Nonlinear Vibration
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Lecture - 19
Floquet theory, Hill's infinite determinant, Resonance in para

Welcome to today class of Non-Linear Vibration. So, today, we will start the module 6, and in this module we are going to study regarding the parametrically excited systems.

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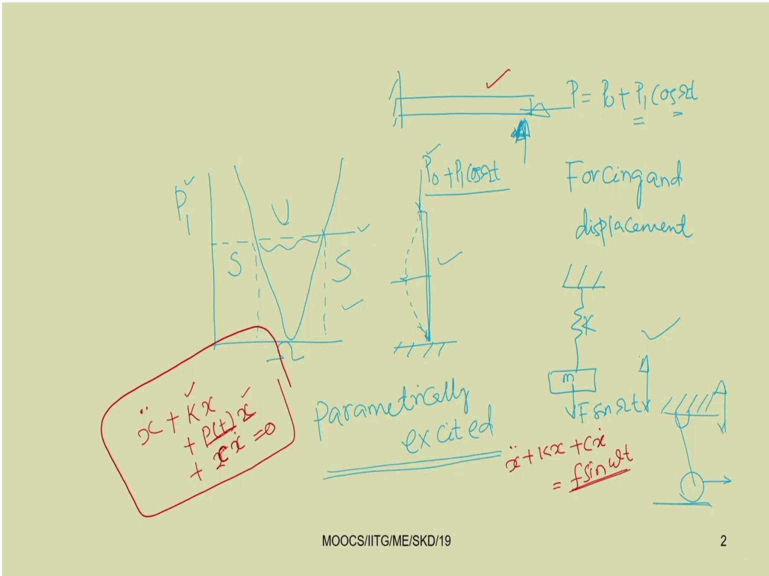

So, already you are familiar with the system. So, where we have taken, so for example, you just take this cantilever beam which is subjected to an excitation force here. So, if it is subjected to an excitation force. So, in this case, the force the forcing and so you can note the forcing. So, you are applying this force here. So, the forcing and the displacement take in the same direction, forcing and displacement, forcing and the displacement takes in the same direction.

Second example, you just take of the spring mass temper system also or spring mass system. Here also if you are applying a force in this direction that is F sin omega t, so here the displacement is also taking place in the same direction. But let us take another example where we have a column in this column. So, let us apply a force in this vertical direction. So, let us apply this buckling force that is instead of a constant force P let us apply force P 0 plus P 1 cos omega t So, let us apply a periodic force.

You can see that, so in this column, so already you know regarding the Euler buckling load. So, in static case, so if you are applying a force constant force then when it exceeds the Euler buckling load, so then it will start to buckle. Cantilever beam will start to buckle, but at these end, at these end there will be 0 slope, ok. So, there will be 0 slope up to certain range you have seen.

So, in case of the Euler buckling load, when we are applying a constant load to the column, so it starts to buckle. So, in this case the displacement takes in a direction, so perpendicular to the direction of application of force. So, here you are applying force in one direction and the displacement is taking place in the other direction.

So, let us take in this cantilever beam also, so if we are applying a load, for example, in these horizontal direction that is P equal to P 0 plus P 1 cos omega t, you can see that if you are applying a force in this horizontal direction, so it will start to buckle or start to bend after for certain value of P 1 and omega which is very very less than that of its critical load.

Similarly, in this case also much below this critical Euler buckling load the beams starts to buckle, so depending on the value of P 0, P 1 and omega. For depending on the value of this P 1 that is the amplitude of the periodic part and omega is the frequency of the periodic part, so depending on this P 1 and omega we can observe that the beam starts to buckle at very very less value in comparison to the critical Euler buckling load. If you can plot these, sometimes you may plot this P 1 for a particular value of a P 0.

So, if you can plot this P 1 versus these omega, so you can see you can see or you can get certain curve like this, where it you can tell beam will buckle. For example, let us take the value of P 1 this is the value of P 1. So, in this case you can see, so this is stable, so this is unstable and this is stable again. So, for these value of omega the system remain stable, so there will be no buckling of the system.

Similarly, for vale greater than this, so there will be no buckling and the system will be stable, but in this range the system is unstable and in this range column starts to buckle. So, we have to find different value of a P 1 and omega for which it will start to buckle. So, this type of excitation when we are applying a force in one direction and displacement is taking in a perpendicular direction are known a parametrically excited system. So, these are known as parametrically excited system.

So, in the previous case, so you have given the direct excitation. So, direct excitation the forcing under displacements are taking place in the same direction. So, this is the direct loading. This is also in case of cantilever beam.

When you have applied a force in transverse direction and the displacement is taking place in transverse direction, so that is you are giving a direct loading. But if you are giving a loading in one direction and displacement is taking in a perpendicular direction, so that time, so the system is known as parametrically excited system.

So, already you have derived the equation of similar system, where for example, you might have you can recall the system what you have taken, so that of a pendulum; let this is a pendulum where the platform is moving, where the platform is moving up and down direction. So, if the platform is moving, so in this case, so you can get a equation of that of a parametrically excited system.

Here the force is applied in this vertical direction and the displacement is taking, so in this horizontal direction. This is a case of a parametrically excited system.

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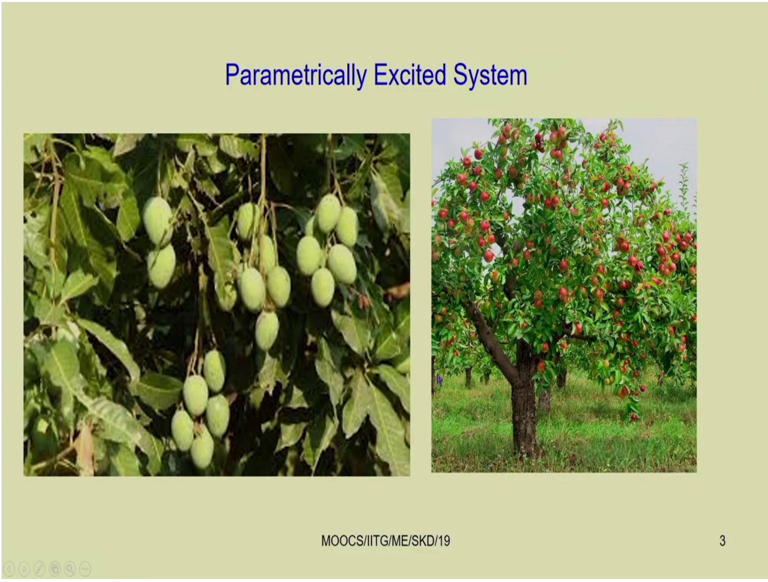

So, you can actually apply this parametrically excited system in many applications. For example, for plucking of these fruits from the tree. So, you can you can shake the tree or you can shake the branch of the tree in one direction, but the fruits can be coming down due to the vibration in a direction perpendicular to that of the direction in which you are applying this force.

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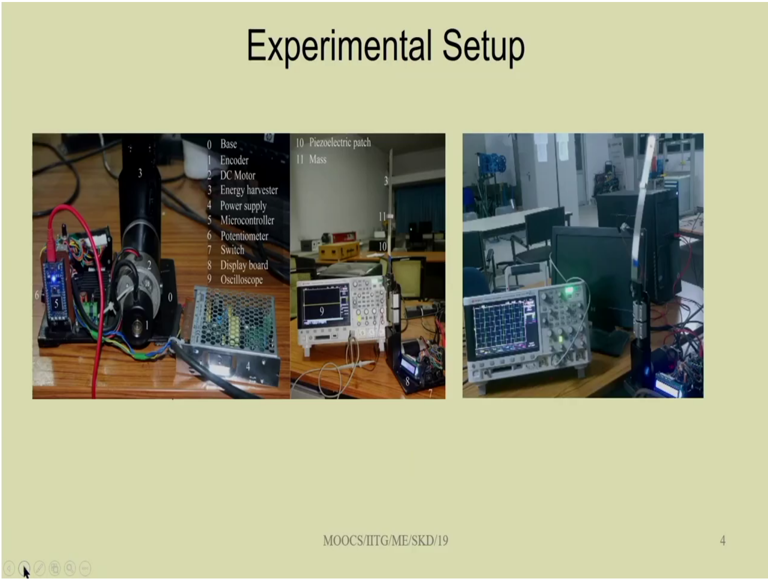

So, there many such applications are there. You can take a base exited. Say for example, this is a base excited cantilever beam. So, this is a beam, cantilever beam and this is a shaker. So, the shaker is developed, so we have developed this shaker in house. So, this is the oscilloscope.

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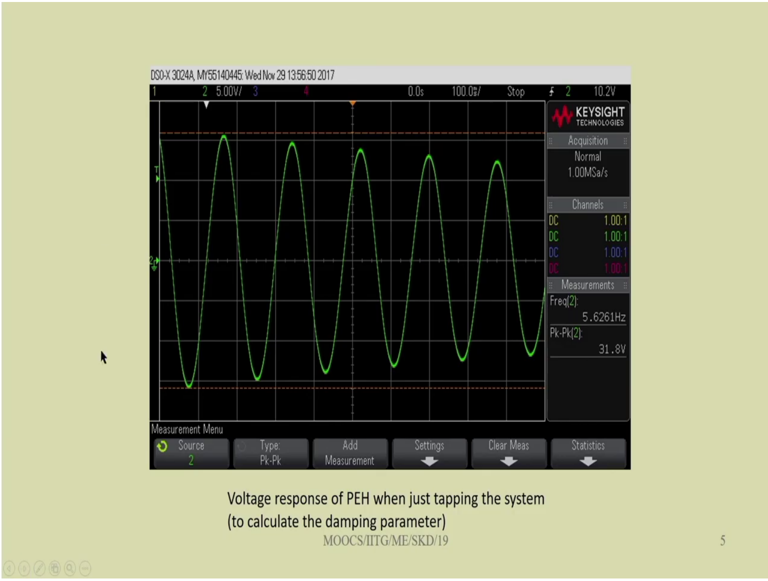

So, you can see, we have we did some experiment and the response is periodic or so, we have put some piezoelectric patch also here. So, you can see some piezoelectric patch are put on the beam. And so, when this beam is moving up and down or when the shaker is given a force in upward direction, so it moves in the transverse direction.

So, due to this transverse direction moment of the beam, so the piezoelectric patches gets strained and due to that thing this voltage is generated. So, if you take this voltage then you can see the voltage from the oscilloscope. This type of oscillations are known as parametrically oscillation in a parametrically excited systems.

So, in this type of systems you can see in the previous case here the equation of motion can be written in this for that is x double dot plus k x plus c x dot equal to f sin omega t. So, this forcing part you can see represent in the right hand side of this equation. And in case of the parametrically excited system, the equation can be written in this form this is x double dot plus k x plus, so you can write this is equal to p t into x. So, plus if you want to add a dumping term.

So, then you can put for example, 2 or you can write this c, you can write equal to c x dot c x dot, so that will be equal to 0. So, in this case it will be equal to 0. The equation, so here you can note or you can see that the response term x, the coefficient of the response term here k is constant. But this part, so in this case this is a time varying term. So, this p t is a time varying term.

So, if the coefficient of this x is a parameter and that is why this is known as a parametrically excited system. So, here p t which is a parameter and is written as the coefficient of the response of the system and that is why this type of systems are known as parametrically excited system.

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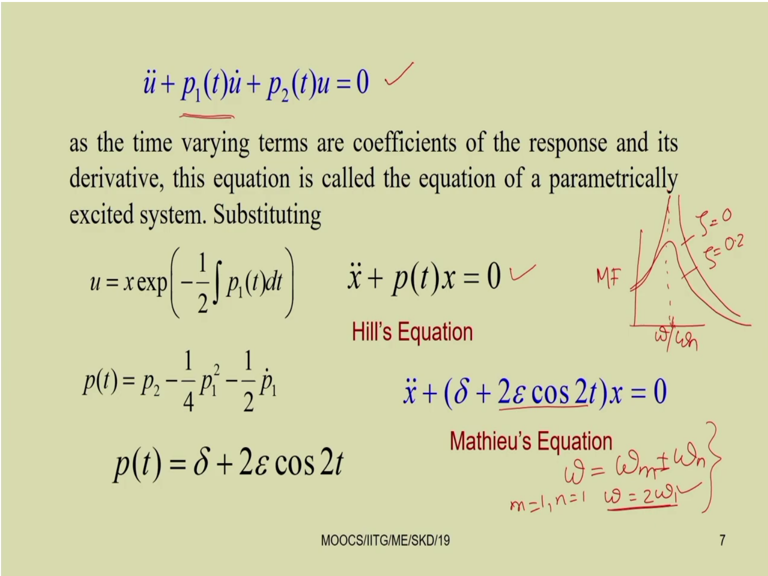

In this parametrically excited system, generally the equation can be written in this form that is u double dot plus p 1 t u double dot plus p 2 t u equal to 0, where this p 1 and p 2 are time varying term. As the time varying terms are coefficient of the response and its derivative, this equation is called the equation of a parametrically excited system. You can write this type of equation and this type of equation can be conveniently reduced to some other forms.

For example, if you take u equal to x e to the power minus half integration p 1 t dt, so you can eliminate this damping term or you can eliminate this u dot term. So, here by putting this u equal to x e to the power minus half integration p 1 t dt, then you can find or you can write this equation in this form that is x double dot plus p t x equal to 0, where p t equal to p 2 minus 1 by 4 p 1 square minus half p 1 dot. This equation where.

So, now, you have reduced this p 1 t u dot term is not there, that is the derivative term is not there and it is simply written by x double dot plus p t into x. So, there is no u dot term here or x dot term in this equation.

So, this equation is known as Hill's equation. If you take this p t equal to delta plus 2 epsilon cos 2 t, then this equation can be written x double dot plus delta x plus 2 epsilon cos 2 t x equal to 0. This equation is known as Mathieu equation. This equation is the Hill's equation.

And the second equation what just now I told that is x double dot plus delta x plus 2 epsilon cos 2 t x is equal to 0, here the periodic term that is 2 epsilon cos 2 t is the coefficient of x. So, that is why this is a equation of a parametrically excited system. So, unlike in case of force vibration or direct excitation you know that the resonance will occur at a frequency when the excitation frequency is equal to the natural frequency of the system.

For example, in case of a single spring mass system, so you have seen, so when the natural frequency is equal to the or when the excitation frequency is equal to the natural frequency you have observed the resonance conditions. If you recall this magnification factor, so magnification factor verses this omega by omega n where omega is the external frequency and omega n is the natural frequency. So, for undammed system, so the response will be like this.

So, at omega equal to omega n that is at value 1, at omega equal to omega n, so the resonance it is the resonance condition or the response tends to infinite. By applying this damming, so this can be reduced to the amplitude can be reduced.

So, that is, so this is your zeta equal to 0 that is damping equal to 0. So, you can put some certain value of damping and you can have a relation like this or response like this. But in case of the parametrically excited system, you can observe that when the frequency external frequency is away from the natural frequency then you have the resonance condition.

For example, so you can have when omega equal to omega m plus omega n. So, you can have the resonance condition. So, what is m and n? So, m and n are the, so for example, m equal to 1, n equal to also 1, so these becomes 2. So, when m equal to 1 and n equal to 1, this omega equal to 2 omega 1. So, omega equal to 2 omega 1. In that case, it is known as principal parametric resonance condition of the first mode.

We have taken m equal to 1, n equal to 1; that means, the first mode we are taking. That means, when the system is oscillating at its first mode then the resonance will occur when this external frequency is twice that of the natural frequency. If you are considering why is the second natural frequency, then it will be principal parametric resonance of second mode.

Similarly, principle parametric resonance of third mode that way we can write the principle parametric resonance. So, when m naught equal to n. So, in that case we will have combination parametric resonance. For example, m equal to 1 n equal to 2. So, in that case, we will have combination parametric resonance of the first and second mode of sum type, we can have sum or difference type also. So, we may have a minus sign here also.

So, taking m equal to 2 and n equal to 1, so we will have omega equal to omega 2 minus omega 1. So, in that case, we will have combination parametric resonance of difference type. In this case, you have seen when the frequency is away from the natural frequency then we are getting the resonance condition.

Particularly, we are finding the resonance condition when it is twice the natural frequency, those are known as principal parametric resonance condition. And when it is combination of different modes also, so it may be sum type or difference types, combination resonance of sum type and combination resonance of difference type.

So, 3 different type of resonance conditions you have observed here. One is principal parametric resonance condition, second one is combination parametric resonance condition of sum type and third one is combination parametric resonance of difference type. All those resonance conditions will see in next class.

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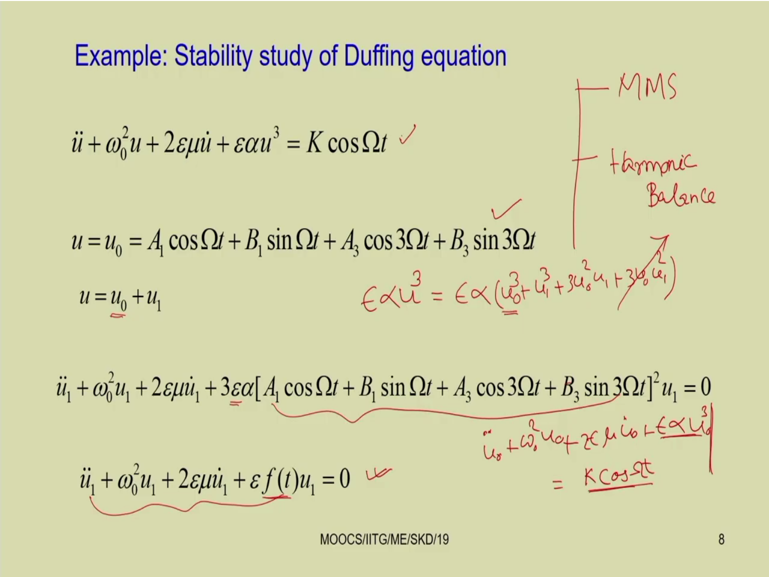

Another example you just see where we can get or where we can find this new type of equation also. For example, we are solving when we are solving these duffing equation, and to solve this duffing equation already we know, so we can use different methods.

For example, previously we have used method of multiple scales and we have used this method of Lindstedt-Poincare technique also. Also harmonic balance method, method of averaging, and different (Refer Time: 16:40) methods we have used.

For example, while we have used this method of multiple scales. So, in that case we got two or we got the reduced equation. By perturbing those reduced equations we got the Jacobian matrix, and by finding the eigen value of the Jacobian matrix we studied the stability.

But if we are going to use for example, use this harmonic balance method. So, if we are going to use harmonic balance method, in this case for example, let us take this harmonic balance method where we assume this solution to be u equal to u 0 equal to A 1 cos omega t plus B 1 sin omega t plus A 3 cos 3 omega t plus B 3 sin 3 omega t.

Now, by substituting this equation in these original equation, and then separating the coefficient of cos omega t, sin omega t, cos 3 omega t, and sin 3 omega t we got a matrix of A and B. By solving that matrix we can get the coefficient A 1, B 1, and A 3, B 3, and we know the solution u equal to u 0. So, in that case we can get this u 0.

So, the solution we got or the equilibrium state we got to study that equilibrium point. So, whether that equilibrium point is stable or not, so we have to perturb this equation. This original equation we have to perturbed around the equilibrium position u 0, we can take this u equal to u 0 plus u 1, where u 1 is the perturbation over this u 0.

We can write u equal to u 0 plus u 1 in the first equation, and you just note that as u 0 is the equilibrium solution. So, it will directly satisfy this first equation. We can write this equation after substituting it in this equation, so we can write this equation in this form. So, it will be u 1 double dot plus omega 0 square u 1 plus 2 epsilon mu u 1 dot plus 3 epsilon alpha A 1 cos omega t plus B 1 sin omega t plus A 3 cos omega 3 t plus B 3 sin omega 3 t square into u 1.

You just see, so this is nothing but this is u 0. This is the expression for u 0 and it is a function of u 0, so as a non-linear terms this u cube non-linear term is present here. You can have this term here in this perturbation; that means, if you have expanding this thing by substituting u equal to u 0 plus u 1 in this epsilon alpha, so for example, epsilon alpha u cube. So, you can write this equal to epsilon alpha, then this becomes u 0 cube plus u 1 cube plus 3 u 0 square u 1 plus 3 u 0 u 1 square.

So, this way you can write, you can expand this thing, and you can see, so this term actually it will go when you are substituting this u equal to u 0 in this original equation. So, if you substitute u equal to u 0 in this original equations, so your equation becomes u 0 double dot plus omega 0 square u 0 plus 2 epsilon mu u 0 dot plus epsilon alpha u 0 cube. So, this will be equal to K cos omega t.

Now, when you are substituting this u equal to u 0 plus u 1, so it has to satisfy this equation as u 0 is the equilibrium solution. And neglecting this term as u 1 is very small, so u 1 square can be neglected. So, you just see this term is already here epsilon u, u 0 cube. This K cos omega t and K cos omega t will cancel the remaining terms will be this were you can write this equation in this form.

And this can be reduced or it can be reduced in this following form also. So, that is u 1 double dot plus omega 0 square u 1 plus 2 epsilon mu u 1 dot plus. So, this whole term these are time varying term. So, that thing I can put equal to f t. So, epsilon f t into u 1. So, as u 1 is, so this is epsilon term, so 3 alpha into this u 0 square can be written as f t. So, this equation can be written in this form.

So, here you just note or you can see that this equation is written u 1 double dot plus omega 0 square u 1, where omega 0 is the frequency natural frequency of the system plus 2 epsilon u 2 epsilon mu u dot. So, this is the dumpling term. Plus you have a forcing term or epsilon f t u 1, where this f t if this time varying term is the coefficient of u 1 which the response of the system. So, so this equation is reduced to that of a Mathieu, Hill type of equation.

So, now, what you have seen while studying the stability of the duffing equation, so if you are not using the method like this method of multiple scale or the averaging method, so where directly you were getting this equilibrium solution and the stability. But if you are using some method like this harmonic balance method, then after getting this solution you must have to perturb the original equation to study its stability about the equilibrium position. So, here the equilibrium position is u 0.

Now, you have perturb this u 0 or you have given a perturbation u 1 about u 0. So, that you have written this u equal to u 0 plus u 1 and you have landed with this equation which is similar to that of a Mathieu, Hill type of equation.

So, by soling this equation, so you can know what is the solution of this thing. So, if the solution, if the response is growing; that means, if u 1 is growing with time, then the systems becomes unstable and if it is remaining within certain limit certain bound then the response is bounded or the response is stable.

So, we can study, so there are different methods to study this stability or different methods are there to study this type of system. Here also we can use this perturbation method also to study the response of the system and also to find the parametric instability region and study the system behavior.

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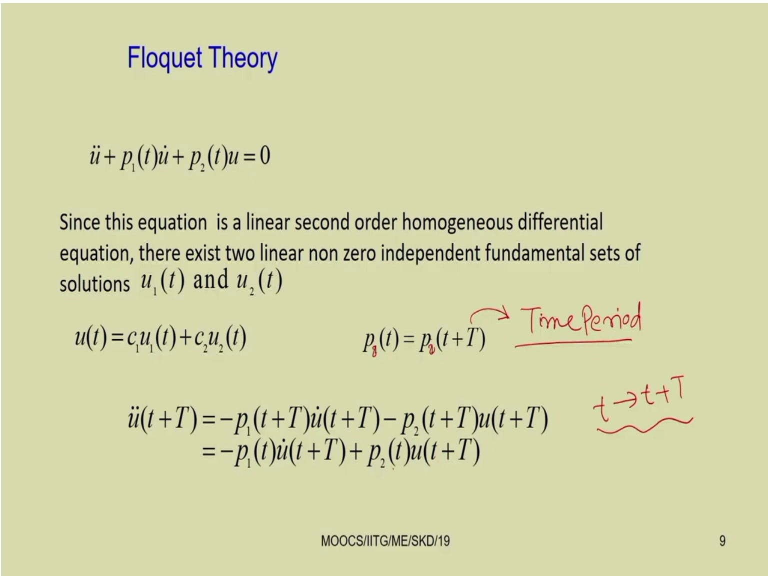

Let us see one theory. So, this is the Floquet theory which is generally used to study the stability of such type of system. So, here you have seen as the system contain, so this is a coefficient is periodic. So, we are expecting to get the periodic solution out of this equation.

So, let us see. Let us consider the system u double dot plus p 1 t u dot plus p 2 t u equal to 0. Since, this equation is a linear second order differential equation, in this case, there must and this is a homogenous differential equation, right hand side equal to 0. The second order, it must contain or it there exist to two linear nonzero independent fundamental sets of solution in this case. So, we have two fundamental sets of solution. So, for example, let us take this u 1 t and u 2 t as the two fundamental set of solutions.

Now, you can write this u t equal to as these are fundamental set of solutions. So, a combination of linear combination of these two solutions are also a solution. So, we can write this u t equal to c 1 u 1 t plus c 2 u 2 t. So, we are assuming here this p 1 t and p 2 t are periodic function of time. So, if their periodic function of time; that means, p 1 t equal to p 1 t plus T, where T is the time period. So, this T is the time period. So, after one period this value must be same.

So, if we are assuming this p 1 t and p 2 t, so I can write this p i t also, so i equal to 1 and 2. So, i equal to 1 and 2. So, p i t equal to p i t plus T. So, in that case, so I can write this equation u double dot. Now, substituting so after one cycle let us see so what will be the response. So, then we can replace this t by t plus T. So, t plus capital T, capital T is the time period. So, then u double dot t plus T can be written.

So, in this original equation, we are substituting t equal to t plus T. So, we are replacing this t by t plus T. So, to check what will happen after one cycle, so u double dot t plus T will be equal to we will have taken this two terms to the right hand side that is why it will be minus p 1 t plus T and u dot t plus T minus p 2 t plus T into u t plus T.

So, already you know that p 1 t plus T equal to p 1 t and p 2 t plus T equal to p 2 t, so in this case. So, we can write this equation equal to minus p 1 t u dot t plus T plus p 2 t u t plus T. So, you just compare this original equation with this equation, they are same.

So, here you can observe that this if u t is a solution, so u t plus T is also a solution of the system. So, here what you have taken? So, you have taken two fundamental set of solution that is u 1 t and u 2 t, and you have now observed that if u t is a solution, then u t plus T is also a solution of the system.

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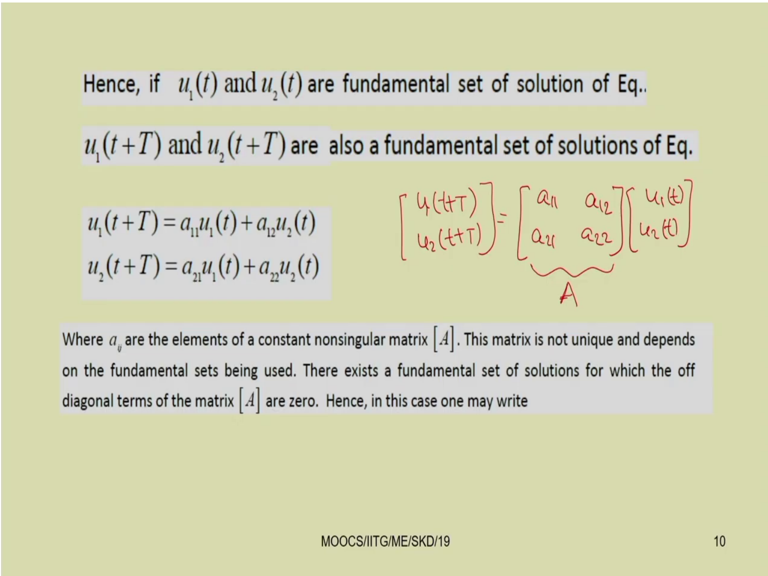

So, as u t plus T is solution, hence if u 1 t and u 2 t are fundamental set of solutions of that equation, then u 1 t plus T and u 2 t plus T are also fundamental set of solution of that equation. If u 1 t plus T and u 2 t plus T are fundamental set of solution, we can write this equal to a 11 u 1 t plus a 12 u 2 t, and then u t plus T can be written as a 21 u 1 t plus a 22 u 2 t or you can write this thing also.

So, for example, so you can write this u 1 t plus T and u 2 t plus T. This vector can be written in this matrix form also. So, this will be a 11, a 12, a 21, a 22, into u 1 t and u 2 t. Clearly you just observe that this u 1 and u 2 are determined after one cycle starting from t, for example, t equal to 0 we can find what is the response at after one cycle, then knowing that response we can write down this u t plus T and u 2 t plus T as a matrix A. So, this is a matrix A.

So, this is a non-singular matrix, you can take this a ij are element of a constant non-singular matrix A. So, starting with this u 1 t and u 2 t with some initial condition, so we can find what is the response after one cycle.

So, knowing that response we can write down this in this form. And then we can find. So, this matrix is you just see this matrix is not unique and depends on the fundamental sets being used. We can use different set of this u 1, u 2, and we can have different different sets of A for finding this response.

We can chose this u 1 and u 2 in such way that we can get this a matrix diagonal then this equation be will be uncoupled. For example, so if this a 12 and a 21 are 0, then we can write this u 1 t plus T will be equal to a 11 u 1 t, and u 2 t plus T will be equal to a 22 u 2 t. As these are not unique, so we can get a get another set of u 1, u 2, for another set of fundamental solutions for which these diagonal it will be a will be a diagonal matrix or the off diagonal terms will be 0.

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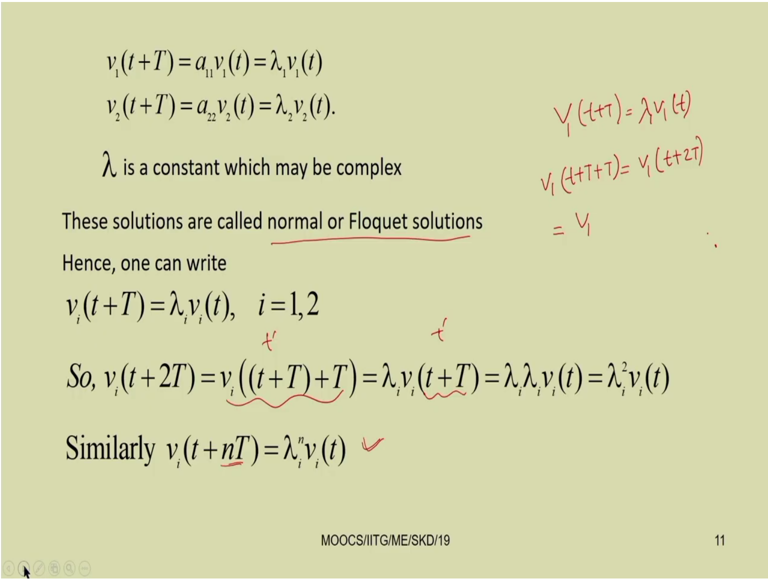

We will have only the diagonal matrix this off diagonal terms will be 0. Let us name them as v 1 and v 2. So, then v 1 and v 2 we can write in this ways. So, v 1 t plus T equal to a 11 v 1 t. So, we can write this as lambda 1 v 1 t also.

So, then v 2 t plus T we can write this is equal to a 2 to v 2 t or you can write this as lambda 2 v 2 t. That means, we can get a set of solution for which the A matrix will be only diagonal. This off diagonal terms are 0, and in that case we can write this v 1 t plus T equal to lambda 1 v 1 t and v 2 t plus T equal to lambda 2 v 2 t.

So, here this lambda is a constant which may be complex also. So, these solutions are known as the normal or Floquet solution. The set of solution for which the A matrix is diagonal are known as Floquet solution or normal solution.

So, in that case, we can write this v i t plus T equal to lambda i v i t. So, i equal to 1. And 2 we can substitute, so that we can have this v 1 t plus T equal to lambda 1 v 1 t and v 2 t plus T equal to lambda 2 v 2 t plus T.

Now, let us add one more cycle. So, for example, will; so, this v, now you have seen this for example, let us take only v 1. So, v 1 t plus T, so you have seen v 1 t plus T equal to lambda 1 v 1 t. Now, if I will add one more cycle for example, and write this v t plus T plus T. Another T let me add, so then this becomes what? So, then this becomes t plus 2 T. So, this is nothing but v 1 t plus 2 T.

So, in that case, what I will write, so I can write this equal to this thing can be written as v 1. So, it is v 1 t plus T plus T. So, this is equal to; so, as already you know v 1 t plus T equal to, v 1 t plus T equal to lambda 1 v 1 t you can write, so for the first case it will be lambda 1 v 1 t. And then again you have to multiply, so this thing v i t plus 2 t can be written as v i t plus T plus T. So, this is written as lambda i. This part you can write equal to conveniently you can write this as lambda i.

For example, let us you take this part as t dash. So, this t plus T equal to t dash. So, v i t dash plus T will be nothing, but lambda i v i t dash. So, this part we have written t dash, so that you will not get confuse. So, v i t plus T plus t can be written as v i t plus T plus T. So, this t plus T if I am putting as t dash. So, you know v i t dash plus T will be equal to lambda i v i t dash. Again, you just see this v i t dash is nothing, but v i t plus T. So, in that case, it will again reduce to lambda i v i t.

So, in this way, you are getting lambda i square v i t. Here v i t plus 2 t equal to lambda i square v i t. Similarly, if you take v i t plus nT, n times time period after n time period if you want to find what should be the response. So, in that case, it can be written as lambda i to the power n v i t, so lambda i to the power n v i t in this way. So, you can write this v i t plus nT equal to lambda i to the power n v i t.

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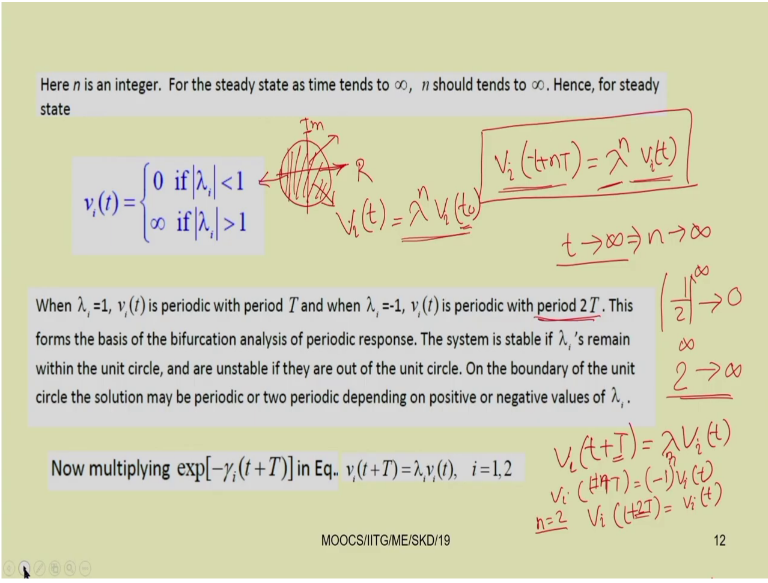

Here n is the integer 1, 2, 3, 4, n will be 2, 3, 4. So, that we have seen. So, t plus 2 T, t plus 2 T here you have lambda square. And nT, so it will have lambda to the power n. Here what you have seen for steady state as t tends to time t tends to infinite. So, n should tends to infinite also so; that means, so if you are writing for example, this v i t plus nT, so this is equal to lambda to the power n v i t. So, you have seen this equation.

So, here, so for steady state that is t tends to t tends to infinite. So, how t will tends to infinite? So, when this n t tends to infinite t will tends to infinite or this n tends to infinite this becomes n infinite. So, implies n tends to n tends to infinite.

So, you just see if n tends to infinite here you just see lambda to the power n. So, you have this lambda to the power n v i t. It must be so for steady state, so we can write this v i t equal to 0. So, v i t will be equal to 1 by lambda, v i t you can write though that is for steady state. So, that time will write this in terms of t.

So, it will be, so this side you just see, so v i t can be written as lambda to the power n v i t. This is the v i initial t 0 I can put or I can write let me let me write this way. So, v i t will be equal to lambda to the power n. So, starting from v i t 0, so to avoid confusion, so you just write the starting point is t 0. So, after t tends to infinite that is n tends to infinite. So, you can write this way if this lambda i mod lambda i less than 1.

So, if mod lambda i less than 1, so in that case, you just see this v i t will tends to 0. If lambda i; that means, if you have a fraction, so less than 1 means. So, you have a fraction. So, if have a fraction, so for example, let us take 1 by 2, 1 by 2 to the power infinite, so 2 to the power infinite, so 1 by 2 to the power infinite, so it tends to 0 v i t will be 0; that means, the response will comes to the trivial state that is 0.

So, if lambda i less than 1, less than 1 means this lambda i is a fraction. So, if it is a fraction only, so in that case as t tends to infinite, the response also tends to the 0. And if it is greater than 1, so you just see if lambda i greater than 1, then for example, let us take 2, so 2 to the power infinite will tends to infinite. The response will be unbounded that is the response will be unstable or the system will be unstable.

So, if this lambda i mod lambda i greater than 1, we can draw; that means, we can draw a circle unit circle. So, we can draw the unit circle and in this unit circle, so this is 1. So, we can draw a unit circle. So, for example, this lambda i is a complex number. So, we can have the real part and imaginary part. So, we can draw this unit circle and if the roots or the lambda are inside this unit circle then the system is stable. So, if it is outside the unit circle, then the system is unstable.

So, from here we have understood that this lambda must be less than 1 or it must lie within this unit circle, so that the system will be stable. So, when lambda i equal to 1, so v i, so you can see when lambda i equal to 1, so v i t is periodic with period t and if lambda i equal to minus 1 v i is periodic with period 2 T. So, that thing you can observe from here.

So, because we are writing this v i t plus T, v i t plus T equal to lambda v i t, so from this thing or n T, so if you add n T, so from this thing you can see this. So, if lambda equal to 1. So, for example, lambda equal to 1, so v i t plus T equal to v i t. So, it has a period T. So, now, if let us take this lambda equal to minus 1, so v i t plus T, t plus T, so if you are taking this lambda equal to minus 1, so it will be equal to this.

If we are putting this n. So, let us put this n T. So, v i plus n T. So, you just see for n equal to 2 only, so this will becomes v i t. So, for n equal to 2, so for n equal to 2 v i t plus. So, n T we are taking, so 2 T will be equal to v i t. So, that is why if lambda equal to minus 1, so you can see if lambda i equal to minus 1, then v i is periodic with period 2 T.

So, here you have seen this v i, if lambda i is equal to minus 1, then the system as a 2 periodic, 2 periodic system. So, it must have a period 2 T. Depending on the lambda value from this equation or from this equation you can decide, so what is the least value of lambda for which this will be periodic or two periodic. This form the basic of the bifurcation analysis for periodic response.

So, later we will study the stability of the periodic solution. So, in that case, we will see different type of bifurcation. Actually, these lambda equal to 1 for in the transition curve. So, it forms the transition, so below, so from stable to unstable region. So, later will see, so if the roots of the lambda that they are roots of the monodromy matrix also later will see.

So, if they are coming out of these region, so either it can come to this point, it can come to this point or it can come as a complex conjugate. So, depending on this things, so you have different bifurcation. And those bifurcations will study in next few classes. So, the system is stable if lambda i's remain within the unit circle, and are unstable if they are outside the unit circle. On the boundary of the unit circle, the solution may be periodic or to periodic depending on the positive or negative value of lambda i.

So, depending on the negative or positive value of, so if it positive, so if it is positive then it is periodic and if it is negative if lambda i is negative then it is of two periodic depending on the positive and negative value of lambda i. We can instead of using lambda, so we can use another we can write it in another form also.

Now, let us multiply this e to the power minus gamma i t plus T in this equation v i t plus T equal to lambda i v i t. So, both the side if you multiply this minus e to the power minus gamma i t plus T, then we can see e to the power minus t plus T, v i t plus T equal to lambda i. So, previously we have this lambda i lambda.

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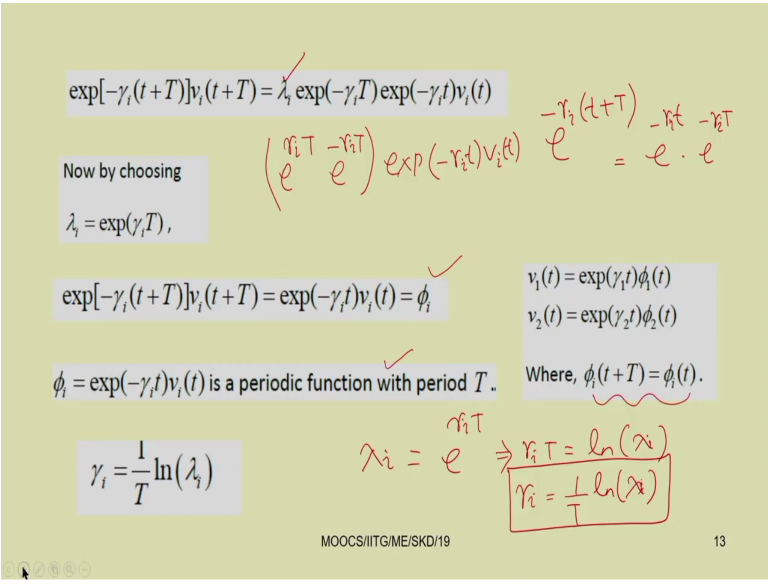

So, you just see, the previously you have this lambda i v i t. So, now you have multiply this by e to the power minus gamma i T. So, this becomes lambda i. e to the power minus gamma i T into e to the power minus gamma i T v i t; e to the power, so you can write this e to the power minus gamma i T plus T equal to e to the power minus gamma i T into e to the power minus gamma i capital T.

Now, expanding, so you have expanded this in the right hand side. So, you have written this lambda i e to the power minus gamma i T and e to the power minus gamma i T v i t. So, now, what we can do? So, we can chose this lambda i in such a way that this lambda i equal to e to the power gamma i T.

So, if we can write this lambda i equal to e to the power gamma i T. Now, what you can see? So, lambda i we have written e to the power gamma i T and so, here also we have another term that is e to the power minus gamma i T. So, we can multiply these two which will give raise to 1. So, this is e to the power, the remaining thing is e to the power minus gamma i T into v i t. This lambda i by taking these things. So, you can write in this form.

So, but you just note this e to the power minus gamma i T plus T, so what we have observed here e to the power minus gamma i T plus T into v i t plus T is nothing but e to the power minus gamma i T into v i t. These term you can write as phi i t also. If this is v 1 t is a solution, so then and v 1 t and v 2 are two solution. Then, this multiplication of this e to the power minus gamma i T v i t which can be written as phi i t will also be a solution. And these, so from this we you can write this v i t equal to phi i into e to the power gamma i T.

So, v 1 t can be written as e to the power gamma 1 t into phi 1 t and v 2 t can be written as e to the power gamma 2 t phi 2 t. So, where we can write, so from this thing easily we can write this phi i t plus T as v i t plus T equal to v i t. So, we can write this phi i t plus T equal to phi i t also. This way also, so phi i t, phi i equal to e to the power minus gamma i T v i t is a periodic function with period T.

So, we can take this as a periodic function with period T, where from this equation; so, lambda i as a lambda i we have taken equal to e to the power gamma i T. This implies this gamma i T equal to l n lambda i.

So, from this thing, so we can write this gamma i equal to 1 by T ln lambda i. So, this is known as the Floquet multiplier. You can find this Floquet multiplier also. So, gamma i equal to 1 by T ln lambda i. You know that this lambda i must be within this unit circle to make the system stable.

So, for this purpose you just see this ln lambda i, so if it is we can have different value of this lambda and we can see.

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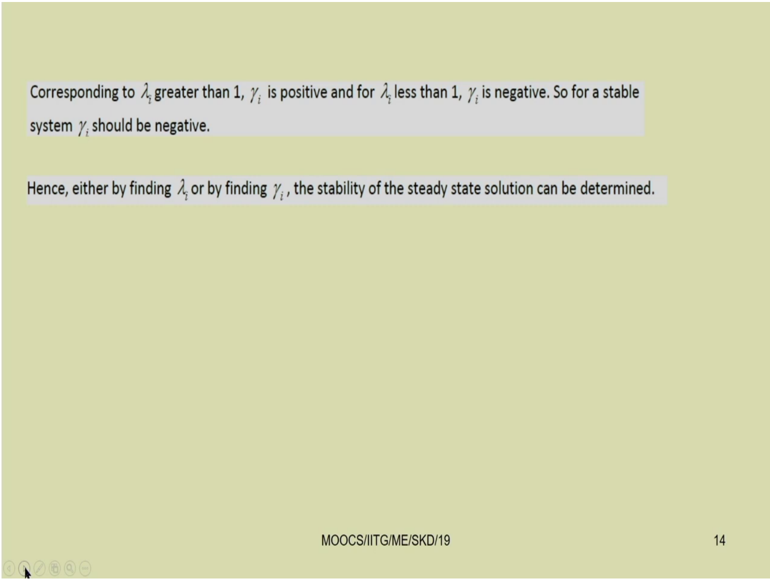

So, corresponding to lambda i greater than 1. So, if it is greater than 1, so in that case gamma i is positive and if lambda i is less than 1 then gamma i is negative. So, if it is point for example, let us take this 0.1, 0.2 for lambda i, so then this becomes negative, gamma is negative. And if we are taking a value greater than 1, then it is positive.

By checking whether this gamma i is positive or negative, so we can tell the system is stable or not. So, for a stable system, so gamma i must be negative because for that system this lambda i must be within this unit circle.

When lambda i is within this unit circle, so gamma i is negative. Now, in this way either by checking this lambda i or by checking this Floquet multiplier gamma i, you can study the stability of a system. This way you can use the Floquet theory to study the stability of a periodic system.

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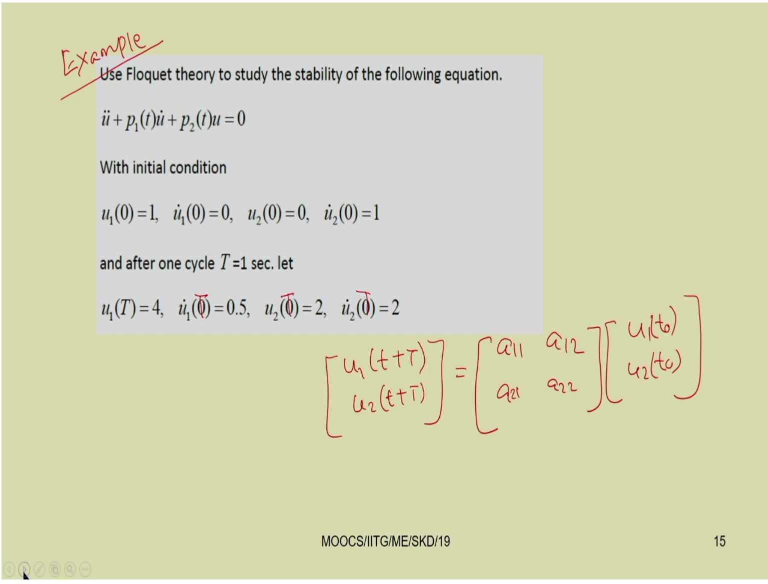

So, let us take one example. So, this is an example which also you can find in my NPTEL course. So, in this example, let us use this Floquet theory to study the stability of the following system. So, let us take u double dot plus u 1 t u dot plus p 2 t u equal to 0. So, this initial conditions are given to us that is u 1 0 equal to 1, u 1 dot 0 equal to 0 and u 2 0 equal to 0 and u 2 dot 0 equal to 1. These are the initial condition known to us.

So, what is u 1 and u 2? So, u 1 and u 2 are two fundamental sets of solution for this equation. And after one cycle, so we have been given that u 1 t equal to 4, u 1 dot t u 1 dot. So, all these things will be equal to t u 1 dot t equal to 0.5, u 2 t equal to 0.5. So, after one cycle, so the values are given to us. So, what you can do? So, we can write. So, already we known this equation, u 1 t plus T u 1 u t plus T and u 2 t plus T. So, we can write. So, these equal to a 11, a 12, a 21, a 22, into this is u 1 and u 2, u 1 t 0 and u 2 t 0

We can write for example, in this case. So, all the things are been given to us. So, u 1 t plus T, u 2 t plus T, we are writing in this way. And from this we can find. So, by differentiating this equations also we can write down this equation again and from that thing, so we can find what is a 1, a 2, gamma 1, gamma 2.

So, you just see we have 4 unknown. So, a 11, a 12, a 21, a 22. So, we required 4 parameters. So, we have taken two parameter u 1 t, u 2 t, and another two parameter we can take u 1 dot t and u 2 dot t. And from that thing, we can find this equation.

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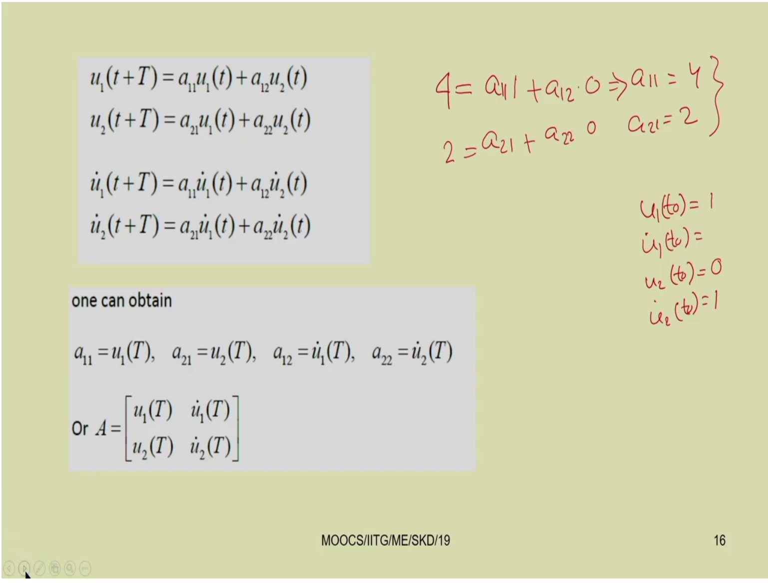

So, the solutions we can write this way u 1 t plus T equal to a 11 u t a 12 u t, u 2 t plus T equal to u 1 t plus u 22 u 2 t. Similarly, u 1 dot t plus T equal to a 11 u 1 dot t plus a 12 u 2 dot t and these u 2 dot t plus T equal to a 21 u dot t plus a 22 u 2 dot t. So, you just see 4 equation 4 unknowns. So, by solving this thing we can find this.

So, by solving this, so we can find, for example let us take u 1 t plus T after one cycle. So, u 1 value is given to be 4. So, 4 equal to, so you can write in this case. So, your equation becomes 4 equal to a 11. So, u 1 T, so u 1 T is given to be at t equal to 0, so u 1 value equal to 1. So, you can put this is equal to 1, so a 1 into 1, so plus a 12 into u 2 T.

So, you just note that u 2 T equal to, u 2 T, u 2 T 0 at t equal to 0. So, this is equal to 0. So, I can put equal to 0, a 2 into 0. Let us write it here. So, we have u 1 t 0 equal to u 1 t 0 equal to

1, u 1 dot t 0 we have to write equal to, u 2 t 0 already we have seen this is equal to 0, and u 2 dot t 0. So, these values are given. So, let us substitute it here.

So, from this thing we have seen u 1 dot 0 equal to 0, u 2 0 equal to 0 and u 2 0. So, u 2 0 equal to 0 and this is equal to 1, u 2 dot equal to 1. And u 1 dot is also given to be, u 1 dot is also given to be 1. And these, other values are u 1 t equal to 4 and u 2 t equal to 2. So, let us now write the second equation.

So, in second equation 2 equal to u 2. So, 2 will be equal to a 21, a 21 into u 1 T, u 1 T equal to 1, so a 21 it becomes and plus a 22 into u 2 T, u 2 T equal to 0, so it is multiplied by 0. So, from this thing, so what you have observed? So, you observed that a 11 is nothing but it is equal to 4 and a 21 equal to 2 or in other words. So, you can write this way also a 11 equal to u 1 T and a 21 equal to u 2 T, a 12 equal to u 1 dot T and a 22 equal to u 2 dot T.

So, this way you can; similarly another two equations you can write and from that thing also you can find the value and you can write this A matrix. So, A equal to u 1 T, u 1 dot T, u 2 T and u 2 dot T.

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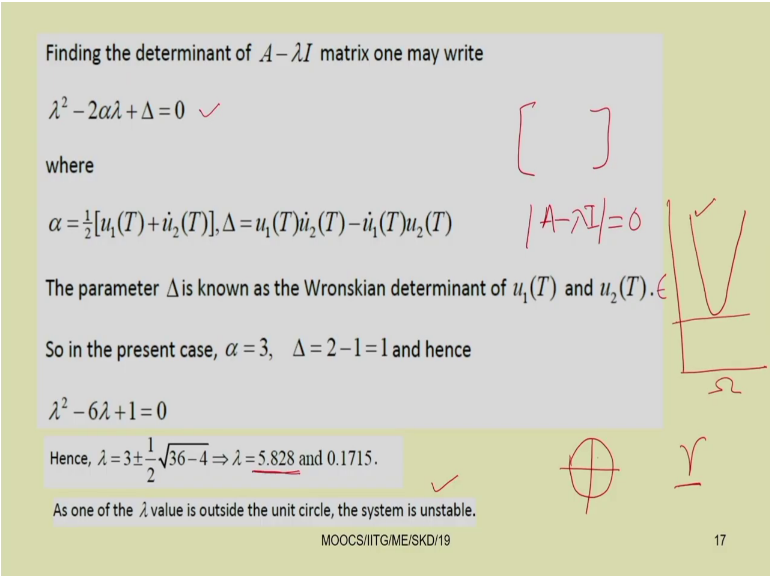

So, after finding this A matrix, now you can find. So, now, A minus lambda i, so you have to find this lambda. So, A minus lambda i, so you just write determinant of A minus lambda i equal to 0. Now, you can get this thing. So, that means, lambda square minus 2 alpha lambda plus delta equal to 0. So, in this form you can write, so where alpha equal to, you just see where alpha equal to half u and T plus u 2 dot T and delta equal to delta equal to u 1 T, u 2 T minus u 1 dot T and u 2 dot T.

So, this is the determinant and this is the trace of that matrix. That way you can write also or directly from these things, so writing this a after writing this A matrix, determinant of A minus lambda i you can do determinant of A minus lambda i equal to 0; put determinant of A minus lambda i equal to 0.

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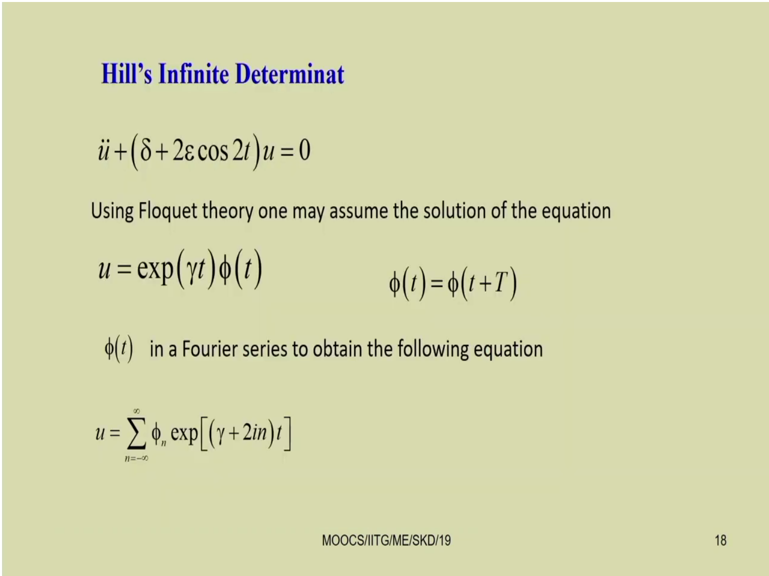

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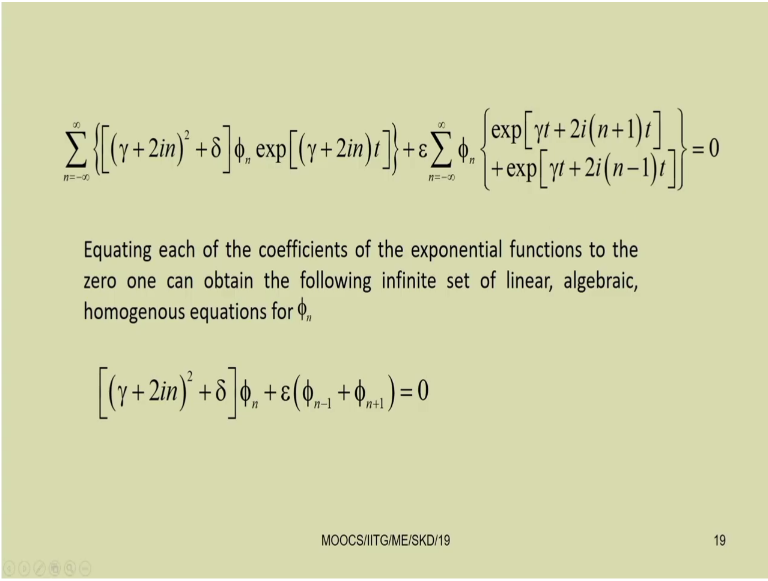

Otherwise, you can directly find the eigen values of A matrix also. So, from that thing you can get this lambda. In this case, for the given parameter, so delta is known as the Wronskian determinant and of u 1 and u 2 T.

So, in the present case alpha equal to 3, delta equal to 2 minus 1 that is equal to 1 and hence, so lambda square minus 6 lambda plus 1 equal to 0. Hence lambda you can get, so two value of lambda you have got. So, lambda equal to 5.828 and 0.1715.

So, you just see as one of the value that is 5.828 is outside the unit circle the system is unstable. So, you have seen the system is unstable as one value of lambda is outside the, outside the unit circle. So, this way you can use this Floquet multiplier and in this way you can use this Floquet theory to determine this lambda to find the response of the system.

So, instead of finding this lambda value we can find the Floquet multiplier gamma also and you can study this stability. So, tomorrow class or next class we are going to use this Floquet multiplier to study the other different stability of other different systems. Also, will study different different other methods to find the instability region. So, here you just see, so you have to find the value of the system parameter for which it is the system is moving from a stable state to unstable states.

So, this will give rise to a boundary and so, it will give rise to a boundary for different system parameter. For example, in this case for epsilon and omega if you want to plot, so you can get a boundary, so this boundary is known as parametric instability boundary or transition curve. This curve is also known as the transition curve. And we can find the minimum value.

So, for example, here you can find the minimum value of epsilon for which the system is always stable. So, this way we can study different boundary or different transition curve for different equations of parametrically excited system. Next class we will take the Hill's equation and find the Hill's determinant also. Also, will use this method of multiple scale and other different methods to find the parametric instability region.

Further, we will use different continuous systems and we will try to reduce those continuous system to that of a single degree of freedom system and multi-degrees of freedom system to find the parametric instability region and the response of the system.

Thank you.