Nonlinear Vibration
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Lecture - 27
Cantilever beam based piezoelectric based energy harvester

So, welcome today class of Non-linear Vibration. So, last few classes we are studying the application of this non-linear vibration and particularly we have taken different systems for examples we have these cantilever beam. So, this cantilever beam can be base excited.

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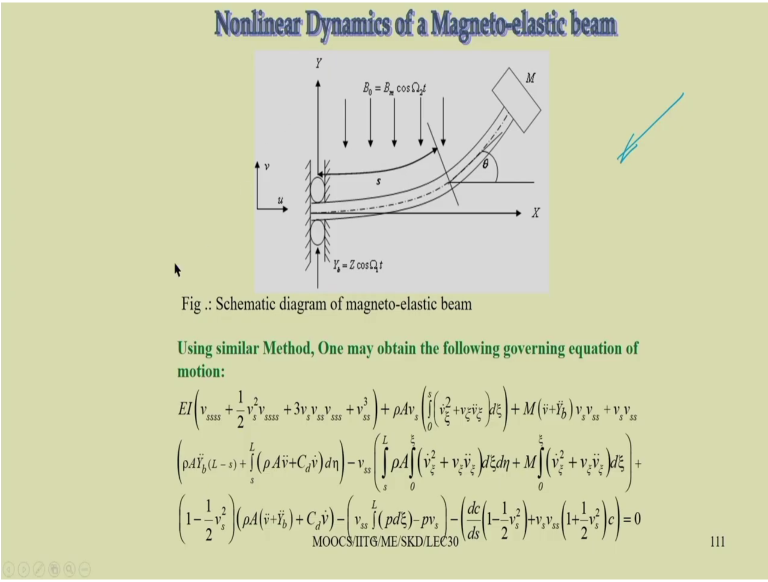

So, for example, last time we have taken this magneto elastic beam also. So, we have taken this beam as visco-elastic beam, magneto elastic beam or the elastic beam. In all this cases we have studied the non-linear dynamics of the system in some cases also we have applied this load.

So, we have applied a axial load, which produce actually or gives raise to parametrically excited system. Also we have taken in all this cases these based to be excited in all these cases we have studied the non-linear dynamics by fast deriving this spatio-temporal equation.

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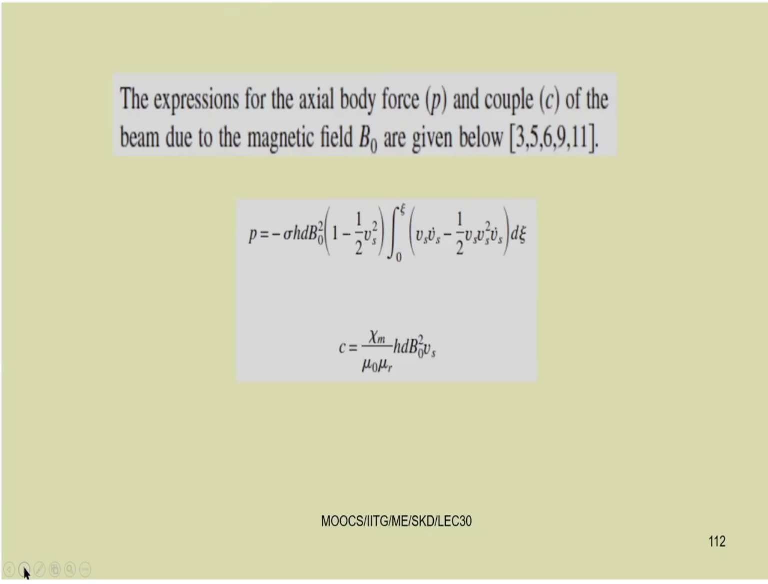

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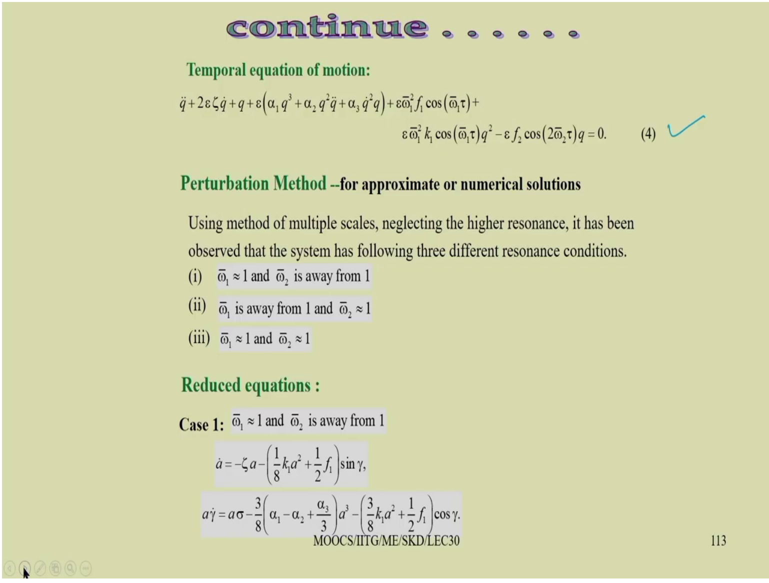

And then converting that spatio temporal equation to its temporal form by applying Galerkin method; generalized Galerkin method. And in this generalized Galerkin method so, we have used this assumed mode space and for example, this is one of the temporal equation we have derived.

After getting the temporal equation then we have used this perturbation method particularly this method of multiple scales and consider different resonance conditions. By considering different resonance conditions we have shown when the system will be stable unstable and also different bifurcation points. For example, so, we have studied the pitchfork bifurcation point. So, it may be super critical or sub critical pitchfork bifurcation points, then this saddle node bifurcation and then Hopf bifurcation points.

So, in Hopf also it may be super critical or sub critical bifurcation and previous two while previous two that is the pitchfork and saddle node bifurcation points are static bifurcations this Hopf bifurcation is a dynamic bifurcation, which gives raise to periodic response. So, later we have seen how this periodic response also bifurcates or it leads to period doubling and period doubling route to chaos.

So, by doing this Poincare section or by using this Poincare section, so, we have characterized the fixed point periodic, quasi periodic and chaotic responses for example, in case of periodic responses. So, we have in a single period. So, we have only single dot point in the phase portrait while we are drawing the Poincare section.

Similarly, for two periodic, we will have only two points for multiple multi periodic then it will have multiple points on the phase portraits, but if the response is chaotic, then it will fill up the whole space and if it is quasi periodic then we have seen that we can have a close loop in the phase portrait.

So, that way we have studied different routes to chaos. For example, period doubling route to chaos, torus breakdown route to chaos then torus doubling route to chaos also we have seen this crisis for example, this attracted margin crisis, interior crisis, exterior crisis and also we have seen this intermittency root to chaos in the simple cantilever beam when it is base excited and where also we have considered in addition to the external resonance conditions.

So, we have considered the internal resonance conditions. So, in internal resonance condition particularly we have taken this one is to 3. Internal resonance conditions 1 is to 3 is to 5 internal resonance condition, when we are considering 3 modes and 1 is to 3 is to 9 also in case of 3 mode interactions we can find.

Similarly, in case of the viscoelastic beam. So, we have use this concept of loss factor and there we have seen how we can decrease or increase the amplitude of response by changing different system parameter similarly in case of the magneto elastic beam. So, by applying magnetic field we have seen the parametric instability region. So, today class we are going to study or we will extend that same system by putting a piezoelectric patch on that thing. So, which can be used for the energy harvesting purpose.

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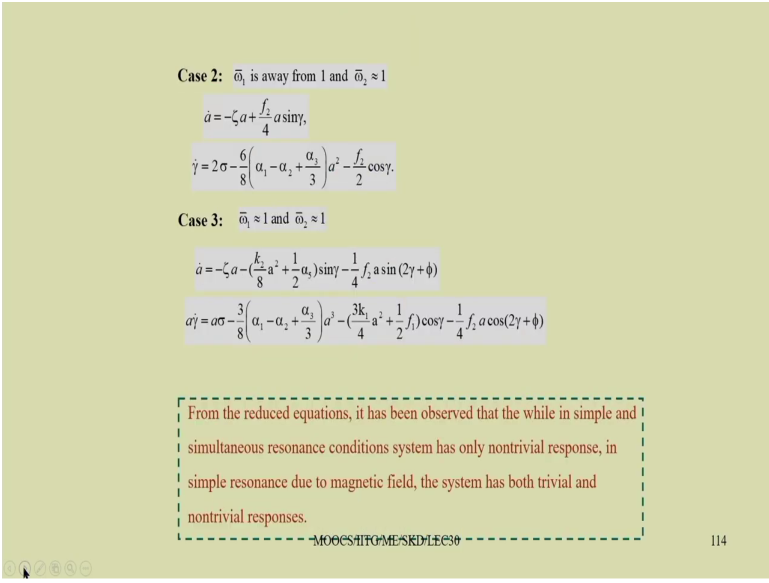

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Results and Discussions

Physical parameters	Values
Length (L)	0.500 m
Width (d)	0.005 m
Height (h)	0.001 m
Mass Density (m)	0.04 kg
The permeability of the vacuum ( )	1.26×10^{-6} Hm^{-1}

Case 1: Simple resonance case due to support motion

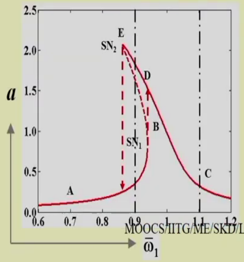

Fig. : Frequency response curves
$M = 0.02$ kg, $C_d = 0.01$ N-s/m^2, $Z = 0.025$ m,
$B_m = 0.30 Am^{-1}$.

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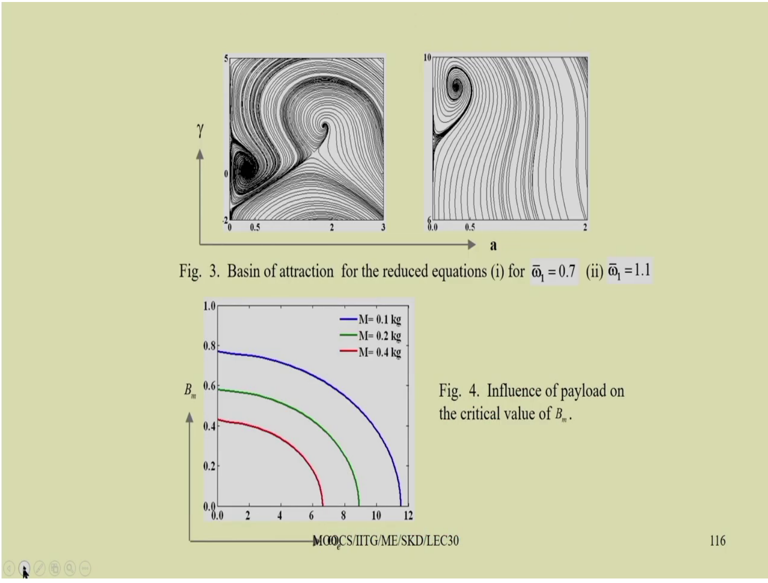

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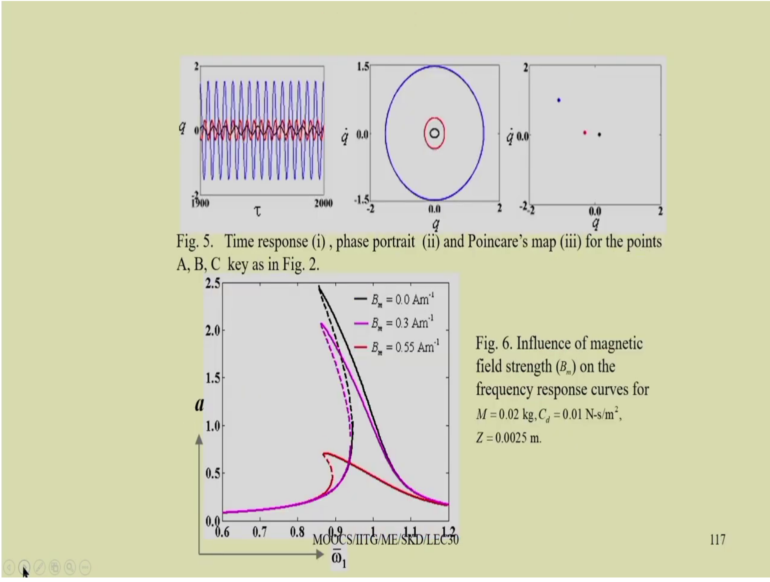

Today class particularly we will see this energy harvesting and in addition to that, we will see a sandwich beam also. So, how we can analyze a sandwich beam that thing we can see. So, in this magneto elastic beam. So, applying different magnetic field. So, already we have seen how the frequency response can be controlled or the how the response amplitude can be controlled.

So, for example, without magnetic field. So, you have seen this black frequency response curve. Now, by applying a magnetic field of 0.3, this pink color and then finally, by applying a higher order magnetic field. So, it has reduced further to this thing if we have a magneto elastic beam then by easily applying magnetic field. So, we can control the vibration of the system.

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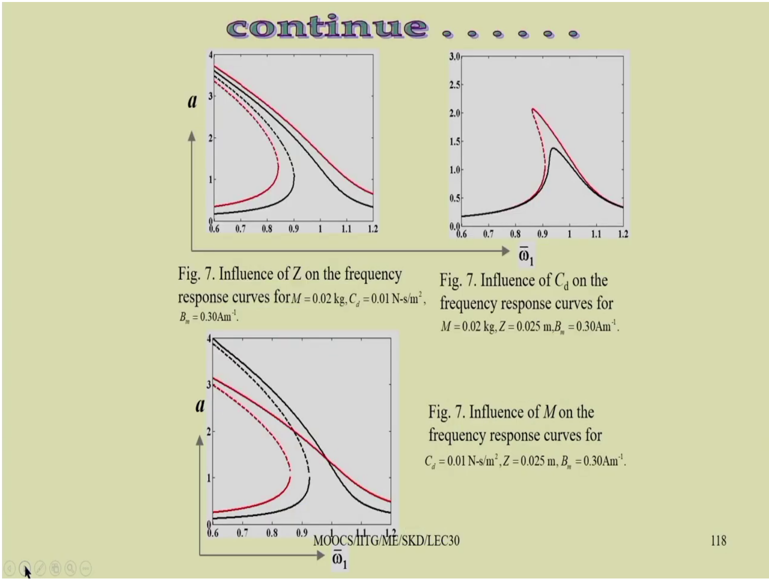

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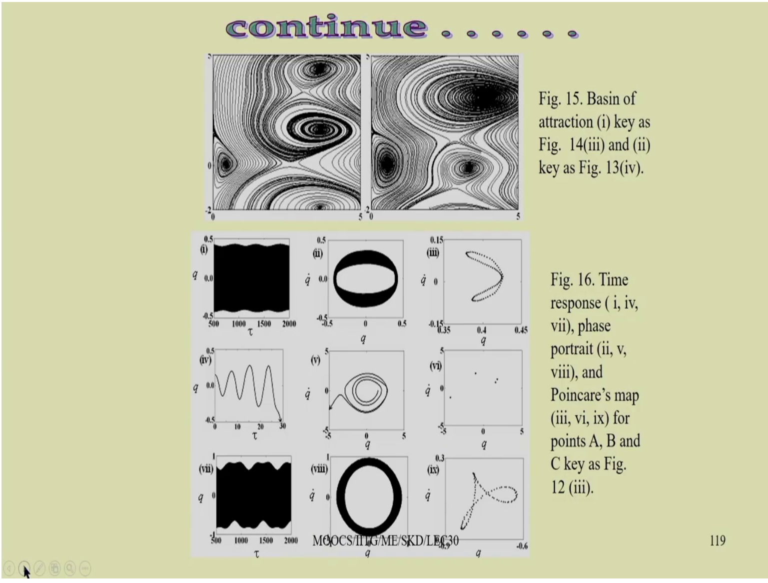

So, this is happening as the stiffness of the structure we can controlled by applying this magnetic field.

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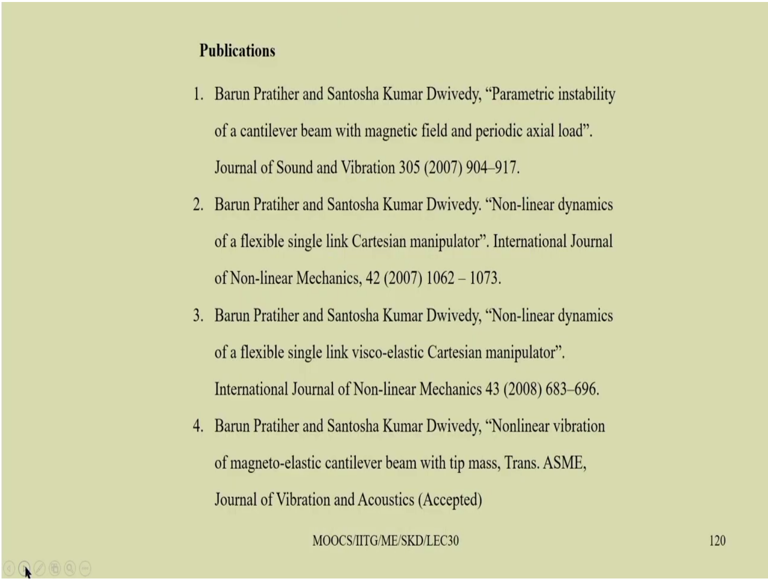

When we have multiple solutions. So, by using this basin of attractions we can study the different responses and these are the publications by my PhD student Barun Pratiher. So, related to elastic viscoelastic and magneto elastic beams.

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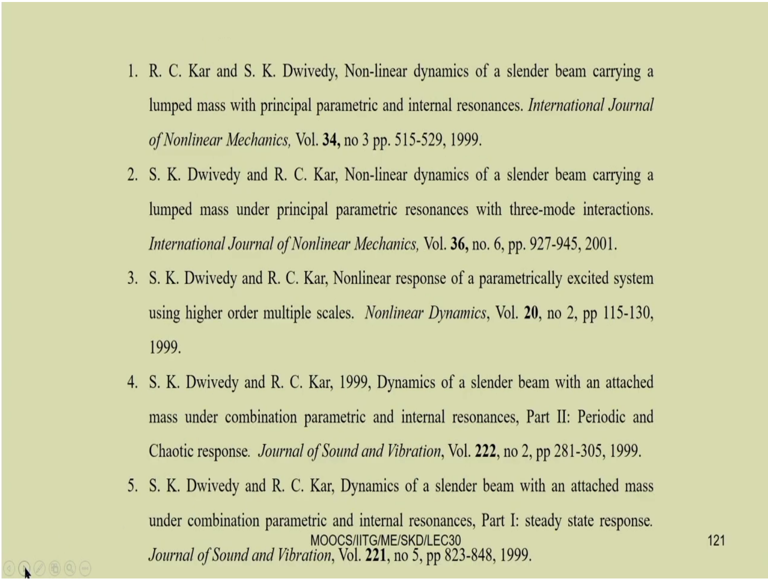

So, in all these cases you can see the single link manipulator how it can be used for vibration control. So, these are the papers by Professor R. C Kar and myself. So, these are on this internal resonance conditions. So, several papers are there published in general of non-linear mechanics, non-linear dynamics then general of sound and vibration all this papers you can refer to have a better understanding how we can convert a continuous system to that of a discrete system and then how to analyze to get the complex nature these non-linear vibration

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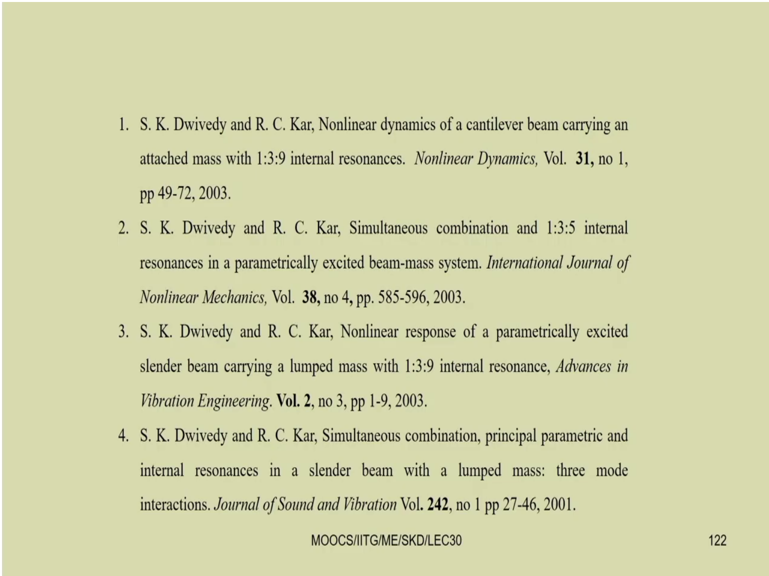

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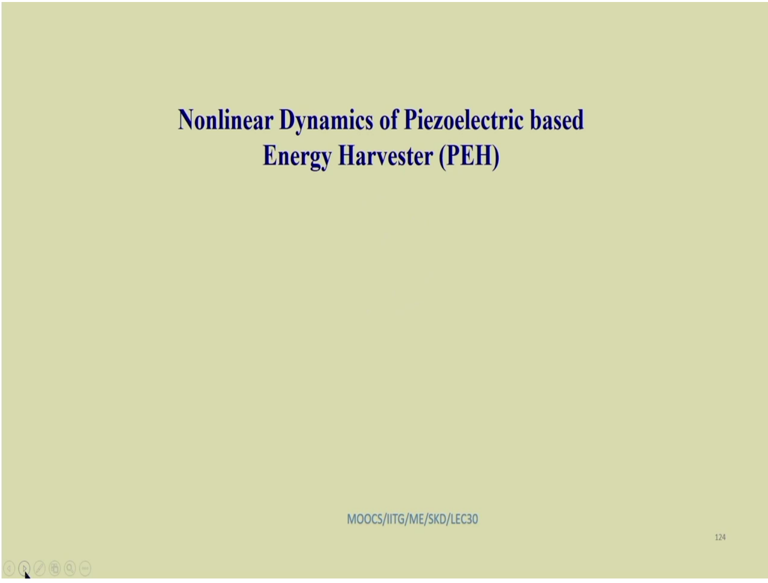

Let us come to today class where we are going to discuss regarding the non-linear dynamics of piezoelectric based energy harvester.

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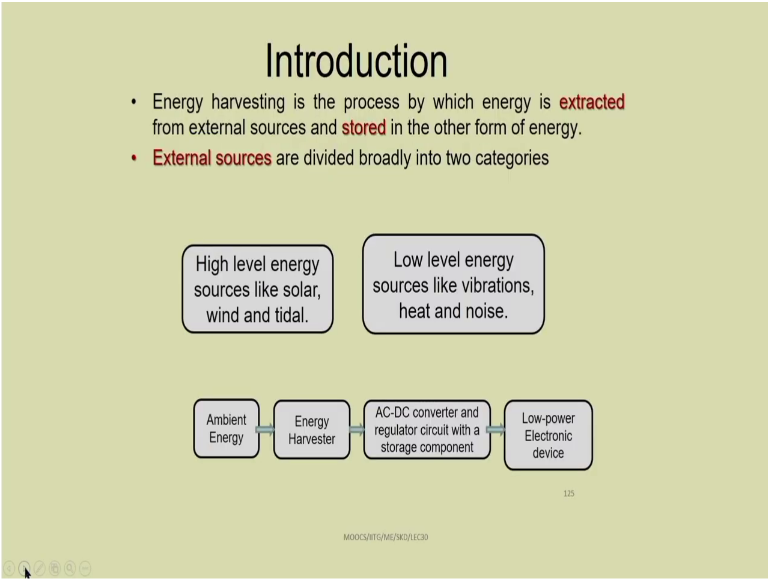

Here the same system we are taking, but by applying this piezoelectric patch on that thing. So, we will see how it can be used as the harvester. Energy harvesting is the process by which energy is extracted from the external source and stored in the other form of energy.

This external sources are divided broadly into two category one is this high level energy source like the solar, wind and tidal and this low level energy source like this vibration, heat and noise from the ambient energy. So, we can extract the energy by using this energy harvester and then we can convert that thing to AC-DC converter and regulate; regulator circuit with a storage component then that low power electronic device we can use to utilize this harvested energy.

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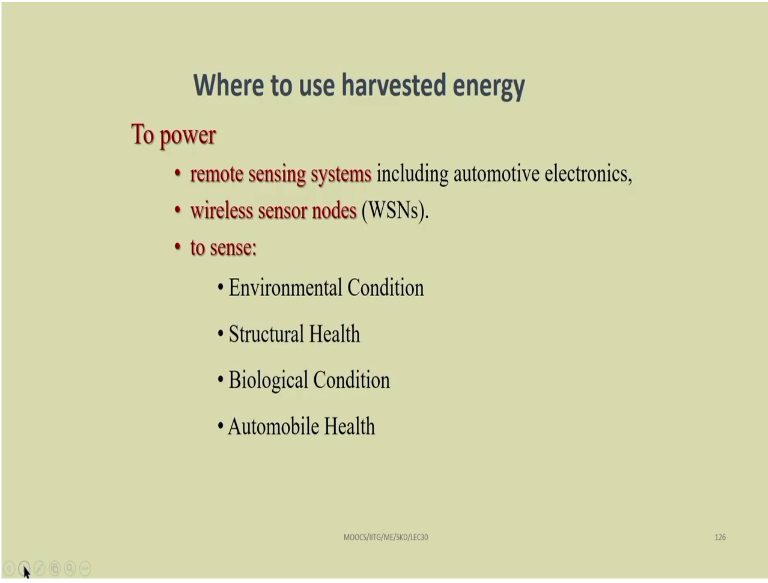

As these are the low power energy then we can particularly use them in remote sensing applications including automobile electronics wireless sensor nodes to sense environmental conditions, structural health, biological condition and automobile health and many other applications also it can be used.

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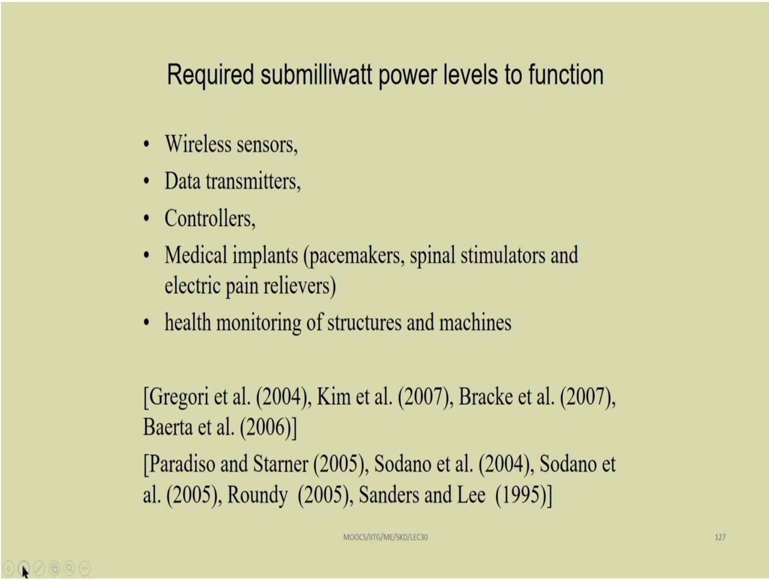

These are the cases where we required this sub milliwatt power level to function for example, this wireless sensors data transmitter, controllers, medical implant particularly this pacemakers, spinal stimulator and electric pain reliever. So, here we required very less power. So, in all these cases or health monitoring of structure and machines. So, you can see all this papers, where this applications are clearly written.

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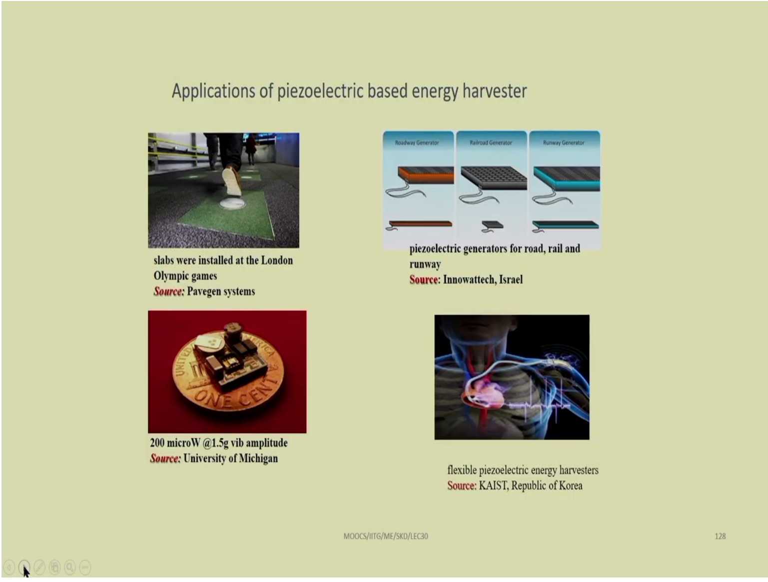

There many structures where this sub milli watt power can be utilized sensing purpose, this energy harvester can be used utilized their very effectively for example, this is a pacemaker. So, here also while walking this slabs were in slabs were installed at the London Olympic games, where by walking we can generate this energy harvesting. So, these are the piezoelectric generator for road, rails and runway.

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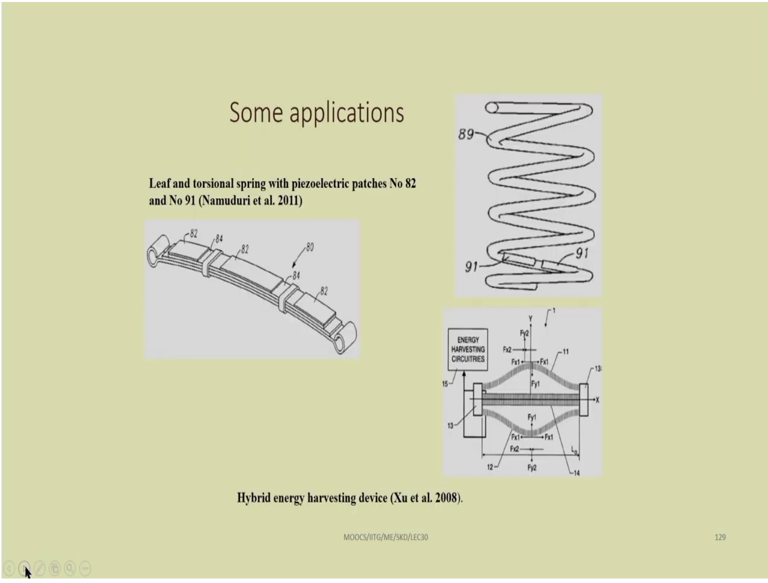

So, we can utilize here also the piezoelectric energy harvester for getting these energy. Similarly, in the leaf spring of a automobile we can put the piezoelectric patch as the leaf spring is subjected to this bending here this piezoelectric patch will be.

So, piezoelectric patch can sense that vibration and or sense that bending and due to this bending this voltage will be generated and that voltage can be harvested. Similarly, in other type of spring in shock observer or this energy harvesting circuit also you can use a energy harvesting circuit.

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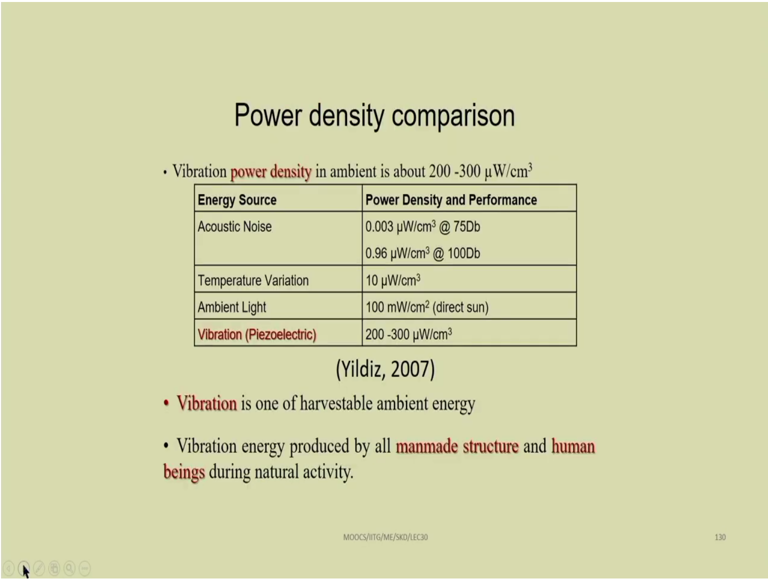

So, you can use this hybrid energy harvesting device that several energy harvesting devices are there. So, you may use this piezoelectric type energy harvesting or magneto elastic type energy harvesting. So, we will see one by one and so, this power density comparison you should know.

So, the vibration power density in ambient is about 200 to 300 micro watt per cm cube, acoustic noise contain this 0.003 micro watt per cm cube at 75 Db and similarly 0.96 micro watt per cm cube at 100 Db.

So, you just see by increasing this noise. So, we can increase the power density. Similarly, temperature variations so, for temperature variations we can get 10 micro watt per cm cube, ambient light can give 100 milli watt per cm square. So, direct sun we can get this 100 milli watt per cm square.

So, then vibration piezoelectric base we can generate 200 to 300 micro watt per cm cube. So, this things you can find in this paper Yildiz, 2007. So, in the last portion of this presentation I will show you the references where you can get the details of this reference. So, vibration is one of the harvestable ambient energy. Vibration energy produced by all manmade structure and human beings during natural activity can be used for the harvest for harvesting the energy.

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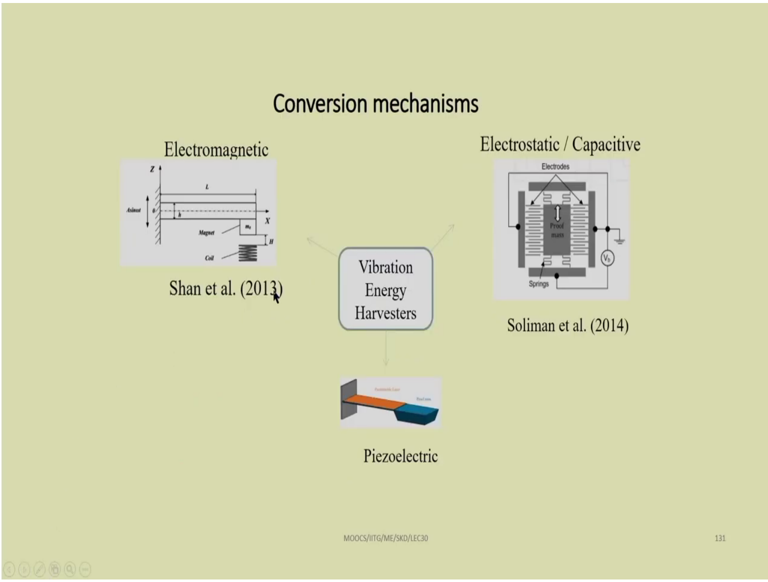

There are several conversion mechanism for example, this electromagnetic conversion one can do this electromagnetic conversion we can use easily this Faraday concept, Faraday principle to generate this thing. For example, so, at the end of the cantilever let us have a magnet and we have a coil here. So, when the beam is vibrating this magnetic field is changing.

So, the plugs caught inside this coil is changing and due to the rate of change of this plugs in the coil emf will be generated. So, this emf can be harvested or can be taken for different application purpose. Similarly, we can use this electrostatic or this capacity type of conversion mechanism where we have a proof mass.

So, proof mass is supported by this spring, as this proof mass is moving up and down the gap between these two capacitive or capacitor can be changed. So, you can see with this proof mass there are some additional structures are there. So, by the movement of the proof mass there will be variation in the gap between the capacitor plate. So, that will give raise to emf and that emf also can be harvested we can have this electromagnetic wave, piezoelectric magnetic wave or this electrostatic or capacitive wave of generating these or harvesting the energy.

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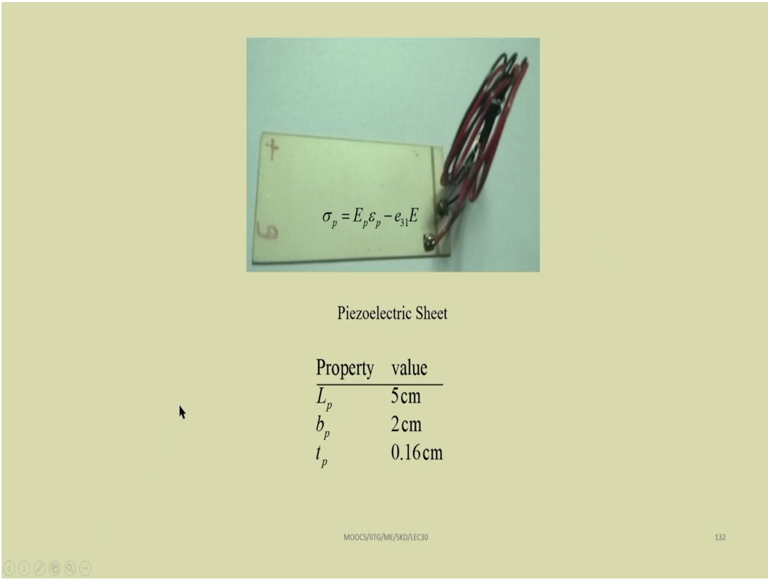

So, if are taking. So, this is typical piezoelectric slab. So, in this piezoelectric sheet the property for example, this as a length of 5 cm, it is width is 2 cm and thickness equal to 0.16 cm. So, this patch we have purchased and it can be it has been used in our experimental setup for harvesting this energy.

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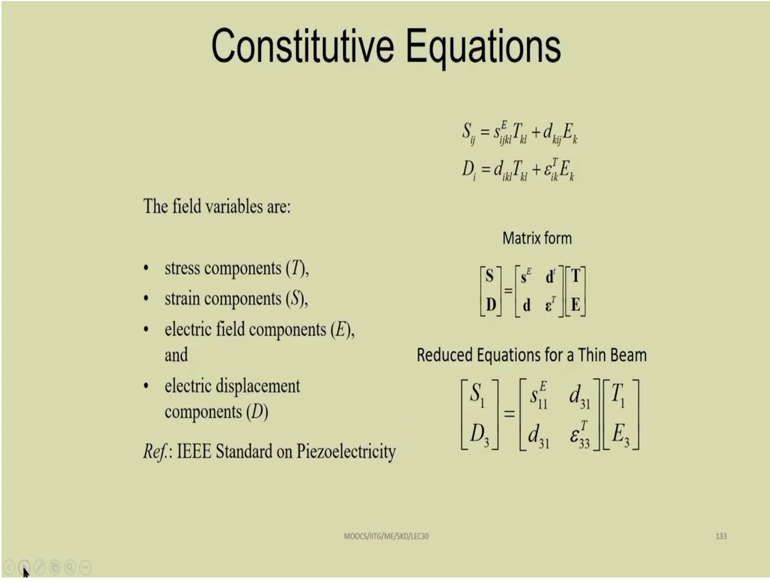

While using this piezoelectric patch. So, we should know that in this piezoelectric patch. So, this strain is due to both electric and due to this direct bending of the beam. So, one is due to direct bending of the beam and another part is due to this electric part. So, the strain can be written. So, this is due to stress component and this is due to electric component.

So, we can find this constitutive equation that is reduced equation here you can write this between strain. So, stress component is T here, strain component is S, electric field component is E and electric displacement component is D. So, by using this thing the stress strain relation can be. So, this is the strain component and this is the stress component can be written by using this matrix and by using this one. So, we can easily derive this equation of motion ok.

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So, you can use for example, this PZT lead zirconate titanate, coupling mode will be d31 or d33, then PVDF, then direction of force and poling directions we can see then MFC micro-fiber composite. So, this is the MFC patch you can see, then we can we required AC to DC converter also for all this conversion mechanism.

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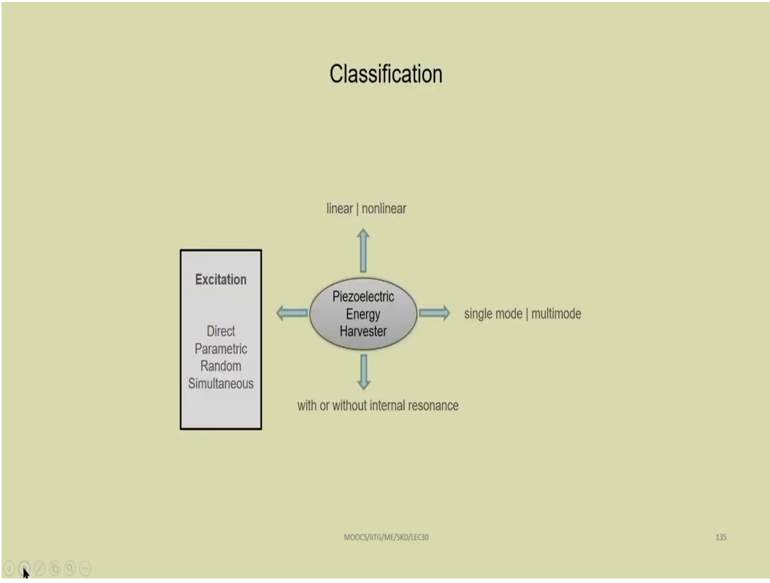

So, classification; so, we can have a excitation. So, direct parametric run it can be direct excitation, it can be parametric excitation, it can be random or simultaneous of all this excitations also can be applied to a system to harvest this energy.

So, in case of direct for example, we can give this wind force or we can give a we can put it on a secure or a vibrating structure and in case of parametrically excited system the direction of the force and the direction of the response are taking place in orthogonal direction.

And in case of random excitation while we may have a deterministic so, we can have a random excitation like the earthquake type of excitation or sometimes there are many other phenomena, which contribute to this random excitation, this wind excitation also sometimes is this random excitation.

So, we can add all this things and we can have a simultaneous combination of all this direct parametric random or direct and parametric excitation particularly for this easy analysis purpose, we generally take this direct and parametrically excited system and also random excitation can taken separately for this analysis purpose or this random excitation can be converted to equivalent direct and parametric excitation to study the phenomena.

So, we have a energy harvesting system then. So, the system study may be linear though actually the system is non-linear, many times it can be studied as a linear system sometimes or few researchers have taken this as a non-linear systems. So, with or without internal resonance conditions also it can be studied.

Generally, when we are taking this as a continuous systems. So, the for analysis purpose it can be converted to a single mode or multi-mode like what we have seen in previously when we have considered the internal resonance conditions. So, if we are not considering internal resonance conditions in that case only single mode approximation may be possible.

But, when we are considering internal resonance condition then multi-mode approximation we may go for this multi-mode approximation and take the number of appropriate modes particularly when we are modeling the system using a finite element methods. So, we can easily go for multi-mode approximation and we can analyze the system for different resonance conditions.

Already you are familiar with the resonance condition for example, we have the simple resonance condition, then principle parametric resonance condition of first mode, second mode, third mode so, or nth mode. Then combination parametric resonance conditions so, where we can have combination of first mode second mode, combination of second mode third mode, combination of first and third mode.

So, that way so, we can have the combination parametric resonance of some type different types and also in addition to this we can due to the presence of nonlinearity. So, we can have some harmonic resonance condition and also super harmonic resonance conditions.

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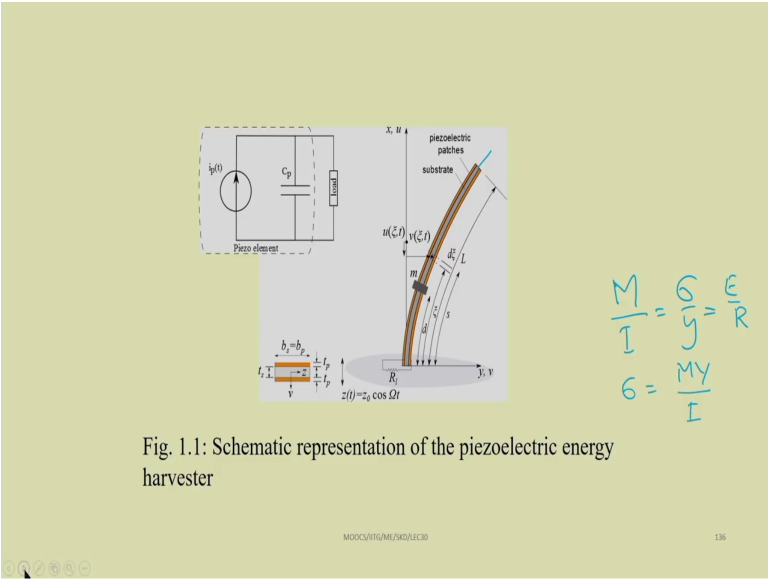

Fig. 1.1: Schematic representation of the piezoelectric energy harvester

So, let us come to our system. So, this schematic representation of a piezoelectric energy harvester. So, this is the substrate that is the main body and we are. So, this is the substrate part, this inside part is the substrate part, then this piezoelectric patches are put on both the sides of this thing, this substrate and then we have a attached mass.

So, the mass can be attached at any arbitrary position. So, previously we have seen by putting this mass at any arbitrary position, we will be able to generate internal resonance conditions. So, we can adjust this mass in such way that between the ratio of the frequency between any two modes can be controlled or can be taken in the form of integer relation.

So, in that case we can have the internal resonance conditions and here. So, at the base side the movement will be large, as the movement will be larger the base side. So, the strain will be more stress will be more as we knows M by I equal to sigma by y. So, M by I that is M by I equal to sigma by Y equal to E by R. So, in case of bending we know this relation. So, our sigma will be equal to M Y by M Y by I.

So, bending movement will be maximum at this free end fix end. So, bending movement is maximum at the fix end. So, sigma will be maximum there and at the sigma is maximum their strain also will be maximum there. So, we can generate the voltage more voltage at this position. So, we can put a circuit here to generate or tap that energy, which is produced by this piezoelectric patch. Here we have a capacity we can have a load or we can have capacity type of circuit to store this energy or utilize this energy.

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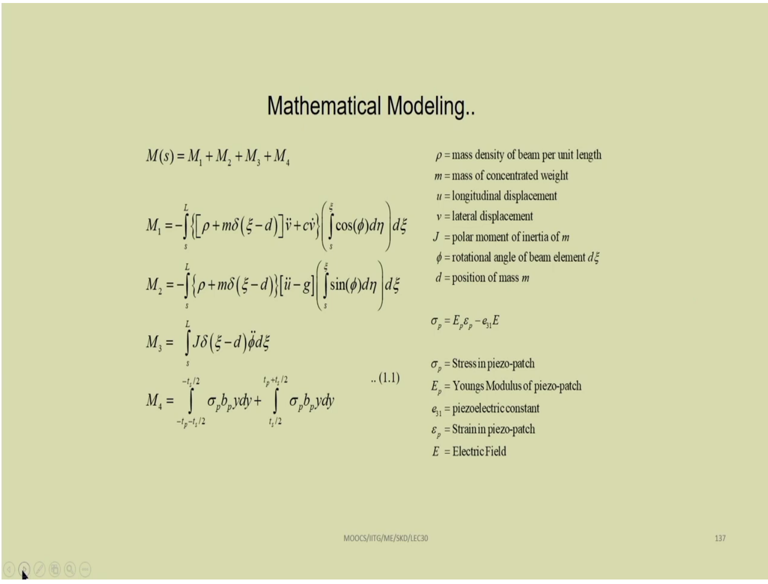

So, now like in the previous case here also we can derive this equation motion by either using this Newton second law or by using this Hamilton principle or the energy based principle like

LaGrange principle also by using this Newton or Dlm principle. So, we can write M equal to that is bending movement at a 6 on s can be written as M 1 plus M 2 plus M 3 plus M 4.

So, that thing we have seen previously at any position s. So, this is position s this mass is at a distance d. So, we have taken a domain position zeta domain position zeta we have taken and then this is d zeta a small element d zeta we have taken. So, like previous case. So, we can derive this M 1 M 2. So, M 1, M 2, M 3 p is in the transverse direction.

So, we have taken this transverse direction then axial direction. So, axial direction displacement is u and this transverse direction displacement is v. So, first we have to take the inertia of force in the transverse direction. So, this M 1 is due to bending due to the transverse direction forces. So, this is equal to rho plus m delta zeta minus d v double dot plus c v dot.

So, into that distance. So, that distance equal to s to zeta integration s to zeta cos phi d eta. So, this d eta is the eta is the dummy variable in to d zeta. So, this is M 1. Similarly, M 2 is due to u double dot that is in. So, that is in the axial direction. So, in axial direction also you have the weight. So, that g component is coming here. So, rho plus m delta zeta minus d into u double dot minus z into the bending part. So, this is the force acting into distance will be the bend will give the movement.

So, s to zeta phi d eta d zeta, then due to this rotary inertia part this M 3 will be there. So, j delta zeta minus d phi double dot d zeta. So, this is the rotary inertia part due to the attached mass then finally, for the piezoelectric patch. So, we have another equation that is M 4. So, M 4 will be equal to sigma p into b p ydy. So, then this is sigma p b p ydy for the piezoelectric patch. So, we can have. So, we can divide that into two part and we can derive this equation motion.

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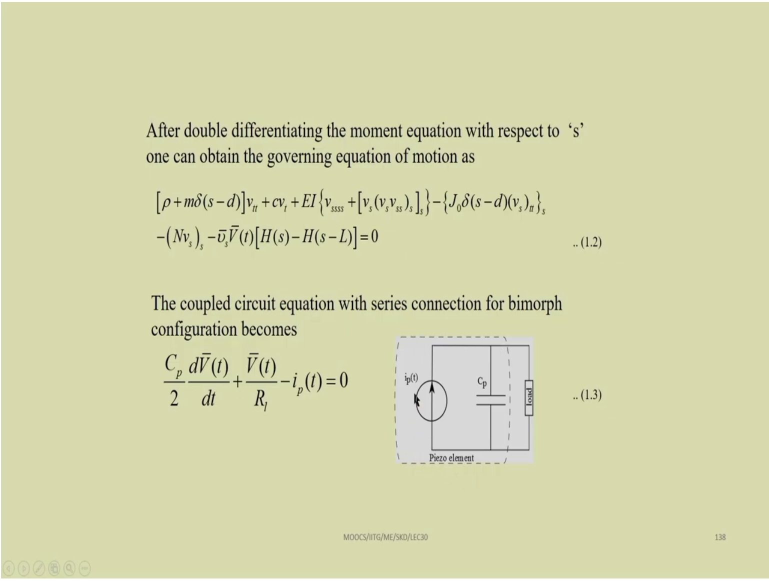

Actually, if we double differentiate this movement equation, then we can get this forcing equation. So, differentiating twice that equation so, we can get the equation in this form. So, then for the circuit. So, this is one equation we got.

So, now for the circuit. So, we can draw the circuit diagram and for the piezoelectric patch. So, we can write the equation motion. So, this equation is C p by 2 dV t by dt plus V t by R minus i p t equal to 0. So, this is using this Kirchhoff's voltage law. So, we can easily derive this one.

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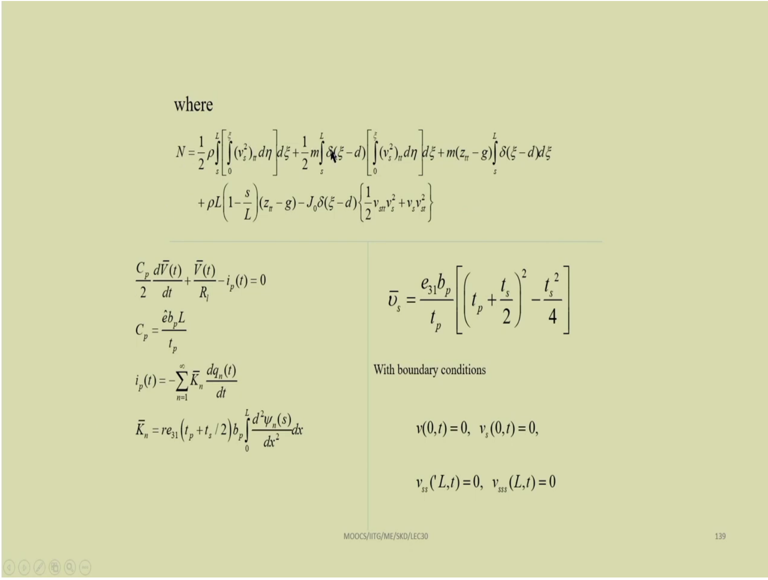

Then where N equal to this and all this equations are you can easily derive. So, these are the boundary conditions also we obtain. So, when we are applying this Hamilton principle the boundary conditions we can get it along with the equation motion.

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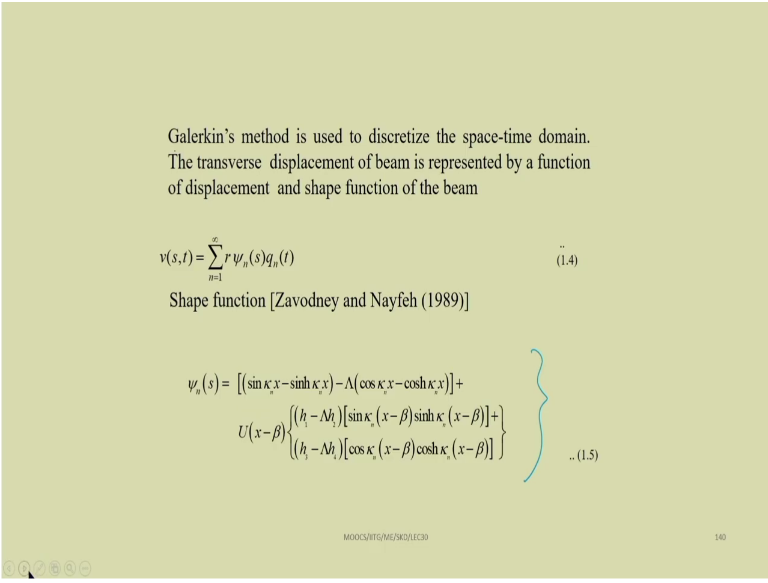

So, now generalize Galerkin method can be used. So, if you compare this equation with the previous equation what you have derived. So, here due to the presence of the piezoelectric patch. So, you have we have the electromechanical component present in this case. So, now, by taking this BST equal to that is this displacement it is a function of both s and t that is space and time we can write it equal to n equal to 1 to infinite r psi n s q n t, where r is the scaling factor and psi n s is the save functions and q n is the time modulation. So, the psi n we can take it from that of the Zavodney and Nayfeh, where it is derived for the mass art are we derive by keeping the mass at some position. So, this psi n s can be written here.

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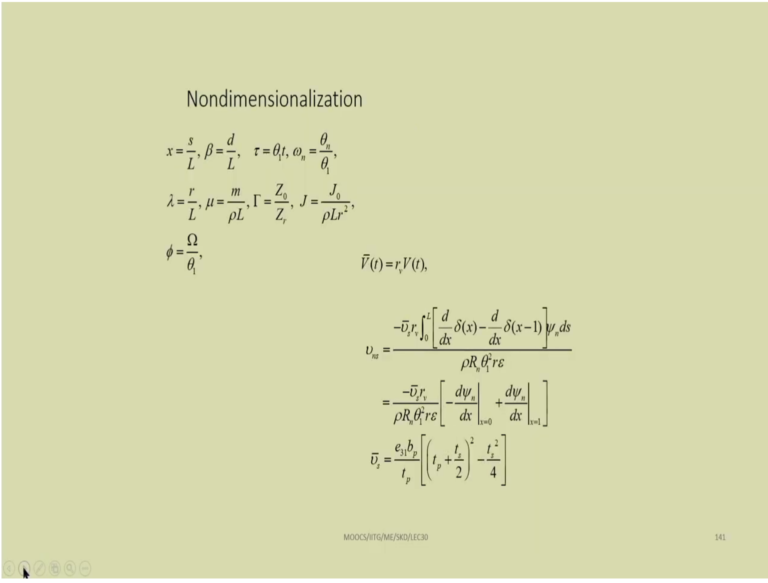

Then using this non-dimensional term for example, this non-dimensional distance equal to s by L, then non-dimensional position beta equal to d by L tau equal to theta 1 t where theta 1 is the frequency of the first mode, tau equal to theta 1 t, omega n equal to theta n by theta 1, lambda equal to r by L, mu equal to mu m by rho L, mu is the mass ratio and capital gamma equal to Z 0 by Z r.

So, Z 0 is the base excitation amplitude of base excitation and Z r is the reference amplitude we have taken. So, J equal to J 0 by rho L r square and phi equal to omega by theta 1. So, this way by using different non-dimensional parameter and using this single mode one can use multi-mode approximation or single mode approximation. So, for the displacement similarly for voltage we can write voltage V t equal to r V into V t V bar t so, we have use this non-dimensional term.

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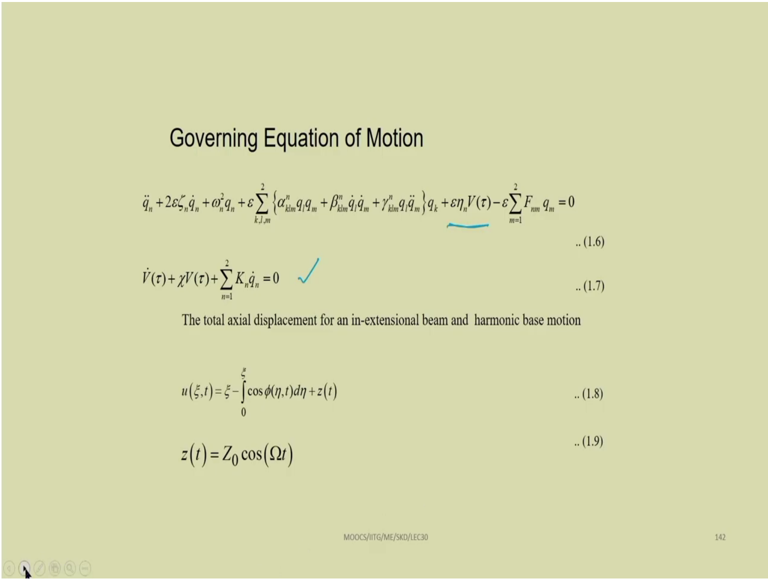

Now, by putting all this non-dimensional term. So, we can derive this equation motion which can be written in it is temporal form in this way that is q n double dot plus 2 epsilon zeta n q n dot plus omega n square q n plus epsilon k j m. So, if we are taking only two more, then this k j m will vary from 1 to 2 alpha k element q l q m then beta klm n q l dot q m dot then gamma klm n. So, q 1 q m double dot into q k.

So, outside this q k is written this shows that is a cubic non-linear terms we are taking, in this the first term q l q m and into q k that is geometric non-linear term and then q k into q l dot into q m dot is the inertia non-linear term, due to the multiplication of two velocity term. Similarly, this gamma n klm n into q l into q k into q m double dot this is the inertia non-linear term.

Similar to the previous paper we have both geometric and inertia nonlinearity here. So, plus epsilon eta V b t. So, this is the voltage term you just see in the first equation we have a coupled electromechanical system, electromechanical coupling. So, due to the presence of this term that is epsilon eta n V tau. So, this is the voltage term, electric term.

So, it is coupling both these equation that is this mechanically excited term or mechanically vibrating term that is q with this v minus this is the forcing minus epsilon m equal to 1 to 2 F nm q m equal to 0. So, similarly this is the voltage equation. So, voltage equation equal to V dot tau plus psi V tau plus n equal to 1 to 2 K n q n dot equal to 0.

We can apply the inextensibility conditions. So, here so, you can note that we have applied this inextensibility condition that is u zeta t equal to zeta minus 0 to zeta integration 0 to zeta cos phi eta t d eta plus z t. So, we have used this inextensibility condition to derive this equation motion also we have taken these z t equal to Z 0 cos omega t in this analysis.

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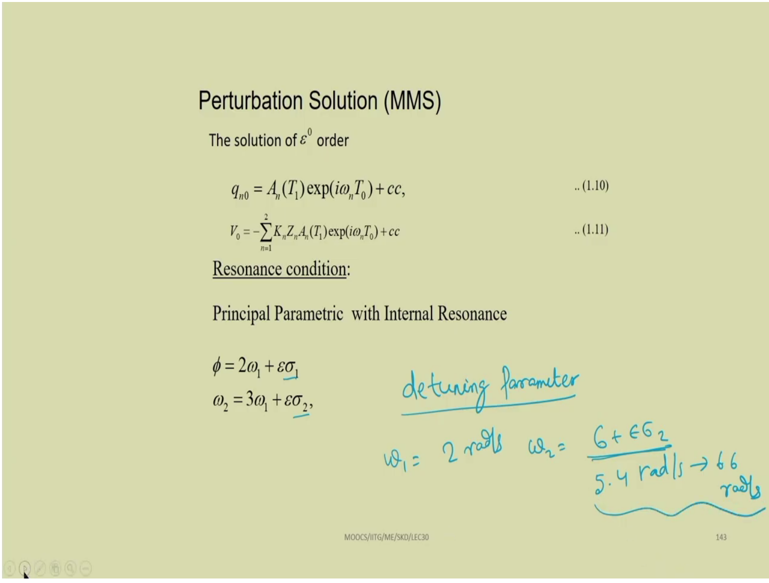

So, now after getting this governing equation in temporal form. So, we can apply the perturbation solution. For example, here we are we can use many other method for example, one may use this normal form method, harmonic balance method, linstead Poincare method, KBM method, this homotopy methods so, many methods are available or simply by using this RK 4 method, Runge Kutta fourth order Runge Kutta method.

So, one can find the response of the solution particularly if you are using this Runge Kutta fourth order fifth order Runge Kutta method, then you can get the response of the system for particular system parameter. For example, so, if one has to obtain the frequency response plot from the time response one has to spend use memory space and also time to get that simple frequency response plot.

For this purpose so, one has to go for the perturbation method so, that easily one can plot the frequency response, force response or study the variation of different system parameters with the given system parameter. Now, applying this perturbation technique for example, taking this method of multiple scale, we can write down this q n 0 equal to A n e to the power i omega n T 0 plus e c and then V 0 equal to minus n equal to 1 to 2 K n Z n A n T 1 e to the power i omega n T 0 plus cc, where cc is represent the complex conjugate of the preceding term. Whatever terms are there before the cc all the terms have the complex conjugate the cc represents the complex conjugate of the preceding terms.

Here we can consider the internal resonance condition and external resonance condition particularly if you are interested for principle parametric resonance condition. So, we can have this phi equal to 2 omega 1plus epsilon sigma 1, and omega 2 equal to 3 times we are taking this internal resonance condition, omega 2 equal to 3 omega 1 plus epsilon sigma 2.

So, where sigma 1 and sigma 2 are detuning parameter detuning. So, detuning parameter. So, gives the nearness of the external frequency to the twice of the natural frequency similarly this sigma 2 gives the nearness of the second mode frequency to 3 times of the first mode frequency for example, let omega 1 equal to let omega 1 equal to 2 hertz so, 2 radiant per second.

So, in that case omega 2 will be equal to 3 times that is 6. So, plus minus epsilon sigma 2. By taking ten percent of this 6 for then we can go. So, we can vary it from for example, 5.4 to 5.4 radiant per second to 6.6 radiant per second. So, we can vary this thing and we can find the response near to the principle parametric resonance conditions.

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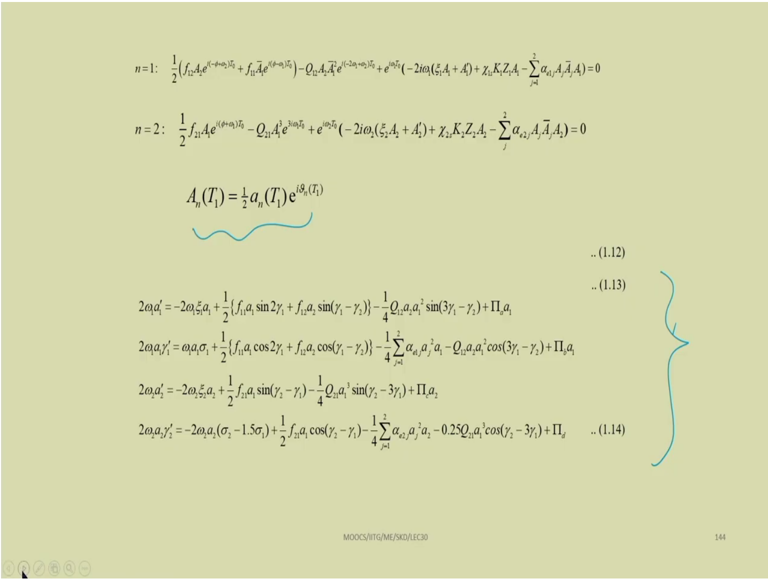

So, here by substituting n equal to 1. So, as we are going for two more interaction. So, we can substitute n equal to 1, n equal to 2 we will get actually 5 produced equation in this case, proceeding in the previous case by using this method of multiple scale. So, we can get 4 equations for this amplitude and his motion of the beam plus one equation for the voltage generated voltage.

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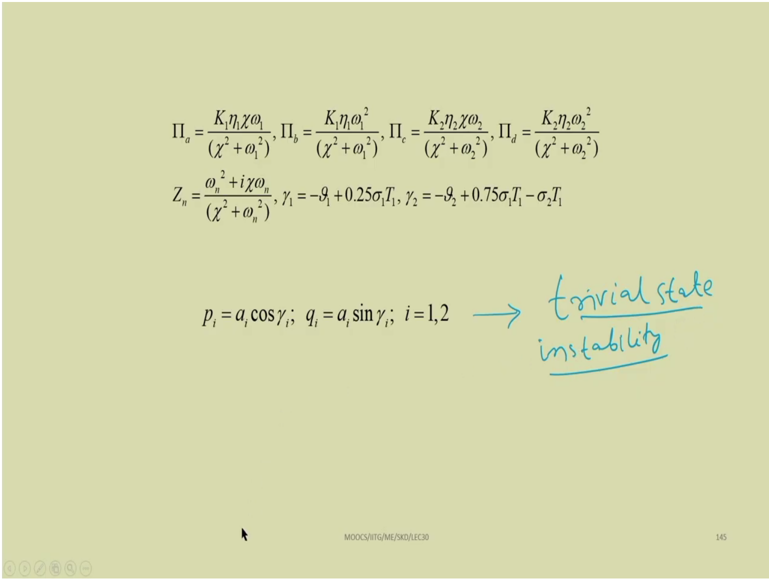

So, we will get five equations and solving those five equations. So, we can find the response of the system. Similar to the previous case here you can make the transformation p i equal to a i cos gamma i and q i equal to a i sin gamma i.

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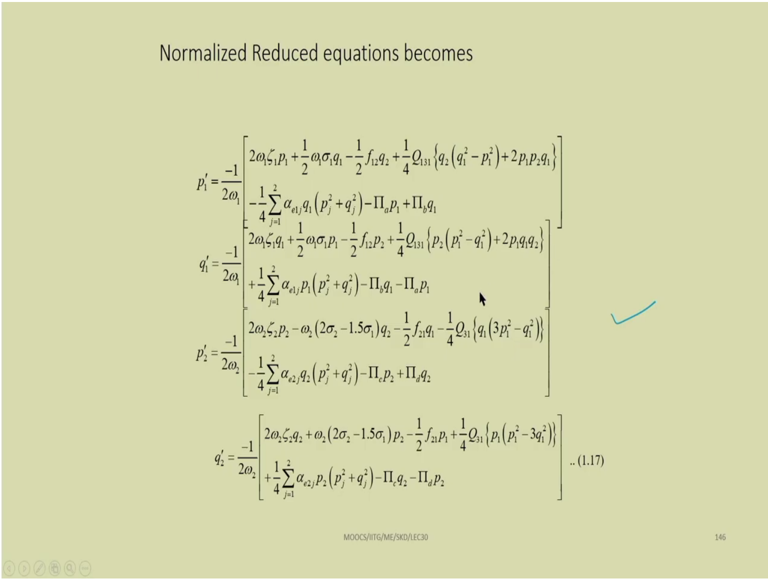

So, where i equal to 1 to 2. So, we have this four points. So, this p i q i conversion is taking place to have the trivial state in stability region. So, to study the trivial state in stability region generally, we convert this thing. So, to study the trivial state in stability region then we will convert this to p i and q i form these are the equation one can obtain using this p i and q i form. So, p 1 dash q 2 dash p 2 dash and q 2 dash.

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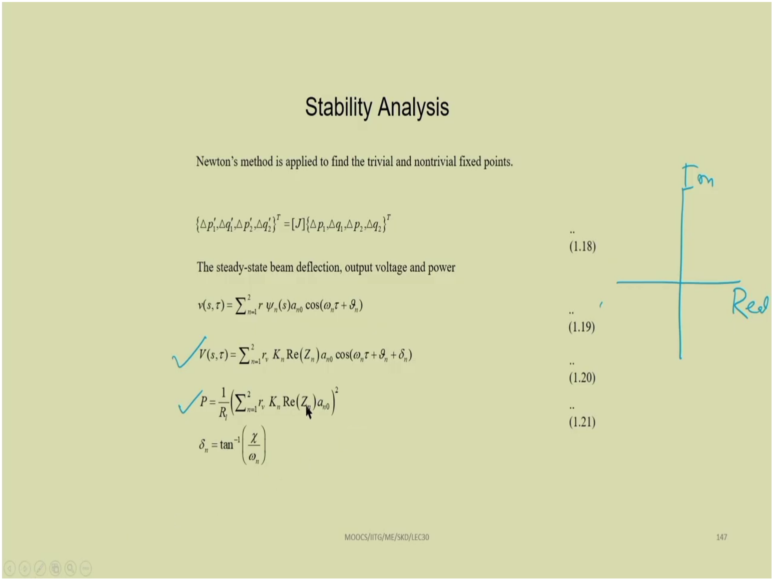

And then one can do the stability analysis by perturbing this equations. So, by perturbing this equations one can find this Jacobean matrix and by finding this Jacobean matrix then one can find the eigen values of the Jacobean matrix. By finding this eigen value of the Jacobean matrix if the real part of the eigen value lies in the left hand side of the s plane. So, left hand side of the s plane then the system is stable. So, this is real. So, this is imaginary.

So, if it is lying in the left hand side of the s plane then it will be stable otherwise it is unstable one can study the stability and bifurcations of the response. So, one can convert after getting this displacement one can find the voltage equation in this way, after getting this voltage equation one can find the power and so, this is the power expression. So, depending on the resistive, capacitive and inductive circuit the power will be different and this is the expression for power one can get.

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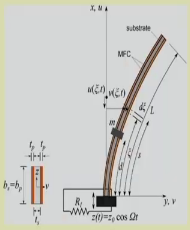
Results and Discussion



Properties	Piezo	Substrate
Young'smodulus,GPa	$E_p = 15.86$	$E_b = 190$
Density,kgm^{-3}	$\rho_p = 5440$	$\rho_b = 7800$
Length,m	$L_p = 12.54 \times 10^{-2}$	$L_b = 12.54 \times 10^{-2}$
Width,m	$b_p = 1.12 \times 10^{-2}$	$b_b = 1.22 \times 10^{-2}$
Thickness,m	$t_p = 1.25 \times 10^{-4}$	$t_b = 3.7 \times 10^{-4}$
Permittivity,nFm^{-1}	$\hat{e} = 19.36$	–
Piezoelectric constant,Cm^{-2}	$e_{31} = -19.84$	–

Table 1.1: Geometric and material properties of substrate with piezoelectric patch

Let us see some of the results. For example, so, if we are taking material property what is taken for example, for piezoelectric patch the material property E p can be taken Young's modulus can be taken 15.86 GPa for the substrate like this aluminium so, it is 190 GPa. So, you just see this piezoelectric patch has very very less Young's modulus is density of this piezoelectric patch is 5,440 and for the substrate equal to 7800. Similarly, length is taken so, 12.54 into 10 to the power minus 2. So, this is in meter.

So, if you are considering the full length of the beam then we can take both L p and L b same width also 1.12 into 10 to the power minus 2 meter. So, width is width of the substrate is taken slightly more this is 1.22 into 10 to the power minus 2, thickness of the substrate part is taken 1.25 into 10 to the power minus 4 and then this is for the piezo patch and for the substrate it is equal to 3.7 into 10 to the power minus 4 meter.

The permittivity is taken 19.36, piezoelectric constant coulomb for meter square e 31 is taken minus 19.84 these parameters are similar to those available in many literature.

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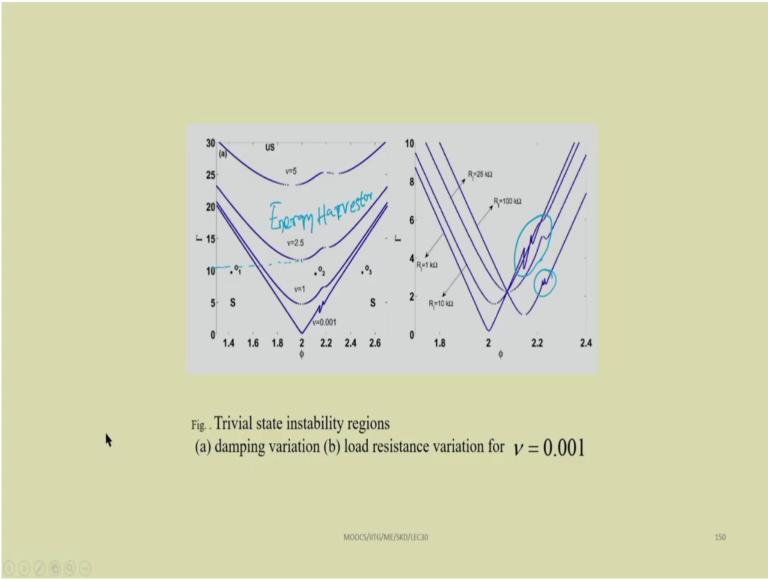

So, taking all this parameter. So, one can easily plot the instability region. So, one can see these are different instability region and with different value of this nu that is damping parameter. So, one can see the instability region moves of; that means, for example, if you are taking this nu equal to 2.5, then below this gamma equal to 10 always whatever may be the value of the excitation frequency pi, the system will remain stable.

Only we should be bothered or it can be used as a energy harvester only if the excitation amplitude is greater than 10 otherwise for nu equal to 2.5 it cannot use as a energy harvester.

Actually, this unstable position when the system becomes unstable, this is the position where we are using these as a energy harvester this is the.

So, instable region actually it cannot be used as a energy harvester, because it will not vibrate when it is trivially unstable. So, that configuration we have to take to use this as a energy harvester. You can see by using this internal resonance conditions for example, here or here. So, you can see we have multiple regions.

So, multiple regions where it can change from stable to unstable and we are particularly looking for the region when this system becomes parametrically unstable and we can use that region only for these vibration or energy harvester purpose. So, when it is in this region s trivial is stable so that means, it is not vibrating. So, if it is not vibrating. So, it cannot be used as a energy harvester.

So, in the previous cases we have seen or we have checked the condition where the system will be stable or we are interested to know when the system will be stable, but in this case we are interested when the system becomes unstable when the system becomes trivially unstable because this trivial unstable configuration will gives raise to these energy harvesting condition. So, the system we can harvest the energy when this is inside the instability region or the parametric instability region.

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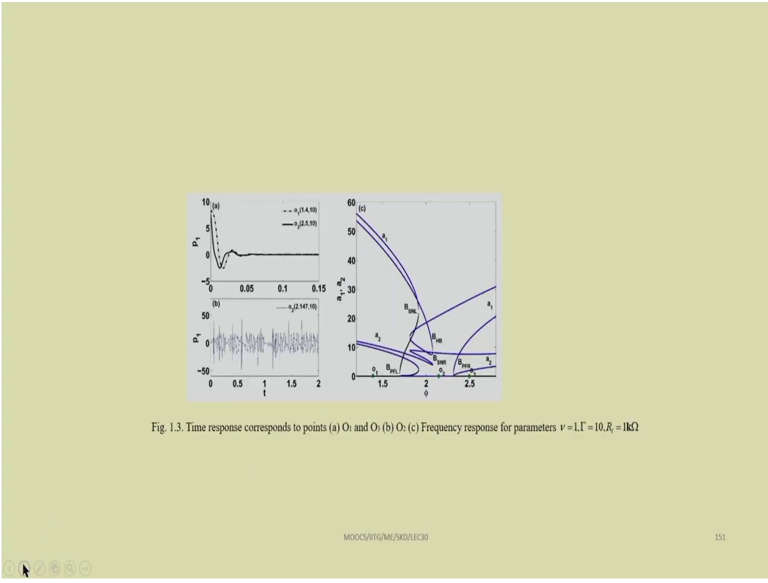

So, you can see these are the time response one can obtain for different value by putting different value O 1, O 2, O 3 what we have seen here? So, this is O 1, O 2, O 3. So, here O 1 and O 3 are stable region.

So, in case of O 1, O 3 we just see it is stable and finally, it is 0. So, it is in trivial state it is not vibrating, but when it is operating in O 2 the system response we can see. So, this is chaotic response and it is system is unstable, but we can generate very high voltage due to the vibration. So, we have a very high amplitude of vibration in this region.

So, you just see for the same system parameter here we are taking O 1, O 2, O 3 have the same value of gamma, but this phi changing. So, O 1 is in the stable range, O 3 is in the stable range and O 2 is in the unstable range parametric instability region for if we are operating this at O 1 and O 3.

So, system will not vibrate in a study state, but if we are operating it at O 2, then the system will vibrate and we can use that as a energy harvester that magnitude of the vibration can be obtained from the frequency response plot.

For example, in this particular case for example, O 2. So, if we are taking phi 2 near to 2.2 phi 2 nearly equal to 2.2 we can find what is the response amplitude. So, the response amplitude is one to be around 20, 20 to. So, it will go on increasing also depending on the other system parameter ok.

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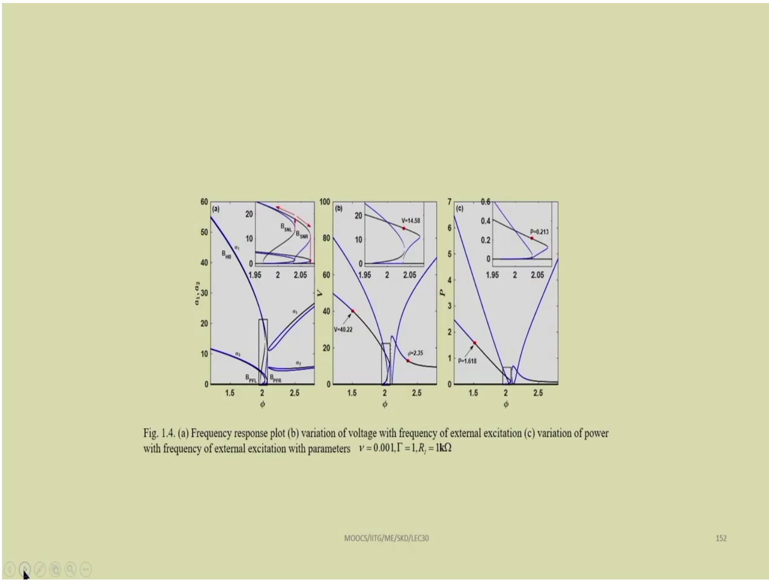

This way we can find the frequency response. So, frequency response actually can be plotted by solving this 4 or 5 equations what we have seen, reduced equation in first order reduced equation. By solving this non-linear equations by using Newton's method. So, we can find or we have plotted these frequency response plot. So, while plotting this one we have taken several initial conditions.

So, there are several methods to plot this thing. So, later you will know or you have known by this time these continuation principle also, continuation technique to get the response. By using different technique you can find the frequency response plot, but in this particular case we have taken a large number of initial conditions and use this Newton's method to find the response of the system.

So, in that way frequency response has been plotted. So, these are the frequency response for different value of this new and gamma. So, for example, nu equal to 0.001, gamma that is the base excitation equal to 1, this is the random initial parameters similarly R load resistance is taken to be 1 kilo ohm, in that case we have obtained this voltage. So, we can see we can obtain very high voltage in this case also.

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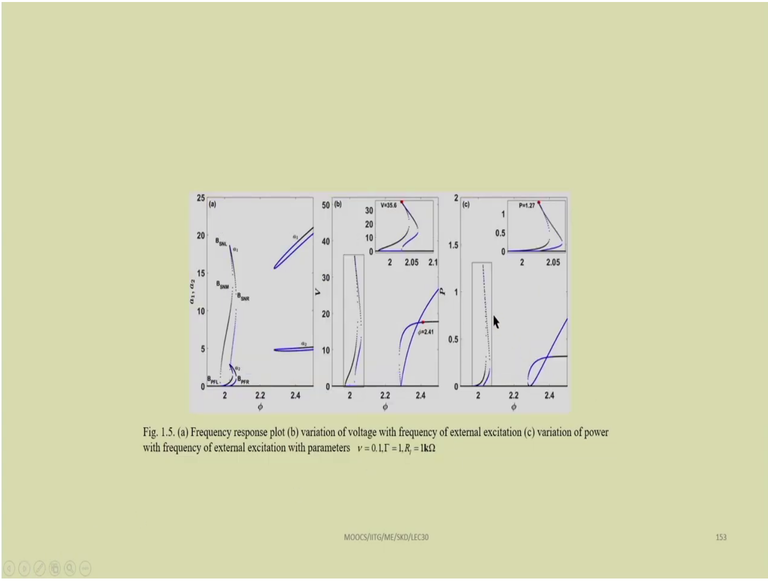

So, the last curve shows the power. So, power we are obtaining some in terms of the watt, few watt, power we are getting in this case.

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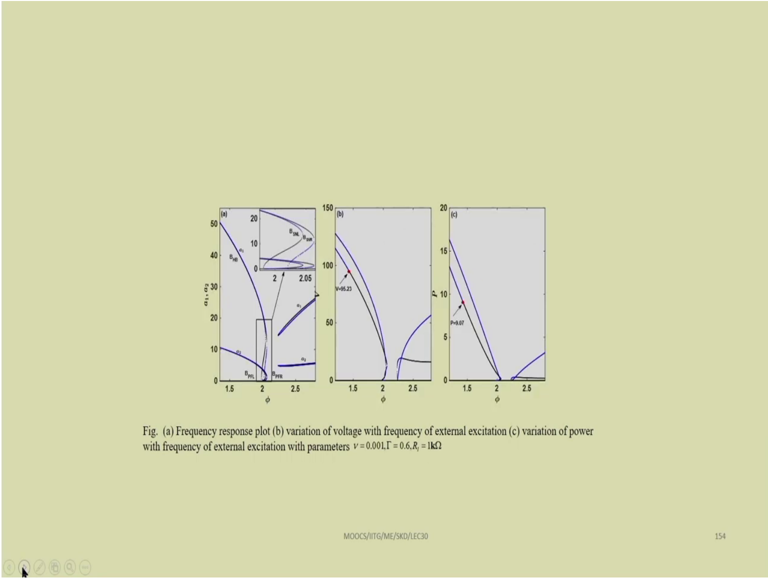

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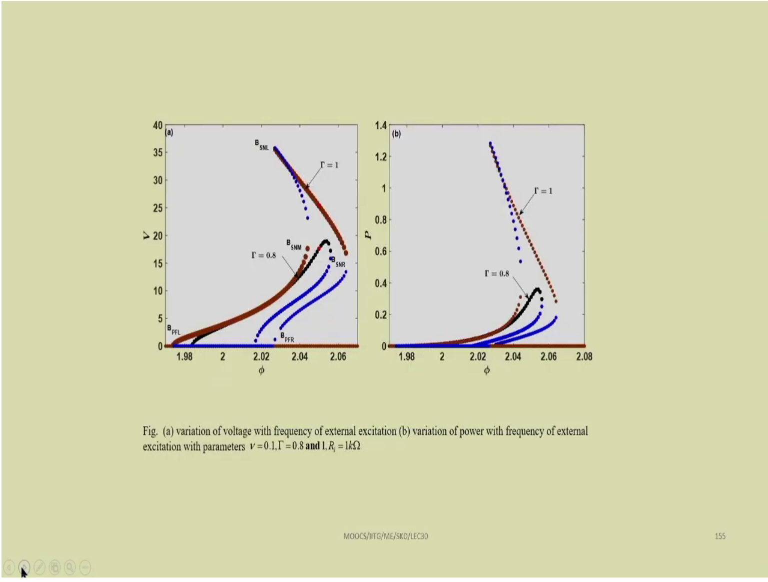

By changing different load resistance or changing different value of gamma and nu we can have different frequency response and from those frequency response we can find exactly how much energy we can harvest or what is the voltage we can get and how much power we can generate.

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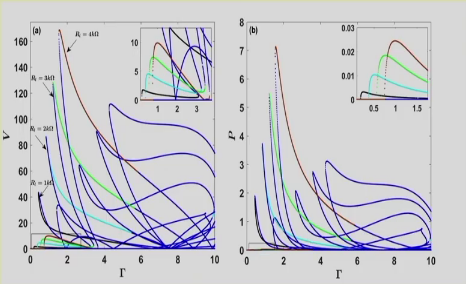

Fig. Variation of Voltage (V) and power (P) with amplitude of base excitation Γ for parameters $\nu = 0.001, \phi = 2$ with varying load resistance $R_l = 1,2,3,4\,\mathrm{k\Omega}$

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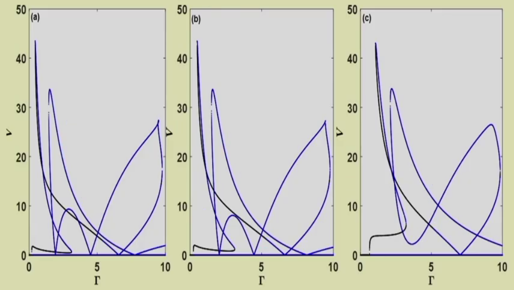

Fig. (a, b, c) Variation of voltage (V) with amplitude of base excitation Γ for parameters $\nu = 0.001, 0.01, 0.1; \phi = 2; R_l = 1\mathbf{k}\Omega$

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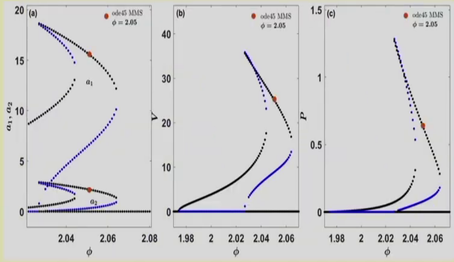

Fig. Time response (using ode45 in MATLAB) of reduced Eqs. (1.14) at $\nu = 0.1, \phi = 2.05, \Gamma = 1, R_l = 1\text{k}\Omega$

For different system parameter taking a large number of system parameters here all this plots are have been plotted, those who are interested to know more regarding this thing. So, they can study the system, they can physically derive these equations and stimulate these equations to find this response. This will be given as an assignment to find the frequency response and force response in case of the energy harvester.

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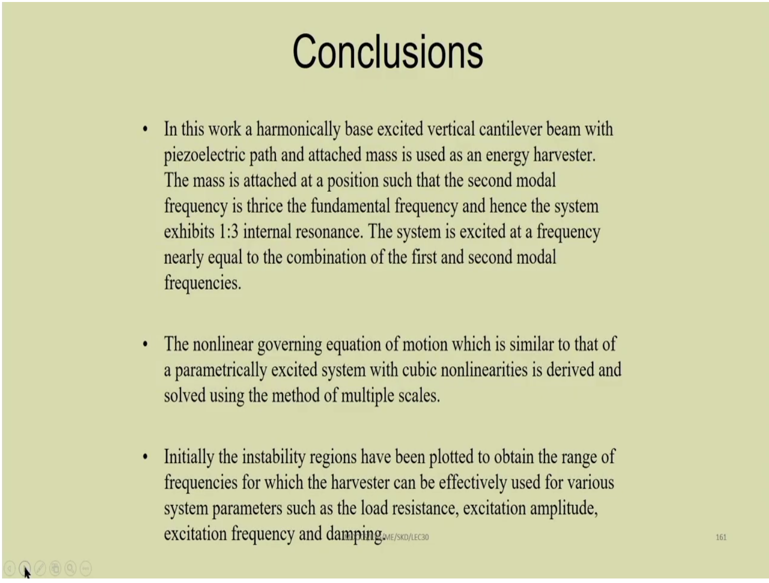

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- From these instability regions it has been observed that there exist critical value of amplitude of base excitation for different value of load resistance and damping below which the harvester can't be used as it leads to trivial state fixed point response.
- Critical bifurcation points such as saddle node, pitch-fork and Hopf bifurcations have been found which will help the designer to choose the system parameters to have optimum voltage or power generation.
- With internal resonance for low damping, softening behaviour is observed which helps in energy harvesting at low frequency of excitation.
- Also with internal resonance the energy transduction is higher for certain frequency range as compared to without internal resonance
- Excitation amplitude effects the critical bifurcation points and hardening behaviour changes to softening behaviour.

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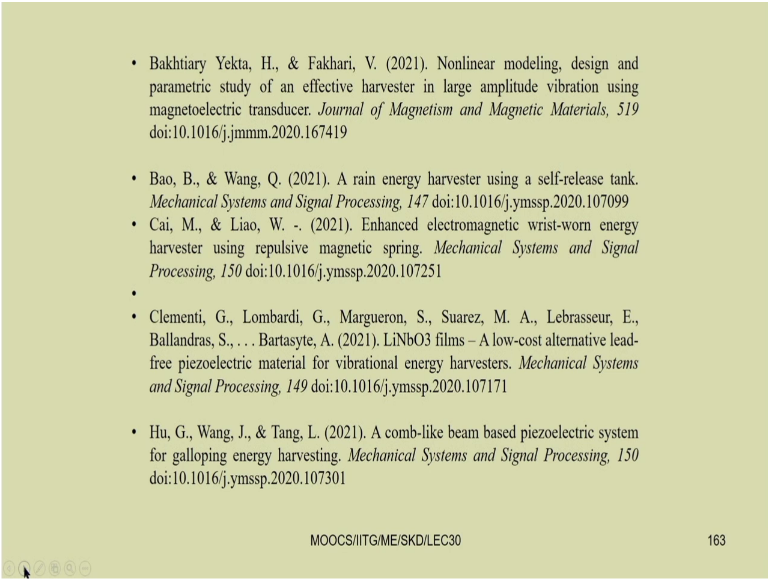

You have seen now. So, we can use this energy harvester for getting this voltage and power and their several papers available and you can particularly see there are several recent papers available on this energy harvesting. So, these are the paper you can published in 2021 for example, this Non-linear modeling, design and parametric study of an effective harvester in large amplitude vibration using magneto electric transducer.

So, here we have used this piezoelectric transducer one can use the magneto electric transducer where one magnet can be used it can be electromagnet or it can be permanent magnet. So, that electromagnet or permanent magnet can be used and a coil will be there. So, due to vibration these field will change and that will help in generating this voltage.

This is a very recent paper you can go through it and this, one more paper by Bao and Wang. A rain energy harvester using a self-release tank so, it is published in Mechanical Systems and Signal Processing. Then Cai and Liao they have published another paper Enhance electromagnetic wrist-worn energy harvester using repulsive magnetic spring.

So, you just see here the harvester is put on the wrist. So, from the body motion also we can harvest the energy or generate the energy. Another paper also we can see a low-cost alternative lead free piezoelectric material for vibration of energy harvester we are using particularly this PVDF or this PZT where we are using this lead.

So, this paper tells about lead free piezoelectric material. Then another paper you can see this is a comb like beam based piezoelectric system for galloping energy harvesting. So, this is very recent paper a number of beams are used for harvesting this energy.

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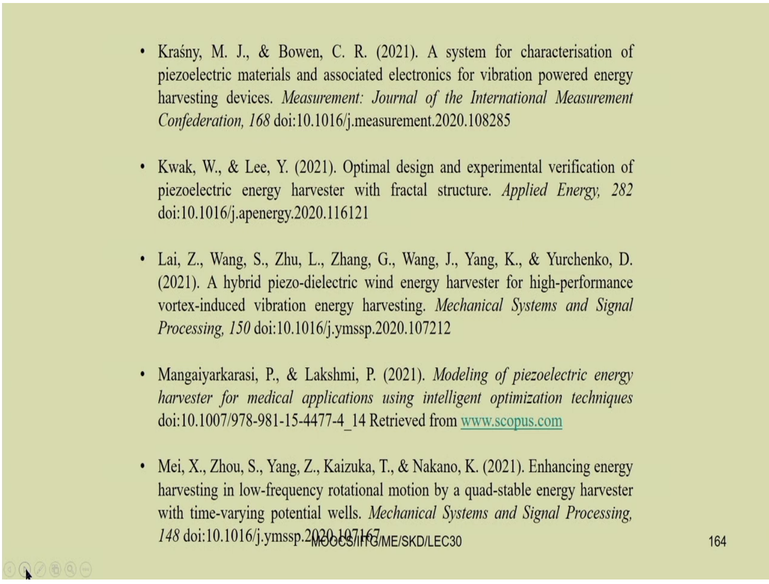

And then a system of characterization of piezoelectric material and associated electronics for vibration powered energy harvesting devices this is also very recent paper. Optimal design and experimental verification of piezoelectric energy harvester with fractal structure this is published in applied energy.

So, you can go through this one also a hybrid piezoelectric wind energy harvester for high performance vertex induced vibration energy harvesting. So, here galloping method is used for generating the or harvesting the energy, then modeling of piezoelectric energy harvester for medical applications using intelligent optimization techniques, then you can see also the enhancing energy harvesting in low frequency rotational motion by a quad stable energy harvester with time varying potential well.

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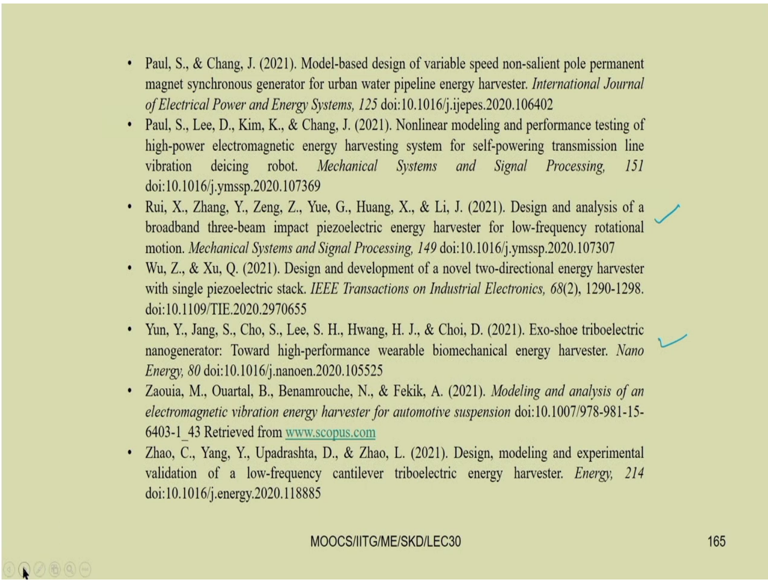

So, these are the recent publications what I am showing, I have taken it from Scopus and just the created this bibliography of the recent paper. So, some of the more recent papers are model based design and variable speed non salient pole permanent magnets synchronous generator for urban water pipeline in energy harvesting.

Then non-linear modeling and performance testing of high power electromagnetic energy harvesting system for self-powering transmission line, vibration, deicing robot then you can have this design and analysis of a broadband three beam impact piezoelectric energy harvester.

So, here we have taken a single beam. So, you can take three beams or you can take multiple beams also to harvest this energy. So, multiple beams can be taken to harvest the energy also then design and development of novel two directional energy harvester with single piezoelectric stack.

Then Exo-shoe triboelectric magnet nanogenerator. So, here you just see these are the variable devices towards high performance variable bio mechanical energy harvester. Then modeling and analysis of an electromagnetic vibration energy harvester for automotive suspension in the beginning I have shown how we can put this piezoelectric patch on the lip spring also how we can put them on the helical spring in the shock observer also.

So, there are some other paper for example, design modeling and experimental validation of a low frequency cantilever triboelectric energy harvester. Similarly, you can have many other you can such many other paper you can see some of the paper related to energy harvesting are shown here.

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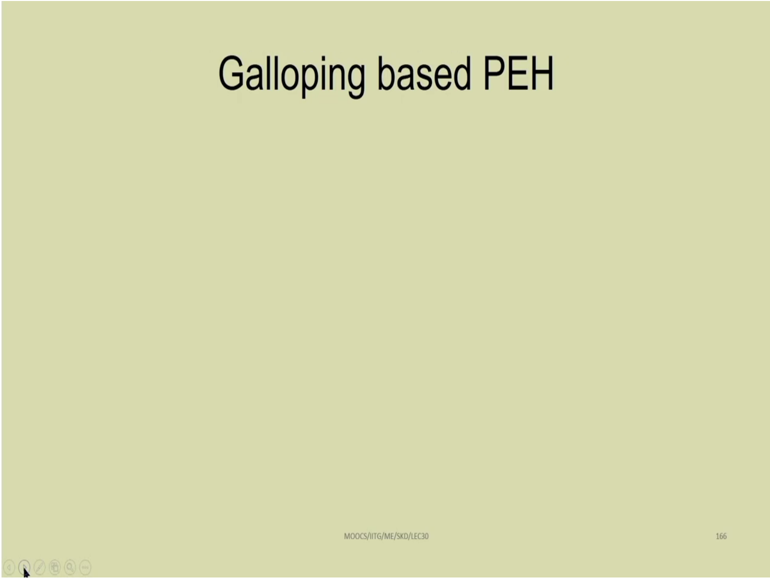

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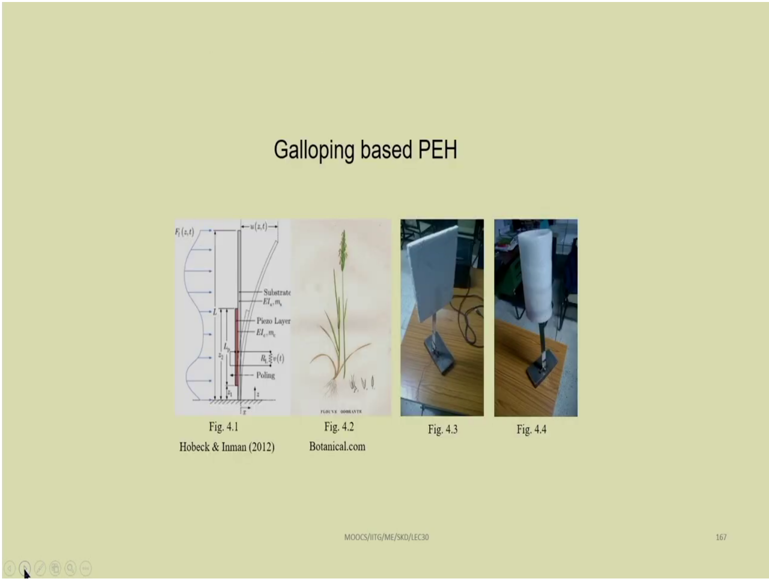

So, you can take any of them and go through it and see this thing. Here also we have developed this galloping based piezoelectric energy harvester. In case of galloping in this same cantilever beam, bottom part we can put the piezoelectric and here we can put a bluff body. When it is put in front of the wind, then due to the obstruction by the bluff body. So, wake formation will be there in the back side and this will give raise to this vertex induced vibration in this case.

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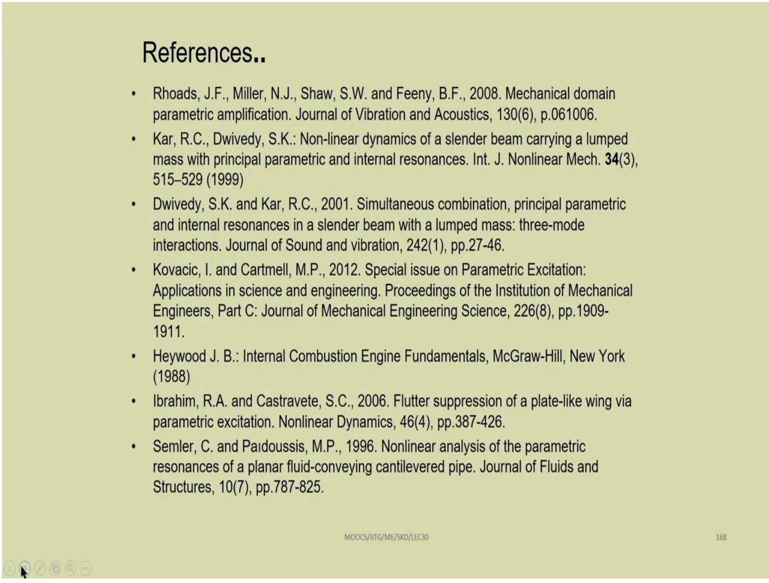

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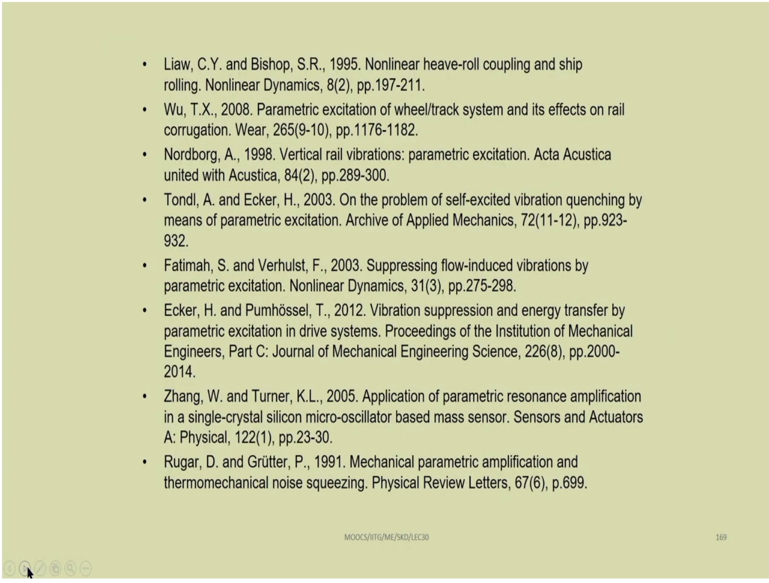

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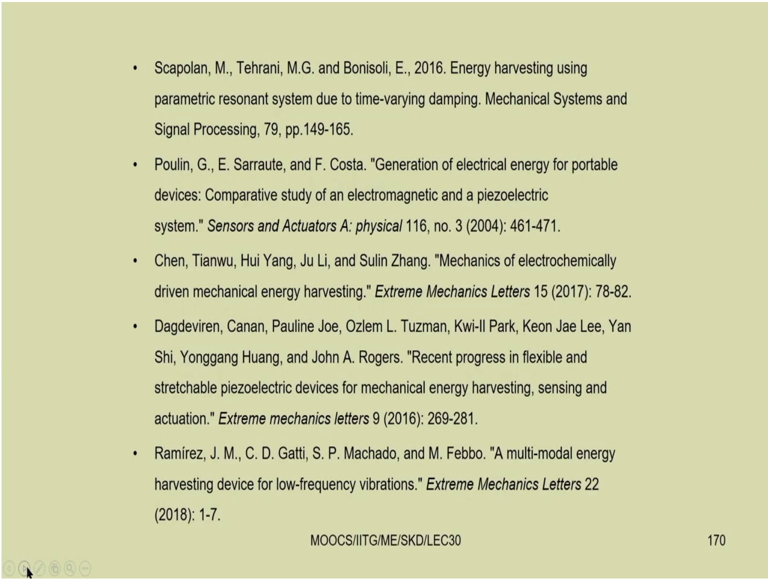

So, these are some of the references related to galloping based energy harvester. One can go through all this papers so, many references have been given the basic principle remaining same, but the forcing part will be different.

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So, in this case due to this wind force forcing term can be taken, in the previous case we have take we have given the a axial loading and base excited system. So, from there forcing term were generated here one can see. So, this is the bluff bodies just we have put in front of the UPS. So, from the ups this air is coming, which is vibrating the structure. So, as it is vibrating. So, you can see energy is generated voltage is generated, which we have shown through this oscillator scope.

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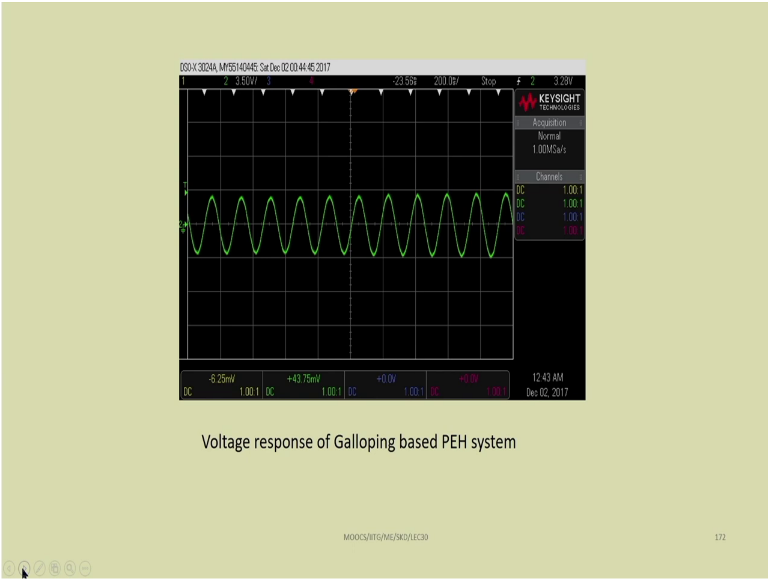

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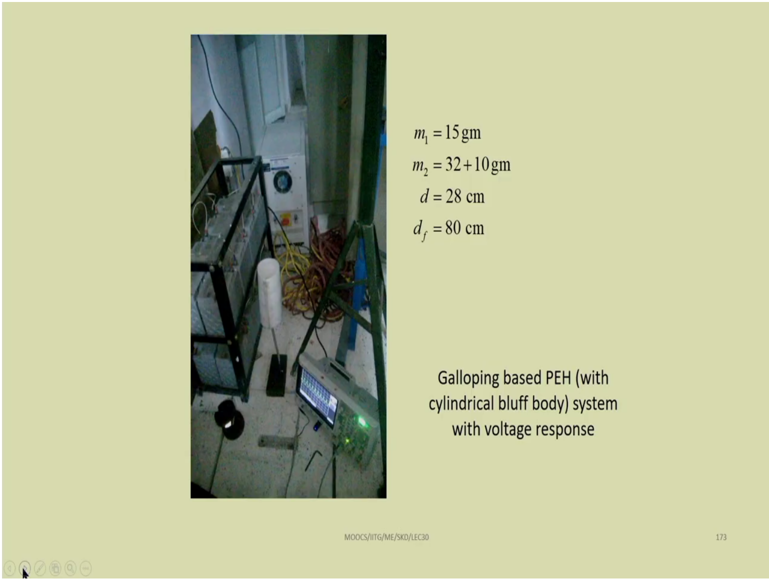

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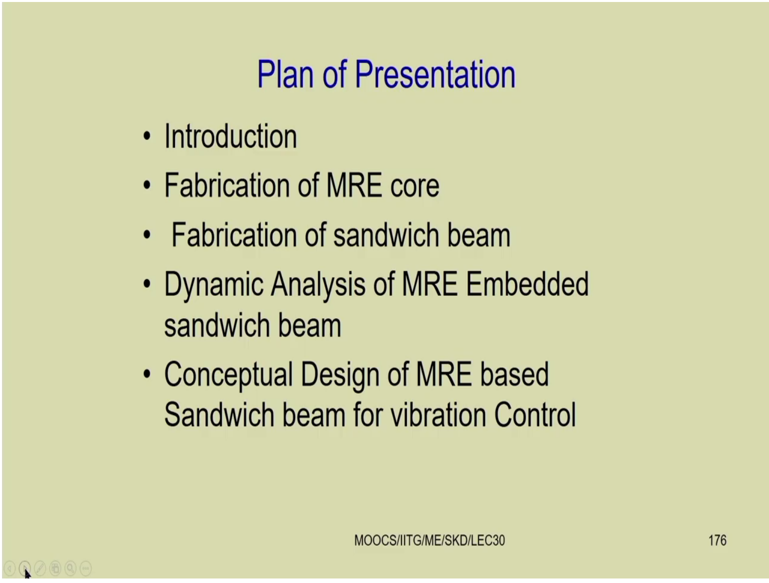

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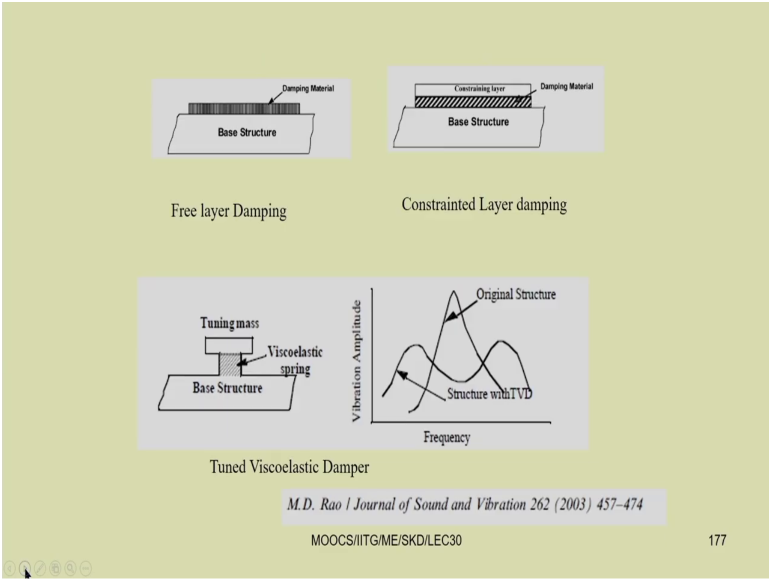

So, experimentally we have shown these voltage generation due to this galloping phenomena. Now, briefly let me tell how you can use similar concept. So, you have seen now we have three layer two piezoelectric layer and in between we have the substrate.

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Automotive applications

Engines and powertrains	Body structures	Brakes, and accessaries
Oil pans	Dash panels	Brake insulators
Valve covers	Door panels	Backing plates
Engine covers	Floor panels	Brake covers
Push rod covers	Wheelhouses	Steering brackets
Transmission covers	Cargo bays	Door latches
Timing belt covers	Roof panels	Window motors
Transfer case covers	Upper cowl	Exhaust shields

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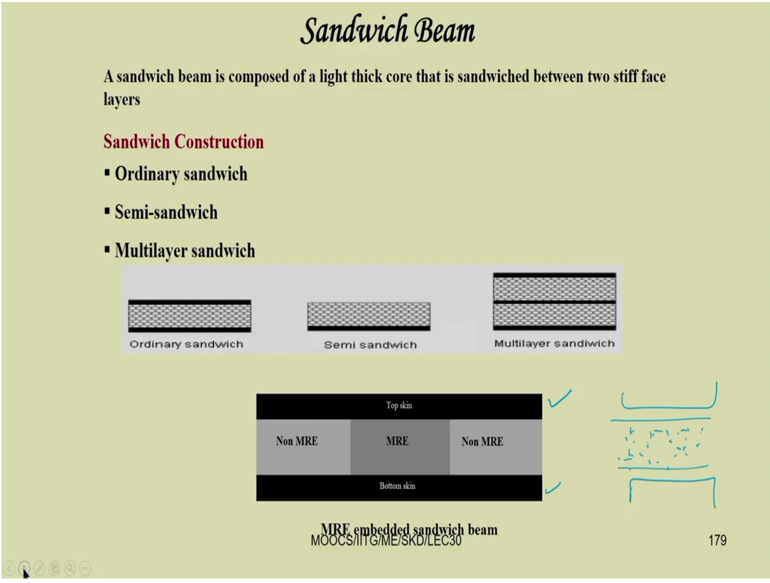

So, similarly, so, those are sandwich structure. So, these are sandwich structure. So, in the sandwich structure so, we can have a top skin also a bottom skin and in between we can have the viscoelastic layer. So, in this viscoelastic layer we can make these magneto rheological elastomer. So, in case of magneto rheological elastomer. So, this is the elastomer that is viscoelastic material.

So, while curing. So, we use to put this iron particle and after it got cured this becomes this MRE Magneto Rheological Elastomer. So, by applying this magnetic field. So, one can find or you can align this magnetic field or this iron particle. Due to this alignment of the iron particle the Young's modulus which is a complex number changes that is the stiffness and the damping property of the material changes.

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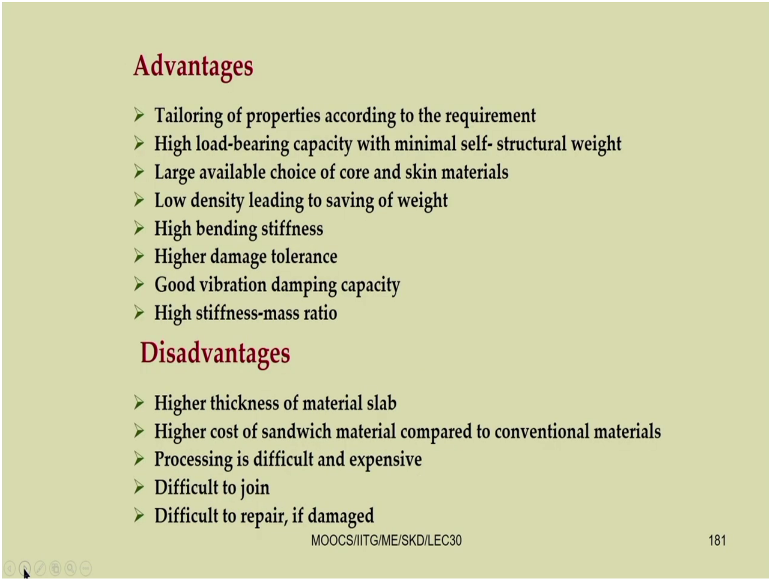

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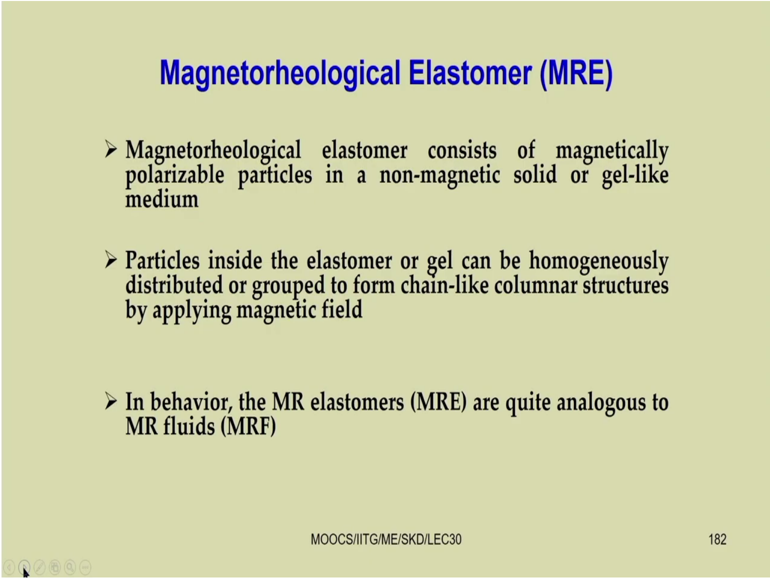

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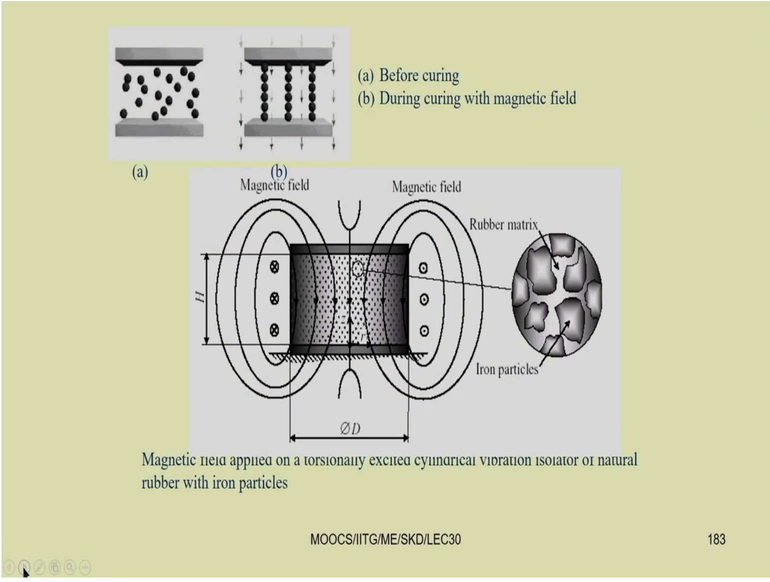

So, we can have different skin material and also different core materials. So, we can go through all this thing. So, there are several advantage and disadvantage of this thing magnetorheological elastomer. So, this is the basic principle already I told ok.

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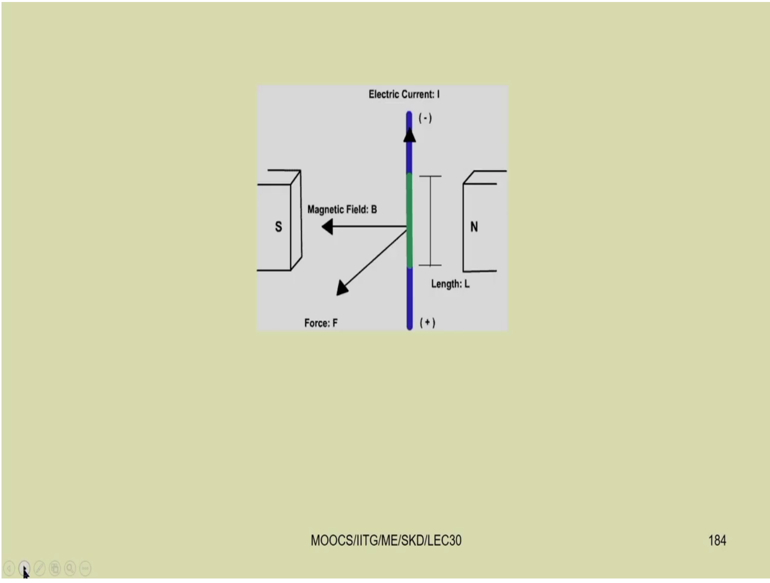

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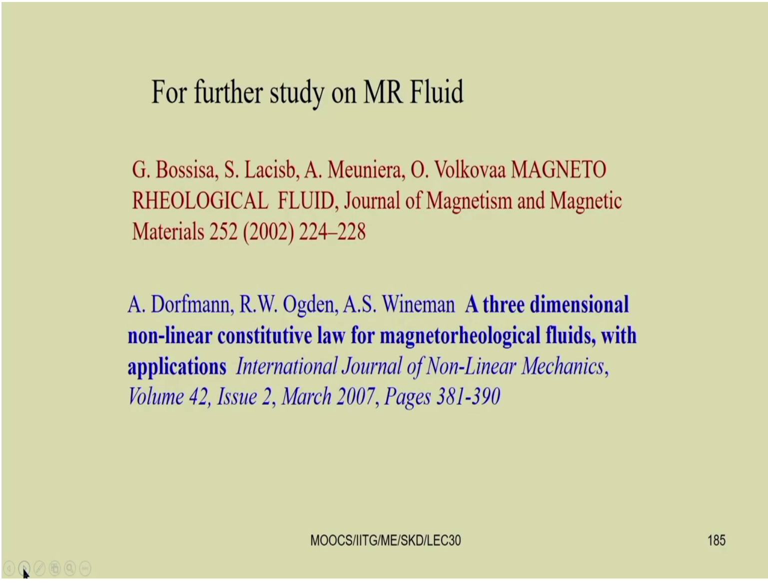

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the stress would have reached this maximum. MR suspensions have first received less attention than electrorheological suspensions mainly because the weight and the space required by the coils to produce the magnetic field were thought to be a severe restriction for practical applications. Furthermore, the response time of the fluid is limited by the rising time $\tau = L/R$ (with L being the inductance and R the resistance of the coils) of the magnetic field which in practice is about 10^{-1}–10^{-2} s. Nevertheless, just comparing the magnetostatic energy density $\mu_0 H^2$ for $H = 3000$ Oe and the electrostatic energy density $\varepsilon_0 E^2$ for a field $E = 3$ kV/mm (close to the breakdown field), it appears that the former is larger by an order of magnitude. This is the main reason why yield stresses obtained with MR fluids are usually much larger than those obtained with ER fluids. Yield stresses close to 100 kPa are obtained with magnetic suspensions made of carbonyl iron [4] whereas it is difficult to attain 10 kPa with an ER fluid.

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Types of MREs

1. Isotropic
2. Anisotropic (Structured)

Difference between Isotropic and Anisotropic MREs

- The major difference between the two types of MREs is the formation of chain like and columnar structures in the matrix along the direction of magnetic field before curing for the structured MREs.
- The relative MR effect for structured MREs is larger than that for isotropic MREs at the same iron particle content
- The loss factor of isotropic MREs is larger than that of structured MREs

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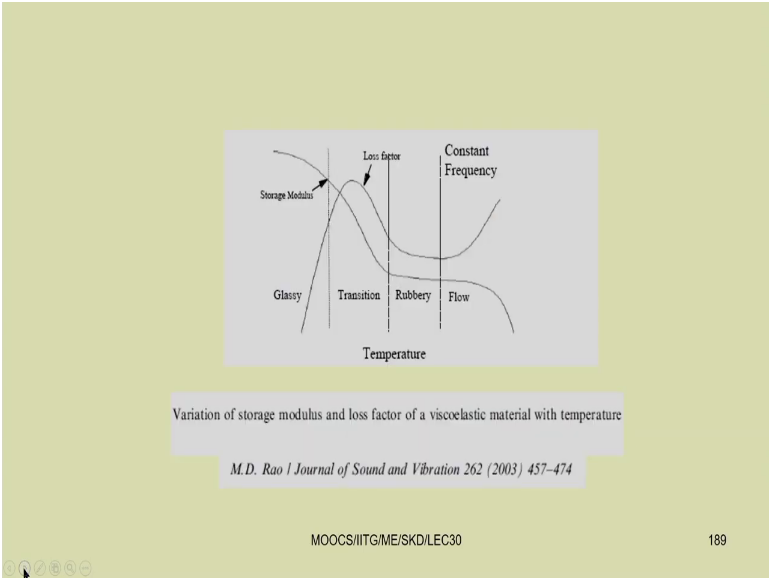

So, there are two different type of MRE, one is the isotropic MRE second one is the anisotropic MRE and you can see by applying temperature we can change this loss factor and storage modulus.

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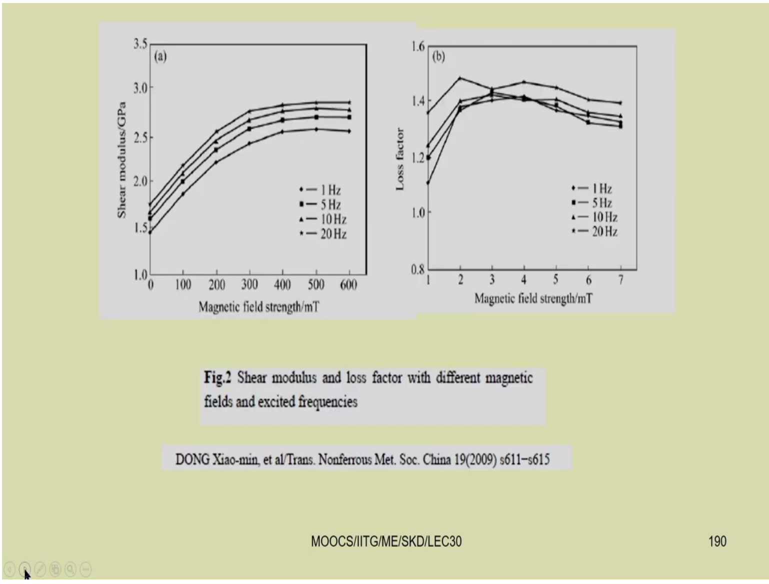

Similarly, by applying magnetic field also we can have the storage differential modulus and loss factor. So, this is the magnetic field strength. So, go on applying we go on applying this magnetic field strength and you can see the shear modulus is changing. So, here the loss factor is also changing.

So, initial loss factor and final loss factor if you see then we can increase it 2 times to 3 times also or 1.5 times. Here it is shown for this case it is 1.5 times here also you can see this is of 1.5 to 2 times also we can increase the shear modulus storage modulus.

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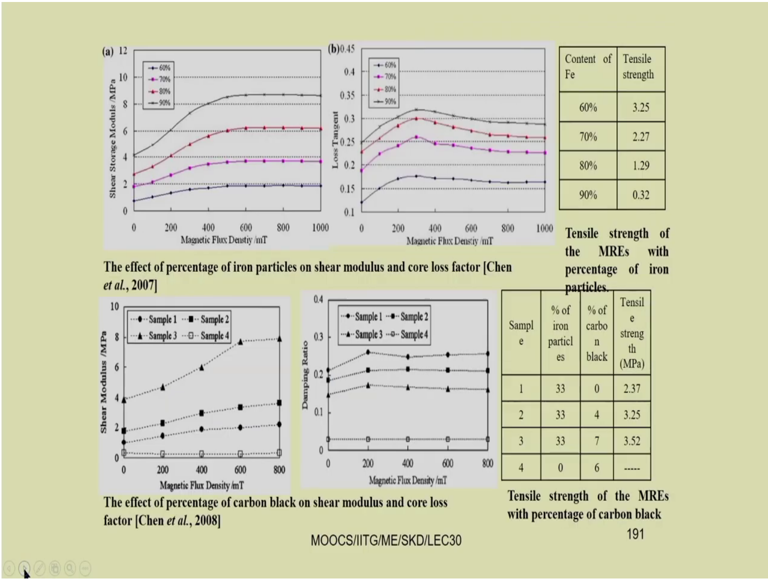

By changing this shear storage modulus and also this loss factor. So, we can control or we can make the sandwich beam as these a smart material.

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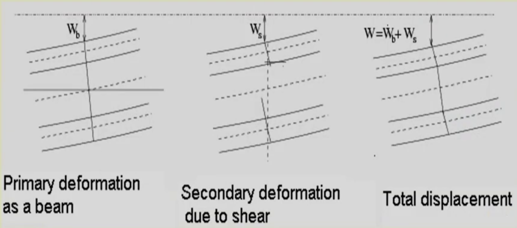
Sandwich Beam Theories

Classical Theory

The deflection of the upper and lower faces are equal to each other, and is known as "Anti-plane" concept.The longitudinal displacement distribution throughout the height of the core is linear. Core is having no strain in transverse direction



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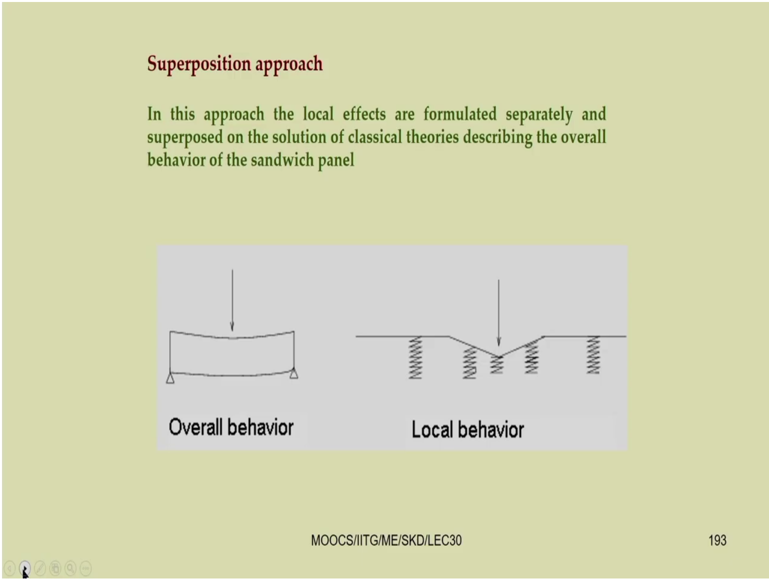

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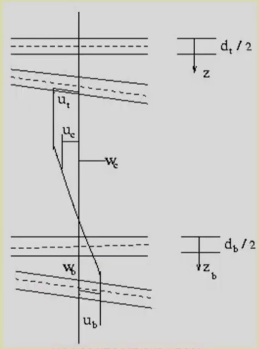
Higher Order Theory

For sandwich panels with core as flexible materials such as foam materials the displacement fields through out the height are non-linear, so the anti-plane approach or classical theory is not applicable



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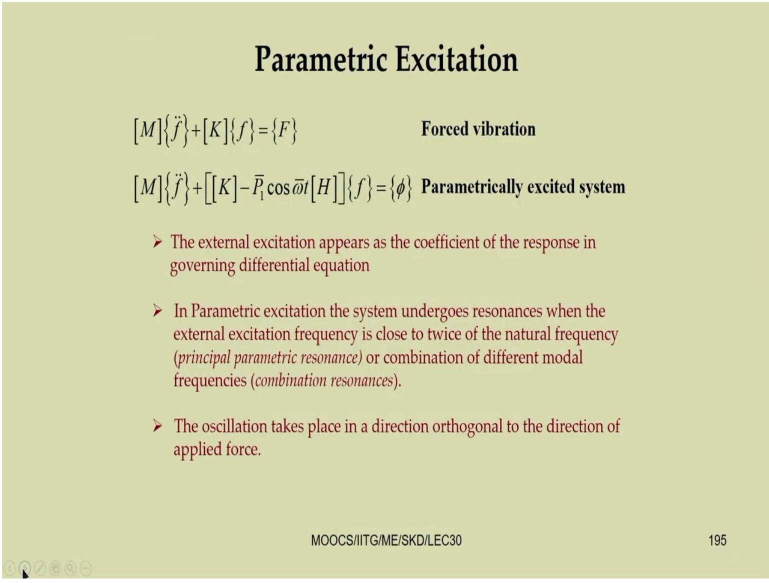

So, here we can use different percentage of iron particle, different percentage of this carbon black and we can change this material property and we can develop our own sandwich structure and we can apply different classical theory or super position of rows or high order theory and you can study the system.

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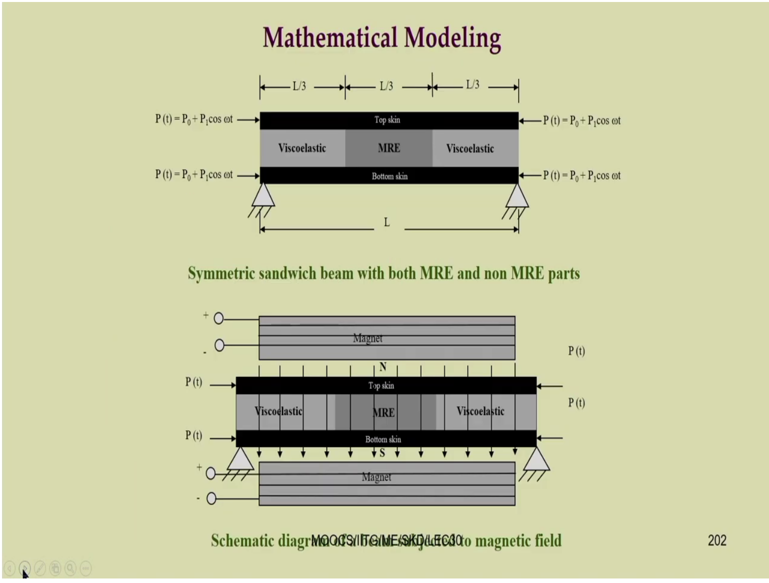

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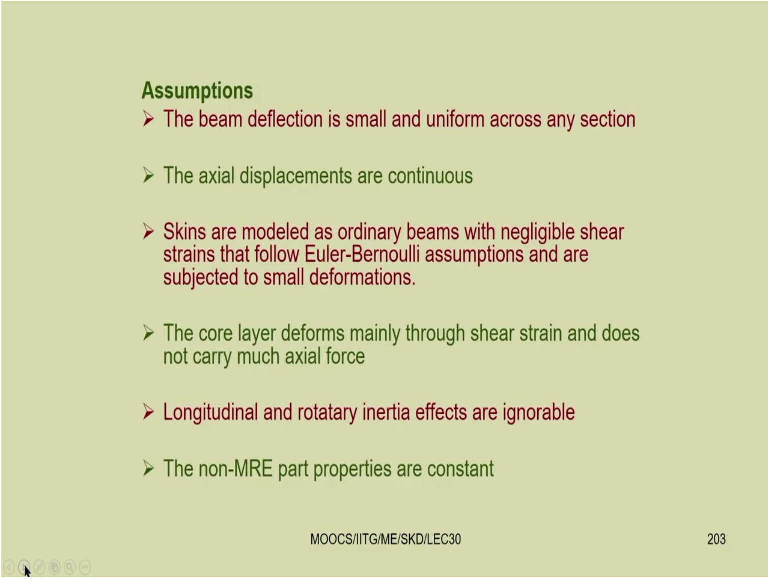

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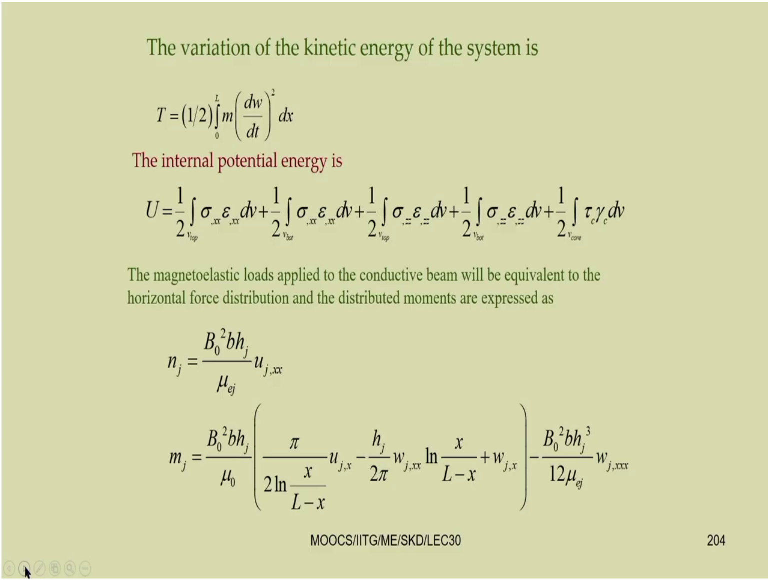

So, if the system is subjected to one axial load. So, there are several literature available here. So, these are literature some of the literature have been shown here. In this particular case let us have a sandwich beam where this is subjected to axial loading and these magnetic field is applied.

So, we can have different boundary conditions. So, for different boundary conditions can derive this kinetic energy, potential energy then these loading due to. So, if you are taking the skin material to be conductive then we can have the Laurence force.

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The non-conservative work is

$$W_{nc} = (1/2)\int_0^L P w_{,x}^2 dx + \int_0^L n_t \delta u_t dx + \int_0^L n_b \delta u_b dx + \int_0^L m_t \delta w_{,x} dx + \int_0^L m_b \delta w_{,x} dx$$

Using extended Hamilton's principle

$$\int_{t_1}^{t_2} (\delta L + \delta W_{nc}) dt = 0,$$

$$\delta L = \delta T - \delta U$$

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The obtained non-dimensional equations of motion are

$$\ddot{\bar{w}}+\left[(1+Y)+Y\frac{B_0^{\,2}}{\mu_e E}-\frac{B_0^{\,2}}{\mu_e E}\right]\bar{w}_{\bar{x}\bar{x}\bar{x}\bar{x}}-\frac{6}{\pi}\left(\frac{B_0^{\,2}}{\mu_0 E}\right)\left(\frac{L}{h_t}\right)\ln\left(\frac{\bar{x}}{1-\bar{x}}\right)\bar{w}_{\bar{x}\bar{x}\bar{x}}+3\left(\frac{B_0^{\,2}}{\mu_0 E}\right)\left(\frac{L}{h_t}\right)^2\bar{w}_{\bar{x}\bar{x}}-\left(Y+Y\frac{B_0^{\,2}}{\mu_e E}\right)\bar{u}_{\bar{x}\bar{x}\bar{x}}+\bar{P}\bar{w}_{\bar{x}\bar{x}}=0$$

$$\bar{w}_{\bar{x}\bar{x}\bar{x}}-\bar{u}_{\bar{x}\bar{x}}+g_c^{\,*}\left(\frac{h_t}{h_c}\right)^2\bar{u}+\frac{B_0^{\,2}}{\mu_e E}\bar{w}_{\bar{x}\bar{x}\bar{x}}-\frac{B_0^{\,2}}{\mu_e E}\bar{u}_{\bar{x}\bar{x}}=0$$

Where,

$$g_c^{\,*}=g_c\left(1+j\eta_c\right),\qquad g_c=\left(\frac{G_c}{2E}\right)\left(\frac{h_c}{h_t}\right)\left(\frac{L}{h_t}\right)^2,\qquad Y=3\left(1+\frac{h_c}{h_t}\right)^2$$

Solutions of Equations of motion are assumed in the form

$$\bar{w}\left(\bar{x},\bar{t}\right)=\sum_{i=1}^{r}q_i\left(\bar{t}\right)w_i\left(\bar{x}\right),\qquad \bar{u}\left(\bar{x},\bar{t}\right)=\sum_{k=r+1}^{s}q_k\left(\bar{t}\right)u_k\left(\bar{x}\right)$$

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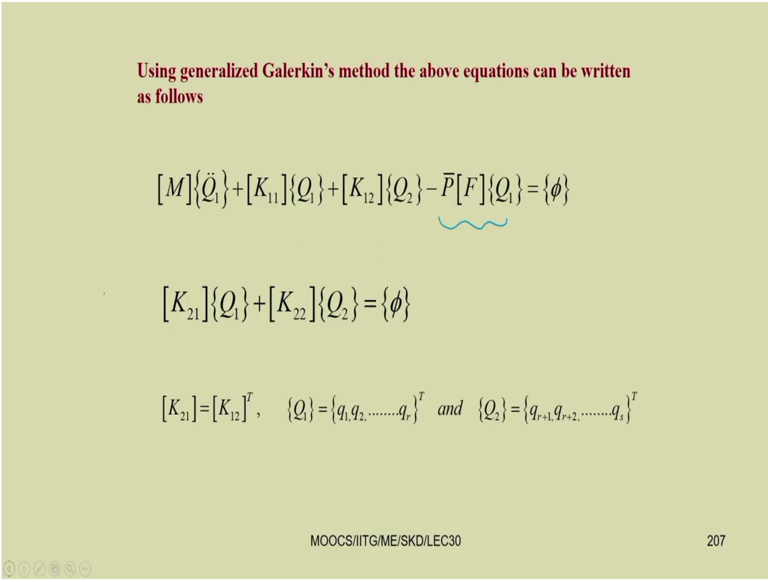

And couple we can find this n and this force and this couple and due to that thing this axial loading will be there and we can find the non-conservative work done and using extended Hamilton principle. So, we can derive this equation motion. So, after getting this equation motion, we can apply equation motion will be in the form it will be spatio temporal equation.

So, now, we can convert that thing to its temporal form by using Galerkin method. So, which will reduce to this form. So, where this M is matrix, K 11 K 12. So, all these are matrix. So, Q 1 2. So, these are the state vectors. So, you can see due to the presence of this term P 1 F Q 1 where Q is coefficient of Q 1 equal to P 1 F. So, which is a time varying function. So, this is a parametrically excited system.

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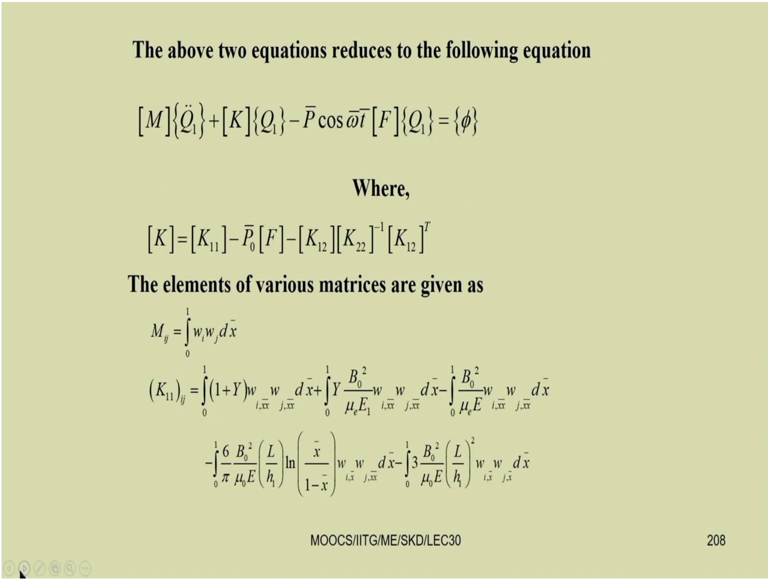

So, in this parametrically excited system you just see this K equal to stiffness part K 11 minus P 0 F minus K 12 K 22 inverse into K 12 transpose, this way you can write down this equation and different elements of different matrix can be found here.

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$$(K_{12})_{ij} = -\int_0^1 Y w_{i,\bar{x}\bar{x}} u_{k,\bar{x}} \, d\bar{x} - \int_0^1 Y \frac{B_0^2}{\mu_e E} w_{i,\bar{x}\bar{x}} u_{k,\bar{x}} \, d\bar{x}$$

$$(K_{22})_{kl} = -\int_0^1 Y u_{k,\bar{x}} u_{l,\bar{x}} \, d\bar{x} + \int_0^1 Y \frac{B_0^2}{\mu_e E} u_{k,\bar{x}} u_{l,\bar{x}} \, d\bar{x} + \int_0^1 Y g^* \left(\frac{h_t}{h_c}\right)^2 u_{k,\bar{x}} u_{l,\bar{x}} H_1 \, d\bar{x}$$

$$+\int_0^1 Y g^* \left(\frac{h_t}{h_c}\right)^2 u_{k,\bar{x}} u_{l,\bar{x}} H_2 \, d\bar{x}$$

$$F_{ij} = -\int_0^1 w_{i,\bar{x}} w_{j,\bar{x}} \, d\bar{x}$$

For the beam MRE placed in the middle of the core

$$H_1 = 1 - H(x - L_1) + H(x - L_2)$$

$$H_2 = H(x - L_1) - H(x - L_2)$$

Where, $L_1 = L/3, L_2 = 2L/3$

Where H is a heaviside function

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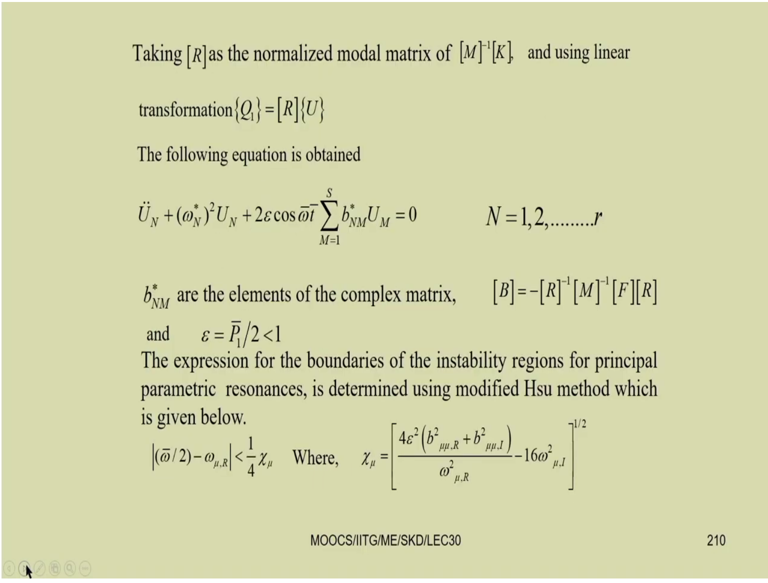

Actually, you can go through detail of this work in the work of Nayak Biswajith Nayak was my PhD student, you can get this reduced equation now using this reduced equation. So, we can use this modified Hsu method to find the instability region.

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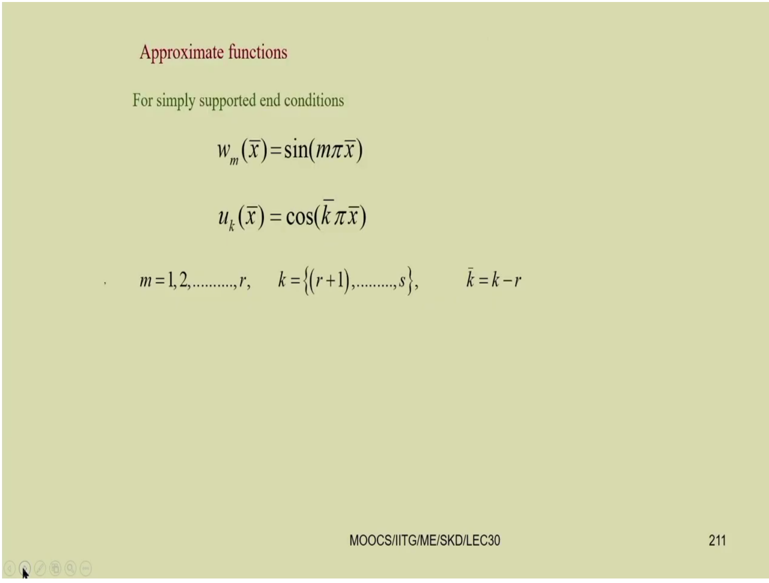

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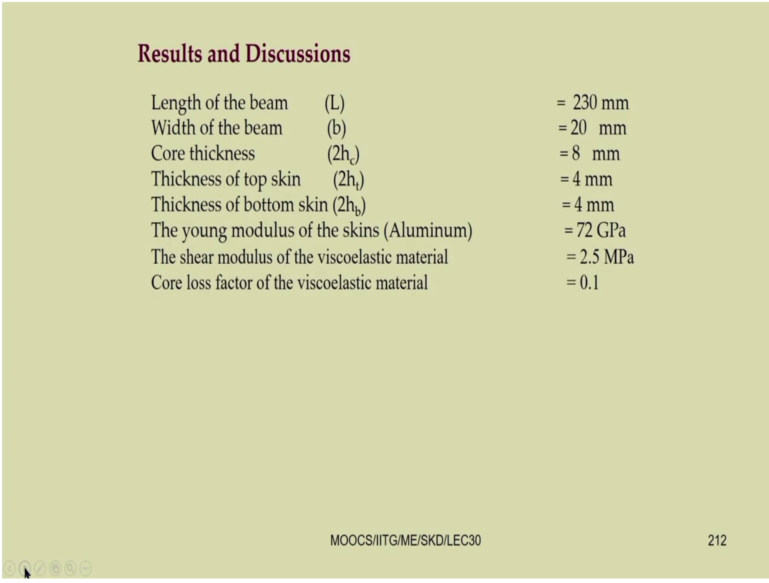

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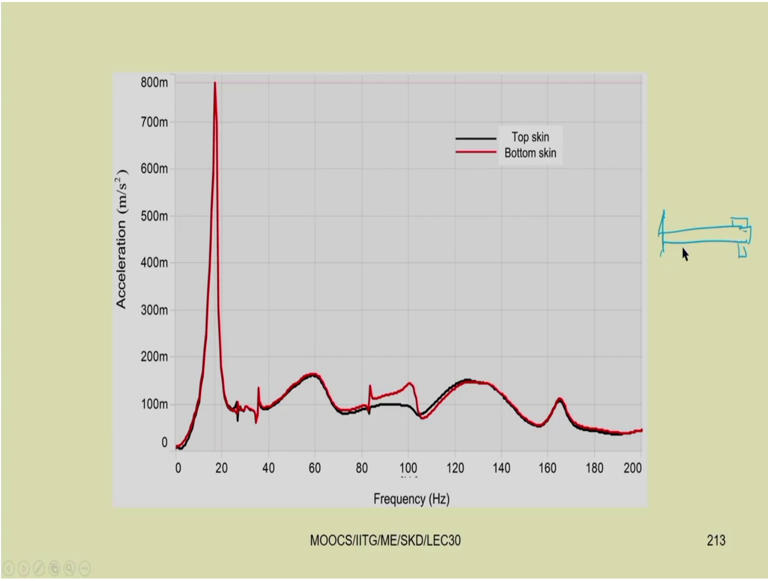

So, we can find this instability region. So, these are the equations already given there in this modified Hsu method. By using this Hsu method, we can find the response of the system and if we can do this experiment for example, let us take the cantilever beam and put accelerometer here and accelerometer here.

For the top accelerometer and bottom accelerometer if you see this acceleration verses frequency you can see here matching. So, one can use classical theory for this type of analysis also. You can see the response also for the top skin and bottom skin.

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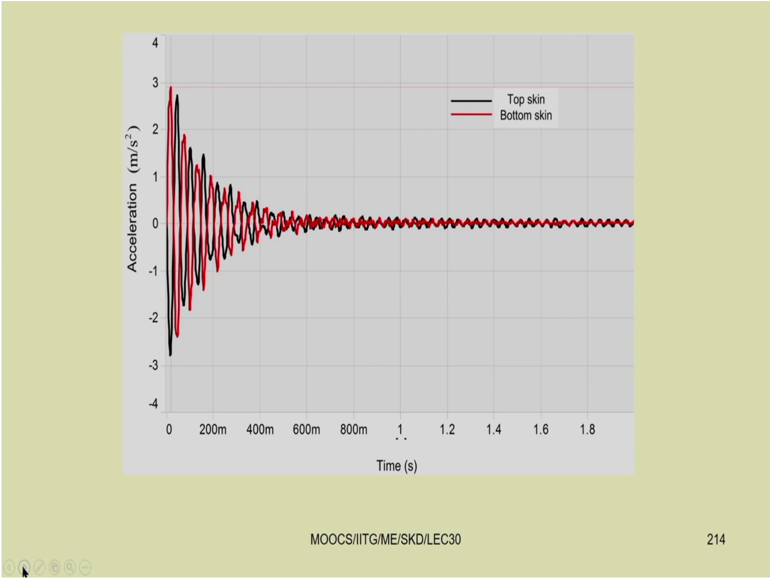

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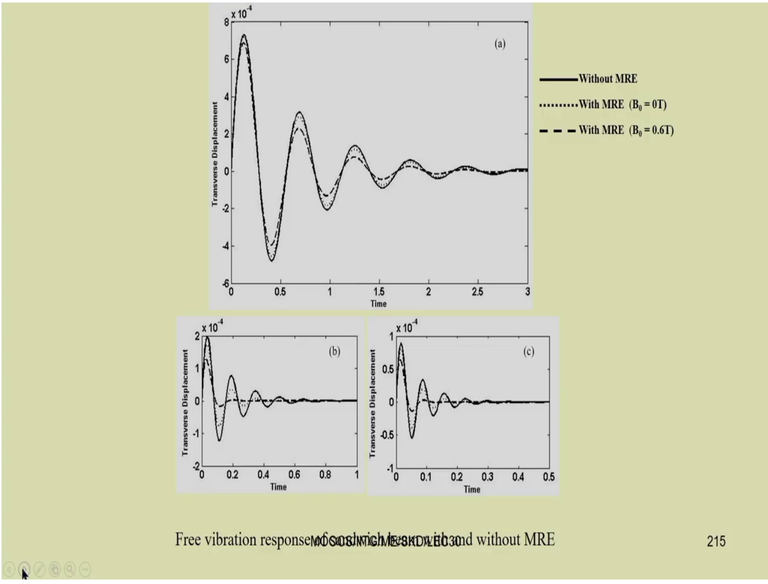

So, now you can see the response without applying voltage and without applying magnetic field and with applying magnetic field. So, it is clearly evident that by applying magnetic field we can reduce the vibration considerably here it is more clear. So, we can see the vibration is reduced and the settling time we can reduce the settling time very much.

So, in the first case while it is. So, without you just see. So, it is vibrating up to this, but when we are taking with a magnetic field, you can see the vibration is settled down here. So, we have a settling and this peak value is also very less without magnetic field peak value is very high and with magnetic field it is very less.

(Refer Slide Time: 56:06)

Table 1

End conditions of the sandwich beam	Natural frequency no.	Present model	Ray and Kar [1995] (From Figures (5,7,8,9and 13))
Simple supported	1	10.16	10
	2	39.77	39.5
	3	89.12	89.0

Table 2

End conditions of the sandwich beam	Natural frequency no.	Present model	Howson and Zare [2005]
Simple supported	1	59.58	57.136
	2	229.03	219.585
	3	485.33	465.172

Table 3

End conditions of the sandwich beam	Natural frequency no.	Present model (0T)	Present model (0.09T)	Experiment (Sun *et al.* [2003]) (0T)	Experiment (Sun *et al.* [2003]) (0.09T)
Simple supported	1	34.016	40.499	31.373	41.176
	2	95.398	110.684	64.706	78.431
	3	187.203	207.192	131.370	164.710

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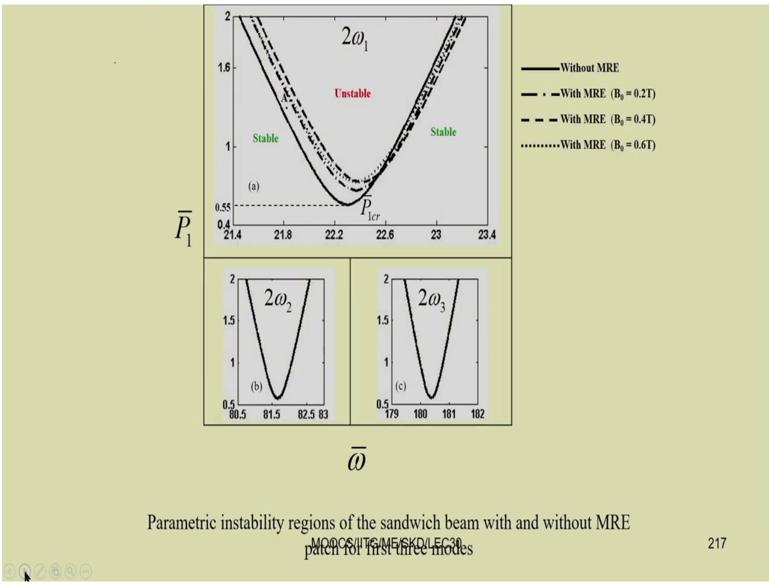

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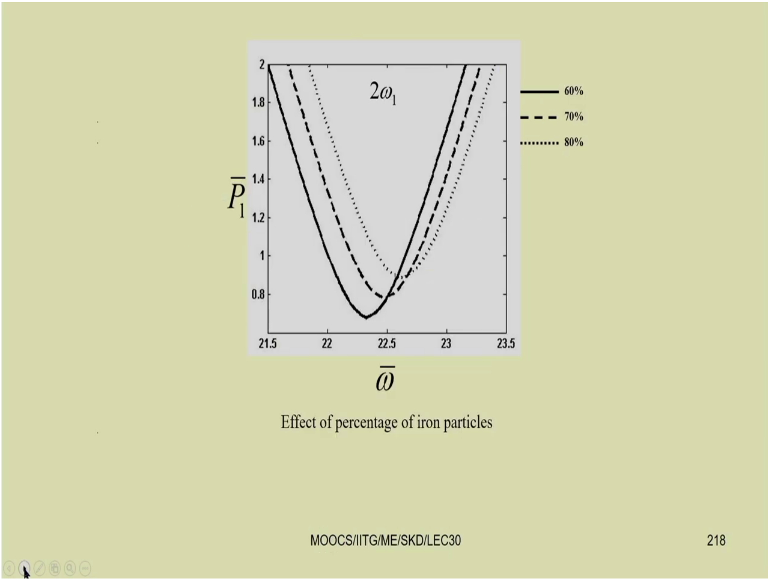

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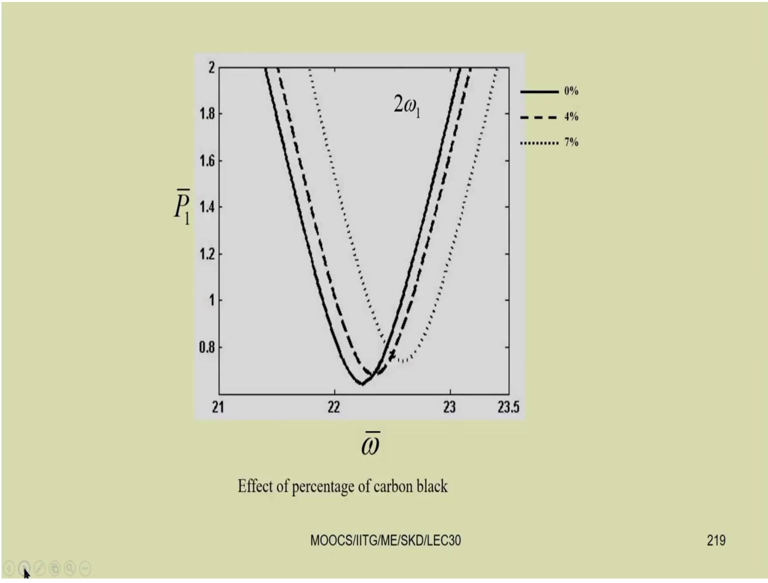

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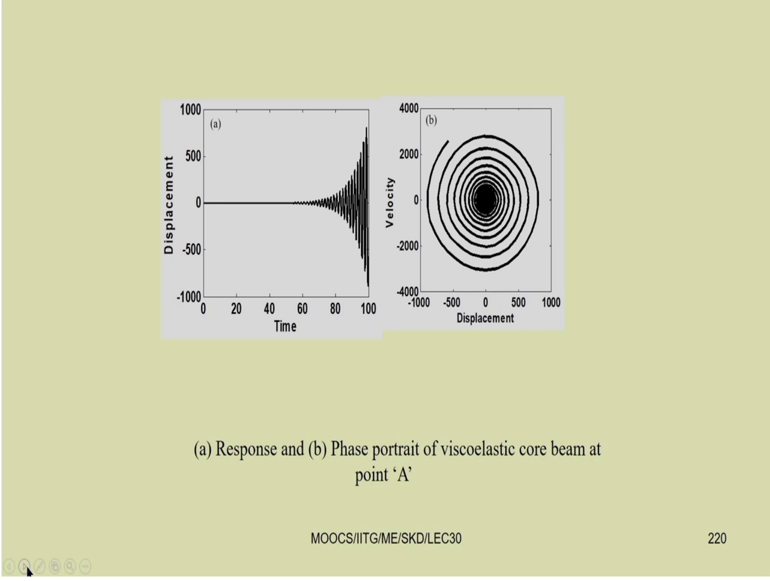

So, this way you can control the vibration of a sandwich beam by applying this magnetic field. So, this is instability region you can plot with different effect different carbon black you have put iron particle and also you have put magnetic field. So, by putting magnetic field different iron particle and different carbon black percentage of carbon black.

So, we can see the response amplitude can be conveniently controlled. So, if it is operating in this instability region inside this thing then one can find the response amplitude will grow.

So, clearly the source the response is growing. So, if you want to use this as a energy harvester, then you can use it inside this unstable region. If you want to use this as a for controlling the vibration purpose then you can use in this stable region. One can observe this biting type of phenomena also.

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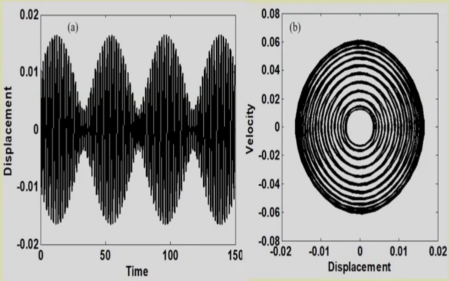

(a) Response and (b) Phase portrait of MRE embedded viscoelastic core beam at point A, subjected to magnetic field of 0.2T

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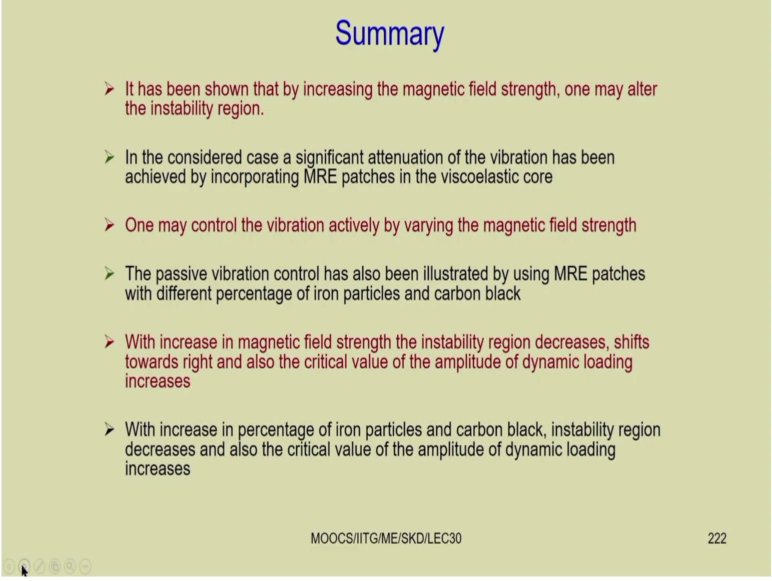

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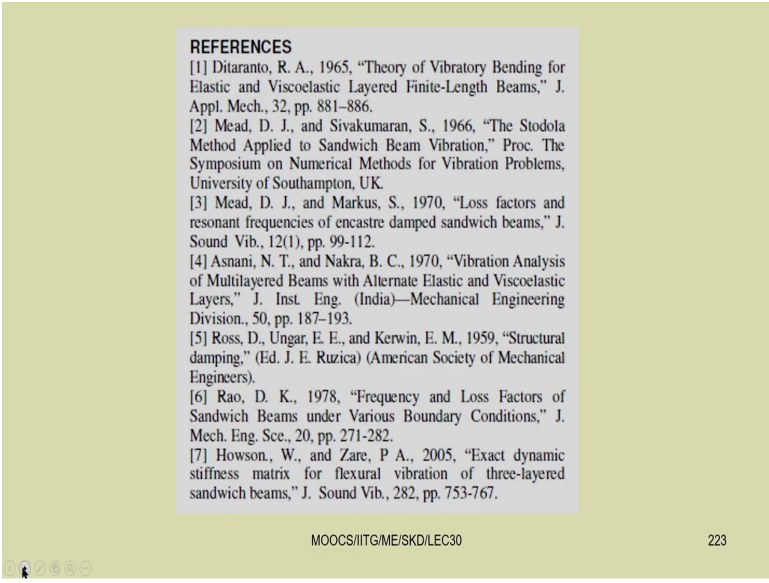

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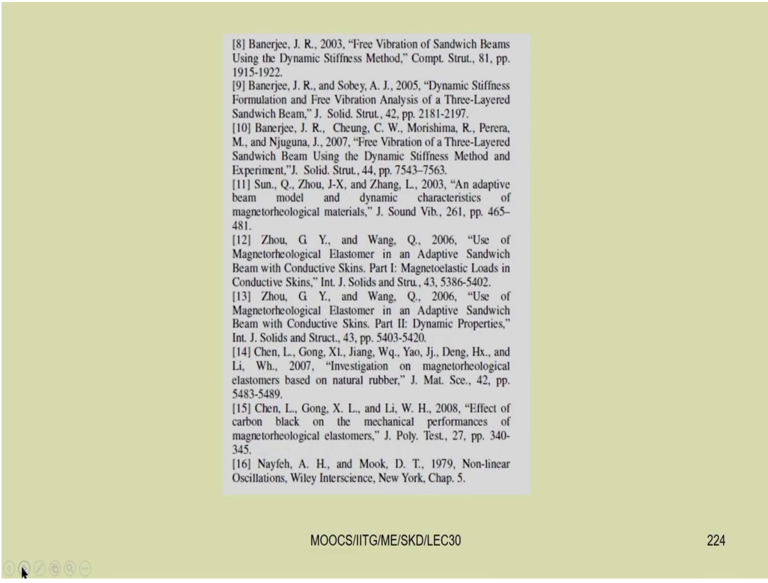

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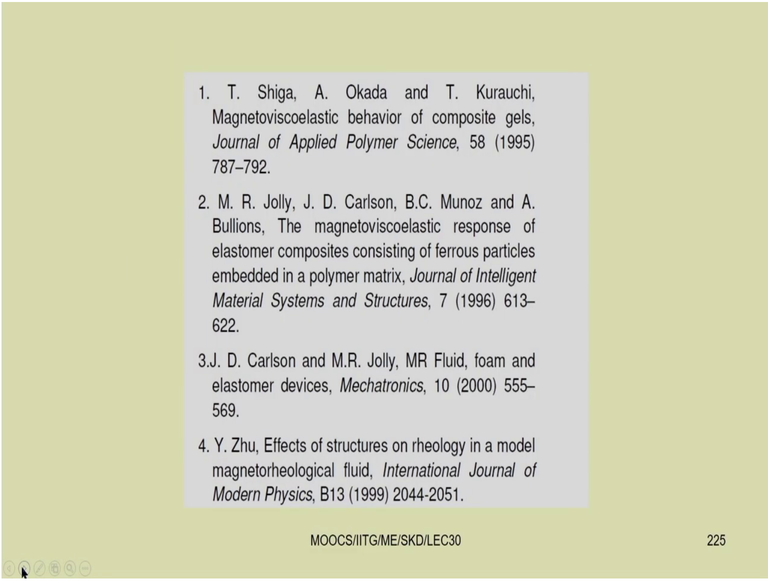

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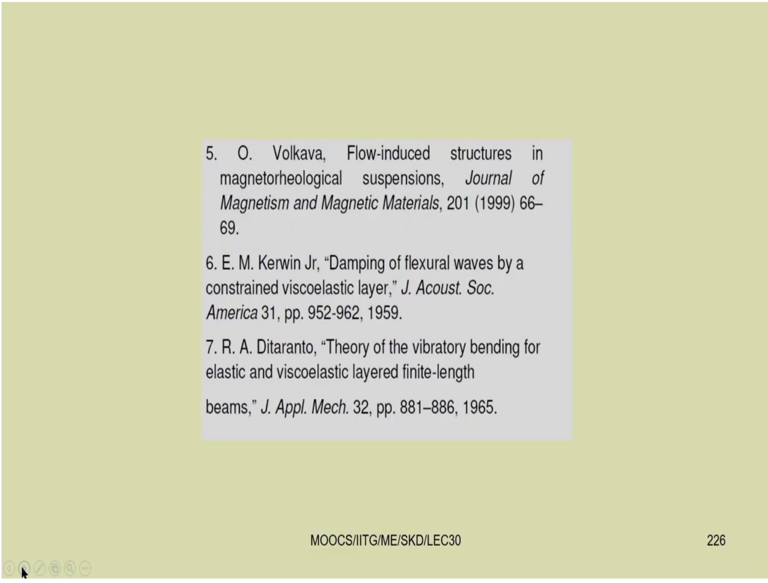

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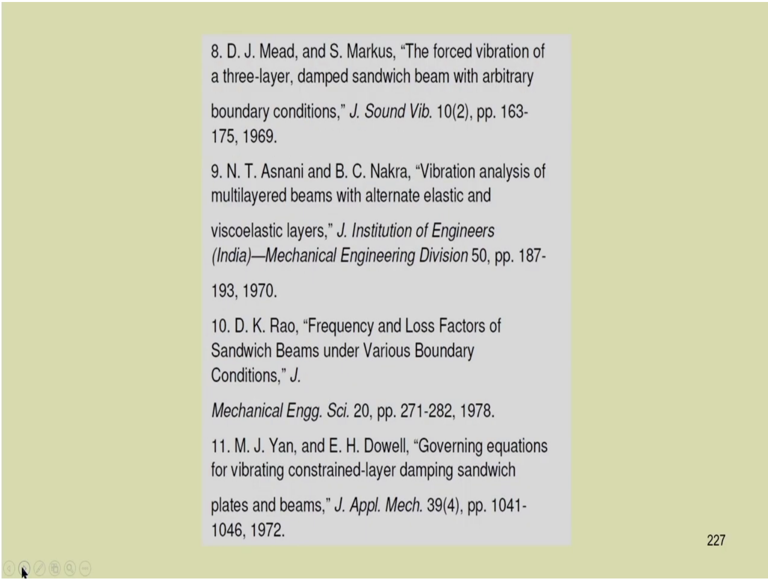

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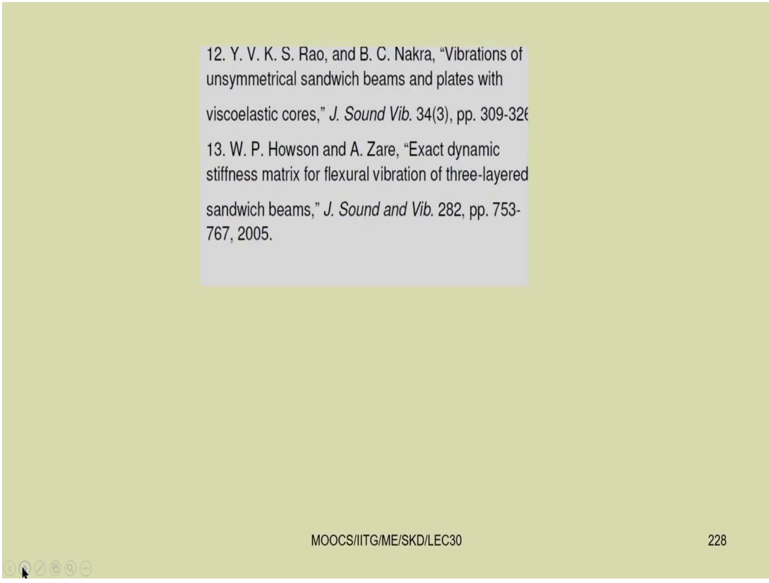

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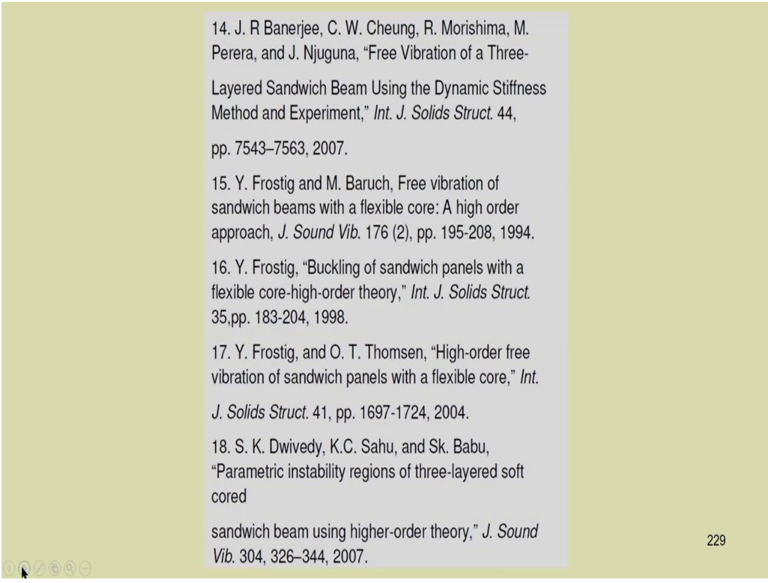

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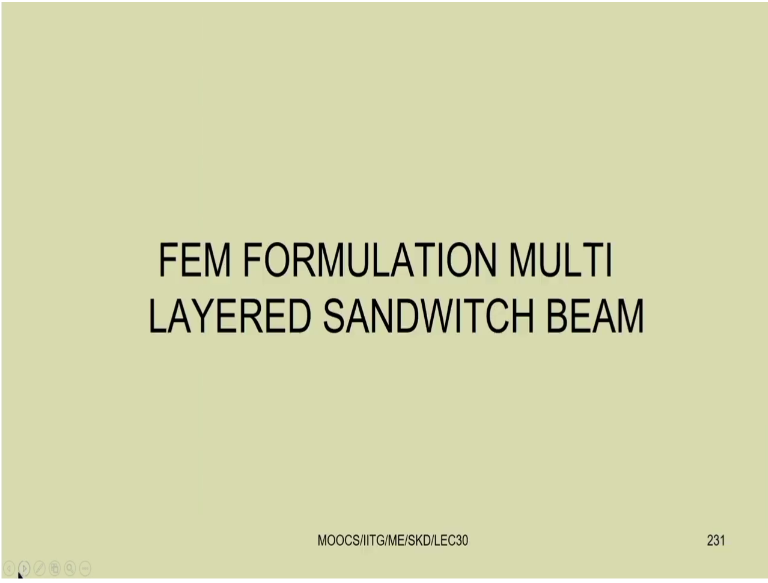

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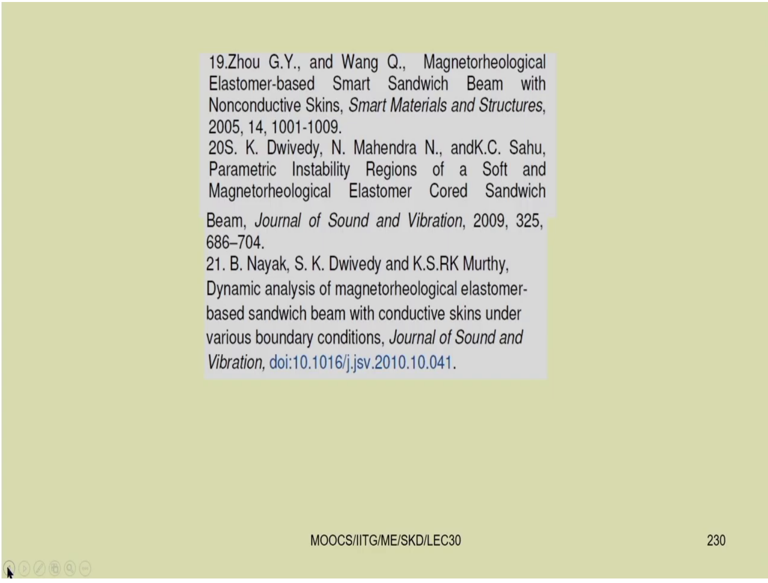

So, these are some of the references you can see with this thing. So, let us conclude the session here. Now, you know how you can use a continuous system for analyzing that as in non-linear system. So, you can reduce that spatio temporal system to its temporal form by applying Galerkin procedure.

So, then getting this temporal equation. So, you can apply different perturbation techniques to find the non-linear response of the system and in this non-linear response you can study different stability and bifurcations and you can apply it for many different purposes as for the requirement.

So, next class we are going to study some more applications for example, we will take a multi degree of freedom system particularly we will take this vibration observer. So, a two degrees of freedom system will take this passive and active vibration observer and apply different control strategy to study the attenuation of vibration or observation of vibration in this vibrating system.

Thank you very much.