

Nonlinear Vibration
Prof. Santosha Kumar Dwivedy
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Lecture - 29

Active vibration absorber with time delay acceleration feedback by HBM

Welcome to today class of Non-linear Vibration. So, in this module we are discussing regarding the applications of non-linear vibration in the first 3 classes.

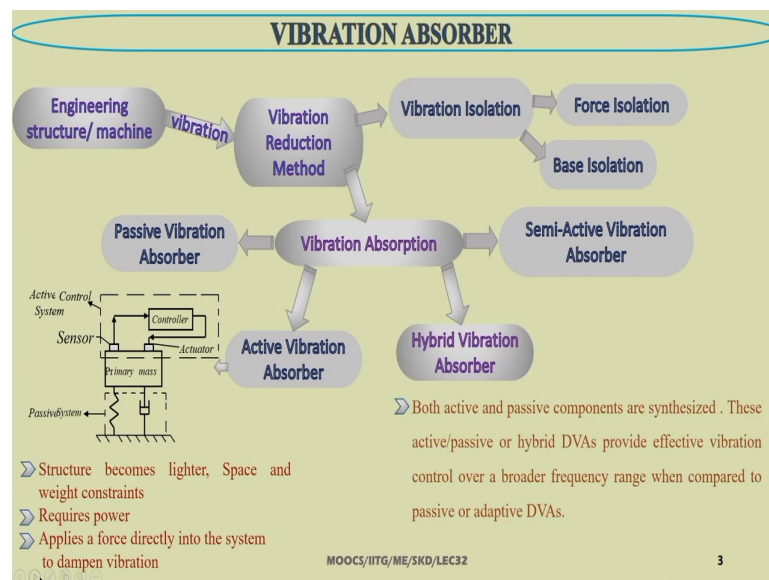
(Refer Slide Time: 00:50)



So, we have discussed regarding the vibration of continuous system where we have discussed regarding the vibration of a beam subjected to arbitrary base excitation or where a mass is attached at an arbitrary position.

So, in the second sets of applications. So, we are discussing regarding the non-linear vibration absorber.

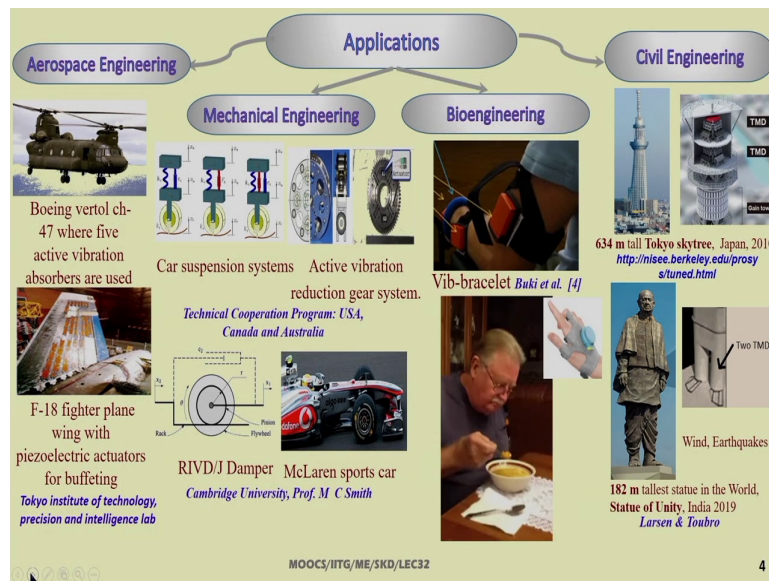
(Refer Slide Time: 01:15)



So, last class we have started this non-linear vibration absorber and here we have discussed the applications of many applications of non-linear vibration absorber also we have discussed regarding how it is different from vibration isolations.

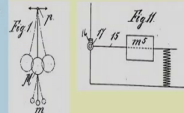
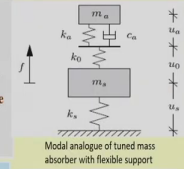
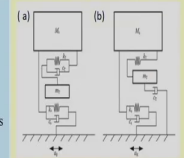
So, what are the passive vibration absorber? Active vibration absorber and this hybrid vibration absorber in case of the passive. So, we have only spring and damper system attached to the system.

(Refer Slide Time: 01:49)



So, we know regarding the tuned vibration absorber when this excitation frequency equal to the natural frequency of the secondary system. So, here you can see several pictures of the vibration absorber what we have discussed last class.

(Refer Slide Time: 02:03)

Literature Review	
Passive vibration absorber (Linear analysis)	
Frahm[1] <i>Device for damping vibrations of bodies</i> US Patent 989958 (1911)	<ul style="list-style-type: none"> Investigated eleven different models of undamped vibration absorber to suppress the resonant vibration of the various primary systems. Designed and developed an undamped vibration absorber to suppress resonant vibration of a ship subjected to periodic force. 
Krenk and Høgsberg [2] <i>Tuned mass absorber on a flexible structure</i> Journal of Sound and Vibration 333.6 (2014): 1577-1595	<ul style="list-style-type: none"> Designed a tuned mass absorber (TMA) on flexible structure for vibration suppression a SDOF spring, mass system. Obtained fully balanced frequency curves by using fixed point theory optimization. Proposed TMA applied on 10 storey building, taut cable and on pedestrian bridge for attenuating vibration of the primary system.  <p>Modal analogue of tuned mass absorber with flexible support</p>
Xiang and Nishitani [7] <i>Optimum design for more effective tuned mass damper system and its application to base-isolated buildings</i> Structural Control and Health Monitoring 21.1 (2014): 98-114.	<ul style="list-style-type: none"> Designed non-traditional tuned mass dampers (TMDs) to suppress vibration of seismic induced base isolated structural system. The optimal non-traditional TMDs suppress vibration in wide bandwidth and requires less TMDs stroke than traditional TMDs. Quasi fixed point theory of optimization is developed to obtain optimal parameters for the absorbers.  <p>Analytical model of (a) traditional mass damper (TMD) system, (b) non-traditional TMD system</p>

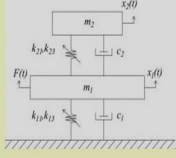
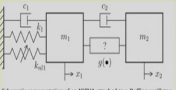
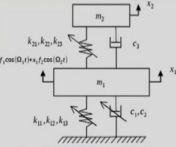
(Refer Slide Time: 02:04)

Active vibration absorber (Linear analysis)		Contd...
<p>Vyhlidal et al.[16] <i>Analysis and design aspects of delayed resonator absorber with position, velocity or acceleration feedback</i> <i>Journal of Sound and Vibration 333.5 (2019): 1331-1343</i></p>	<ul style="list-style-type: none"> Analysed lumped and distributed delayed resonators with acceleration, velocity and position feedback. Proposed design criteria by comparing among various various feedbacks and delay effects. 	
<p>Kucera et al.[16] <i>Extended delayed resonators - Design and experimental verification</i> <i>Mechatronics 41 (2017) 29-44.</i></p>	<ul style="list-style-type: none"> Designed and experimented both delayed and non-delayed acceleration feedback control together for vibration suppression of the system. The operable frequency range is widened by including a non-delayed part to adjust virtually the mass and thus the natural frequency of the active absorber. The properties and performance of the resulting algorithms are compared with the delay free PI (proportional and integral) feedback control law. 	<p>Fig. 1. SDOF Primary Structure (P) with an active vibration absorber (A) to suppress displacement x_p induced by harmonic disturbance force $f(t)$.</p>
<p>Brenan et al. [28] <i>An investigation into the simultaneous use of a resonator as an energy harvester and a vibration absorber</i> <i>Journal of Sound and Vibration 333.5 (2014): 1331-1343</i></p>	<ul style="list-style-type: none"> Investigated and showed the use of an auxiliary system to act as a vibration absorber and an energy harvester simultaneously by providing broad band random excitation and single frequency excitation to the host structure. Different optimizing criteria namely Den Hartog's equal peaks method, H2 norm of minimization of kinetic energy of host structure are compared and studied. 	

(Refer Slide Time: 02:09)

Passive vibration absorber (Nonlinear analysis)		
<p>Vyhlidal et al.[16] <i>Analysis and design aspects of delayed resonator absorber with position, velocity or acceleration feedback</i> <i>Journal of Sound and Vibration</i> 333.5 (2019): 1331-1343</p>	<ul style="list-style-type: none"> Analysed lumped and distributed delayed resonators with acceleration, velocity and position feedback. Proposed design criteria by comparing among various various feedbacks and delay effects. 	
<p>Kucera et al.[16] <i>Extended delayed resonators - Design and experimental verification</i> <i>Mechatronics</i> 41 (2017) 29-44.</p>	<ul style="list-style-type: none"> Designed and experimented both delayed and non-delayed acceleration feedback control together for vibration suppression of the system. The operable frequency range is widened by including a non-delayed part to adjust virtually the mass and thus the natural frequency of the active absorber. The properties and performance of the resulting algorithms are compared with the delay free PI (proportional and integral) feedback control law. 	<p>Fig. 1. SDOF Primary Structure (P) with an active vibration absorber (A) to suppress displacement u induced by harmonic disturbance force $f(t)$.</p>
<p>Brenan et al. [28] <i>An investigation into the simultaneous use of a resonator as an energy harvester and a vibration absorber</i> <i>Journal of Sound and Vibration</i> 333.5 (2014): 1331-1343</p>	<ul style="list-style-type: none"> Investigated and showed the use of an auxiliary system to act as a vibration absorber and an energy harvester simultaneously by providing broad band random excitation and single frequency excitation to the host structure. Different optimizing criteria namely Den Hartog's equal peaks method, H2 norm of minimization of kinetic energy of host structure are compared and studied. 	

(Refer Slide Time: 02:13)

Passive vibration absorber (Nonlinear analysis) Contd...		
<p>Rabelo et al.[22] <i>Numerical analysis of vibration of a nonlinear system with bounded delay under the primary resonances</i> <i>International Journal of Nonlinear Mechanics</i>, 112, (2019), 92-105.</p>	<ul style="list-style-type: none"> Analysed the effects of time delay in the damping on the stability of the response of a two DOF system where the primary system is subjected to external harmonic excitation. demonstrate that the time delay can act as a stability factor to control the vibration amplitude of the main system Used MMS to obtain the reduced equations for the primary and simultaneous resonance conditions. 	
<p>Habib et al. [8] <i>Nonlinear generalization of den hartog's equal-peak method</i> <i>Mechanical Systems and Signal Processing</i>, 52, (2015) 17-28.</p>	<ul style="list-style-type: none"> Obtained the optimum stiffness and damping formulae for the nonlinear passive DVA attached to nonlinear primary system which shows Den Hartog's equal-peaks in the frequency response curves. The performance of the nonlinear tuned vibration absorber showed superior to the classical linear tuned vibration absorber. 	 <p>Schematic representation of an NCTVA attached to a duffing oscillator.</p>
<p>Rabelo M et al. [8] <i>Computational and numerical analysis of a nonlinear mechanical system with bounded delay</i> <i>International Journal of Non-Linear Mechanics</i> 91 (2017) 36-57</p>	<ul style="list-style-type: none"> Analysed the stability of a nonlinear system with two degree of freedom system with time delay in the linear damping. MMS and Fourth Order Runge-Kutta Method is used to obtain solution of the system. Modified Routh-Hurwitz criterion is developed to study the stability of the system. 	 <p>Fig. 3. Two degree-of freedom damped system.</p>

(Refer Slide Time: 02:16)

Contd...

Active vibration absorber (Nonlinear analysis)

<p>Sun and Xu [32] <i>Vibration control of nonlinear Absorber-Isolator-Combined structure with time-delayed coupling</i> <i>International Journal of Non-Linear Mechanics</i> 83 (2016) 48–58.</p>	<ul style="list-style-type: none"> ❑ A novel application of internal resonance in vibration suppression of an Absorber-Isolator-Combined (AIC) structure with time-delayed coupling control at the resonance frequency band. ❑ MMS is used to obtain the reduced equations. ❑ Vibration suppression effectiveness, control mechanisms and stability of the steady states for different internal resonances is achieved. ❑ For 1:2 internal resonance increasing time delay reduces the resonance peak about 40% and for 1:3 internal resonance the resonance peak is reduced about 65% more than the case of 1:2
<p>Lu et al. [28] <i>Nonlinear dissipative devices in structural vibration control: A review</i> <i>Journal of Sound and Vibration</i> 423.9 (2018): 18–49.</p>	<ul style="list-style-type: none"> • Reviewed 296 recent articles on the state-of-the-art technologies of nonlinear dissipative devices and absorbers. • Discussed various nonlinear vibration absorbers, namely nonlinear viscous damper, nonlinear energy sink, nonlinear dampers, particle impact damper etc and characterized their wide frequency band of vibration attenuation and high robustness.

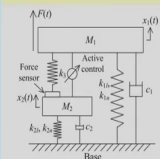


Fig. 6. The proposed AIC structure with time-delayed coupling control.

MOOCs/IITG/ME/SKD/LEC32
8

We made some literature review regarding the active, passive and hybrid vibration absorber and so, we have started the derivation of non-linear equation of motion or derivation of equation of motion of different vibration absorber.

(Refer Slide Time: 02:21)

Objective

✓ The objective of the present work is to investigate vibration suppression and study the nonlinear dynamics of single, multi DOF and continuous primary system under external harmonic, parametric, and base excitations by a modified design of piezoelectric based active nonlinear vibration absorber (ANVA).

❖ To achieve the main objective six different works have been carried out in this work.

1. Linear and nonlinear analysis of ANVA by displacement and acceleration feedback to suppress vibration of the SDOF spring, mass, damper primary system under external harmonic and parametric excitations.
 - **Methodology:** Laplace transformations and 1st order method of multiple scales (MMS).
 - **Resonance conditions:** Primary and principal parametric
2. Nonlinear dynamics of ANVA with time delay in acceleration feedback to suppress vibration of SDOF spring, mass, damper primary system under external multi-hard harmonic and parametric excitations.
 - **Methodology:** 1st and 2nd order method of multiple scales (MMS).
 - **Resonance conditions:** Primary, principal parametric, superharmonic, subharmonic, 1:1, and 3:1 internal resonance.

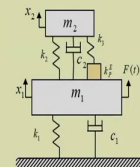
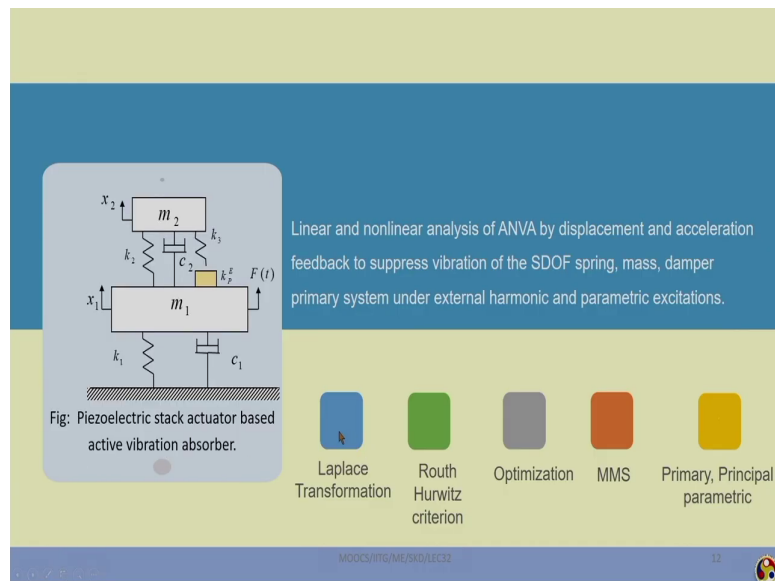


Fig: Piezoelectric stack actuator based active vibration absorber.

(Refer Slide Time: 02:30)

3. ANVA with time delay in acceleration feedback is used to suppress vibration of SDOF spring, mass, damper primary system under external harmonic and base excitations, and obtained Den Hartog's equal peaks.
 - **Methodology:** Modified harmonic balance method (HBM).
 - **Resonance conditions:** Primary
4. Linear and nonlinear analysis of traditional and non-traditional ANVA with the combination of time delay in displacement, velocity, and acceleration feedback to suppress vibration of SDOF spring, mass, primary system under external harmonic and parametric excitations.
 - **Methodology:** Weighted modal approach, 1st order MMS, 2nd order MMS and HBM.
 - **Resonance conditions:** Primary, principal parametric.
5. Nonlinear dynamics of traditional and non-traditional ANVA considering nonlinear damper with the combination of time delay in displacement, velocity, and acceleration feedback to suppress vibration of SDOF spring, mass primary system under external hard harmonic, parametric excitation, and base excitations.
 - **Methodology:** Weighted modal approach, 1st order MMS, 2nd order MMS.
 - **Resonance conditions:** Primary, principal parametric, superharmonic, subharmonic, 1:1, and 3:1 internal resonance.

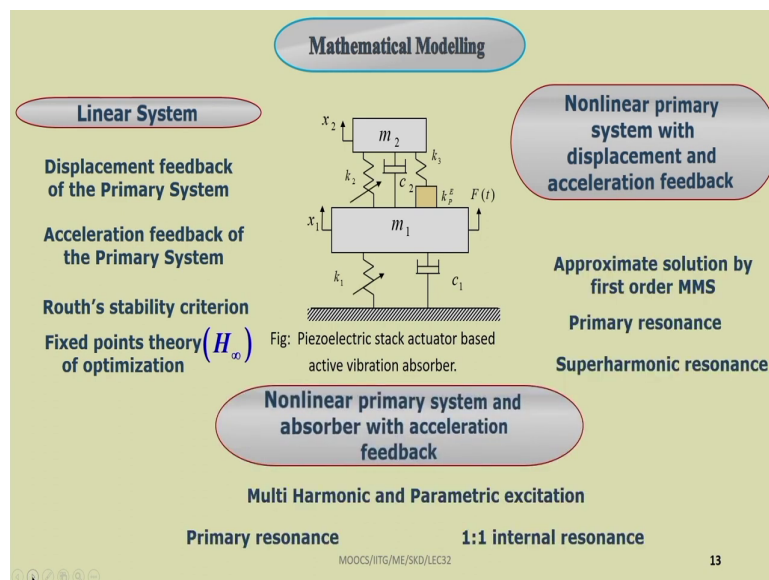
(Refer Slide Time: 02:33)



In this vibration absorber we are particularly interested for this piezoelectric stack actuators and here we have discuss regarding the linear and non-linear analysis of active non-linear vibration absorber by displacement and acceleration feedback to suppress the vibration of single degree of freedom system, spring mass damper primary system under external harmonic and parametric excitation.

Already you know what is harmonic excitation if the forcing is in the form of sine and cosine, then it is known as harmonic excitation and in case of the parametrically excited system, you can find a time varying term which is coefficient of the excitation term.

(Refer Slide Time: 03:23)



So, initially we apply this Laplace transform or initially we have derived this equation of motion of the system and then apply this Laplace transform to write down the characteristic equation. So, this is the systems we have taken. So, here the primary system has mass m_1 , secondary system has mass m_2 .

So, the primary system is supported by a spring stiffness case spring k_1 and damper c_1 and the primary is connected to the secondary by a spring k_2 damper c_2 and a piezoelectric stack actuator with stiffness K_{PE} , also it is connected to the mass 2 by another spring k_3 .

So, we can find the equivalent stiffness of this k_3 and k_{PE} and these force in the stack actuator or the displacement in the stack actuator can be written using δ . So, δ will be

equal to $n d^3 v$. So, where n is the number of stacks in the stack actuator and v is the voltage applied and d^3 is the material property of the actuator material.

So, that way we can write down the force applied by the stack actuator and we can derive this equation of motion.

(Refer Slide Time: 04:41)

Mathematical Modelling

Linear System Analysis

The equations of motion of the system in the Fig. can be written as

$$m_1 \ddot{x}_1 + k_1 x_1 + k_2 (x_1 - x_2) + c_1 \dot{x}_1 + c_2 (\dot{x}_1 - \dot{x}_2) = F(t) - F_c$$

$$m_2 \ddot{x}_2 + k_2 (x_2 - x_1) + c_2 (\dot{x}_2 - \dot{x}_1) = F_c$$

$$F_c = k_r (x_1 + \delta_0 - x_2), \quad \text{where} \quad k_r = \frac{k_p k_3}{k_p + k_3}$$

Recasting Eqs into respective non-dimensional forms one writes,

$$\left. \begin{aligned} \ddot{X}_1 + X_1 + 2\xi_1 \dot{X}_1 - \mu \Omega_2^2 X - 2\xi_2 \mu \Omega_2 \dot{X} &= f(\tau) - k\lambda v \\ \ddot{X} + \Omega_2^2 X + 2\xi_2 \Omega_2 \dot{X} &= -\dot{X}_1 + \frac{k\lambda v}{\mu} \end{aligned} \right\}$$

The non-dimensional parameters used in Eq.(1.5) and Eq.(1.6) are

$$\checkmark \quad X_1 = \frac{x_1}{x_0}, \quad X_2 = \frac{x_2}{x_0}, \quad X = X_2 - X_1, \quad k = \frac{k_1}{k_1}, \quad \mu = \frac{m_2}{m_1}, \quad v = \frac{v}{v_0}, \quad a_1 = \sqrt{\frac{k_1}{m_1}}, \quad a_2 = \sqrt{\frac{k_2 + k_3}{m_2}}, \quad \Omega_2 = \frac{\omega_2}{a_2}$$

$$\xi_1 = \frac{c_1}{2m_1 a_1}, \quad \xi_2 = \frac{c_2}{2m_2 a_2},$$

x_0 and v_0 are reference displacement and voltage quantity and the $\tau = \omega_0 t$

*dot' denotes differentiation with respect to the non-dimensional time

14

Fig: Piezoelectric stack actuator based Hybrid vibration absorber.

So, in this already we have studied this part. So, here we have derived this equation of motion and in this equation motion particularly let me repeat. So, you can combine this k_3 and k_{PE} to have a equivalent stiffness which is equal to $k_{PE} \cdot k_3$ by k_{PE} plus k_3 .

So, this equation that is for the you can draw the free body diagram for the primary system and the secondary system and applying Newton second law or d'Alembert principle you can derive this equation of motion.

So, now by dividing m_1 in the first equations, the equation can be reduced m_1 in the first equation we have to divide and then by taking this k_1 root over k_1 by m_1 equal to k_1 by ω_0 k_1 by m_1 root over equal to ω_0 .

And taking a scaling taking a non-dimensional time taking a non-dimensional time τ equal to $\omega_0 t$ the equation can be non dimensionalized by also taking a non-dimensional displacement parameter X_1 equal to x_1 by x_0 and X_2 by x_2 by x_0 and X equal to X_2 minus X_1 , k equal to k_r by k_1 and μ equal to m_2 by m_1 v equal to v by v_0 v is the voltage applied.

So, v_0 is the reference voltage you can take and then ω_0 equal to root over k_1 by m_1 and ω_2 equal to root over k_2 k_r by m_2 and this external frequency ω_2 equal to ω small ω_2 by ω_0 . So, the damping parameter ζ_2 equal to c_2 by $2 m_2 \omega_2$ and ζ_1 equal to c_1 $2 m_1 \omega_0$. So, that way by taking all these parameter.

So, you can reduce to these two equation. So, here you can see this λ parameter. So, λ equal to $n d^3 v_0$ by x_0 . So, this is due to the piezoelectric property of the stack actuator. So, you got this parameter λ . So, this is the piezoelectric property due to piezoelectric material.

(Refer Slide Time: 07:03)

Contd...

Taking the Laplace transformations on both sides of Eqn

$$(s^2 + 2\xi_p s + 1)Z_p(s) - (\mu\Omega_a^2 + 2\xi_a\mu\Omega_a s)Z(s) = F(s) - k\lambda V(s), \text{ and } (s^2 + 2\xi_a\Omega_a s + \Omega_a^2)Z(s) + s^2 Z_p(s) = \frac{k\lambda V(s)}{\mu}$$

Acceleration feedback of the Primary System

Providing a negative feedback to the primary system with controller gain k_c is given by $V = -k_c \ddot{X}_1$ ✓

The transfer function of the primary mass is obtained as

$$G(s) = \frac{X_1(s)}{F(s)} = \frac{s^2 + 2\xi_2\Omega_2 s + \Omega_2^2}{b_4 s^4 + b_3 s^3 + b_2 s^2 + b_1 s + b_0} \quad \checkmark$$

where the coefficients b_4, b_3, b_2, b_1 and b_0 are expressed as

$$\alpha = k/k_c, b_1 = 1 - \alpha, b_2 = 2\xi_2\Omega_2 + 2\xi_1 + 2\xi_2\Omega_2\mu, b_3 = \Omega_1^2 + 4\xi_1\xi_2\Omega_2 + \mu\Omega_1^2 + 1, \\ b_4 = 2\xi_1\Omega_1^2 + 2\xi_2\Omega_2 \quad b_0 = \Omega_1^2$$

Figure 2: Block diagram for acceleration feedback of the primary mass

MOOC5/IITG/ME/SKD/LEC32 15

So, then in the system. So, what we can do? So, we can apply these control force also we can use a controller. So, you have a primary system this primary system. So, the output is given to be X_1 double dot. So, here we are taking acceleration feedback. So, if we are taking a acceleration feedback. So, we have the accelerometer we can put. So, it can sense this acceleration and it will give the acceleration to the controller.

So, in the controller. So, we can write down this control force. So, you just see in controller we can write down the control force. So, this control force or in terms of voltage can be written v equal to minus $k_c X_1$ double dot. So, this is k_c multiplied by X_1 double dot will give you the voltage.

So, taking this voltage as the. So, taking this as the feedback now it can be fed to the actuator k_c . So, we can get. So, from this k_c into d_0 that is the displacement that will give you the

actuator force. So, getting the actuator force then and these external forcing external forcing is $F(t)$. So, it will act on the primary system and we will get the response in this way. So, this control loop one can draw the block diagram of the control loop.

And you can study the system. So, here the transfer function if one write the equation. So, as the system is linear. So, we can write in using this Laplace transform. So, the transfer function from the primary system which is the output displacement by the input force Laplace of input force equal to $a(s)$ and output displacement is $X(s)$. So, we write this $G(s)$ equal to $X(s)$ by $a(s)$.

So, it can be written in this form and you can see this denominator contain a fourth order term that is $b_4 s^4 + b_3 s^3 + b_2 s^2 + b_1 s + b_0$ where this coefficient can be expressed in this following way. So, here this alpha parameter one can take this alpha parameter equal to k_λ / k_c already we have defined k in the previous slide.

So, b_4 equal to $1 - \alpha$. So, actually these to define this b_1, b_2, b_3, b_4 we are using this parameter alpha which is a function of the actuator for ce actuator parameters. So, alpha is the function of the actuator parameter like this k_λ and k_c k_c is the control gain. So, you have seen here v equal to $-\ddot{X}$. So, by taking this way these non-dimensional parameter.

And so, from this characteristic equation. So, one can use this Routh Hurwitz criteria to find the stability of the system.

(Refer Slide Time: 09:54)

Contd...

The passive part of the absorber is first optimized according to the standard procedure [2] and accordingly, the absorber frequency and damping parameters are set as $\Omega_2 = 1/(1+\mu)$ and $\xi_2 = \sqrt{3\mu/(8(1+\mu))}$

Using this optimal tuning ratio, damping ratio the stability of the system is studied by Routh's stability criterion be stable for

$$\begin{array}{r|l}
 s^4 & b_1 \qquad \qquad \qquad b_3 \qquad \qquad b_5 \\
 s^3 & b_2 \qquad \qquad \qquad b_4 \\
 s^2 & \frac{b_2 b_3 - b_4 b_1}{b_2} \qquad \qquad b_5 \\
 s^1 & \frac{b_2 b_3 b_4 - b_1 b_4^2 - b_2^2 b_5}{b_2 b_3 - b_4 b_1} \\
 s^0 & b_5
 \end{array}$$

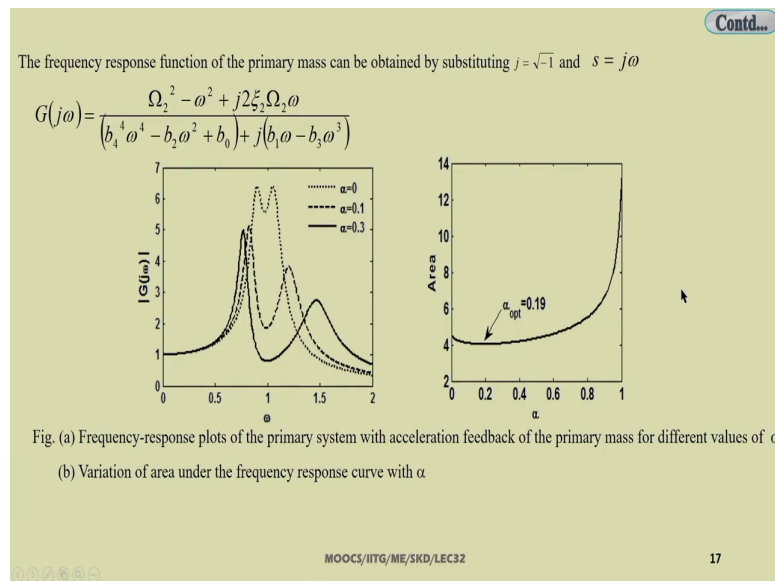
The system is shown to be stable for $-\mu < \alpha < 1$

↖

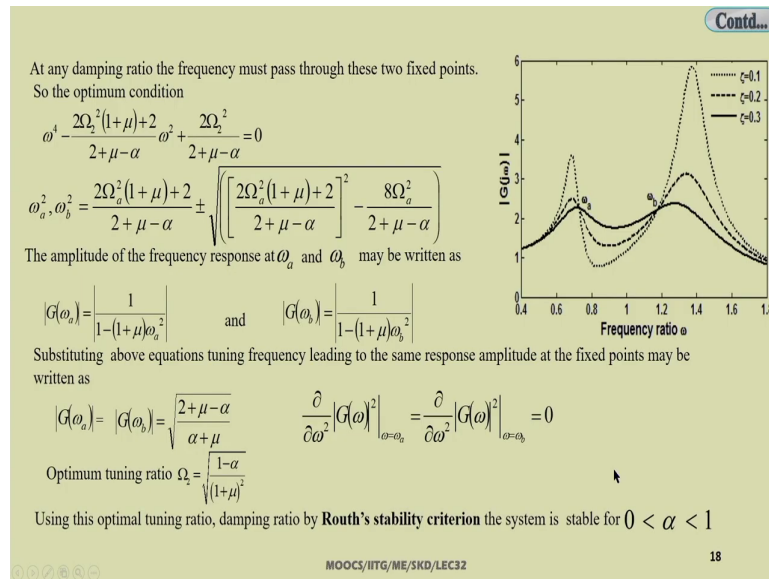
MOOC5/IITG/ME/SKD/LEC32 16

So, it can be shown that the system is stable. So, when this minus mu less than alpha less than 1, mu is the mass ratio parameter.

(Refer Slide Time: 10:08)



(Refer Slide Time: 10:20)



So, if one plot this response. So, last class we have seen how we are getting the frequency response and also the variation of the area under the frequency response curve with alpha parameter.

So, from this thing one can get the optimal parameter one can observe these two fixed point and taking these two fixed point theory. So, one can find the optimum parameter and from that thing one can find the optimum parameter of zeta or optimum parameter to have the further study.

(Refer Slide Time: 10:42)

Contd...

Displacement feedback of the Primary System

Providing a negative feedback to the primary system with controller gain k_c is given by $v = -k_c X_1$

$$G(s) = \frac{X_1(s)}{F(s)} = \frac{s^2 + 2\xi_2 \Omega_2 s + \Omega_2^2}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}$$

where the coefficients a_4, a_3, a_2, a_1 and a_0 are expressed as

$$a_4 = 1, a_3 = 2\xi_1 \Omega_1 + 2\xi_2 + 2\mu\xi_2 \Omega_2, a_2 = \Omega_1^2 - \alpha + 4\xi_1 \xi_2 \Omega_1 + \mu\Omega_2^2 + 1, a_1 = 2\xi_1 \Omega_1^2 + 2\xi_2 \Omega_2 \quad \text{and} \quad a_0 = \Omega_1^2$$

The optimum tuning ratio and damping ratio are obtained by

$$\Omega_2 = \sqrt{\frac{2 + \alpha(1 + \mu)}{2(1 + \mu)^2}} \quad \text{and} \quad \xi_2 = \sqrt{\frac{6\mu - \alpha(\mu^2 + 7\mu + 6)}{16(1 + \mu)(1 + \alpha\mu)}}$$

using the Routh's stability criterion, values the stability region for the control gain may be obtained as

$$-\frac{1}{\mu + 1} < \alpha < \frac{\mu}{\mu + 1}$$

MOOC5/ITG/ME/SKD/LEC32
19

One can take the displacement feedback of the primary system also. So, in case of displacement feedback. So, previously we have taken the acceleration feedback also we can take displacement feedback where this v can be written equal to minus $k_c X_1$ and we can perform the analysis similarly we can find the parameter of α for which the system is stable.

(Refer Slide Time: 11:08)

Nonlinear primary system with displacement feedback

$$\ddot{X}_1 + (1 + \mu\Omega_2^2 - \alpha)\dot{X}_1 + (2\xi_1 + 2\mu\xi_2\Omega_2)\dot{X}_1 - \mu\Delta\Omega_2^2\dot{X}_2 - 2\mu\xi_2\Omega_2\dot{X}_2 + qX_1^3 = F(r)\cos(\Omega t)$$

$$\ddot{X}_2 + \left(\frac{\alpha}{\mu} - \Omega_2^2\right)\dot{X}_1 + \Omega_2^2\dot{X}_2 + 2\xi_2\Omega_2(\dot{X}_2 - \dot{X}_1) = 0$$

ordering Eq. by using book keeping parameter ε the corresponding equation of motions may be written as

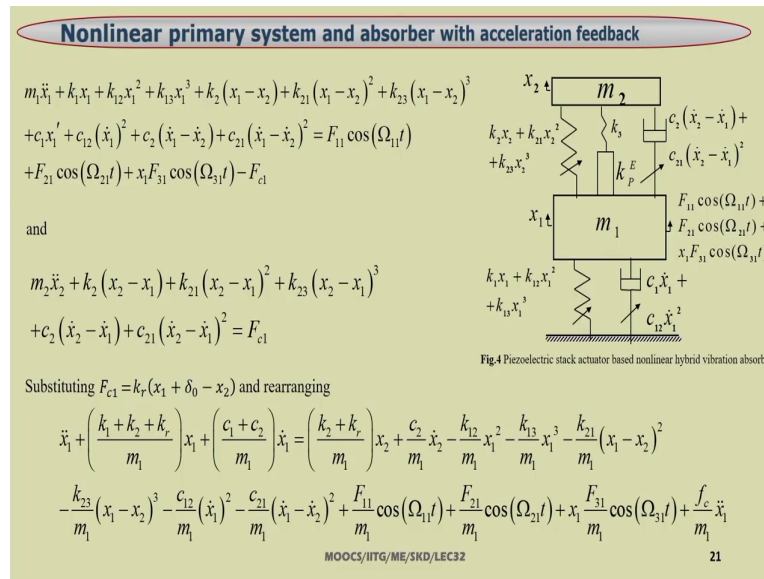
$$\ddot{X}_1 + \omega_0^2 X_1 + \varepsilon z_1 \dot{X}_1 + \varepsilon q_1 X_1^3 - \varepsilon z_2 \dot{X}_2 - \varepsilon^2 r X_2 = \varepsilon F(r)\cos(\Omega t)$$

$$\ddot{X}_2 + \Omega_2^2 X_2 + \varepsilon z_2 \dot{X}_2 - \varepsilon z_1 \dot{X}_1 = \frac{1}{\varepsilon} h X_1$$

For ordering the Eq. the following values are taken as $\mu = 0.05$, $q = 0.1$, $\alpha = \alpha_{gr} = -0.7$, $\Omega_2 = \sqrt{\frac{2 + \alpha(1 + \mu)}{2(1 + \mu)}}$

$$\omega_0^2 = 1 + \mu\Omega_2^2 - \alpha, z_1 = \frac{2\mu\xi_2\Omega_2}{\varepsilon}, q_1 = \frac{q}{\varepsilon}, r = \frac{\mu\Delta\Omega_2^2}{\varepsilon^2}, z_2 = 2\xi_2\Omega_2, \xi_1 = 0 \quad \text{and} \quad \xi_2 = \frac{6\mu - \alpha(\mu^2 + 7\mu + 6)}{16(1 + \mu)(1 + \alpha + \mu)}$$

(Refer Slide Time: 11:14)



So, then we can plot this responses also. So, previously we have seen when we are taking the system to be when the systems are linear. So, we can take the non-linear system also. So, non-linear primary system with displacement feedback also we can take. So, in that case the equation will reduce to this form and one can use different methods. So, here you just see the non-linear parameter $q \times 1$, q is added to the previous equation as you know this superposition theory cannot be applied to this non-linear system. So, you cannot extend the idea of linear system here to predict the non-linear response.

So, one has to study properly the non-linear response of the system. So, one can apply method of multiple scale or any other method to study the response of the system. So, here one can take also the spring to be non-linear in the secondary case, previously we have taken spring to

be non-linear in the primary case only. So, by taking the spring to be non-linear in the secondary case.

Similarly, we can derive this equation of motion. So, here this F_c that is the control force can be written equal to k_r into x_1 plus delta 0 minus x_2 and following the similar way we can have the non-linear equation of motion. So, after getting this non-linear equation of motion.

(Refer Slide Time: 12:38)

Contd...

Assuming $\omega_1^2 = \frac{k_1 + k_2 + k_r}{m_1}$ and $\tau = \omega_1 t$

$$\omega_1^2 \frac{d^2 x_1}{d\tau^2} + \omega_1^2 x_1 + \left(\frac{c_1 + c_2}{m_1} \right) \omega_1 \frac{dx_1}{d\tau} = \left(\frac{k_2 + k_r}{m_1} \right) x_2 + \frac{c_2}{m_1} \omega_1 \frac{dx_2}{d\tau} - \frac{k_{12}}{m_1} x_1^2 - \frac{k_{13}}{m_1} x_1^3 - \frac{k_{21}}{m_1} (x_1 - x_2)^2 - \frac{k_{23}}{m_1} (x_1 - x_2)^3$$

$$- \frac{c_{12}}{m_1} \omega_1^2 \left(\frac{dx_1}{d\tau} \right)^2 - F_{c1} - \frac{c_{21}}{m_1} \omega_1^2 \left(\frac{dx_1}{d\tau} - \frac{dx_2}{d\tau} \right)^2 + \tilde{F}_1 \cos(\Omega_1 \tau) + \tilde{F}_2 \cos(\Omega_2 \tau) + x_1 \tilde{F}_3 \cos(\Omega_3 \tau) + F_{c1} \omega_1^2 \frac{d^2 x_1}{d\tau^2}$$

Non dimensionalizing the Eq.

$$\ddot{X}_1 + X_1 + h_{1c} \dot{X}_1 = \omega_{2c}^2 X_2 + h_{2c} \dot{X}_2 - \alpha_{11c} X_1^2 - \alpha_{13c} X_1^3 - \alpha_{21c} (X_1 - X_2)^2 - \alpha_{23c} (X_1 - X_2)^3$$

$$- h_{12c} \dot{X}_1^2 - h_{21c} (\dot{X}_1 - \dot{X}_2)^2 + F_1 \cos(\Omega_1 \tau) + F_2 \cos(\Omega_2 \tau) + x_1 F_3 \cos(\Omega_3 \tau) + F_{c1} \ddot{X}_1$$

and

$$\ddot{X}_2 + F_{c2} \ddot{X}_1 + \omega_r^2 X_2 + h_2 \dot{X}_2 = h_2 \dot{X}_1 - h_{21} (\dot{X}_2 - \dot{X}_1)^2 - \alpha_{21} (X_2 - X_1)^2 - \alpha_{23} (X_2 - X_1)^3 + \omega_r^2 X_1$$

where

$$h_{1c} = \left(\frac{c_1 + c_2}{m_1 \omega_1} \right), \omega_{2c}^2 = \left(\frac{k_2 + k_r}{m_1 \omega_1^2} \right), h_{2c} = \frac{c_2}{m_1 \omega_1}, \alpha_{11c} = \frac{k_{12} x_0}{m_1 \omega_1^2}, \alpha_{13c} = \frac{k_{13} x_0^2}{m_1 \omega_1^2}, \alpha_{21c} = \frac{k_{21} x_0}{m_1 \omega_1^2},$$

$$\alpha_{23c} = \frac{k_{23} x_0^2}{m_1 \omega_1^2}, h_{12c} = \frac{c_{12} x_0}{m_1}, h_{21c} = \frac{c_{21} x_0}{m_1}, X_1 = \frac{x_1}{x_0}, X_2 = \frac{x_2}{x_0}, F_1 = \frac{\tilde{F}_1}{\omega_1^2 x_0}, F_2 = \frac{\tilde{F}_2}{\omega_1^2 x_0}, F_3 = \frac{\tilde{F}_3}{\omega_1^2 x_0}$$

MOOCs/IITG/MESKD/LEC32

22

(Refer Slide Time: 12:51)

Contd...

$$\omega_r = \left(\frac{\omega_2}{\omega_1} \right)^2, \omega_2 = \sqrt{\left(\frac{k_2 + k_r}{m_2} \right)}, F_c = \frac{k_r k_c m d_{33}}{m_2}, h_2 = \frac{c_2}{m_2 \omega_1}, h_{21} = \frac{c_{21} x_0}{m_2}, \alpha_{21} = \frac{k_{21} x_0}{m_2 \omega_1^2}, \alpha_{23} = \frac{k_{23} x_0^2}{m_2 \omega_1^2}$$

$$\Omega_1 = \frac{\Omega_{11}}{\omega_1}, \Omega_2 = \frac{\Omega_{21}}{\omega_1}, \Omega_3 = \frac{\Omega_{31}}{\omega_1}, \tilde{F}_1 = \frac{F_{11}}{m_1}, \tilde{F}_2 = \frac{F_{21}}{m_1}, \tilde{F}_3 = \frac{F_{31}}{m_1}, F_{c1} = \frac{f_c}{m_1}$$

considering a small book keeping parameter ϵ in the corresponding equations obtained as Eq. (86) and (87)

$$\begin{aligned} & \ddot{X}_1 + X_1 + \epsilon z_1 \dot{X}_1 = \epsilon \bar{\omega} X_2 + \epsilon z_2 \dot{X}_2 - \epsilon (\bar{\alpha}_{11} + \bar{\alpha}_{21}) X_1^2 - \epsilon (\bar{\alpha}_{13} + \bar{\alpha}_{23}) X_1^3 - \epsilon^3 \bar{\alpha}_{21} X_2^2 \\ & + 2\epsilon^3 \bar{\alpha}_{21} X_2 X_1 + \epsilon^3 \bar{\alpha}_{23} X_2^3 + 3\epsilon^2 \bar{\alpha}_{23} X_1^2 X_2 - 3\epsilon^2 \bar{\alpha}_{23} X_1 X_2^2 - \epsilon (h_{12} + z_{21}) \dot{X}_1^2 - \epsilon^2 z_{21} \dot{X}_2^2 \\ & + 2\epsilon^3 z_{21} \dot{X}_1 \dot{X}_2 + \epsilon \bar{F}_1 \cos(\Omega_1 \tau) + \epsilon \bar{F}_2 \cos(\Omega_2 \tau) + \epsilon X_1 \bar{F}_3 \cos(\Omega_3 \tau) + \epsilon \bar{F}_{c1} \dot{X}_1 \end{aligned}$$

and

$$\begin{aligned} & \ddot{X}_2 + F_{c2} \dot{X}_1 + \omega_2 X_2 + \epsilon h_2 \dot{X}_2 = \epsilon \bar{h}_2 \dot{X}_1 - \epsilon \bar{h}_{21} \dot{X}_2^2 - \epsilon \bar{h}_{21} \dot{X}_1^2 + 2\epsilon \bar{h}_{21} \dot{X}_1 \dot{X}_2 - \epsilon \bar{\alpha}_{21} X_2^2 - \epsilon \bar{\alpha}_{21} X_1^2 \\ & + 2\epsilon \bar{\alpha}_{21} X_1 X_2 - \epsilon \bar{\alpha}_{23} X_2^3 + 3\epsilon \bar{\alpha}_{23} X_2^2 X_1 - 3\epsilon \bar{\alpha}_{23} X_2 X_1^2 + \epsilon \bar{\alpha}_{23} X_1^3 + \omega_r^2 X_1 \end{aligned}$$

where

$$z_1 = \frac{h_{12}}{\epsilon}, \bar{\omega} = \frac{\omega_{21}^2}{\epsilon}, z_2 = \frac{h_{12}}{\epsilon}, \bar{\alpha}_{11} = \frac{\alpha_{111}}{\epsilon}, \bar{\alpha}_{13} = \frac{\alpha_{113}}{\epsilon}, \bar{\alpha}_{21} = \frac{\alpha_{211}}{\epsilon}, \bar{\alpha}_{23} = \frac{\alpha_{213}}{\epsilon}, h_{12} = \frac{h_{12}}{\epsilon},$$

$$z_{21} = \frac{h_{12}}{\epsilon}, \bar{F}_1 = \frac{F_{11}}{\epsilon}, \bar{F}_2 = \frac{F_{21}}{\epsilon}, \bar{F}_3 = \frac{F_{31}}{\epsilon}, \bar{F}_{c1} = \frac{F_{c1}}{\epsilon}, \bar{h}_2 = \frac{h_{21}}{\epsilon}, \bar{h}_{21} = \frac{h_{21}}{\epsilon}, \bar{\alpha}_{21} = \frac{\alpha_{211}}{\epsilon}, \bar{\alpha}_{23} = \frac{\alpha_{213}}{\epsilon}$$

23

So, we can further study this equations to find the response. So, here we may apply the method of multiple scales to derive this equation of motion in this case you can check the forcing term is of the order of epsilon it is taken.

So, here it is epsilon F 1. So, if you are taking multiple forces. So, in the case we can take multiple forces in the primary system if multi component of the forces you just see this is omega 1. So, 3 frequency components are there omega 1, omega 2, omega 3. So, multi frequency excitation. So, they can be independent, they can dependent also on each other.

So, it depends on the user how to set this parameter or it also depend on the applications where we are taking multiple number of frequencies acting multiple number of forcing acting having different frequencies in addition to that. So, you we have 3 different forcing term and in addition to that. So, we have this control force here control force is also taken of the order

of epsilon. So, it is F_c into X double dot. So, this is the acceleration feedback taken that way one can find the equation motion.

(Refer Slide Time: 14:11)

Primary resonance Contd...

when $\Omega_1 = (1 + \varepsilon \sigma_1) \omega_r$, $\Omega_2 = (1 + \varepsilon \sigma_2) \omega_r$, $\Omega_3 = (2 + \varepsilon \sigma_3) \omega_r$, $\omega_r \gg 1$

$$a_1' = \frac{z_2 a_1 (\omega_r^2 - F_{c2})}{2(\omega_r^2 - 1)} - \frac{(h_{12} + z_{21}) a_1}{2} + \frac{\bar{F}_1}{2} \sin(\gamma_1) + \frac{\bar{F}_2}{2} \sin(\gamma_{12}) + \frac{\bar{F}_3 a_1}{4} \sin(\gamma_{13})$$

$$a_1 \gamma_1' = \sigma_1 a_1 + \frac{\bar{\omega} (\omega_r^2 - F_{c2}) a_1}{2(\omega_r^2 - 1)} - \frac{3}{8} (\bar{a}_{13} + \bar{a}_{23}) a_1^3 + \frac{\bar{F}_1}{2} \cos(\gamma_1) + \frac{\bar{F}_2}{2} \cos(\gamma_{12}) + \frac{\bar{F}_3 a_1}{4} \cos(\gamma_{13}) + \frac{\bar{F}_{c1}}{2} a_1$$

$$a_2' \omega_r = -\bar{h}_2 \omega_r \frac{a_2}{2} + \frac{\omega_r^2 z_2 a_2}{2(-\omega_r^2 + 1)} - \frac{3}{4} \tilde{a}_{23} a_1^2 a_2 \frac{(\omega_r^2 - F_{c2})}{\omega_r^2 - 1} \sin(2\gamma_2)$$

$$-a_2 \gamma_2' = \frac{F_{c2} \omega_r^2 (\bar{\omega} - z_2) a_2}{2(-\omega_r^2 + 1)} + \frac{\omega_r^2 \bar{\omega} a_2}{2(-\omega_r^2 + 1)} - \frac{3}{4} \tilde{a}_{23} a_1^2 a_2$$

$$+ \frac{3}{4} \tilde{a}_{23} a_1^2 a_2 \frac{(\omega_r^2 - F_{c2})}{\omega_r^2 - 1} + \frac{3}{4} \tilde{a}_{23} a_1^2 a_2 \frac{(\omega_r^2 - F_{c2})}{\omega_r^2 - 1} \cos(2\gamma_2) - \frac{3 \tilde{a}_{23} a_2^3}{8}$$

24

So, now after writing this equation motion. So, then one can apply different resonance conditions for example, one can study the primary resonance condition. So, in case of the primary resonance condition as we have taken 3 frequency term. So, we can take ω_1 equal to $1 + \varepsilon \sigma_1$ and ω_2 also can be taken as $1 + \varepsilon \sigma_2$ and ω_3 also can be taken.

So, ω_1 can be taken equal to $1 + \varepsilon \sigma_1$. So, this is $1 + \varepsilon \sigma_2$, and $1 + \varepsilon \sigma_3$ where ω_r we are taking very very away from that frequency ratio is taken very very away from 1. So, here you can see this non dimensional frequency of the primary system is 1 that is why this ω_1 is taken to be $1 + \varepsilon \sigma_1$. So, if

you see this equation the governing equation you can see the coefficient of x_1 equal to 1, and m_1 equal to 1 coefficient of the first mass equal to 1.

So, when you have non dimensionalize this thing the non-dimensional frequency parameter which is coefficient of x_1 in this case taking a unit mass it will be equal to 1. So, that is why we have taken the resonance condition when this external non-dimensional frequency becomes 1.

So, in this case the external non dimensional frequency equal to 1 so, that is why taking this detuning parameter. So, one can write ω_1 equal to $1 + \epsilon \sigma_1$ ω_2 equal to $1 + \epsilon \sigma_2$. So, if we are considering. So, in case of 3 you just see in case of 3 it is taken to be $2 + \epsilon \sigma_2 + \epsilon \sigma_3$.

So, the third forcing term if you see you just take the third forcing term. So, in the third forcing term we can see this is parametric, this is parametrically excited as the coefficient of x_1 is the time varying term $F_3 \cos \omega_3 T$. It will give rise to resonance condition. So, when ω_3 is nearly twice the natural frequency of the system. So, that is why this ω_3 is taken to be twice ω_1 . So, ω_1 is taken to be 1. So, that is why ω_3 is taken to be $2 + \epsilon \sigma_3$.

So, this is the principal parametric resonance condition. So, we are studying then this primary resonance due to all these conditions. So, applying this method of multiple scale following the similar principle what we have studied before. So, as we have two equations. So, it will yield 4 first order differential equation, these differential equations are the reduced equation.

So, this a_1 , γ_1 , a_2 and γ_2 equation can be obtained in this way. So, you just see these equations are not simple equation what you have studied in case of the duffing oscillator. So, in this case you cannot find the solution or you cannot find a closed form solution like in case of the linear system.

So, here you have to so, for steady state this a_1 , γ_1 , a_2 and γ_2 will be equal to 0 and we will have a set of algebraic and transcendental equation which

you can solve to get this a 1, a 2, gamma 1, gamma 2 which will give the response amplitude of the primary and secondary systems and phase of the primary and secondary systems.

(Refer Slide Time: 18:16)

1:1 internal resonance
Contd...

Considering when $\omega_r = 1 + \varepsilon\sigma$ then $\frac{(\omega_r^2 - F_{c2})}{\omega_r^2 - 1} A_1 \exp(i\tau_0) = 0$

$$2iD_1 A_1 = \bar{\omega} A_2 \exp(i\sigma\tau_1) + z_2 (iA_2 \exp(i\sigma\tau_1)) - i(h_{12} + z_{21}) A_1 - 3(\bar{\alpha}_{13} + \bar{\alpha}_{23}) A_1^2 \bar{A}_1$$

$$+ \bar{F}_1 \left(\frac{\exp(i\sigma_1\tau_1)}{2} \right) + \bar{F}_2 \left(\frac{\exp(i\sigma_2\tau_1)}{2} \right) + \bar{A}_1 \bar{F}_3 \left(\frac{\exp(i\sigma_3\tau_1)}{2} \right) + \bar{F}_{c1} A_1$$

and

$$2i\omega_r D_1 A_2 = 2iF_{c2} D_1 A_1 \exp(-i\sigma\tau_1) + i\bar{h}_2 (\omega_r - \varepsilon\sigma) A_1 \exp(-i\sigma\tau_1) - i\bar{h}_2 A_2 - 3\bar{\alpha}_{23} A_1^2 \bar{A}_2 + 3\bar{\alpha}_{23} A_1^2 \bar{A}_1 \exp(-i\sigma\tau_1)$$

$$- 3\bar{\alpha}_{23} (A_1^2 \bar{A}_2 \exp(-2i\sigma\tau_1) + A_1 \bar{A}_1 A_2) + 3\bar{\alpha}_{23} (A_2^2 \bar{A}_1 \exp(i\sigma\tau_1) + A_2 \bar{A}_2 A_1 \exp(-i\sigma\tau_1))$$

Assuming polar form $A_1 = \frac{1}{2} a_1 e^{i\beta_1}$ and $A_2 = \frac{1}{2} a_2 e^{i\beta_2}$ and substituting autonomous solution by assuming

$$\sigma_1\tau_1 - \beta_1 = \gamma_1, (\sigma_2 - \sigma_1)\tau_1 + \gamma_1 = \gamma_{12}, \gamma_{13} = (\sigma_3 - 2\sigma_1)\tau_1 + 2\gamma_1 \text{ and } \kappa = \beta_1 - \beta_2 - \sigma\tau_1$$

$$a_1' = -\frac{\bar{\omega} a_2}{2} \sin(\kappa) - \frac{z_2 a_2}{2} \sin(\kappa) - \frac{(h_{12} + z_{21}) a_1}{2} + \frac{\bar{F}_1}{2} \sin \gamma_1 + \frac{\bar{F}_2}{2} \sin \gamma_{12} + a_1 \frac{\bar{F}_3}{4} \sin(\gamma_{13})$$

$$a_1 \gamma_1' = \sigma_1 a_1 + \frac{\bar{\omega} a_2}{2} \cos(\kappa) + \frac{z_2 a_2}{2} \cos(\kappa) - \frac{3}{8} (\bar{\alpha}_{13} + \bar{\alpha}_{23}) a_1^3 + \frac{\bar{F}_1}{2} \cos \gamma_1 + \frac{\bar{F}_2}{2} \cos \gamma_{12} + \frac{a_1 \bar{F}_3}{4} \cos(\gamma_{13}) + \frac{\bar{F}_{c1}}{2} a_1$$

MOOC5/ITG/ME/SKD/LEC32
25

So, now considering omega r equal to 1 plus epsilon sigma, then we can have this term that is omega r square minus F c 2 by omega r square minus 1 A 1 e to the power i T 0 equal to 0.

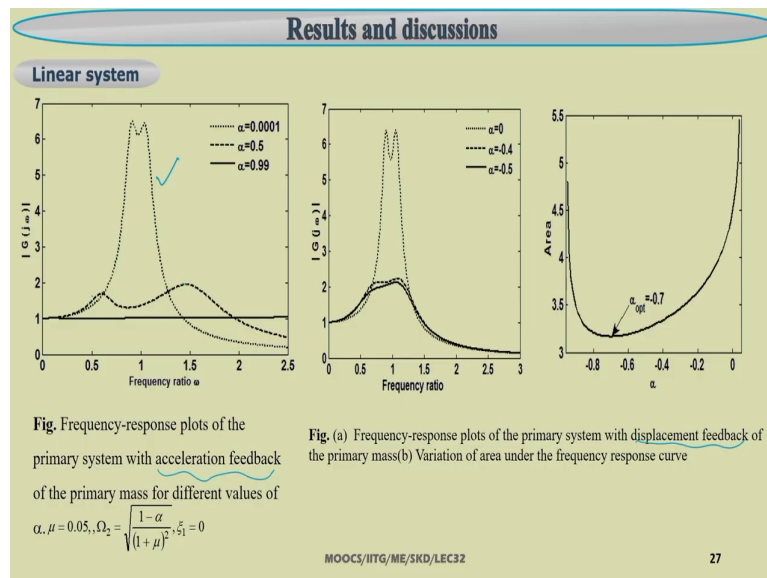
Similarly we will get this equation taking the polar form A 1 equal to half a 1 e to the power i beta 1 and A 2 e to the power i beta 2 and substituting in this autonomous equation. So, we can study the response of the system. So, if we are considering this internal resonance condition reduced equation will contain the additional term taking that additional term.

(Refer Slide Time: 19:04)

$$\begin{aligned}
 a_2' \omega_r = & F_{c2} z_2 \frac{a_2}{2} - (h_{12} + z_{21}) F_{c2} \frac{a_1}{2} \cos(\kappa) - 3F_{c2} (\bar{\alpha}_{13} + \bar{\alpha}_{23}) \frac{a_1^3}{8} \sin(\kappa) \\
 & + \frac{F_{c2} \bar{F}_1}{2} \sin(\kappa + \gamma_1) + \frac{\bar{F}_2}{2} F_{c2} \sin(\gamma_{12} + \kappa) + \frac{\bar{F}_{c1}}{2} F_{c2} \sin(\kappa) + \bar{h}_2 \frac{a_1}{2} \cos(\kappa) - \bar{h}_2 \frac{a_2}{2} \\
 & + 3\tilde{\alpha}_{23} \frac{a_1^3}{8} \sin(\kappa) - 3\tilde{\alpha}_{23} \frac{a_1^2 a_2}{8} \sin(2\kappa) - 3\tilde{\alpha}_{23} \frac{a_2^2 a_1}{8} \sin(\kappa) + 3\tilde{\alpha}_{23} \frac{a_2^2 a_1}{8} \sin(\kappa)
 \end{aligned}$$

$$\begin{aligned}
 a_2 \gamma_1' \omega_r + a_2 \kappa' \omega_r = & (\sigma_1 - \sigma) a_2 + \bar{\omega} F_{c2} \frac{a_2}{2} - (h_{12} + z_{21}) F_{c2} \frac{a_1}{2} \sin(\kappa) - 3F_{c2} (\bar{\alpha}_{13} + \bar{\alpha}_{23}) \frac{a_1^3}{8} \cos(\kappa) \\
 & + \frac{\bar{F}_1}{2} F_{c2} \cos(\kappa + \gamma_1) + \frac{\bar{F}_2}{2} F_{c2} \cos(\kappa + \gamma_{12}) + \frac{\bar{F}_{c1}}{2} F_{c2} \cos(\kappa) - \bar{h}_2 \frac{a_1}{2} \sin(\kappa) - 3\tilde{\alpha}_{23} \frac{a_2^3}{8} \\
 & + 3\tilde{\alpha}_{23} \frac{a_1^3}{8} \cos(\kappa) - 3\tilde{\alpha}_{23} \frac{a_1^2 a_2}{8} \cos(2\kappa) - 3\tilde{\alpha}_{23} \frac{a_1^2 a_2}{8} \cos(\kappa) + 3\tilde{\alpha}_{23} \frac{a_2^2 a_1}{8} \cos(\kappa) + 3\tilde{\alpha}_{23} \frac{a_2^2 a_1}{8} \cos(\kappa)
 \end{aligned}$$

(Refer Slide Time: 19:04)



So, again we can derive this condition we have another set of equations. So, now you can see. So, in case of the linear systems we have already observed that if we are considering alpha equal to 0.0001. So, this is the response we are getting frequency response we are getting and by increasing this control parameter alpha 2.5 or alpha 2.99.

So, you can see drastically the frequency the response amplitude decreases, but here one can absorb the resonance peak. So, is slightly high also, but further increasing this alpha 2.99. So, you can see the response amplitude is flattened and one can get very less value of response amplitude.

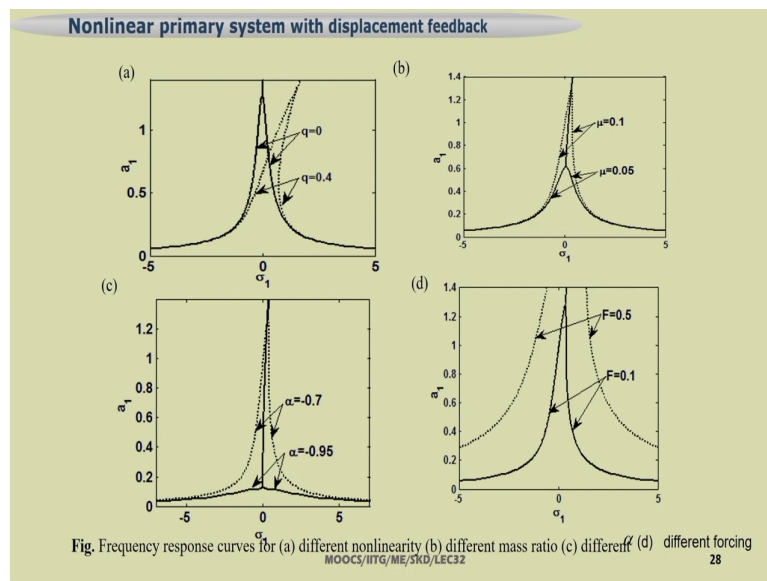
For less control parameter you have seen the response is very high and by applying this control. So, we can easily reduce the response amplitude. So, this part we have seen when we

have applied this acceleration feedback and when you are applying this displacement feedback.

So, you just see. So, we have taken this v equal to minus k_c into x double dot, but when we are taking this v equal to minus k_c into \dot{x} . So, in that case you can see by applying this control gain. So, it is though it is reducing though it is reducing the response amplitude, but the reduction is not. So, high as compared to that in acceleration feedback.

So, acceleration feedback is giving a better result than the displacement feedback what you have observed here.

(Refer Slide Time: 20:58)

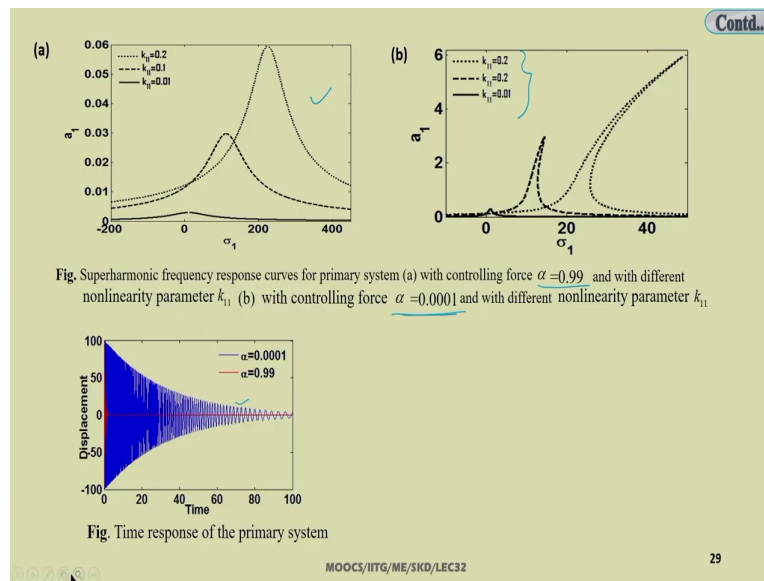


So, that is in the case of the linear case. So, if we have the non-linear primary system with a displacement feedback. So, now, you can see the bending of the bending of the response curve.

So, initially q equal to 0. So, this is the response plot similar to that of a linear system and in case of the non-linear system it is tilting towards right. So, in this case we can have a broadband of frequency for which the response amplitude is very high. Previously, we have a less. So, you have a less range of frequency for which we have the response amplitude to be very high but when the system becomes non-linear.

So, we have a larger frequency range for which the system become non-linear or the system response amplitude is very high. By changing different control parameter we can find or we can absorb. So, what will be the response amplitude of the system.

(Refer Slide Time: 22:08)



So, the response amplitude of the system can be controlled then by possibly by controlling those system parameter or actively by applying this voltage and controlling the piezoelectric material property. So, here you can see. So, these are the frequency response for different value of k_{11} . So, we have taken a sub harmonic resonance condition also sub harmonic.

So, in case of the sub harmonic frequency response curve for primary system. So, this is the sub harmonic case. So, the previously you have seen the primary resonance case and this is the sub harmonic resonance case. So, this is also for the sub harmonic response. So, when α is taken to be 0.99 in this case the response α is taken the control force α is taken 0.0001.

By taking different k_{11} value. So, we can absorb how shifting of the frequency response takes place. So, as we are increasing this k_{11} . So, our natural frequency non dimensional

natural frequency of the system increases. So, that is why there is a shifting of the frequency response towards right. One can plot the time response it can easily be seen that when α equal to 0.0001.

So, one has very large amplitude of oscillation and it takes a large time to settle. So, for example, here till 100 also it is not settled what if we are taking very high value of α . So, you can see it settled down very quickly to its final value. By taking proper control parameter and acceleration feedback here we have shown that the response amplitude can be conveniently controlled in case of the vibration absorber or it can be absorbed.

(Refer Slide Time: 24:01)

Conclusions

- Optimum parameters for the absorber configuration obtained using **fixed point theory**. From the linear analysis it is found that the amplitude of the primary system without active force is 6.45 to the static deflection but its **amplitude reduces to 1.15 times the static deflection (82% amplitude) when controlling force is applied** within the stability region of operation.
- Nonlinear analysis also investigated by considering a cubic nonlinear stiffness along with the linear stiffness in the primary system. Comparing to the linear analysis in the **nonlinear analysis the maximum amplitude of the primary system reduced to 0.1036 for the primary resonance condition** when harmonic force is acting on the primary system.
- One case of superharmonic response is also studied for the primary system which shows more hardening effect in the frequency response when less controlling force is applied in the primary system. **Time responses** shows the effectiveness in settling the vibration of primary system and absorber quickly.
- A nonlinear hybrid vibration absorber with quadratic and cubic nonlinearity in primary system and absorber is considered with acceleration feedback of primary system. The **mass ratio of 0.01 between the absorber and primary system is considered for the analysis**. In the nonlinear analysis the maximum non-dimensional amplitude of the primary system is found to be 0.8 for the primary resonance condition with controlling force, when multi harmonic force and parametric excitation force are acting on it.

MOOC5/IITG/ME/SKD/LEC3230

We have seen this optimum parameter for the absorber configuration obtained using fixed point theory. So, from the linear analysis it is found that the amplitude of primary system without active force is 6.45 to the static deflection, but its amplitude reduces to 1.15 times to

the static deflection, 82 percent amplitude is controlled in this case, similarly non-linear analysis also investigated by considering a cubic non-linear stiffness along with linear stiffness in the primary system.

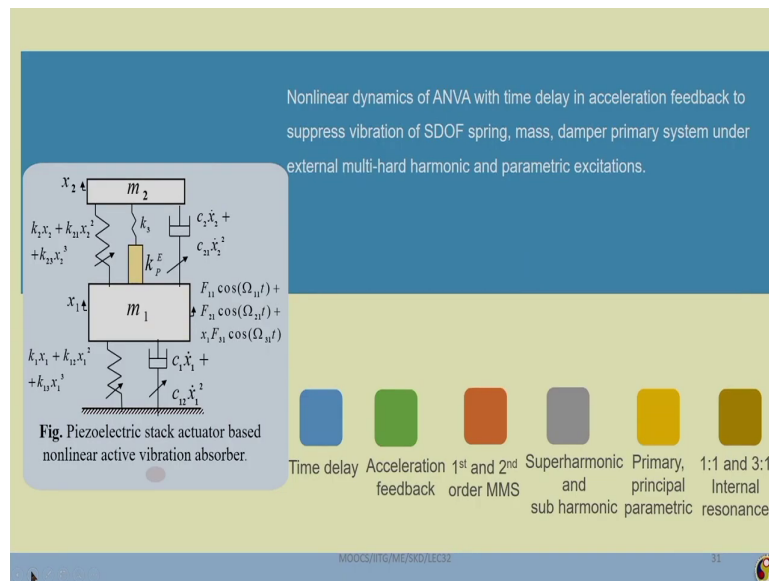
So, comparing the linear analysis in the in non-linear analysis, the maximum amplitude of the primary system reduces 0.1036 for the primary resonance condition when harmonic force is acting on the primary system. One case of super harmonic response is also studied.

So, for the primary system which shows more hardening effect. So, you have seen. So, it is tilted towards right we have observed the hardening effect in the frequency response. When less controlling force is applied so, time response shows the effectiveness of those settling time of the primary system when the absorber is used.

A non-linear hybrid vibration absorber with quadratic and cubic nonlinearity in primary system also is studied and here the mass ratio of 0.01 between the absorber and primary system is considered for the analysis. So, you can note that the in general literature a mass ratio of 1 is to 20 is considered, but in this case you can see you have taken a mass ratio up to 100 also 0.01; that means, so, m_2 by m_1 equal to 0.01 that is 1 by 100.

So, 100 times so we are able to control the vibration of the system by putting a mass 1 by 100 times that of the primary mass. By using this non-linear analysis you have seen we can control the vibration or we can absorb the vibration of a non-linear vibration absorber by putting a mass 1 by 100 of the of that of the primary system. So, this is the advantage of using the non-linear vibration absorber.

(Refer Slide Time: 26:29)



So, let us see another similar problems, where we have taken this non-linear dynamics of this active non-linear vibration absorber with time delay. So, generally when we have apply the control force. So, suddenly it will not react and it will take some time to react that is why there will be some delay in the response.

If we are considering that time delay. So, these analyses will be modified. So, in this case we are going to study the response of the system if there is a time delay in this acceleration feedback to suppress the vibration of a single degree of freedom spring mass damper primary system under external multi hard harmonic and parametric excitation.

So, previously we have taken weak forcing. So, the forcing term has written epsilon time by using the bookkeeping parameter epsilon which signifies that we have taken a weak forcing

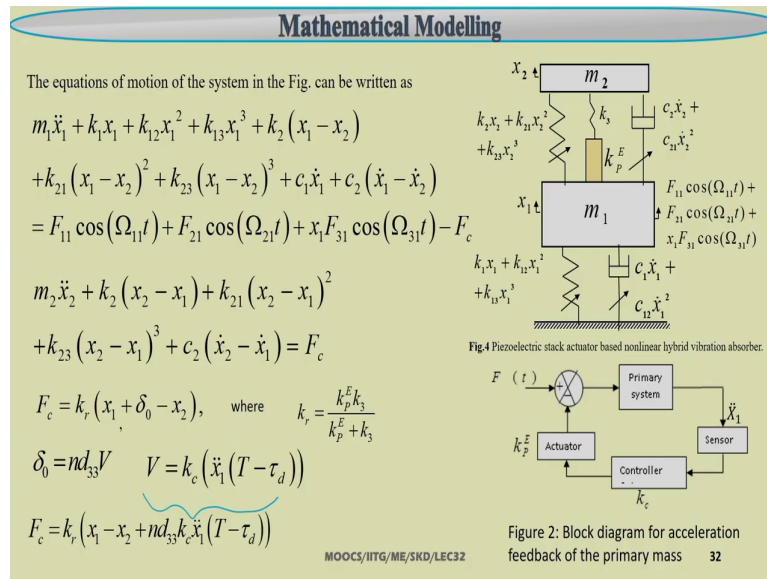
bought in this present case, now we will take a hard harmonic excitation so; that means, this excitation term will be of the same order as that of the linear part of the equation of motion.

And also we will consider this parametric excitation. So, excitation term will take in such a way that, the time varying forcing term will be the coefficient of the response term that is x . So, the we have shown the same system. So, here this m is the primary system. So, you can take any vibrating system actually and you can represent this using a spring and damper system.

So, here the damper and the spring may or may not be linear. So, we may consider a non-linear spring and non-linear damping also. So, this non-linear spring and damping. So, it is up to you. So, up to what order you are taking. So, in this case the damping is shown to be quadratic and the spring stiffness is shown to be cubic order.

So, by taking this quadratic damping in both the primary system and the secondary system and this stiffness parameter or the forcing due to the spring stiffness up to cubic order, then we can write down this equation of motion.

(Refer Slide Time: 28:59)



So, after writing this equation of motion here we can apply control force similar to the previous case here the control force is taken equal to. So, the control force F_c will be equal to taking these acceleration feedback it will be k_c will be equal to k_c into \ddot{x}_1 the equation of motion can be written in this way.

So, here δ_0 that is the displacement of the piezoelectric stack actuator, δ_0 equal to $n d_{33} V$ like previous case here also we have taken the same way, but this control force here it is written equal to k_r into x_1 plus δ_0 minus x_2 . So, δ_0 is the displacement of this stack actuator x_1 is the displacement of the primary system.

If we are not considering the stack actuator, then it would have been different. So, now, by considering this stack actuator. So, we have to add this δ_0 . So, total displacement will be x_1 plus δ_0 minus x_2 . So, this F_c is written equal to k_r into x_1 plus δ_0 minus x_2 .

So, where k_r equal to t combined series stiffness of k_{PE} and k_3 . We can take this control law in this way. So, now, we are taking a delay time delay feedback. So, taking τ_d as the time delay. So, we can write V equal to k_c into x_1 double dot T minus τ_d .

So, here you just see here we have taken this time delay as τ_d . So, in this acceleration feedback. So, we are assuming there is a delay of τ_d . So, when you are applying the respon[se] when we are applying or sensing the response and there will be a time lag when it will actually actuate. So, for the actuation.

So, we can take V equal to k_c into x_1 double dot into T minus τ_d by taking this way. So, we can write down this equation of motion in its non-dimensional form, this control force now can be written F_c equal to k_r into x_1 minus x_2 plus n_d 3 three k_c into x_1 double dot into T minus τ_d .

(Refer Slide Time: 31:29)

Contd...

Assuming $\omega_1^2 = \frac{k_1 + k_2 + k_r}{m_1}$ and non-dimensional time $\tau = \omega_1 t$, the Eq can be modified as

$$\begin{aligned} \frac{d^2 x_1}{d\tau^2} + \omega_1^2 x_1 + \left(2\xi_1 + \frac{\omega_r}{\mu} 2\xi_2 \right) \frac{dx_1}{d\tau} &= \left(\frac{\omega_r^2}{\mu} \right) x_2 + 2\xi_2 \frac{\omega_r}{\mu} \frac{dx_2}{d\tau} - \alpha_{12} x_1^2 - \alpha_{13} x_1^3 - \alpha_{21} (x_1 - x_2)^2 \\ &- \alpha_{23} (x_1 - x_2)^3 + F_1 \cos(\Omega_1 \tau) + F_2 \cos(\Omega_2 \tau) + x_1 F_3 \cos(\Omega_3 \tau) - F_{c1} \left(\frac{d^2 x_1 (\tau - \tau_d)}{d\tau^2} \right) \\ \frac{d^2 x_2}{d\tau^2} - \omega_r^2 x_1 + \omega_r^2 x_2 + \omega_r^2 \frac{k_{21}}{m_2 \omega_2^2} (x_2 - x_1)^2 &+ \omega_r^2 \frac{k_{23}}{m_2 \omega_2^2} (x_2 - x_1)^3 + \omega_r 2\xi_2 \left(\frac{dx_2}{d\tau} - \frac{dx_1}{d\tau} \right) \\ &= \mu F_{c1} \left(\frac{d^2 x_1 (\tau - \tau_d)}{d\tau^2} \right) \end{aligned}$$

MOOCS/IITG/ME/SKD/LEC32

33

The governing equation of motion can be written in this form. So, that is taking these τ_1 equal to τ equal to $\omega_1 t$ into $\omega_1 t$ into $\omega_1 t$ we have taken in this case.

So, as we have taken this is equal to $\omega_1 t$. So, you can write down this equation in this form $\frac{d^2 x_1}{d\tau^2} + \omega_n^2 x_1 + 2\zeta_1 \omega_n \frac{dx_1}{d\tau} + \mu^2 \zeta_2 \frac{dx_1}{d\tau} = \omega_r^2 \mu^2 x_2 + 2\zeta_2 \omega_r \frac{dx_2}{d\tau} - \alpha_2 x_1^2$.

So, this is the quadratic nonlinearity we have taken and α_3 cubic non-linearity it is taken and then this is due to the difference in that $\alpha_2 x_1^2$ minus x_2^2 whole square similarly cubic order non-linearity $\alpha_3 x_1^3$ minus x_2^3 whole cube plus this forcing we have taken three different type of forcing we have taken.

So, 2 forcing that is $F_1 \cos \omega_1 \tau + F_2 \cos \omega_2 \tau$. So, plus so this is the parametric forcing term this is $F_3 \cos \omega_3 \tau$ which is coefficient of x_1 . So, minus $F_c \frac{d^2 x_1}{d\tau^2} - \tau \frac{dx_1}{d\tau}$. So, here you just see this forcing are not taken of the order of epsilon. So, in that case we can tell that is to be hard excitation.

(Refer Slide Time: 33:19)

Contd...

where

$$\omega_{n1} = 1, 2\xi_1 = \frac{c_1}{m_1\omega_1}, \omega_r = \left(\frac{\omega_2}{\omega_1}\right), \omega_2 = \sqrt{\frac{k_2 + k_r}{m_2}}, \omega_1 = \sqrt{\frac{k_1 + k_2 + k_r}{m_1}}, \mu = \frac{m_1}{m_2}, 2\xi_2 = \frac{c_2}{m_2\omega_2}, \alpha_{12} = \frac{k_{12}}{m_1\omega_1^2},$$

$$\alpha_{13} = \frac{k_{13}}{m_1\omega_1^2}, \alpha_{21} = \frac{k_{21}}{m_2\omega_2^2}, \alpha_{23} = \frac{k_{23}}{m_2\omega_2^2}, F_1 = \frac{\tilde{F}_1}{\omega_1^2 x_0}, F_2 = \frac{\tilde{F}_2}{\omega_1^2 x_0}, F_3 = \frac{\tilde{F}_3}{\omega_1^2 x_0}, F_{cl} = \frac{k_r k_c n d_{33}}{m_1},$$

$$\Omega_1 = \frac{\Omega_{11}}{\omega_1}, \Omega_2 = \frac{\Omega_{21}}{\omega_1}, \Omega_3 = \frac{\Omega_{31}}{\omega_1}$$

A small book keeping parameter \mathcal{E} is considered for ordering Eq. as follows

Variable	ξ_1	ξ_2	$\frac{1}{\mu}$	α_{12}	α_{13}	α_{21}	α_{23}	F_{cl}	F_1	F_2	F_3
Scaling	\mathcal{E}	\mathcal{E}	\mathcal{E}	\mathcal{E}^2	\mathcal{E}^2	\mathcal{E}^2	\mathcal{E}^2	\mathcal{E}^2	\mathcal{E}	\mathcal{E}	\mathcal{E}

MOOCs/IITG/ME/SKD/LEC32 34

Similarly, we can write down this equation for x_2 , after writing this equation for x_1 and x_2 we can non dimensionalize that thing.

So, by first by dividing this m_1 and then writing you just see this ω_r is taken to be ω_2 by ω_1 and ω_2 equal to k_2 plus k_r by m_2 root over and ω_1 equal to root over k_1 plus k_2 plus k_r by m_1 and this μ equal to m_1 by m_2 $2\xi_2$ equal to c_2 by $m_2\omega_2$ α_{12} equal to k_{12} by $m_1\omega_1^2$.

Similarly, α_{13} equal to k_{13} by $m_1\omega_1^2$, α_{21} equal to k_{21} by $m_2\omega_2^2$, α_{23} equal to k_{23} by $m_2\omega_2^2$ and this forcing term are non dimensionalized as F_1 by $\omega_1^2 x_0$, F_2 by $\omega_1^2 x_0$, and F_3 by $\omega_1^2 x_0$.

Similarly, F_c can be written as $k_c k_r k_c n d$ 33 by m 1 where ω_1 non dimensional first frequency equal to ω_1 by ω_1 ω_2 equal to ω_{21} by ω_1 and ω_3 equal to ω_{31} by ω_1 . So, you can note that the equations what we have written is not unique.

So, you can write the equation in a different form also you can write the linear part using this matrix form and you can then apply these modal analysis method to reduced this mass matrix and stiffness matrix to the uncoupled form and then one can write down these equation separately.

So, where the linear part will be uncoupled and the non-linear part and some forcing part will be coupled. So, that will give or that will yield another set of equations, you may also use this weighted model matrix. So, instead of this model matrix p one can use this weighted model matrix.

So, in that case the resulting mass matrix will have the unit vector. So, I will be equal to. So, I will be equal to identity matrix and this coefficient of this x that is the displacement will contain the eigenvalues of the system. So, it may be ω_1^2 0 0 ω_2^2 .

So, that way also one can write another set of equations the writing of this equation is not unique. So, you should understand that the writing of these equations are not unique, but you can reconvert these equations to its original form after finding the response of the system. One can use this modal analysis method to rewrite these equations also actually depending on the applications you can choose or you can check what will be the order of this damping.

So, for example, in this particular case this order of damping are taken to be epsilon order. So, then this α_{12} , α_{13} , α_{21} , α_{23} F_c 1. So, these are taken to be order of epsilon square and F_1 , F_2 order of epsilon and F_1 , F_2 , F_3 are taken to be order of epsilon.

(Refer Slide Time: 37:00)

The final equation of motion after ordering can be written as

$$\begin{aligned} \frac{d^2 x_1}{d\tau^2} + \omega_{n1}^2 x_1 + \left(2\varepsilon \xi_1 + \varepsilon^3 \frac{\omega_r}{\mu} 2\xi_2 \right) \frac{dx_1}{d\tau} &= \varepsilon^2 \left(\frac{\omega_r^2}{\mu} \right) x_2 + \varepsilon^3 2\xi_2 \frac{\omega_r}{\mu} \frac{dx_2}{d\tau} - \varepsilon^2 \tilde{\alpha}_{12} x_1^2 - \varepsilon^2 \tilde{\alpha}_{13} x_1^3 - \varepsilon^2 \tilde{\alpha}_{21} (x_1 - x_2)^2 \\ &- \varepsilon^2 \tilde{\alpha}_{23} (x_1 - x_2)^3 + \varepsilon F_1 \cos(\Omega_1 \tau) + \varepsilon F_2 \cos(\Omega_2 \tau) + \varepsilon x_1 F_3 \cos(\Omega_3 \tau) - \varepsilon^2 F_{cl} \left(\frac{d^2 x_1 (\tau - \tau_d)}{d\tau^2} \right) \\ \frac{d^2 x_2}{d\tau^2} - \omega_r^2 x_1 + \omega_r^2 x_2 + \varepsilon^2 \omega_r^2 \alpha_{21} (x_2 - x_1)^2 + \varepsilon^2 \omega_r^2 \alpha_{23} (x_2 - x_1)^3 + \varepsilon 2\xi_2 \omega_r \left(\frac{dx_2}{d\tau} - \frac{dx_1}{d\tau} \right) \\ &= \mu F_{cl} \left(\frac{d^2 x_1 (\tau - \tau_d)}{d\tau^2} \right) \end{aligned}$$

Approximate solution by MMS

$$\begin{aligned} x_1 &= x_{10}(\tau_0, \tau_1) + \varepsilon x_{11}(\tau_0, \tau_1) + \varepsilon^2 x_{12}(\tau_0, \tau_1) \\ x_1(\tau - \tau_d) &= x_{10}(\tau_0 - \tau_d, \tau_1 - \varepsilon \tau_d, \dots) + \varepsilon x_{11}(\tau_0 - \tau_d, \tau_1 - \varepsilon \tau_d, \dots) + \varepsilon^2 x_{12}(\tau_0 - \tau_d, \tau_1 - \varepsilon \tau_d, \dots) \\ x_2 &= x_{20}(\tau_0, \tau_1) + \varepsilon x_{21}(\tau_0, \tau_1) + \varepsilon^2 x_{22}(\tau_0, \tau_1) \end{aligned}$$

MOOCs/IITG/ME/SKD/LEC32 35

In this way. So, you can non dimensionalize and write down the equation of motion by ordering it with the use of different bookkeeping parameter. So, epsilon. So, it may be of the order of epsilon, it may be some terms may be of the order of epsilon square and some of them may be of the order of epsilon to the power 0, epsilon to the power 1 and epsilon to the power 2.

So, you can you may go higher order also, but if you go for higher order then the number of equations will go on increasing without much increase in the precision value of the response. So, up to second order you may go. So, if you are going for higher order.

So, you may go for the symbolic software tools or use the symbolic software tools to write down the equation of motion or find the equation of motion. We can get a final set of

equation of motion non dimensional set of equation of motion. So, here it is written as $d^2 x_1 / d\tau^2 + \omega_n^2 x_1$.

So, here you just see ω_n is not taken to be 1. So, 1 may take it take that thing equal to 1. So, if this ω_n is taken to be equal to ω_n . So, depending on how we are taking this non dimensional parameter, the coefficients of our equation will change.

So, here we have 3 this 3 forcing that is $\epsilon F_1 \cos \omega_1 \tau$, $\epsilon F_2 \cos \omega_2 \tau$, $\epsilon F_3 \cos \omega_3 \tau$ minus $\epsilon^2 F_c d^2 x_1 / d\tau^2$. So, here in the forcing. So, we have use the acceleration feedback. So, time delay acceleration feedback is taken here.

So, similarly the second equation can be written using the time delay acceleration feedback. So, $d^2 x_2 / d\tau^2$ is a function of τ and $\tau - \tau_d$ is the time delay function. So, then one can solve this equation by using this method of multiple scale. So, it can be taken x_1 equal to $x_{10} + \epsilon x_{11} + \epsilon^2 x_{12}$.

And you just see how the delay term is written. So, delay term is written simply by using the same expression of this one. So, where this τ is replaced by $\tau_0 - \tau_d$, and this τ_1 is replaced by $\tau_1 - \epsilon \tau_d$, τ_0 , τ_1 , τ_2 . So, they are of the different time scale.

So, here for the delay. So, this time scales also has to be modified. So, for example, this τ_0 will be $\tau_0 - \tau_d$, τ_1 which is $\epsilon \tau_0$ it can be written $\tau_1 - \epsilon \tau_d$, similarly the higher order terms also can also be written this $x_1 \tau - \tau_d$.

So, the d is a delay by using the delay term one can write the displacement x_1 in this form. Similarly, displacement x_2 can be written $x_{20} \tau_0 + \epsilon x_{21} \tau_0 + \epsilon^2 x_{22} \tau_0$ and the corresponding time delay term also can be written.

(Refer Slide Time: 40:41)

Contd...

The secular term obtained for the primary, principal parametric and 1:1 internal resonance condition

$$\Omega_1 = \omega_{n1} + \varepsilon\sigma_1, \Omega_2 = \omega_{n1} + \varepsilon\sigma_2, \Omega_3 = 2\omega_{n1} + \varepsilon\sigma_3$$

where σ_1, σ_2 are the detuning parameter for the primary resonance condition for excitation force F_1 and F_2 and σ_3 is the detuning parameter for the parametric excitation force F_3

The secular term obtained by considering 1:1 internal resonance condition i.e. $\omega_r = \omega_{n1} + \varepsilon\sigma$ where σ is the detuning parameter for the internal resonance.

$$2i\omega_{n1}D_1A_1 = -2i\omega_{n1}\xi_1A_1 + \frac{F_1 \exp(i\sigma_1\tau_1)}{2} + \frac{F_2 \exp(i\sigma_2\tau_1)}{2} + \frac{\bar{A}_1 F_3 \exp(i\sigma_3\tau_1)}{2}$$

$$2i\omega_r D_1 B_1 = 2i\omega_r (\omega_r - \varepsilon\sigma) \xi_2 A_1 \exp(-i\sigma\tau_1) - 2iB_1 \omega_r^2 \xi_2 +$$

$$\mu F_{c1} \exp\left(i(-\omega_r \tau_d - \varepsilon\sigma\tau_0 + \varepsilon\sigma\tau_d)\right) \left(\frac{-2i\omega_{n1}\xi_1 A_{1d} + \frac{F_1 \exp(i\sigma_1(\tau_1 - \varepsilon\tau_d))}{2}}{2} + \frac{F_2 \exp(i\sigma_2(\tau_1 - \varepsilon\tau_d))}{2} + \frac{\bar{A}_1 F_3 \exp(i\sigma_3(\tau_1 - \varepsilon\tau_d))}{2} \right)$$

MOOCs/IITG/ME/SKD/LEC32 36

So, by taking these primary principal parametric and 1 is to 1 internal resonance condition. So, here if we are taking omega 2 equal to omega 1 that is the frequency of the first mode equal to the frequency of the second mode.

So, in that case we can write this equation in this form by using this detuning parameters, we can write omega 1 equal to omega n 1 plus epsilon sigma 1 omega 2 equal to omega n 2 plus epsilon sigma 2 and omega 3 equal to 2 omega n 1 plus epsilon sigma three.

Now, following the previous procedures for internal resonance condition. So, you can take this omega r equal to omega n 1 plus epsilon sigma, now we can write the reduced or we can write the terms which give rise to secular term. So, this is the term which gives rise to secular term. So, this term has to be eliminated to get the response of the system.

(Refer Slide Time: 41:46)

Now, Equations (16) and (24) can be combined to describe the modulation of the complex amplitude to the second nonlinear order with respect to the original time scale τ using

$$\frac{dA_1}{d\tau} = \varepsilon D_1 A_1 + \varepsilon^2 D_2 A_1$$

$$\frac{dB_1}{d\tau} = \varepsilon D_1 B_1 + \varepsilon^2 D_2 B_1$$

Finally the four equations are given for the steady state equations can be written as

The autonomous solution of the steady state equations from Eq. (31) to (34) can be written as by assuming

$$\gamma_1 = \sigma_1 \tau_1 - \beta_1, \gamma_2 = \beta_2 + (\sigma - \sigma_1) \tau_1,$$

$$1.01 \gamma_1 = \sigma_2 \tau_1 - \beta_1, 2\gamma_1 = \sigma_3 \tau_1 - 2\beta_1,$$

$$\gamma_2 + \gamma_1 = \beta_2 - \beta_1 + \sigma \tau_1, \gamma_1 - \gamma_2 = -\beta_2 - \beta_1 + (2\sigma_1 - \sigma) \tau_1 = -\beta_2 - \beta_1 + (\sigma_3 - \sigma) \tau_1$$

$$(\beta_1 (\tau_1 - \varepsilon \tau_d) - \omega_{n1} \tau_d - \beta_1) = (-\omega_{n1} + \sigma_1) \tau_d$$

MOOCs/IITG/ME/SKD/LEC32 37

So, here we can take this $\frac{dA_1}{d\tau}$ equal to $\varepsilon D_1 A_1 + \varepsilon^2 D_2 A_1$. So, similarly $\frac{dB_1}{d\tau}$ equal to $\varepsilon D_1 B_1 + \varepsilon^2 D_2 B_1$ finally, we can get a set of reduced equations and those reduced equation can be solved to find the response one can use non autonomous form.

So, previously we have taken in terms of. So, you just see a equal to half a capital A equal to half a ε to the power i beta generally we take and then we can convert that beta to gamma form or to remove the time terms in this equation of motion to make the equation autonomous.

So, we can use that one. So, we have to use this γ_1 equal to $\sigma_1 T_1$ minus β_1 γ_2 equal to β_2 plus σ minus σ_1 into τ_1 by properly writing these terms

we can make the equations autonomous here this time terms will be eliminated in this study state.

(Refer Slide Time: 42:57)

In this section **MMS version II** is proceeded, i.e. the terms $D_1 A_1$, $D_1^2 A_1$, $D_1 B_1$, $D_1^2 B_1$ present in the equation with order of ε^2 are made zero, whereas in MMS version I, these terms are kept which is derived in the above equations. Hence, MMS version I contains some additional terms. So the secular term and the solution obtained in the Eq. (24) and (25) are modified as

$$\begin{aligned}
 2i\omega_{n1} D_2 A_1 &= F_{c1} \omega_{n1}^2 A_1 \exp(i(-\omega_{n1} \tau_d)) + \frac{\omega_{n1}^2 B_1 \exp(i(\varepsilon \sigma) \tau_0)}{\mu} - 3\alpha_{13} A_1^2 \bar{A}_1 - 3\alpha_{23} A_1^2 \bar{A}_1 \\
 &+ 3\alpha_{23} A_1^2 \bar{B}_1 \exp(-i(\varepsilon \sigma)) + 6\alpha_{23} A_1 \bar{A}_1 B_1 \left(\exp(i(\varepsilon \sigma) \tau_0) \right) - 6\alpha_{23} A_1 B_1 \bar{B}_1 \\
 &- 3\alpha_{23} \bar{A}_1 B_1^2 \exp(i(2\varepsilon \sigma) \tau_0) + 3\alpha_{23} B_1^2 \bar{B}_1 \exp(i(\varepsilon \sigma) \tau_0) - \frac{A_1 F_s^2}{4(8\omega_{n1}^2 + \varepsilon(\varepsilon \sigma_s^2 + 6\omega_{n1} \sigma_s))} \\
 2i\omega_{n1} D_2 B_1 &= 3\alpha_{23} \omega_{n1}^2 A_1^2 \bar{A}_1 \exp(-i\varepsilon \sigma \tau_0) - 3\alpha_{23} \omega_{n1}^2 (A_1^2 \bar{B}_1 \exp(-2i\varepsilon \sigma \tau_0) + A_1 \bar{A}_1 B_1 + A_1 \bar{A}_1 B_1) + \\
 &3\alpha_{23} \omega_{n1}^2 (\bar{A}_1 B_1^2 \exp(i\varepsilon \sigma \tau_0) + A_1 B_1 \bar{B}_1 \exp(-i\varepsilon \sigma \tau_0) + \bar{A}_1 B_1 \bar{B}_1 \exp(-i\varepsilon \sigma \tau_0)) - 3\alpha_{23} \omega_{n1}^2 B_1^2 \bar{B}_1 \\
 &+ \mu F_{c1} \exp(i(-\omega_{n1} \tau_d - \varepsilon \sigma \tau_0 + \varepsilon \sigma \tau_d)) \left(\frac{F_{c1} \omega_{n1}^2 A_1 \exp(-i\omega_{n1} \tau_d) + \frac{\omega_{n1}^2 B_1 \exp(i(\varepsilon \sigma) \tau_0)}{\mu} - 3\alpha_{13} A_1^2 \bar{A}_1}{4(8\omega_{n1}^2 + \varepsilon(\varepsilon \sigma_s^2 + 6\omega_{n1} \sigma_s))} \right. \\
 &\quad \left. - 3\alpha_{23} A_1^2 \bar{A}_1 + 3\alpha_{23} A_1^2 \bar{B}_1 \exp(-i(\varepsilon \sigma)) + 6\alpha_{23} A_1 \bar{A}_1 B_1 \left(\exp(i(\varepsilon \sigma) \tau_0) \right) \right. \\
 &\quad \left. - 6\alpha_{23} A_1 B_1 \bar{B}_1 - 3\alpha_{23} A_1 B_1^2 \exp(i(2\varepsilon \sigma) \tau_0) + 3\alpha_{23} B_1^2 \bar{B}_1 \exp(i(\varepsilon \sigma) \tau_0) \right) \\
 &\quad \left. - \frac{A_1 F_s^2}{4(8\omega_{n1}^2 + \varepsilon(\varepsilon \sigma_s^2 + 6\omega_{n1} \sigma_s))} \right)_{SKD/LEC32}
 \end{aligned}$$

So, one can use this method of multiple scale version 1 or version 2. So, in version 2. So, the normal way when we are deriving that is version 1. So, in case of the version 2 the term $D_1 A_1$, $D_1^2 A_1$, $D_1 B_1$, $D_1^2 B_1$ present in the equation with order ε^2 are made zero, whereas in MMS version 1 these terms are kept which is derived in the above equation.

Hence method of multiple scale version 1 contain some additional term the secular term and the solution obtained using those equations are modified. So, if you want to use method of multiple scale version 2 proposed by Rahman and Burton. So, you can write down these equations in this form.

So, here the term with $D^1 A^1 D^1$ square A^1 and then $D^1 B^1 D^1$ square B^1 are eliminated while studying the terms while studying the term which gives rise to secular terms.

(Refer Slide Time: 44:15)

Results and discussions

In this section a parametric study is undertaken to study the effects of controlling force and stiffness α_{2c} . Non-dimensional parameters involved in the problem are assumed as follows.

- Harmonic excitation force $F_1 = 0.1$
- Parametric excitation force $F_3 = 0.01$
- Mass ratio between primary mass to absorber mass $\mu = 100$
- Damping for the primary mass and the absorber $h_{1c} = 0.002$ and $h_{2c} = 0.0004$
- The stiffness at the juncture of PZT actuator and absorber mass k_c is varied from 0.001 to 0.1
- Quadratic and cubic nonlinearity stiffness is considered to be 3% and 4% of the linear stiffness for the primary system and the absorber respectively
- The controlling force F_{c1} is varied from 0 to 0.002

$\omega = 1$

Newton's method
 → initial condition

MOOCS/IITG/ME/SKD/LEC32

39

So, these are the equation we obtained now solving this equation. So, you just see we have a set of equation. So, we can solve these equations by using numerical method particularly you can use this Newton's method which actually required which actually required the initial condition initial condition to find the response of the system. So, here to find this initial condition you may solve this equation by using ODE 4-5 or by using this Runge Kutta method.

And then taking those response as the initial condition one can make further study to find the response of the system. Otherwise one can use this continuation technique to find the response of that system for a wide range of frequency. We have studied the how you can use

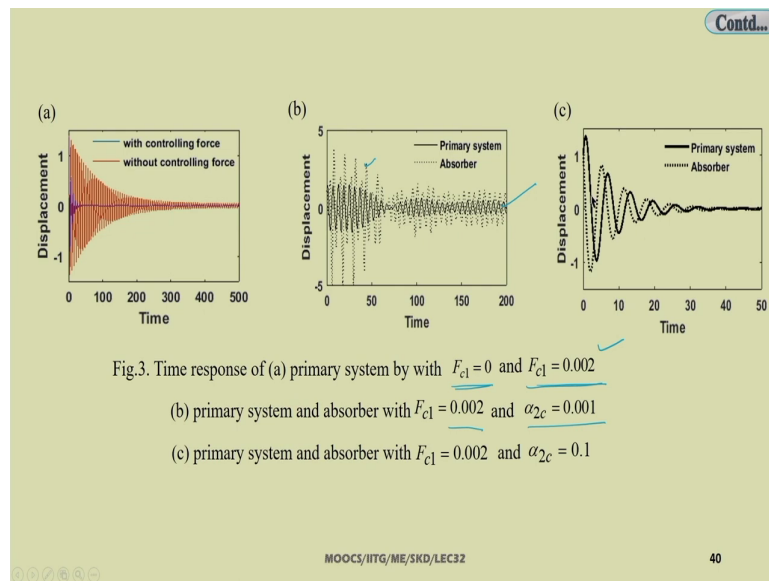
this controlling force and in this numerical analysis we will see by taking a harmonic force of F_1 equal to 0.1 here you just see we have taken this ω equal to 1.

So, that is why that ω is the coefficient of x_1 to be 1. So, that is why to make the forcing term to be non-weak forcing or hard forcing. So, depending on that thing. So, we have to choose this F_1 and F_3 . So, here F_1 is taken to be 0.1 and F_3 equal to 0.01.

So, mass ratio of the primary system to absorber is taken. So, already I told you that it is taken to be 100, previous literature limit their analysis of 2 mass ratio of 20. So, damping factor damping parameter of the mass and absorber is taken to be 0.002 and 0.004. So, the stiffness at the junction of PZT actuator and the absorber a mass k_c is varied from 0.001 to 0.1.

So, quadratic and cubic nonlinearity stiffness are considered to be 3 percent and 4 percent of the linear stiffness of the primary and of the primary system and the absorber respectively. So, controlling force F_{c1} is varied from 0 to 0.002. So, taking this numerical values. So, one can study the response of the system.

(Refer Slide Time: 46:58)



So, here the response of the system. So, initially we have plotted the time response. So, clearly you can see the time response without control and with control. So, with control. So, the line you can see clearly it is it shows that it required very less time to control the response of the system with a control force, but without control force it will take a very large time to obtain the steady state response.

So, in the first figure you have seen this F_{c1} equal to 0, and F_{c2} you can take to be 0.002. Similarly, in the primary system F_{c1} equal to 0 point. So, in the first case. So, in this first figure we have taken F_{c1} equal to 0. So, which is without control force and F_{c1} by applying a very very small F_{c1} that is control force equal to 0.002.

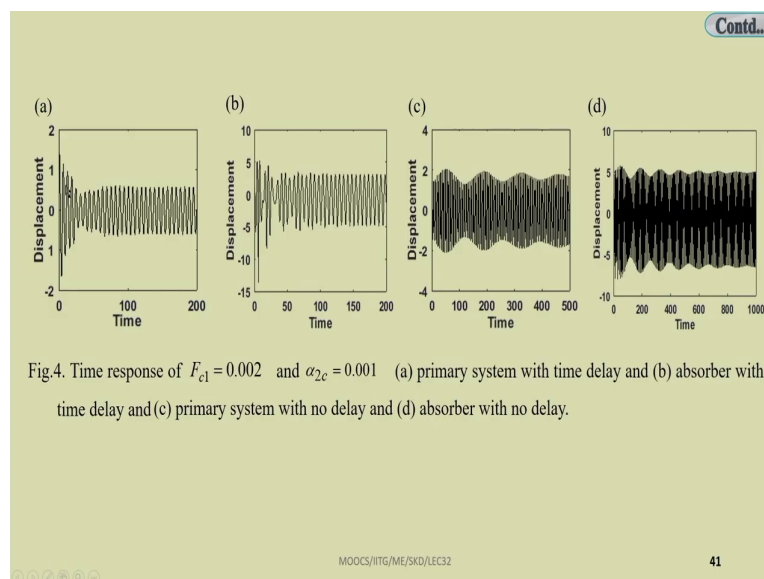
So, we can see the settling time is very very less. So, quickly it reduces to its equilibrium position by applying this control force. So, similarly primary system absorber with F_{c1} equal

to 0.002. So, the first case it is F_{c1} equal to 0.002 and α_{2c} equal to 0.001, here the primary system and absorber are shown this is the primary system response you can absorb that primary system this is the primary system response and this is the absorber response.

So, the primary response is reduced significantly what the absorber response increases in case of the vibration absorber. So, that takes place. So, generally we control the vibration of the primary system and the secondary system oscillate and to control that oscillation of the secondary system.

So, generally. So, damping is used in the system. The system can be made non-linear or the damping can also be used in the third figure it can be clearly shown the displacement and the time. So, primary system. So, this is the primary system and this is the absorber.

(Refer Slide Time: 49:23)

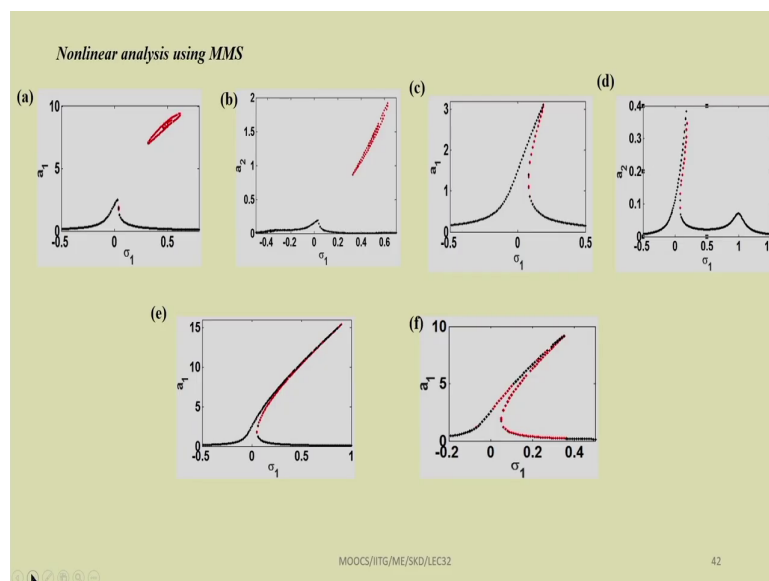


So, now by taking this time delay. So, time response of $F < 1$. So, this is the time response of $F < 1$. So, primary system with time delay. So, this is primary system and absorber with time delay primary system with no time delay. So, here in this case there is no time delay in the primary system. In c primary system with no time delay absorber with no delay.

So, it is without delay and first two figures are with delay you can easily see with delay. So, we can control actually here you can absorb some beating type of phenomena also. In case of the systems without delay and here with delay you can easily see the response amplitude is reduced the response amplitude is reduced.

So, the delay is acting. So, you have observed that the delay is acting as a damper to the system.

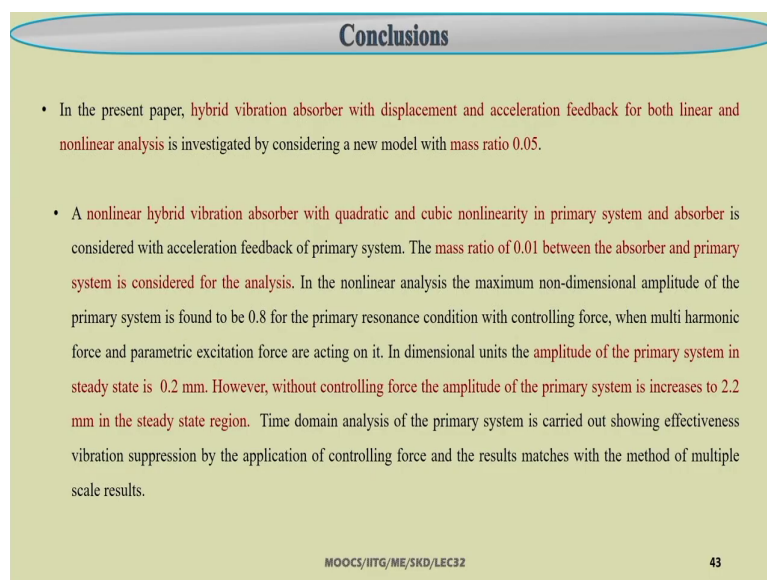
(Refer Slide Time: 50:25)



Using this non-linear analysis, using method of multiple scale. If you find the response it is showing this hardening type of effect with different bifurcations. So, you just see here we have this hopf bifurcation and here you have the saddle node bifurcation.

So, these are the different responses we have observed in this case. So, in the present work. So, we have seen this hybrid vibration absorber with displacement and acceleration feedback for both linear and non-linear analysis.

(Refer Slide Time: 50:43)



Conclusions

- In the present paper, hybrid vibration absorber with displacement and acceleration feedback for both linear and nonlinear analysis is investigated by considering a new model with mass ratio 0.05.
- A nonlinear hybrid vibration absorber with quadratic and cubic nonlinearity in primary system and absorber is considered with acceleration feedback of primary system. The mass ratio of 0.01 between the absorber and primary system is considered for the analysis. In the nonlinear analysis the maximum non-dimensional amplitude of the primary system is found to be 0.8 for the primary resonance condition with controlling force, when multi harmonic force and parametric excitation force are acting on it. In dimensional units the amplitude of the primary system in steady state is 0.2 mm. However, without controlling force the amplitude of the primary system is increases to 2.2 mm in the steady state region. Time domain analysis of the primary system is carried out showing effectiveness vibration suppression by the application of controlling force and the results matches with the method of multiple scale results.

MOOC5/IITG/ME/SKD/LEC32 43

When the mass ratio is taken to be of the order of 0.05 that is the conventional a work that is up to 20, we have gone also here mass ratio of to 100 and we have seen the response of the system for different response of the system for different mass ratio and different thing.

(Refer Slide Time: 51:17)

Mathematical Modelling

The equations of motion of the system in the Fig. can be written as

$$\begin{aligned} m_1 \ddot{x}_1 + k_1 x_1 + k_{12} x_1^2 + k_{13} x_1^3 + k_2 (x_1 - x_2) \\ + k_{21} (x_1 - x_2)^2 + k_{23} (x_1 - x_2)^3 + c_1 \dot{x}_1 + c_2 (\dot{x}_1 - \dot{x}_2) \\ = F_{11} \cos(\Omega_{11} t) + F_{21} \cos(\Omega_{21} t) + x_1 F_{31} \cos(\Omega_{31} t) - F_c \end{aligned}$$

$$\begin{aligned} m_2 \ddot{x}_2 + k_2 (x_2 - x_1) + k_{21} (x_2 - x_1)^2 \\ + k_{23} (x_2 - x_1)^3 + c_2 (\dot{x}_2 - \dot{x}_1) = F_c \end{aligned}$$

$$F_c = k_r (x_1 + \delta_0 - x_2), \quad \text{where} \quad k_r = \frac{k_p^E k_3}{k_p^E + k_3}$$

$$\delta_0 = n d_{33} V \quad V = k_c (\ddot{x}_1 (T - \tau_d))$$

$$F_c = k_r (x_1 - x_2 + n d_{33} k_c \ddot{x}_1 (T - \tau_d))$$

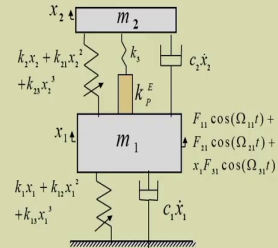
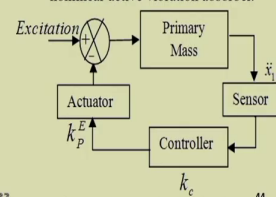


Figure 1 Piezoelectric stack actuator based nonlinear active vibration absorber.



MOOC5/ITG/ME/SKD/LEC32

Fig. Block diagram with acceleration feedback of primary system.

(Refer Slide Time: 51:34)

Assuming $\omega_1^2 = \frac{k_1 + k_2 + k_r}{m_1}$ and non-dimensional time $\tau = \omega_1 t$, the Eq can be modified as

A small book keeping parameter ε is considered for ordering Eq. as follows

Variable	ξ_1, ξ_2, F_3	$\mu^{-1}, \alpha_{12}, \alpha_{13}, \alpha_{21}, \alpha_{23}, F_{c1}$
Scaling	ε	ε^2

The final equation of motion after ordering can be written as

$$\begin{aligned} \frac{d^2 x_1}{d\tau^2} + \omega_{n1}^2 x_1 + \left(2\varepsilon\xi_1 + \varepsilon^3 \frac{\omega_r}{\mu} 2\xi_2 \right) \frac{dx_1}{d\tau} = \varepsilon^2 \left(\frac{\omega_r^2}{\mu} \right) x_2 + \varepsilon^3 2\xi_2 \frac{\omega_r}{\mu} \frac{dx_2}{d\tau} - \varepsilon^2 \alpha_{12} x_1^2 \\ - \varepsilon^2 \alpha_{13} x_1^3 - \varepsilon^2 \alpha_{21} (x_1 - x_2)^2 - \varepsilon^2 \alpha_{23} (x_1 - x_2)^3 + F_1 \cos(\Omega_1 \tau) + \\ F_2 \cos(\Omega_2 \tau) + \varepsilon x_1 F_3 \cos(\Omega_3 \tau) - \varepsilon^2 F_{c1} \left(\frac{d^2 x_1 (\tau - \tau_d)}{d\tau^2} \right) \\ \frac{d^2 x_2}{d\tau^2} - \omega_r^2 x_1 + \omega_r^2 x_2 + \varepsilon^2 \mu \alpha_{21} (x_2 - x_1)^2 + \varepsilon^2 \mu \alpha_{23} (x_2 - x_1)^3 + \varepsilon 2\xi_2 \omega_r \left(\frac{dx_2}{d\tau} - \frac{dx_1}{d\tau} \right) \\ = \mu F_{c1} \left(\frac{d^2 x_1 (\tau - \tau_d)}{d\tau^2} \right) \end{aligned}$$

MOOCS/IITG/ME/SKD/LEC32 45

(Refer Slide Time: 51:39)

Contd...

where

$$\omega_{n1} = 1, 2\xi_1 = \frac{c_1}{m_1\omega_1}, \omega_r = \left(\frac{\omega_2}{\omega_1}\right), \omega_2 = \sqrt{\frac{k_2 + k_r}{m_2}}, \mu = \frac{m_1}{m_2}, \omega_1 = \sqrt{\frac{k_1 + k_2 + k_r}{m_1}},$$

$$2\xi_2 = \frac{c_2}{m_2\omega_2}, \alpha_{12} = \frac{k_{12}}{m_1\omega_1^2}, \alpha_{13} = \frac{k_{13}}{m_1\omega_1^2}, \alpha_{21} = \frac{k_{21}}{m_1\omega_1^2}, \alpha_{23} = \frac{k_{23}}{m_1\omega_1^2}, F_1 = \frac{F_1}{m_1\omega_1^2},$$

$$F_2 = \frac{F_2}{m_1\omega_1^2}, F_3 = \frac{F_3}{m_1\omega_1^2}, F_{c1} = \frac{k_r k_c n d_{33}}{m_1}, \Omega_1 = \frac{\Omega_{11}}{\omega_1}, \Omega_2 = \frac{\Omega_{21}}{\omega_1}, \Omega_3 = \frac{\Omega_{31}}{\omega_1}$$

Approximate solution by MMS

$$x_1 = x_{10}(\tau_0, \tau_1) + \varepsilon x_{11}(\tau_0, \tau_1) + \varepsilon^2 x_{12}(\tau_0, \tau_1)$$

$$x_1(\tau - \tau_d) = x_{10}(\tau_0 - \tau_d, \tau_1 - \varepsilon \tau_d, \dots) + \varepsilon x_{11}(\tau_0 - \tau_d, \tau_1 - \varepsilon \tau_d, \dots) + \varepsilon^2 x_{12}(\tau_0 - \tau_d, \tau_1 - \varepsilon \tau_d, \dots)$$

$$x_2 = x_{20}(\tau_0, \tau_1) + \varepsilon x_{21}(\tau_0, \tau_1) + \varepsilon^2 x_{22}(\tau_0, \tau_1)$$

MOOCS/IITG/ME/SKD/LEC32

46

(Refer Slide Time: 51:43)

Contd...

The secular term obtained for the super harmonic, subharmonic, principal parametric and 1:1 internal resonance conditions

$$\Omega_1 = \frac{\omega_{n1}}{3} + \varepsilon\sigma_1, \quad \Omega_2 = 3\omega_{n1} + \varepsilon\sigma_2, \quad \Omega_3 = 2\omega_{n1} + \varepsilon\sigma_3$$

where σ_1, σ_2 are the detuning parameter for the primary resonance condition for excitation force F_1 and F_2 and σ_3 is the detuning parameter for the parametric excitation force F_3

The secular term obtained by considering 1:1 internal resonance condition i.e. $\omega_r = \omega_{n1} + \varepsilon\sigma$ where σ is the detuning parameter for the internal resonance.

$$2i\omega_{n1}D_1A_1 = -2i\omega_{n1}\xi_1A_1 + \frac{\bar{A}_1F_3 \exp(i\sigma_3\tau_1)}{2}$$

$$2i\omega_r D_1B_1 = 2i\omega_r (\omega_r + \varepsilon\sigma)\xi_2A_1 \exp(i\sigma\tau_1) - 2iB_1\omega_r^2\xi_2 + \left(-2i\xi_1A_1 + \frac{\bar{A}_1F_3 \exp(i\sigma_3\tau_1)}{2\omega_{n1}} \right) \mu F_{c1}\omega_{n1} \exp(i(\sigma\tau_1 - \omega_{n1}\tau_d))$$

MOOCS/IITG/ME/SKD/LEC32

47

So, if we can consider the. So, now, also we can consider other different system that is sub harmonic and super harmonic resonance condition. So, in sub harmonic and super harmonic resonance conditions. So, we can see the response one can follow the similar procedure to find the solution.

So, in case of super harmonic resonance condition. So, you just see the forcing term the frequency can be taken in a different way. So, this omega 1 can be taken as omega n 1 by 3 plus epsilon sigma 1 and omega 2 is taken to be 3 times omega n 1 in case of super harmonic.

So, this is the condition in the super harmonic that is omega 1 equal to omega n 1 by 3 and sub harmonic this is the condition for sub harmonic that is omega 2 equal to 3 into omega n 1

plus epsilon sigma 2 and it is parametric also principal parametric omega 3 equal to 3 2 omega 1 plus epsilon sigma 3 internal resonance conditions also one can consider.

So, in that case omega r is considered to be omega n 1 plus epsilon sigma 1 and proceeding in the similar way.

(Refer Slide Time: 52:42)

Now, Equations can be combined to describe the modulation of the complex amplitude to the second nonlinear order with respect to the original time scale τ using

$$\frac{dA_1}{d\tau} = \varepsilon D_1 A_1 + \varepsilon^2 D_2 A_1 \Rightarrow 2i\omega_{n1} \frac{dA_1}{dt} = \varepsilon 2i\omega_{n1} D_1 A_1 + \varepsilon^2 2i\omega_{n1} D_2 A_1$$

$$\frac{dB_1}{dt} = \varepsilon D_1 B_1 + \varepsilon^2 D_2 B_1 \Rightarrow 2i\omega_r \frac{dB_1}{dt} = \varepsilon 2i\omega_r D_1 B_1 + \varepsilon^2 2i\omega_r D_2 B_1$$

Assuming polar form $A_1 = \frac{1}{2} a_1 e^{i\beta_1}$ and $B_1 = \frac{1}{2} a_2 e^{i\beta_2}$ the autonomous solution of the steady state equations can be written as by assuming

where

$$\gamma_1 = \sigma_1 \tau_1 - \beta_1, \quad \gamma_2 = (\sigma_1 + \sigma) \tau_1 - \beta_2 \quad \text{and} \quad \sigma_1 = \sigma_2 / 3 = \sigma_3 / 2$$

separating real and imaginary part from the Eq. the final steady state autonomous equation is expressed as

MOOCs/IITG/ME/SKD/LEC32 48

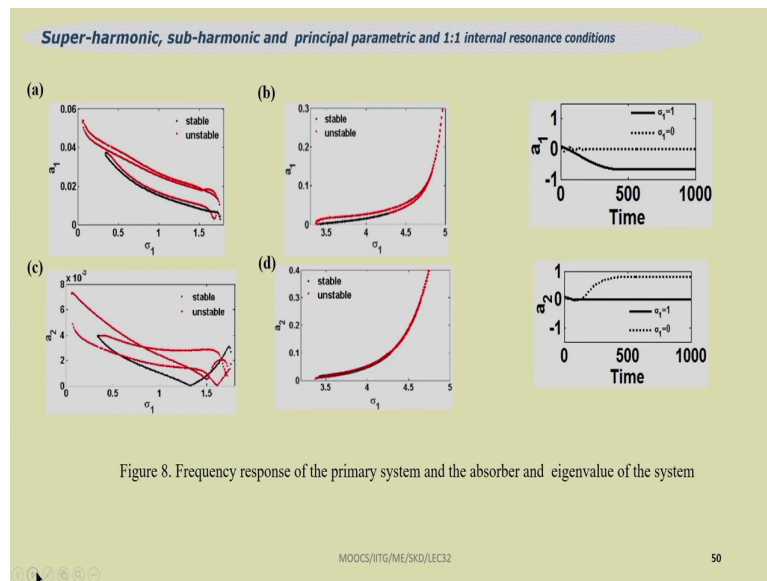
(Refer Slide Time: 52:45)

Results and discussions

In this section a parametric study is undertaken by obtaining the solution of steady state equation of **MMS-II** by **Newton's method** to study the effects of frequency response and time response of the system.

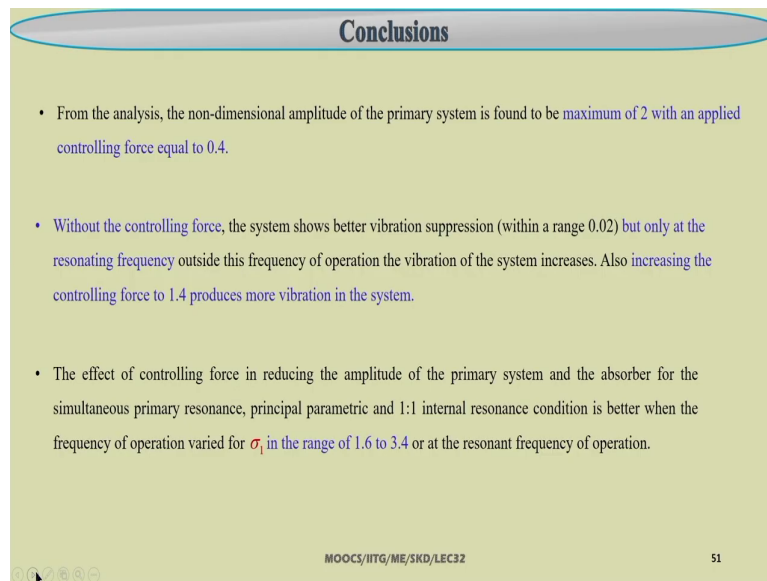
- Harmonic excitation force $F_1 = 0.9$, $F_2 = 0.9$
- Parametric excitation force $F_3 = 0.5$
- Mass ratio between primary mass to absorber mass $\mu = 100$
- Damping ratio for the primary mass and the absorber $\xi_1 = 0.04$ and $\xi_2 = 0.08$
- Quadratic and cubic nonlinearity stiffness is considered to be 0.5% and 0.8 % of the linear stiffness for the primary system and the absorber respectively
- The controlling force F_{c1} is varied from 0 to 0.8
- ordering parameter $\varepsilon = 0.1$
- The detuning parameters σ , σ_2 and σ_3 values are considered to be 1, 1 and 2 respectively with a time delay of 0.1.

(Refer Slide Time: 52:48)



So, we can get the reduced equation and after solving this reduced equation we can study the response amplitude. So, here you can see several response amplitudes are been plotted.

(Refer Slide Time: 52:54)



Conclusions

- From the analysis, the non-dimensional amplitude of the primary system is found to be maximum of 2 with an applied controlling force equal to 0.4.
- Without the controlling force, the system shows better vibration suppression (within a range 0.02) but only at the resonating frequency outside this frequency of operation the vibration of the system increases. Also increasing the controlling force to 1.4 produces more vibration in the system.
- The effect of controlling force in reducing the amplitude of the primary system and the absorber for the simultaneous primary resonance, principal parametric and 1:1 internal resonance condition is better when the frequency of operation varied for σ_1 in the range of 1.6 to 3.4 or at the resonant frequency of operation.

MOOC5/IITG/ME/SKD/LEC32 51

So, you can observe that. So, the non-dimensional amplitude of the primary system is found to be maximum of 2 with an applied controlling force equal to 0.4.

So, without the controlling force the system shows better vibration suppression with a range of 0.02, but only at the resonating frequency outside the frequency of operation the vibration of the system increases also increasing the controlling force to 1.4 produces more vibration in the system.

So, the effect of controlling force in reducing the amplitude of the primary system and the absorber for the simultaneous primary resonance principal parametric and 1 is to 1 internal resonance condition is better when the frequency of operation is worried for σ_1 in the

range of 1.6 to 3.4 or at resonant frequency of operation. By controlling this detuning parameter.

So, in this case we have seen. So, we have applied three condition simultaneously. So, we have the super harmonic, sub harmonic as we have taken 3 frequency terms. So, we have we can apply sub harmonic, super harmonic and principal parametric resonance condition simultaneously.

(Refer Slide Time: 54:21)

Contd...

Mathematical Modelling

Assuming $\omega_1^2 = \frac{k_1 + k_2 + k_r}{m_1}$ and non-dimensional time $\tau = \omega_1 t$, the Eq can be modified as

$$\frac{d^2 x_1}{d\tau^2} + \omega_1^2 x_1 = \varepsilon \mu \omega_1^2 x_2 - \varepsilon \alpha_{13} x_1^3 - \varepsilon^2 \alpha_{23c} (x_1 - x_2)^3 - \varepsilon^2 (z_1 + z_{12}) \frac{dx_1}{d\tau} + \varepsilon^2 z_{12} \frac{dx_2}{d\tau} + F_1 \cos(\Omega_1 \tau) + F_2 \cos(\Omega_2 \tau) + \varepsilon x_1 F_3 \cos(\Omega_3 \tau) - \varepsilon^2 F_{c1} \left(\frac{d^2 x_1 (\tau - \tau_d)}{d\tau^2} \right)$$

$$\frac{d^2 x_2}{d\tau^2} + \omega_2^2 x_2 = \varepsilon \frac{F_{c1}}{\mu} \left(\frac{d^2 x_1 (\tau - \tau_d)}{d\tau^2} \right) + \omega_2^2 x_1 - \varepsilon \alpha_{23} (x_2 - x_1)^3 - \varepsilon z_2 \left(\frac{dx_2}{d\tau} - \frac{dx_1}{d\tau} \right)$$

where

$$\omega_1 = \sqrt{\frac{k_1 + k_2 + k_r}{m_1 \omega_{n1}^4}}, \mu = \frac{m_2}{\varepsilon m_1 \omega_{n1}^2}, \omega_2 = \sqrt{\frac{k_2 + k_r}{m_2 \omega_{n1}^4}}, \alpha_{13} = \frac{k_{13}}{\varepsilon m_1 \omega_{n1}^2}, \alpha_{23c} = \frac{k_{23}}{\varepsilon^2 m_1 \omega_{n1}^2}$$

$$\alpha_{23} = \frac{k_{23}}{\varepsilon m_2 \omega_{n1}^2}, z_1 = \frac{c_1}{\varepsilon^2 m_1 \omega_{n1}}, z_{12} = \frac{c_2}{\varepsilon^2 m_1 \omega_{n1}}, z_2 = \frac{c_2}{\varepsilon m_2 \omega_{n1}}, F = \frac{F_{111}}{m_1 \omega_{n1}^2}$$

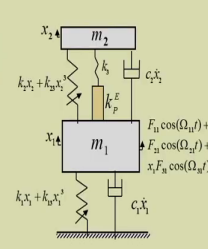
$$F_2 = \frac{F_{21}}{m_1 \omega_{n1}^2}, F_{31} = \frac{F_3}{\varepsilon m_1 \omega_{n1}^2}, F_{c1} = \frac{k_r k_{c1} n d_{33}}{\varepsilon^2 m_1 \omega_{n1}^2}, \Omega_1 = \frac{\Omega_{11}}{\omega_{n1}}, \Omega_2 = \frac{\Omega_{21}}{\omega_{n1}}, \Omega_3 = \frac{\Omega_{31}}{\omega_{n1}}$$


Figure 1. Piezoelectric stack actuator based nonlinear active vibration absorber.

52

(Refer Slide Time: 54:27)

Resonance cases

- i-primary resonance ($\Omega_j \cong \omega_1$)
- ii-sub-harmonic resonance $\Omega_2 \cong 3\omega_1$
- iii-superharmonic resonance $3\Omega_1 \cong \omega_1$
- iv- principal parametric resonance $\Omega_3 \cong 2\omega_1$
- v- internal resonance $\omega_2 \cong m\omega_1$, $\omega_1 \cong m\omega_2$ for $m = 1, 3$
- vi- Simultaneous resonance

The secular term obtained for the superharmonic, subharmonic, principal parametric and 3:1 internal resonance conditions

$$3\Omega_1 = \omega_1 + \varepsilon\sigma_1, \Omega_2 = 3\omega_1 + \varepsilon\sigma_2, \Omega_3 = 2\omega_1 + \varepsilon\sigma_3$$

where σ_1, σ_2 are the detuning parameter for the primary resonance condition for excitation force F_1 and F_2 and σ_3 is the detuning parameter for the parametric excitation force F_3

The secular term obtained by considering 3:1 internal resonance condition i.e. $\omega_2 = 3\omega_1 + \varepsilon\sigma$

where σ is the detuning parameter for the internal resonance.

MOOCs/ITG/ME/SKD/LEC32 53

If we are taking only single frequency then this is not possible. One can do this further analysis. So, there are several other analysis one can do. So, for example, ω_j equal to ω_1 can take this is primary resonance conditions one can take then in this case sub harmonic ω_2 equal to $3\omega_1$, super harmonic $3\omega_1$ equal to ω_1 can be taken.

And then principal parametric resonance condition ω_3 equal to $2\omega_1$ and internal resonance conditions ω_2 equal to $m\omega_1$, ω_1 equal to $m\omega_2$ many different conditions can be taken. So, one can take this 3 is to 1 internal resonance conditions also.

(Refer Slide Time: 55:16)

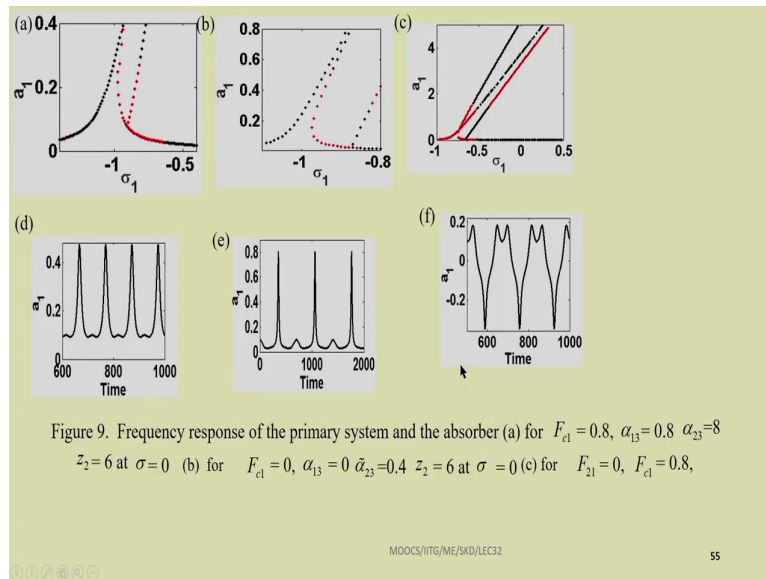
Results and discussions

Superharmonic, Subharmonic, principal parametric and 3:1 internal resonance conditions

In this section a parametric study is undertaken by obtaining the solution of steady state equation of MMS by **Newton's method** to study the effects of frequency response and time response of the system.

- Harmonic excitation force $F_1 = 0.4$, $F_2 = 0.38$
- Parametric excitation force $F_3 = 0.05$
- Mass ratio between primary mass to absorber mass $\mu = 0.02$
- Quadratic and cubic nonlinearity stiffness is considered to be 4% and 8 % of the linear stiffness for the primary system and the absorber respectively.
- The controlling force F_{c1} is varied from 0 to 0.008
- It can be observed that while a small variation of the primary system ω_1 occurs from 1 rad/s to 1.18 rad/s but that of the absorber ω_2 changes from 1 rad/s to 3 rad/s by changing the stiffness k_r from 0 to 0.176 N/m.
- The detuning parameters σ , σ_2 and σ_3 values are varied for a time delay of 0.1.

(Refer Slide Time: 55:21)

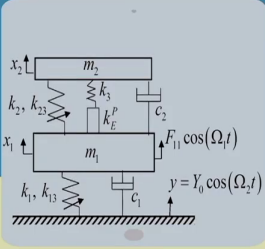


(Refer Slide Time: 55:27)






Conclusions

- In the present paper, hybrid vibration absorber with time delayed acceleration feedback force was investigated by considering a new model with mass ratio 50. The proposed model comprises a conventional dynamic vibration absorber in tandem with a piezoelectric stack actuator to provide the active control force and by which the frequency of the absorber can be changed actively. The primary system is subjected to multi harmonic hard excitation and parametric excitation. Method of multiple scale (MMS) is used to study the frequency responses and time responses of the system. The analysis is conducted for simultaneous resonance conditions.
- The cubic nonlinear stiffness in the primary system reduces the amplitude of the primary system than without considering nonlinearity in the stiffness at the resonating frequency of operation.
- The hard excitation on primary system produces higher amplitude and instability region for the mass ratio of 50, so the external forcing of the order ε^0 to ε^2 is more effective for studying the effect of actuating force on the primary system.

(Refer Slide Time: 55:33)

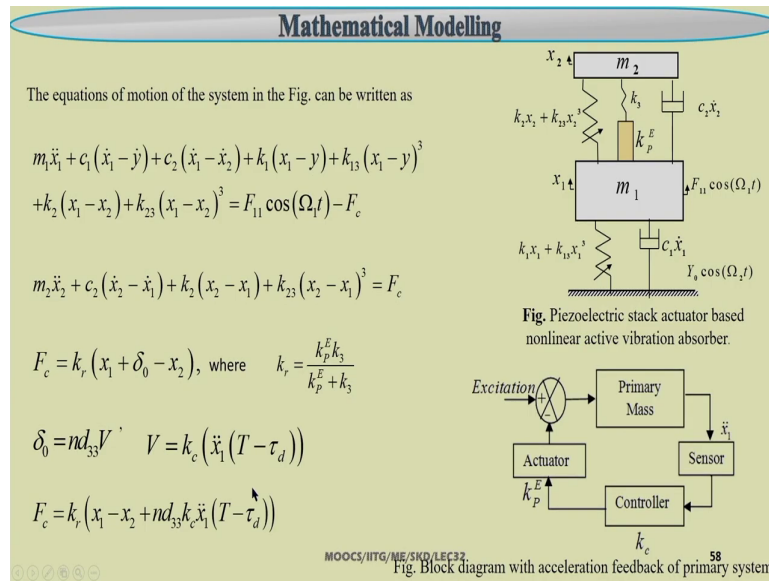


ANVA with time delay in acceleration feedback is used to suppress vibration of SDOF spring, mass, damper primary system under external harmonic and base excitations, and obtained Den Hartog's equal peaks.

				
Time delay	Acceleration feedback	Modified HBM	Den Hartog's equal peaks Base excitations	Primary resonance

MOOCs/IITG/ME/SKD/LEC32 57

(Refer Slide Time: 55:37)



So, by taking all these internal resonance conditions and external resonance condition. So, one can study this absorber and by studying this absorber so, one can find the results and one can see the different frequency response and time response of the system and in this way so, one can study the non-linear vibration absorber also one can study the non-linear vibration absorber by using in this harmonic balance method.

(Refer Slide Time: 55:41)

Contd...

The Eq. (1) and (2) are rewritten below in non-dimensional form by assuming non-dimensional time $\tau_1 = \omega_1 t$ where, $\omega_1 = \sqrt{k_1 / m_1}$.

The final equation of motion after ordering can be written as

$$\ddot{u}_1 + 2\xi_1 \dot{u}_1 - 2\xi_2 \dot{u}_2 + u_1 + \alpha_{13c} u_1^3 - (\alpha + \alpha_r) u_2 - \beta u_2^3 = F_1 \cos \Omega \tau_1 + Y \cos(\Omega \tau_1 - \gamma)$$

$$+ \alpha_{13c} (Y \cos(\Omega \tau_1 - \gamma))^3 + 3\alpha_{13c} \left(u_1^2 Y \cos(\Omega \tau_1 - \gamma) - u_1 (Y \cos(\Omega \tau_1 - \gamma))^2 \right) - F_{c1} \ddot{u}_1 (\tau_1 - \tau)$$

$$\mu \ddot{u}_2 + 2\xi_2 \dot{u}_2 + (\alpha + \alpha_r) u_2 + \beta u_2^3 = F_{c1} \ddot{u}_1 (\tau_1 - \tau) - \mu \ddot{u}_1$$

where

$$u_1 = x_1 / x_0, u_2 = (x_2 - x_1) / x_0, \mu = \frac{m_2}{m_1}, \xi_1 = \frac{c_1}{2m_1 \omega_1}, \xi_2 = \frac{c_2}{2m_1 \omega_1}, \alpha = \frac{k_2}{k_1}, \alpha_r = \frac{k_r}{k_1}, Y = Y_0 / x_0, \alpha_{13c} = \frac{k_{13} x_0^2}{k_1},$$

$$\beta = \frac{k_{23} x_0^2}{k_1}, F_1 = \frac{F_{11}}{m_1 \omega_1^2 x_0}, F_{c1} = \alpha_r k_c n d_{33}, \Omega = \frac{\Omega_1}{\omega_1}, \frac{\Omega_2}{\omega_1} = \Omega - \gamma, \gamma = \text{phase}, x_0 = \text{reference length}$$

MOOCs/IITG/ME/SKD/LEC32

59

(Refer Slide Time: 55:44)

Contd...

Mathematical Analysis by HBM

Harmonic Balance Method with slowly varying parameter is employed to analyze the steady-state dynamics of the system

$$u_1(\tau) = A(\tau) \cos(\Omega\tau + \varphi_1(\tau))$$

$$u_1(\tau - \tau_d) = A(\tau_d) \cos(\Omega(\tau - \tau_d) + \varphi_1(\tau - \tau_d))$$

$$u_2(\tau) = B(\tau) \cos(\Omega\tau + \varphi_2(\tau))$$

where $A(\tau)$, $B(\tau)$, $\varphi_1(\tau)$ and $\varphi_2(\tau)$ are slowly-varying functions of time τ such that one can neglect the following terms: \ddot{A} , \ddot{B} , $\ddot{\varphi}_1$, $\ddot{\varphi}_2$, $\dot{\varphi}_1^2$, $\dot{\varphi}_2^2$, $\dot{A}\dot{\varphi}_1$, $\dot{B}\dot{\varphi}_2$

equating the co-efficient of $\sin \Omega t$ and $\cos \Omega t$ terms separately to zero, yields the following algebraic equations.

$$\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} \begin{bmatrix} \dot{A} \\ \dot{B} \\ \dot{\varphi}_1 \\ \dot{\varphi}_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

a_1 to a_{16} and b_1 to b_4 are given in Appendix-I

60

Just let me briefly show how harmonic balance method can be used in this way. In this case you can have the same equation motion what you can consider this u_1 equal to $A \cos(\Omega\tau + \varphi_1)$ and $u_1(\tau - \tau_d) = A(\tau_d) \cos(\Omega(\tau - \tau_d) + \varphi_1(\tau - \tau_d))$ and u_2 equal to $B \cos(\Omega\tau + \varphi_2)$.

So, here this A , B what we have considered are not constant, but slowly varying function of time in the previous harmonic balance method. So, we have taken these as constant, but here by taking these terms as slowly varying function of time. So, one can initially get this A , B and φ_1 , φ_2 and by perturbing that thing.

So, one can study the stability simultaneously to find the response plot. So, here by substituting this way one can get by collecting the coefficient of $\sin \Omega t$ and $\cos \Omega t$.

So, these equations can be obtained and from these thing one can get this $A_1 \dot{B}_1 \dot{\psi}_1 \dot{\psi}_2$.

So, for steady state. So, this will be equal to 0 and to study the stability. So, one can find the eigenvalue of the Jacobean matrix and you can study like similar to that of the that we have study in case of the method of multiple scale.

In this way so, one can study the vibration absorber. So, whether it is a linear vibration absorber, non-linear vibration absorber, it is subjected to single frequency multi frequency with time delay, without time delay with internal resonance condition, without internal resonance condition and also with 1 is to 1 internal resonance condition with 1 is to 3 internal resonance condition.

So, several different combinations are possible. So, one can use required vibration absorber depending on the application of the system. Next class we are going to see one more application. So, to a different system. So, there we can take cutting tool vibration.

So, particularly we can take a lathe machine vibration, vibration of the tool and workpiece during turning and we will use a different method to solve the non-linear governing equation to obtain the instability region.

Thank you.