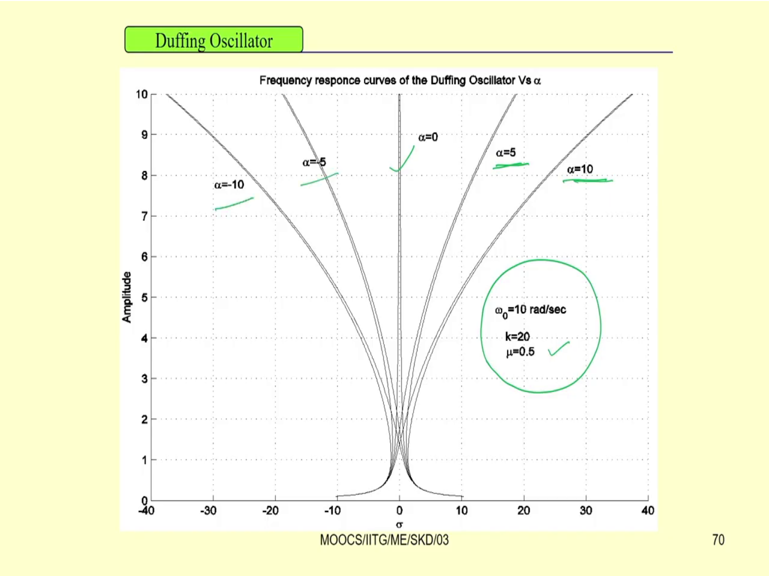

So, in this case now dividing this m in this equation so, you can write this equation in this clearly in this form that is equal to x double dot. This K by m is nothing but the square of the natural frequency that is omega n square x and then c by m can be written as 2 zeta omega n x double dot. Then this alpha 1 by m can be written as alpha x cube and then this F cos omega t F by m can be written as epsilon F.

So, here it is written in terms of a small using a small book keeping parameter otherwise simply it can be written as F cos omega t where later by ordering you can put this compare the coefficient of this cos omega t or this amplitude of the forcing term with that of the linear term omega n square and then order it. So, this is the duffing equation.

So, in this duffing equation you can write this equation in this form that is x double dot plus omega n square x plus 2 zeta omega n x dot plus alpha x cube equal to epsilon f cos omega t. So, using a cubic non-linear spring so, you are writing this way. So, sometimes you may be using a quadratic non-linear spring also.

So, in that case you can add the term beta x square to this equation and let us see. So, already you are familiar for the response of the system. So, when a simple spring mass system is subjected to a force f sin omega t. So, you are familiar with the response already we have plotted this magnification factor x by x 0 x by x 0 verses omega by omega n. So, this equation this plot already you have seen.

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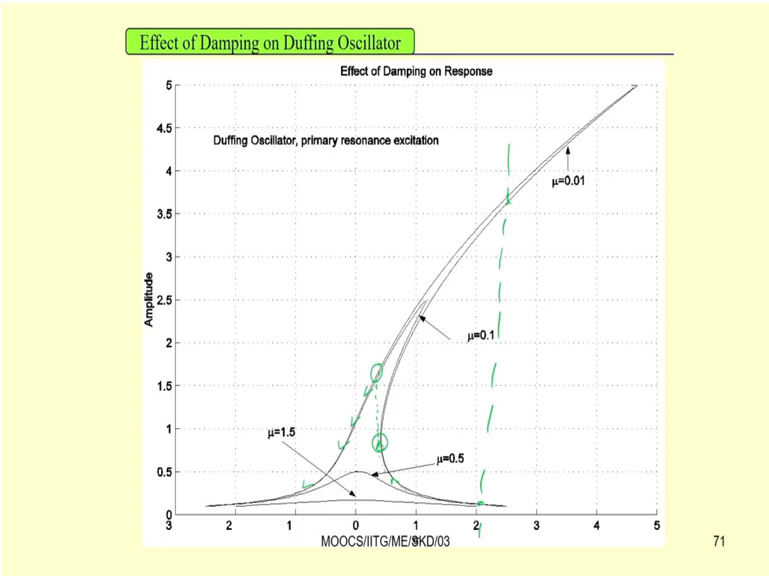

So, here at omega by omega n when omega by omega n nearly equal to 1, so, you have seen the response is maximum almost maximum and for undamped system when there is no damping so, this term this response goes to infinite. So, this is the; this is the things already you have studied.

Now, for a non-linear system you can see for a non-linear system cubic nonlinearity taking the parameter equal to omega 0 equal to 10 radian per second k equal to 20 and mu equal to 0.5 that is damping equal to 0.5. So, you can plot this frequency response. So, later we will know how to get this frequency response.

So, we will find some closed form solution and if you plot those closed form solution so, you can see the response of the system. So, for a different value of alpha these are non-linear terms different value of alpha. So, for example, alpha equal to 10, alpha equal to 5, alpha equal to 0, alpha equal to 0 there is no nonlinearity.

So, the response goes to infinite and for different value of alpha, so, you can plot the response of the system. So, similarly you can study what is the effect of damping. So, for a different value of damping so, you can study the response. And, as you have seen in this linear case in the presence of damping the response amplitude decreases, here also response amplitude decreases also.

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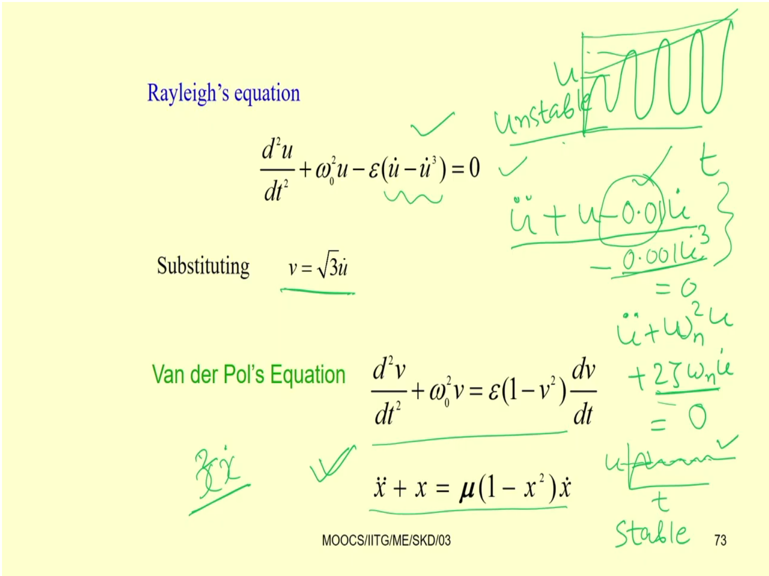

Here you can clearly observe so; you can have multiple solutions unlike single solution in case of the linear case. So, in this particular case you just see so, you have a solution here, you have a solution here, you have three solutions for a particular value of omega by omega n for a particular value of a frequency ratio you have multiple solutions. So, this is the characteristic of non-linear equations where you can have multiple solutions in the system and you can observe.

So, for example, let us decrease the frequency ratio. So, at this point so, when we are going reducing this thing. So, this response amplitude will go on increasing. So, it is increasing increasing, but at this position what will happen? So, it will jump up to this position. So, there is a jump up phenomenon. So, you can easily observe a jump up phenomenon. So, it will jump from this position to this position if it has a stable solution.

So, we will see later we will see what you mean by stable solution, unstable solution or stability of the response. So, here what we have observed. So, as there is no. So, there is no response as it is not continuous here. So, or the response is not there. So, it will jump from this position to this position.

So, while we will study the response of the system we will study more regarding this jump up jump down or this type of response. So, here by reducing this frequency ratio so, you have seen initially it will continue along this line and here it will jump to this position and later it will go down. So, it will follow this path, this will follow this path and the response will decrease.

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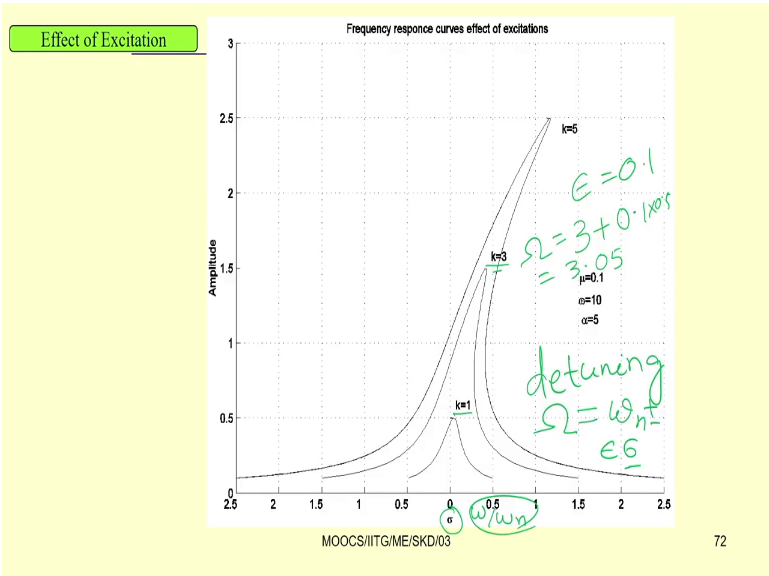

So, let us see what is the effect of so, you can see what is the effect of this response, so, excitation; so, excitation that F cos omega t, that F is written in terms of k here. So, if you have different value of response frequency amplitude of the. So, if you have different value of excitation amplitude of excitation so, you can plot this or this plot amplitude verses this. So, later I will tell this thing.

So, this is omega y you can plot this thing using this omega y omega n frequency ratio or by using some parameter later I will introduce that thing that is known as detuning parameter detuning. So, detuning parameter. So, what do you mean by detuning parameter? So, the response frequency for example, we are writing response frequency we can write this response frequency equal to omega n plus minus epsilon sigma. So, here sigma is the detuning parameter.

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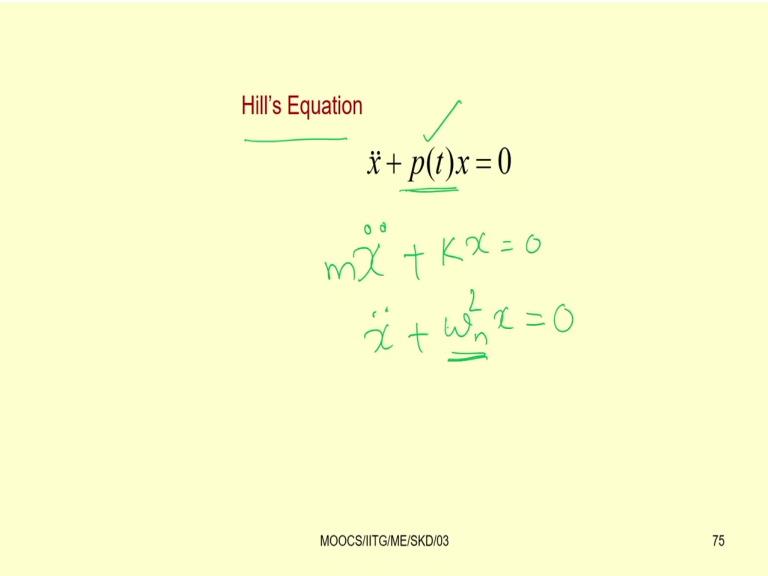

So, detuning parameter tells us regarding the closeness regarding the closeness of the response, regarding the closeness of the frequency excitation frequency to the natural frequency of the system. So, here omega equal to that is the external frequency equal to the natural frequency plus minus epsilon sigma.

So, for example, if the natural frequency is 3 we can study near about this 3 Hertz we can start from 2.7 Hertz to 3.3 Hertz and see in the near neighborhood of this natural frequency what is happened to the system. So, detuning parameter, when detuning parameter equal to 0 it is exactly matching with the external amplitude that is omega capital omega that is frequency external frequency equal to the natural frequency omega n of the system.

And, this sigma equal to 0.5. So, for a epsilon equal to 0.2 epsilon equal to 0.1. So, if we are taking a epsilon equal to. So, let us take epsilon equal to 0.1. So, if epsilon equal to 0.1 so, in that case so, if epsilon equal to 0.1 so, in that case so, capital omega will be equal to capital omega we have taken omega n equal to 3; 3 plus 0.1 into 0.5. So, then this becomes; so, then this becomes 3 point. So, this becomes 3.05.

The frequency external frequency will be equal to so, the external frequency will be equal to 3.05 Hertz. So, if I taking this omega in Hertz then it will be in Hertz; if I taking that is in radian per second then it will be in radian per second. So, we will study in details regarding this duffing oscillator and also the effect of different damping excitation amplitude of excitation then frequency of excitation in module 3 of this course. So, let us see some other commonly used equations. So, one such equation is the relay equation.

So, this relay equation can be written in this form that is d square u by dt square plus omega 0 square u minus epsilon into u minus u dot minus u dot cube. So, here you just see this two terms are damping term. So, here you have a non-linearity in damping. So, u dot is the damping and u dot cube is nonlinearity in damping.

So, the relay equation contains a nonlinearity in damping and also you can observed here this equation the damping term minus epsilon u dot so, that is negative. So, that is the coefficient of u dot equal to minus epsilon; for example, you can write this equation this form u double dot plus.

For example, let me write using some number numerical value. So, u coefficient of u equal to 1 then this becomes let me write this is equal to 0.01 u dot minus. So, point let me ok. So, you can write this epsilon u dot equal to. So, let me write 0.001 u dot cube so, this equal to 0. So, here you just see. So, if you take up to this part only u double dot plus u minus point 0.01 u dot here the coefficient of u dot; coefficient of u dot is negative unlike this spring mass damper system what we have seen.

So, there your equation is u double dot plus e omega square or omega n square u plus 2 zeta omega n u dot equal to for free vibration equal to 0 or for force vibration it is equal to f sin omega t. Here the coefficient of 2 zeta omega n what you have studied is positive.

So, if you have a negative coefficient so, in that case the response will be unstable. So, if you plot this u versus time. So, if you plot this u versus time later in this class today class. I will tell you how you can solve this equation using a numerical method. So, up to this linear equation you know the solution.

So, you know the solution if it is under damped un damped over damped or critically damped and in this case. So, if you plot this u versus t. So, you know that the response will decrease and finally, it will come to. So, it will be due to the presence of damping. So, it will reach the steady state that is it will be 0.

So, this line is 0, so, but if it is negative so, in that case so, if it is negative so, in that case if you plot it then the response no longer will be. So, the response you can so the response will it will grow with time. So, it will grow with time. So, if you plot this u versus t then the response you can see to grow with time.

Here it is the response is decreasing and final coming to a steady state value that is 0. So, it is attaining a steady state, but in this case the response amplitude is. So, here the response amplitude is this, now with time the response amplitude is increasing it may increase exponentially also. So, the response amplitude may increase exponentially also. So, or the response amplitude is increasing with time.

So, these type of systems are known as unstable system unstable. So, no longer it is stable. So, this is unstable, but here the system response is stable. So, the system has a stable response in this case, but in this case the system has an unstable response. So, now, you are adding in addition to that. So, you are adding some non-linearity cubic non-linearity to the system.

So, this type of system later will simulate and we can see, but will be the response of the system and if you substitute this u so, v equal to if you substitute v equal to root 3 u dot. So, if you substitute v equal to root 3 u dot so, these equation actually will reduce to this form it will reduce to this form that is d square v by dt square plus omega square v equal to epsilon into 1 minus v square into dv by dt or you can write it in this form that is x double dot plus x equal to mu into 1 minus x square into x dot.

So, here what you can see so, though you have a non-linearity so, that non-linearity is not cubic in damping. So, you have a non-linear term that is x square. So, this non-linear term is x square into x dot. So, this non-linear term is x square in. So, this is x square into x dot. So, here you have a cubic damping non-linearity, but here you have a x square into x dot type of non-linearity present in the system.

So, this type of equation are commonly known as Van der Pol's equation. So, in case of Van der Pol's equation we will see that the response always gives rise to a limit cycle. So, later we will study what do you mean by limit cycle.

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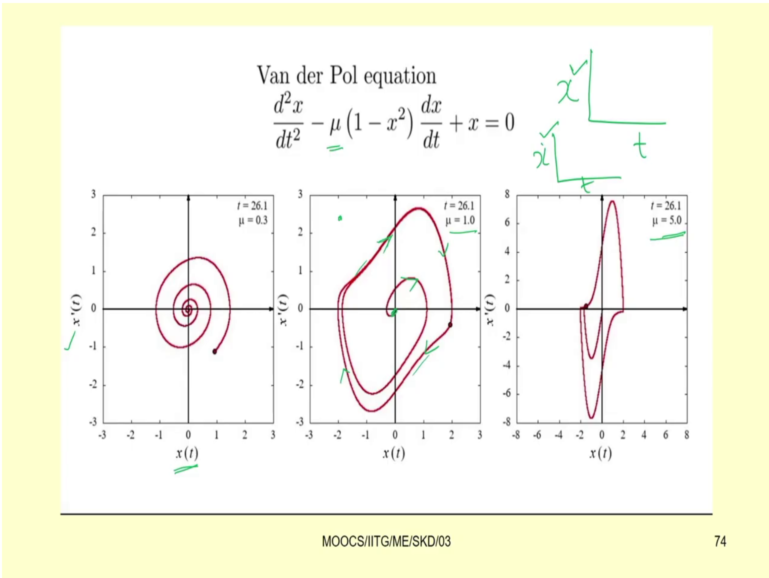

So, let us see some other ok. So, here we have plotted this thing also. So, if you plot this Van der Pol equation that is d square x by dt square minus mu into 1 minus x square dx by dt plus x equal to 0 so, you just see. So, with time you can see so, for example, taking this mu equal to 0.3.

So, you can see mu equal to 0.3 at time t equal to 26.1. So, if you plot x versus if you plot x versus x dash t. So, this is known as phase portrait or phase plane. So, if you are plotting this phase portrait you have a time response. So, in time response you are plotting x versus t you can also plot these x dot versus t and combinedly if you plot this x versus x dot, then that is known as your phase portrait or phase plane plot.

So, if you draw the phase portrait so, you can see the response will grow the response will grow and it is this position. So, now, you take mu equal to point 1.0 so, you can take mu equal to 1.0 and plot it. So, what you can see? So, it is reaching a so, starting from this position so, it is reaching a limit cycle; that means, no longer it will go away a from this thing; always it will move on this line.

So, this line so, it will move on this line or in this trajectory. So, the response will be like this ok. So, if you take different you take different value by taking different value of mu. So, you can plot these things and see the phase portrait clearly you see the time response, displacement versus time, velocity versus time and also you just plot. So, if you take a point outside this thing instead of taking starting point from the origin so, if you take a point outside of the this thing also, you can see with time it will come to this position.

This co orbital it will come to this limit cycle. So, you can have different limit cycle by changing this value of this mu and you can see that thing clearly. So, later we will study how you can solve this equation and find the response or phase portrait time response. So, another term also I will tell you. So, that is known as point cross sections after some time.

So, similarly we have some other equations. So, here in this case you just see unlike this case of the spring mass system we are we have written x double dot plus Kx equal to 0 or we have written x double dot plus or mx double dot plus Kx equal to 0 or x double dot plus omega square omega n square x equal to 0.

Here the omega n square is a constant number. So, instead of a constant number, if it is a function of or time varying function or time varying parameter, then this type of equation is known as Hill's equation. And, previously I have introduced this equation to be that of a parametrically exited system.

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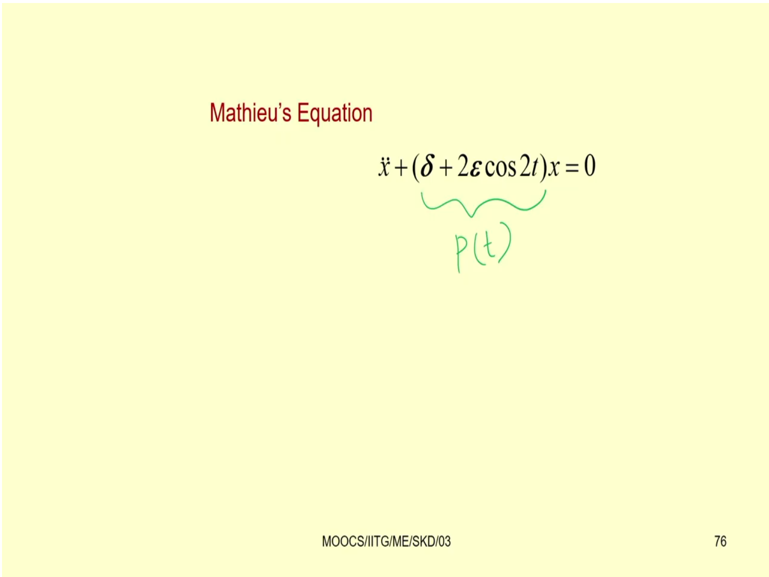

So, in this Hill equation so, if you replace this pt that is the time varying term by this particular form that is delta plus. So, this p t if you write p t equal to delta plus 2 epsilon cos 2t, then this type of equation is known as Mathieu equation. So, you have seen both Hill's equation and Mathieu's equation are the equation of a parametrically exited system.

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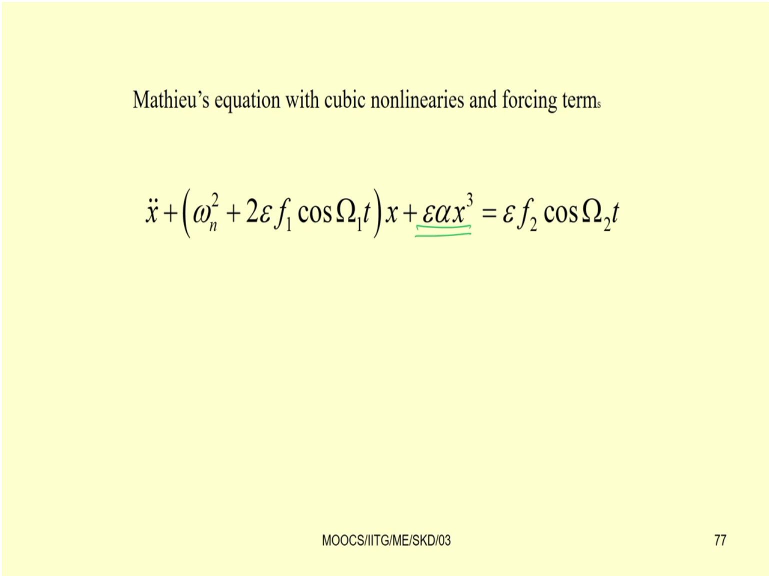

So, in this parametrically exited system or in this Mathieu's equation, so, we can add also this cubic non equilibrium term or we can add many different types of non-linear terms and we can write the equation in different forms and we can see so, what will be the response in this case. So, one thing you may remember here as the superposition rule is not applicable to non-linear system.

So, we cannot predict what will happen to the response of the system if we are changing the changing some of the parameter of the non-linear system. So, each non-linear response of each non-linear equation or non-linear systems are unique. And, one has to study those separately and one cannot predict or one should not predict the response by knowing the response of a similar system which will leads to erroneous assumptions or erroneous results.

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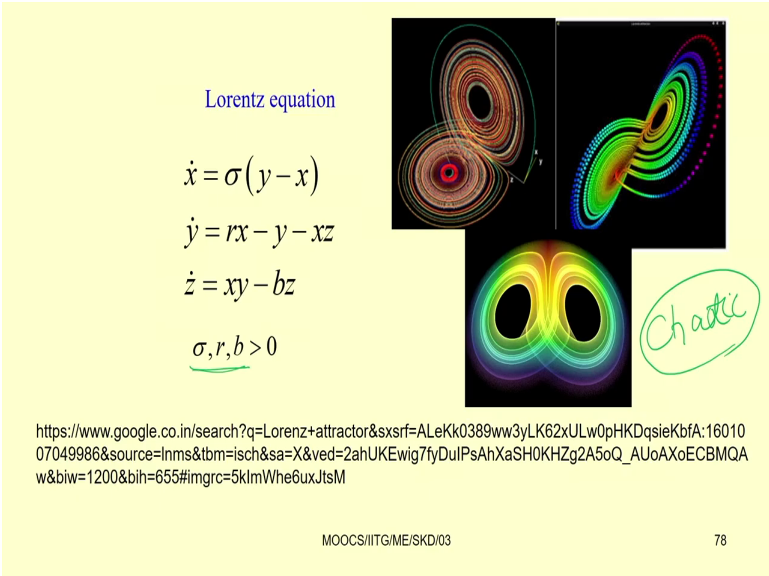

So, you may be you might have heard about another type of equations. So, here we have I have we have written three first order equation x double dot x dot equal to sigma into y minus x, y dot equal to rx minus y minus xz and z dot equal to xy minus bz. Here we have three parameters so, sigma, r, b.

By changing the sigma, r, b you can see many interesting phenomena you can observe or many interesting curves you can observe. So, these types of curves are known as chaotic curve. So, later in this course we are going to study what you mean by chaotic c h a o t i c chaotic response. So, these responses are known as chaotic response.

So, I will study in detail regarding chaotic response also and how to distinguish this chaotic response that thing also we are going to study. So, now, we have seen so, we have different type of equations one simple equations we have seen that is duffing equation where we have a cubic or quadratic non-linear term.

Similarly, you have seen another equation that is your Rayleigh's equation, Van der Pol's equation, also Hill's equation, Mathieu equation and combining this non-linearity in different type of equations we can generate a number of equations also.

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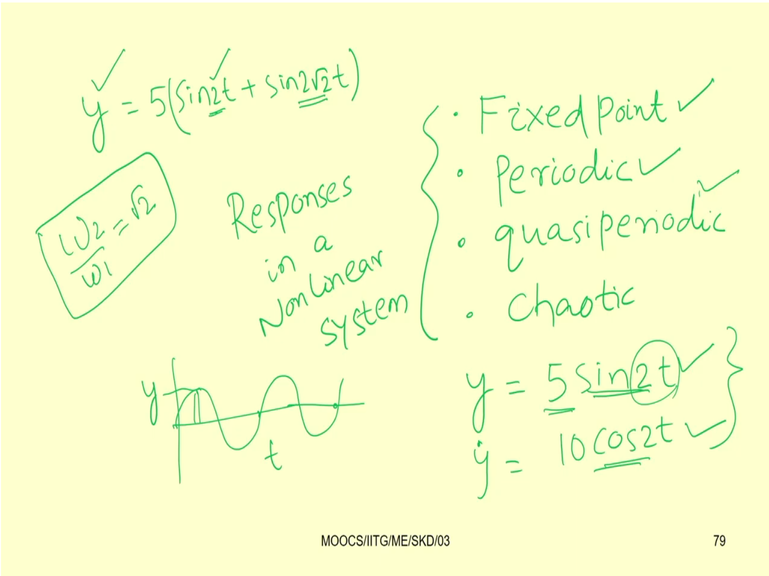

And, particularly these non-linear equations when we are studying so, we can have different type of response particularly we can study about fixed point response. The response can be written in this way – one is the fixed point response we will study what you mean by fixed point response; then the response may be periodic; the response may be quasi periodic; the response may be quasi periodic the response may be chaotic also. So, these are different responses you can get responses in a non-linear system.

So, these are responses in a non-linear system the response may be fixed point that is a fix number so, you will get a fix point. So, you will see then the response may be periodic. So, for periodic for example, you know if you know this y equal to let writing this thing y equal to 5 sin 2t. So, in this case y dot will be equal to 10 cos 2t. Yes or no? So, this y is a function of sin and y dot is a function of cos.

So, these are the periodic response. You know already these are harmonic response harmonically harmonic terms, sine and cosine can be retained as harmonic in term. So, these responses which repeats it is self with time are known as periodic response. So, here if you plot the sin so, you know after 2 pi so, it will repeat.

So, this is one cycle, this is another cycle. So, it repeats after each cycle. So, if it repeats if the response repeats, then that type of response this is if you plot this is x verses t or y verses you can plot this is y verses t. So, y verses t you plot in case of this thing so, amplitude your amplitude is equal to 5 and as your plotting the sin 2t.

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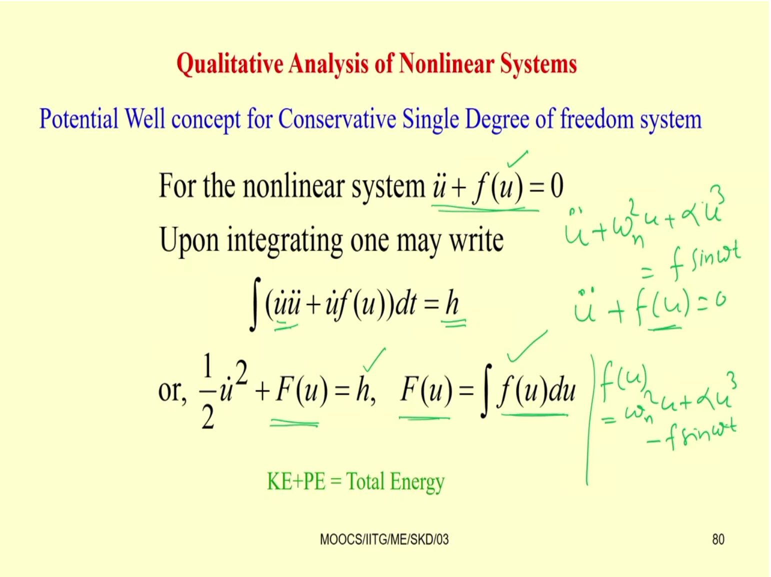

So, you can find what is the what is the value of t for which you will get one period. So, this value equal to your 5. So, it is upto 180 degrees in 180 degree. So, 2t will become 360. So, here so, it will repeat after every after every pi. So, similarly your response may be quasi periodic. So, we will see what you mean by quasi periodic.

So, in case of quasi periodic so, you can write your y for example, let me write this y equal to if I will write y equal to 5 sin 2t plus let me give a brackets sin 2 root 2t. So, here the omega here so, this is sin omega t or sin omega 1 t sin omega 2t. So, this is so, this is a 2 periodic 2 in this case you have seen it contains this frequency 2 frequency response.

So, here the frequency is 2. So, here the frequency root 2, 2 root 2. So, that means, if you have a ratio that is omega 2 by omega 1, so, is an irrational number. So, in this case it is equal to root 2. So, if omega 2 by omega 1 is an irrational number, then the response what you will get so, that will be quasi periodic response. So, you will get a quasi periodic response if the ratio between these two are irrational number.

So, later we will see with example so, what you mean by chaotic response. So, I will tell you this period doubling bifurcation leads to chaos. So, those things I will tell after few minutes. So, let us see let us have a qualitative analysis of a non-linear system. So, for example, we can write the non-linear equation is this form that is u double dot plus f u equal to 0.

So, in this case for example, u double dot plus omega n square u let me write this equation alpha x cube equal to f sin omega t. So, no this is u cube. So, instead of x, this is u cube this is the equation. So, you can write this equation equal to u double dot plus f u equal to 0. So, where f u will be equal to.

So, here f u this function of u equal to omega n square u plus alpha u cube minus f sin omega t. So, this way you can write f u. So, all the equations you can write in this form that is u double dot plus f u equal to 0. So, now, if you integrate these things so, what you can see so, you can write this integration in this form just you multiply u dot in this equation left side and right side and integrate it. So, these becomes a constant because the differentiation of a constant is 0.

So, integration of this thing will be a constant. So, equation you can write in this form u dot into u double dot plus u dot F u dt equal to h or you can write integration of this u dot u double dot is nothing but your u dot square and this integration of this u dot f u it can be written as capital F u. So, that is equal to h.

So, if you see this capital F u, so, this is equal to u dot f u dt. So, this u dot you can write in form of du by dt. So, this dt dt cancels and your F u can be written as F u small F u into du so, but what is this things? So, this term is potential energy of the system worked down into displacement in the potential energy and this half u dot square is nothing but the kinetic energy.

So, this is written in terms of the energy form; that means, your kinetic energy plus the potential energy equal to constant energy the total energy of the system. So, you can write half u dot square plus F u equal to h or what I can write? So I can write is half u dot square equal to h minus f u or u dot square equal to 2 into h minus f u or u dot equal to root over 2 into h minus f u.

So, I have an expression between this u dot and u that is between the displacement and velocity which will give me and relation between this displacement and velocity that is known as your phase portrait. So, you can plot the phase portrait.

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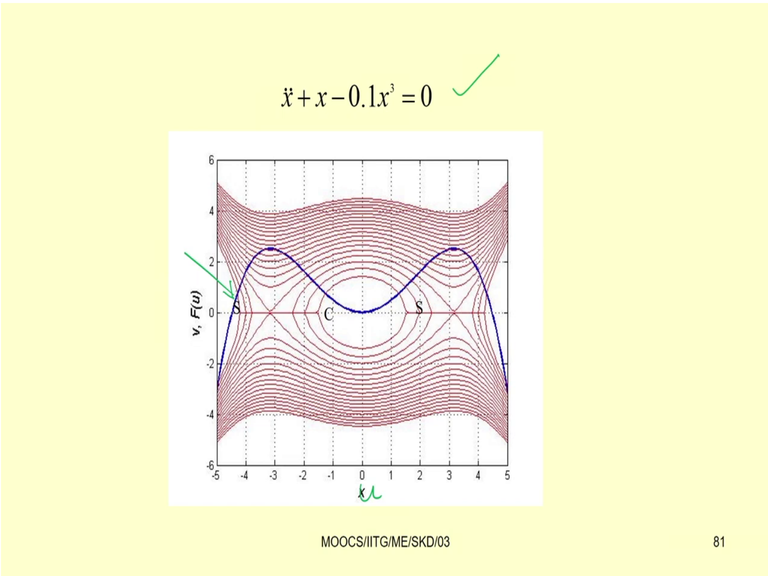

So, for example, if you take this equation so, these curves shows the. So, these curves shows these variation of these a F u verses x a F u verses u, not x ok. So, it is written in terms of x. So, then it will be x.

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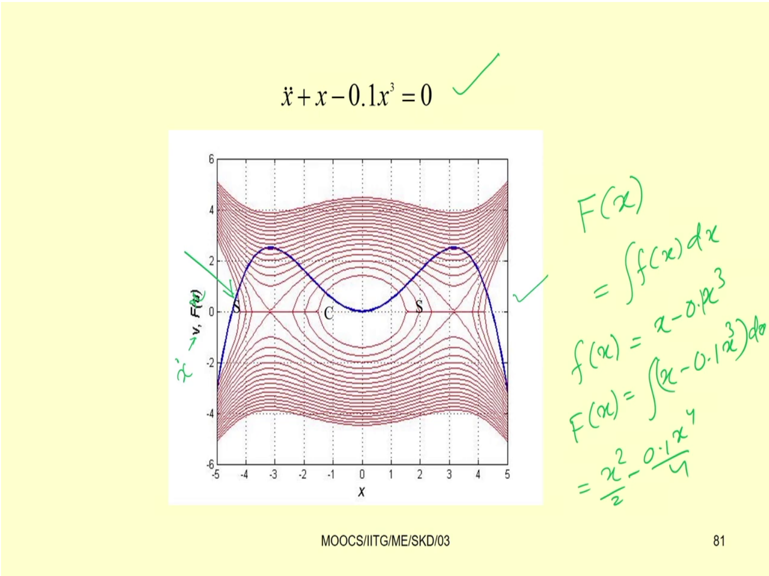

And, this instead of F u so, this will be F x. So, F x. So, these curves plots the F x versus x and if you plot the velocity versus x that is F v is nothing, but your x dot. So, if you plot x dot versus x you can get this type of curve. So, we will study more on this type of curve and what is the meaning of this curve after or in the second or third module of this course.

Now, for the time being you should know to plot this curve. So, first what you have to do? So, first find F x find F x. So, F x is nothing but your integration of f x into dx. So, what is small f x here? So, the small f x equal to x minus 0.1 x cube. So, capital F x is nothing but capital F x will be integration of x minus 0.1 x cube dx. So, you just integrate these thing and by integrating so you will get this is equal to x square by 2 minus 0.1 x to the power 4 by 4. So, this is F x.

So, these versus x easily you can plot. So, here now ok. So, let me show you how you can use. You can easily plot this thing using different software like this MATLAB software you can use or you may use some many open access softwares are available like one such software is your GeoGebra or simply you can use the excel file to plot this thing also.

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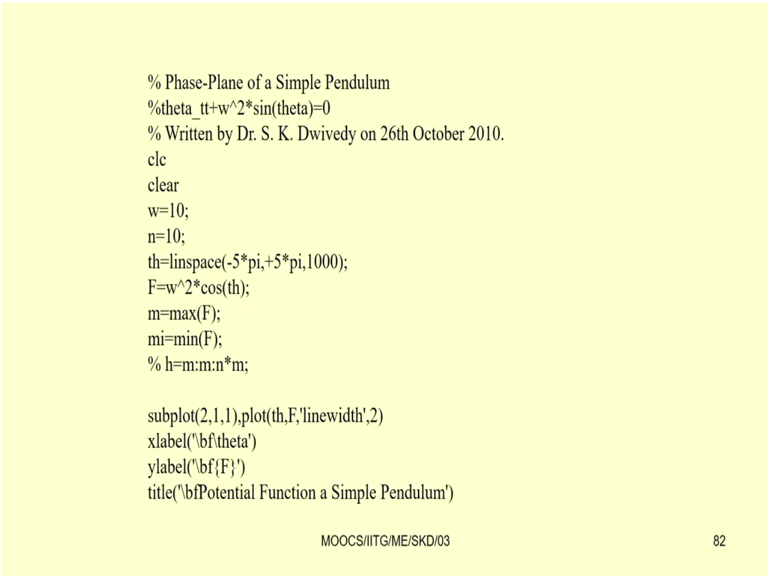

So, I will show you one example.

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```
for i=0:15
h=((m-mi)/n)*i+mi;
thdot=sqrt(h-F);
r=real(thdot);
subplot(2,1,2),plot(th,real(thdot),'k','Linewidth',);

hold on
subplot(2,1,2),plot(th,-
real(thdot),'k','Linewidth',2)
end

grid on
hold off
xlabel('\bf\theta')
ylabel('\bf{V}')
title('\bfPhase Plot of a Simple Pendulum')
```

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So, this is this code is written using MATLAB to plot this equation.

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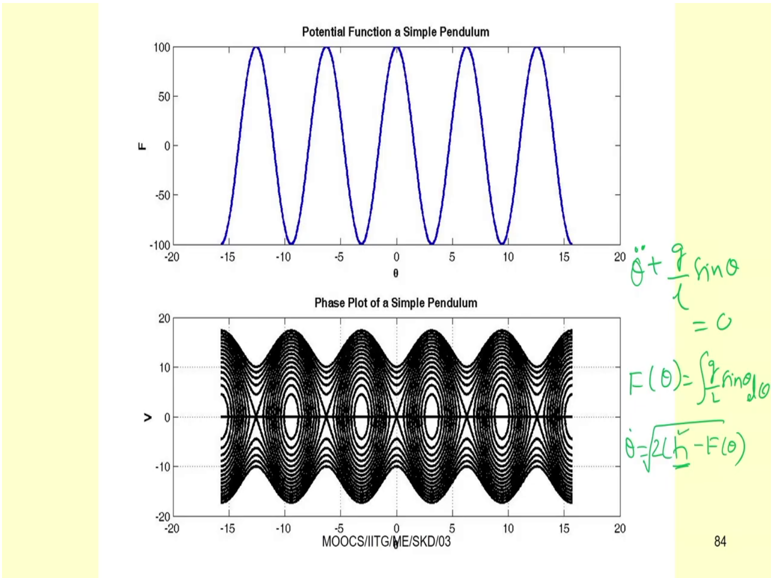

So, you can see. So, this is your F versus this theta you can put. So, for a simple pendulum this is written this curve is written for a simple pendulum. Simple pendulum equation already you know. So, the simple pendulum equation is theta double dot plus g by l sin theta equal to 0.

So, here this F theta will be equal to; so, small f theta equal to g by l sin theta. So, it will be equal to g by l sin theta d theta. So, you can integrate it easily and you can plot it. So, now, so, you know your velocity or theta dot that is v is written here will be equal to 2 into root over 2 into h minus F u F theta. So, giving different value of h giving different value of h so, you can plot different lines. So, these different lines actually is for different value of energy level. So, for different energy level so, you can plot this thing.

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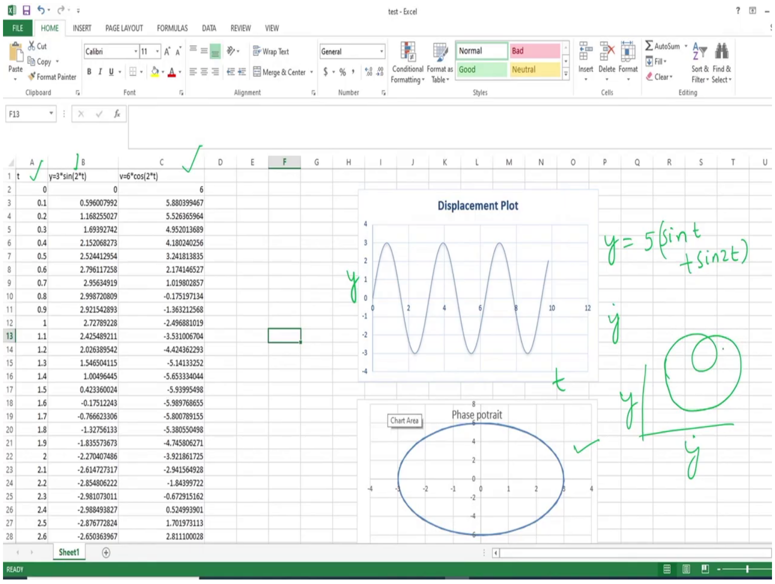

Similarly, you can simply use one excel file to plot also to the response I can show you easily. So, this is the or I will open this excel file and you can see for example, you want to plot this is t. So, you just give different value of time t and then you write y equal to 3 sin 2t.

And, it is if y equal to 3 sin 2t your velocity will be 6 cos 2t. So, physically you can write this t, t equal to 0. Let for with increment of point one even to plot. So, then so, you just write 0.1. So, how will you write? So, let me show you by using this exactly using.

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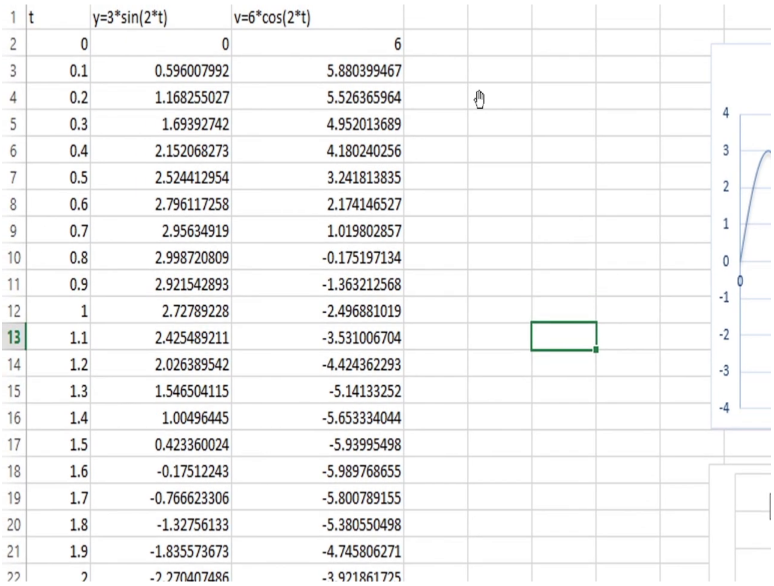

	t	y=3*sin(2*t)	v=6*cos(2*t)
1	t	y=3*sin(2*t)	v=6*cos(2*t)
2	0	0	6
3	0.1	0.596007992	5.880399467
4	0.2	1.168255027	5.526365964
5	0.3	1.69392742	4.952013689
6	0.4	2.152068273	4.180240256
7	0.5	2.524412954	3.241813835
8	0.6	2.796117258	2.174146527
9	0.7	2.95634919	1.019802857
10	0.8	2.998720809	-0.175197134
11	0.9	2.921542893	-1.363212568
12	1	2.72789228	-2.496881019
13	1.1	2.425489211	-3.531006704
14	1.2	2.026389542	-4.424362293
15	1.3	1.546504115	-5.14133252
16	1.4	1.00496445	-5.653334044
17	1.5	0.423360024	-5.93995498
18	1.6	-0.17512243	-5.989768655
19	1.7	-0.766623306	-5.800789155
20	1.8	-1.32756133	-5.380550498
21	1.9	-1.835573673	-4.745806271
22	2	-2.270407486	-3.921861725

So, here you can write this is equal to 0.1. So, then 0 so, here you will write equal to so, this equation in excel file you can write this equation you go to that part, just write equal to. So, let us go back and use this excel file; use this excel file and see this thing. I will show using this excel file ok I will show you how to use this excel file just after few minutes.

So, you just see by using this excel file. So, you can plot this displacement versus time. So, this is displacement versus time y versus t. Similarly, you can plot this y dot versus t. So, that is this column and this column; this column first column a and c column you can plot, then you can plot using this second column and third column. So, this will give you the phase portrait.

So, this way so, you have seen. So, if you have a single period that is here the frequency equal to t 2. So, you have got a single curve. So, if you are taking for example, let you take y equal to 5 sin t plus sin 2t. So, you can find easily what is u y dot if you plot this y versus y dot. So, this is y versus y dot, then you can get a curve which is like this. So, here you will get two loops. So, I will show you that thing.

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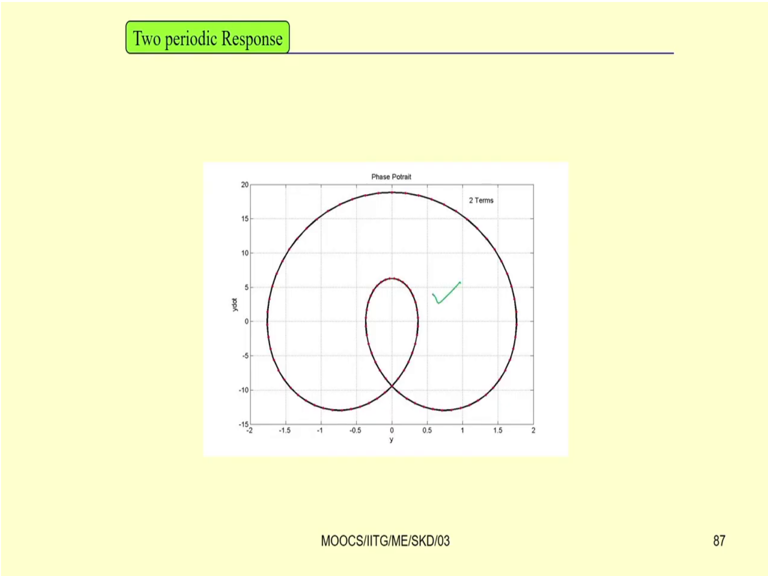

So, here you are getting two loops. So, if you have two terms, then you are getting two loops.

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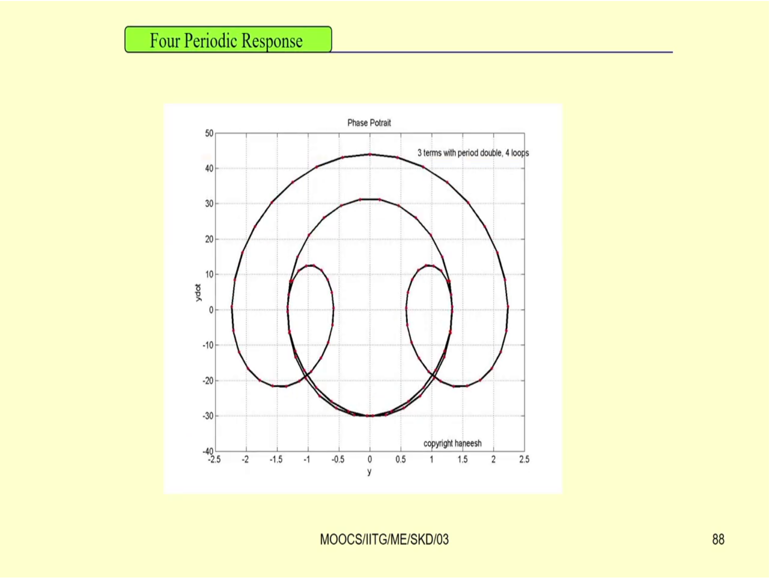

Similarly, if you have four, then you can get four loops in this curve. So, this is known as 4 periodic. So, if you have more than that thing so, you can get 8 periodic.

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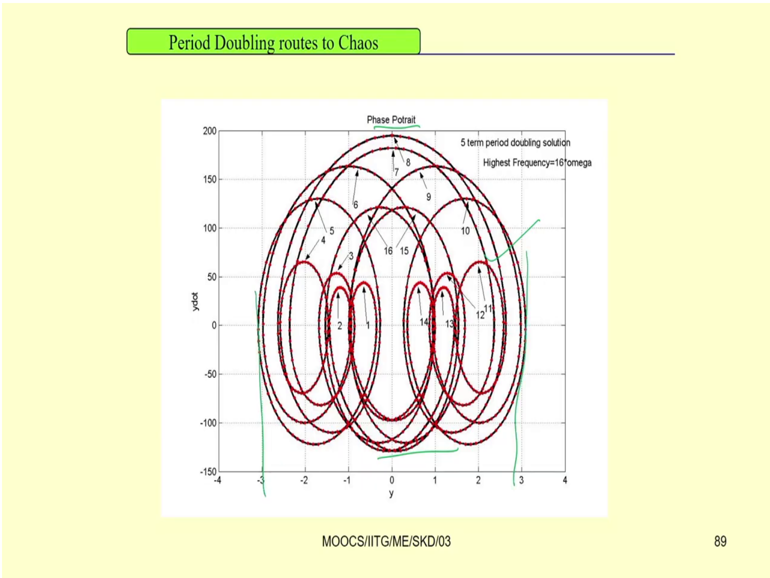

And, similarly 16 periodic you go on increasing the number of terms then you can get that this period doubling route to chaos. So, these type of curve where it is deterministic that amplitude is fixed, amplitude is bounded so, it is varying between this and this.

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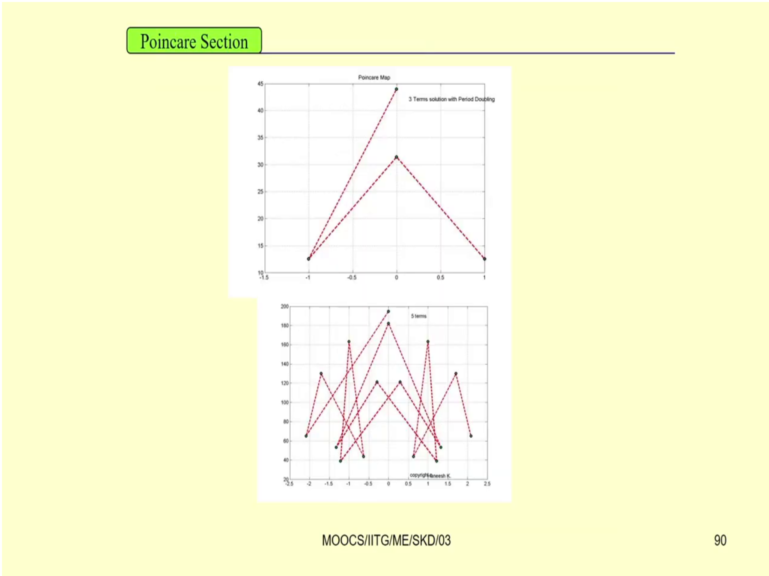

So, your do, this is y versus y dot. So, y is varying between y is bounded. So, y is bounded between this. So, you have a bounded response and you can find this.

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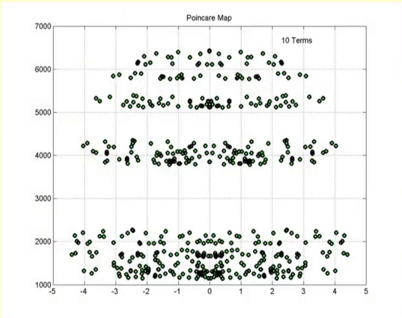
Poincare's Section



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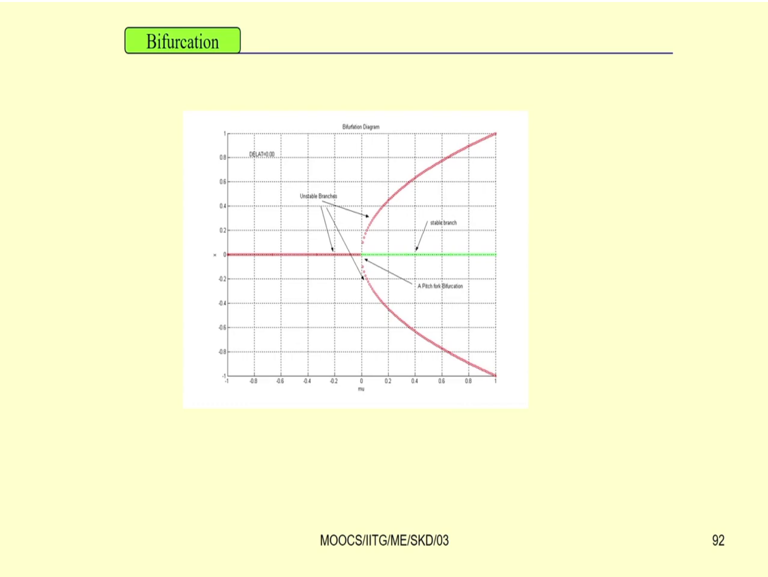

So, later I will tell you regarding this Poincare map and by using this Poincare map you can see. So, you can find the find how with system parameter the response is varying. So, we can study the stability and bifurcation of the system. So, from next class we are going to study regarding the how to derive this equation of motion.

So, before that thing I will tell briefly regarding bifurcation diagram and types of bifurcation. So, this will be part of commonly used equation. Just now I will just demonstrate how you can use this excel file also for your simulation purpose.

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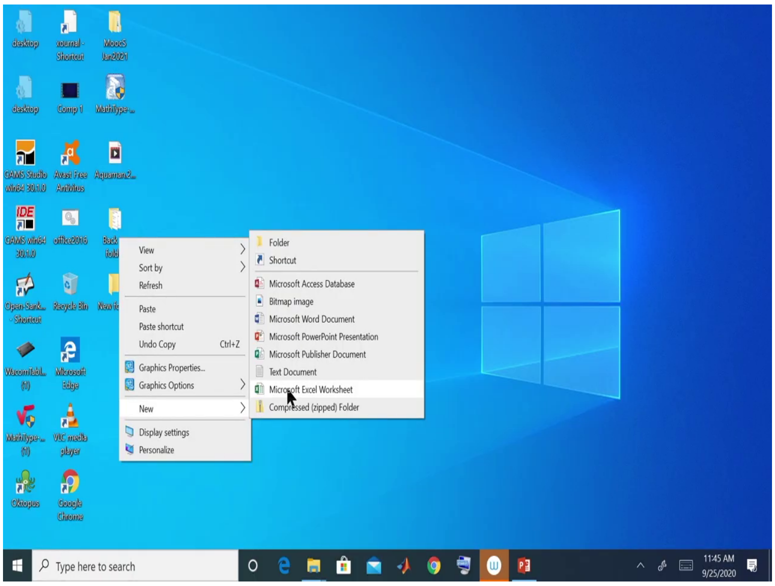

So, let me open one excel file. So, you can open. So, you go to new. So, here you can see. So, you have a excel file here.

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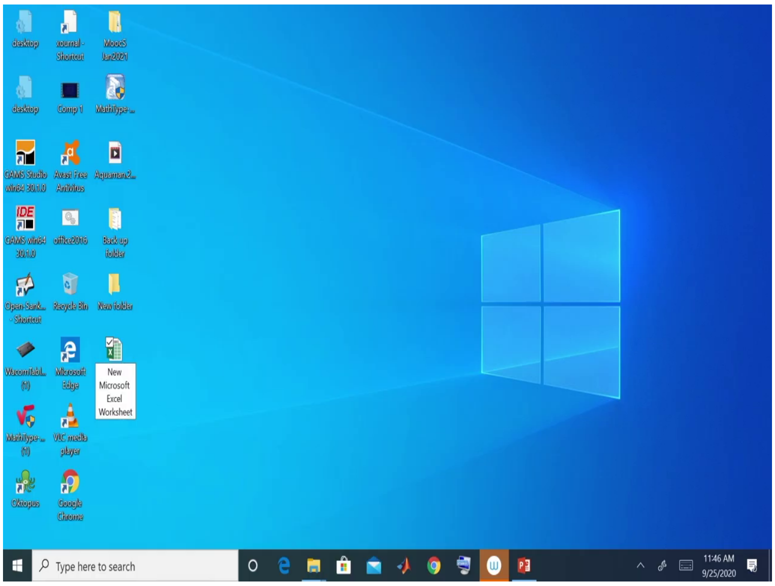

So, let me give some name to this excel file. For example, let me write a new excel file ok.

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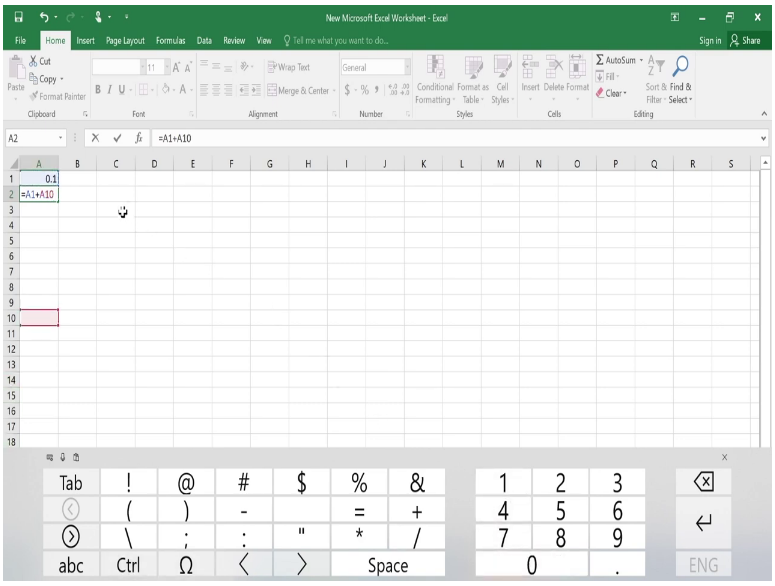

So, this is the excel file you have seen. So, let me write ok. So, this first column let us write this is equal to 0.1. So, this second column you can write this is equal just put this is equal to so, this first column plus 0.1. So, let me put the increment ok. First let me start this is equal to 0. So, let first is 0.

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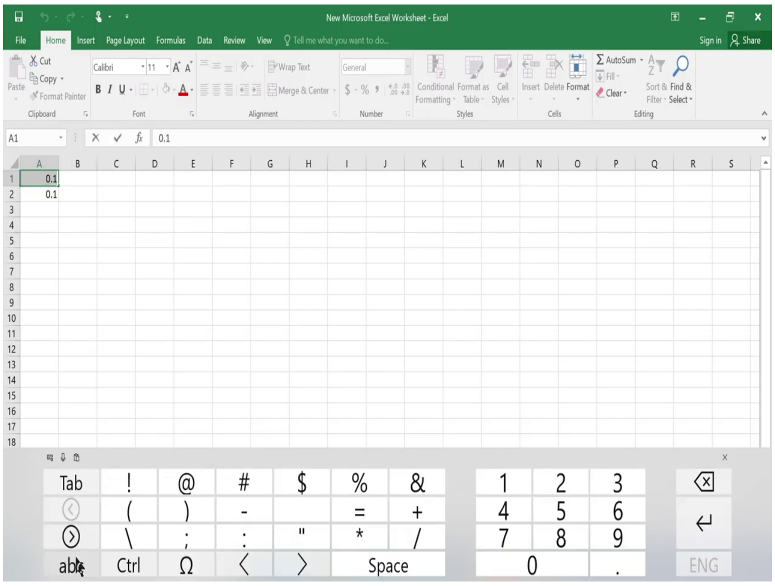

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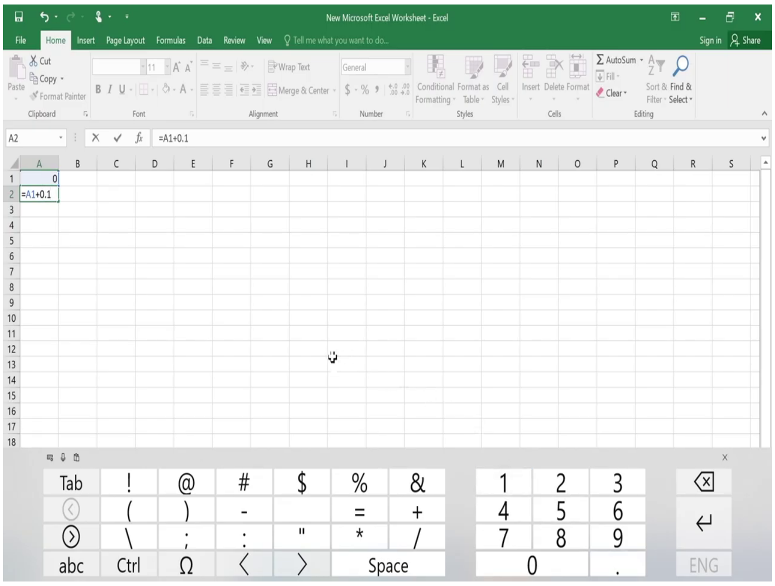

This one let us delete this one and write this is equal to this is equal to 0. This is 0, now this will write these equal to simply you write equal to this first row plus 0.1.

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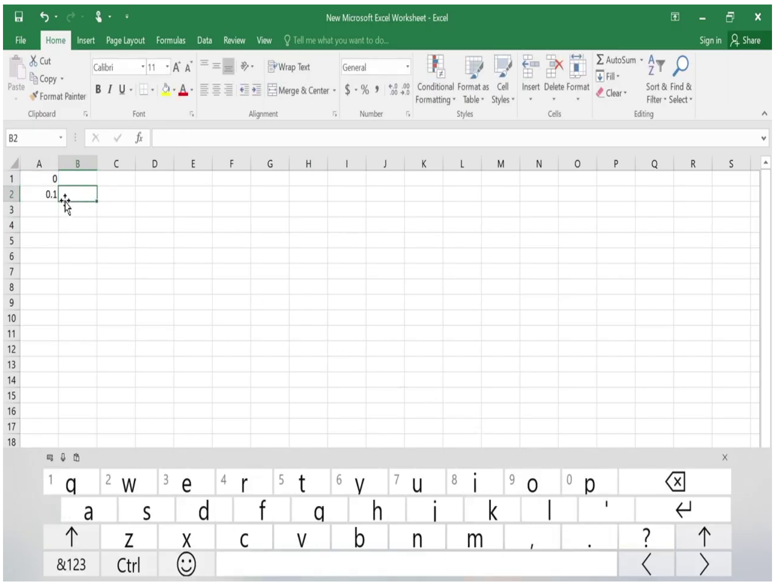

So, now, you copy this one. So, control C.

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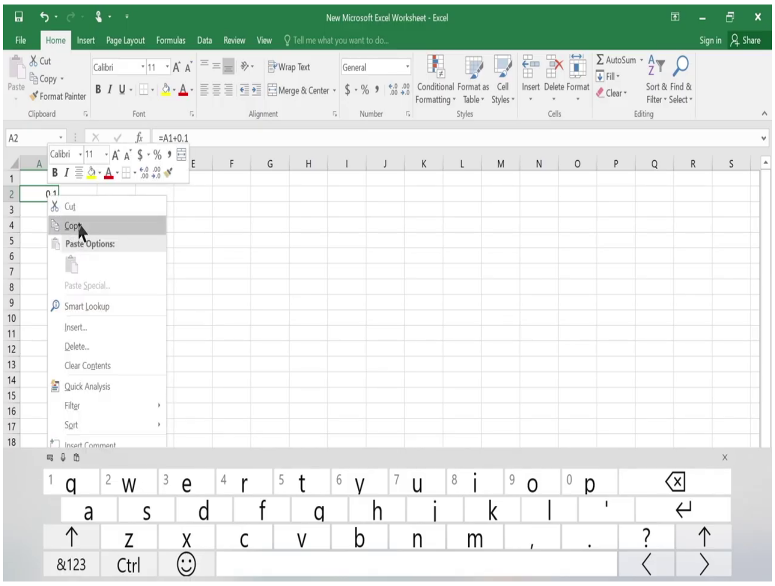

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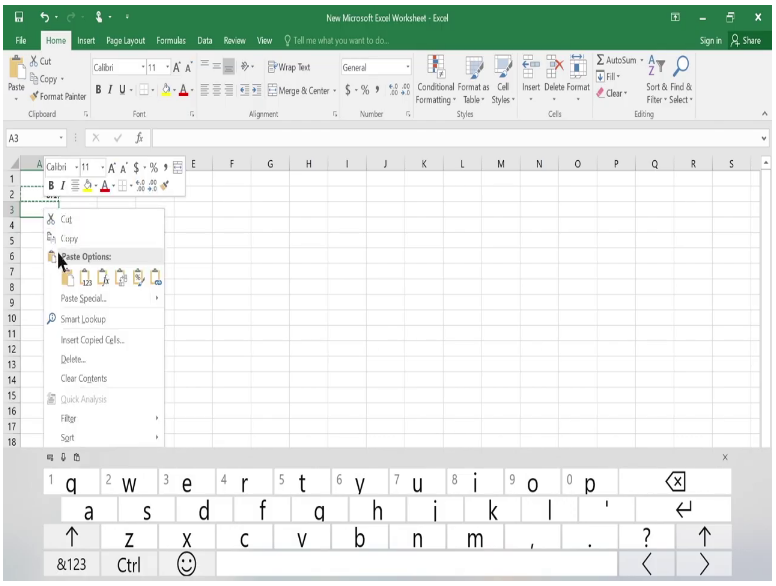

So, you can copy, right click copy. Now, you just take this line and go. So, for example, up to these thing let us put. So, let paste it. So, simply you can. For example, let me copy and paste. So, this thing if you paste so, you have seen so, just paste it.

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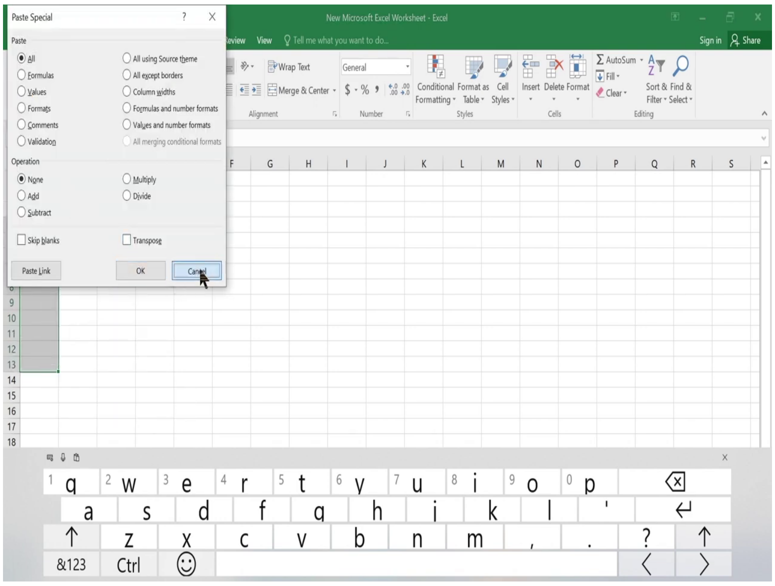

So, you can find not special.

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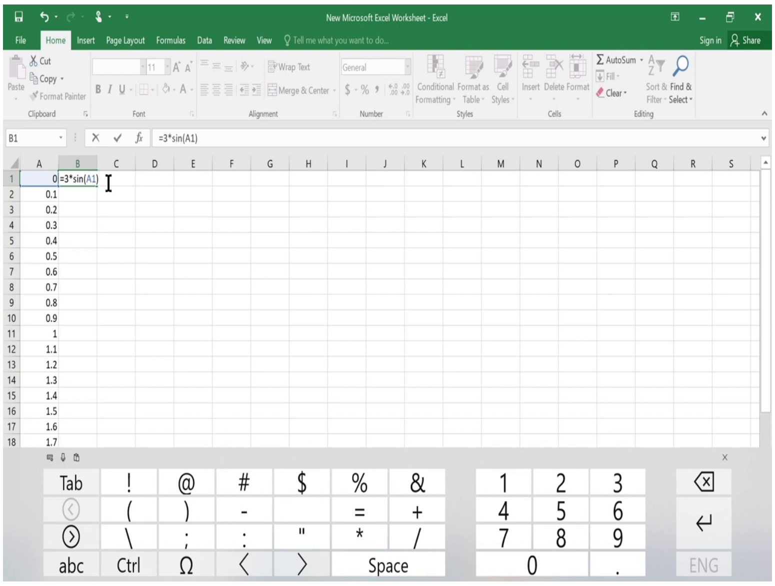

So, paste, similarly go on pasting. So, automatically you can generate.

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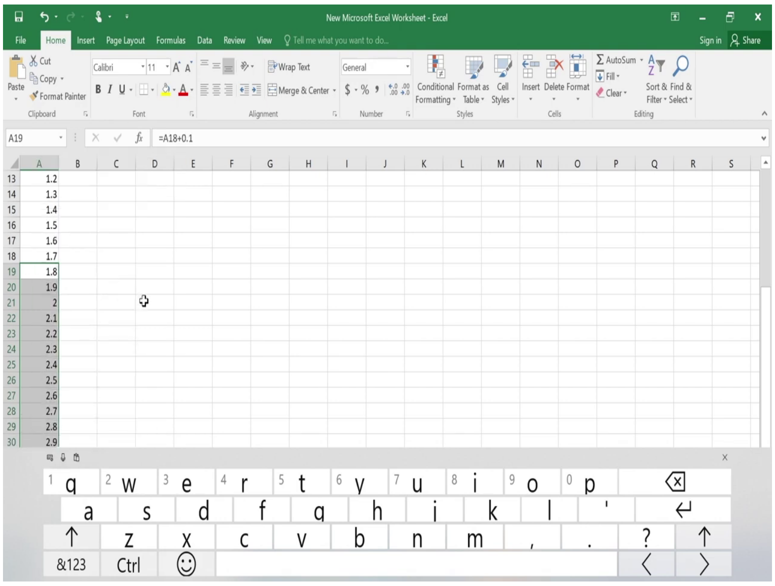

So, you have seen. So, your able to use this excel file for generating your first column that is your time. So, let time you have started with 0 with increment of 0.1, 0.2, 0.3; 0.1 increment you have written. So, in this excel file you are able to generate this one. Now, in the B column so, let you want to put. So, this is equal to 3 sin t. So, you want to write. So, this is equal to 3 sin t.

So, come to this one. So, this is equal to so, this is equal to 3, let me put this is 3 ok. So, first you have to write equal to. So, you just put equal to 3 into. So, put the into sin 3 into sin. So, you have to write sin omega t; so, sin sin sin sin you just put a bracket sin omega t.

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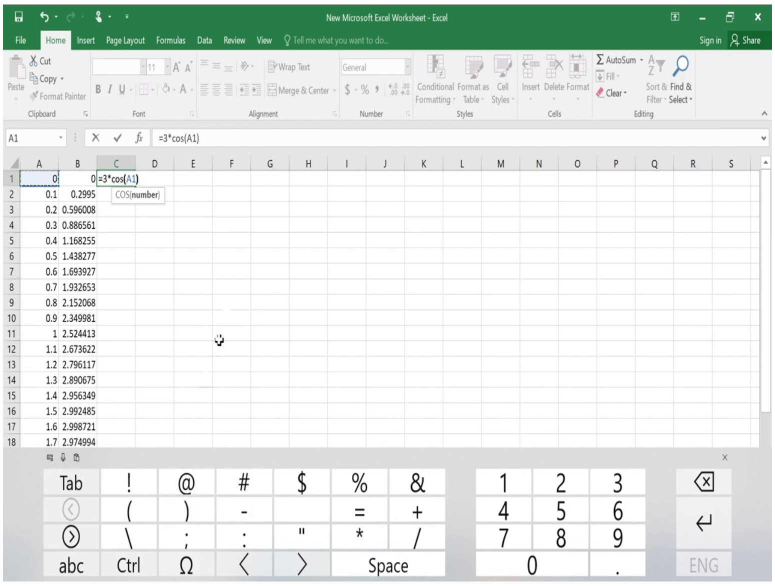

So, here sin t simply you want write t. So, here t will be the first column and you close this thing. So, if you enter so, you just see just you enter you can see the number you will get the number. Now, again you copy and paste. So, you copy that thing now you paste for the whole thing of to what you have written. So, upto 2.9 for example, you have written so, you paste it here.

So, you have seen so, for time t and this is 3 sin t you have got. So, velocity already you know what is the expression for velocity. So, it will be 3 cos t. So, this thing you can write equal to 3 cos t. So, let me write 3 cos t. So, equal to 3 into cos t cos you just put a bracket. So, t for t you have to first A1. So, it will be A1. So, you close this thing. So, you got it.

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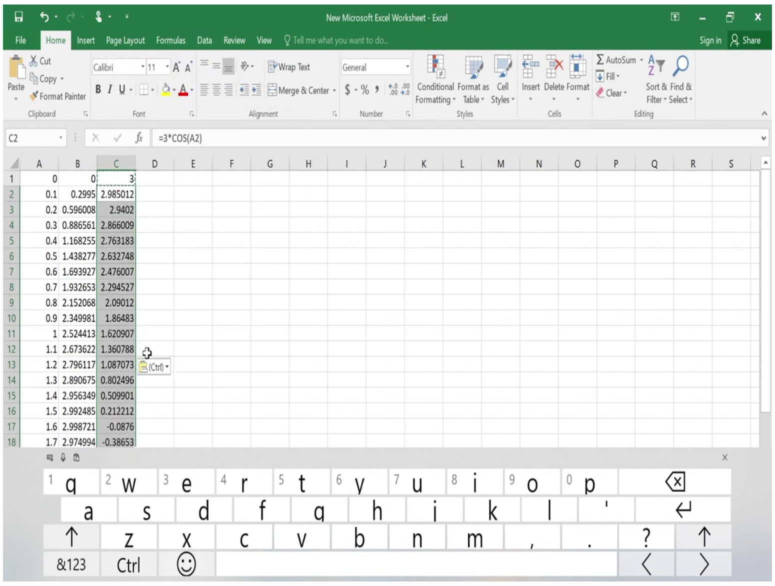

So, now you just simply enter. So, you got the value. So, now, you copy and paste this one. So, copy. So, in your computer you can control c control v you can do also paste this thing. So, you got all the number.

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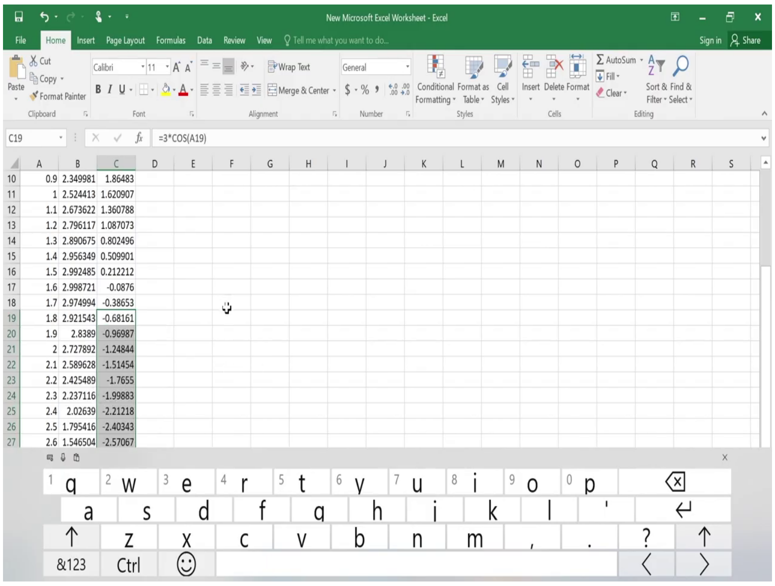

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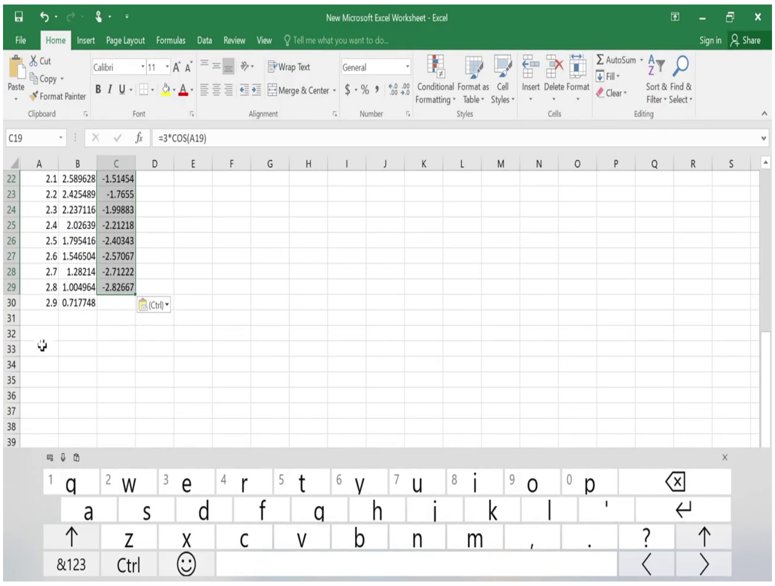

So, now, go on pasting this thing ok. So, let us complete this for example, one cycle; one cycle will be 3 ok. So, up to. So, here t you are writing. So, omega equal to so, it should be 2 pi. So, up to 6 point something you have to put. So, control c, copy.

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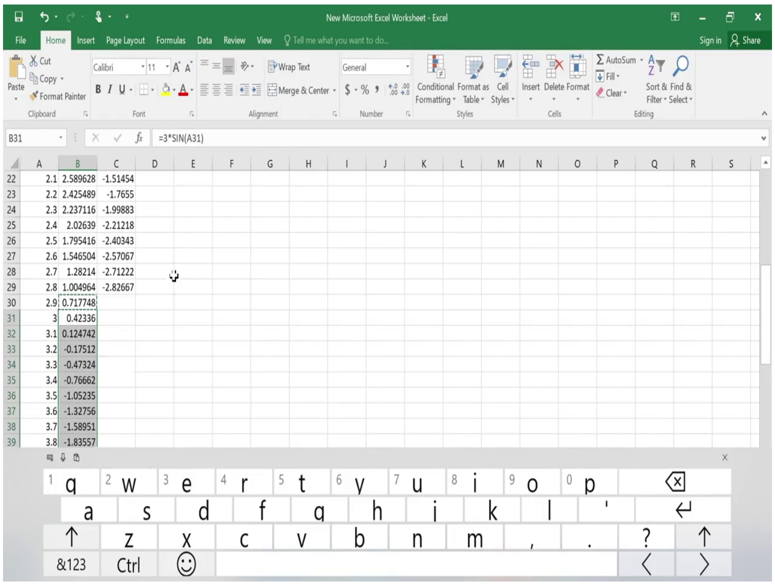

So, now, let us extend this thing paste.

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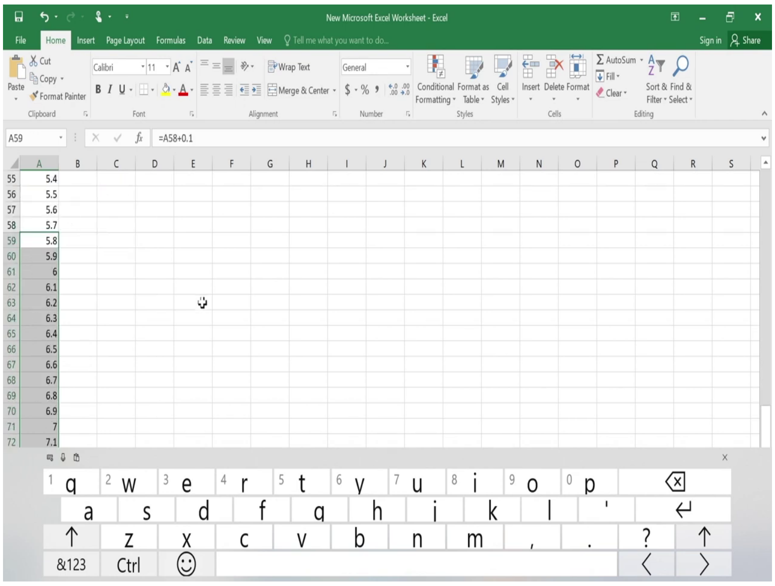

Similarly, go on pasting this thing ok.

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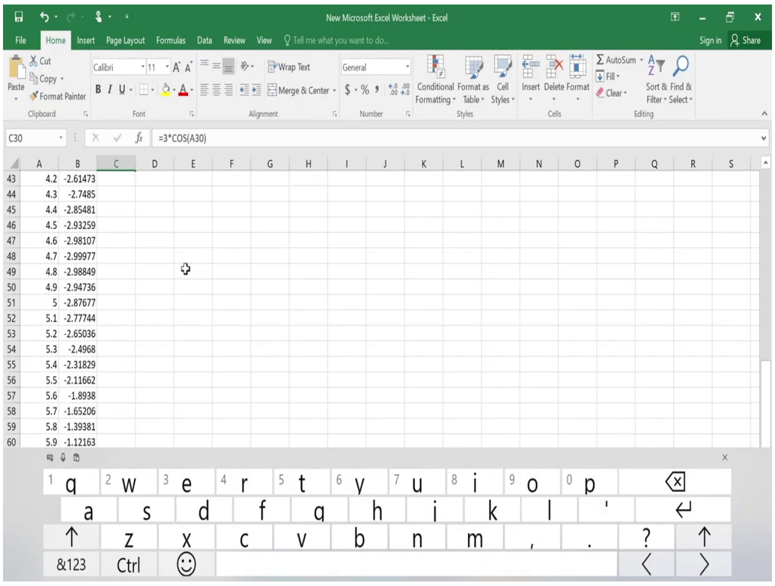

So, this way you can generate the t column now you generate the that x column and then x dot column. So, copy, now paste it. So, you just see by using simple excel file also you should be able to solve your problem. So, if you do not MATLAB; in MATLAB easily you can easily you can do it by writing only two lines.

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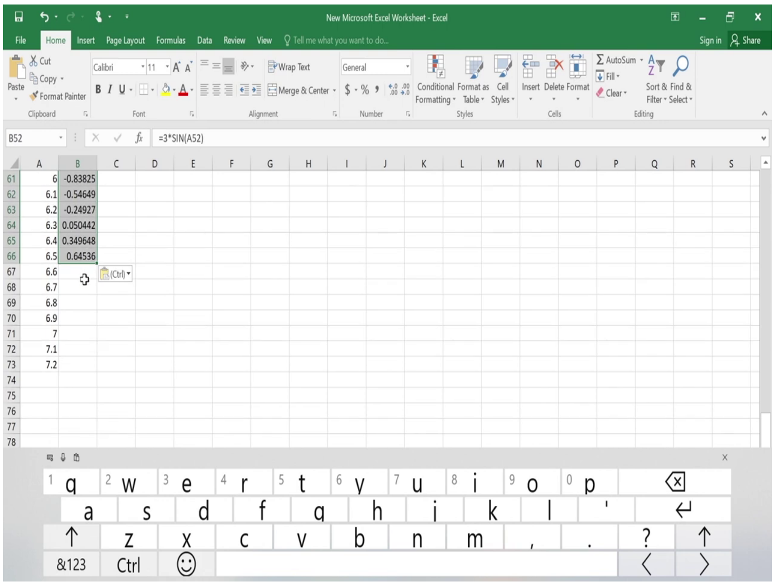

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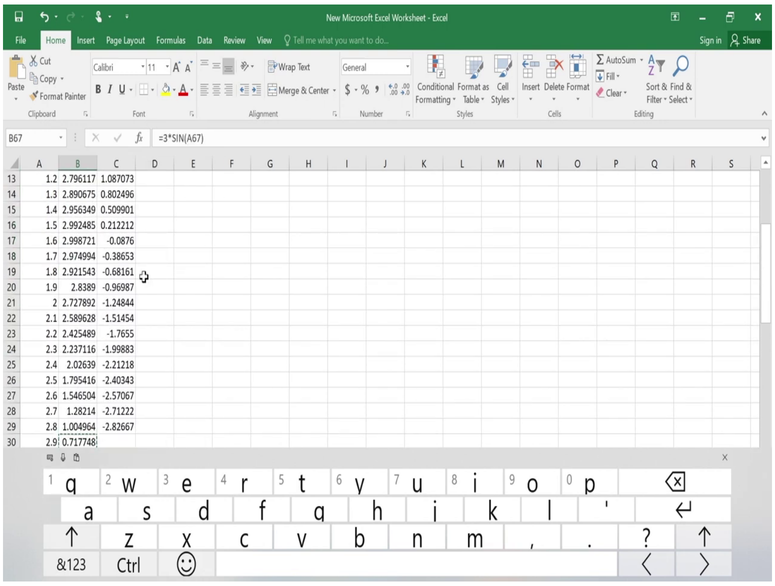

So, you just see you have generated the t and x; now, let me generate the x dot also, just I am doing copy and paste nothing else which you can easily do it, yes.

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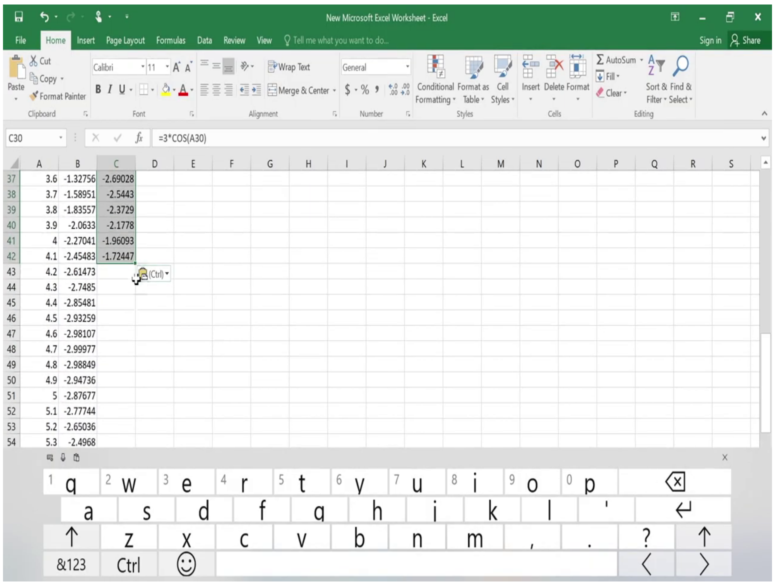

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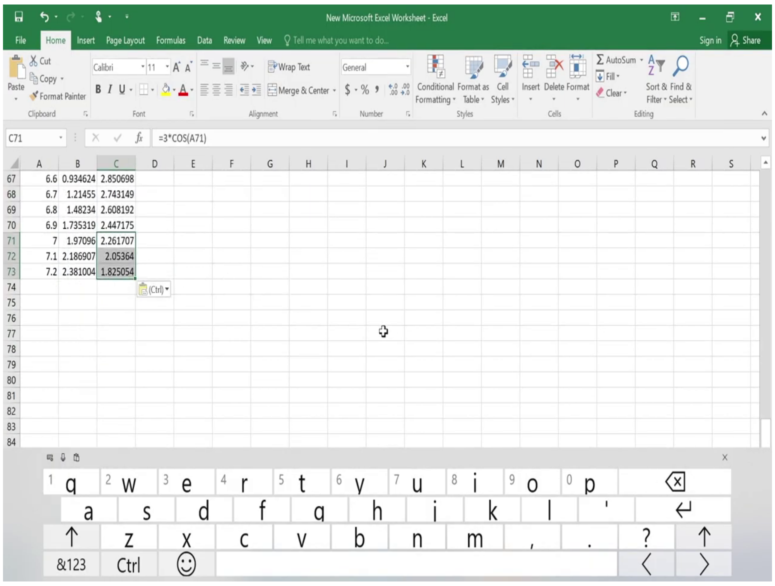

So, now you have plotted for example, you have ploted this t 3x equal to 3 sin t and x dot equal to 3 cos t. Now, you want to plot x versus time. So, to plot x versus time just you mark these two column. So, A, B you have just mark just move your cursor, so that it will move to the end of this thing end of your this one. So, first two columns you have seen. Then what you will do? So, you just go to Insert ok.

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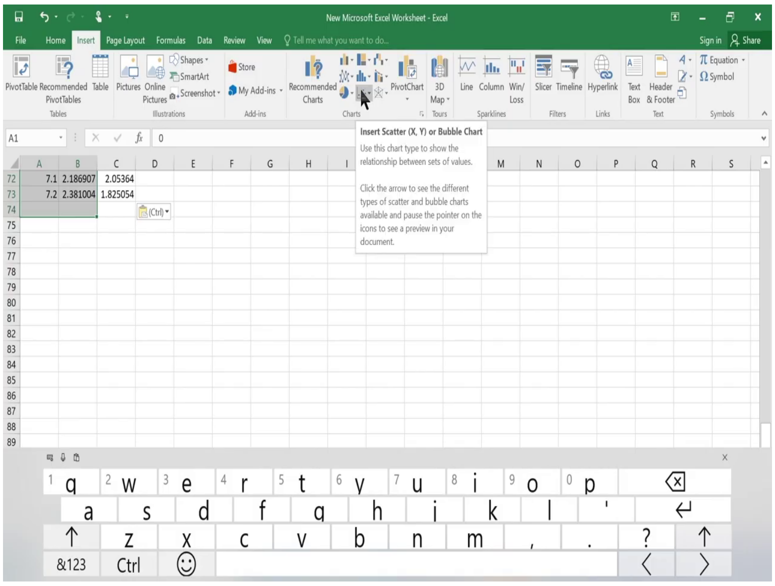

So, in insert you can see there are several option are there. So, this option is for your plotting the X, Y plot. So, click this one.

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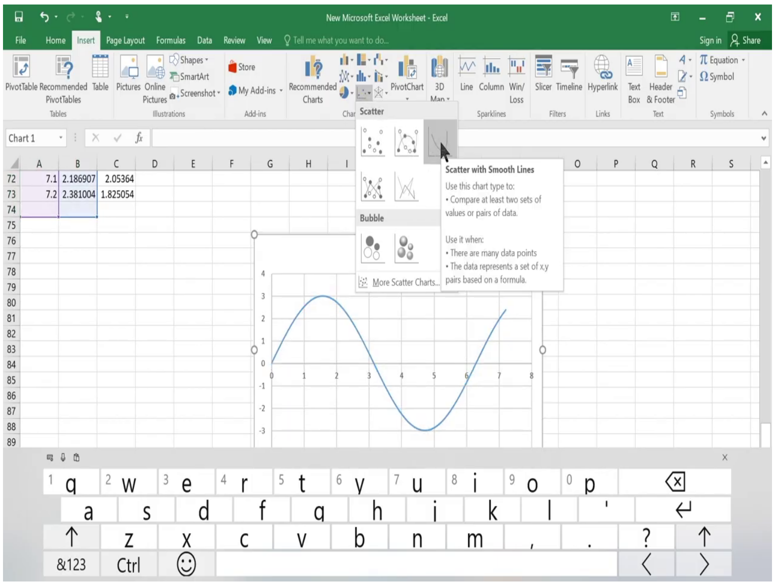

So, you see all these thing. So, clearly you have seen so, you have a curve.

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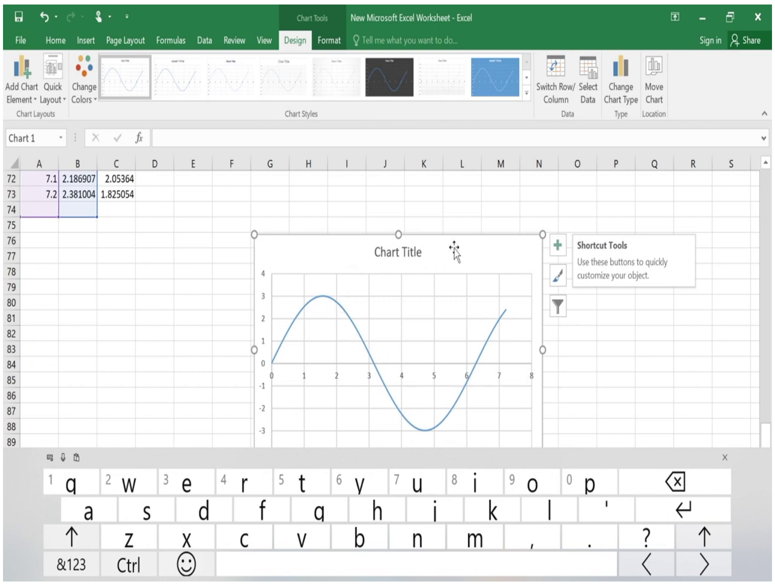

So, now, you have these x versus time. So, you have plotted this x versus time plot now. So, now, you want to plot y dot versus time. So, for example, you want to plot this y dot versus time. So, in that case so you have to plot the first column and the third column. So, you have to plot the first and third column. So, let you want to plot the phase portrait. So, in that case in case of phase portrait you can do it.

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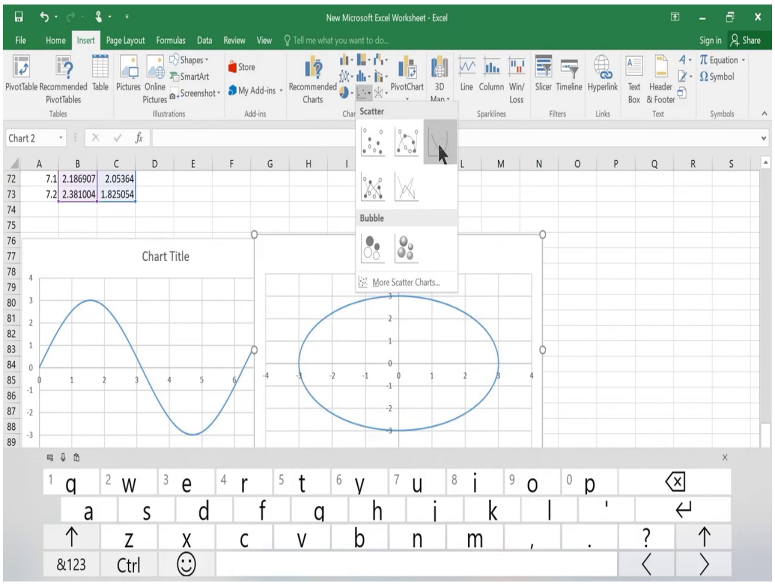

So, this is phase portrait I want to show you. So, again so, you have marked it these two column you have marked, then Insert again you go the here and click it.

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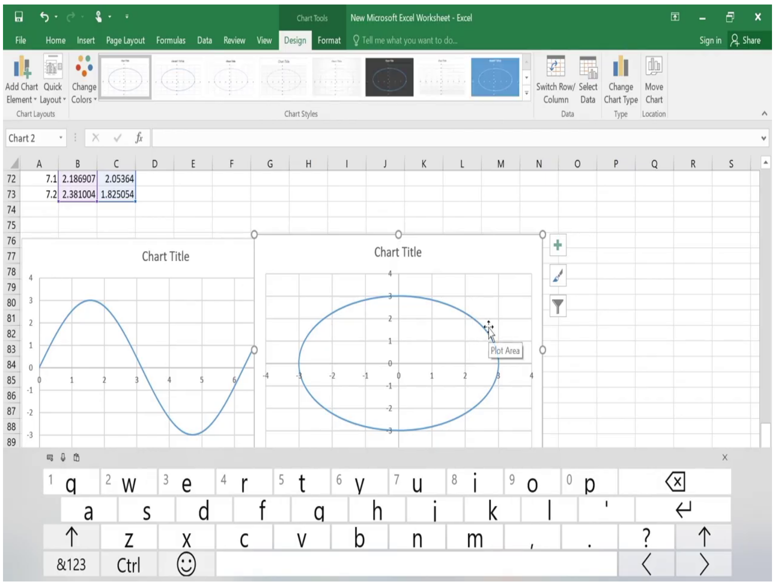

So, this is the phase portrait you have plotted. So, this way by simply using this excel file also you can plot. So, now, you just plot take this assignment to plot Y equal to 2 sin t plus 2 sin root 2t.

Thank you, thank you very much.