Nonlinear Vibration

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Lecture - 21
Parametric instability of sandwich plate

Welcome to today class of Non-Linear Vibration. So, today, we are going to study different applications of parametrically excited system. So, here we will derive the equation of motion of a parametrically excited plate. And this finite element method also we will use in this plate theory, so that you can know how the equation of motion of a parametrically excited sandwich plate can be derived.

And after getting the equation of motion of the parametrically excited system using this finite element method, so we will see how we can find the parametric instability region using HSU method. One method that we will use that is known as HSU method to determine the parametric instability region.

In the last class, you have used this Floquet theory and also you have used this method of multiple scale to find the instability region. So, here we will take a multi-degrees of freedom system, and later one continuous system to study the instability region and also the response of the parametrically excited system.

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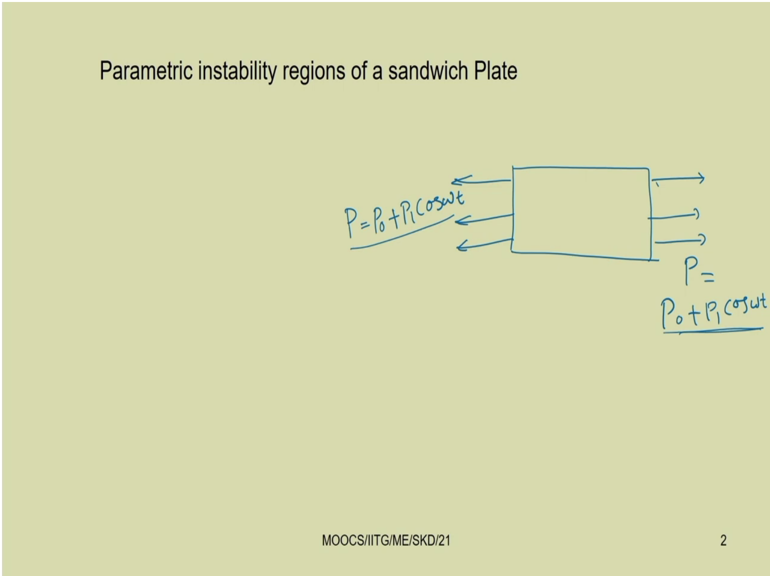

So, let us start deriving the equation of motion of a parametrically excited sandwich plate. So, already you know a beam or plate it will be parametrically excited for example. So, in case of this plate if it is subjected to; so, let us apply this force.

We can apply this force for example, the force P equal to P 0 plus P 1 cos omega t. So, we can apply this force both the side P equal to P 0 plus P 1 cos omega t. So, you have seen by applying a periodically varying force, so the system will be subjected to a equation which is similar to that of a Mathieu equation.

When you are taking a continuous system or multi-degrees of freedom system, so we will instead of having a single equation, so we will have now multiple equations. And we have to solve these equations simultaneously to find the governing equation of motion of or governing equation of motion and we will use the HSU method to find the instability region.

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So, this is sandwich plate. So, a normal plate also you can use for this purpose. But here we are using a sandwich plate which find many applications particularly in space industry or in automobiles or in transportation engineering. So, in many places these sandwich plates find their applications.

So, in a simple sandwich plate, so there are 3 layers. So, the top and bottom they are known as skin layer, top and bottom are known as skin layer, and the middle one is the core layer. So, middle one is the core layer. Generally, if a beam or plate is subjected to bending, so the upper and lower plates are subjected to more stress.

So, more material or thick material can be placed at the top and bottom skins and the middle part can be placed with some lightweight material. So, in this case, the core material can be a light weight material. For example, one can use this viscoelastic material viscoelastic material. So, this viscoelastic material may be, so this may be rubber like material one can put this rubber like material or one can put this thermocol PVC, all those type of material which are light weight can be put in the core layer.

In this core layer also, one can use this magnetorheological elastomer MRE, magnetorheological elastomer. And by applying this magnetic field, so one can change this stiffness and damping property, so whenever it is required. So, either one can have a sandwich plate which may be passive or active. So, today class we are going to study the sandwich plate with this passive type of sandwich plate, so where we are going to use the leptadenia pyrotechnica based viscoelastic core.

In particularly, in this viscoelastic core we use rubber like material. So, in this rubber material, we have added this leptadenia pyrotechnica powder and we have prepared this core material and after preparing the core material then this sandwich plate have been made and by using that sandwich plates.

So, we have done this analysis. So, let us, so particularly we will be interested to know this formulation today class. Those who are interested to know more regarding this fabrication and the experimental parts, so they may read more papers related to the sandwich plate.

So, this work is carried out by my PhD student Doctor Rajesh Kumar Ojha, Rajesh Kumar Ojha. So, you can see the paper by Doctor Rajesh Kumar Ojha which is a faculty member in UCET Bikaner.

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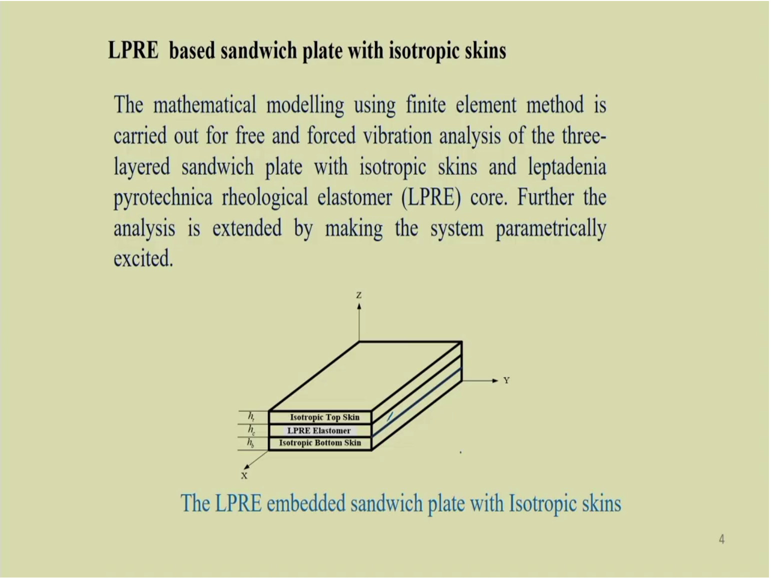

So, now, let us see the formulation. So, in this leptadenia pyrotechnica based elastomor core. So, this is the core layer in this plate. So, this is the top isotropic and bottom isotropic plates are there.

So, you may replace actually this top and bottom skin by many different material. For example, so you may use the composite material or you may use this functionally graded material also. The basic formulation will be remaining same, but they are matrix different type of matrix will be used in their formulation or their elastic properties will be different.

And so, if you know the formulation of this isotropic material then you can expand that thing to study composite or functionally graded material also. So, let us see by using a isotropic material how you can derive this equation of motion. So, already you know the equation of motion can be derived by using this Lagrange principle or Hamilton principle.

To derive this equation of motion using this energy based principle that is Lagrange or Hamilton principle, first we must find the kinetic energy and potential energy. When this plate is vibrating these different layers are there, these different layers are connected by adhesive. Some basic assumptions has to be made for deriving this equation of motion.

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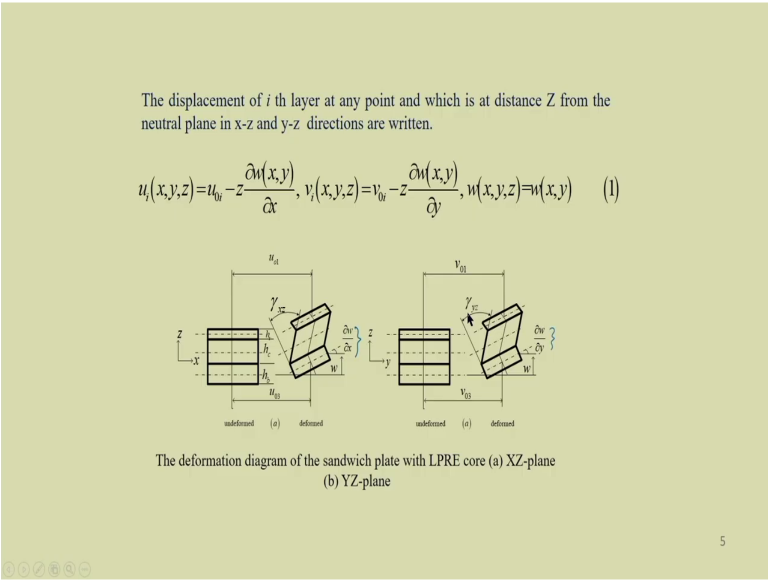

Derive or let us see that the displacement of ith layer of any point and which is at a distance Z from the neutral plane, so in x z and y z direction can be written as u x, y, z equal to u 0 i minus z into del w x, y by del x. That means, so you can see with respect to, so this is the neutral axis. So, this is before deformation that is on deformed and this is after deformation that is deformed.

So, you can see this is u 0 3 and here this is v 0 3. So, u in axial direction and v in the other direction you can take. So, here u i u can replace it by u 0 i, u 0 i that is in the neutral axis minus z. So, how far it is from the z axis? This is the z direction. If you take a layer a distance at a distance x from the neutral axis, then this in axial direction deformation can be written by using this equation that is u 0 i minus z into del w by del x.

Similarly, v i you can write equal to v 0 i minus z into del w by del x, so del w by del x. So, this is del w by del y and this part is your del w by del x del w by del x. So, w is the displacement in the transverse direction. We may assume this classical theory. So, here we are assuming that in z direction, there is no change in the reflection. So, that means, at any point along the z direction the displacement is same displacement is considered to be same.

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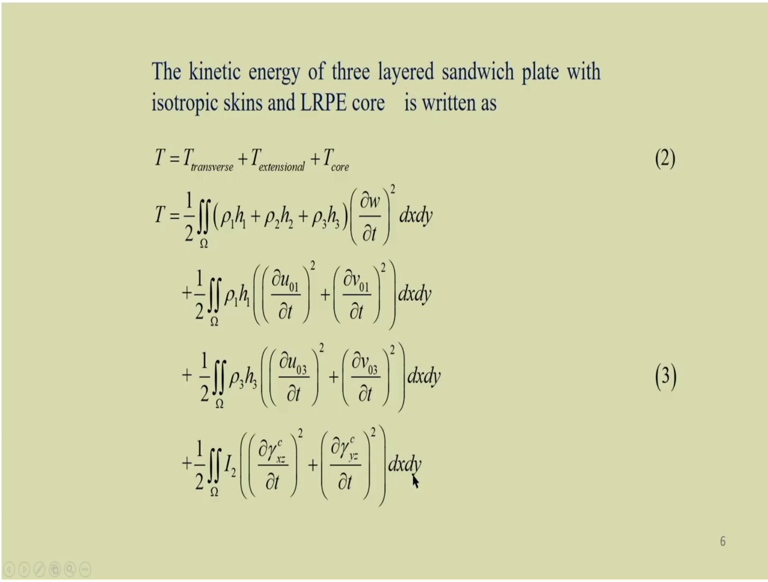

So, the kinetic energy can be, so as we have 3 layer, the kinetic energy you can write it equal to the kinetic energy of the top layer, from the bottom layer and for the core part also by taking. So, kinetic energy then can be written. So, as we have taken the displacement in both x, y, and z direction. So, we can have this kinetic energy this way it can be written. So, it will be half rho 1 h 1, rho 2 h 2, rho 3 h 3 into del w by del t square dxdy.

Actually you can take this is mass, so half mv square formed. The kinetic energy can be written in this form of half m v square, so that m at mass, so mass per. So, you can take this rho 1 and h 1. So, rho 1 is the mass per unit volume. So, and h 1 by multiplying this thickness, and then this dxdy is the area. So, h 1 into dxdy that will give you the volume. So, rho 1 h 1 into dxdy into, so del w by del t, so that is the kinetic energy in the z direction.

Similarly, kinetic energy in or the direction can be also obtained rho 1 h 1 del u 01 by del t square plus del 0 v 01 by del t square dxdy. Similarly, so this is for the, so rho 1 h 1 that is for the top layer and for the rho 3 h 3 for the bottom layer and then so, we can take this here also shear strain due to this shear action that is the rotary inertia also we can take into account that is i theta dot square.

So, this term is half m v square that is translational then we can have the rotational thing half i theta dot square which is can be written in terms of gamma, del gamma x z by del t square, del gamma z t yz by del t. So, this is for the core layer in dxdy. So, this way the kinetic energy is written.

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The potential energy of isotropic top and bottom skins caused by the tensile and bending is written.

$$U_{skins} = \frac{1}{2}\iint_{\Omega} \{\varepsilon_{ip}\}^T [D_{ip}] \{\varepsilon_{ip}\} dxdy + \frac{1}{2}\iint_{\Omega} \{\varepsilon_{ib}\}^T [D_{ib}] \{\varepsilon_{ib}\} dxdy \qquad (4)$$

where,

$$\{\varepsilon_{ip}\} = \begin{bmatrix} \frac{\partial u_{0i}}{\partial x} & \frac{\partial v_{0i}}{\partial y} & \frac{\partial u_{0i}}{\partial y} + \frac{\partial v_{0i}}{\partial x} \end{bmatrix}^T ; \; [D_{ip}] = \frac{E_i}{1-\nu_i^2} \begin{bmatrix} 1 & \nu_i & 0 \\ \nu_i & 1 & 0 \\ 0 & 0 & \frac{1-\nu_i}{2} \end{bmatrix} \qquad (5)$$

$$\{\varepsilon_{ib}\} = \begin{bmatrix} \frac{\partial^2 w}{\partial x^2} & \frac{\partial^2 w}{\partial y^2} & 2\frac{\partial^2 w}{\partial x \partial y} \end{bmatrix}^T ; \; [D_{ib}] = \frac{E_i h_i^3}{1-\nu_i^2} \begin{bmatrix} 1 & \nu_i & 0 \\ \nu_i & 1 & 0 \\ 0 & 0 & \frac{1-\nu_i}{2} \end{bmatrix} \qquad (6)$$

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Similarly, we can have the potential energy. So, the potential energy due to all the 3 layers we can write. So, for the skin layer, so this is for the skin layer that is, so we know the potential energy can be written half stress into strain into d v or it can be written stress into strain into d v or in terms of strain also we can write. So, as stress by strain equal to D, D matrix or E. So, in this case. So, it can be written half integral epsilon that is strain transpose of strain matrix into D into epsilon ip into dxdy.

Similarly, it can be written. So, we have we can write for the bottom layer and top layer, so this is the skin part. So, for the skin part we can write using these two. Then, this epsilon i P can be written as del u 0 i by del x del v 0 i by del y del. So, this is epsilon x epsilon y and gamma x y. So, that is del u by del x, del v by del x and for del gamma it will be del u by del y plus del v by del x. So, that way it can be written del ip.

Similarly, D ip can be written using these matrix. So, E i by 1 minus nu square, 1, nu i, 0, nu i, 1, 0, 0, 0, 1 minus nu i by 2. So, where nu i is the Poisson's ratio and E is the Young's modules of the material. Similarly, for the bottom layer. So, this is for the top layer. This is for the bottom layer. Similar way it can be written.

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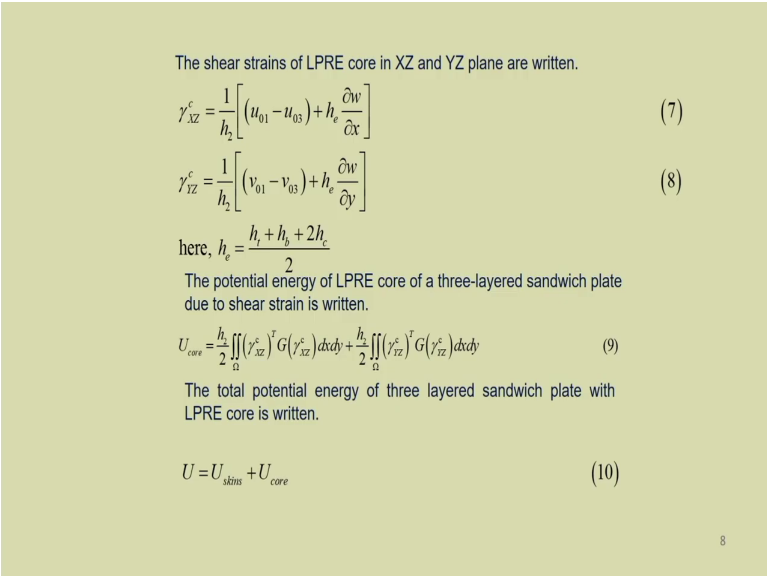

Then, we can write the total strain energy the shear strain energy also for LPRE in XZ and YZ plane can be written. So, this way it can be written. So, now the total the potential energy of LPRE core for the 3-layered sandwich plate due to shear strain is written in this form. So, gamma for the direct stress we have written in terms of epsilon.

So, epsilon e into e or epsilon transpose e into e. Similarly, for the shear it can be written gamma c xz transpose G gamma c xz, so dxdy plus h 2 by 2 gamma c yz transpose G gamma yz c dxdy. The total potential energy of the 3 layered sandwich plate with LPRE core is written as U equal to U skin plus U core.

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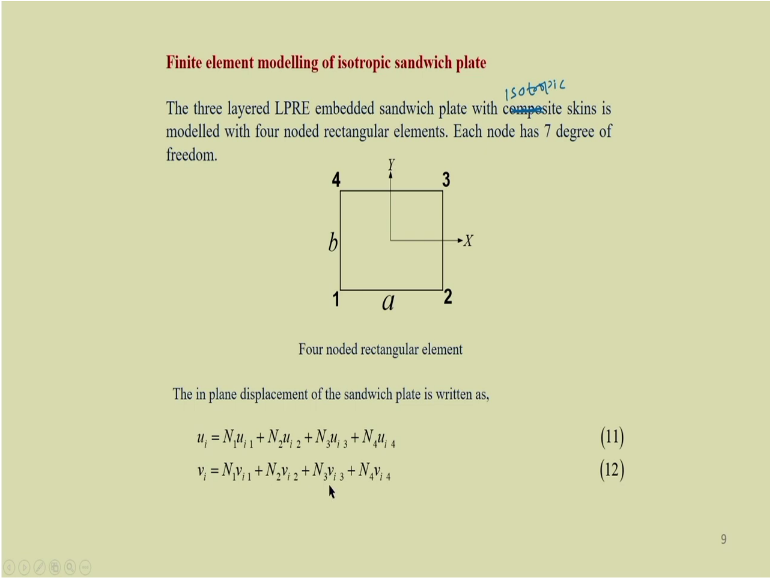

So, knowing this U skin that U core, so now, we can go for the FEM formulation. So, in this FEM formulation the 3-layered LPRE embedded sandwich plate with isotropic skin. So, with isotropic skin can be modelled. So, let us take this isotropic. So, this isotropic skin is modelled with a 4 noded rectangular element. So, each node has 7 degrees of freedom. We have taken each node with 7 degrees of freedom.

And here the in plane displacement of the sandwich plate is given by, so u i we can write that N 1 u 1 i plus N 2 u i 2, N 3 u i 3 plus N 4 u i 4. Similarly, v can be written. So, u is the axial direction, v is in the transpose direction. So, that is N 1 v 1; or u is in x direction, v is in y direction, and w will be in z direction. So, v direction also we have N 1 v i 1, similarly N 2 v i 2, N 3 v i 3, and N 4 v i 4.

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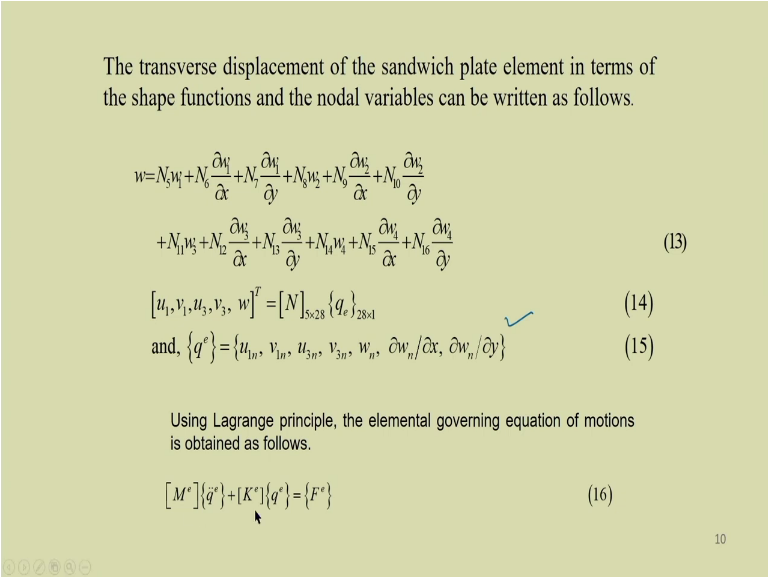

So, this way it can be written w in the transverse direction. So, can be written N 5. So, up to 4 you have written there. So, now, say function with 5. So, w equal to N 5 w 1, N 6 del w 1 by del x, N 7 del w 1 by del y, N 8 w 2 plus N 9 del w by del x plus N 10 del w 2 by del y plus N 11 del w 3, N 12 del w 3 by del x plus N 13 del w 3 by del y, N 14 del w 4 and plus N 15 del w 4 by del x plus N 16 del w 4 by del y.

So, this way we can write, so u 1, v 1, u 3, v 3, w. So, u 1, v 1 is the is for the top layer u 1, v 1; then u 3, v 3 for the bottom layer, and then w, is the in the transverse direction. So, transpose can be written as N 5 into 2. So, you just see, so it will be 5 into 28. So, here you have 5, 1, 2, 3, 4, 5; u 1, v 1, u 3, v 3 and w. So, for the core you just see that we have not considered the axial this deformation in x and y direction.

So, we have written this thing in terms of these 5 parameters then we can write this N 5 28 into q e, so this is 28 cross 1. So, here this q e, q e per node you just see these are the 7 degrees of freedom. So, that is u 1 n, v 1 n, u 3 n, v 3 n, w n. So, these are these 5 are displacement along with that. So, you can take to slope or theta. So, del w n by del x, del w n by del y.

So, this way we have taken 7 degrees of freedom. So, q contain the 7 degrees freedom. So, that is u 1 n, v 1 n, that is for the top layer; then u 3 n, v 3 n for the bottom layer, then w n in the transverse direction. So, in addition to that variation of w along x that is theta x you can write or you can write these in terms of del w n by del x comma del w n by del y. So, using Lagrange principle the elemental governing equation of motion is obtained as; so, now, it can be written in this form M e q e double dot plus K e q e equal to F e. So, here the parameters can be found easily.

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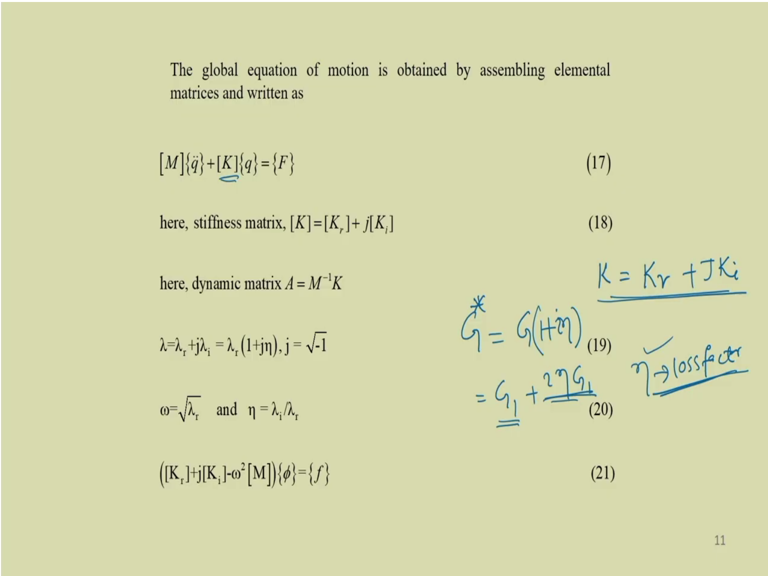

The global equation of motion can be obtained by assembling these matrix. And finally, it can be written in this form that is M q double dot plus K q equal to F. Now, here you note that, so one can apply different boundary conditions. So, in this case, so several different boundary conditions can be taken.

For example, the sides may be simply supported, the side may be clamped, the side may be free (Refer Time: 18:12) or a combination of all these 3 conditions. So, one side it may be fixed, other side it may be roller supported or it may be free; or one side may be clamped and other side may be simply supported. So, there will be many different boundary conditions and combination of those boundary conditions may be considered for the analysis purpose.

In case of parametrically excited system. So, till now we have not considered the axial force. So, now, we will consider in the next step we will consider the axial force to make the equation parametrically excited. So, now, you just see these equation that is 17, M q double dot plus K q equal to F. So, this is similar to that of a equation of motion of a multi-degree of freedom system subjected to force vibration.

So, here this q contains 7 elements. So, already we have seen these 7 elements that is u 1, v 1, u 3, v 3, w, and del w by del x, and del w by del y. That way depending on the number of elements, so we can have different number of, so the size matrix size will goes on increasing.

So, here the stiffness matrix K, here it can be noted that this K, so as we are considering this viscoelastic material. So, this viscoelastic material has a property that is the stiffness property this K stiffness will be equal to. So, it will have a real part and it will have a complex part K r plus J K i, because this elasticity modulus or E or G what we are considering for in case of the viscoelastic material

For example, the G shear modulus. G can be actually G star it is written. So, G star equal to G plus or we can write G star equal to 1 plus i eta or it can be written this is equal to G 1 plus i, this is equal to 1 plus i eta, so this is i eta G or G 1. So, this is the storage modulus and this part is known as the loss modulus. And this eta is known as the loss factor. So, eta is known as the loss factor.

Before doing this theoretical analysis one must know by performing experiment one can find the loss factor of the material. And as it contains this loss factor which is as an imaginary term or it can be this G can be written as a complex number. So, this stiffness matrix contain both real part and the imaginary part. So, K can be written K r plus j K i, i or j you can write. So, that is root over minus 1.

So, now the dynamic matrix can be found by finding this A equal to M inverse K. So, if you just see the eigenvalue of A will contain both real part and imaginary part. So, this lambda i equal to lambda r plus j lambda i or it can be written as lambda r into 1 plus j eta, so where this eta is known as the loss factor j equal to root over minus 1. So, here omega that is the natural frequency of the systems can be obtained by having the square root of lambda r and eta equal to lambda imaginary by lambda real part.

So, this equation can be written in this form also. So, K r plus j K i minus omega square M into phi equal to F. This way also one can write this equation. And solving this equation knowing this K or iterative solver by using iterative solver one can find this omega. Or one can find the eigenvalue of this a matrix to find the eigenvalues and from that thing frequency n loss factor can be obtained.

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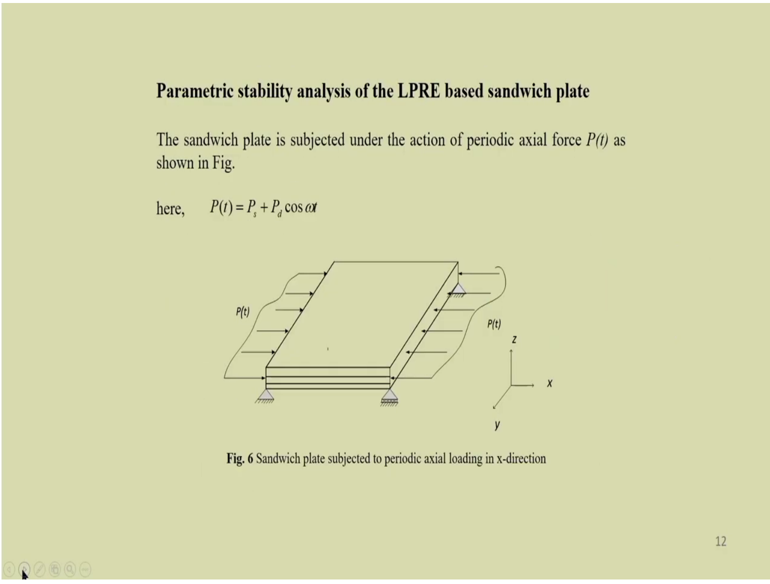

So, now let us see, so if it is subjected to this axial loading, compressive loading or tensile loading. So, let us apply compressive loading in this sandwich plate. So, it is simply supported. So, let us apply a compressive load which is time varying. So, P t equal to P s plus P d cos omega t.

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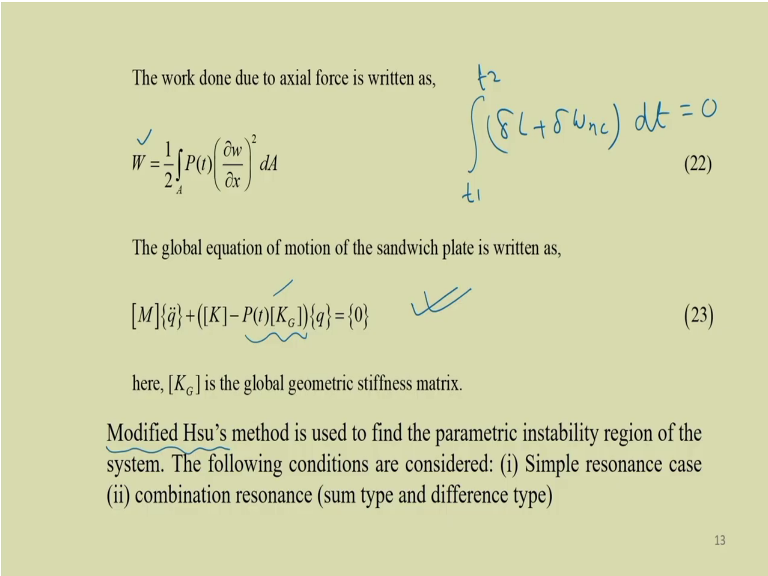

So, if it is applied then in the equation motion what we have derived just now, so there we can put a work done due to this axial force which can be written as half integration P t into del w by del x square d A. So, from this thing by taking this w and by applying this equation motion or you can confine this thing. So, this is the this work done you got it. So, after knowing this work done, so you can apply this Hamilton principle also. Hamilton principle if you recall, so this is equal to integration t 1 to t 2, it is del l plus del w n c dt, del w n c dt equal to 0.

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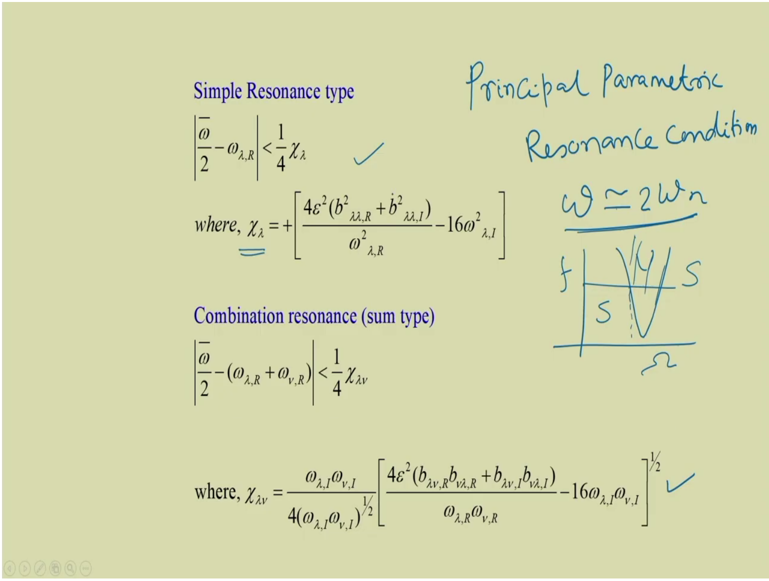

So, by using this Hamilton principle, so one can derive this governing equation of motion. So, the governing equation of motion can be obtained. So, this governing equation motion can be written in this form. And here it can be noted that in this governing equation the coefficient of q is a time varying term, that is P t into K G, P t into K G is the time varying term. So, that is why this equation is known as a that of a parametrically excited system.

So, if this term is not there that is for the free or force vibration case previously what we have seen, the equation can be written M x, M q double dot plus K q equal to 0 or M q double dot plus K q equal to force. But if this compressive loading case is considered in this case, so then this equation will be reduced to this form. So, these form is similar to that of a Mathieu, Hill type of equation.

So, here to find the parametric instability region. So, one can use this modified HSU method. So, one can use this modified HSU method to find the parametric instability region. So, one can study more regarding this modified HSU method. So, this method is nothing but this is the method of averaging. So, by using this method of averaging one can solve this equation and one can find the instability region.

So, previously I told how we can use this method of multiple scales also to find these resonance conditions in case of parametrically excited system and how you can find the response. Further also, today class further also we will see in another system how we can find the instability region by using this method of multiple scales.

In case of HSU method, so they have given a very simple equation to find the instability region. These instability regions are simple resonance conditions or this is principal parametric resonance conditions also, this is also known as principal parametric resonance condition.

In case of principal parametric resonance condition, this external frequency is nearly equal to twice the natural frequency of the system and we can obtain the instability region near to 2 n. So, this way we can find this instability region. So, here it can be this frequency and here it can be the forcing parameter f, clear.

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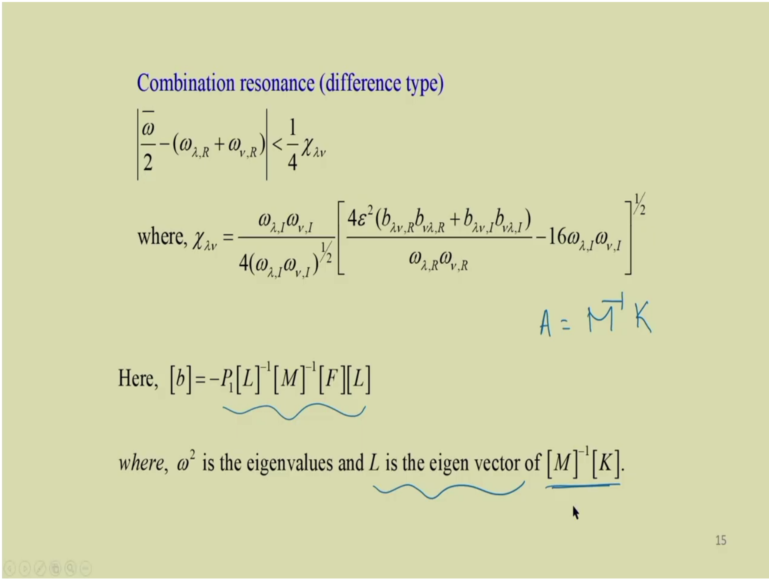

So, by plotting this thing one can find the instability region. So, to find this transition core or this instability region. So, you can use this equation what is this equation for principal parametric region and case you can find this equation that is omega bar by 2 minus omega lambda, R, that is omega lambda, R is the real part of the real part of this square take the square root of the eigenvalue. So, the real part of that is omega lambda, R.

So, if this omega bar by 2 minus omega lambda, R, mod of that thing is less than 1 by x mu x lambda. So, where this x lambda is written in this form, x lambda is nothing but 4 epsilon square b square lambda lambda, R plus b square lambda lambda I divided by omega square lambda, R minus 16 omega square lambda I, where this b matrix can be obtained or b is the element of the matrix this one.

Actually, first, so now, first you have the M matrix K matrix. So, from that M matrix and K matrix first find M inverse K, so that is the A matrix. Now, find the eigenvalue and eigenvector. So, taking these eigenvalue and eigenvector, so here L 1 is the, L is the eigenvector of M inverse K. So, this P 1 that is the forcing term you know, so this b equal to minus P 1 into l transpose M transpose F L.

So, where this L is the eigenvector of M inverse K, L is eigenvector of A inverse K. So, that way you can find the v matrix.

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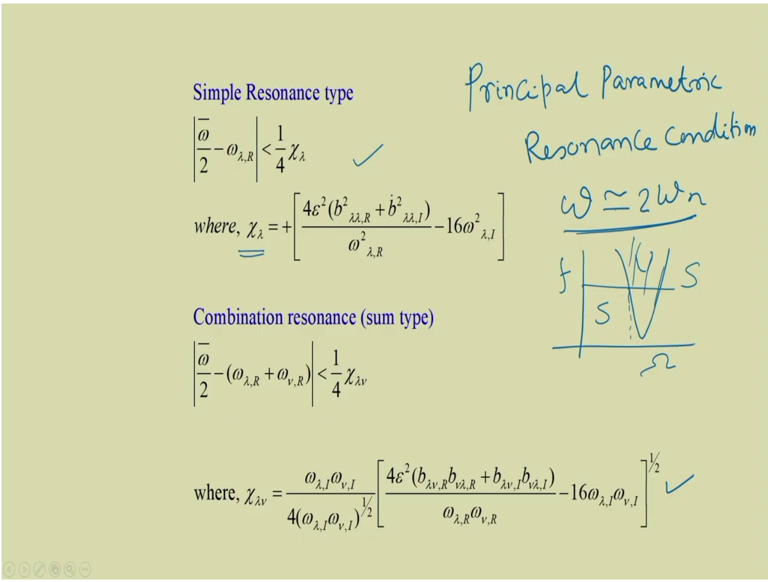

So, after getting this v matrix. So, you can now you can find this x lambda. And by using this expression here, so you can plot or one can plot the instability region or the transition curve.

So, this transition curve means for a particular value of f, so up to this thing. So, the system is stable, then the system becomes unstable and after that the system is stable.

So, if one to have vibration free system then you must operate the system in a zone where it is for both value of f and mu f and omega it is stable. So, if you are operating in this zone, so then the system is unstable, the system response becomes unstable. So, this is known as parametric instability region.

So, this way, so similarly for combination resonance. So, already we know that combination resonance can be of two type, so one is the sum type and other one is the difference type. So, in case of sum type, one can write this omega bar by 2 minus omega lambda, R plus omega nu, R. So, here you have to take two frequency.

So, omega lambda is one frequency and omega nu is another frequency. That is why this is combination parametric resonance of sum type. So, omega bar by 2 minus omega lambda, R plus omega nu, R should be less than 1 by 4 xi lambda v. So, where this xi lambda v can be written in this form omega lambda, I. So, omega lambda, I is the imaginary part of omega.

So, omega already I told you, so this is the square root of lambda. So, this omega lambda, I into omega v, I divided by 4 omega lambda, I into omega v, I, so to the power half into 4 epsilon square b lambda, v, R; b lambda, R plus b lambda, v, I and b v lambda, I; omega lambda, R; omega v, R. So, this lambda and v, R nothing but they correspond to two natural frequency of the system.

So, it is the for example, lambda I can take 1 and v I can take 2, so it will be omega 1 R that is the real part of the eigenvalue for the first mode and omega 2 R that is the real part of the eigenvalue, square root of the eigenvalue for second mode. So, that way using these two expressions, so you can note down these expressions and you can write your own code to derive this instability region. So, you can plot this thing, so for which it becomes a transition curve.

Similarly, for combination resonance of difference type. So, for difference type also you can find this way. For the sum type and difference type you can find and you can write down this equation, so where you can get this b, expression for b also.

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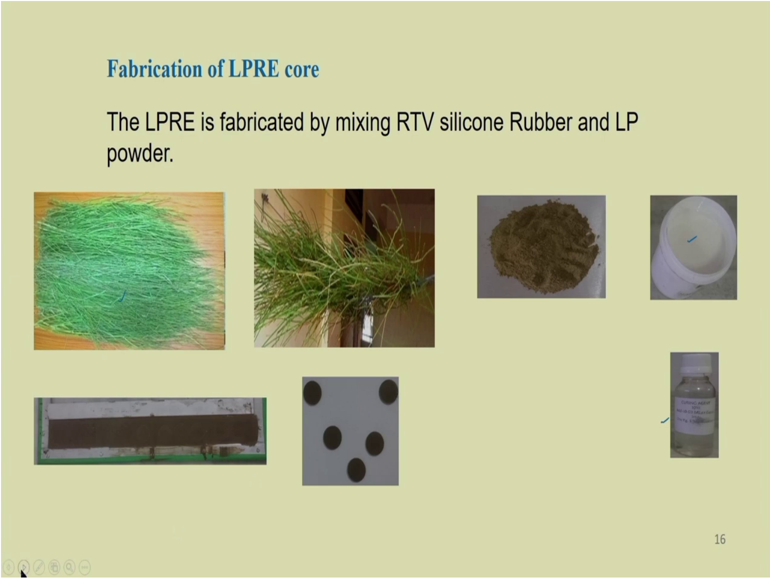

So, in these particular work, so we have taken this leptadenia pyrotechnica. So, this is a this is a grass like, hobs available plentily in Rajasthan desert. We have taken these leptadenia pyrotechnica, then we have dried it, and after drying it, then this powder has been made.

So, then it has been mixed with the rubber, the synthetic rubber or natural rubber, this is synthetic rubber with these additives. So, then this put in a mold and it is cured. So, after curing that thing this sandwich core has been formed. So, these are the samples taken for this microstructure analysis and also for getting the material property of this leptadenia pyrotechnica best sandwich core material.

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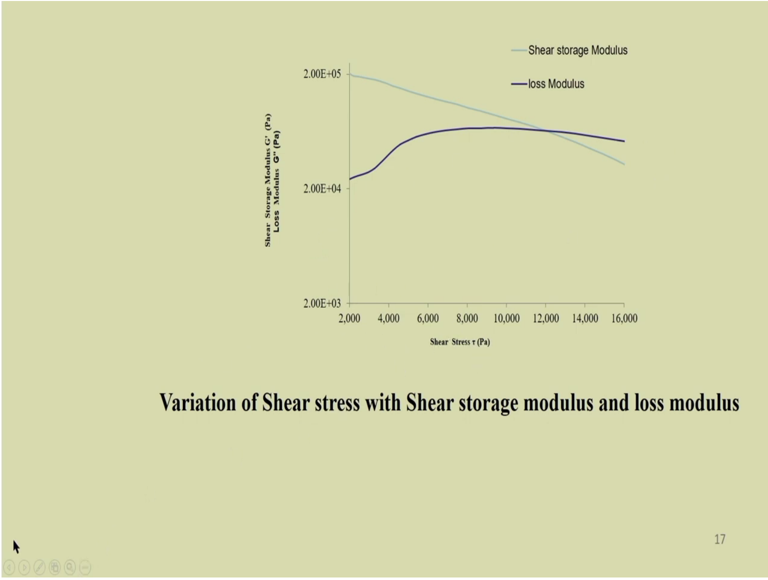

This thing this lost factor and the shear storage modulus and loss storage modulus have been found out using this instrument. It can be plotted with this shear different shear stress.

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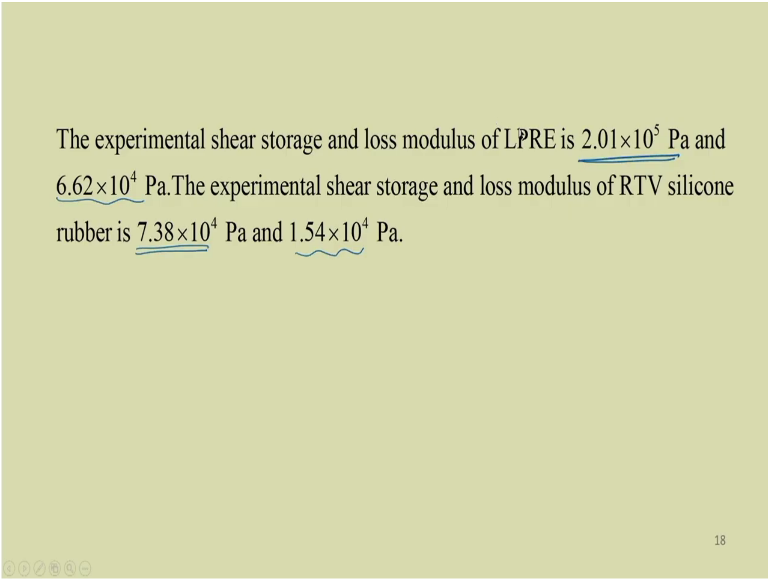

And then, so it can be observed that the experimental shear storage and loss modulus of LPRE is found to be 2 into 10 to the power 2.01 into 10 to the power 5 Pascal. So, you just see for leptadenia pyrotechnica it is 2.01 into 10 to the power 5 Pascal. But the for RTV silicone it is 7.38 into 10 to the power 4, so 7.38 into 10 to the power 4 it is 2.01 into 10 to the power 5, so that it is almost 3 times. So, by putting this leptadenia pyrotechnica powder, we have increased the strength of the sandwich panel or sandwich plate by 3 times or the core material strength can be increased by 3 times.

Similarly, the damping property can be checked from this thing from the loss factor. So, the loss factor is 6.62 in to 10 to the power 4 Pascal, but for the RTV silicone it is 1.54 into 10 to the power 4. So, you just see, so the loss factor is also increased almost 4 times in this case.

So, both the loss factor and the stiffness parameter that is the stiffness factor are increased by using this leptadenia pyrotechnica elastomer core.

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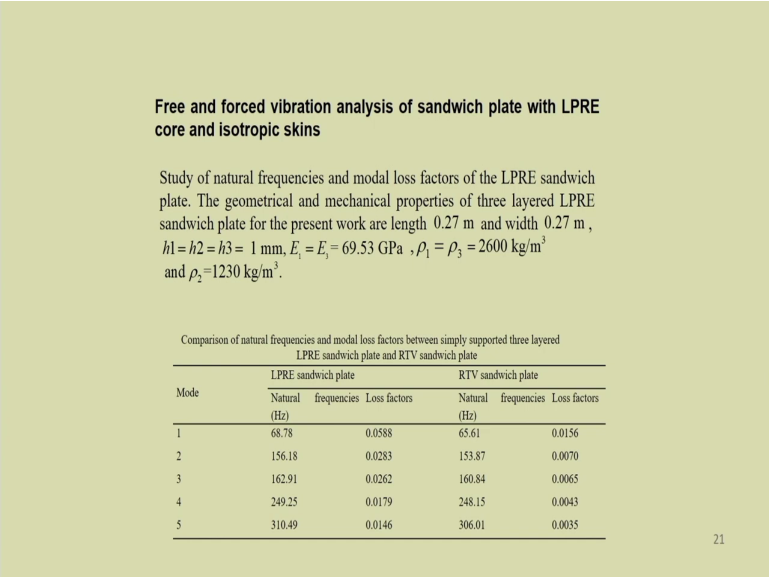
Free and forced vibration analysis of sandwich plate with LPRE core and isotropic skins

Study of natural frequencies and modal loss factors of the LPRE sandwich plate. The geometrical and mechanical properties of three layered LPRE sandwich plate for the present work are length 0.27 m and width 0.27 m, $h1 = h2 = h3 =$ 1 mm, $E_1 = E_3 =$ 69.53 GPa, $\rho_1 = \rho_3 = 2600$ kg/m^3 and $\rho_2 = 1230$ kg/m^3.

Comparison of natural frequencies and modal loss factors between simply supported three layered LPRE sandwich plate and RTV sandwich plate

Mode	LPRE sandwich plate		RTV sandwich plate	
	Natural frequencies (Hz)	Loss factors	Natural frequencies (Hz)	Loss factors
1	68.78	0.0588	65.61	0.0156
2	156.18	0.0283	153.87	0.0070
3	162.91	0.0262	160.84	0.0065
4	249.25	0.0179	248.15	0.0043
5	310.49	0.0146	306.01	0.0035

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We can study those things for a different boundary conditions. So, here these free and forced initially the free and forced vibration things have been studied. By developing a code in MATLAB. So, here the natural frequency and loss factor are determined. So, by finding the eigenvalue of the M inverse K matrix.

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The Effect of LPRE and top layer thicknesses on natural frequencies and modal loss factors of three layered LPRE sandwich plate with CCCC boundary conditions have been investigated,

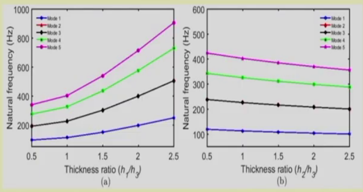

Fig. Variation of the modal frequencies with thickness ratios of the sandwich plate: (a) the effect of the thickness ratio (h_1/h_3) (b) the effect of the thickness ratio (h_2/h_3)

Loss factors of three layered sandwich plate with isotropic skins and LPRE core

Thickness	Loss factors				
ratio (*h2/h3*)	Mode I	Mode 2	Mode 3	Mode 4	Mode 5
0.5	0.0232	0.0140	0.0140	0.0103	0.0091
1	0.0258	0.0154	0.0154	0.0117	0.0095
1.5	0.0270	0.0161	0.0161	0.0123	0.0099
2	0.0290	0.0174	0.0174	0.0132	0.0107
2.5	0.0314	0.0189	0.0189	0.0144	0.0116

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Now, we can see how the natural frequency is varying with different thickness ratio h 1 by h 3. So, similarly the natural frequency, so for different boundary conditions effect of the thickness ratio, so h 2 by h 3 is considered in this case. And the effect of thickness ratio h 1 by h 3 is considered in this the first case; h 1 by h 3 that is top and bottom layer and h 2 by h 3 that is for the core material, core to the bottom material. So, that way it can be studied.

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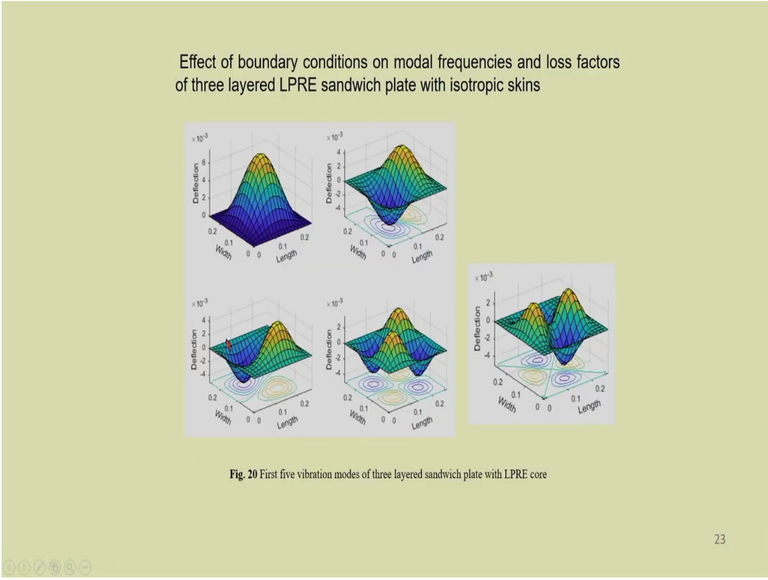

You can see different this different modes of first mode, second mode, third mode. So, different mode of vibration of the sandwich plate.

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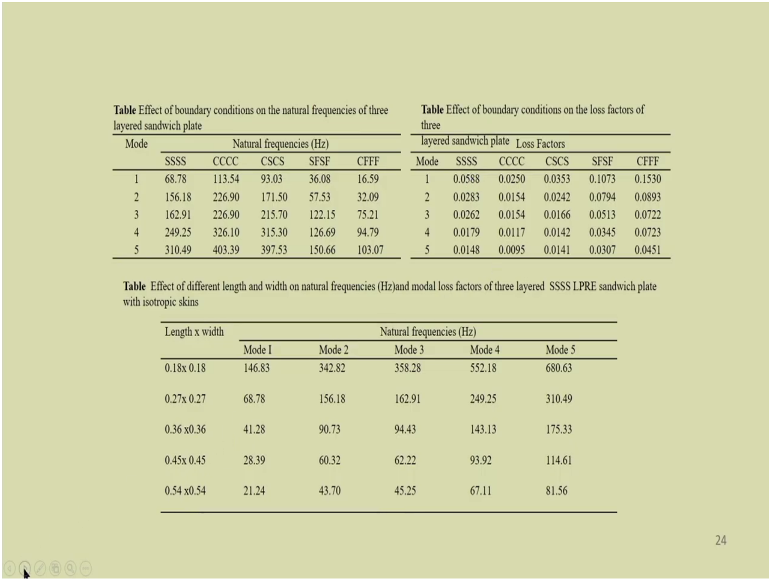

Table Effect of boundary conditions on the natural frequencies of three layered sandwich plate

Mode	Natural frequencies (Hz)				
	SSSS	CCCC	CSCS	SFSF	CFFF
1	68.78	113.54	93.03	36.08	16.59
2	156.18	226.90	171.50	57.53	32.09
3	162.91	226.90	215.70	122.15	75.21
4	249.25	326.10	315.30	126.69	94.79
5	310.49	403.39	397.53	150.66	103.07

Table Effect of boundary conditions on the loss factors of three layered sandwich plate

Loss Factors					
Mode	SSSS	CCCC	CSCS	SFSF	CFFF
1	0.0588	0.0250	0.0353	0.1073	0.1530
2	0.0283	0.0154	0.0242	0.0794	0.0893
3	0.0262	0.0154	0.0166	0.0513	0.0722
4	0.0179	0.0117	0.0142	0.0345	0.0723
5	0.0148	0.0095	0.0141	0.0307	0.0451

Table Effect of different length and width on natural frequencies (Hz)and modal loss factors of three layered SSSS LPRE sandwich plate with isotropic skins

Length x width	Natural frequencies (Hz)				
	Mode I	Mode 2	Mode 3	Mode 4	Mode 5
0.18x 0.18	146.83	342.82	358.28	552.18	680.63
0.27x 0.27	68.78	156.18	162.91	249.25	310.49
0.36 x0.36	41.28	90.73	94.43	143.13	175.33
0.45x 0.45	28.39	60.32	62.22	93.92	114.61
0.54 x0.54	21.24	43.70	45.25	67.11	81.56

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These are the mode shapes different mode shapes one can see. Now, the effect of boundary conditions also one can study. So, for example, SSSS, that means, 4 side are simply supported CCCC that is 4 side are clamped, then CSCS one side clamped, next side is simply supported, then the other side is clamped then simply supported. Then F is here free, so SFSF, so one side simply supported other side free, so SFSF. Then CFFF, this is a cantilever type, so one side is clamped and other 3 sides are free.

So, this way one can study the natural frequency effect of natural frequency for different boundary conditions. Those who are ware interested they can study more on this thing, that is how the natural frequency changing with different thickness, also how the core factor loss factor is changing with the different thickness ratio. So, all those things one can study.

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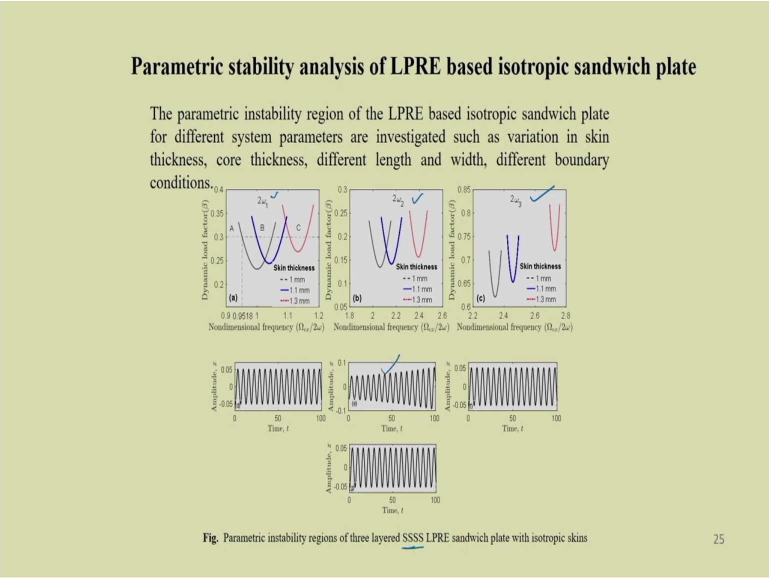

But our purpose is to show if the parametric instability region. So, here you can see parametric instability region is also plotted. So, in this case instead of using method of multiple scale we have used the HSU method the formula what I have shown you there to plot the instability region. So, here these instability region are plotted for 3 value of skin thickness. For example, skin thickness of 1 millimeter, 1.1 millimeter and 1.3 millimeter.

You can see for different skin thickness. Now, as these stiffness is changing by changing this skin thickness the stiffness parameter of the skin will change. So, this give raise to higher value of this core. So, the natural frequency is shifted to right, that is why this instability region also starts or moves towards right.

So, one can plot or one can find the region for which the system is stable and when it is unstable. So, this is instability region for the first mode, that is principal parametric resonance condition of the first mode, twice omega 1, this is principal parametric resonance of second mode twice omega 2, and this is principal parametric resonance condition for third mode that is two omega 3.

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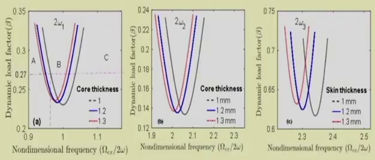

S. No	h_2 (mm)	Mode-1		Mode-2		Mode-3	
1	1	0.2320	1	0.1337	2.06	0.6167	2.34
2	1.2	0.2334	0.98	0.1354	2.02	0.6247	2.30
3	1.3	0.2360	0.97	0.1371	2.00	0.6320	2.28

Fig. Parametric instability regions of three layered SSSS sandwich plate with isotropic skins and core layer

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So, this way one can plot these instability region for different conditions and different boundary conditions also. First case it is plotted for different. So, you can see for this is for SSSS all side simply supported. So, here you can verify that. So, in this case you just see in this case the response is growing, this is for this middle point.

So, a, b, c if you have taking 3 point. So, this is for point a when the response is stable, and this is for point c when the response is stable also; a and c for these two curves it is stable. But for b the response grows and the system becomes unstable. So, this is, it is clearly showing the system is unstable. The response amplitude goes on increasing.

So, this way one can study the instability region of a sandwich plate. With different conditions one can consider also different boundary conditions one can consider. So, by considering different boundary conditions, so in the fem formulation those boundary conditions can be applied. And after applying these boundary conditions, so one can find the these eigen values, and eigenvector, and one can study by using HSU method this instability regions.

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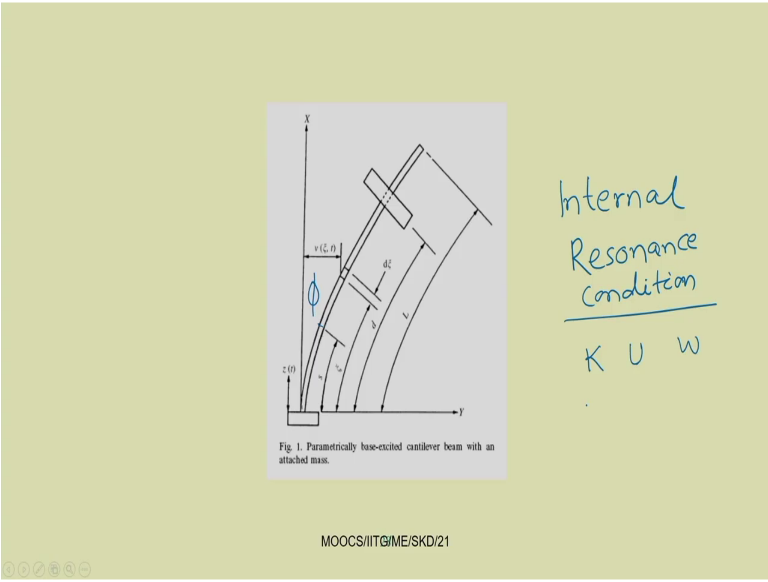

Now, let us see another example. Let us take a parametrically excited that is base excited cantilever beam with an attached mass. In this case particularly, we are going to introduce one more resonance condition which is known as internal resonance condition, internal resonance condition.

So, here we have a cantilever beam. So, the cantilever is moving up and down. So, when it is moving up and down then due to slenderness, so this beam moves in the transverse direction. So, if we are taking this angle is phi you can see that this sin phi can be written, so if we are taking these distance s it can be written as, so dv by ds. So, sin phi will be equal dv by ds.

From that thing this term cos phi can be obtained and one can write the movement. So, one can derive this equation motion either by using this energy method or by using this method movement based method, force and movement based method that is the Newton second law or d'Alembert principle.

So, here taking a small element different forces can be taken. So, the forces for example, it will be m u double dot, then m u double dot in the axial direction. Let the axial direction deflection is u, then in transverse direction it will be m v double dot. We can write these m v double dot and m u double dot, so that is the inertia force.

So, then by taking the movement at a distance s from here we can get this equation for this movement. Now, differentiating twice this movement equation, so one can get the equation of motion of the system, otherwise one can find first the kinetic energy, then the potential energy, and then this work function W and then writing these or this kinetic energy can be written as T.

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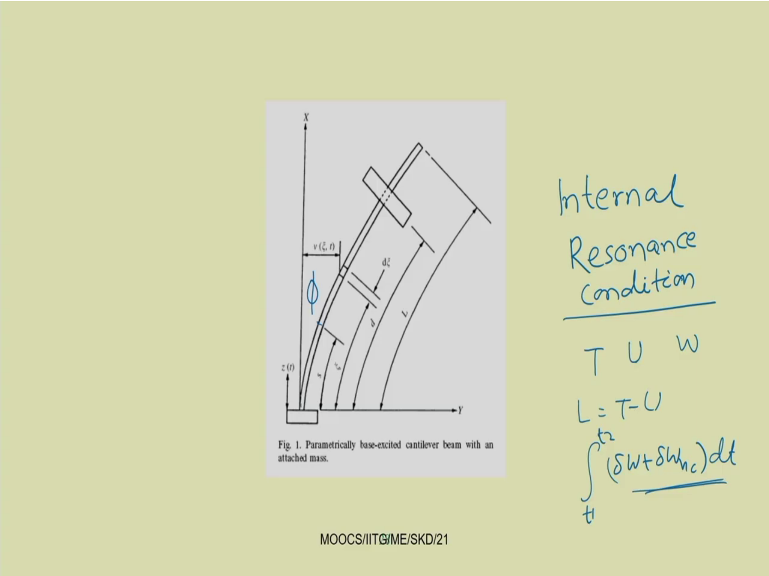

So, now writing this L equal to T minus U, using this Hamilton principle one can write this is t 1 to t 2 delta L plus delta W nc dt equal to 0. So, one can find the equation motion and the boundary conditions.

The advantage of using this extended Hamilton principle is that one can get the equation motion and the boundary condition simultaneously. By using this Lagrange principle, so one can obtain the equation motion conveniently, but the boundary conditions one has to decide later.

But by using this extended Hamilton principle, one can get both these equation of motion and the boundary conditions.

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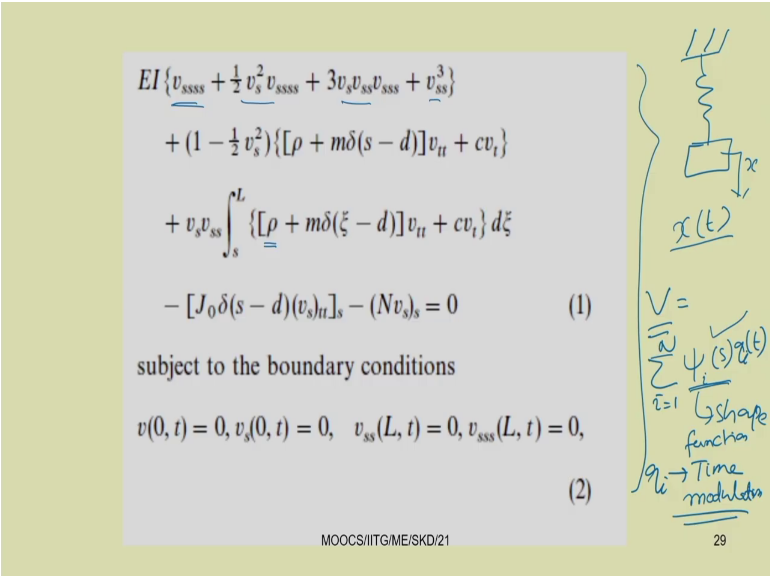

In this case the equation of motion can be written in this form. So, if you just see, in this equation, so if you compare this equation with that of the Euler Bernoulli beam equation, in Euler Bernoulli beam equation only this term is there.

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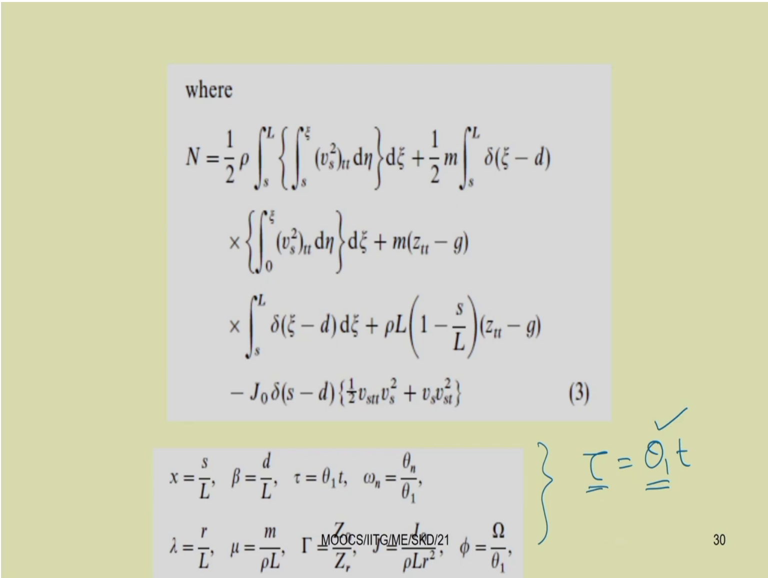

So, one more term also will be there, this correspond to EI del 4th w by del s forth. So, this term was there. And another term also you can get which will be equal to in terms of rho a; so rho. So, this term also was there rho you can have another term that is rho v double dot also will be there.

But as we are taking this non-linear equation of motion, so we are assuming it to be non-linear because this phi is, so this phi we are considering not to be very small. So, this phi is not small. If this phi is very small then you can consider the non-linear terms will vanish.

But for higher value of phi, when the sin phi is not equal to phi, but it will contain other terms. So, that time you must have to expand this equation and you can get the these equations. So, you just see, due to presence of all these terms the equation motion is a non-linear equation. So, these also contain the non-linear equation.

Now, by taking these parameter that is non-dimensional parameter x equal to s by L, beta equal to d by L, tau equal to theta 1 t. So, morning, in last class also I told you how to use this non-dimensional time parameter. So, tau is the non-dimensional time parameter which is equal to written here as theta 1 t. Theta 1 is nothing but the coefficient of the coefficient of the linear term that is q.

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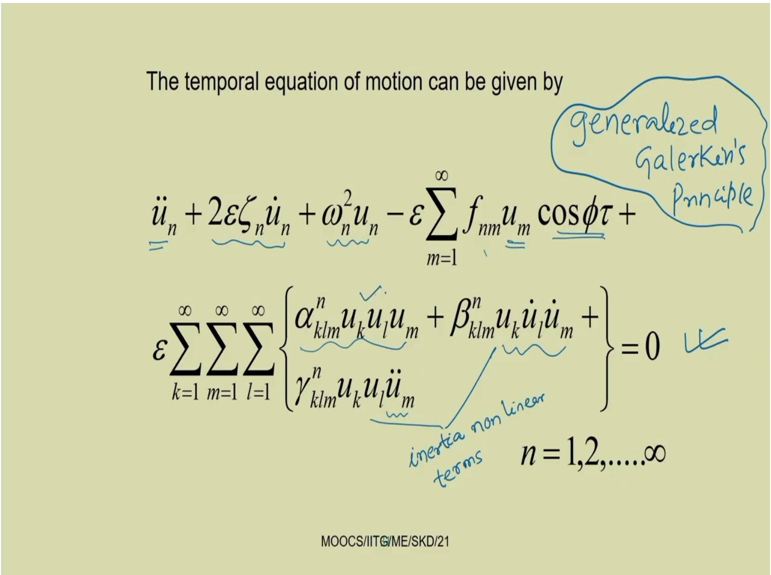

This way by taking these all these parameters, so one can get this temporal equation of motion by applying this generalized Galerkin's method. So, one has to apply the generalized Galerkin's principle or generalized Galerkin's principle or method. Here in this generalized Galerkin's method one may consider the equation q in this form.

Here the equation is written in terms of v, so all the terms are written in terms of v. So, one can write this v equal to you just see this continuous system and v is a function of both space that is s and time t.

So, in case of the spring mass system or the muscle parameter quad, today we have seen in all those cases the equation motion is written in terms of a time parameter that is this x, x is a function of only t, x is a function of only t. But in this case v is a function of both space that is s and time t.

By using the separable variation method type of thing, so we can write this v equal to summation. So, it will be psi i s into q i t, i equal to 1 to n. So, we can take one to infinite also, but limiting these number of modes we can write i equal to 1 to n, where the psi i s is the shape function, psi i is the shape function and q i is known as the time modulation, u i is known as time modulation.

Now, to apply this generalized Galerkin method. So, in this governing equation which is written in space and time, so we can substitute these psi i s and q i t, where the psi i s is the mode shape or it can be any admissible function. So, by putting this admissible function, so as you know that admissible function only satisfy the geometric boundary conditions, it does not satisfy all the boundary conditions, it also does not satisfy the governing equation also. So, in that case by substituting this equation we will have some residue.

Now, by minimizing this residue, how you can minimize this residue? So, initially we have to substitute this equation in original equation, then we have to multiply weight function and integrate it over the length of the beam and equated to 0 to find the equation which will be reduced to that of a time functions only. So, it will be written in its temporal form then. So, by using this generalized Galerkin method, this equation of motion is reduced to this form.

So, here you just see this equation contain this u n double dot, then this is the inertia term, and this is the damping term. This is omega n square u n, then this is the forcing term. Here you can node that this forcing term this u m, that is the response the coefficient of response equal to f nm cos phi t. So, that is a time varying term is the coefficient of u m, that is why the system is a parametrically excited equation.

Here you just see the number of non-linear term, so this contain the non-linear term u k u l u m. So, this is known as geometric non-linear parameter. And here this term u k u l dot and u m dot, so product of two velocity term is also an acceleration term.

Here u double dot is there, so this is also an acceleration term. So, this and this, so they are known as inertia non-linear term. So, these are inertia non-linear term. You have here in this case two inertia non-linear terms, and one geometric non-linear term. This way you can write the equation of motion.

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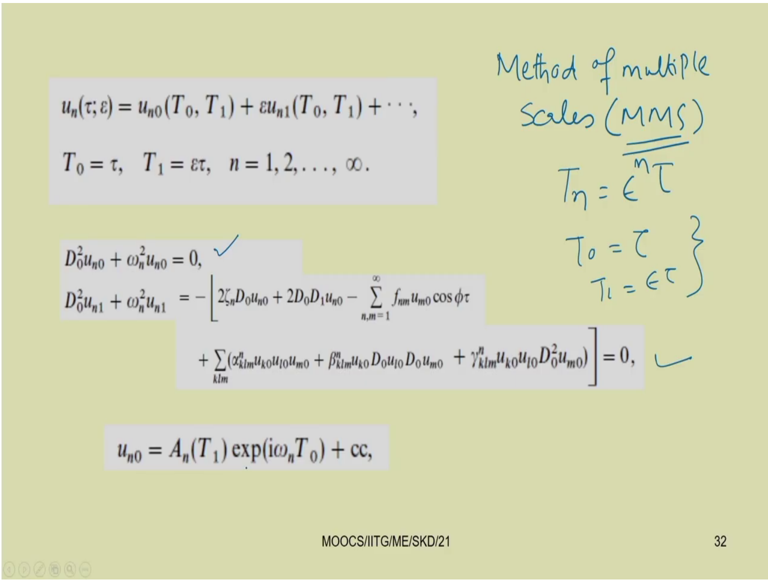

So, after writing this equation of motion you can solve this equation motion by using many methods. So, here let us use this method of multiple scale, method of multiple scales to derive the solution and the stability equation.

So, here to apply this method of multiple scale. So, we can write this u n tau epsilon equal to u n 0 T 0, T 1 plus epsilon u n 1 T 0, T 1. Here we are taking, so already you know that this T n equal to epsilon to the power n T. So, here you are taking non-dimensional time this tau, so this T 0 equal to epsilon to the power 0 that is equal to 1 that is tau and T 1 equal to epsilon tau, so we are taking up to two time scale. So, T 0 equal to tau and T 1 equal to epsilon tau.

So, now substituting this equation in the original equation and separating the terms with different order of epsilon, so we can have these two equation. So, this is epsilon to the power 0 and this is epsilon to the power 1. So, now, the solution of the first one can be written in this form u n 0 equal to A n T 1 e to the power i omega n T 0 plus cc, cc is nothing but the complex conjugate of the preceding term.

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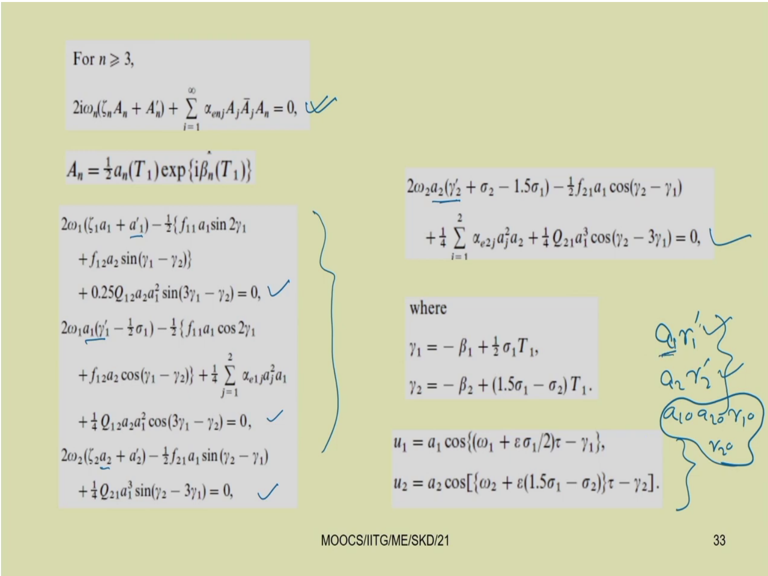

So, then for n equal to, so you just see for n equal to 1 and 2 you can write in this form, but if n greater than 3, so you just see the equation can be written 2 i omega n zeta n A n. Now, after getting this u n equal to A n e to the power i omega n T 0 plus cc. So, you can substitute it in this equation, and after substituting this equation, so you can eliminate the term or eliminate the secular term to get the response.

For n greater than 3, so we can see that due to the presence of damping. So, the system response will die down. So, it is not required to then consider for n greater than 3. For n less than 3, so these are the secular term to eliminate the secular term. So, we can write down this equation and here this A n can be written in its polar form that is half, A n e to the power i beta n. So, then to eliminate this secular term. So, we have these terms. And here we have substituted this.

You just see we have now 4 equations. So, one, so this is the second; this is the first equation, second equation, third equation, and fourth equation. So, two in terms of, so as you are taking two mode, so in this case you note that we are taking two mode. So, you can see another paper by (Refer Time: 50:14) and (Refer Time: 50:15), where they have considered the same system by considering single mode approximation.

And here we have taken two mode approximation and that is why we have 4 equation and they have 1 equations. And the solution can be written in this form. The solution can be written in this form.

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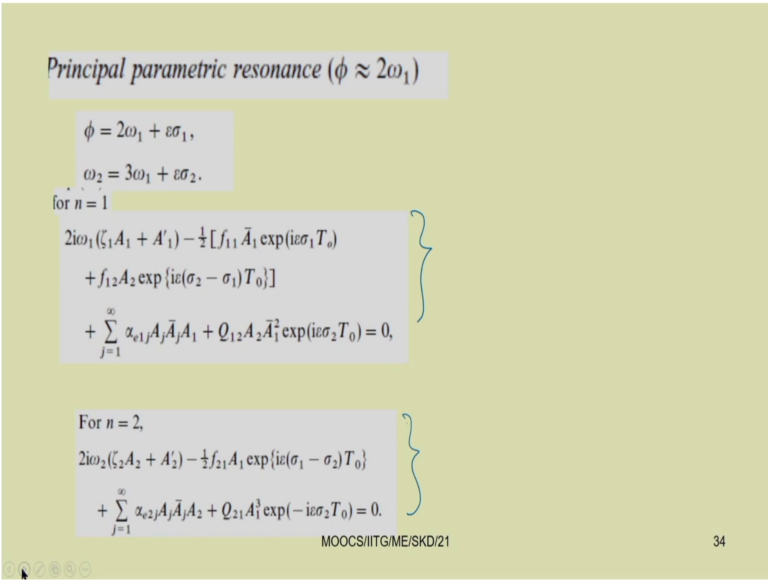

And for principal parametric resonance condition, so we can take this phi nearly equal to 2 omega 1. So, phi equal to 2 omega 1 plus epsilon sigma 1. And omega 2, here we are considering the internal resonance condition. So, internal resonance condition is nothing, but when different modes have integer relationship, then we generally called that system to be internally resonated.

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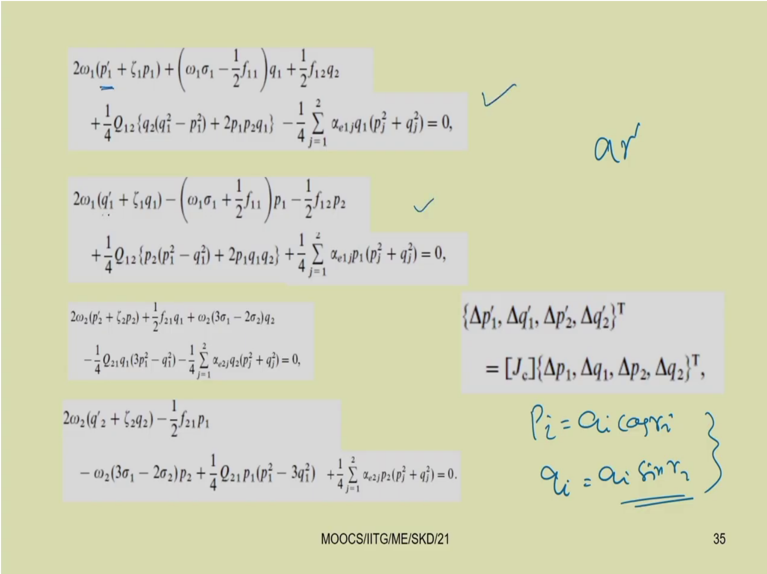

So, in this particular case, so we are taking this condition that we are taking the condition that second mode is 3 times that of the first mode.

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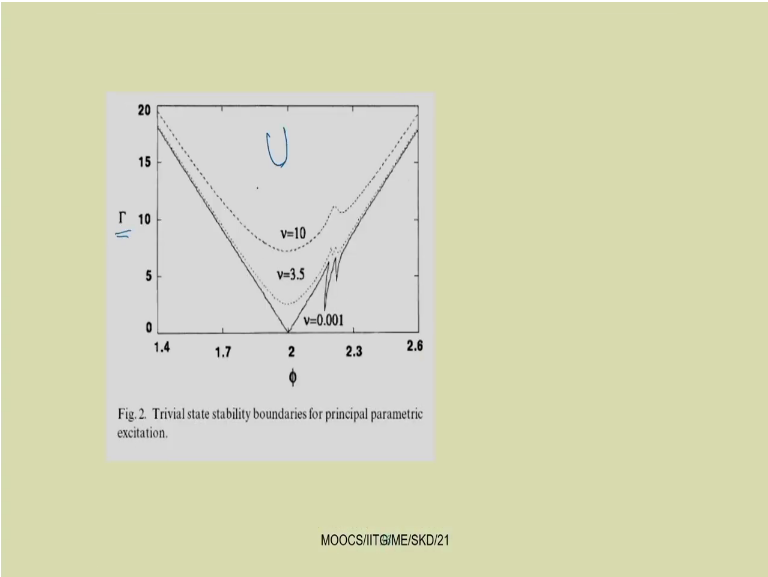

So, if the second mode is 3 times that of the first mode, putting these equations actually in the previous equation, so we can see, so there are some terms which are mixed secular term and they must have to be also eliminated in addition to this secular term.

So, the resulting equations you can find these. So, these for n equal to 1 this is the equation, and for n equal to 2 this is the equation, and here by substituting A n equal to half, A n e to the power i beta, we can write the reduced equations. And after writing the reduced equation, so then we can find actually we can check. So, these are the reduced equation actually. So, these are the reduced equation. These 4 are the reduced equation we got.

And for n greater than 3 we can see that the due to presence of damping, so these response tends to 0. So, so there will be no response for n greater than equal to 3 and there will be interaction between first mode and second mode, due to the presence of this internal resonance condition that is omega 2 equal to 3 omega plus epsilon sigma 2. Here sigma 1 and sigma 2 are detuning parameter.

One can observe that due to the presence of the term a gamma dash. So, if you just see, can show these reduced equation. So, in this reduced equation you just see the term a 1 gamma 1 dash; in the first case we have a 1 dash, second case a 1 gamma 1 dash, third case a 2 and fourth case a 2 gamma 2 dash.

So, due to presence of these a 1 gamma 1 dash and a 2 gamma 2 dash, if we are considering for example, a 10, a 20, gamma 10 and gamma 20 are the steady state response. So, if we perturb these things, so you can see for trivial state, so these perturbation of this one and this one will no longer be there.

So, due to the presence of this (Refer Time: 53:22) term a 10 and a 20, so if you perturb this thing then it will be a first term will be delta a 1 into gamma 1 dash and second term will be a 10 into gamma 10 plus delta gamma 1 dash. So, for that thing this as it is multiplied with this one, this term will be 0. This way as these will not contain this trivial. For trivial state as these terms will no longer be there. So, we have to put some transformation.

So, here we can put a transformation like this p i and q i, p i equal to a i cos gamma i and q i equal to a i sin gamma i. So, by putting this perturb; this transformation, so we can write down these 4 equation and easily you can perturb these equations. So, you can see these equations are written p 1 dash q 1 dash and p 2 dash q 2 dash. So, by perturbing these equations. So, you can find the instability region now. One such instability region is shown here.

So, clearly you can see for different value of response amplitude and frequency the instability region.

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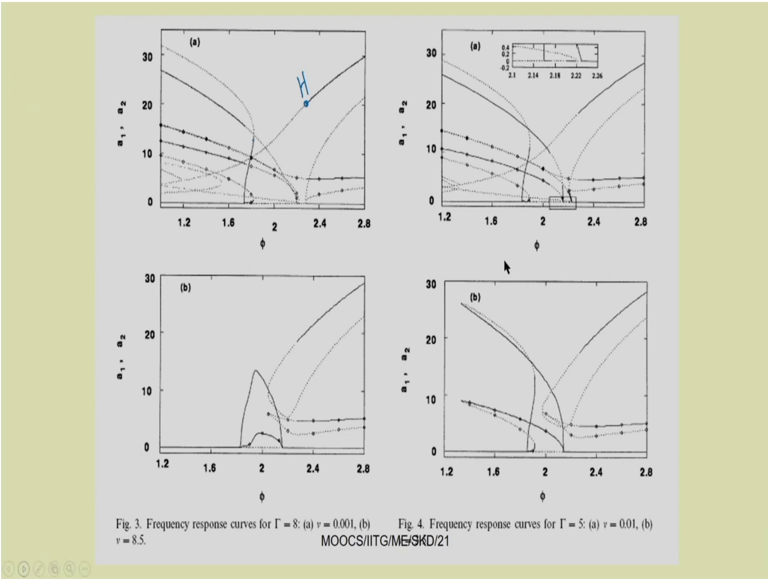

So, in this case inside this thing the system response is unstable and outside the system response is stable. So, you can plot various responses also. So, you can find this steady state response by solving those 4 equations those 4 equations can be solved by using these by using different methods you can use, but particularly you may be interested to use this Newton's method. So, by using this Newton's method for multiple number of algebraic equations, so you can find the solution.

So, here you can observe many different type of bifurcations. For examples this point is of bifurcation and this point. So, you can have a saddle node bifurcation, here also a saddle node bifurcation. So, many different type of bifurcations or all sorts of bifurcation points you can find responses. This is with respect to frequency, so this is frequency response plot.

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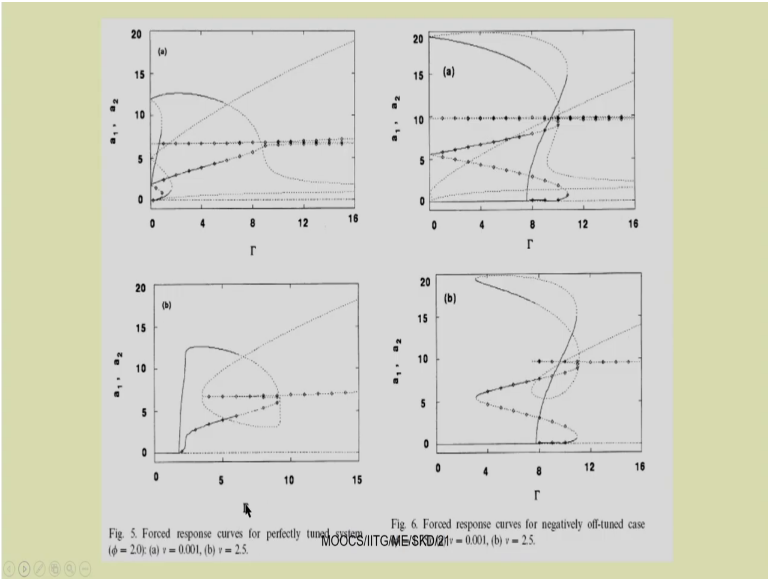

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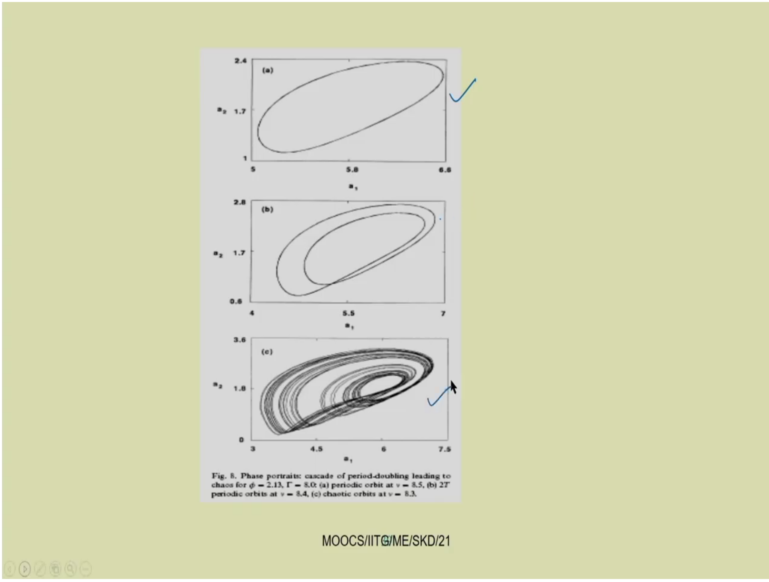

Also you can plot the forced response plot. And the near the bifurcation, so you can see by varying the system parameter you can see the system has a periodic, then two periodic, and if you go on changing the system parameter finally, it lands with the chaotic response.

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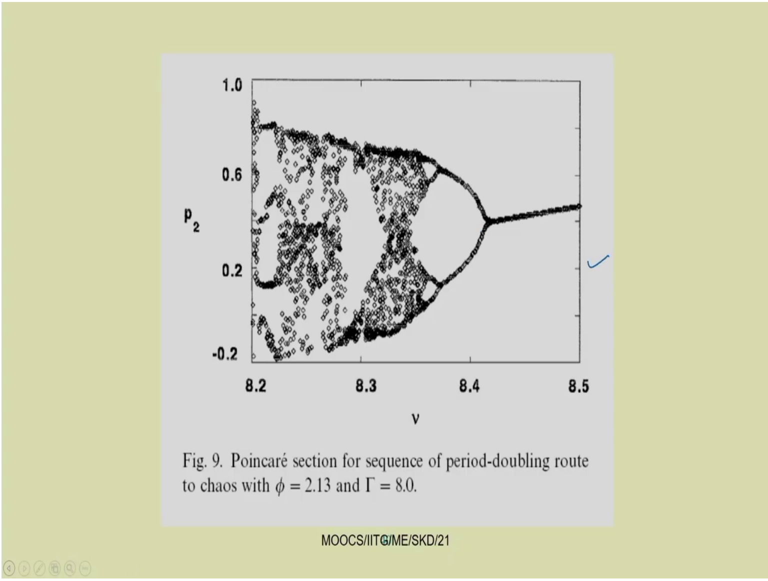

Fig. 9. Poincaré section for sequence of period-doubling route to chaos with $\phi = 2.13$ and $\Gamma = 8.0$.

So, if you plot the Poincare section, so for periodic it is a single point, for two period it is two point.

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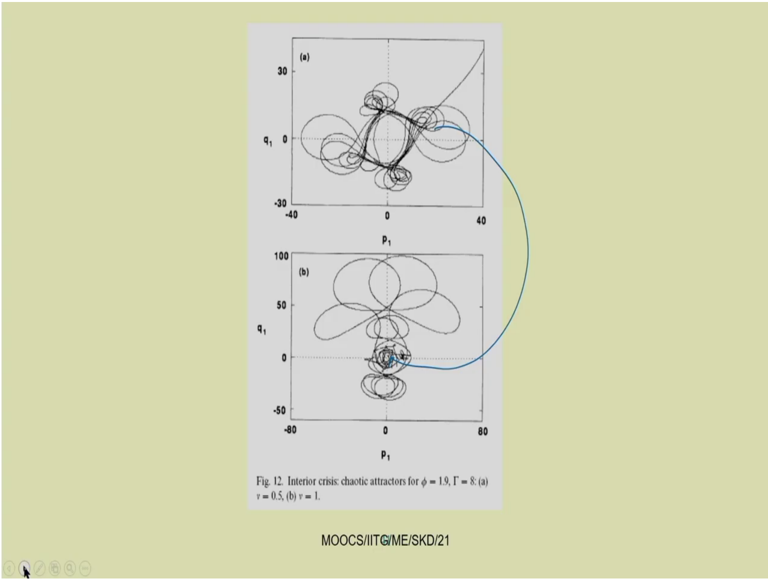

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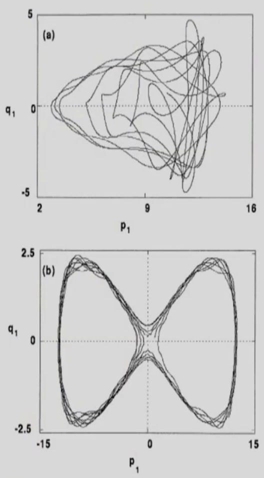

Fig. 13. Attractor merging crisis: $\Gamma = 8$, $v = 0.01$: (a) $\phi = 1.81$, (b) $\phi = 1.79$.

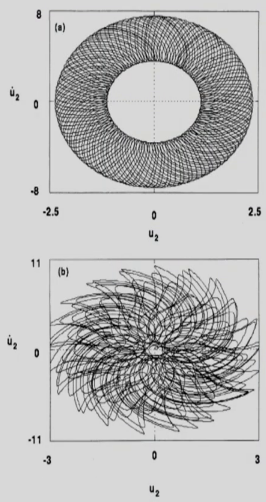

Fig. 14. Cascade of torus doubling route to chaos: [illegible] $\Gamma = 8$: (a) T_2 orbits at $v = 8.5$, (b) chaotic orbits at $v = 8.3$.

q_i ($i = 1, 2$) by Eq. (21a) and (21b). These equations

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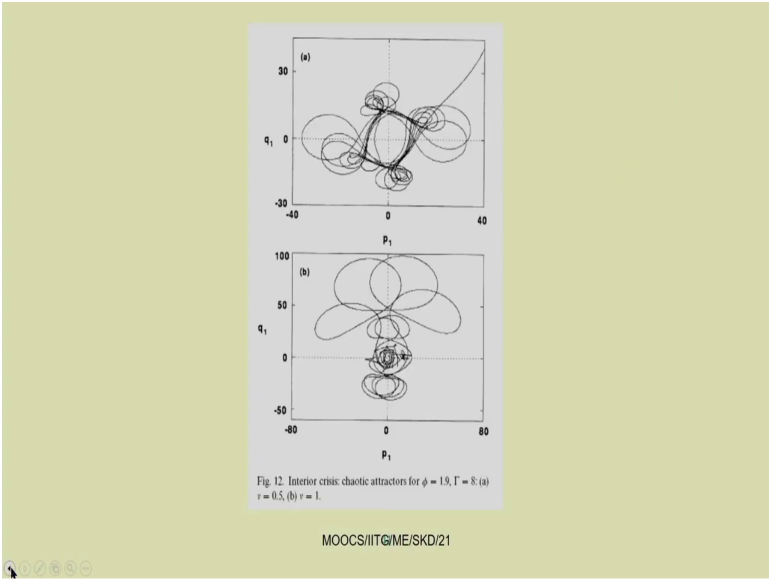

If you goes on doing the Poincare section for different system parameters, so you can conveniently see that the system response becomes chaotic. So, this is period doubling leads to chaos. So, period doubling leads to chaos type of phenomena you can observe in this case.

Similarly, you can observe other different type of this chaotic response also. So, here one interior crisis you can observe. So, initially this is the attractor. So, when it is come in contact with the unstable fixed point response, so you can see that this attractor is now inside this bigger attractor.

So, this is known as interior crisis. Similarly, you can have this attractor merging, so single attractor. So, now this attractor merging crisis is occurring in the system. So, these are the cascade of torus doubling route to chaos also we can find. So, initially the response is this is quasi periodic or torus. Now, by changing the system parameter you can see this is the torus doubling route to chaos.

In this way, so you can study different type of instability region, different type of response, and also this chaotic response, crisis, all these things you can study in a simpler system like a beam just excited by excited from the base. Unlike the direct excitation, the parametric excited systems are more complex and their analysis are more and more interesting. And you may study more different types of parametrically excited system and know more about them.

So, thank you.