Nonlinear Vibration

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Lecture - 20

Parametrically excited pneumatic artificial muscle

Welcome to today class of Non-linear Vibration. Today, we are going to study more on this Floquet theory and also application of this parametrically excited system. So, we will take one example of a pneumatic artificial muscle which can be model as a parametrically excited system and we can find the response of the system. Last class we have discussed regarding a parametrically excited system and given some examples.

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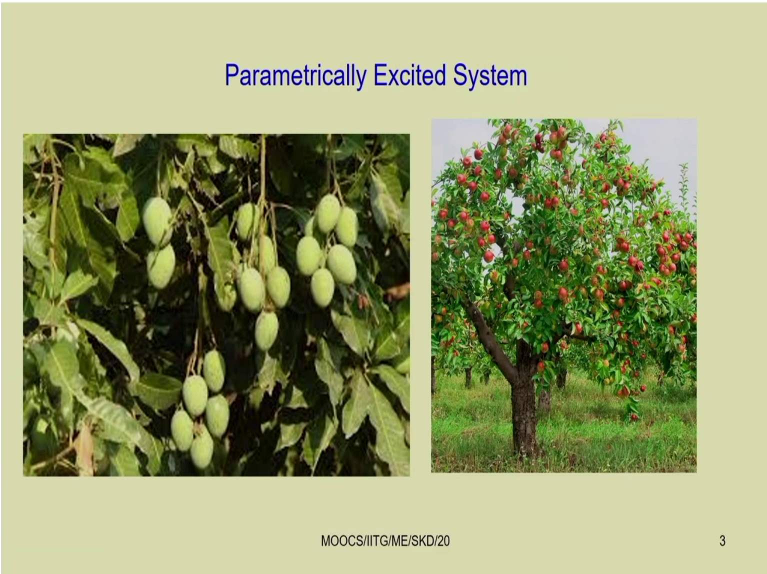

For example, for flocking the fruits in the tree so, you can shake the tree so, that the fruits can come down. So, here you are exciting the branch in such a way that the application of force and the response are taking in orthogonal direction.

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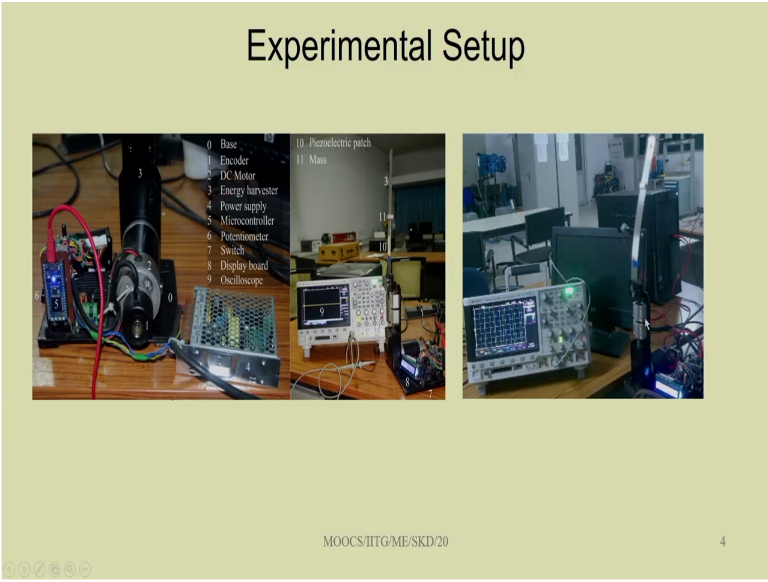

In particularly in case of parametrically excited system, we have seen this example that is a basic sited cantilever beam so, in this basic sited cantilever beam when the base is moving up and down. So, we can see the cantilever beam will move in the transverse direction. Here the application of force and the direction of motion are taking place in perpendicular directions, that is a typical case of a parametrically excited system.

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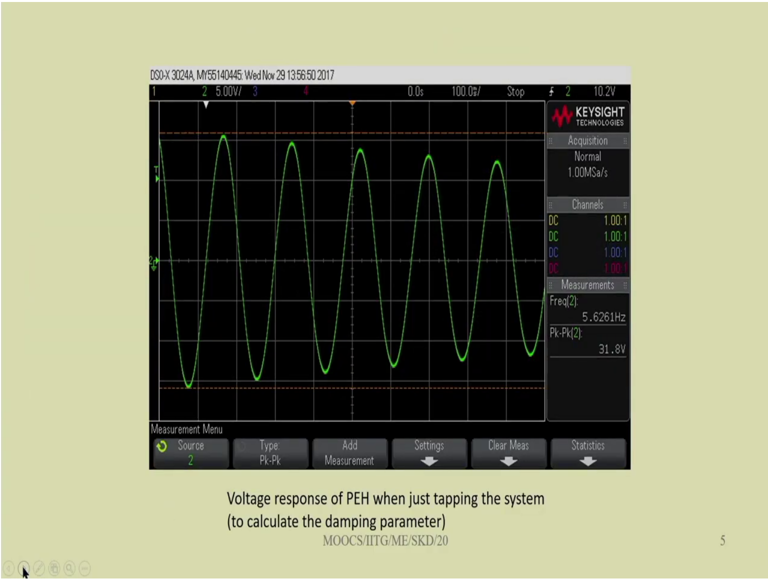

So, you have seen in parametrically excited system the force application of force, and for example. So, let us take this cantilever beam.

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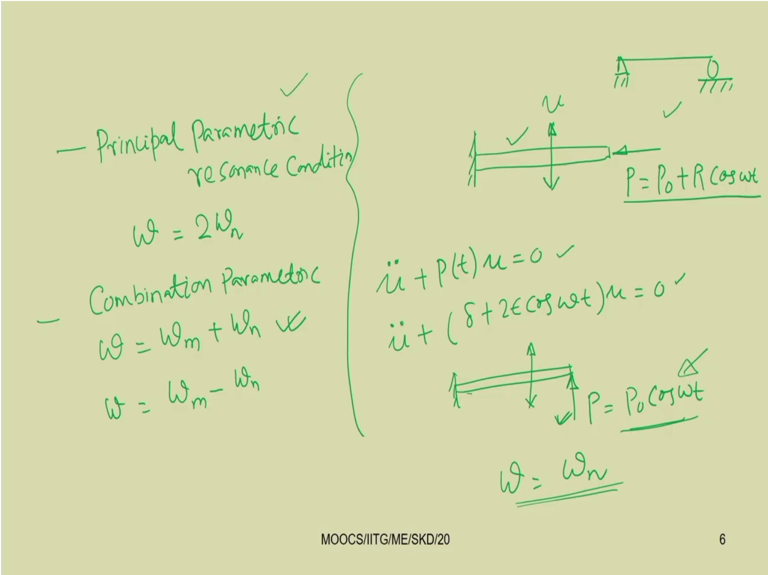

So, here the force is applied let the force is applied in the axial direction, and the displacement is taking place in the perpendicular direction. So, here also you can note that this force P is equal to P 0 plus P 1 cos omega t. So, here we are giving a time bearing force so, in this time bearing force. So, if you write down the equation of a parametrically excited system, it can be typically written in this form.

So, u double dot plus P t u equal to 0. So, this is known as the Hill's equation and in this Hill's equation. So, you have already seen. So, if we are writing this P t in these form delta plus 2 epsilon cos omega t into u equal to 0. So, this is known as Mathieu equation. So, we know this Hills equation and Mathieu equation. So, if you compare this equation with the system for example, you take the system so, which is directly excited.

So, for example, here you have applied a force for example, this is P equal to P 0 cos omega t. So, this is the force you have given and the system is moving in this transverse direction also. So, here the direction of force and the direction of application direction of force and the direction of response are in the same direction, but in this case the force is applied in this horizontal direction and the displacement is taking place in the transverse directions.

So, this is the difference between this parametrically excited system, and the direct force excited system in physical sense. Also the resonance conditions, we have discussed yesterday. So, in this case in direct excitation when this omega that is the external frequency equal to the natural frequency omega equal to omega n. So, we are getting the resonance condition, but in this case in case of parametrically excited systems.

So, we have discussed regarding three different type of resonance condition. So, one is known as principal parametric resonance condition, principal parametric resonance condition.

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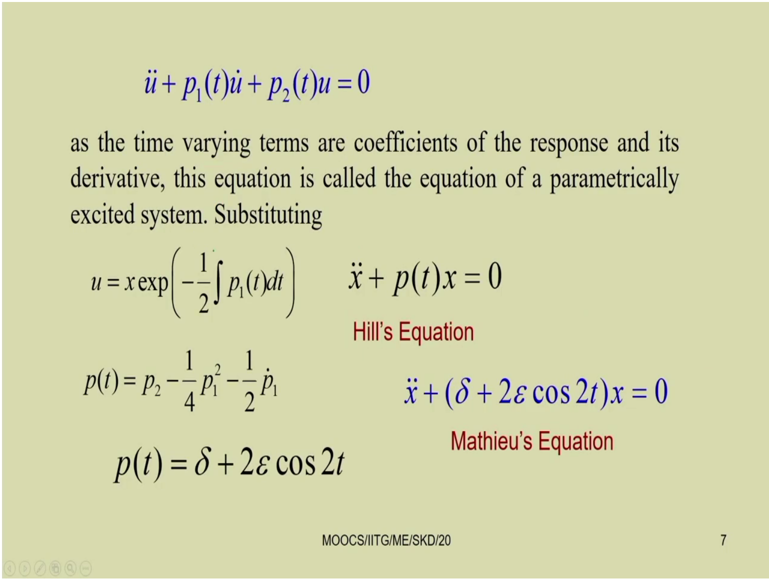

So, in this condition so, omega that is the external frequency can be written as twice the natural frequency so, in this condition so it is written it is equal to twice the natural frequency. So, depending on the first mode for example, let us take a continuous system like this cantilever beam or a simply supported beam. So, in these cases so we have infinite number of natural frequency, because these are the continuous system.

So, in case of continuous system so, we have infinite number of natural frequency we will have infinite number of parametric resonance condition. For example, it may be at omega equal to 2 omega 1 omega equal to 2 omega 3. So, here omega 1, omega 2, omega 3 are the natural frequency of the system. Also we have discussed or I told. So, we can have the resonance condition that is known as combination parametric resonance condition combination parametric.

So, in case of combination parametric resonance condition, we have this omega equal to that is the external frequency will be equal to external frequency omega will be equal to omega m plus minus omega n. So, this is combination parametric resonance of some type and another one we can have also omega equal to omega m minus omega n.

So, in this case so, it is combination parametric resonance of different types. So, this m and n can take any number that is 1, 2, 3, 4 depending on the natural frequency of the system.

Here so, you have observed that in case of parametrically excited system, the resonance conditions occur away from the natural frequency it may occur twice, of the natural frequency which is known as principal parametric, or it can occur at a frequency which may be some or difference type of different natural frequency.

But, in case of direct excitation resonance will occur at a frequency near to the natural frequency of the system.

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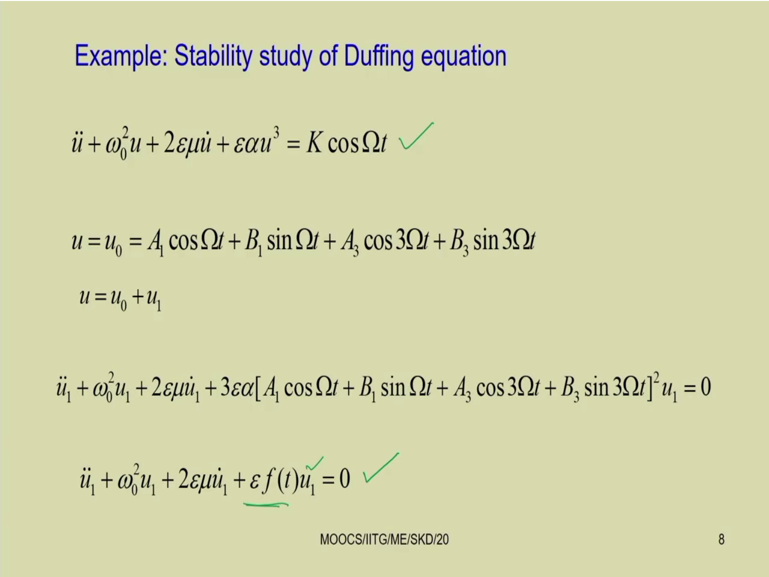

Further we have studied for example, so we have a system like this Duffing equation. So, the for studying the stability when we perturb these equation, these equation actually yield a equation which is similar to that of a parametrically excited system. Here, we have taken u equal to u 0 plus u 1 u is the equilibrium solution. So, taking this equilibrium solution so, by substituting it in these original equation.

So, we got this u 1 double dot plus omega 0 square u 1 plus 2 epsilon mu u 1 dot plus epsilon f t u 1 equal to 0, in this parametrically excited system. So, you have seen the coefficient of the response coefficient of response your responses u 1. So, the coefficient of response is a parameter which is periodic in nature.

So, this f t is a periodic function. So, if the coefficient of u that is the response is a periodic function, then we call these type of equations are parametrically excited system equation for a parametrically excited system.

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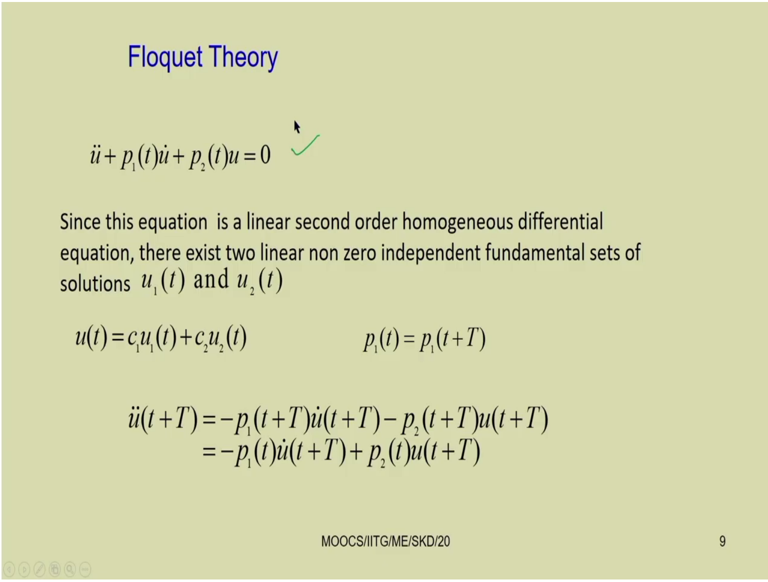

So, here we have applied last class we have seen how to apply this Floquet theory. So, in case of Floquet theory we have studied that for a second order differential equation. So, we can have two fundamental sets of solution. For example, let us take two fundamental setup solution that is u 1 t u 2 t. So, the combination of these fundamental solution is also a solution that thing we know.

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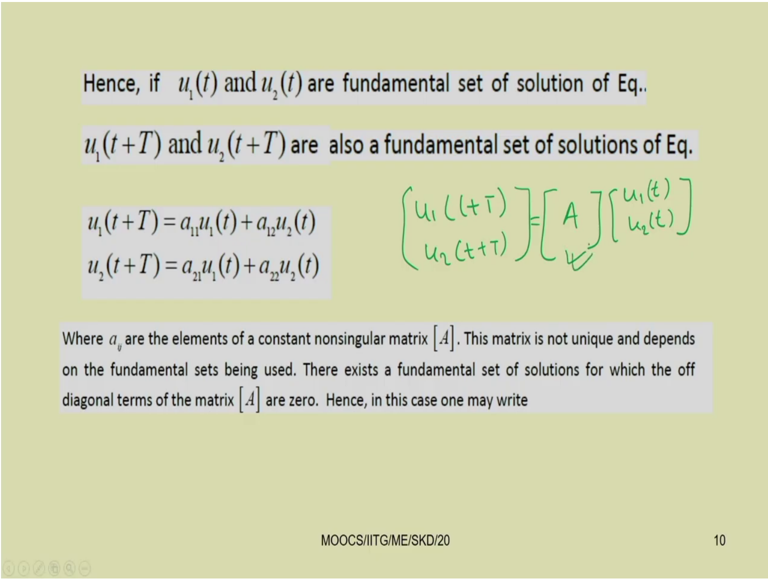

So, from that thing we have proceed and we have found that, if we can find the roots of the monodromy matrix.

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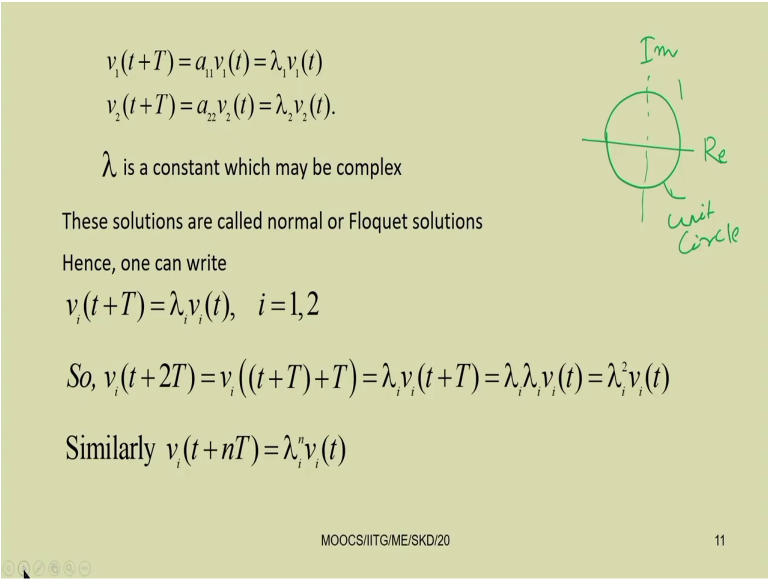

If it is within the unit circle so, it must be within the unit circle. So, if it is within the unit circle then it is stable. So, if it is outside the unit circle then it is unstable. So, how to do that thing? So, we have taken for example, in this case. So, for solving these equation this u 1 double dot plus p 1 t u dot plus p 2 t u equal to 0. So, we initially assume two solution u 1 t and u 2 t. So, we have seen that u 1 t plus t is also periodic and that thing can be written as in this form.

So, u 1 t 1 plus T can be written as a 11 u 1 t plus a 12 u 2 t similarly u 2 t plus T can be written as a 2 1 u 1 t plus a 22 u 2 t and so, that you can write a matrix. So, this u 1 t plus T and u 2 t plus T so, we can write a matrix actually with A matrix which is equal to A into u 1 t and u 2 t. So, this way we can write a matrix.

And by finding this eigenvalue of these matrix. So, we can tell whether the system is stable or not. So, by finding the eigenvalue of that matrix, if the roots or the eigenvalues are within these limit cycle unit cycle, then it will be stable. So, if it is outside this unit circle then it is unstable. So, this is the real part and this is the imaginary part. So, this way we have studied this Floquet theory.

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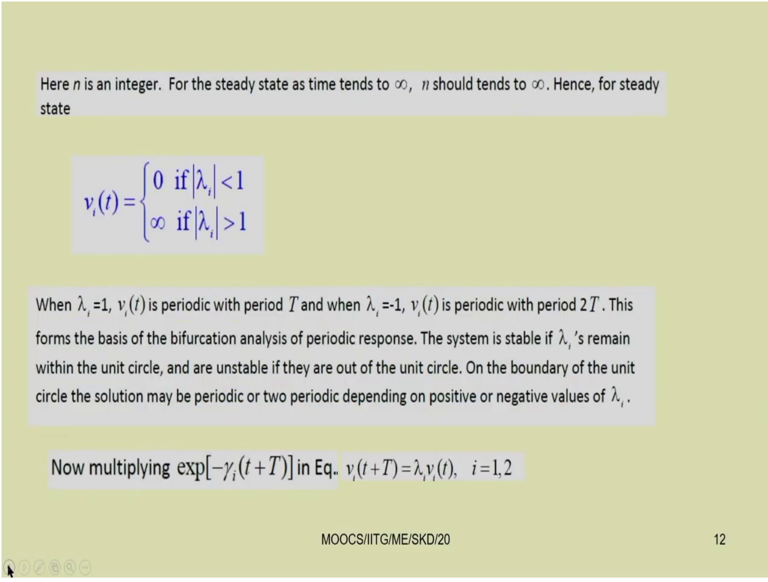

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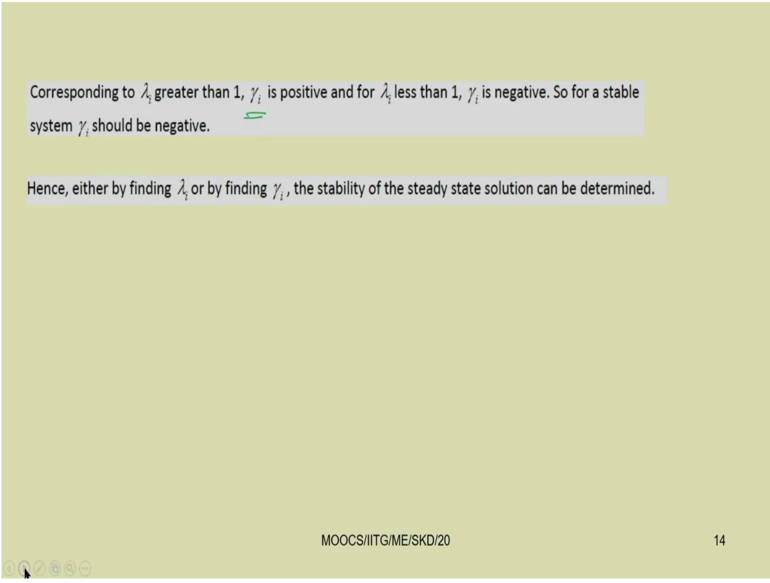

So, corresponding if lambda i greater than 1, we can find if lambda i greater than 1, then gamma i is also positive. So, either we can find the roots lambda i to study the stability otherwise we can find these Floquet multiplier gamma i also to find whether the system is stable or not. So, if gamma i is positive then we know that the system is unstable and if gamma i is negative, then you can tell the system is stable.

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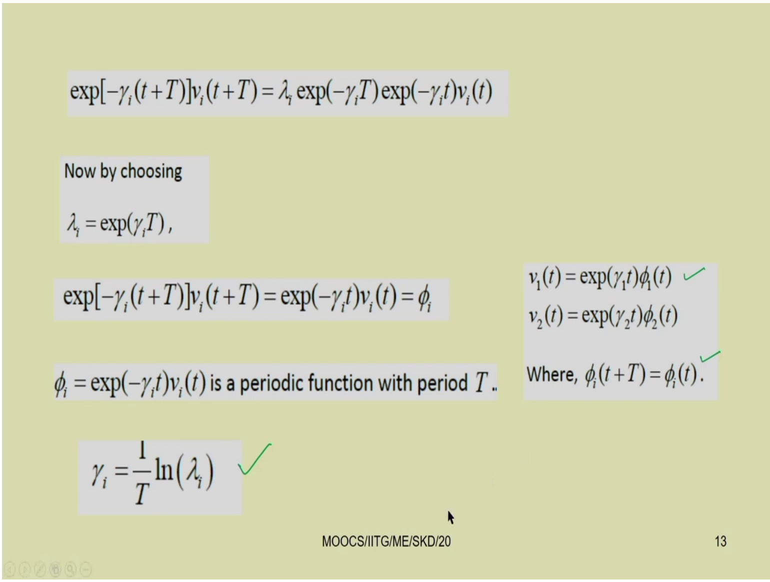

And the relation between lambda i and gamma i is these. So, gamma i equal to 1 by T ln lambda i. So, either you find these gamma i or lambda i and you can find the solution. So, to find these gamma I so, you may initially take a solution in this form in the form of phi; that means, v 1 t if you take equal to the solution in this form e u to the power gamma 1 t phi 1 t and v 2 t equal to e to the power gamma 2 t phi 2 t.

So, where phi pi i t plus T equal to pi i t, then directly you can get gamma. So, instead of getting lambda so, if you get gamma so, in case of those are Floquet multipliers. So, knowing this Floquet multiplier, you can tell whether the system is stable or unstable. So, last class we have seen one example.

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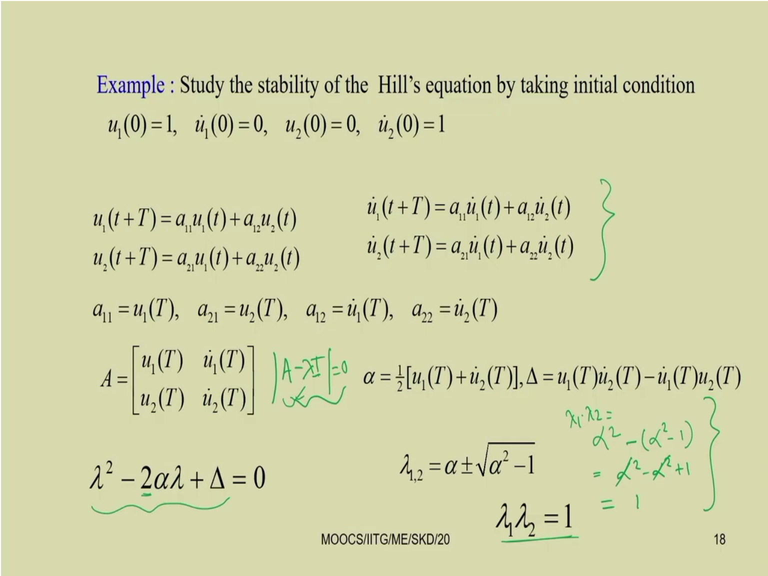

So, let us see another example for example, let us study the stability of Hill's equation by taking this initial condition u 1 equal to 1, u 1 dot equal to u 1 dot 0 equal to 0, u 2 equal to 0 and u 2 dot equal to 1. Like previous class here also we can take these u 1 t plus T equal to a 11 u 1 t plus a 12 u 2 t. Similarly, u 2 t plus T equal to a 21 u 1 t plus a 22 u 2 t. Similarly, u 1 dot t plus T equal to a 11 u 1 t plus a 12 u 2 dot t and u 2 dot t plus T equal to a 21 u 1 t plus a 22 u 2 dot t by substituting these initial conditions.

So, we can find these a 11 a 12 a 21 a 22, where it can be written a 11 can be found as u 1 T a 21 can be found as u 2 T. So, you just see first you substitute this in this equation. So, t equal to 0 so, u 1 0 plus t that is u 1 t so, u 1 t is nothing, but a 11 into u 1 t equal to 1. So, this becomes a 11 so you got a 11 equal to u 1 t. Similarly a 21 to find a 21 so, you just see this equation second equation here by substituting t equal to 0.

So, u 2 t equal to a 21 into 1 that is a 21 plus a 22 into u 2 t as u 2 t equal to 0, you have taken so, these a 21 becomes u 2 T. Similarly from these two equations also we can get a 1 2 equal to so, you just see here you are getting u 1 t plus T equal to a 11 u 1 only, similarly uncoupled. So, now, you are not getting the coupled term because this here this a u 2 or a you are taking these initial condition and from these initial condition.

So, 11 term becoming 0. So, directly you are getting the coefficients, in third case also you got the directly you got the coefficient a 12 a 12 equal to a u 1 dot T and a 22, similarly u dot equal to u 2 dot T. So, these a matrix now can be reconstructed equal to u 1 T, u 1 dot T u 2 T u 2 dot T. We can find the determinant of these a minus lambda i equation so, to find the eigenvalue.

So, either you directly find the eigenvalue or you just find the determinant of this thing and equate to 0. This is a 2 degree of 2 is to 2 matrix, then easily you can do it by hand that is why simply you can find this determinant of A minus lambda I equal to 0 so, which will give rise to this equation that is lambda square minus 2 alpha lambda plus delta equal to 0.

So, where alpha equal to half of the trace u 1 plus u 2 dot and then so, half of u 1 plus u 2 dot, because we are taking 2. Otherwise you can write this lambda square minus the trace into lambda plus delta I equal to 0. So, where delta equal to u 1 T into u 2 dot T minus, this is the determinant of A u 1 T into u 2 dot T minus u 1 dot T minus u 2 T.

After getting this alpha and lambda so, you can write down this thing. So, you can find this lambda square so, from these thing you can find the root. So, the roots are lambda 1 2 equal to so, you can use this formula minus b plus minus root over b square minus 4 u c by 2 a. And, you can find this lambda 1, 2 equal to alpha plus minus root over alpha square minus 1. So, from these thing you can note this lambda 1 and lambda 2. So, if you multiply this 2 then this becomes you just see this is alpha.

So, a square minus b square s o, it will be so let alpha equal to a and this part is b. So, this is a plus b into a minus b. So, a plus b in to a minus b is square minus b square. So, your lambda 1 into lambda 2 equal to a square plus b square in case of b square it is equal to alpha square minus 1 only a square minus b square so, it is a square minus b square.

So, this becomes alpha square minus alpha square plus 1. So, these cancels and this becomes 1. So, here you have observed that this lambda 1 into lambda 2 is always equal to 1. So, if one of the root is less than 1. So, for example, let lambda 1 equal to half, then lambda 2 must be equal to 2. If one of the root is greater than 1 so, the other root must be equal less than 1.

So, in that case one of the root is lying outside the unit circle as lambda 1 and lambda 2 for a stable system must stay within the unit circle. So, that is why to have a stable system, this lambda 1 and lambda 2 either must be equal to 1 or minus 1. So, if it is 1 then we can have the transition curve.

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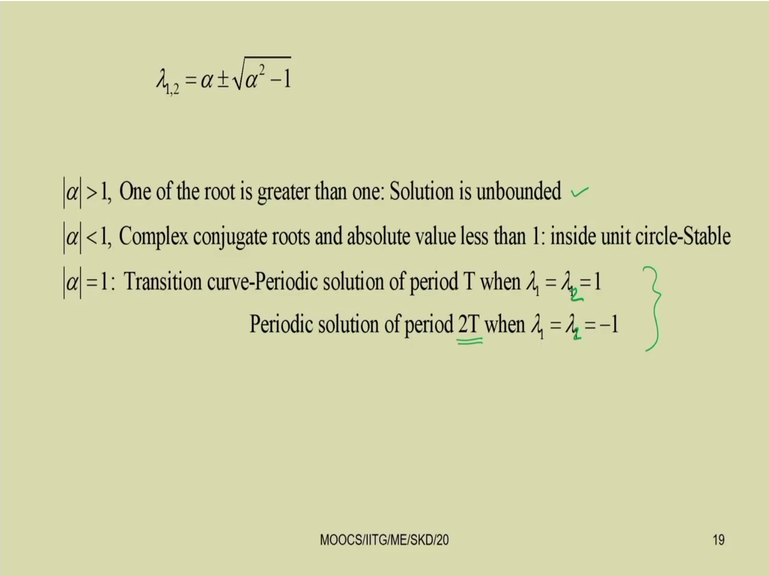

So, from here so you can observe that we have three conditions from this lambda 1 and lambda 2 so, you observe lambda 1 equal to alpha plus minus root over alpha square minus 1. So, from this thing so we can observe that if alpha is greater than 1. So, you just see this equation that is alpha plus minus root over alpha square minus 1. So, if alpha is greater than 1 then this alpha square minus 1 is positive.

So, we have a positive root here then this alpha plus this positive part will be greater than 1. And, then that minus part will leads to less than 1 so; that means, if alpha is greater than 1. So, we have a unstable solution the solution will be on bounded as one of the lambda is greater than 1.

Let us see if alpha is less than 1. So, let us take a number where alpha is less than 1. So, if it is less than 1. So, this part this alpha square minus 1 is negative. So, we have a imaginary term here.

So, we have a complex number here complex conjugate so, if alpha is less than 1. So, we will have a complex conjugate term. So, if we have a complex conjugate term alpha less than 1. So, we have a complex conjugate roots, but the absolute value is less than 1. So, here the absolute value will be less than 1.

So, it will lie inside the unit circle. So, the system will be stable or the periodic solution will be stable. But, if alpha equal to 1, then this root over alpha square minus 1 equal to 0 so, we have two a real root so, alpha if alpha equal to 1. So, we have 2 real root that is an equal root. So, that is lambda 1 equal to lambda 2 equal to either it may be.

So, it can be 1 or minus 1, we have a transition curve periodic solution of period T when lambda 1 equal to lambda 2 lambda 1 equal to lambda 2 equal to 1. Similarly periodic solution of period 2T, when lambda 1 equal to lambda 2 equal to minus 1. So, you just see in both the cases. So, the product is lambda 1 into lambda 2 equal to 1 here also minus 1 into minus 1 also 1. So, when it is minus 1. So, previously we have seen that it must be of period 2T.

So, this way we can study in given any system given any periodic solution. So, we can study whether the solution is stable or not, you can recall in case of a fixed point response. So, generally we find the Jacobian matrix and the eigenvalues of Jacobian matrix. But, in this case we are taking another matrix that a matrix which is generally known as the monodromy matrix.

So, here we are finding the eigenvalue of the monodromy matrix, but in case of the fixed point response we used to find the eigenvalue of the Jacobian matrix. The these two this difference you must clearly understand.

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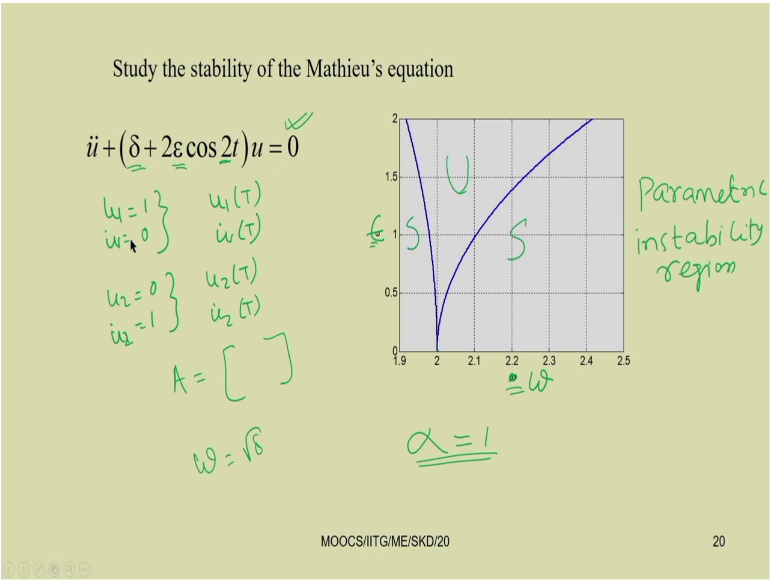

So, let us take in the case of the Mathieu equation. So, in this case you just note that here this equation can be written in this form Mathieu equation. So, that p t we are replacing it by delta plus 2 epsilon cos 2t u so, taking these initial condition u 1 and u 2. So, after one cycle we can find what is the response.

So, after finding the response for two initial condition, for example let us take u 1 equal to starting with u 1 equal to 1 and u 1 dot equal to 0. So, we can find what is u 1 T and u 1 dot T we can find. Similarly starting from u 2 equal to 0 and u 2 dot equal to 1 so, we can find what is u 2 T and u 2 this is u 2 dot so, u 2 dot T.

So, after finding these four parameters so, this thing we can get it from by solving this equation. So, this equation we can use this Runge Kutta method to write in so, we can write this equation into first order differential equation. And after writing these as first two to first order differential equation so, by taking this initial condition u 1 equal to 0 and u 1 dot equal to 0 so, you can find these solution.

After one cycle similarly by taking this initial condition 0 and 1 that is u 2 equal to 0 and u 2 dot equal to 1. So, we can get u 2 after one cycle that is u 2 T and u 2 dot T. So, you just note here that this now we can construct these A matrix.

So, now, constructing these A matrix, we can note that this A matrix is a function of this delta and epsilon. So, delta and epsilon now by taking these omega equal to root over delta; root over delta. So, we can plot this omega and epsilon.

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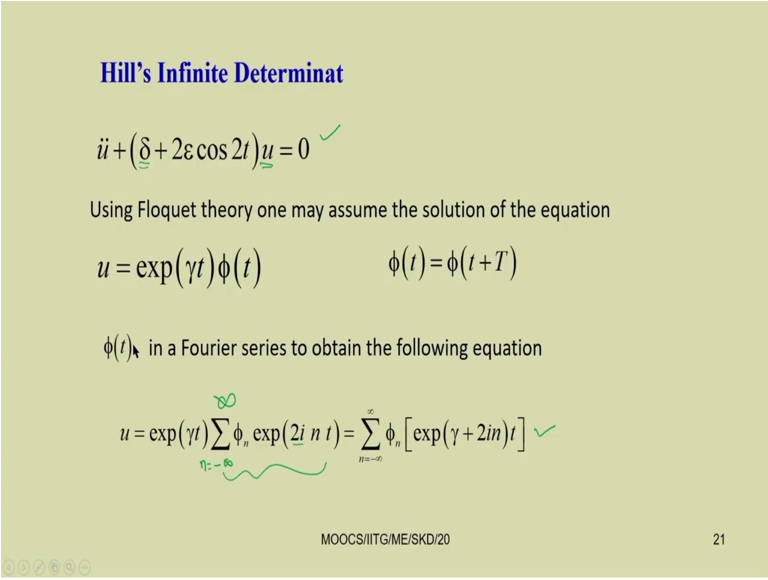

So, we can plot this omega and epsilon and find the value of alpha, for which alpha equal to 1 we can find the value of alpha for which this is equal to 1. So, if 1 can plot these thing that is epsilon versus this is epsilon, versus this omega epsilon versus omega, then 1 can get a curve like this. Where this part is stable and this part is unstable and again this is stable.

So, this is known as the instability region or this is otherwise known as parametric instability region parametric instability region. So, on the transition on the transition curve the response is periodic. So, outside these thing this is stable this is unstable region and again this S is the stable region. So, this way one can physically study any given equation.

So, whether it is Mathieu equation or any other given equation so first find. So, if it is periodic. So, then you can find this way so, here you just note this cos 2 t you just see at omega equal to 2 so, at epsilon equal to 0. So, omega equal to 2. So, the instability region start from this position 2.

So, here this delta you can see so, that when omega equal to 2 so, you can get this curve. So, for any value of epsilon you can take any value of epsilon and you can see or you can find the solution and check whether the solution is stable or not, by solving this equation by using Runge Kutta method ok, or you can use any perturbation method to solve this equation and study that thing.

So, let us see another method to solve this or to find the instability region in case of Mathieu equation. So, here we can use these Hill's determinant Hill's infinite determinant method to find the stability conditions in case of the Mathieu equation. So, this is the Mathieu equation. So, now, we know by using this Floquet theory we can write this u equal to e to the power gamma t into phi t this phi t also you know that this phi t is periodic that is phi t equal to phi t plus T.

So, are this is a periodic function. So, we can expand this phi t by using a Fourier series. So, let us use a Fourier series to expand this phi t. So, that thing can be written as phi ne to the power 2i n t, where i equal to root over minus 1. Now, this is n equal to 1 to infinite you can take or n equal to minus infinity to minus infinity to n equal to minus infinity to plus infinity n equal to minus infinite to minus infinite to plus infinite.

So, you are taking e to the power 2i n t so, for n equal to 1. So, you have this phi 1 e to the power 2i t similarly n equal to 2 you have 4i t. So, that way you can go on expanding so, put different value of n and you can find. So, as it is from minus infinity to infinite. So, you can have large number of exponential function, now by substituting this equation.

So, you just see e to the power gamma t and we have expanded this phi t in terms of its Fourier series. Then, we can write these thing equal to n equal to minus infinite to infinite phi n e to the power gamma. So, you can e to the power gamma t into e to the power 2in t, it will be equal to e to the power gamma plus 2in into t.

Now, by substituting this u in these equation so, if you differentiate it twice now you can differentiate it twice so, this becomes this is gamma. So, this term will come out. So, this is gamma plus 2in gamma plus 2in. So, twice you are differentiating. So, it will be gamma plus 2in whole square into the same term will be there.

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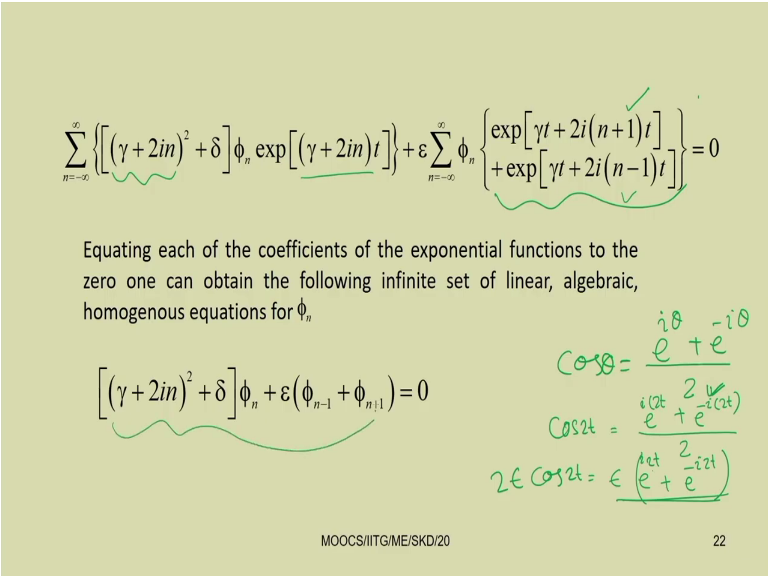

So, now, you just substitute it in this original equation. So, you got this one so, it is minus n equal to minus infinite to infinite, summation n equal to minus infinite to infinite. So, this is the term square term due to differentiation. So, gamma plus 2in whole square plus, another term is there delta u n so, as you have this delta u term so, delta into u so, you have delta into u. So, from that thing also you can get one term that term also you can take it.

So, then this becomes gamma plus 2in whole square plus delta into phi n e to the power gamma plus 2in t so, plus then for the other term. So, for the other term you just see the other term is 2 epsilon cos 2 t into u 2 epsilon cos 2 t into u so, this cos 2 t you can write in exponential form also. And, then you can multiply that thing with u n. So, the resulting things will be so, you know cos phi you can write or cos theta, you can write equal to cos theta equal to e to the power i theta plus e to the power minus i theta by 2.

So, by putting these thing so, we have a two term there 2 cos theta or 2 cos omega t. So, 2 2 will cancel so you will have this e to the power i theta plus e to the power minus i theta your theta is nothing, but your gamma plus theta will be equal to so, in this case. So, this is cos 2 t you are taking. So, you are taking cos 2 t. So, it will be we have taken cos 2 t. So, then it will be e to the power i into 2 t plus e to minus i into 2 t divided by 2 this is cos 2 cos.

So, we have 2 cos we have a term 2 epsilon cos 2 t so, 2 epsilon cos 2 t will be so 2 2 will cancel. So, it becomes epsilon e to the power 2t i into 2 t plus e to the power minus i 2t. So, now, we can multiply this phi which is summation of n equal to minus infinite to infinite e to the power 2i n t. So, if we multiply this with this term. So, then the resulting things will be this.

So, in one case it is added by so this term so, that is gamma t is there in addition to in terms of 2 so, this is 2 will be added. So, if you take these 2 common. So, this becomes 2i n plus 1 t another term so for this 2i t. So, this is the n plus 1 term you got and for this minus 2i t so, by taking these 2 common. So, this becomes 2i n minus 1 t. So, you just see so we have a term e to the power gamma plus 2i n t, we have another term 2i n plus 1 t and here another term. So, that is 2i n minus 1 t.

So, now we can equate the terms coefficient of exponential function to be 0. So, now, we can take different exponential function and equate it to 0 for example, by taking n equal to 1. So, we will have gamma plus 2i here. So, similarly here you just see this is a term n plus 1 and here is a term n minus 1, we have to take a lower term here to have the same exponential form.

So, if we are taking the for example, mth term here so m. So, in that case it will be m minus if you will take n equal to m minus 1 then this becomes m. This is m minus 1 plus 1. So, this becomes m similarly here we have to take n equal to m plus 1 so, that this will also give rise to m. So, this way we have to choose the proper number to find the exponential function to collect the coefficient.

So, because this is a infinite so you just see n equal to minus infinite to infinite. So, we can get or we can collect a infinite determinant from this equation. So, to write down this equation and from this equation, if we collect the coefficients of exponential function will have a infinite number of equations. So, the nth equation can be written in this form, you just see the coefficient of nth equation can be written in this form that is gamma plus 2in square plus delta.

So, this is the term gamma plus 2in square plus delta into phi n. So, you just see everywhere this e to the power gamma plus 2in t can be taken common and so, it can be taken out decide to have this e to the power gamma plus 2in. So, here n must be so we have to take n n minus 1. So, if it is n minus 1 so, then n minus 1 plus 1 will be n. So, that is why it will be epsilon into phi n minus 1.

Similarly, for this one we will get only one term that is n we have to replace it by n plus 1. So, that is why this becomes phi n plus 1. So, this is gamma plus 2in whole square plus delta into phi n plus epsilon into phi n minus 1 plus phi n plus 1 into e to the power gamma 2i n t equal to 0 as exponential function cannot be 0. So, this must be equal to 0. So, we can find this must be equal to 0.

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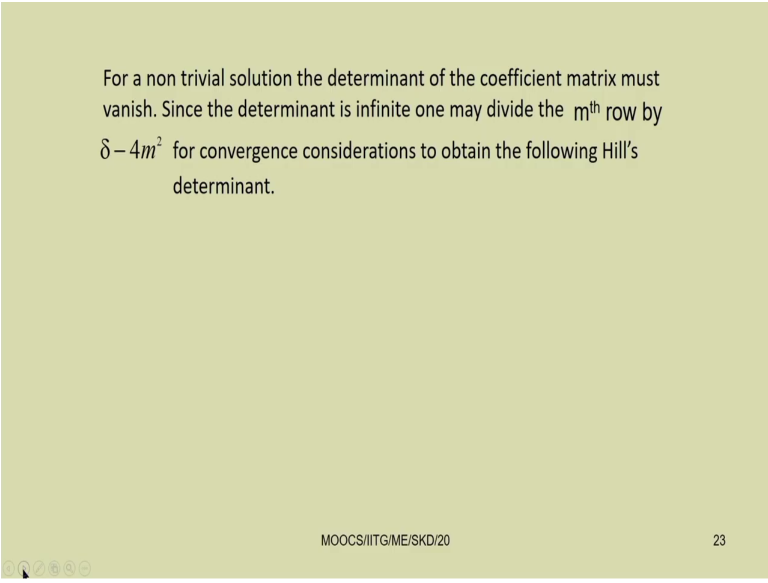

Now, from these equation so, we can for a nontrivial solution. So, when the solution is non trivial that is non zero, the determinant of the coefficient matrix must vanish since the determinant is infinite. So, here we have infinite number of equations. So, that is why we have this infinite determinant. So, since the determinant is infinite when may divide the mth row by delta minus 4m square.

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$$\Delta(\gamma)=\begin{vmatrix}\cdots & . & . & . & . & . & . & . & . & . & \cdots\\ \cdots & 0 & \frac{\varepsilon}{\delta-4^2} & \frac{\delta+(\gamma-4i)^2}{\delta-4^2} & 0 & 0 & 0 & 0 & 0 & 0 & \cdots\\ \cdots & 0 & 0 & \frac{\varepsilon}{\delta-2^2} & \frac{\delta+(\gamma-2i)^2}{\delta-2^2} & \frac{\varepsilon}{\delta-2^2} & 0 & 0 & 0 & 0 & \cdots\\ \cdots & 0 & 0 & 0 & \frac{\varepsilon}{\delta} & \frac{\delta+\gamma^2}{\delta} & \frac{\varepsilon}{\delta} & 0 & 0 & 0 & \cdots\\ \cdots & 0 & 0 & 0 & 0 & \frac{\varepsilon}{\delta-2^2} & \frac{\delta+(\gamma-2i)^2}{\delta-2^2} & \frac{\varepsilon}{\delta-2^2} & 0 & 0 & \cdots\\ \cdots & 0 & 0 & 0 & 0 & 0 & \frac{\varepsilon}{\delta-4^2} & \frac{\delta+(\gamma-4i)^2}{\delta-4^2} & \frac{\varepsilon}{\delta-4^2} & 0 & \cdots\end{vmatrix}$$

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For convergence consideration to obtain the following Hill’s determinants so, we can get this Hill’s determinant actually by dividing by so, by dividing by delta minus 4m square. So, by taking m equal to for example, m equal to 1 m equal to 1 this is delta minus 4 so, m equal to 1 we can take so, this becomes epsilon by delta minus 4 square so, then delta plus gamma minus 4i whole square by delta minus 4 square.

So, other terms will be 0 similarly we can get all these equations. So, from the original the equation what we have derived here. So, from this equation we can find all these terms.

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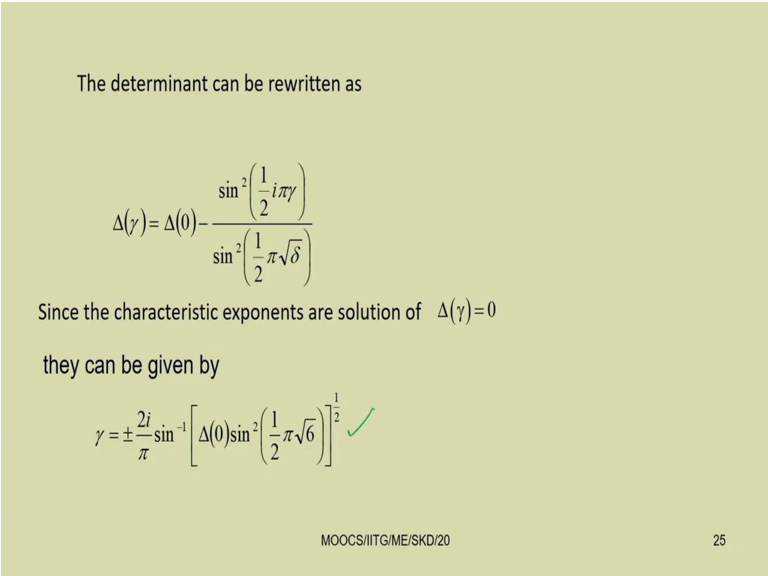

The determinant can be re written as delta gamma equal to delta 0 minus sin square half i pi gamma by sin square half pi root over delta. So, from these thing since the characteristic exponent of the solution delta gamma equal to 0. So, from this thing we can find this gamma. So, gamma equal to gamma is Floquet multiplier.

So, this gamma equal to plus minus 2 i by pi sin inverse delta 0 sin square half pi root over 6 to the power half. So, from this thing so, you know so when gamma is positive the system is unstable and when gamma is negative the system is stable. So, you have to find the value of gamma for which it is 0. So, as to find the transition curve or you can find the lambda value. So, if the lambda is laying in the unit circle then it will be stable.

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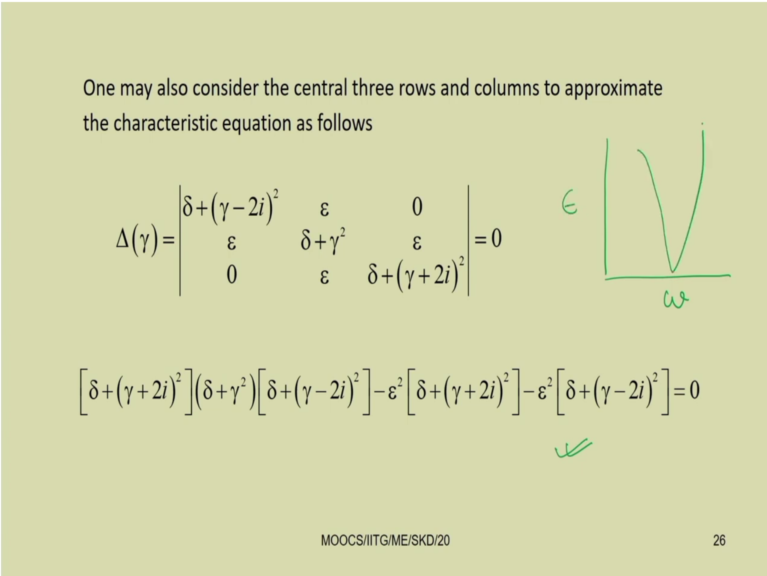

So, you can also further you can consider only the central three rows and three columns. So, from these things so, you can consider the central three rows so, three rows and three columns you can consider and from that thing also you can write these three equation that is delta plus gamma minus 2i square epsilon 0, epsilon delta plus gamma square epsilon. Then, 0 epsilon delta plus gamma plus 2i whole square and make it equal to 0.

So, from these thing finding these determinant. So, this is the determinant from this determinant, you can find the or you can plot this delta versus either delta versus epsilon or omega versus epsilon, where omega is root over delta. So, you can plot this omega versus epsilon to find the instability region.

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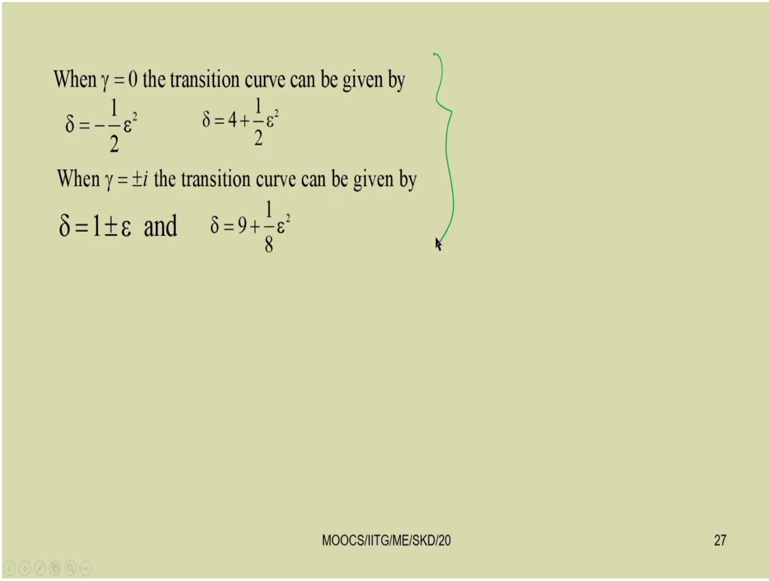

So, you just see when gamma equal to 0, then you can get the transition curve so, from these things. So, you got the condition that delta equal to minus half epsilon square or delta equal to 4 plus half epsilon square, when gamma equal to plus minus i transition curve, we can give it like this that is delta equal to 1 plus epsilon and delta equal to 9 plus 1 by 8 epsilon square.

So, you can plot this delta versus epsilon and can get the transition curves. So, this is as an assignment. So, you can plot these delta either delta versus epsilon or this omega versus epsilon curve to find the transition curve.

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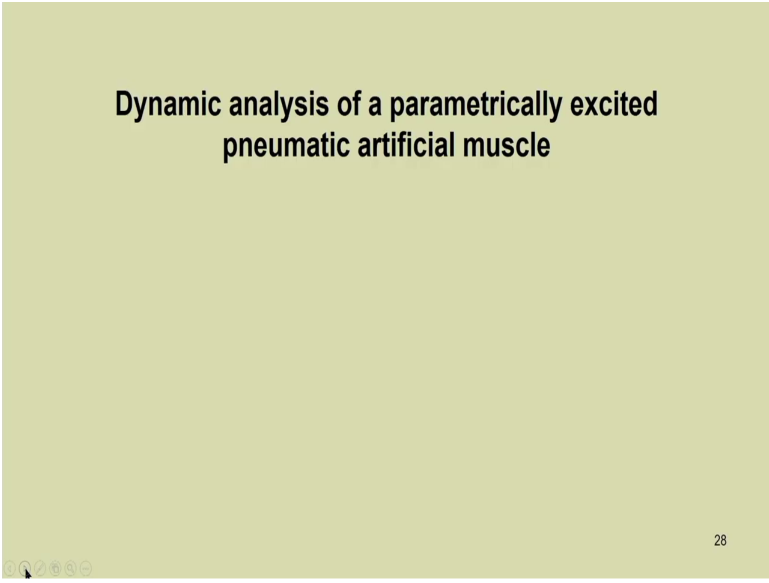

So, let us take one example for example, we are going to take a parametrically excited pneumatic artificial muscle. And, here we will study the response of the system which is similar to that of a parametrically excited one.

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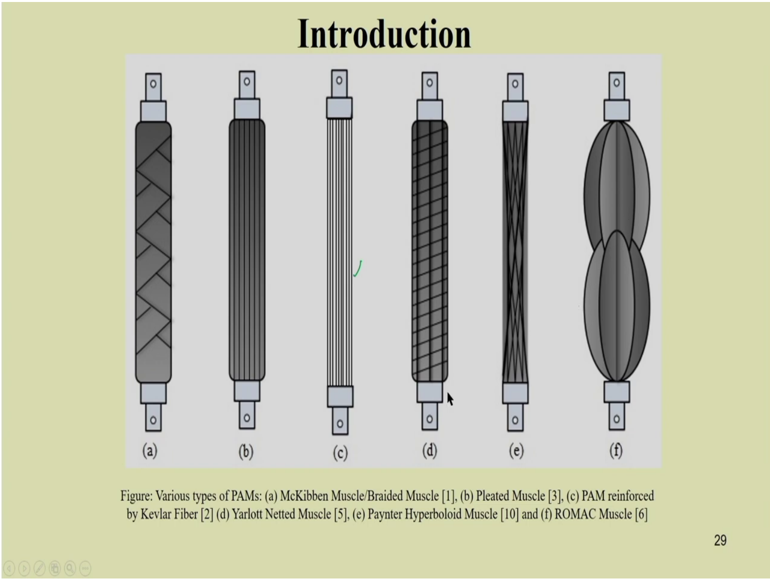

So, why this becomes parametrically excited also we will see. So, before that thing you must know what is artificial muscle? So, these are the different artificial muscles have been shown. So, various type of PAMs so, the first one so, this one is known as McKibben muscle or braided muscle.

So, then this is pleated muscle, then this one is PAM reinforced with Kevlar fiber. So, fiber is embedded in this thing. So, then you can have the Yarlott netted muscle, this one Yarlott netted muscle, then Paynter hyperboloid muscles there. Then, you can this is hyperboloid muscle you can see this hyperbolic thing, then we have this ROMAC muscle. So, there are several different pneumatic artificial muscles are there.

So, here you are inserting this air pressure and due to air pressure. So, as the length as the length is constraint so, it cannot go beyond a certain length so, then it will bulge. So, as it is bulging then there will be contraction in this muscle. So, due to this contraction. So, it can pull to do some work.

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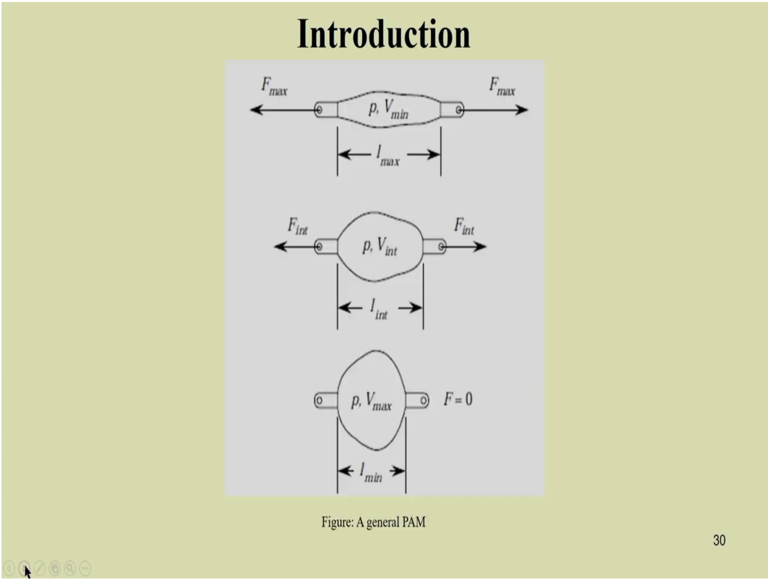

So, that way you can use it for different purpose. So, so this is the l max this is the l intermediate and this is the l minimum. So, this force equal to 0. So, you have a intermediate force here, then here is the maximum force you are applying. So, this is for a general PAM in this way you can do the study.

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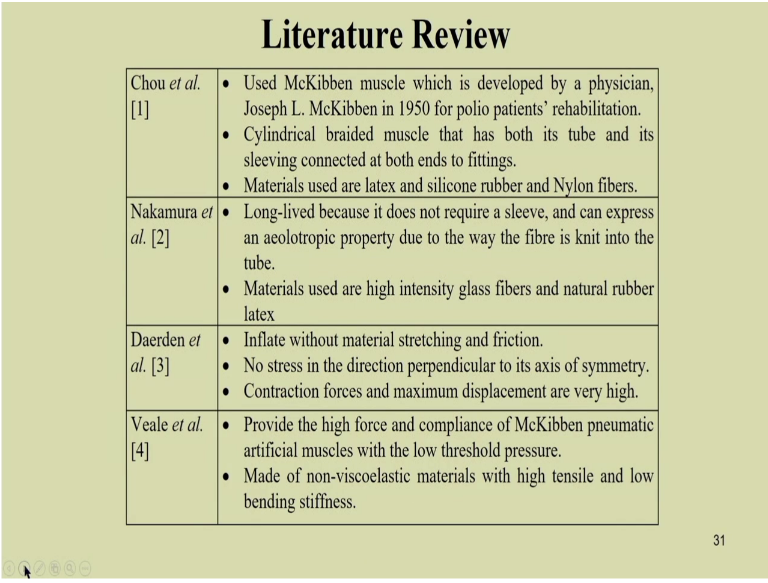

Literature Review

Chou *et al.* [1]	• Used McKibben muscle which is developed by a physician, Joseph L. McKibben in 1950 for polio patients' rehabilitation. • Cylindrical braided muscle that has both its tube and its sleeving connected at both ends to fittings. • Materials used are latex and silicone rubber and Nylon fibers.
Nakamura *et al.* [2]	• Long-lived because it does not require a sleeve, and can express an aeolotropic property due to the way the fibre is knit into the tube. • Materials used are high intensity glass fibers and natural rubber latex
Daerden *et al.* [3]	• Inflate without material stretching and friction. • No stress in the direction perpendicular to its axis of symmetry. • Contraction forces and maximum displacement are very high.
Veale *et al.* [4]	• Provide the high force and compliance of McKibben pneumatic artificial muscles with the low threshold pressure. • Made of non-viscoelastic materials with high tensile and low bending stiffness.

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So, there are several literature available on this pneumatic artificial muscle. So, some of the literature are given here so, for example, these Chou et al. So, use this McKibben muscle which is developed by a physician Joseph L McKibben in 1950 for polio patient rehabilitation purpose.

So, that is a cylindrical braided muscle that has both its tube and its sleeving connected at both ends to fittings. This material used are latex and silicone rubber and nylon fiber, then Nakamura et al so, long lived because it does not require a sleeve and can express an aeolotropic property due to the way the fiber is knit into the tube. Material used are high intensity glass fiber and natural rubber latex.

Then Daerden et al the inflate without material stretching and friction. So, no stress in the direction perpendicular to the axis of symmetry. Contraction forces and maximum displacement are very high. This Veale et al so, provide the high force and compliance of McKibben pneumatic artificial muscle with low threshold pressure so, made up of non viscoelastic material with high tensile and low bending stiffness.

So, there are many literature available at the end also a reference from that thing, you can know what are the paper available in this field.

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So, some of the papers have been written here this is the McKibben type of muscle. So, you can see the photograph of the McKibben type. So, this is the FESTO McKibben type of PAM, PAM is pneumatic artificial muscle or pneumatically actuated muscles also. So, you can put PAMs reinforced by straight glass fiber. So, here it is this glass fiber you can see so, there are several rings are also there.

So, this is a rubber tube. So, rubber tube with these glass fiber embedded in this, then you have the terminal from where you can use these pipes for this pneumatic supplying this air.

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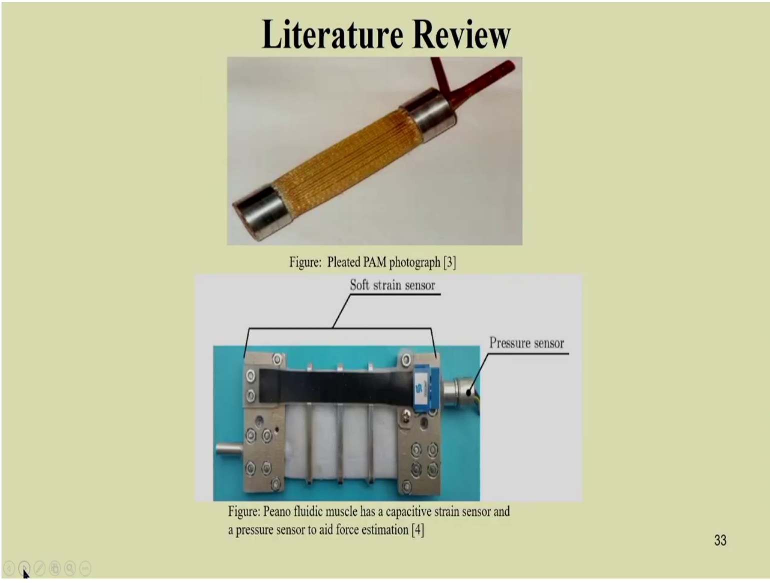

So, there are several other related PAM photographs also you can find. So, this is Peano muscle, Peano fluidic muscle has capacitive strain sensor, and a pressure sensor to aid force estimation. So, you can put a pressure sensor and you can put a force sensor also.

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So, there are several others muscles available. So, some of them have been listed here. For example, this is Yarlott muscle ROMAC muscle, Kukoji muscle, Kukoji muscle and Morin muscle, then Baldwin muscle, Paynter hyperboloid muscle. So, that several different muscles are available.

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And these mussels you can see they are particularly used for this exoskeleton type of thing so, here this figure of a nurse robot suit. So, she has put a robot suit. So, which is pneumatically actuated this is also power assist wear by D Sasaki. So, these references I will give at the end of the presentation or this lecture.

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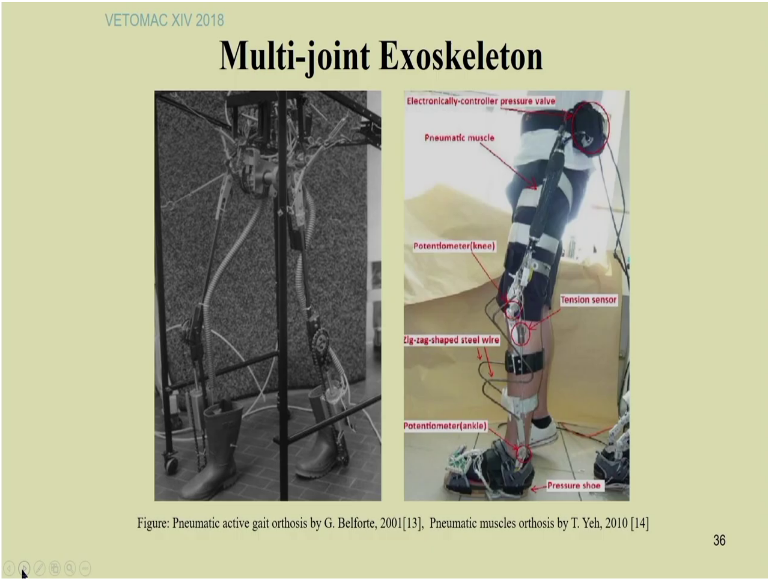

So, we have multi joint exoskeleton. So, these are the exoskeleton system where you can use this type of muscles.

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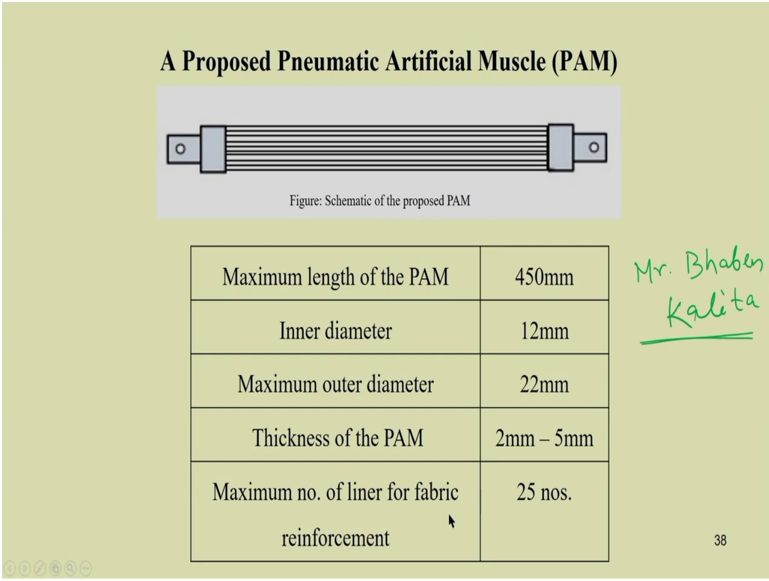

And it can be single joint multi joint here, we are proposing one artificial muscle actually this work is the PhD work of my PhD student Mr. Bhaben Kalita. So, here we have developed one artificial muscle by using this Muga silk and Pat silk the muscle what we are showing here. So, the maximum length of the PAM is 450 mm. So, inner diameter is 12 mm maximum outer diameter is 22 mm.

So, thickness of the PAM is 2 mm to 5 mm so, maximum number of linear for fabric reinforcement 25 fabrics have been put here.

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You can fabricate this thing by using the silicon rubber.

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And, then so this is the fabricated muscle. So, these are the fabricated so, this is the Muga silk or Pat silk. So, we have made one mold so, in this mold we have fabricated this muscle.

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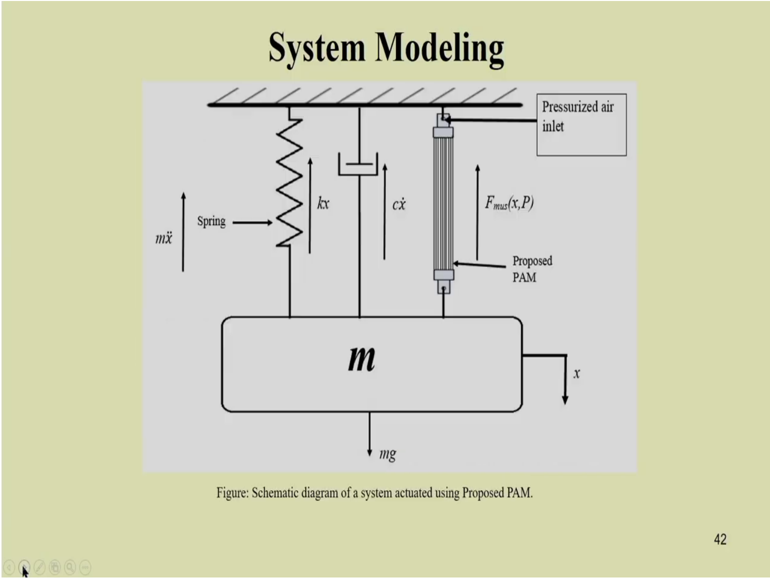

And, we have characterized these muscles also to find the property. So, here you can see we can use this mussel. So, for example, this is the mussel let us use this thing for the analysis. So, here we put a spring one side a spring and other side spring and damper and other side this muscle.

So, this is the pressurized air inlet and outlet. So, you will have inlet and outlet. So, this is the mass it is hanging. So, this is single degree of freedom system. So, if you remove this artificial muscle. So, this is a spring mass damper system only, but now by putting this artificial muscle here.

So, and putting this air so, we will see how the equation of motion will change. So, if you draw the free body diagram of this one. So, it will be subjected to mg force downward that is the weight, then we have a spring force kx plus delta, then we have the mx double dot that is the inertia force. And in addition to that we have these muscle force F. So, this muscle force is a function of the displacement x and pressure P.

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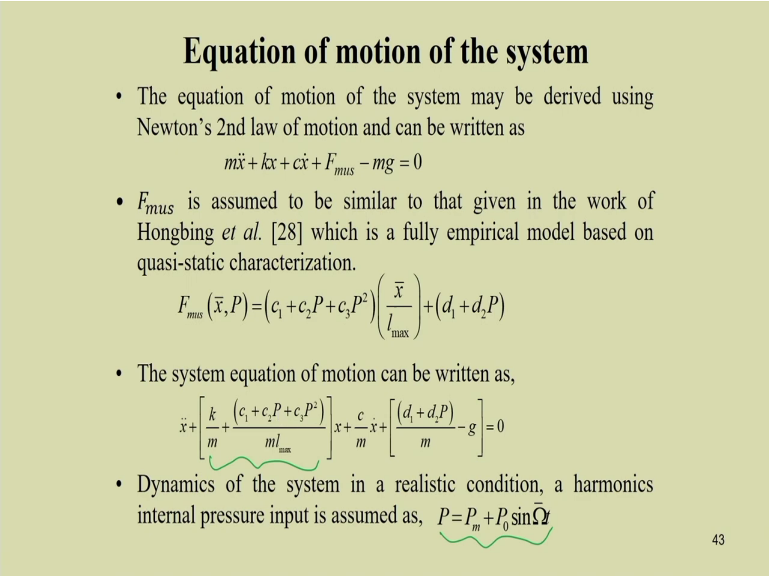

This way we can write now we can write the equation of motion of the system. So, you just see mx double dot plus kx plus cx dot plus F muscle minus mg equal to 0. So, we have to take this F muscle in this form. So, it is followed from the work of Hongbing et al. So, F muscle x bar P equal to c 1 plus c 2 P plus c 3 P square into x bar by l plus d 1 plus d 2 P. So, here a 5 parameter muscle model has been considered here.

So, the c 1, c 2, c 3, d 1, d 2 actually can be obtained from the experiment and performing direct experiments. So, you can get all these system parameter, then you can write this x F muscle equal to in this form, now substituting this f muscle in this equation. So, we can write this equation x double dot plus k by m plus c 1 plus c 2 P plus c 3 P square by m l max x into c by m x dot plus d 1 plus d 2 P by m minus g equal to 0.

Now, let us assume that the P that force that pressure what you are supplying is in this form that is P m plus P 0 cos omega bar t. So, that is periodically bearing pressure so, if it is periodically varying with time.

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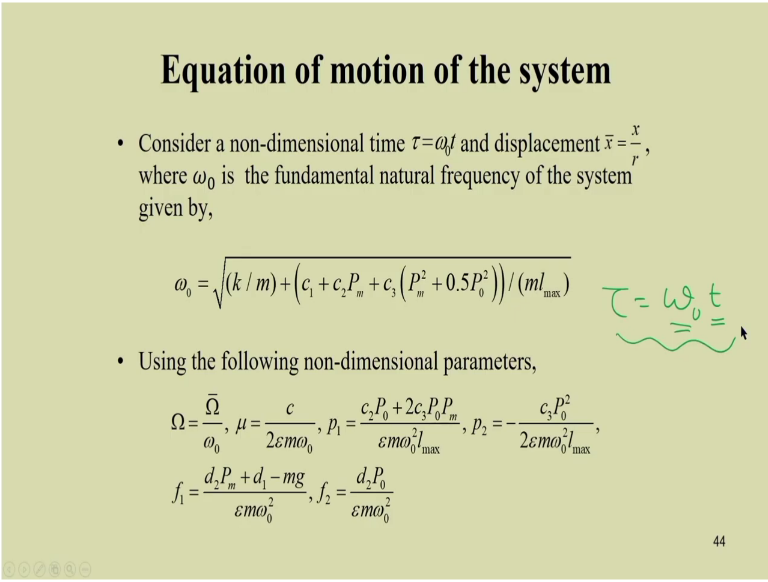

Then, we can write down this equation in this form. So, here you just see in this equation so, we can this is dividing by the this m. So, this part is the square of the natural frequency omega n square. So, this part will so if we will write this equation in this form for example, x double dot plus omega n square x plus 2 zeta omega n x dot plus some term equal to 0.

So, in that case this omega n square will be this and you just see out of these thing this P, P is a constant term and P also contain a periodic terms as P equal to P m plus P 0 sin omega t. So, here so by taking this constant term we will have 1 constant coefficient of x. So, that will contribute for the natural frequency.

And another part which will be a periodic function so, that will be responsible for the parametric excitation of the system. So, by using the parameter omega equal to omega bar by omega 0 mu equal to c by 2 epsilon m omega 0 P 1 equal to c 2 P 0 plus 2 c 3 P 0 P m by epsilon m omega 0 square l max and P 2 equal to minus c 3 P 0 square by 2 epsilon m omega 0 square l max. And f 1 equal to so, d 2 P m plus d 1 minus m g by epsilon m omega 0 square and f 2 equal to d 2 P 0 by epsilon m omega 0 square.

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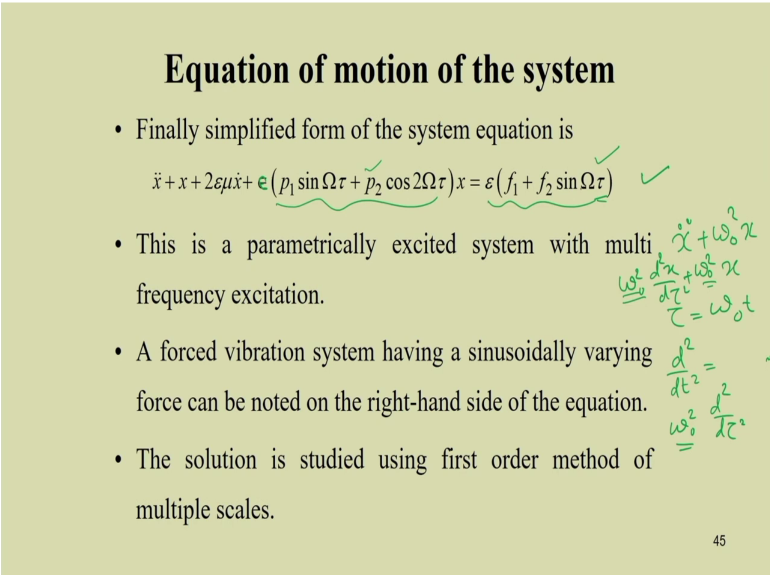

So, we can write down this equation in this form. So, you just see now this equation complicated equation is reduced to a simplified form, which equal to x double dot plus x plus 2 epsilon omega x dot plus epsilon into so, this is epsilon. So, plus epsilon into p 1 sin omega t plus p 2 cos 2 omega t or tau. So, here the time is written in terms of tau is the non dimensional time.

So, you can observe this non dimensional time is written in this form that is you can take this tau equal to omega 0 t tau equal to omega 0 t. So, t is the time in second so, omega 0. So, it is radiant for second so, second cancel. So, this tau becomes non dimensional. So, you can take a non dimensional time tau equal to omega 0 t taking that non dimensional time.

So, we can write this equation that is why so, x double dot plus omega n square x. So, you just see you have a term, let me take only these two term x double dot plus omega 0 square x. So, here we are taking tau equal to omega 0 t. So, the differentiation d square by dt square will be nothing, but omega 0 square d square by d tau square.

So, it will be omega 0 square let me write it again. So, it will be equal to omega 0 square into d square by d tau square. So, you have a omega 0 square term in the beginning, then plus omega. So, this thing will be then written as omega 0 square d square x by d tau square plus omega 0 square x omega 0 square x. Now, you can take this omega 0 square common and divide it everywhere. So, it is divided everywhere that is why in all the coefficient. So, you can see a term of omega 0 square in the denominator.

So, from that way so you can find this equation so, here you just note this equation that is x double dot plus x plus 2 epsilon mu x dot plus epsilon p 1 sin omega t plus p 2 cos 2 omega t into x the coefficient of x is time varying term that is similar to p t. So, we have seen so, this can be reduced to that of a Mathieu Hill type of equation.

But in the right hand side so, you just see the difference between the Mathieu equation or Hill's equation what we have studied and in the right hand side it was 0. But, in this present case in the right hand side we have a term epsilon f 1 plus f 2 sin omega t. So, this is direct excitation term and this is parametrically excited system. So, the system is both parametrically are subjected to both parametric and direct excitation and you just see the frequency also.

So, this is here omega sin omega t here the term is cos 2 omega t and here we have a term with sin omega t. So, we have multi frequency excitation. So, here the excitation one is omega and another one is 2 omega so, we have two frequency excitation in this case. A force vibration system having sinusoidally varying force can be noted in the right hand side also and the solution is studied by so, let us use this first order method of multiple scale to study this equation.

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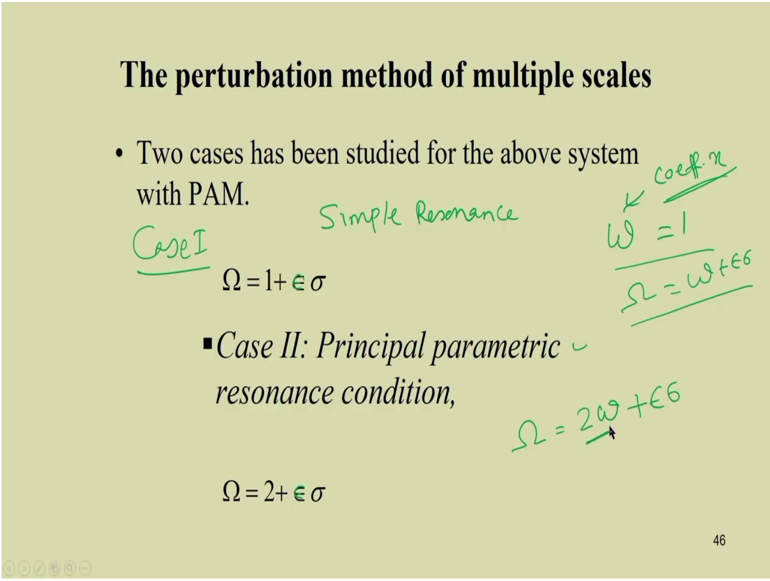

Here, you just see as it is forced and parametric both the things are there so, we can study two different case. So, first case is the simple resonance condition. So, we can have a simple a resonance condition and we can have parametric resonance condition. So, we have case I and case II so in case I. So, we have simple resonance condition. So, when omega equal to 1 plus epsilon sigma 1 plus epsilon sigma.

So, here you just see omega 0 equal to 1 that is why it is written 1 plus epsilon sigma, otherwise you can put this omega equal to omega 0 plus epsilon sigma also. So, in this particular case omega 0 equal to 1 so, that is why it is written that the coefficient this omega 0 so, that is the coefficient of x that is not the omega 0 term what we have written there.

So, this is the coefficient of x; coefficient of x term so, that is 1 in the non dimensional form this is the non dimensional natural frequency that is equal to 1. So, that is why we can write this omega equal to 1 plus epsilon sigma and in the second case, we can write the principal parametric resonance condition. So, principal parametric resonance conditions occur, when it is equal to twice the natural frequency.

Let me write it omega to avoid your confusion so, twice 2 omega plus epsilon sigma. So, here it is twice the natural frequency and by using twice the natural frequency, we can write this omega equal to 2 plus epsilon sigma.

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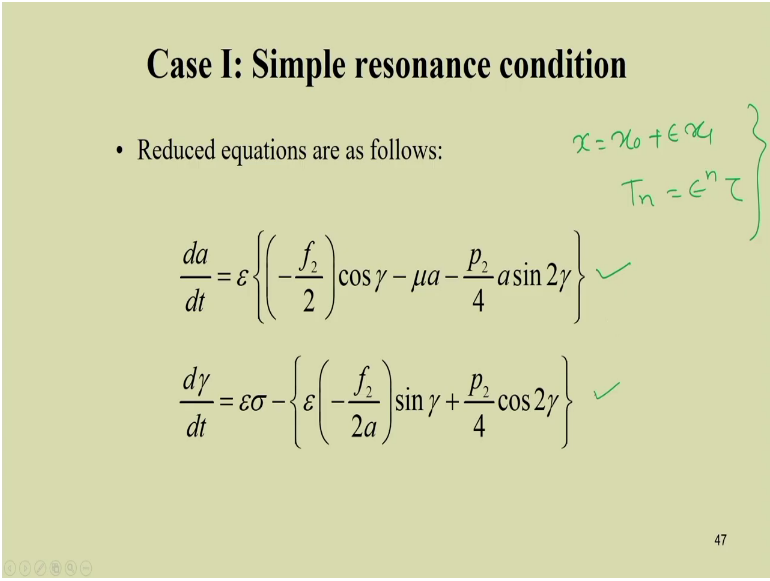

So, now, we can get the reduced equation by using this method of multiple scale. So, you can follow the standard procedure of method of multiple scale by assuming, these u equal to or u r x we have written you just see this equation is written in terms of x. So, you can assume the solution to be so, x equal to x 0 plus epsilon x 1 and using different time scale tau n equal to epsilon to the power n t.

So, here we are writing in terms of tau epsilon to the power n tau and following the procedure similar procedure. So, we can get two reduced equation so, these are the reduced equation we got so, da by dt equal to this and d gamma by dt equal to this so, from this thing. So, we can find the solution that is a and gamma ok.

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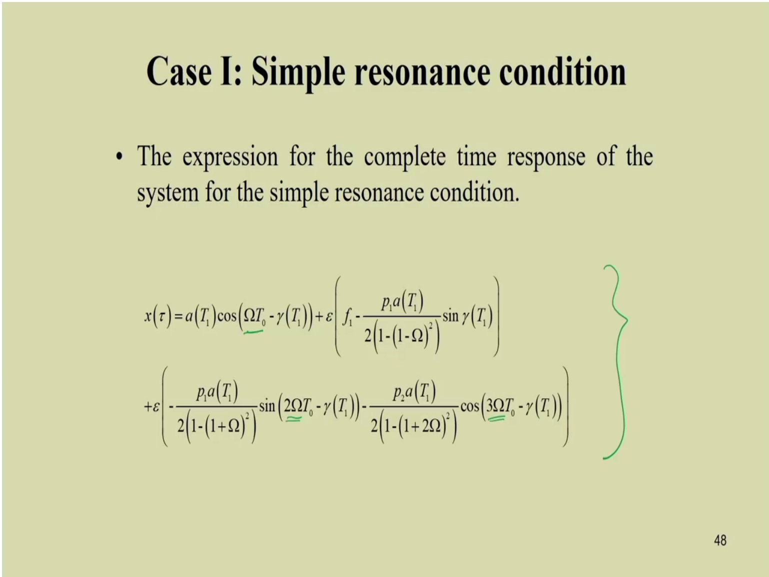

In this case this x tau can be written as a T 1 cos omega T 0 minus gamma T 1 plus epsilon f 1 minus p 1 a T by 2 into 1 minus 1 minus omega whole square sin gamma T 1 plus epsilon into p 1 a T 1 by 2 into 1 minus 1 plus omega whole square sin 2 omega T 0 minus gamma T 1 minus p 1 p 2 a T 1 by 2 into 1 minus 1 plus 2 omega 0 square cos 3 omega T 0 gamma minus 1.

So, you just see so, it contain the frequency for example, omega T 0 here we have 2 omega T 0. And another term that is 3 omega T 0 the solution contain 3 frequency here. So, this way you can find the response of the system and by plotting, this response you can check the response of a pneumatically artificial muscle, this artificial muscle is a function of this muscle parameter.

So, muscle parameter are written in terms of this f 1, f 2, f 3 muscle parameter that is c 1, c 2, c 3, d 1, d 2, and also the pressure by changing this pressure. So, actively you can control at actively you can control the motion of the (Refer Time: 55:29) activator.

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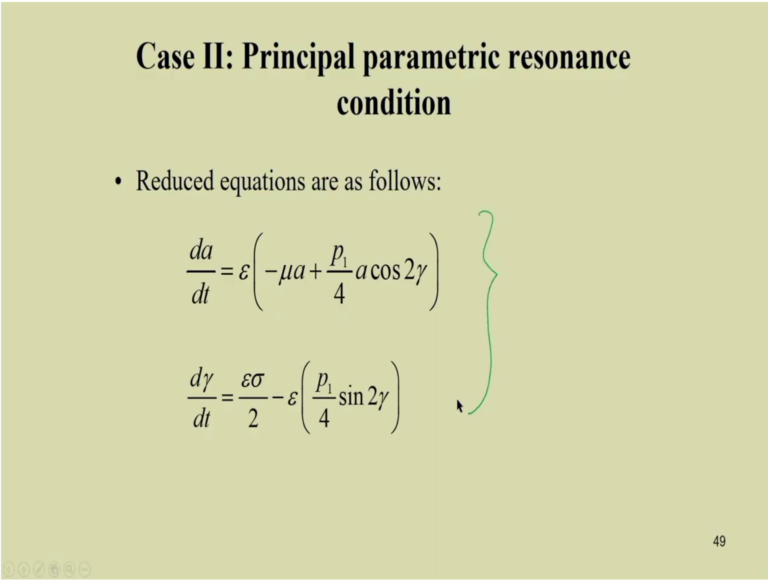

So, this thing can be used as a actuator and you can control the a motion by actively changing this pressure. But, if you do not want, but if you want to change the muscle itself also you can control by changing this muscle itself by changing the c 1, c 2, c 3 and d 1, d 4; that means, by changing this muscle.

By changing this muscle you mean that the length of the muscle can be changed, the thickness of the muscle can be changed. The number of strings you are putting inside this muscle, or the number of strains you are putting inside the muscle has to be changed. So, in that way by changing this muscle parameter; that means, by constructing another muscle so, you can have different type of response of the system.

So, in case of principal parametric resonance conditions also so, by using this method of multiple scale you can have these reduced equation.

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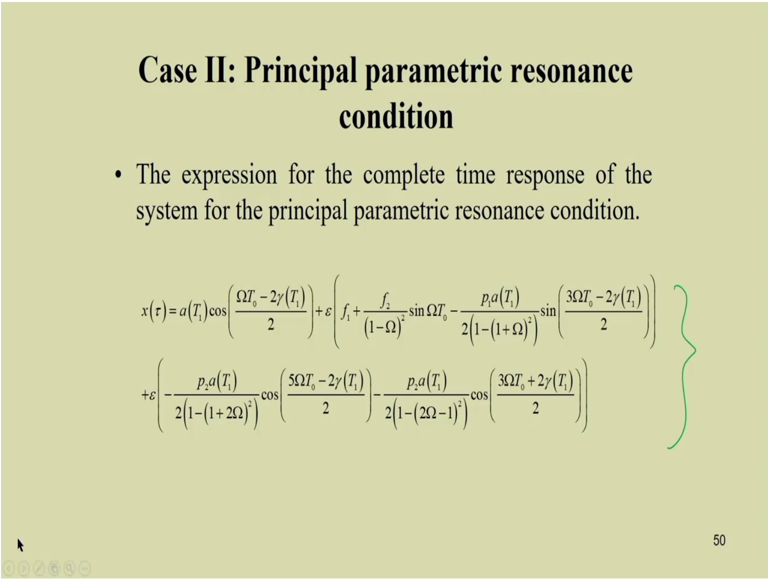

And from these reduced equation so, you can find the solution in this form. So, this is the solution in case of the principle parametric resonance condition. And, in case of combination parametric resonance condition so, you have already seen the response the response will be different.

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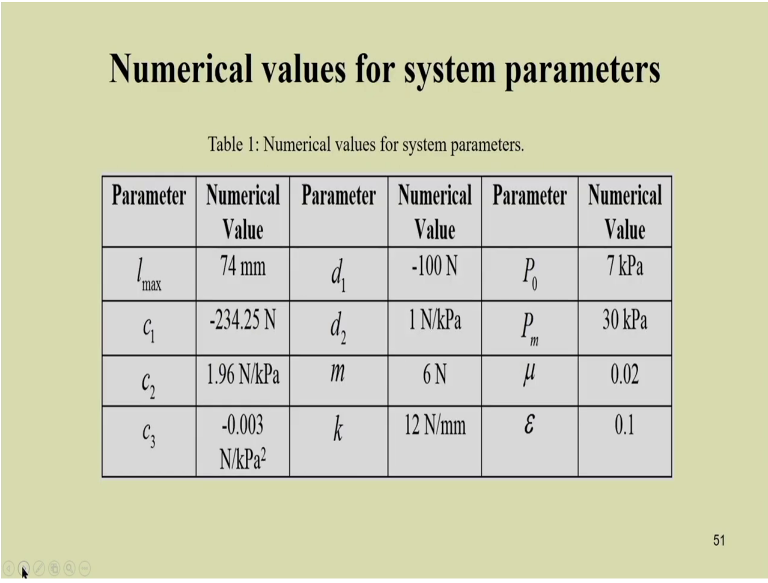

Parameter	Numerical Value	Parameter	Numerical Value	Parameter	Numerical Value
l_{max}	74 mm	d_1	-100 N	P_0	7 kPa
c_1	-234.25 N	d_2	1 N/kPa	P_m	30 kPa
c_2	1.96 N/kPa	m	6 N	μ	0.02
c_3	-0.003 N/kPa^2	k	12 N/mm	ε	0.1

And, you can see these so, for example, these are the numerical value we have taken for our analysis.

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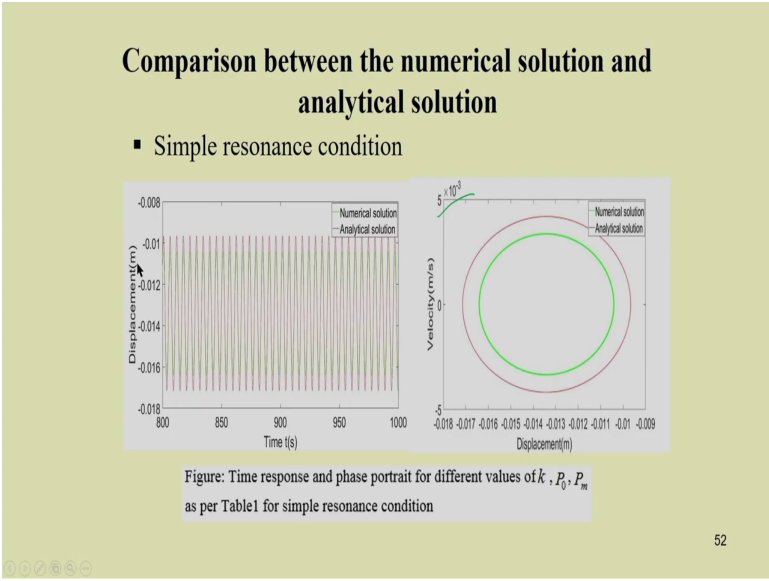

Figure: Time response and phase portrait for different values of k, P_0, P_m as per Table1 for simple resonance condition

And, we have plotted these simple resonance condition. So, numerical and analytical solutions you can plot so; that means, numerical solution that is by using ODE for 5 the original equation you can solve. And then by using method of multiple scale you can get. So, you just see you can note the response is 5 into 10 to the power minus 3.

So, very small so; that means, though it is exaggerated in actual physical sense the difference is not much. So, the numerical solution and the these solution obtain by this method of multiple scale are very very close. So, you can see here so, this is in meter it is written. So, if you convert in mm so, they are in the decimal differences in decimal term.

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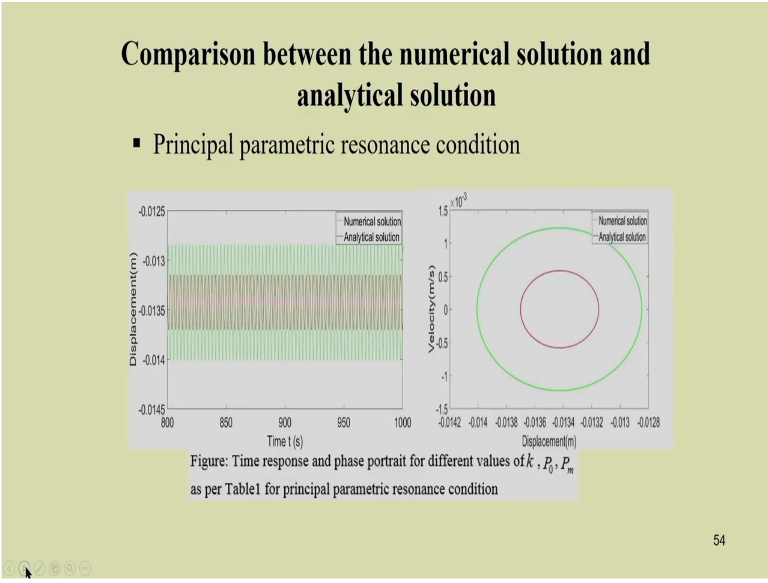

Similarly, in case of principle parametric resonance condition.

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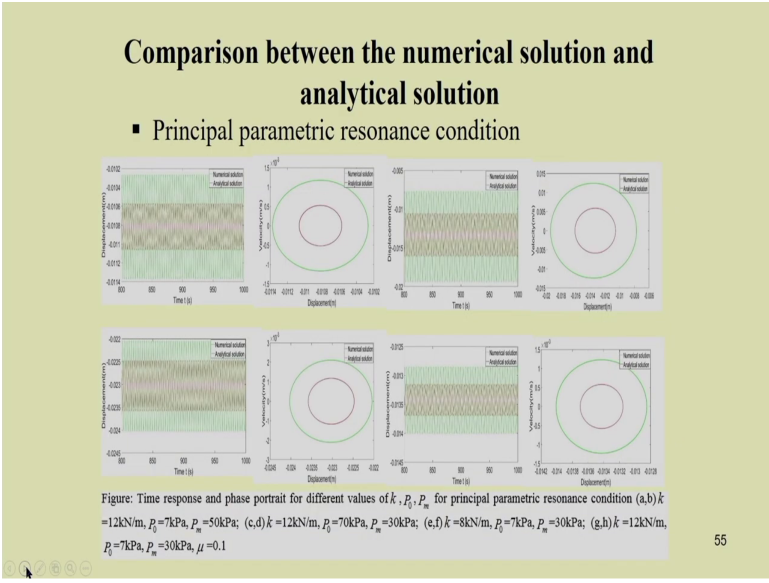

So, you can study the responses so, for different system parameters.

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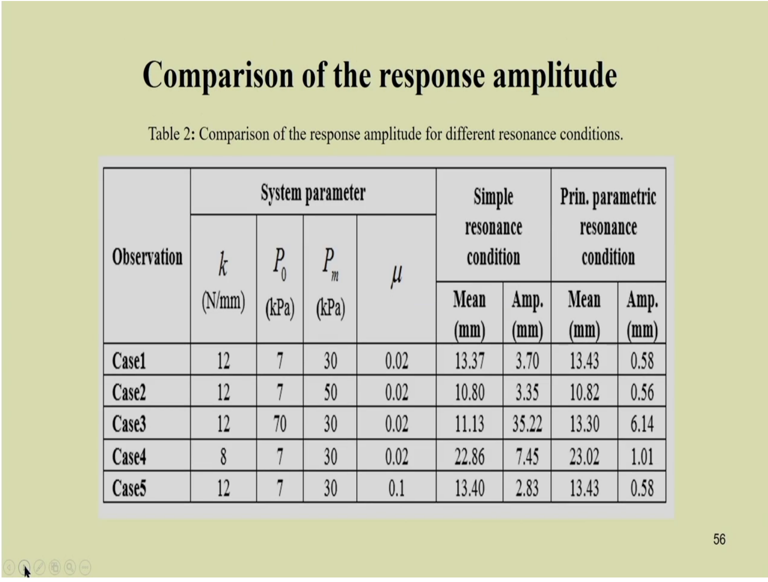

Comparison of the response amplitude

Table 2: Comparison of the response amplitude for different resonance conditions.

Observation	System parameter				Simple resonance condition		Prin. parametric resonance condition	
	k (N/mm)	P_0 (kPa)	P_m (kPa)	μ	Mean (mm)	Amp. (mm)	Mean (mm)	Amp. (mm)
Case1	12	7	30	0.02	13.37	3.70	13.43	0.58
Case2	12	7	50	0.02	10.80	3.35	10.82	0.56
Case3	12	70	30	0.02	11.13	35.22	13.30	6.14
Case4	8	7	30	0.02	22.86	7.45	23.02	1.01
Case5	12	7	30	0.1	13.40	2.83	13.43	0.58

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So, you can compare the simple resonance and parametric resonance condition.

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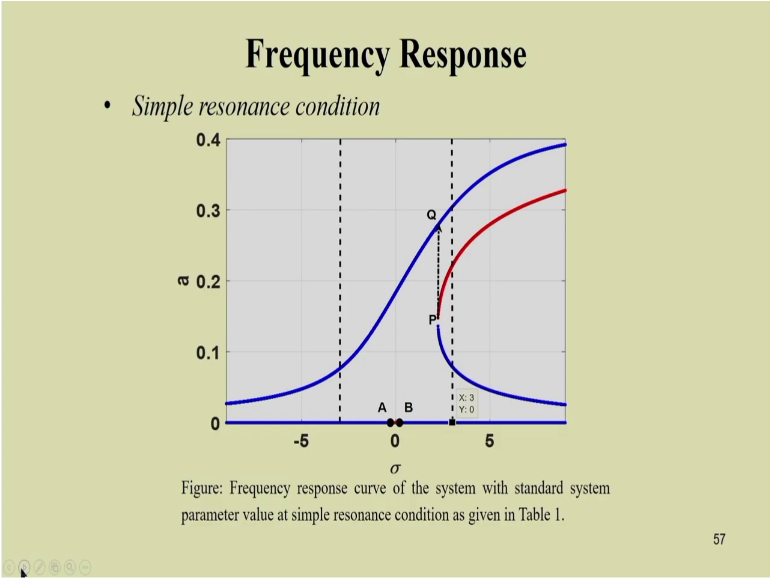

And in case of simple resonance so, this is the response curve what you can get. So, here at P point you away. So, you just see this P point you have a; you have a saddle node bifurcation, because the transient at the transient you just see the transient is same and here. So, this is a saddle node bifurcation from a stable branch it is going to an unstable branch.

So, if you lower the frequency. So, that sigma is the detuning parameter. So, you can absorb that from this point P so, it will jump two point Q. So, you will have a jump of phenomena if you reduce the pressure. So, when you are reducing the frequency of the system that ends by changing your pressure also we can change the frequency. So, that time you can see there is a jump of phenomena, but initially by increasing the frequency. So, you can go on increasing the response amplitude.

So, depending on the required amplitude so, you can set your frequency and you can get the required length of the or required deflection of the artificial muscle. So, you just see as you have a number of solution for example, let us take this. So, you have three stable state so, this you just see the 0 line is also a solution and it is stable also. So, this 0 line as it is a solution. So, you have three stable solution so, this is a tristable system and with on one unstable response.

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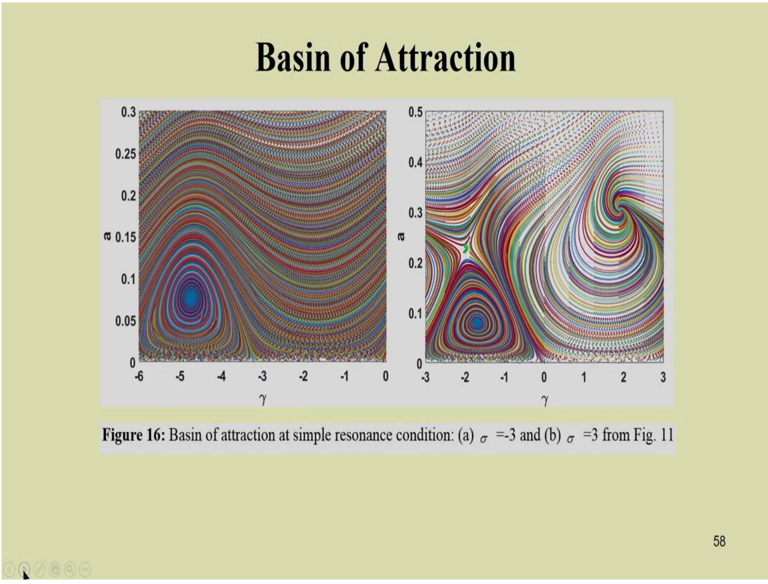

Figure 16: Basin of attraction at simple resonance condition: (a) σ =-3 and (b) σ =3 from Fig. 11

So, three stable and one unstable and beforehand before so, for example, at this position so for sigma less than this A value. So, you have only one solution. So, if you draw the basin of attraction. So, you can see you have one solution here. And in the other case you have four solution so, here 1 2 3 and this point is the unstable point. So, this is the unstable point so or saddle point.

So, in addition to this you have. So, this is the trivial state you got. So, similar to this here is the trivial state and another two nontrivial state. So, this way by studying the basin of attraction so, you can check the feasibility of the solution or for what initial conditions; you are getting what response you can study.

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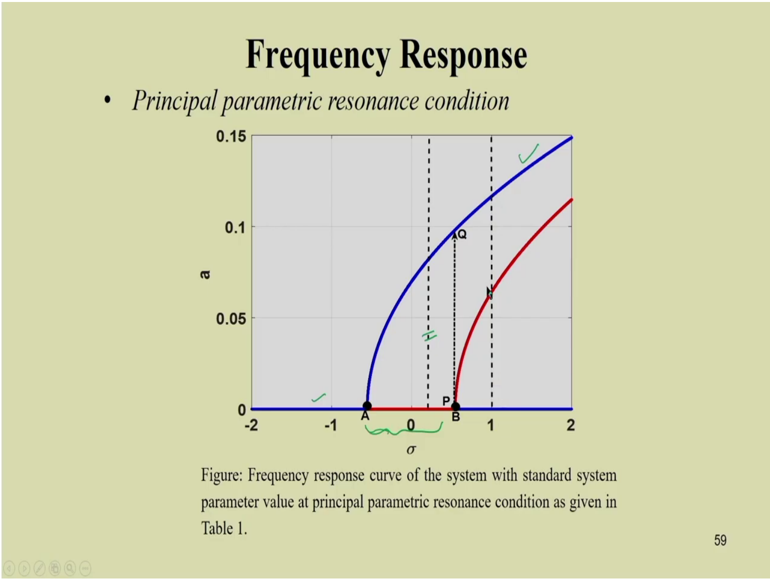

In case of the principle parametric resonance condition so, you just see the bifurcation is different. So, here you have a so here you have pitch for type of bifurcation. So, here the of 2 minus 2 2 A so, this trivial state is stable. So, only you have a trivial state; that means, before A.

So, whatever may be the force or the frequency the system is stable; that means, it will be in its trivial state that will be there will be no inflation. So, there will be no expansion of the artificial muscle. So, only the artificial muscle will work in this range of A to B, the artificial muscle will work in the range of A to B from O to A or 0 to A, it will not work, it will work only from A to B and after B also it may work, but after B it as two state.

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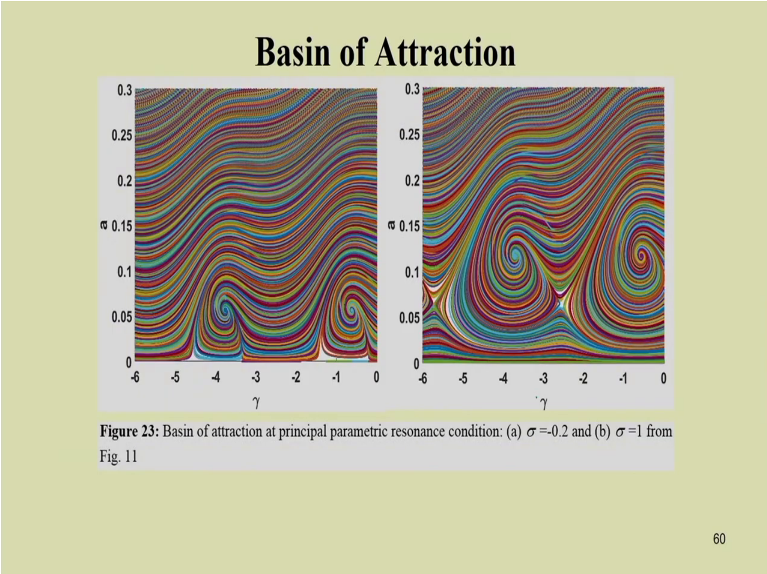

Figure 23: Basin of attraction at principal parametric resonance condition: (a) σ =-0.2 and (b) σ =1 from Fig. 11

So, one trivial state and one non trivial state so, that thing clearly you can see from the basin of attraction. So, at two points the basin of attraction have been plotted for example, at this point so basin of attraction is plotted here one on is here. So, two point basin of attraction have been plotted. So, in one case you have the saddle point so, you can observe the saddle point. So, this is unstable and one stable.

So, here two stable by stable and one unstable region is there. So, based on that thing so, you just see these are the unstable point. So, this is the saddle point so, two stable point. So, that way you can study from the basin of attraction so, in this way. So, you have observed that in case of a artificial muscle.

So, you can have both the simple resonance condition and parametric resonance condition. And by using this method of multiple scale, we have seen how the response will behave I have shown you the basin of attraction. So, this basin of attraction you can plot by taking different initial conditions. So, later I will tell how to plot these basin of attractions also.

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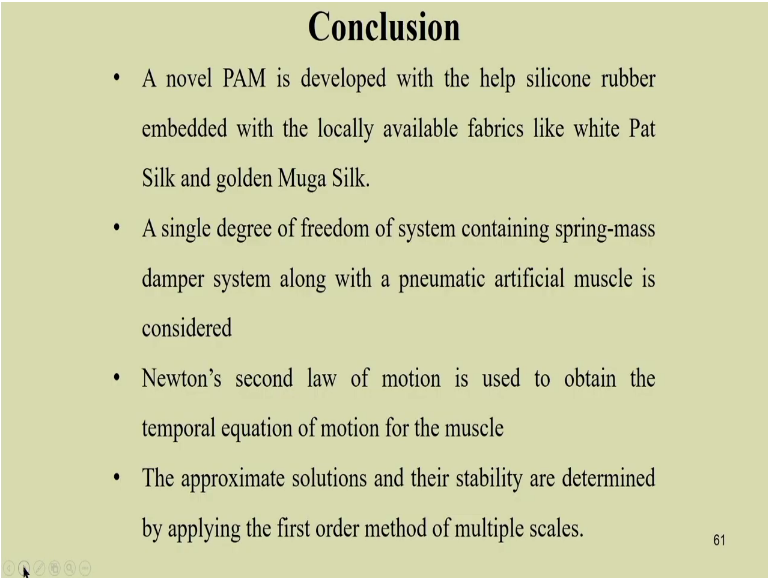

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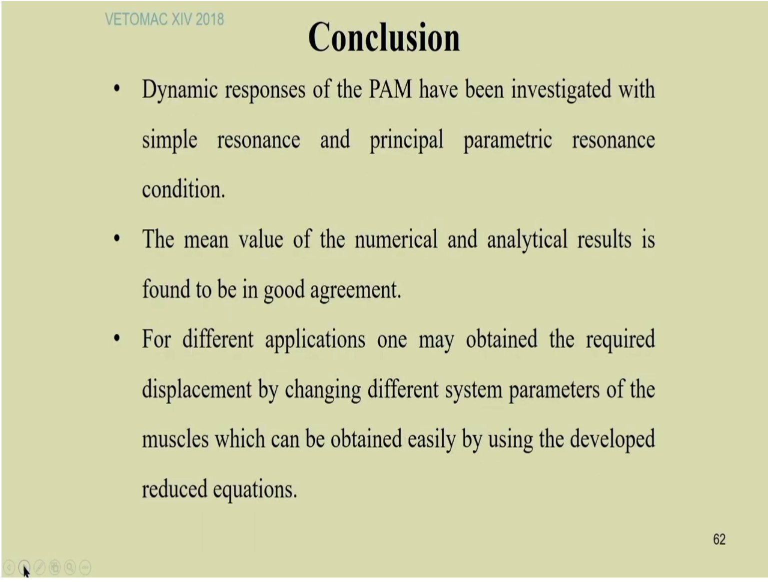

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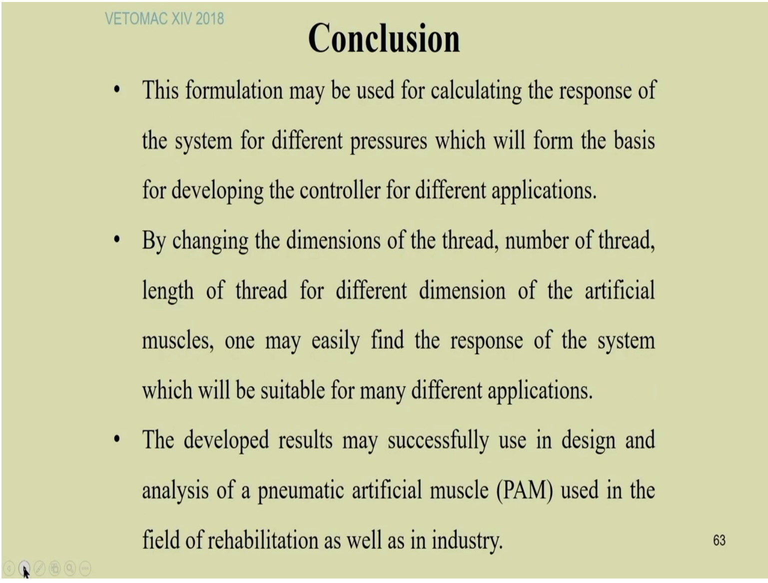

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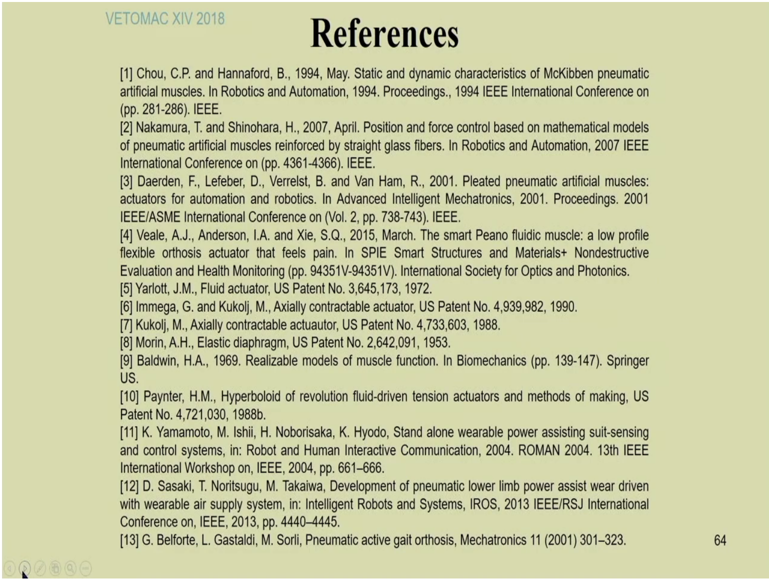

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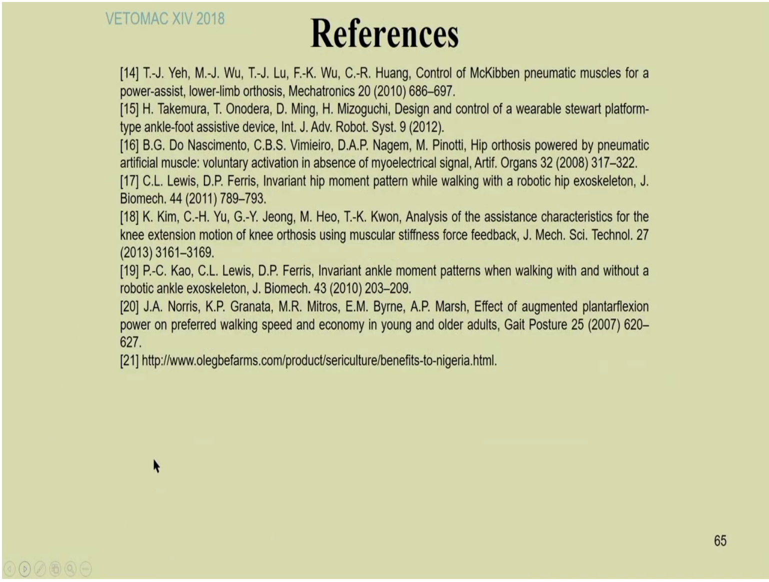

So, the there are many references what I have shown you. So, these are the references actually a part of this work was presented in this VETOMAC conference in 2018. So, these are the references, you can study these reference some of these references for artificial muscle.

So, next class we will study another example so, where we will take a continuous system. So, here we have taken a single degree of freedom system, but later we will take a continuous system in that continuous system. So, we will study how by using different methods. So, we can get this resonance conditions, the parametric instability region and we can study also how it can find many applications for example, one such application I will tell that is used in this energy harvester purpose.

Thank you very much.