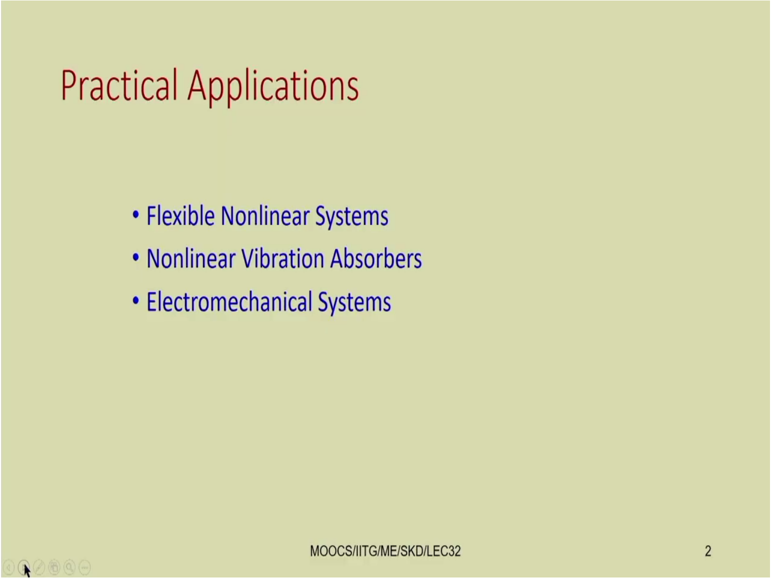
Nonlinear Vibration

Prof. Santosha Kumar Dwivedy
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Lecture - 29
Active vibration absorber with time delay acceleration feedback by HBM

Welcome to today class of Non-linear Vibration. So, in this module we are discussing regarding the applications of non-linear vibration in the first 3 classes.

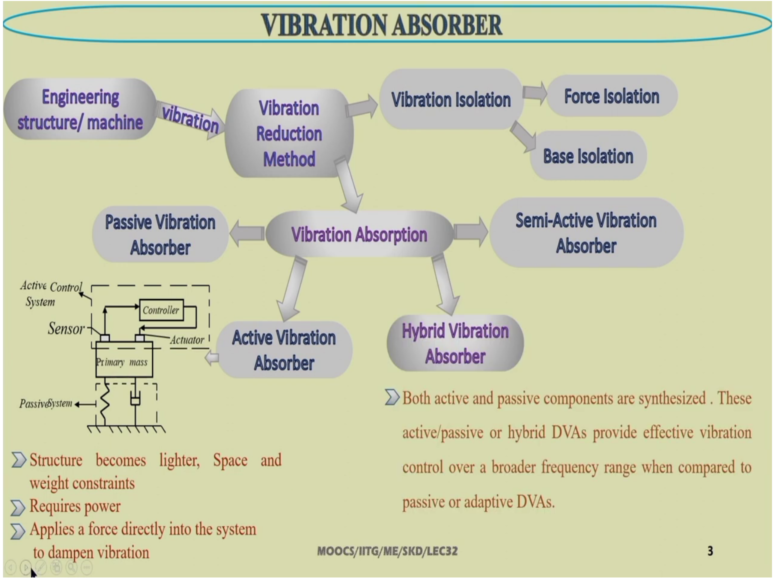
(Refer Slide Time: 00:50)



So, we have discussed regarding the vibration of continuous system where we have discussed regarding the vibration of a beam subjected to arbitrary base excitation or where a mass is attached at an arbitrary position.

So, in the second sets of applications. So, we are discussing regarding the non-linear vibration absorber.

(Refer Slide Time: 01:15)



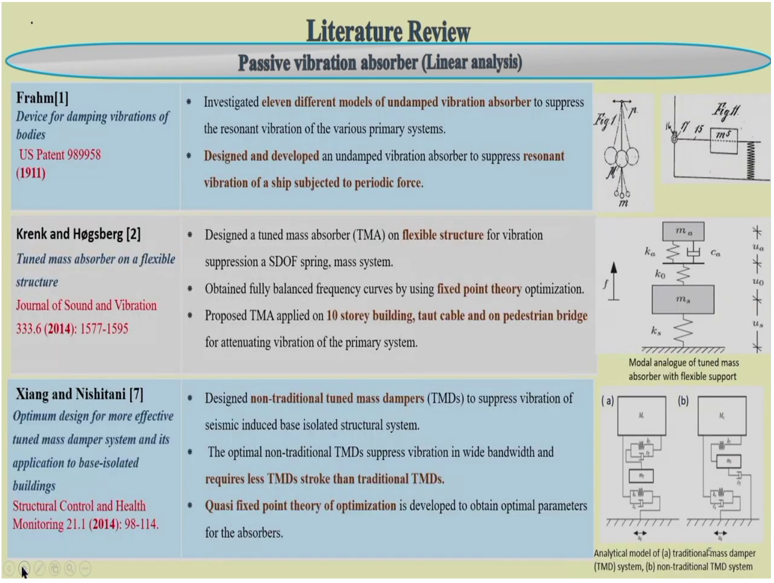
So, last class we have started this non-linear vibration absorber and here we have discussed the applications of many applications of non-linear vibration absorber also we have discussed regarding how it is different from vibration isolations.

So, what are the passive vibration absorber? Active vibration absorber and this hybrid vibration absorber in case of the passive. So, we have only spring and damper system attached to the system.

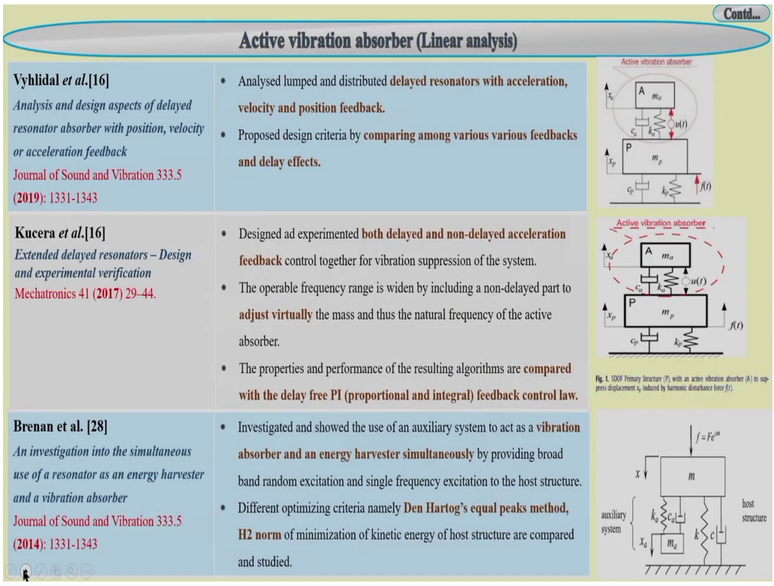
(Refer Slide Time: 01:49)

So, we know regarding the tuned vibration absorber when this excitation frequency equal to the natural frequency of the secondary system. So, here you can see several pictures of the vibration absorber what we have discussed last class.

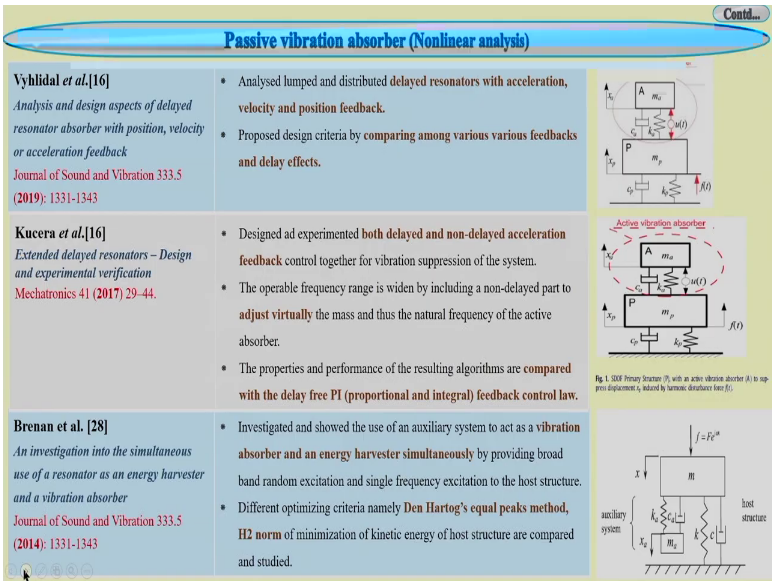
(Refer Slide Time: 02:03)



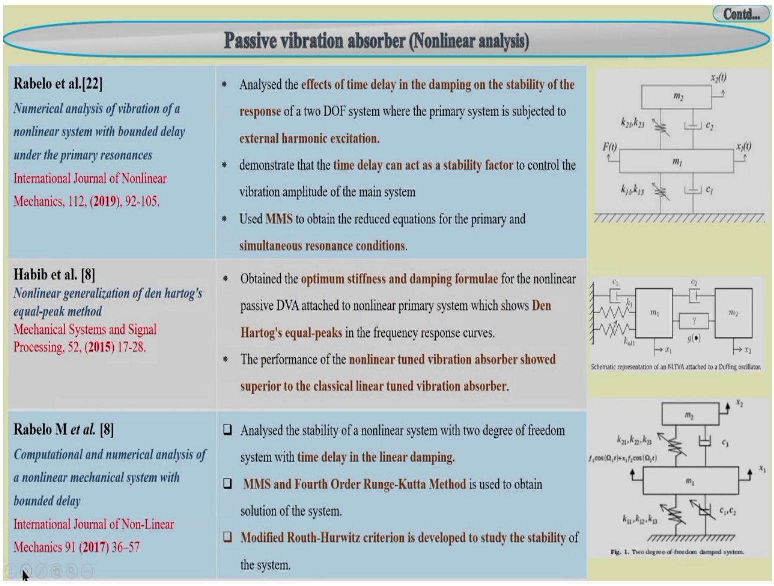
(Refer Slide Time: 02:04)



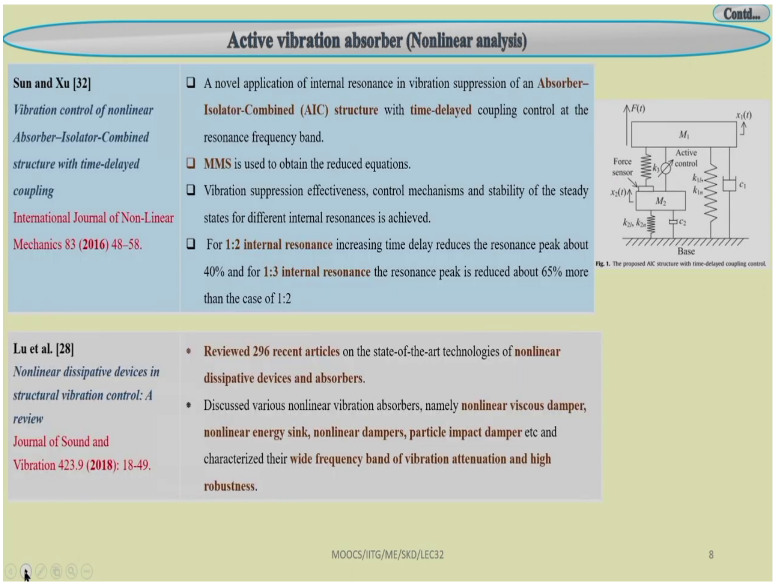
(Refer Slide Time: 02:09)



(Refer Slide Time: 02:13)

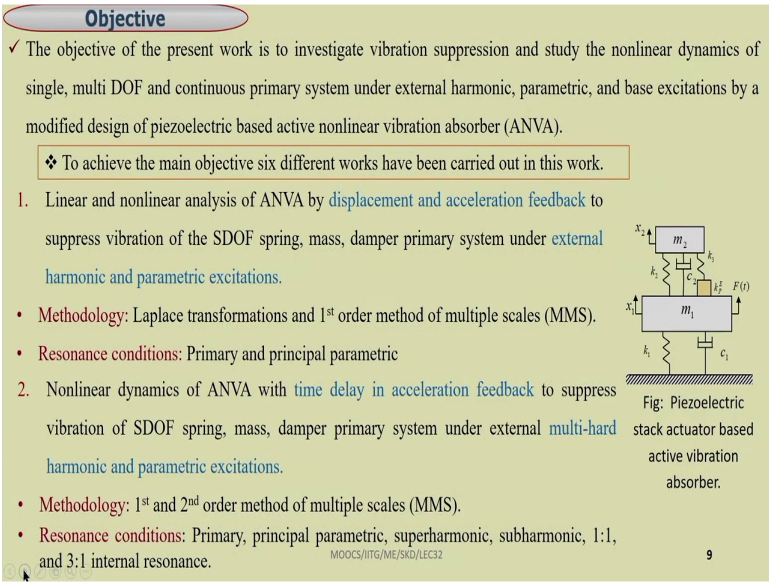


(Refer Slide Time: 02:16)

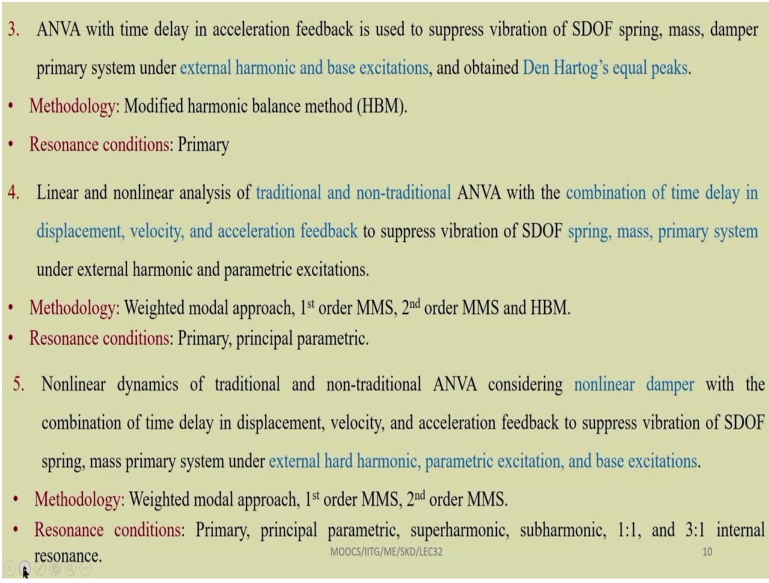


We made some literature review regarding the active, passive and hybrid vibration absorber and so, we have started the derivation of non-linear equation of motion or derivation of equation of motion of different vibration absorber.

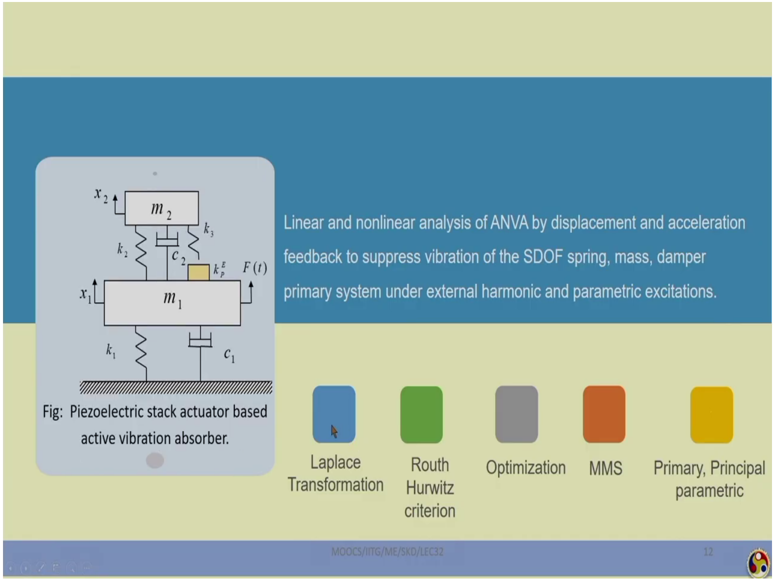
(Refer Slide Time: 02:21)



(Refer Slide Time: 02:30)



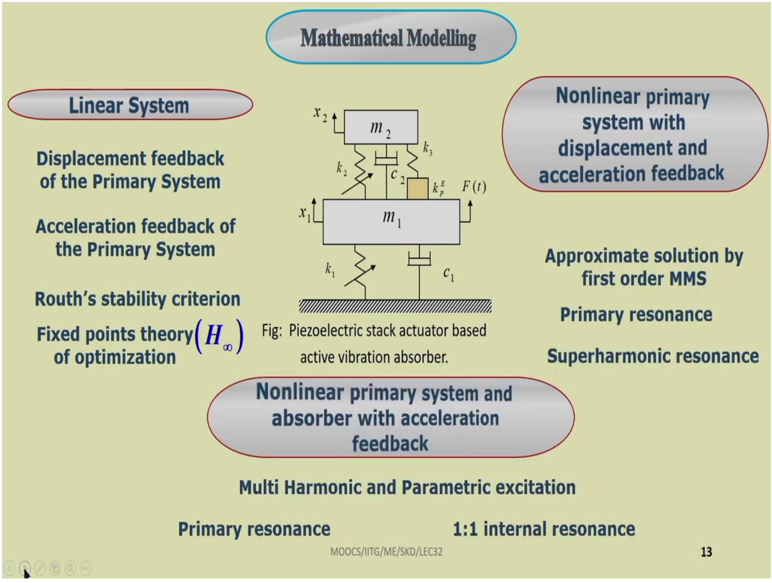
(Refer Slide Time: 02:33)



In this vibration absorber we are particularly interested for this piezoelectric stack actuators and here we have discuss regarding the linear and non-linear analysis of active non-linear vibration absorber by displacement and acceleration feedback to suppress the vibration of single degree of freedom system, spring mass damper primary system under external harmonic and parametric excitation.

Already you know what is harmonic excitation if the forcing is in the form of sine and cosine, then it is known as harmonic excitation and in case of the parametrically excited system, you can find a time varying term which is coefficient of the excitation term.

(Refer Slide Time: 03:23)



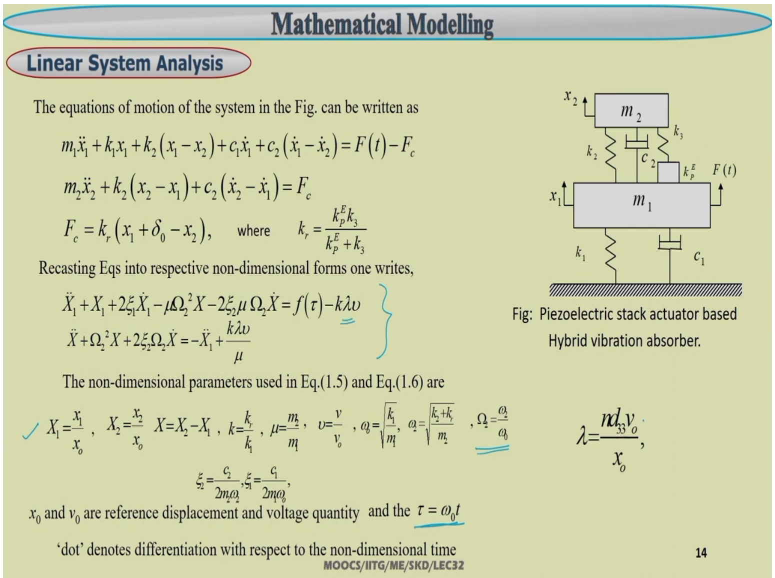
So, initially we apply this Laplace transform or initially we have derived this equation of motion of the system and then apply this Laplace transform to write down the characteristic equation. So, this is the systems we have taken. So, here the primary system has mass m 1, secondary system has mass m 2.

So, the primary system is supported by a spring stiffness case spring k 1 and damper c 1 and the primary is connected to the secondary by a spring k 2 damper c 2 and a piezoelectric stack actuator with stiffness KPE, also it is connected to the mass 2 by another spring k 3.

So, we can find the equivalent stiffness of this k 3 and k P E and these force in the stack actuator or the displacement in the stack actuator can be written using delta. So, delta will be equal to n d 3 3 v. So, where n is the number of stacks in the stack actuator and v is the voltage applied and d 3 3 is the material property of the actuator material.

So, that way we can write down the force applied by the stack actuator and we can derive this equation of motion.

(Refer Slide Time: 04:41)



So, in this already we have studied this part. So, here we have derived this equation of motion and in this equation motion particularly let me repeat. So, you can combine this k 3 and k P E to have a equivalent stiffness which is equal to KPE, k 3 by k P E plus k 3.

So, this equation that is for the you can draw the free body diagram for the primary system and the secondary system and applying Newton second law or d Alembert principle you can derive this equation of motion.

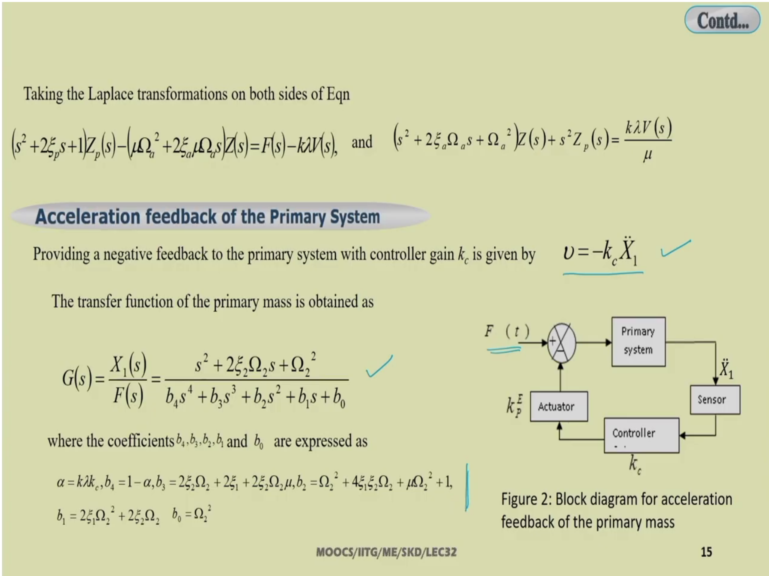
So, now by dividing m 1 in the first equations, the equation can be reduced m 1 in the first equation we have to divide and then by taking this k 1 root over k 1 by m 1 equal to k 1 by omega k 1 by m 1 root over equal to omega 0.

And taking a scaling taking a non-dimensional time taking a non-dimensional time tau equal to omega 0 t the equation can be non dimensionalized by also taking a non-dimensional displacement parameter X 1 equal to x 1 by x 0 and X 2 by x 2 by x 0 and X equal to X 2 minus X 1, k equal to k r by k 1 and mu equal to m 2 by m 1 v equal to v by v 0 v is the voltage applied.

So, v 0 is the reference voltage you can take and then omega 0 equal to root over k 1 by m 1 and omega 2 equal to root over k 2 k r by m 2 and this external frequency omega 2 equal to omega small omega 2 by omega 0. So, the damping parameter zeta 2 equal to c 2 by 2 m 2 omega 2 and zeta 1 equal to c 1 2 m 1 omega 0. So, that way by taking all these parameter.

So, you can reduce to these two equation. So, here you can see this lambda parameter. So, lambda equal to n d 3 3 v 0 by x 0. So, this is due to the piezoelectric property of the stack actuator. So, you got this parameter lambda. So, this is the piezoelectric property due to piezoelectric material.

(Refer Slide Time: 07:03)



So, then in the system. So, what we can do? So, we can apply these control force also we can use a controller. So, you have a primary system this primary system. So, the output is given to be X 1 double dot. So, here we are taking acceleration feedback. So, if we are taking a acceleration feedback. So, we have the accelerometer we can put. So, it can sense this acceleration and it will give the acceleration to the controller.

So, in the controller. So, we can write down this control force. So, you just see in controller we can write down the control force. So, this control force or in terms of voltage can be written v equal to minus k c X 1 double dot. So, this is k c multiplied by X 1 double dot will give you the voltage.

So, taking this voltage as the. So, taking this as the feedback now it can be fed to the actuator k e. So, we can get. So, from this k e into d 0 that is the displacement that will give you the actuator force. So, getting the actuator force then and these external forcing external forcing is F t. So, it will act on the primary system and we will get the response in this way. So, this control loop one can draw the block diagram of the control loop.

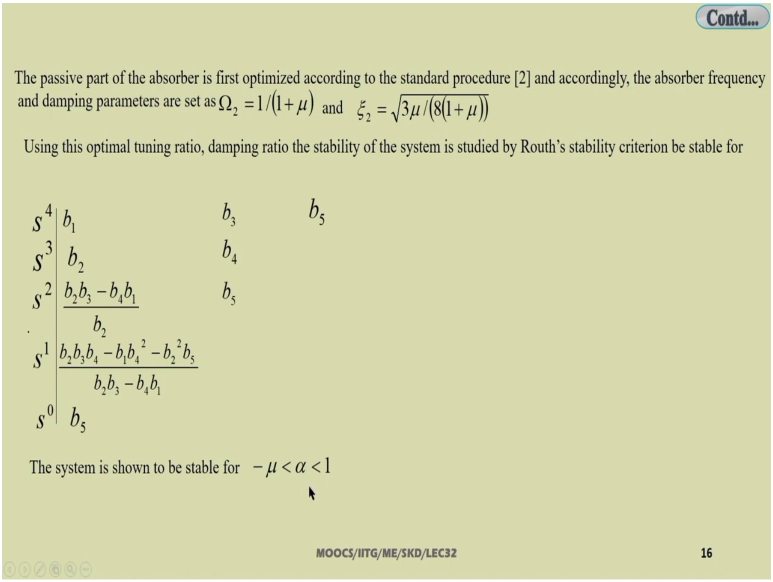
And you can study the system. So, here the transfer function if one write the equation. So, as the system is linear. So, we can write in using this Laplace transform. So, the transfer function from the primary system which is the output displacement by the input force Laplace of input force equal to a F s and output displacement is X 1. So, we write this G s equal to X 1 by X s.

So, it can be written in this form and you can see this denominator contain a fourth order term that is b 4 s 4 b 3 s q b 2 s square b plus b 1 s plus b 0 where this coefficient can be expressed in this following way. So, here this alpha parameter one can take this alpha parameter equal to k lambda k c k lambda k c already we have defined k in the previous slide.

So, b 4 equal to 1 minus alpha. So, actually these to define this b 1 b 2 b 3 b 4 we are using this parameter alpha which is a function of the actuator for ce actuator parameters. So, alpha is the function of the actuator parameter like this k lambda and k c k c is the control gain. So, you have seen here v equal to minus k c into X 1 double dot. So, by taking this way these non-dimensional parameter.

And so, from this characteristic equation. So, one can use this Routh Hurwitz criteria to find the stability of the system.

(Refer Slide Time: 09:54)



So, it can be shown that the system is stable. So, when this minus mu less than alpha less than 1, mu is the mass ratio parameter.

(Refer Slide Time: 10:08)

Contd...

The frequency response function of the primary mass can be obtained by substituting $j = \sqrt{-1}$ and $s = j\omega$

$$G(j\omega) = \frac{\Omega_2^{\ 2} - \omega^2 + j2\xi_2\Omega_2\omega}{\left(b_4^{\ 4}\omega^4 - b_2\omega^2 + b_0\right) + j\left(b_1\omega - b_3\omega^3\right)}$$

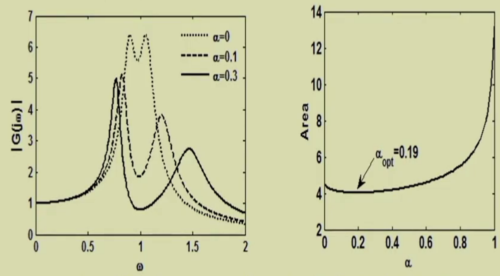
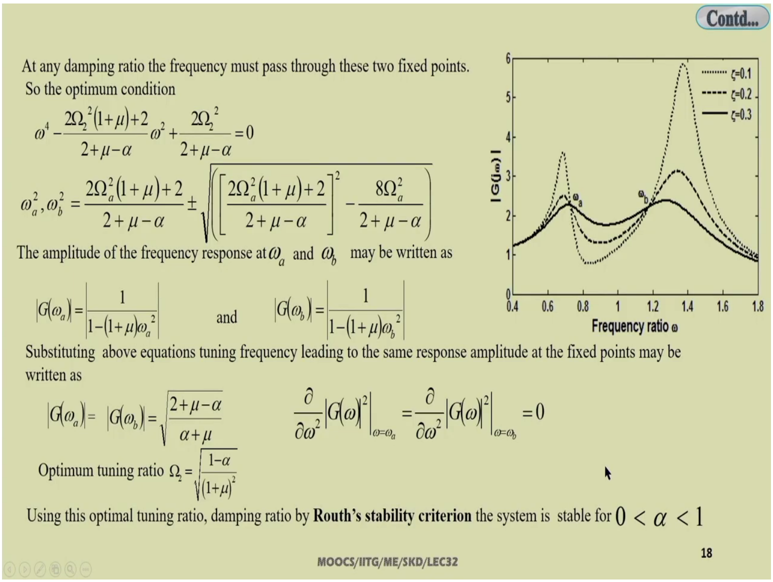


Fig. (a) Frequency-response plots of the primary system with acceleration feedback of the primary mass for different values of α

(b) Variation of area under the frequency response curve with α

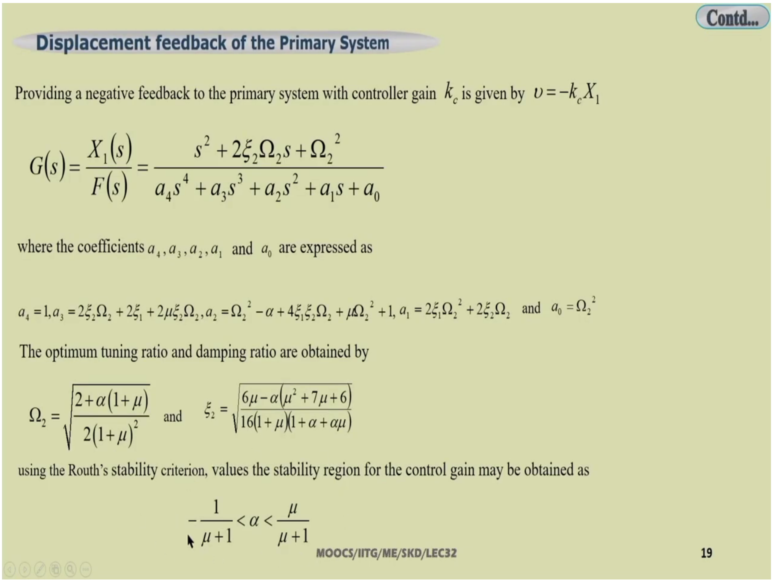
(Refer Slide Time: 10:20)



So, if one plot this response. So, last class we have seen how we are getting the frequency response and also the variation of the area under the frequency response curve with alpha parameter.

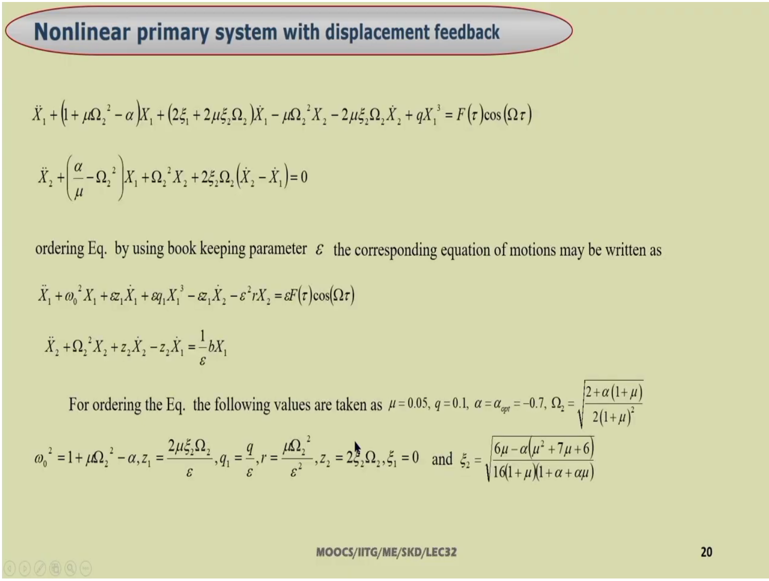
So, from this thing one can get the optimal parameter one can observe these two fixed point and taking these two fixed point theory. So, one can find the optimum parameter and from that thing one can find the optimum parameter of zeta or optimum parameter to have the further study.

(Refer Slide Time: 10:42)



One can take the displacement feedback of the primary system also. So, in case of displacement feedback. So, previously we have taken the acceleration feedback also we can take displacement feedback where this v can be written equal to minus k c X 1 and we can perform the analysis similarly we can find the parameter of alpha for which the system is stable.

(Refer Slide Time: 11:08)



(Refer Slide Time: 11:14)

Nonlinear primary system and absorber with acceleration feedback

$$m_1\ddot{x}_1 + k_1x_1 + k_{12}x_1^2 + k_{13}x_1^3 + k_2(x_1 - x_2) + k_{21}(x_1 - x_2)^2 + k_{23}(x_1 - x_2)^3$$
$$+c_1x_1' + c_{12}(\dot{x}_1)^2 + c_2(\dot{x}_1 - \dot{x}_2) + c_{21}(\dot{x}_1 - \dot{x}_2)^2 = F_{11}\cos(\Omega_{11}t)$$
$$+F_{21}\cos(\Omega_{21}t) + x_1F_{31}\cos(\Omega_{31}t) - F_{c1}$$

and

$$m_2\ddot{x}_2 + k_2(x_2 - x_1) + k_{21}(x_2 - x_1)^2 + k_{23}(x_2 - x_1)^3$$
$$+c_2(\dot{x}_2 - \dot{x}_1) + c_{21}(\dot{x}_2 - \dot{x}_1)^2 = F_{c1}$$

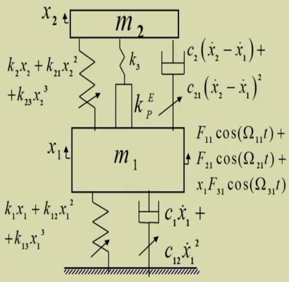


Fig.4 Piezoelectric stack actuator based nonlinear hybrid vibration absorber.

Substituting $F_{c1} = k_r(x_1 + \delta_0 - x_2)$ and rearranging

$$\ddot{x}_1 + \left(\frac{k_1 + k_2 + k_r}{m_1}\right)x_1 + \left(\frac{c_1 + c_2}{m_1}\right)\dot{x}_1 = \left(\frac{k_2 + k_r}{m_1}\right)x_2 + \frac{c_2}{m_1}\dot{x}_2 - \frac{k_{12}}{m_1}x_1^2 - \frac{k_{13}}{m_1}x_1^3 - \frac{k_{21}}{m_1}(x_1 - x_2)^2$$
$$-\frac{k_{23}}{m_1}(x_1 - x_2)^3 - \frac{c_{12}}{m_1}(\dot{x}_1)^2 - \frac{c_{21}}{m_1}(\dot{x}_1 - \dot{x}_2)^2 + \frac{F_{11}}{m_1}\cos(\Omega_{11}t) + \frac{F_{21}}{m_1}\cos(\Omega_{21}t) + x_1\frac{F_{31}}{m_1}\cos(\Omega_{31}t) + \frac{f_c}{m_1}\ddot{x}_1$$

MOOCS/IITG/ME/SKD/LEC32 21

So, then we can plot this responses also. So, previously we have seen when we are taking the system to be when the systems are linear. So, we can take the non-linear system also. So, non-linear primary system with displacement feedback also we can take. So, in that case the equation will reduce to this form and one can use different methods. So, here you just see the non-linear parameter q X 1, q is added to the previous equation as you know this superposition theory cannot be applied to this non-linear system. So, you cannot extend the idea of linear system here to predict the non-linear response.

So, one has to study properly the non-linear response of the system. So, one can apply method of multiple scale or any other method to study the response of the system. So, here one can take also the spring to be non-linear in the secondary case, previously we have taken spring to be non-linear in the primary case only. So, by taking the spring to be non-linear in the secondary case.

Similarly, we can derive this equation of motion. So, here this F c 1 that is the control force can be written equal to k r into x 1 plus delta 0 minus x 2 and following the similar way we can have the non-linear equation of motion. So, after getting this non-linear equation of motion.

(Refer Slide Time: 12:38)

Contd...

Assuming $\omega_1^2 = \frac{k_1 + k_2 + k_r}{m_1}$ and $\tau = \omega_1 t$

$$\omega_1^2 \frac{d^2 x_1}{d\tau^2} + \omega_1^2 x_1 + \left(\frac{c_1 + c_2}{m_1}\right)\omega_1 \frac{dx_1}{d\tau} = \left(\frac{k_2 + k_r}{m_1}\right) x_2 + \frac{c_2}{m_1}\omega_1 \frac{dx_2}{d\tau} - \frac{k_{12}}{m_1} x_1^2 - \frac{k_{13}}{m_1} x_1^3 - \frac{k_{21}}{m_1}(x_1 - x_2)^2 - \frac{k_{23}}{m_1}(x_1 - x_2)^3$$

$$-\frac{c_{12}}{m_1}\omega_1^2 \left(\frac{dx_1}{d\tau}\right)^2 - F_{c1} - \frac{c_{21}}{m_1}\omega_1^2 \left(\frac{dx_1}{d\tau} - \frac{dx_2}{d\tau}\right)^2 + \tilde{F}_1 \cos(\Omega_1 \tau) + \tilde{F}_2 \cos(\Omega_2 \tau) + x_1 \tilde{F}_3 \cos(\Omega_3 \tau) + F_{c1}\omega_1^2 \frac{d^2 x_1}{d\tau^2}$$

Non dimensionalizing the Eq.

$$\ddot{X}_1 + X_1 + h_{1c}\dot{X}_1 = \omega_{2c}^2 X_2 + h_{2c}\dot{X}_2 - \alpha_{11c} X_1^2 - \alpha_{13c} X_1^3 - \alpha_{21c}(X_1 - X_2)^2 - \alpha_{23c}(X_1 - X_2)^3$$

$$-h_{12c}\dot{X}_1^2 - h_{21c}(\dot{X}_1 - \dot{X}_2)^2 + F_1 \cos(\Omega_1 \tau) + F_2 \cos(\Omega_2 \tau) + x_1 F_3 \cos(\Omega_3 \tau) + F_{c1}\ddot{X}_1$$

and

$$\ddot{X}_2 + F_{c2}\ddot{X}_1 + \omega_r^2 X_2 + h_2 \dot{X}_2 = h_2 \dot{X}_1 - h_{21}(\dot{X}_2 - \dot{X}_1)^2 - \alpha_{21}(X_2 - X_1)^2 - \alpha_{23}(X_2 - X_1)^3 + \omega_r^2 X_1$$

where

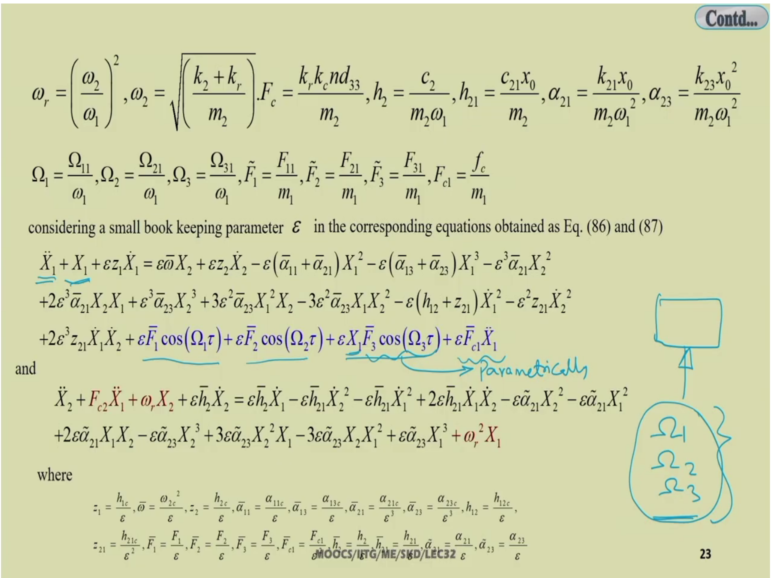
$$h_{1c} = \left(\frac{c_1 + c_2}{m_1 \omega_1}\right), \omega_{2c}^2 = \left(\frac{k_2 + k_r}{m_1 \omega_1^2}\right), \quad h_{2c} = \frac{c_2}{m_1 \omega_1}, \alpha_{11c} = \frac{k_{12} x_0}{m_1 \omega_1^2}, \alpha_{13c} = \frac{k_{13} x_0^2}{m_1 \omega_1^2}, \alpha_{21c} = \frac{k_{21} x_0}{m_1 \omega_1^2},$$

$$\alpha_{23c} = \frac{k_{23} x_0^2}{m_1 \omega_1^2}, h_{12c} = \frac{c_{12} x_0}{m_1}, h_{21c} = \frac{c_{21} x_0}{m_1}, X_1 = \frac{x_1}{x_0}, X_2 = \frac{x_2}{x_0} F_1 = \frac{\tilde{F}_1}{\omega_1^2 x_0}, F_2 = \frac{\tilde{F}_2}{\omega_1^2 x_0}, F_3 = \frac{\tilde{F}_3}{\omega_1^2 x_0}$$

MOOCS/IITG/ME/SKD/LEC32

22

(Refer Slide Time: 12:51)

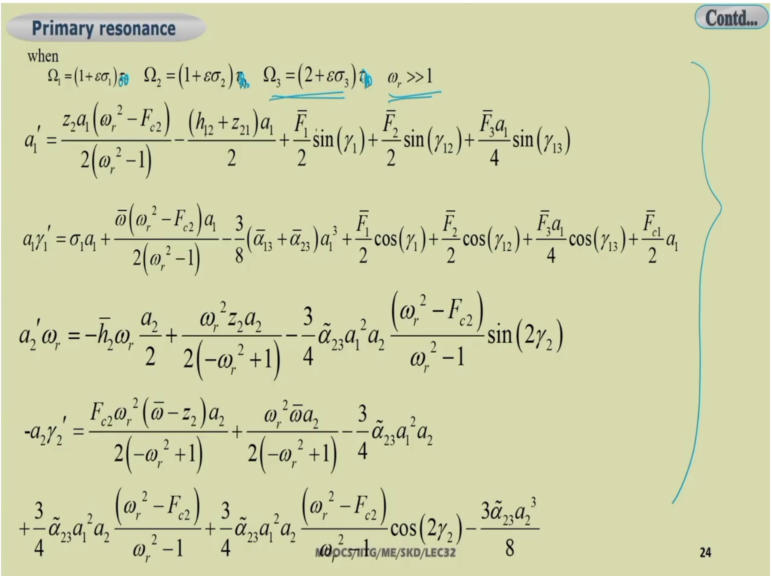


So, we can further study this equations to find the response. So, here we may apply the method of multiple scales to derive this equation of motion in this case you can check the forcing term is of the order of epsilon it is taken.

So, here it is epsilon F 1. So, if you are taking multiple forces. So, in the case we can take multiple forces in the primary system if multi component of the forces you just see this is omega 1. So, 3 frequency components are there omega 1, omega 2, omega 3. So, multi frequency excitation. So, they can be independent, they can dependent also on each other.

So, it depends on the user how to set this parameter or it also depend on the applications where we are taking multiple number of frequencies acting multiple number of forcing acting having different frequencies in addition to that. So, you we have 3 different forcing term and in addition to that. So, we have this control force here control force is also taken of the order of epsilon. So, it is F c 1 into X 1 double dot. So, this is the acceleration feedback taken that way one can find the equation motion.

(Refer Slide Time: 14:11)



So, now after writing this equation motion. So, then one can apply different resonance conditions for example, one can study the primary resonance condition. So, in case of the primary resonance condition as we have taken 3 frequency term. So, we can take omega 1 equal to 1 plus epsilon sigma 1 tau 0, and omega 2 also can be taken as 1 plus epsilon 2 plus 1 plus epsilon 2 and omega 3 also can be taken.

So, omega 1 can be taken equal to 1 plus epsilon sigma 1. So, this is 1 plus epsilon sigma 2, and 1 plus epsilon sigma 3 where omega r we are taking very very away from that frequency ratio is taken very very away from 1. So, here you can see this non dimensional frequency of the primary system is 1 that is why this omega 1 is taken to be 1 plus epsilon sigma 1. So, if you see this equation the governing equation you can see the coefficient of x 1 equal to 1, and m 1 equal to 1 coefficient of the first mass equal to 1.

So, when you have non dimensionalize this thing the non-dimensional frequency parameter which is coefficient of x 1 in this case taking a unit mass it will be equal to 1. So, that is why we have taken the resonance condition when this external non-dimensional frequency becomes 1.

So, in this case the external non dimensional frequency equal to 1 so, that is why taking this detuning parameter. So, one can write omega 1 equal to 1 plus epsilon sigma 1 omega 2 equal to 1 plus epsilon sigma 2. So, if we are considering. So, in case of 3 you just see in case of 3 it is taken to be 2 plus epsilon 2 plus epsilon sigma 3.

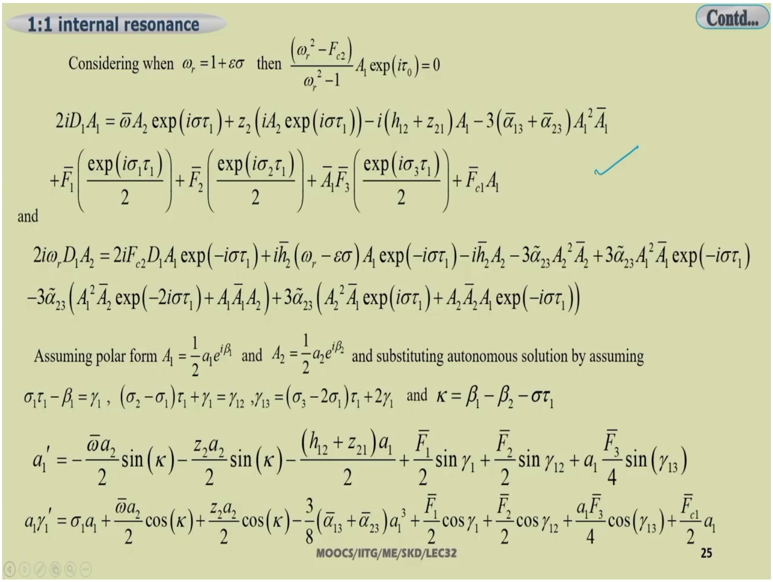
So, the third forcing term if you see you just take the third forcing term. So, in the third forcing term we can see this is parametric, this is parametrically excited as the coefficient of x 1 is the time varying term F 3 cos omega 3 T. It will give rise to resonance condition. So, when omega 3 is nearly twice the natural frequency of the system. So, that is why this omega 3 is taken to be twice omega 1. So, omega 1 is taken to be 1. So, that is why omega 3 is taken to be 2 epsilon sigma 3.

So, this is the principal parametric resonance condition. So, we are studying then this primary resonance due to all these conditions. So, applying this method of multiple scale following the similar principle what we have studied before. So, as we have two equations. So, it will yield 4 first order differential equation, these differential equations are the reduced equation.

So, this a 1 dash, a 1 gamma 1 dash, a 2 dash and a 2 gamma 2 dash equation can be obtained in this way. So, you just see these equations are not simple equation what you have studied in case of the duffing oscillator. So, in this case you cannot find the solution or you cannot find a closed form solution like in case of the linear system.

So, here you have to so, for steady state this a 1 dash gamma 1 dash, a 2 dash and gamma 2 dash will be equal to 0 and we will have a set of algebraic and transcendental equation which you can solve to get this a 1, a 2, gamma 1, gamma 2 which will give the response amplitude of the primary and secondary systems and phase of the primary and secondary systems.

(Refer Slide Time: 18:16)



So, now considering omega r equal to 1 plus epsilon sigma, then we can have this term that is omega r square minus F c 2 by omega r square minus 1 A 1 e to the power i T 0 equal to 0.

Similarly we will get this equation taking the polar form A 1 equal to half a 1 e to the power i beta 1 and A 2 e to the power i beta 2 and substituting in this autonomous equation. So, we can study the response of the system. So, if we are considering this internal resonance condition reduced equation will contain the additional term taking that additional term.

(Refer Slide Time: 19:04)

$$a_2{}'\omega_r = F_{c2} z_2 \frac{a_2}{2} - \left(h_{12} + z_{21}\right) F_{c2} \frac{a_1}{2} \cos\left(\kappa\right) - 3F_{c2}\left(\bar{\alpha}_{13} + \bar{\alpha}_{23}\right) \frac{a_1{}^3}{8} \sin\left(\kappa\right)$$

$$+ \frac{F_{c2}\bar{F}_1}{2} \sin\left(\kappa + \gamma_1\right) + \frac{\bar{F}_2}{2} F_{c2} \sin\left(\gamma_{12} + \kappa\right) + \frac{\bar{F}_{c1}}{2} F_{c2} \sin\left(\kappa\right) + \bar{h}_2 \frac{a_1}{2} \cos\left(\kappa\right) - \bar{h}_2 \frac{a_2}{2}$$

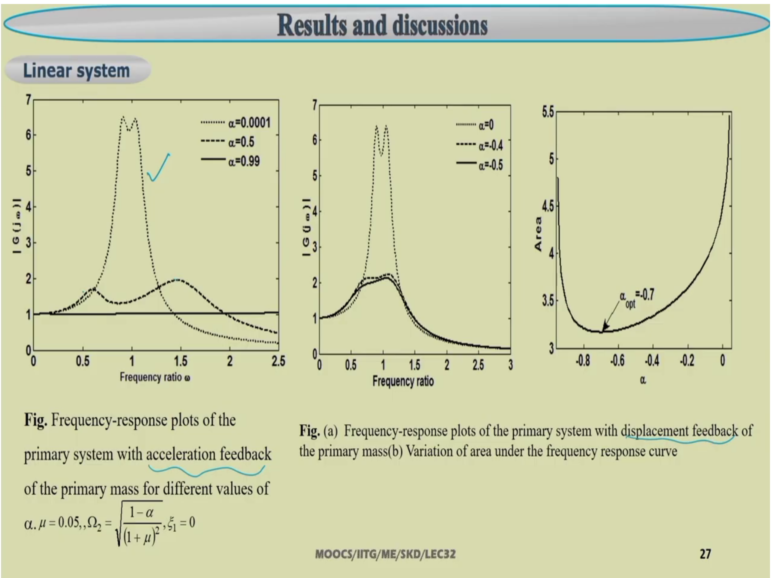
$$+3\tilde{\alpha}_{23} \frac{a_1{}^3}{8} \sin\left(\kappa\right) - 3\tilde{\alpha}_{23} \frac{a_1{}^2 a_2}{8} \sin\left(2\kappa\right) - 3\tilde{\alpha}_{23} \frac{a_2{}^2 a_1}{8} \sin\left(\kappa\right) + 3\tilde{\alpha}_{23} \frac{a_2{}^2 a_1}{8} \sin\left(\kappa\right)$$

$$a_2\gamma_1{}'\omega_r + a_2\kappa'\omega_r = \left(\sigma_1 - \sigma\right) a_2 + \bar{\omega} F_{c2} \frac{a_2}{2} - \left(h_{12} + z_{21}\right) F_{c2} \frac{a_1}{2} \sin\left(\kappa\right) - 3F_{c2}\left(\bar{\alpha}_{13} + \bar{\alpha}_{23}\right) \frac{a_1{}^3}{8} \cos\left(\kappa\right)$$

$$+ \frac{\bar{F}_1}{2} F_{c2} \cos\left(\kappa + \gamma_1\right) + \frac{\bar{F}_2}{2} F_{c2} \cos\left(\kappa + \gamma_{12}\right) + \frac{\bar{F}_{c1}}{2} F_{c2} \cos\left(\kappa\right) - \bar{h}_2 \frac{a_1}{2} \sin\left(\kappa\right) - 3\tilde{\alpha}_{23} \frac{a_2{}^3}{8}$$

$$+3\tilde{\alpha}_{23} \frac{a_1{}^3}{8} \cos\left(\kappa\right) - 3\tilde{\alpha}_{23} \frac{a_1{}^2 a_2}{8} \cos\left(2\kappa\right) - 3\tilde{\alpha}_{23} \frac{a_1{}^2 a_2}{8} + 3\tilde{\alpha}_{23} \frac{a_2{}^2 a_1}{8} \cos\left(\kappa\right) + 3\tilde{\alpha}_{23} \frac{a_2{}^2 a_1}{8} \cos\left(\kappa\right)$$

(Refer Slide Time: 19:04)



So, again we can derive this condition we have another set of equations. So, now you can see. So, in case of the linear systems we have already observed that if we are considering alpha equal to 0.0001. So, this is the response we are getting frequency response we are getting and by increasing this control parameter alpha 2.5 or alpha 2.99.

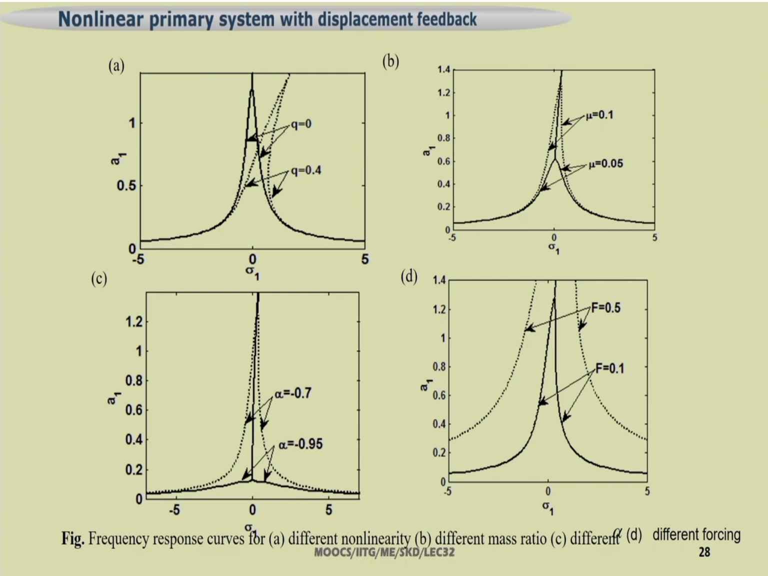
So, you can see drastically the frequency the response amplitude decreases, but here one can absorb the resonance peak. So, is slightly high also, but further increasing this alpha 2.99. So, you can see the response amplitude is flattened and one can get very less value of response amplitude.

For less control parameter you have seen the response is very high and by applying this control. So, we can easily reduce the response amplitude. So, this part we have seen when we have applied this acceleration feedback and when you are applying this displacement feedback.

So, you just see. So, we have taken this v equal to minus k c into x double dot, but when we are taking this v equal to minus k c into x 1. So, in that case you can see by applying this control gain. So, it is though it is reducing though it is reducing the response amplitude, but the reduction is not. So, high as compared to that in acceleration feedback.

So, acceleration feedback is giving a better result than the displacement feedback what you have observed here.

(Refer Slide Time: 20:58)

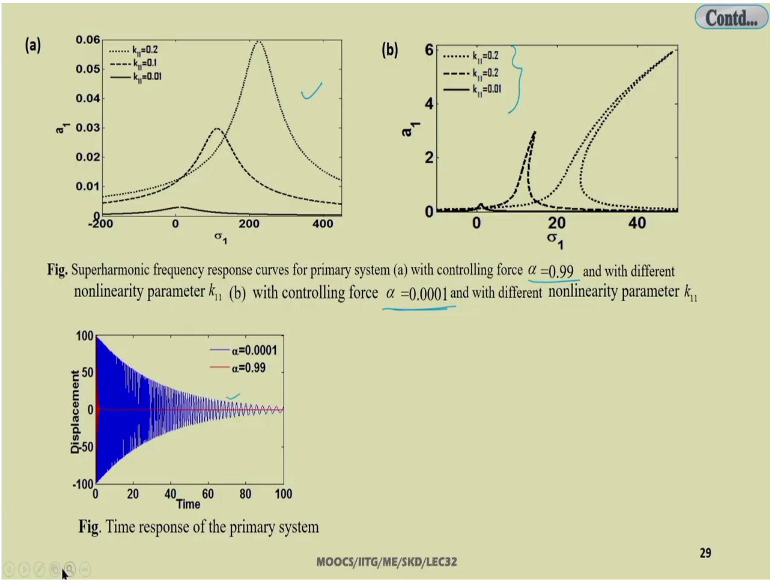


So, that is in the case of the linear case. So, if we have the non-linear primary system with a displacement feedback. So, now, you can see the bending of the bending of the response curve.

So, initially q equal to 0. So, this is the response plot similar to that of a linear system and in case of the non-linear system it is tilting towards right. So, in this case we can have a broadband of frequency for which the response amplitude is very high. Previously, we have a less. So, you have a less range of frequency for which we have the response amplitude to be very high but when the system becomes non-linear.

So, we have a larger frequency range for which the system become non-linear or the system response amplitude is very high. By changing different control parameter we can find or we can absorb. So, what will be the response amplitude of the system.

(Refer Slide Time: 22:08)



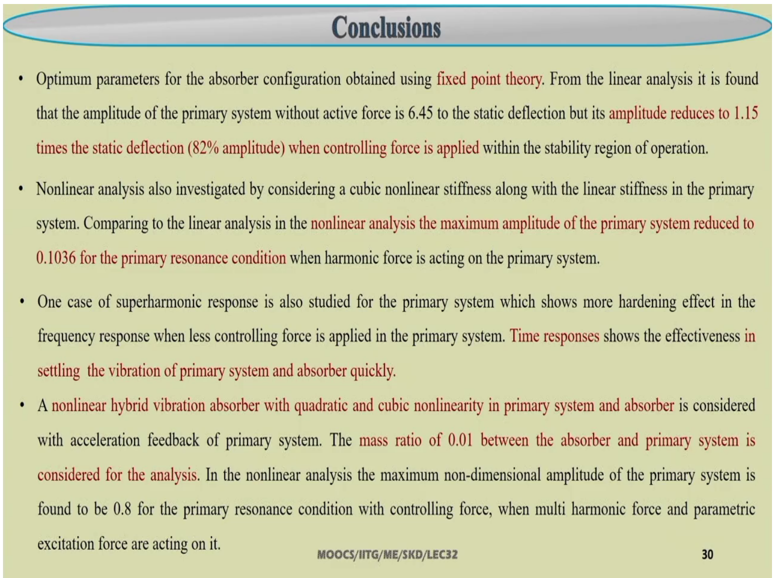
So, the response amplitude of the system can be controlled then by possibly by controlling those system parameter or actively by applying this voltage and controlling the piezoelectric material property. So, here you can see. So, these are the frequency response for different value of k 1 1. So, we have taken a sub harmonic resonance condition also sub harmonic.

So, in case of the sub harmonic frequency response curve for primary system. So, this is the sub harmonic case. So, the previously you have seen the primary resonance case and this is the sub harmonic resonance case. So, this is also for the sub harmonic response. So, when alpha is taken to be 0.99 in this case the response alpha is taken the control force alpha is taken 0.0001.

By taking different k 11 value. So, we can absorb how shifting of the frequency response takes place. So, as we are increasing this k 11. So, our natural frequency non dimensional natural frequency of the system increases. So, that is why there is a shifting of the frequency response towards right. One can plot the time response it can easily be seen that when alpha equal to 0.0001.

So, one has very large amplitude of oscillation and it takes a large time to settle. So, for example, here till 100 also it is not settled what if we are taking very high value of alpha. So, you can see it settled down very quickly to its final value. By taking proper control parameter and acceleration feedback here we have shown that the response amplitude can be conveniently controlled in case of the vibration absorber or it can be absorbed.

(Refer Slide Time: 24:01)



We have seen this optimum parameter for the absorber configuration obtained using fixed point theory. So, from the linear analysis it is found that the amplitude of primary system without active force is 6.45 to the static deflection, but its amplitude reduces to 1.15 times to the static deflection, 82 percent amplitude is controlled in this case, similarly non-linear analysis also investigated by considering a cubic non-linear stiffness along with linear stiffness in the primary system.

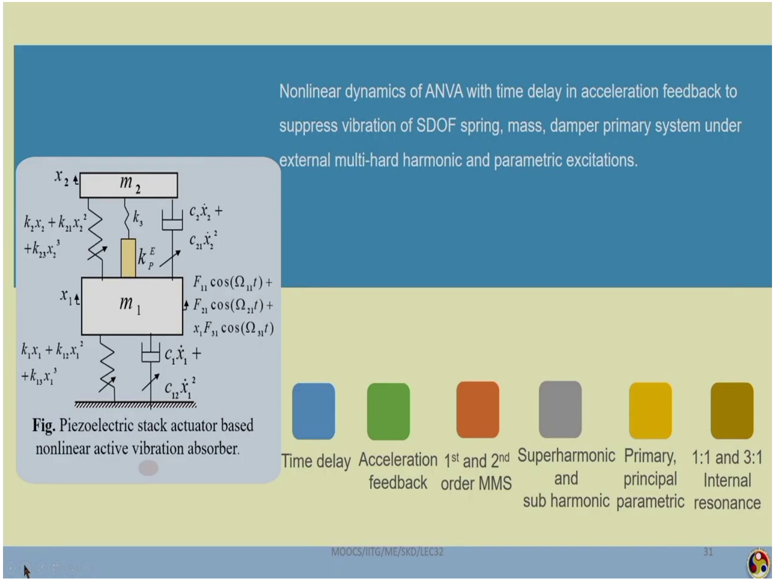
So, comparing the linear analysis in the in non-linear analysis, the maximum amplitude of the primary system reduces 0.1036 for the primary resonance condition when harmonic force is acting on the primary system. One case of super harmonic response is also studied.

So, for the primary system which shows more hardening effect. So, you have seen. So, it is tilted towards right we have observed the hardening effect in the frequency response. When less controlling force is applied so, time response shows the effectiveness of those settling time of the primary system when the absorber is used.

A non-linear hybrid vibration absorber with quadratic and cubic nonlinearity in primary system also is studied and here the mass ratio of 0.01 between the absorber and primary system is considered for the analysis. So, you can note that the in general literature a mass ratio of 1 is to 20 is considered, but in this case you can see you have taken a mass ratio up to 100 also 0.01; that means, so, m 2 by m 1 equal to 0.01 that is 1 by 100.

So, 100 times so we are able to control the vibration of the system by putting a mass 1 by 100 times that of the primary mass. By using this non-linear analysis you have seen we can control the vibration or we can absorb the vibration of a non-linear vibration absorber by putting a mass 1 by 100 of the of that of the primary system. So, this is the advantage of using the non-linear vibration absorber.

(Refer Slide Time: 26:29)



So, let us see another similar problems, where we have taken this non-linear dynamics of this active non-linear vibration absorber with time delay. So, generally when we have apply the control force. So, suddenly it will not react and it will take some time to react that is why there will be some delay in the response.

If we are considering that time delay. So, these analyses will be modified. So, in this case we are going to study the response of the system if there is a time delay in this acceleration feedback to suppress the vibration of a single degree of freedom spring mass damper primary system under external multi hard harmonic and parametric excitation.

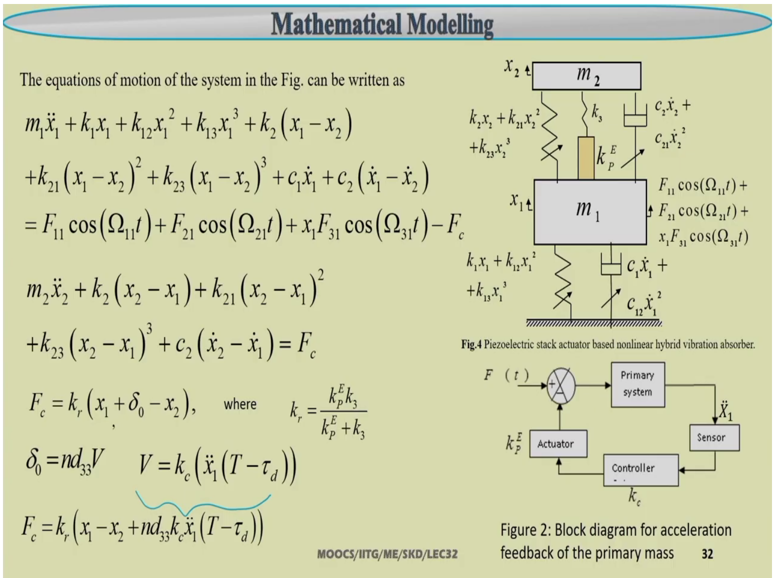
So, previously we have taken weak forcing. So, the forcing term has written epsilon time by using the bookkeeping parameter epsilon which signifies that we have taken a weak forcing bought in this present case, now we will take a hard harmonic excitation so; that means, this excitation term will be of the same order as that of the linear part of the equation of motion.

And also we will consider this parametric excitation. So, excitation term will take in such a way that, the time varying forcing term will be the coefficient of the response term that is x 1. So, the we have shown the same system. So, here this m 1 is the primary system. So, you can take any vibrating system actually and you can represent this using a spring and damper system.

So, here the damper and the spring may or may not be linear. So, we may consider a non-linear spring and non-linear damping also. So, this non-linear spring and damping. So, it is up to you. So, up to what order you are taking. So, in this case the damping is shown to be quadratic and the spring stiffness is shown to be cubic order.

So, by taking this quadratic damping in both the primary system and the secondary system and this stiffness parameter or the forcing due to the spring stiffness up to cubic order, then we can write down this equation of motion.

(Refer Slide Time: 28:59)



So, after writing this equation of motion here we can apply control force similar to the previous case here the control force is taken equal to. So, the control force F c will be equal to taking these acceleration feedback it will be k c will be equal to k c into x 1 double dot the equation of motion can be written in this way.

So, here delta 0 that is the displacement of the piezoelectric stack actuator, delta 0 equal to n d 33 V like previous case here also we have taken the same way, but this control force here it is written equal to k r into x 1 plus delta 0 minus x 2. So, delta 0 is the displacement of this stack actuator x 1 is the displacement of the primary system.

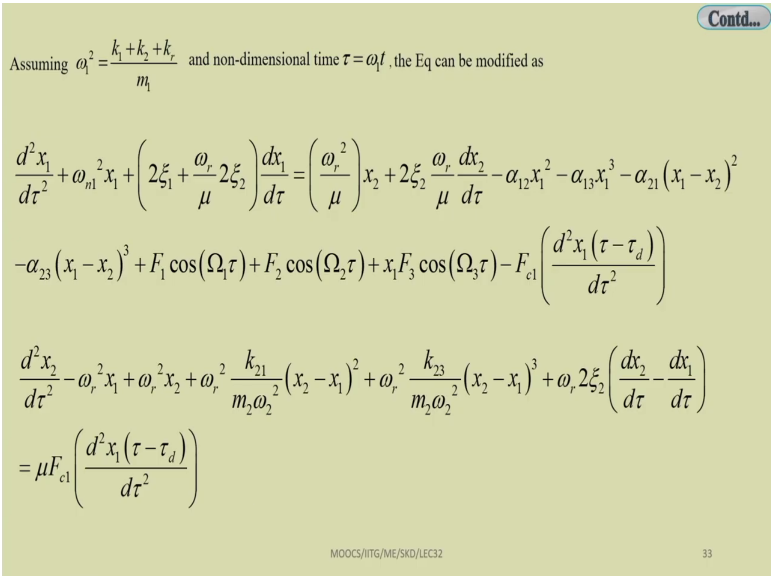
If we are not considering the stack actuator, then it would have been different. So, now, by considering this stack actuator. So, we have to add this delta 0. So, total displacement will be x 1 plus delta 0 minus x 2. So, this F c is written equal to k r into x 1 plus delta 0 minus x 2.

So, where k r equal to t combined series stiffness of k P E and k 3. We can take this control law in this way. So, now, we are taking a delay time delay feedback. So, taking tau d as the time delay. So, we can write V equal to k c into x 1 double dot T minus tau d.

So, here you just see here we have taken this time delay as tau d. So, in this acceleration feedback. So, we are assuming there is a delay of tau d. So, when you are applying the respon[se] when we are applying or sensing the response and there will be a time lag when it will actually actuate. So, for the actuation.

So, we can take V equal to k c into x 1 double dot into T minus tau d by taking this way. So, we can write down this equation of motion in its non-dimensional form, this control force now can be written F c equal to k r into x 1 minus x 2 plus n d 3 three k c into x 1 double dot into T minus tau d.

(Refer Slide Time: 31:29)



The governing equation of motion can be written in this form. So, that is taking these tau 1 equal to tau equal to omega 1 1 into t omega 11 into omega 1 into t we have taken in this case.

So, as we have taken this is equal to omega 1 into t. So, you can write down this equation in this form d square x 1 by d tau square plus omega n 1 square x 1 plus 2 zeta 1 plus omega r by mu 2 zeta 2 d x 1 by d tau equal to omega r square by mu into x 2 plus 2 zeta 2 omega r d x 2 by mu 2 zeta 2 omega r by mu into d x 2 by d tau and minus alpha 12 x 1 square.

So, this is the quadratic nonlinearity we have taken and alpha 13 cubic non-linearity it is taken and then this is due to the difference in that alpha 21 x 1 minus x 2 whole square similarly cubic order non-linearity alpha 23 x 1 minus x 2 whole cube plus this forcing we have taken three different type of forcing we have taken.

So, 2 forcing that is F 1 cos omega 1 tau plus F 2 cos omega 2 tau. So, plus so this is the parametric forcing term this is F 3 cos omega 3 tau which is coefficient of x 1. So, minus F c 1 d square x 1 tau minus tau d by d tau square. So, here you just see this forcing are not taken of the order of epsilon. So, in that case we can tell that is to be hard excitation.

(Refer Slide Time: 33:19)

Contd...

where

$$\omega_{n1}=1, 2\xi_1=\frac{c_1}{m_1\omega_1}, \omega_r=\left(\frac{\omega_2}{\omega_1}\right), \omega_2=\sqrt{\frac{k_2+k_r}{m_2}}, \omega_1=\sqrt{\frac{k_1+k_2+k_r}{m_1}}, \mu=\frac{m_1}{m_2}, 2\xi_2=\frac{c_2}{m_2\omega_2}, \alpha_{12}=\frac{k_{12}}{m_1\omega_1^2},$$

$$\alpha_{13}=\frac{k_{13}}{m_1\omega_1^2}, \alpha_{21}=\frac{k_{21}}{m_2\omega_2^2}, \alpha_{23}=\frac{k_{23}}{m_2\omega_2^2}, F_1=\frac{\tilde{F}_1}{\omega_1^2 x_0}, F_2=\frac{\tilde{F}_2}{\omega_1^2 x_0}, F_3=\frac{\tilde{F}_3}{\omega_1^2 x_0}, F_{c1}=\frac{k_r k_c n d_{33}}{m_1},$$

$$\Omega_1=\frac{\Omega_{11}}{\omega_1}, \Omega_2=\frac{\Omega_{21}}{\omega_1}, \Omega_3=\frac{\Omega_{31}}{\omega_1}$$

A small book keeping parameter ε is considered for ordering Eq. as follows

Variable	ξ_1	ξ_2	$\frac{1}{\mu}$	α_{12}	α_{13}	α_{21}	α_{23}	F_{c1}	F_1	F_2	F_3
Scaling	ε	ε	ε	ε^2	ε^2	ε^2	ε^2	ε^2	ε	ε	ε

Similarly, we can write down this equation for x 2, after writing this equation for x 1 and x 2 we can non dimensionalize that thing.

So, by first by dividing this m 1 and then writing you just see this omega r is taken to be omega 2 by omega 1 and omega 2 equal to k 2 plus k r by m 2 root over and omega 1 equal to root over k 1 plus k 2 plus k r by m 1 and this mu equal to m 1 by m 2 2 zeta 2 equal to c 2 by m 2 omega 2 alpha 1 2 equal to k 1 2 by m 1 omega 1 square.

Similarly, alpha 13 equal to k 13 by m 1 omega 1 square, alpha 23 alpha 21 equal to k 21 by m 2 omega 2, alpha 23 equal to k 23 by m 2 omega 2 square and this forcing term are non dimensionalized as F 1 by omega 1 square x 0, F 2 by omega 1 square x 0, and F 3 by omega 1 square x 0.

Similarly, F c 1 equal to written k c k r k c n d 33 by m 1 where omega 1 non dimensional first frequency equal to omega 1 by omega 1 omega 2 equal to omega 21 by omega 1 and omega 3 equal to omega 31 by omega 1. So, you can note that the equations what we have written is not unique.

So, you can write the equation in a different form also you can write the linear part using this matrix form and you can then apply these modal analysis method to reduced this mass matrix and stiffness matrix to the uncoupled form and then one can write down these equation separately.

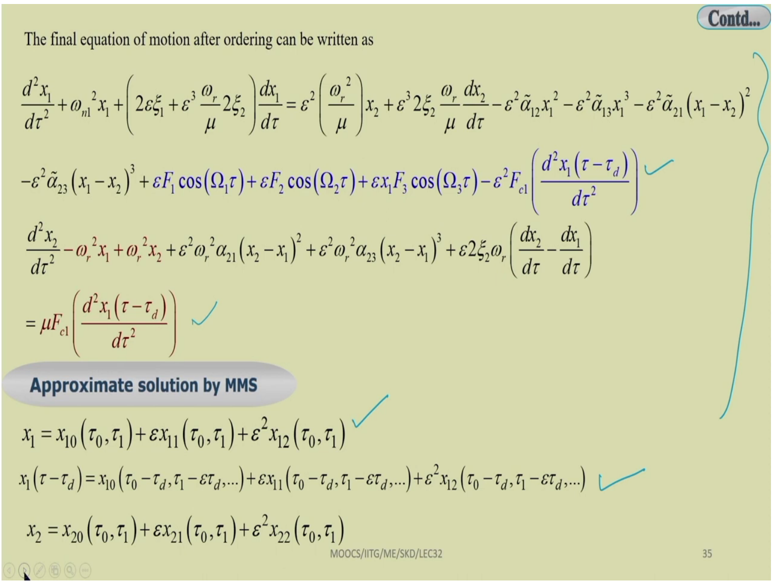
So, where the linear part will be uncoupled and the non-linear part and some forcing part will be coupled. So, that will give or that will yield another set of equations, you may also use this weighted model matrix. So, instead of this model matrix p one can use this weighted model matrix.

So, in that case the resulting mass matrix will have the unit vector. So, I will be equal to. So, I will be equal to identity matrix and this coefficient of this x that is the displacement will contain the eigenvalues of the system. So, it may be omega 1 square 0 0 omega 2 square.

So, that way also one can write another set of equations the writing of this equation is not unique. So, you should understand that the writing of these equations are not unique, but you can reconvert these equations to its original form after finding the response of the system. One can use this modal analysis method to rewrite these equations also actually depending on the applications you can choose or you can check what will be the order of this damping.

So, for example, in this particular case this order of damping are taken to be epsilon order. So, then this alpha 12, alpha 13, alpha 21, alpha 23 F c 1. So, these are taken to be order of epsilon square and F 1, F 2 order of epsilon and F 1, F 2, F 3 are taken to be order of epsilon.

(Refer Slide Time: 37:00)



In this way. So, you can non dimensionalize and write down the equation of motion by ordering it with the use of different bookkeeping parameter. So, epsilon. So, it may be of the order of epsilon, it may be some terms may be of the order of epsilon square and some of them may be of the order of epsilon to the power 0, epsilon to the power 1 and epsilon to the power 2.

So, you can you may go higher order also, but if you go for higher order then the number of equations will go on increasing without much increase in the precision value of the response. So, up to second order you may go. So, if you are going for higher order.

So, you may go for the symbolic software tools or use the symbolic software tools to write down the equation of motion or find the equation of motion. We can get a final set of equation of motion non dimensional set of equation of motion. So, here it is written as d square x 1 by d tau square plus omega n square x 1.

So, here you just see omega n 1 is not taken to be 1. So, 1 may take it take that thing equal to 1. So, if this omega 1 is taken to be equal to omega n 1. So, depending on how we are taking this non dimensional parameter, the coefficients of our equation will change.

So, here we have 3 this 3 forcing that is epsilon F 1 cos omega 1 tau epsilon, F 2 cos omega 2 tau epsilon x 1 F 3 cos omega 3 tau minus epsilon square F c 1 d square x 1 tau minus tau d by d tau square. So, here in the forcing. So, we have use the acceleration feedback. So, time delay acceleration feedback is taken here.

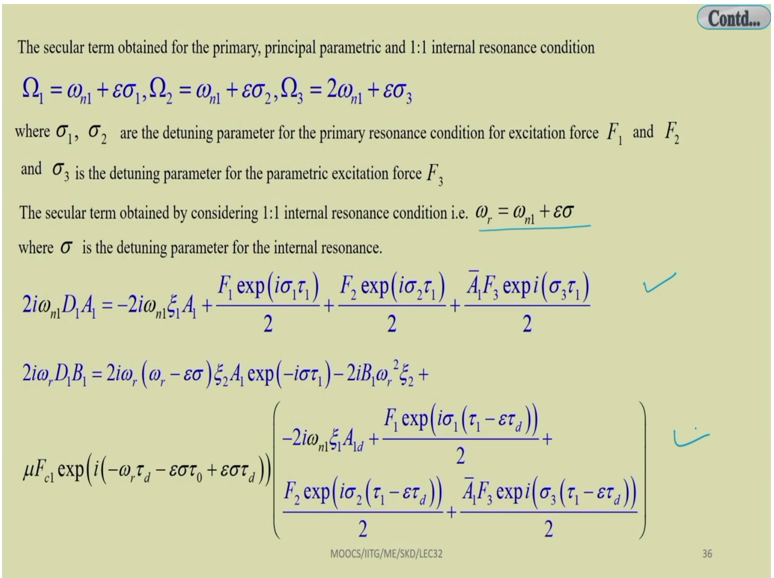
So, similarly the second equation can be written using the time delay acceleration feedback. So, d square x 1 x 1 is a function of tau and tau d tau d is the time delay function. So, then one can solve this equation by using this method of multiple scale. So, it can be taken x 1 equal to x 1 0 plus epsilon x 1 one plus epsilon square x 1 2.

And you just see how the delay term is written. So, delay term is written simply by using the same expression of this one. So, where this tau is replaced by tau 0 minus tau d, and this tau 1 is replaced by tau 1 minus epsilon tau d, tau 0, tau 1, tau 2. So, they are of the different time scale.

So, here for the delay. So, this time scales also has to be modified. So, for example, this tau 0 will be tau 0 minus tau d, tau 1 which is epsilon tau 0 it can be written tau 1 minus epsilon tau d, similarly the higher order terms also can also be written this x 1 tau minus tau d.

So, the d is a delay by using the delay term one can write the displacement x 1 in this form. Similarly, displacement x 2 can be written x 2 0 tau 0 tau 1 plus epsilon x 2 1 tau 0 tau 1 epsilon square x 2 2 tau 0 tau 1 and the corresponding time delay term also can be written.

(Refer Slide Time: 40:41)

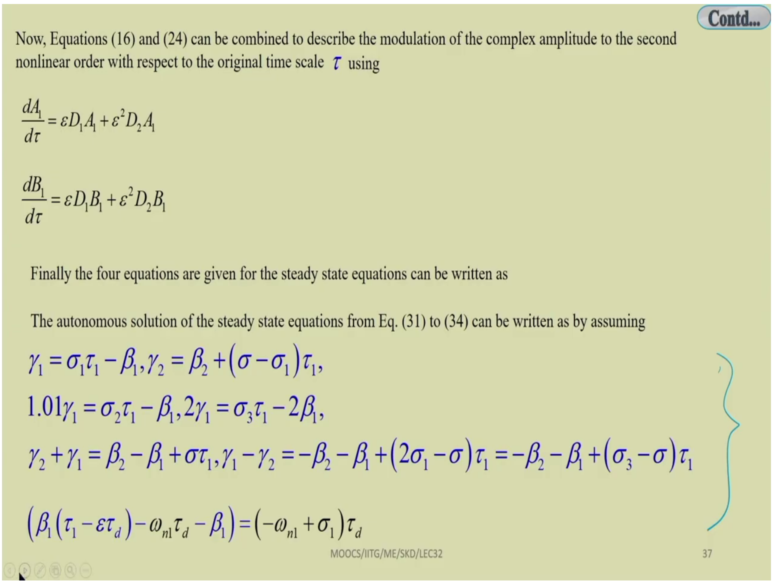


So, by taking these primary principal parametric and 1 is to 1 internal resonance condition. So, here if we are taking omega 2 equal to omega 1 that is the frequency of the first mode equal to the frequency of the second mode.

So, in that case we can write this equation in this form by using this detuning parameters, we can write omega 1 equal to omega n 1 plus epsilon sigma 1 omega 2 equal to omega n 2 plus epsilon sigma 2 and omega 3 equal to 2 omega n 1 plus epsilon sigma three.

Now, following the previous procedures for internal resonance condition. So, you can take this omega r equal to omega n 1 plus epsilon sigma, now we can write the reduced or we can write the terms which give rise to secular term. So, this is the term which gives rise to secular term. So, this term has to be eliminated to get the response of the system.

(Refer Slide Time: 41:46)



So, here we can take this d A 1 by d tau equal to epsilon D 1 A 1 plus epsilon square D 2 A 1. So, similarly d B 1 by d tau equal to epsilon D 1 B 1 plus epsilon square D 2 B 1 finally, we can get a set of reduced equations and those reduced equation can be solved to find the response one can use non autonomous form.

So, previously we have taken in terms of. So, you just see a equal to half a capital A equal to half a e to the power i beta generally we take and then we can convert that beta to gamma form or to remove the time terms in this equation of motion to make the equation autonomous.

So, we can use that one. So, we have to use this gamma 1 equal to sigma 1 T 1 minus beta 1 gamma 2 equal to beta 2 plus sigma minus sigma 1 into tau 1 by properly writing these terms we can make the equations autonomous here this time terms will be eliminated in this study state.

(Refer Slide Time: 42:57)

In this section **MMS version II** is proceeded, i.e. the terms $D_1A_1,\ D_1^2A_1,\ D_1B_1\, D_1^2B_1$ present in the equation with order of ε^2 are made zero, whereas in MMS version I, these terms are kept which is derived in the above equations. Hence, MMS version I contains some additional terms. So the secular term and the solution obtained in the Eq. (24) and (25) are modified as

$$
\begin{aligned}
2i\omega_{n1}D_2A_1 &= F_{c1}\omega_{n1}^2A_1\exp\left(i\left(-\omega_{n1}\tau_d\right)\right)+\frac{\omega_r^2B_1\exp\left(i\left(\varepsilon\sigma\right)\tau_0\right)}{\mu}-3\alpha_{13}A_1^2\bar{A}_1-3\alpha_{23}A_1^2\bar{A}_1\\
&+3\alpha_{23}A_1^2\bar{B}_1\exp\left(-i\left(\varepsilon\sigma\right)\right)+6\alpha_{23}A_1\bar{A}_1B_1\left(\exp\left(i\left(\varepsilon\sigma\right)\tau_0\right)\right)-6\alpha_{23}A_1B_1\bar{B}_1\\
&-3\alpha_{23}\bar{A}_1B_1^2\exp\left(i\left(2\varepsilon\sigma\right)\tau_0\right)+3\alpha_{23}B_1^2\bar{B}_1\exp\left(i\left(\varepsilon\sigma\right)\tau_0\right)-\frac{A_1F_3^2}{4\left(8\omega_{n1}^2+\varepsilon\left(\varepsilon\sigma_3^2+6\omega_{n1}\sigma_3\right)\right)}
\end{aligned}
$$

$$
\begin{aligned}
2i\omega_rD_2B_1 &= 3\alpha_{23}\omega_r^2A_1^2\bar{A}_1\exp\left(-i\varepsilon\sigma\tau_0\right)-3\alpha_{23}\omega_r^2\left(A_1^2\bar{B}_1\exp(-2i\varepsilon\sigma\tau_0)+A_1\bar{A}_1B_1+A_1\bar{A}_1B_1\right)+\\
&3\alpha_{23}\omega_r^2\left(\bar{A}_1B_1^2\exp\left(i\varepsilon\sigma\tau_0\right)+A_1B_1\bar{B}_1\exp\left(-i\varepsilon\sigma\tau_0\right)+\bar{A}_1B_1\bar{B}_1\exp\left(-i\varepsilon\sigma\tau_0\right)\right)-3\alpha_{23}\omega_r^2B_1^2\bar{B}_1\\
&+\mu F_{c1}\exp\left(i\left(-\omega_r\tau_d-\varepsilon\sigma\tau_0+\varepsilon\sigma\tau_d\right)\right)\left(\begin{aligned}&F_{c1}\omega_{n1}^2A_1\exp\left(-i\omega_{n1}\tau_d\right)+\frac{\omega_r^2B_1\exp\left(i\left(\varepsilon\sigma\right)\tau_0\right)}{\mu}-3\alpha_{13}A_1^2\bar{A}_1\\
&-3\alpha_{23}A_1^2\bar{A}_1+3\alpha_{23}A_1^2\bar{B}_1\exp\left(-i\left(\varepsilon\sigma\right)\right)+6\alpha_{23}A_1\bar{A}_1B_1\left(\exp\left(i\left(\varepsilon\sigma\right)\tau_0\right)\right)\\
&-6\alpha_{23}A_1B_1\bar{B}_1-3\alpha_{23}\bar{A}_1B_1^2\exp\left(i\left(2\varepsilon\sigma\right)\tau_0\right)+3\alpha_{23}B_1^2\bar{B}_1\exp\left(i\left(\varepsilon\sigma\right)\tau_0\right)\\
&-\frac{A_1F_3^2}{4\left(8\omega_{n1}^2+\varepsilon\left(\varepsilon\sigma_3^2+6\omega_{n1}\sigma_3\right)\right)}\end{aligned}\right)
\end{aligned}
$$

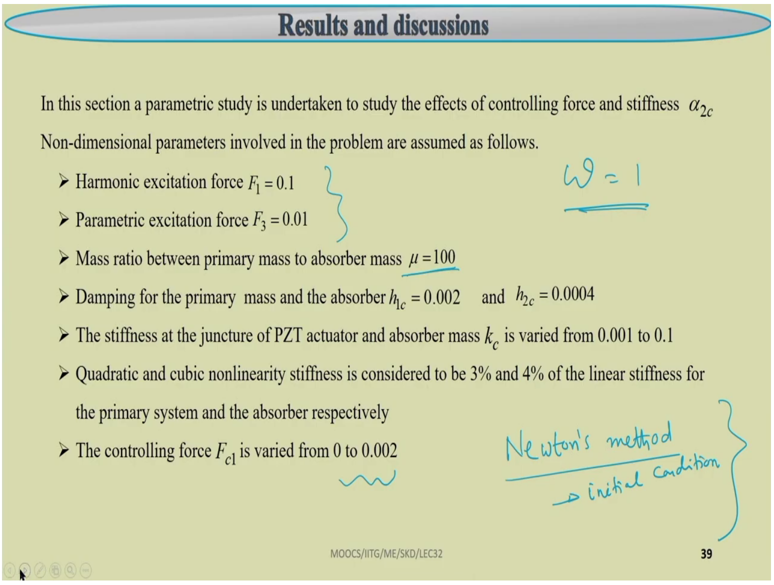
MOOCS/IITG/ME/SKD/LEC32 38

So, one can use this method of multiple scale version 1 or version 2. So, in version 2. So, the normal way when we are deriving that is version 1. So, in case of the version 2 the term D 1 A 1 D 1 square A 1 D 1 B D 1 square B 1 present in the equation with order epsilon square are made zero, whereas in MMS version 1 these terms are kept which is derived in the above equation.

Hence method of multiple scale version 1 contain some additional term the secular term and the solution obtained using those equations are modified. So, if you want to use method of multiple scale version 2 proposed by Rahman and Burton. So, you can write down these equations in this form.

So, here the term with D 1 A 1 D 1 square A 1 and then D 1 B 1 D 1 square B 1 are eliminated while studying the terms while studying the term which gives rise to secular terms.

(Refer Slide Time: 44:15)



So, these are the equation we obtained now solving this equation. So, you just see we have a set of equation. So, we can solve these equations by using numerical method particularly you can use this Newton's method which actually required which actually required the initial condition initial condition to find the response of the system. So, here to find this initial condition you may solve this equation by using ODE 4-5 or by using this Runge Kutta method.

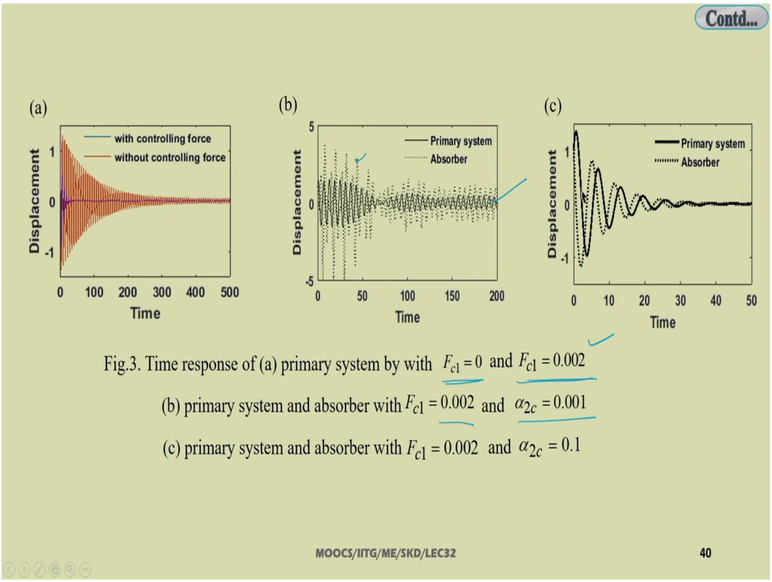
And then taking those response as the initial condition one can make further study to find the response of the system. Otherwise one can use this continuation technique to find the response of that system for a wide range of frequency. We have studied the how you can use this controlling force and in this numerical analysis we will see by taking a harmonic force of F 1 equal to 0.1 here you just see we have taken this omega equal to 1.

So, that is why that omega is the coefficient of x 1 to be 1. So, that is why to make the forcing term to be non-weak forcing or hard forcing. So, depending on that thing. So, we have to choose this F 1 and F 3. So, here F 1 is taken to be 0.1 and F 3 equal to 0.01.

So, mass ratio of the primary system to absorber is taken. So, already I told you that it is taken to be 100, previous literature limit their analysis of 2 mass ratio of 20. So, damping factor damping parameter of the mass and absorber is taken to be 0.002 and 0.004. So, the stiffness at the junction of PZT actuator and the absorber a mass k c is varied from 0.001 to 0.1.

So, quadratic and cubic nonlinearity stiffness are considered to be 3 percent and 4 percent of the linear stiffness of the primary and of the primary system and the absorber respectively. So, controlling force F c 1 is varied from 0 to 0.002. So, taking this numerical values. So, one can study the response of the system.

(Refer Slide Time: 46:58)



So, here the response of the system. So, initially we have plotted the time response. So, clearly you can see the time response without control and with control. So, with control. So, the line you can see clearly it is it shows that it required very less time to control the response of the system with a control force, but without control force it will take a very large time to obtain the steady state response.

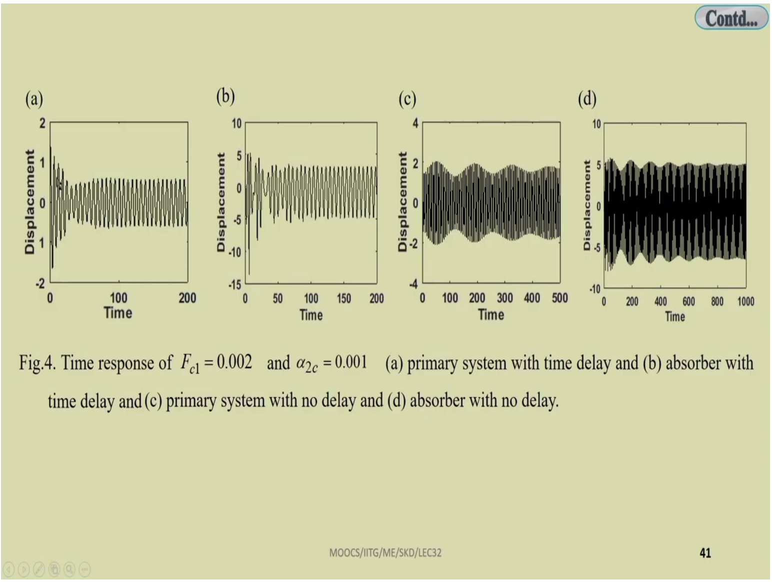
So, in the first figure you have seen this F c 1 equal to 0, and F c 2 you can take to be 0.002. Similarly, in the primary system F c 1 equal to 0 point. So, in the first case. So, in this first figure we have taken F c 1 equal to 0. So, which is without control force and F c 1 by applying a very very small F c 1 that is control force equal to 0.002.

So, we can see the settling time is very very less. So, quickly it reduces to its equilibrium position by applying this control force. So, similarly primary system absorber with F c 1 equal to 0.002. So, the first case it is F c 1 equal to 0.002 and alpha 2 c equal to 0.001, here the primary system and absorber are shown this is the primary system response you can absorb that primary system this is the primary system response and this is the absorber response.

So, the primary response is reduced significantly what the absorber response increases in case of the vibration absorber. So, that takes place. So, generally we control the vibration of the primary system and the secondary system oscillate and to control that oscillation of the secondary system.

So, generally. So, damping is used in the system. The system can be made non-linear or the damping can also be used in the third figure it can be clearly shown the displacement and the time. So, primary system. So, this is the primary system and this is the absorber.

(Refer Slide Time: 49:23)

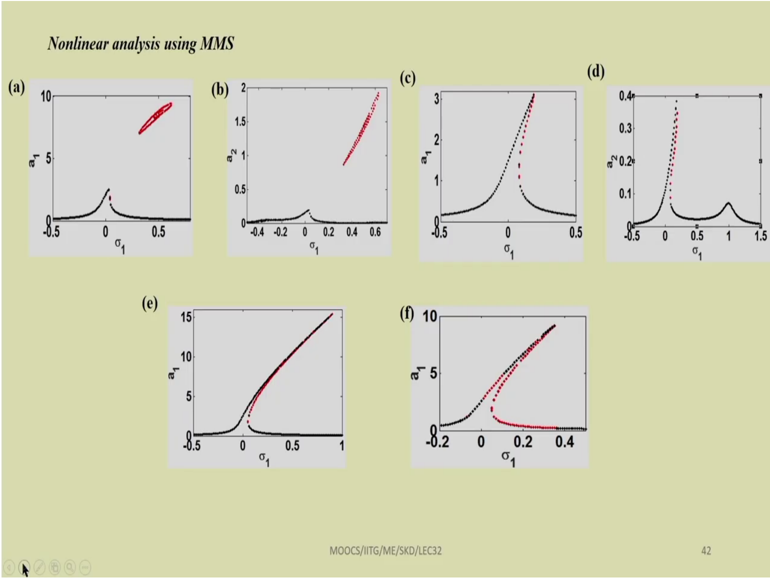


So, now by taking this time delay. So, time response of F c 1. So, this is the time response of F c 1. So, primary system with time delay. So, this is primary system and absorber with time delay primary system with no time delay. So, here in this case there is no time delay in the primary system. In c primary system with no time delay absorber with no delay.

So, it is without delay and first two figures are with delay you can easily see with delay. So, we can control actually here you can absorb some beating type of phenomena also. In case of the systems without delay and here with delay you can easily see the response amplitude is reduced the response amplitude is reduced.

So, the delay is acting. So, you have observed that the delay is acting as a damper to the system.

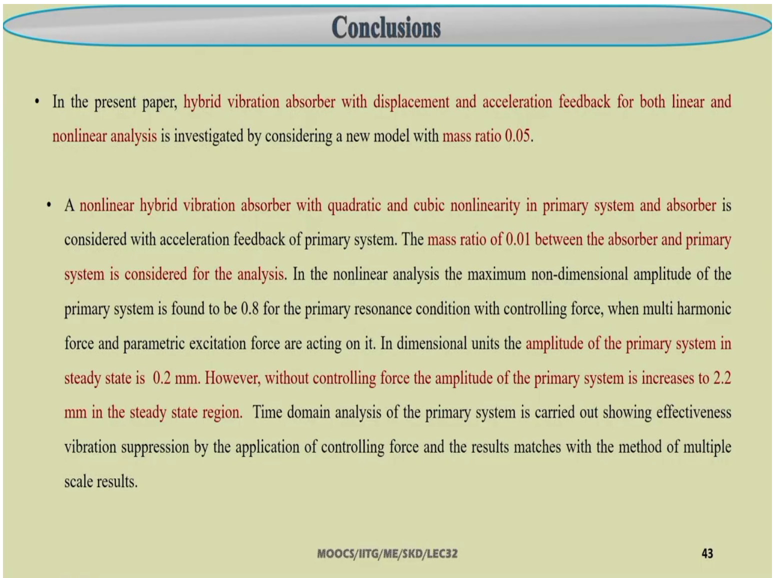
(Refer Slide Time: 50:25)



Using this non-linear analysis, using method of multiple scale. If you find the response it is showing this hardening type of effect with different bifurcations. So, you just see here we have this hopf bifurcation and here you have the saddle node bifurcation.

So, these are the different responses we have observed in this case. So, in the present work. So, we have seen this hybrid vibration absorber with displacement and acceleration feedback for both linear and non-linear analysis.

(Refer Slide Time: 50:43)



When the mass ratio is taken to be of the order of 0.05 that is the conventional a work that is up to 20, we have gone also here mass ratio of to 100 and we have seen the response of the system for different response of the system for different mass ratio and different thing.

(Refer Slide Time: 51:17)

Mathematical Modelling

The equations of motion of the system in the Fig. can be written as

$$m_1\ddot{x}_1 + k_1x_1 + k_{12}x_1^2 + k_{13}x_1^3 + k_2(x_1 - x_2)$$
$$+k_{21}(x_1 - x_2)^2 + k_{23}(x_1 - x_2)^3 + c_1\dot{x}_1 + c_2(\dot{x}_1 - \dot{x}_2)$$
$$= F_{11}\cos(\Omega_{11}t) + F_{21}\cos(\Omega_{21}t) + x_1F_{31}\cos(\Omega_{31}t) - F_c$$

$$m_2\ddot{x}_2 + k_2(x_2 - x_1) + k_{21}(x_2 - x_1)^2$$
$$+k_{23}(x_2 - x_1)^3 + c_2(\dot{x}_2 - \dot{x}_1) = F_c$$

$$F_c = k_r(x_1 + \delta_0 - x_2), \quad \text{where} \quad k_r = \frac{k_P^E k_3}{k_P^E + k_3}$$

$$\delta_0 = nd_{33}V \qquad V = k_c(\ddot{x}_1(T - \tau_d))$$

$$F_c = k_r(x_1 - x_2 + nd_{33}k_c\ddot{x}_1(T - \tau_d))$$

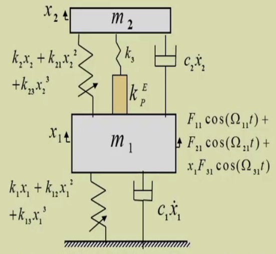


Figure 1 Piezoelectric stack actuator based nonlinear active vibration absorber.

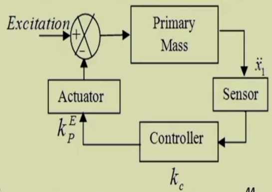
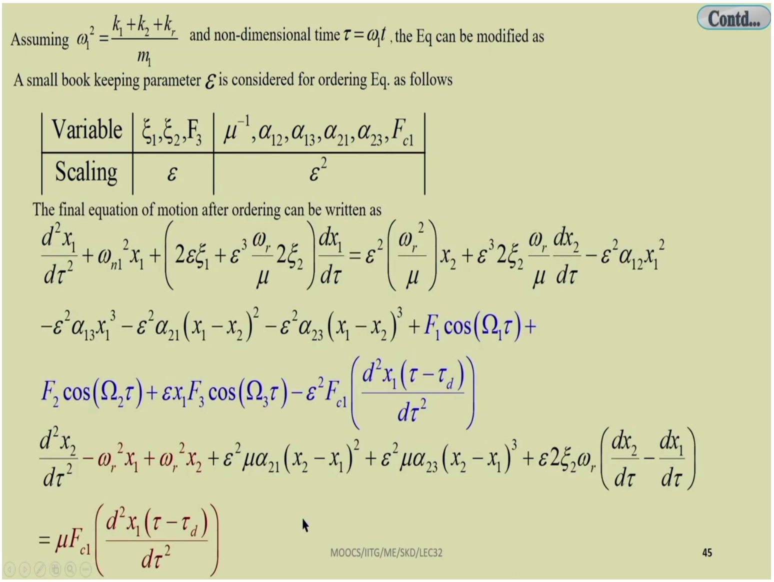


Fig. Block diagram with acceleration feedback of primary system.

(Refer Slide Time: 51:34)



(Refer Slide Time: 51:39)

Contd...

where

$$\omega_{n1} = 1,\ 2\xi_1 = \frac{c_1}{m_1\omega_1},\ \omega_r = \left(\frac{\omega_2}{\omega_1}\right),\ \omega_2 = \sqrt{\frac{k_2 + k_r}{m_2}},\ \mu = \frac{m_1}{m_2},\ \omega_1 = \sqrt{\frac{k_1 + k_2 + k_r}{m_1}},$$

$$2\xi_2 = \frac{c_2}{m_2\omega_2},\ \alpha_{12} = \frac{k_{12}}{m_1\omega_1^2},\ \alpha_{13} = \frac{k_{13}}{m_1\omega_1^2},\ \alpha_{21} = \frac{k_{21}}{m_1\omega_1^2},\ \alpha_{23} = \frac{k_{23}}{m_1\omega_1^2},\ F_1 = \frac{F_1}{m_1\omega_1^2},$$

$$F_2 = \frac{F_2}{m_1\omega_1^2},\ F_3 = \frac{F_3}{m_1\omega_1^2},\ F_{c1} = \frac{k_r k_c n d_{33}}{m_1},\ \Omega_1 = \frac{\Omega_{11}}{\omega_1},\ \Omega_2 = \frac{\Omega_{21}}{\omega_1},\ \Omega_3 = \frac{\Omega_{31}}{\omega_1}$$

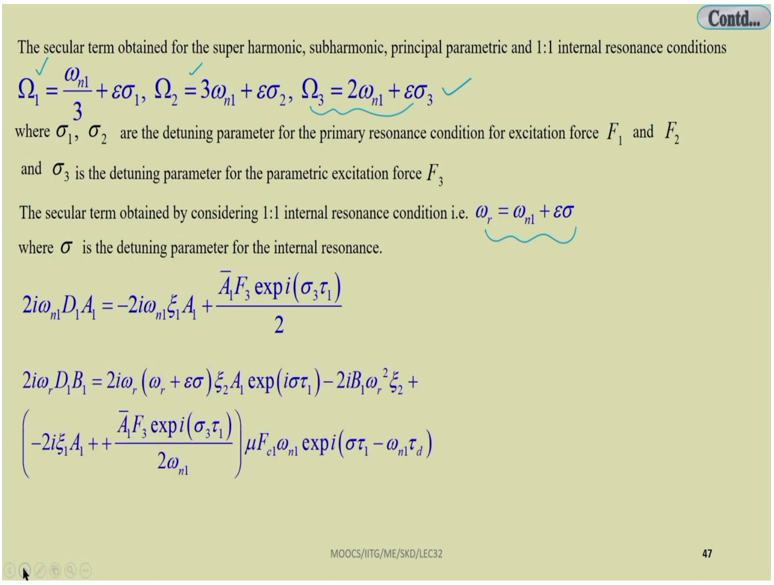
Approximate solution by MMS

$$x_1 = x_{10}(\tau_0, \tau_1) + \varepsilon x_{11}(\tau_0, \tau_1) + \varepsilon^2 x_{12}(\tau_0, \tau_1)$$

$$x_1(\tau - \tau_d) = x_{10}(\tau_0 - \tau_d, \tau_1 - \varepsilon\tau_d, \ldots) + \varepsilon x_{11}(\tau_0 - \tau_d, \tau_1 - \varepsilon\tau_d, \ldots) + \varepsilon^2 x_{12}(\tau_0 - \tau_d, \tau_1 - \varepsilon\tau_d, \ldots)$$

$$x_2 = x_{20}(\tau_0, \tau_1) + \varepsilon x_{21}(\tau_0, \tau_1) + \varepsilon^2 x_{22}(\tau_0, \tau_1)$$

(Refer Slide Time: 51:43)



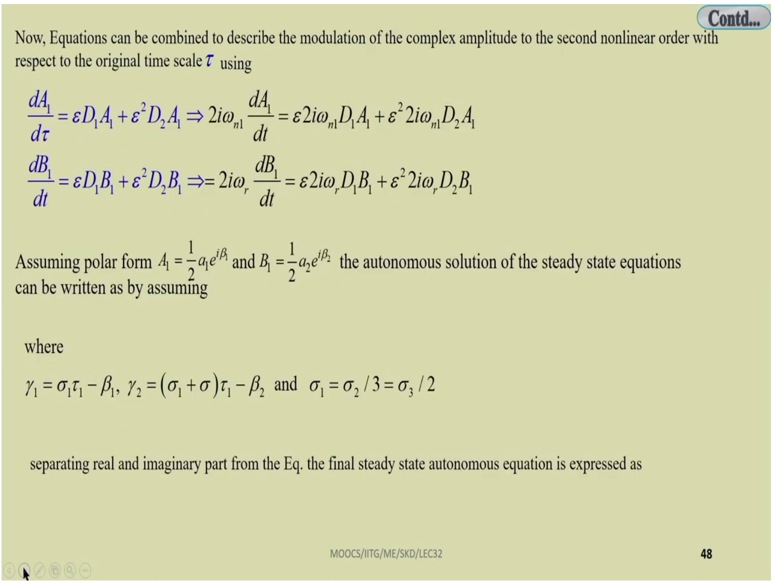
So, if we can consider the. So, now, also we can consider other different system that is sub harmonic and super harmonic resonance condition. So, in sub harmonic and super harmonic resonance conditions. So, we can see the response one can follow the similar procedure to find the solution.

So, in case of super harmonic resonance condition. So, you just see the forcing term the frequency can be taken in a different way. So, this omega 1 can be taken as omega n 1 by 3 plus epsilon sigma 1 and omega 2 is taken to be 3 times omega n 1 in case of super harmonic.

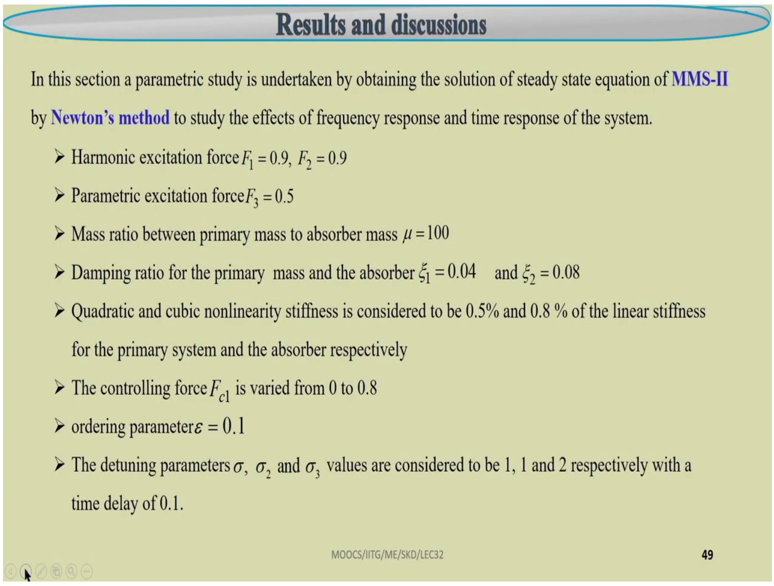
So, this is the condition in the super harmonic that is omega 1 equal to omega n 1 by 3 and sub harmonic this is the condition for sub harmonic that is omega 2 equal to 3 into omega n 1 plus epsilon sigma 2 and it is parametric also principal parametric omega 3 equal to 3 2 omega 1 plus epsilon sigma 3 internal resonance conditions also one can consider.

So, in that case omega r is considered to be omega n 1 plus epsilon sigma 1 and proceeding in the similar way.

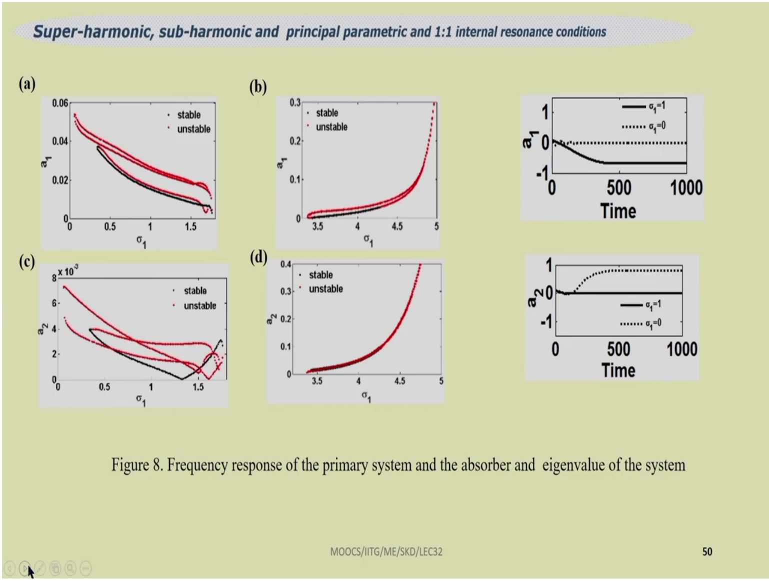
(Refer Slide Time: 52:42)



(Refer Slide Time: 52:45)

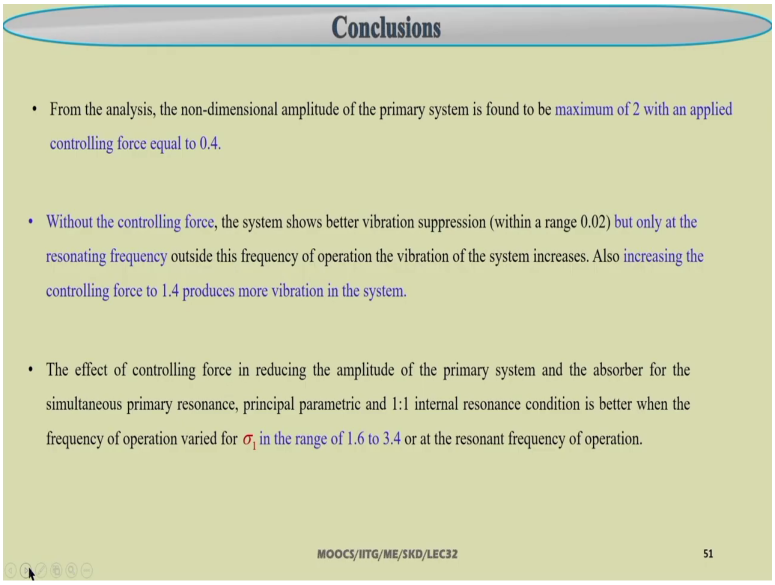


(Refer Slide Time: 52:48)



So, we can get the reduced equation and after solving this reduced equation we can study the response amplitude. So, here you can see several response amplitudes are been plotted.

(Refer Slide Time: 52:54)



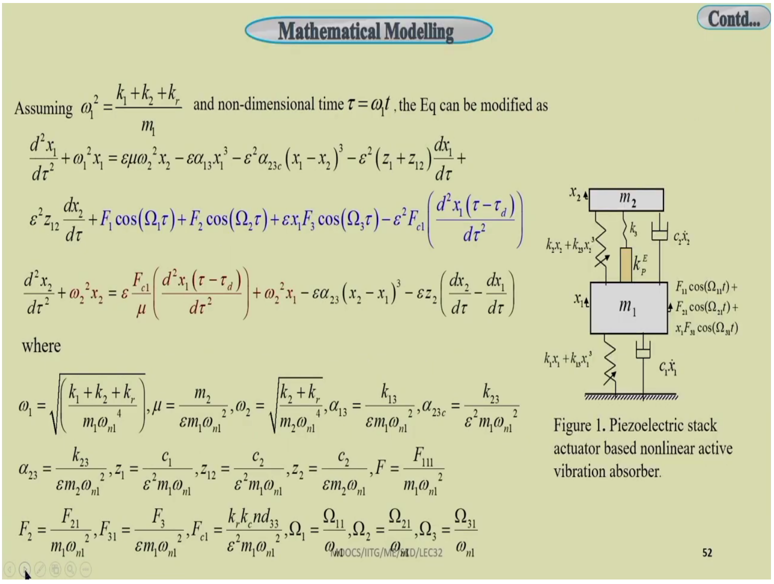
So, you can observe that. So, the non-dimensional amplitude of the primary system is found to be maximum of 2 with an applied controlling force equal to 0.4.

So, without the controlling force the system shows better vibration suppression with a range of 0.02, but only at the resonating frequency outside the frequency of operation the vibration of the system increases also increasing the controlling force to 1.4 produces more vibration in the system.

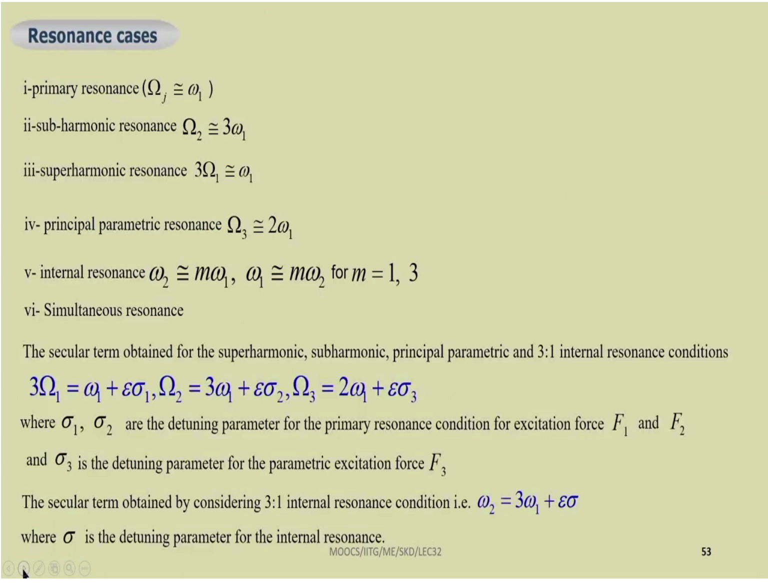
So, the effect of controlling force in reducing the amplitude of the primary system and the absorber for the simultaneous primary resonance principal parametric and 1 is to 1 internal resonance condition is better when the frequency of operation is worried for sigma 1 in the range of 1.6 to 3.4 or at resonant frequency of operation. By controlling this detuning parameter.

So, in this case we have seen. So, we have applied three condition simultaneously. So, we have the super harmonic, sub harmonic as we have taken 3 frequency terms. So, we have we can apply sub harmonic, super harmonic and principal parametric resonance condition simultaneously.

(Refer Slide Time: 54:21)



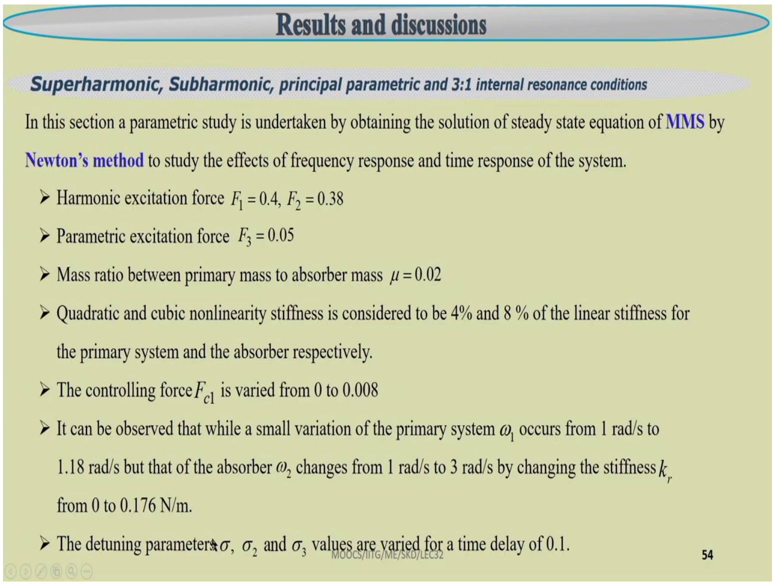
(Refer Slide Time: 54:27)



If we are taking only single frequency then this is not possible. One can do this further analysis. So, there are several other analysis one can do. So, for example, omega j equal to omega 11 can take this is primary resonance conditions one can take then in this case sub harmonic omega 2 equal to 3 omega 1, super harmonic 3 omega 1 equal to omega one can be taken.

And then principal parametric resonance condition omega 3 equal to 2 omega 1 and internal resonance conditions omega 2 equal to m omega 1, omega 1 equal to m omega 2 many different conditions can be taken. So, one can take this 3 is to 1 internal resonance conditions also.

(Refer Slide Time: 55:16)



(Refer Slide Time: 55:21)

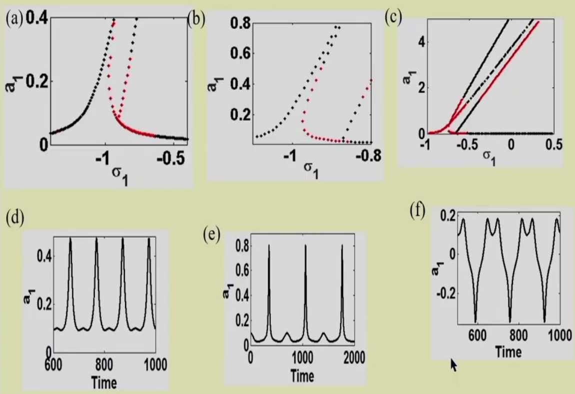
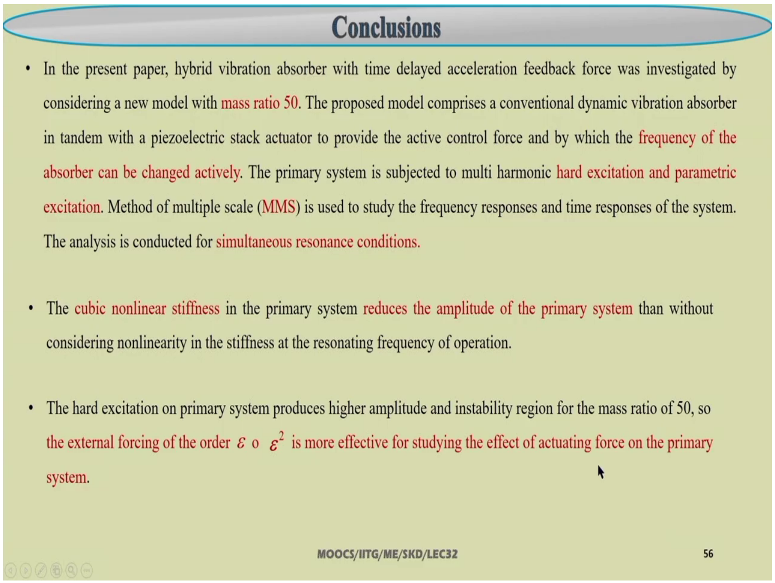


Figure 9. Frequency response of the primary system and the absorber (a) for $F_{c1} = 0.8$, $\alpha_{13} = 0.8$ $\alpha_{23} = 8$ $z_2 = 6$ at $\sigma = 0$ (b) for $F_{c1} = 0$, $\alpha_{13} = 0$ $\tilde{\alpha}_{23} = 0.4$ $z_2 = 6$ at $\sigma = 0$ (c) for $F_{21} = 0$, $F_{c1} = 0.8$,

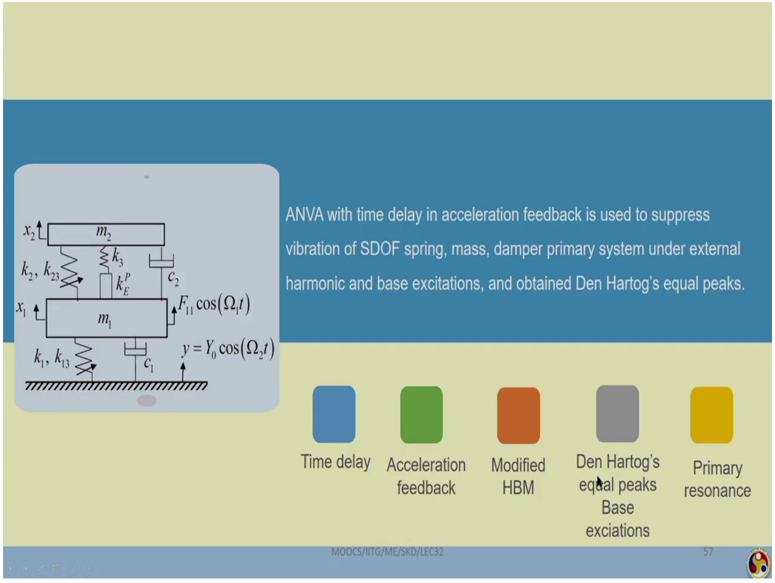
MOOCS/IITG/ME/SKD/LEC32

55

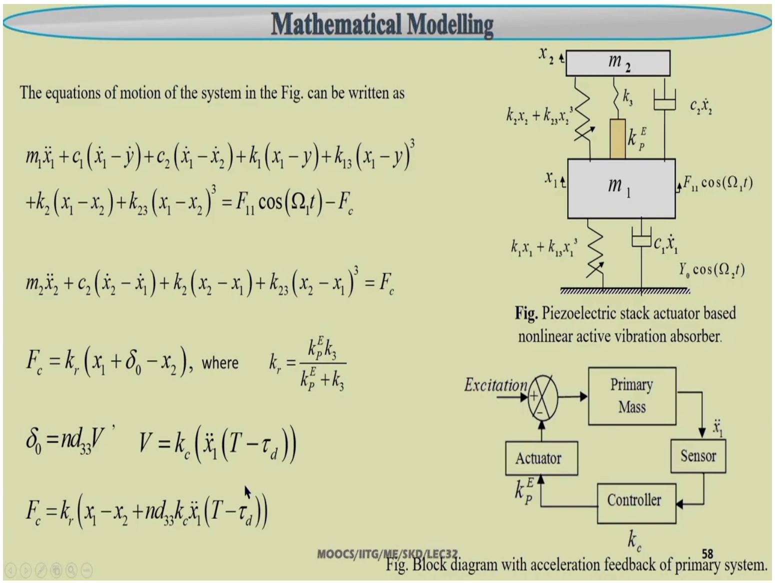
(Refer Slide Time: 55:27)



(Refer Slide Time: 55:33)



(Refer Slide Time: 55:37)



So, by taking all these internal resonance conditions and external resonance condition. So, one can study this absorber and by studying this absorber so, one can find the results and one can sees the different frequency response and time response of the system and in this way so, one can study the non-linear vibration absorber also one can study the non-linear vibration absorber by using in this harmonic balance method.

(Refer Slide Time: 55:41)

Contd...

The Eq. (1) and (2) are rewritten below in non- dimensional form by assuming non-dimensional time $\tau_1 = \omega_1 t$ where, $\omega_1 = \sqrt{k_1 / m_1}$.

The final equation of motion after ordering can be written as

$$\ddot{u}_1 + 2\xi_1\dot{u}_1 - 2\xi_2\dot{u}_2 + u_1 + \alpha_{13c}u_1^3 - (\alpha + \alpha_r)u_2 - \beta u_2^3 = F_1\cos\Omega\tau_1 + Y\cos(\Omega\tau_1 - \gamma)$$

$$+\alpha_{13c}\left(Y\cos(\Omega\tau_1 - \gamma)\right)^3 + 3\alpha_{13c}\begin{pmatrix} u_1^2 Y\cos(\Omega\tau_1 - \gamma) - \\ u_1\left(Y\cos(\Omega\tau_1 - \gamma)\right)^2 \end{pmatrix} - F_{c1}\ddot{u}_1(\tau_1 - \tau)$$

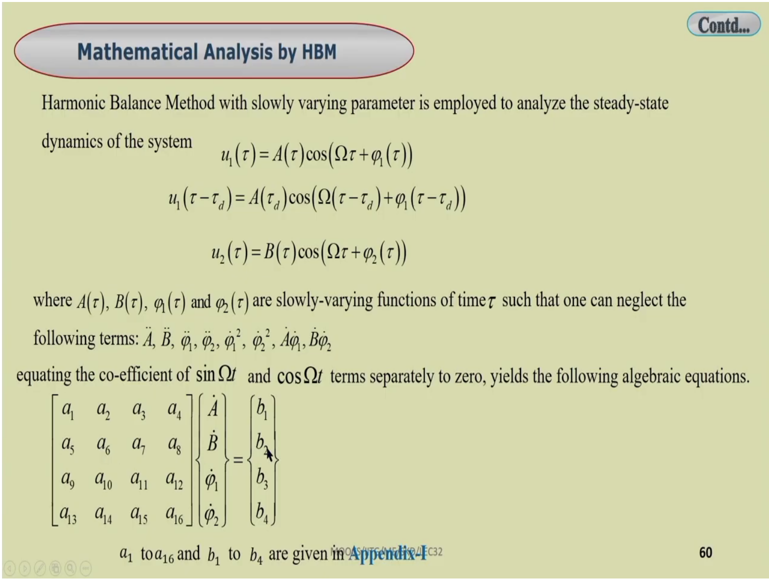
$$\mu\ddot{u}_2 + 2\xi_2\dot{u}_2 + (\alpha + \alpha_r)u_2 + \beta u_2^3 = F_{c1}\ddot{u}_1(\tau_1 - \tau) - \mu\ddot{u}_1$$

where

$$u_1 = x_1 / x_0,\ u_2 = (x_2 - x_1)/x_0,\ \mu = \frac{m_2}{m_1},\ \xi_1 = \frac{c_1}{2m_1\omega_1},\ \xi_2 = \frac{c_2}{2m_1\omega_1},\ \alpha = \frac{k_2}{k_1},\ \alpha_r = \frac{k_r}{k_1},\ Y = Y_0 / x_0,\ \alpha_{13c} = \frac{k_{13}x_0^2}{k_1},$$

$$\beta = \frac{k_{23}x_0^2}{k_1},\ F_1 = \frac{F_{11}}{m_1\omega_1^2 x_0},\ F_{c1} = \alpha_r k_c n d_{33},\ \Omega = \frac{\Omega_1}{\omega_1},\ \frac{\Omega_2}{\omega_1} = \Omega - \gamma,\ \gamma = \textit{phase},\ x_0 = \textit{reference length}$$

(Refer Slide Time: 55:44)



Just let me briefly show how harmonic balance method can be used in this way. In this case you can have the same equation motion what you can consider this u 1 tau equal to A tau cos omega tau plus psi 1 tau u 1 tau minus tau d equal to A tau d cos omega tau minus tau d plus psi 1 tau minus tau d u 2 tau equal to B tau cos omega tau plus psi 2 tau.

So, here this A B what we have considered are not constant, but slowly varying function of time in the previous harmonic balance method. So, we have taken these are constant, but here by taking these terms as slowly varying function of time. So, one can initially one can get this A B and psi 1, psi 2 and by perturbing that thing.

So, one can study the stability simultaneously to find the response plot. So, here by substituting this way one can get by collecting the coefficient of sin omega t and cos omega t.

So, these equations can be obtained and from these thing one can get this A 1 dot B 1 dot psi 1 dot psi 2 dot.

So, for steady state. So, this will be equal to 0 and to study the stability. So, one can find the eigenvalue of the Jacobean matrix and you can study like similar to that of the that we have study in case of the method of multiple scale.

In this way so, one can study the vibration absorber. So, whether it is a linear vibration absorber, non-linear vibration absorber, it is subjected to single frequency multi frequency with time delay, without time delay with internal resonance condition, without internal resonance condition and also with 1 is to 1 internal resonance condition with 1 is to 3 internal resonance condition.

So, several different combinations are possible. So, one can use required vibration absorber depending on the application of the system. Next class we are going to see one more application. So, to a different system. So, there we can take cutting tool vibration.

So, particularly we can take a lathe machine vibration, vibration of the tool and workpiece during turning and we will use a different method to solve the non-linear governing equation to obtain the instability region.

Thank you.