Finite Element modeling of Welding processes
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Lecture - 09
Elastic stress analysis-II and Potential energy method

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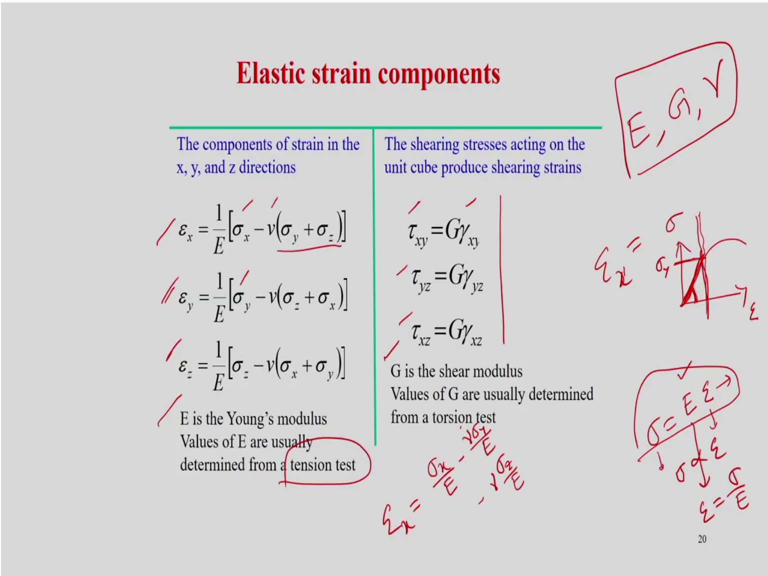

Hello everybody. Today, we will discuss that Elastic stress component, what way we can formulate using the finite element base method and actually, once we look into this finite element formulation, then before that we need to understand that what way we can relate the stress and strain.

But very simple way, if we look into the elastic component or the elastic stress analysis part, then how we can integrate; that means, what way we can find the relationship between the stress and strain. The different form different matrix form, we can find out and accordingly, we can relate between these two components.

So, to do that first, we look into the elastic strain component. So, we know that the components of the strain in individual x and y and z direction and we know that epsilon x the strain in direction x equal to there are three components are there. So, in if we look into that elastic stress versus elastic strain and that is related by the Young’s modulus and in and when there is a acting of the one directional stress or one directional strain.

So, in that case, we can simply stress equal to we can write that E into strain. E is the Young’s modulus or elastic modulus and this is the stress and this is the strain and that is correct in case of assuming that there is a unidirectional load or unidirectional stress is acting.

But this stress versus strain that we can see the we know the stress is proportional to the strain within the elastic limit and then, if we look into the once you need elastic stress will come some part of is the elastic and then, it decreases the plastic zone. So, plastic zone is the non-uniform relation between the stress and strain. We assume this is the strain axis, this is the stress axis. So, this path is linear.

So, when this path is linear, then we can simply relate between the stress and strain Young’s modulus and see it is one-dimensional then we can use this particular relation, the stress the relation between the stress and strain. Now, if the even it is elastic stress or elastic strain components associate with the all x, y and z direction, then what way the strain in one direction can be relate to the with the strain component in other direction in terms of the Poisson’s ratio.

So, the lateral strain by linear strain. So, that is ratio can be defined as a Poisson’s ratio. So, therefore, we can link between the two different direction strain in using the Poisson’s ratio value. So, that comes into this picture. So, if we assuming the strain component, but strain component is the normal strain component; so, normal strain epsilon x.

So, you can say that epsilon x having the one component with the sigma x by E. So, here that we are getting the stress is here, the strain component is basically sigma by E. So, if x directions strain is acting. So, sigma x by E is the one of the strain component and that strain is induced because of the stress along the x direction with the sigma x. Now, other direction what may be the strain?

So, in other cases the strain lateral contraction may happen in the other direction. So, this Poisson's ratio into sigma y divided by E and that other sigma z by E is the basically strain component; but when you multiply by the Poisson's ratio, it can it indicates what is the effective value of the strain along the x axis.

So, effective strain along the x axis, while considering the effect of the all three different stress components can be written effectively is like that; one sigma x the positive because along this direction we assume the strain is a positive value.

And then, Poisson's ratio we introduced and then, we can use the sigma y and sigma z minus of that divided by E. All indicates the strain because of the sigma y and sigma z and that is acting along the x direction that is the along the normal takes care of the strain components along the x axis.

So, this is the effective strain accounting all these three stress components and then, which is modified by the this Poisson's ratio, we can get the effective strain along the x axis. Similarly, along y axis, what is the strain value? The sigma y by E minus of the strain components because of sigma x and sigma z and that is the effective strain along the y axis.

So, similarly epsilon z also, we can estimate. The effective strain sigma z by E minus of we can take the that other strain components because of the stress sigma x and sigma y. So, that is why effectively, we can write out this three different strain component along x, y and z direction is epsilon x, epsilon y and epsilon z.

Now, here E is the Young's modulus and values of E are usually determined from the tension test. So, Uniaxial Tensile Testing, the data if we get and from there, we can find out what is the value of the Young's modulus it is basically simply indicates the slope. Slope means we can if this is the strain value and this is the corresponding stress value, their yielding start at which point.

And that from there the data, we can estimate what is the value of the Young's modulus and a particular uniaxial tension testing of a particular sample. Now, apart from this normal strain component, there are also shear strain component also there. So, shear strain component is directly relate to the shear stress value in terms of the shear modulus.

G is the shear modulus and but value of G usually determined from a torsion test. So, if we look in the pure torsion test from the torsion test, we can estimate what is the value of this value of the G or shear stress versus shear strain or T versus the torque applied and T versus theta; from there also, we can estimate what is the value of the this shear modulus.

Now, shear modulus also within the last limit, the shear stress is proportional to the shear strain. So, stress is proportional to the shear strain and constant of proportionality G is the known as the shear modulus; so, similarly tau x y equal to G gamma x y, tau y z equal to G gamma y z and tau x z equal to G gamma y z.

So, that is why this the relationship between the strain and stress and if we look into overall, the relation between the stress and strain is established using these three different; the Young's modulus is one constant and the shear modulus and the Poisson's ratio. These three components, we have considered to find the relation between the stress and strain.

And stress can strain can be both normal strain components as well as the shear strain component and stress can be both normal stress as well as the shear stress component. So, once we get this relation between the stress and strain, but only thing is that whatever we can arrange this material properties. Basically Young's modulus shear modulus and Poisson's ratio, these are the it is a material property.

So, different material the values of all can be different also. So, therefore, that is well defined or maybe we can well established properties of the for particular material and because this material properties has been already derived from some testing, from some from experiments actually.

Now, once we know this value, though we try to relate between the stress and strain in the vector form or in the by introducing the matrix form. We can relate between the stress strains and that we will see how we can relate between these two.

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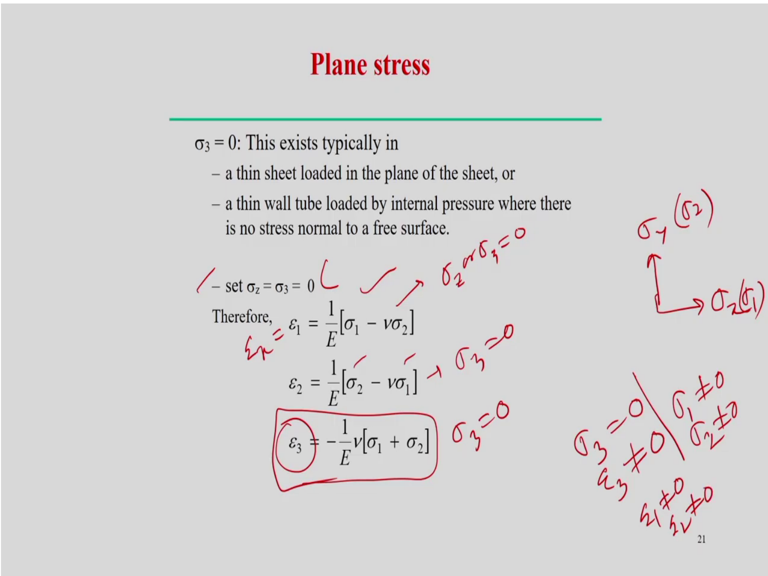

Before that, two usual condition that is the plane stress and plane strength quantities sometimes used to solve in (Refer Time: 08:28) of the particular engineering problem also. For example, we look into the plane stress condition. Plane stress condition is simply plane stress condition, we understand that one of the component may be sigma 3 equal to 0 or sigma z equal to 0.

So, one stress component equal to 0, but it practically applicable if when we are handling very thin sheet, load is applied on a particular thin sheet; in a plane of the sheet, if load is applied or very thin wall tube and it is subjected to some sort of internal pressure. So, in that case, we can find out the there is a stress components; only two stress components that means, the stress components is acting, but or sigma 2 or sigma 1 is acting.

But normal to the sheet, the stage is basically 0 and this is the practical aspect because and this is a quite good assumptions; the plane stress condition, when you are handling very thin sheet also and with particular loading condition. So, that situation may also arise, but we will try to look into that if this is the plane stress condition, how we can relate between the stress and strain in the matrix form or that we will try to look into that.

So, definitely that set sigma z equal to sigma 3 equal to 0 and this is the condition for the plane stress condition. That means, one of the stress components become 0; but it is not necessary, the strain will be 0 in that particular direction; there may be, we will see that. So, once set the sigma 3 or sigma z equal to 0, then epsilon 1 or epsilon x is something like that. Here in this case, we just consider the previous expression that for within the elastic components of the strain along x axis.

So, in this case, we simply consider sigma z or sigma 3 equal to 0 and then, we can get the relation between the strain and strain using this expression. Similarly, epsilon 2 or epsilon y, we can find out the relation in this case also sigma 3 equal to 0. And then, we can get in sigma 2 and sigma 1 in terms of that.

And here, you can see that epsilon 3 is also there. That means strain, but although sigma 3 equal to 0; that means, stress components equal to 0 along the z axis that normal stress components, but there may exist also normal strain component along this particular axis. So, therefore, we can see that from the expression also that epsilon 3, the strain component direction, here if we put the sigma 3 equal to 0, then we can get this value.

It means that although sigma 3 equal to 0, strain component bar epsilon 3 is not equal to 0, some values will be there. And here, you can see that sigma 1 not equal to 0, non-zero component; sigma 2 not equal to 0. I mean we can see also epsilon 1 not equal to 0; epsilon 2 not equal to 0.

So, that means, non-zero components. So, therefore, these are the expression for particular the relation between the shear tension in plane stress condition. So, one we say the plane stress condition.

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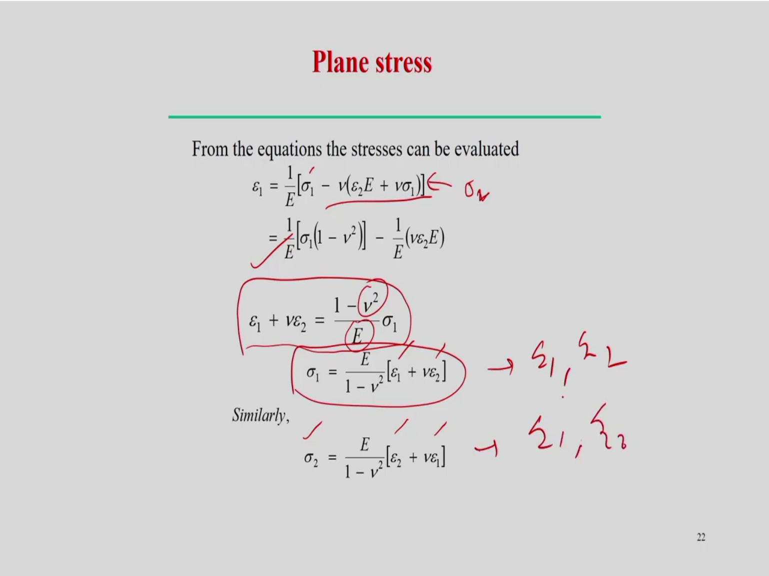

So, from the equation, the stress value can be evaluated this is simply transformation from the strain. We have we put the expression that strain equal to as a function of stress; now, we have convert this variable in such a way the stress equal to in terms of the strain. To do that, some equation we can arrange in such a way that simply that sigma 1 and the first expression which is the value of the; I think sigma 2 value.

So, sigma 2; put the sigma 2 value here and then, that will be represents in terms of the epsilon 2 and sigma 1. So, finally, we are getting this expression. I think even if we put the sigma in term sigma 1 is there, 1 minus nu square and epsilon 2 also in terms of the E. So, this is the expression, we can get.

Now, from here, we can find out epsilon 1 plus nu into epsilon 2 that is the in terms of the epsilon in terms of the sigma 1 also; but other material properties are also included here in this particular expression. Then, we can find out that sigma 1 equal to from this expression, we can sigma 1 equal to in terms of this value. Sigma 1 equal to E by 1 by nu square epsilon epsilon 1 and epsilon 2 and the other material parameters or may be material properties.

Similarly, we can estimate the sigma 2 equal to in the in terms of the epsilon 1 and epsilon 2 and relating to the other parameter. So, it is a simply conversation of the stress in terms of the strain component and we can see that sigma 1 and sigma 2, it is associated only on the these two cases. The it is associated with the epsilon 1 and epsilon 2; this is also expression epsilon 1 and epsilon 2. These two in terms of these two variables, we represent this thing; sigma 1 and sigma 2.

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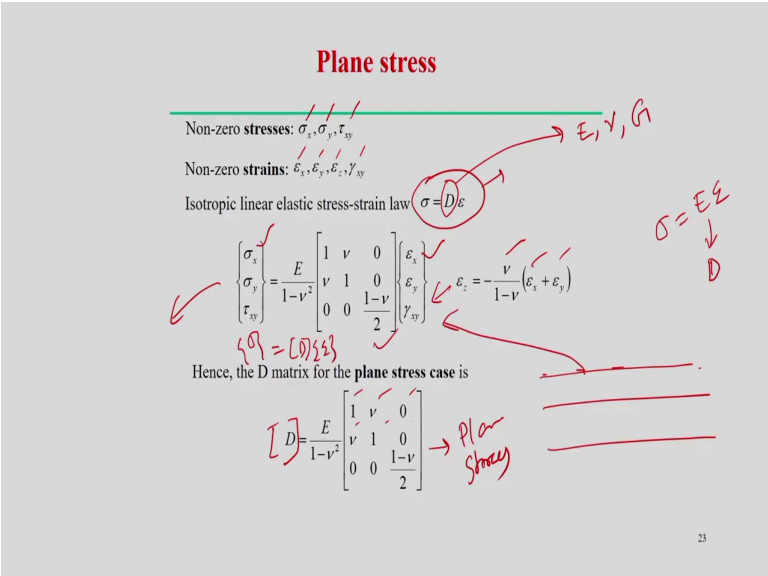

Now, we can see overall that non-zero stresses in this in plane stress condition, what are the non-zero stress component. So, sigma x or sigma 1, sigma y or sigma 2 and tau x y, these are the or tau 1 2, these are the non-zero stress components exist in case of the plane stress condition. Now, non-zero strain epsilon x or epsilon 1, epsilon y or epsilon 2, epsilon z or epsilon 3 and gamma x y or gamma 1 2, these are the typical non-zero strain components in case of the plane stress conditions.

Now, we will try to look in the other form also. Because in finite element, we it is necessary to represents the relation with the stress and strain in the in the form of a matrix formation or using the column vector; in the vector form, column vector form we can write out all these expression. That is the objective to look into all this expression, the equation in the form of a matrix equation. Now, in general, we can say that stress equal to D into epsilon.

So, stress equal to D into epsilon, this is the general expression between the stress and strain. But if we look into that in actual practical cases, the stress and stress components can be more also depending upon whether it is plain stress condition, whether it is a three-dimensional stress state or whether it is plane stress condition, the values of the sigma in the column vector can be different.

So, that is we will try to look into that; but here D is the matrix that can be relate, D actually represents or we can say the as a function of the all the material properties. So, D can be in the form of a the Young's modulus, this Poisson's ratio as well as the shear modulus. So, D in that form.

So, basically it is equivalent to the sigma equal to E into epsilon. If it is one-dimensional stress state, a one-dimensional stress is there or uniaxial tensile testing, if we consider. In that case, stress is relation stress is related to the strain for single value of the stress and then, simply is the Young's modulus.

But when this is three-dimensional state of the stress or different plane stress or plane strain condition, then it will be replaced by the D because D account the different material properties in the matrix form, that we will see. Now, here if we look into that in case of plane stress condition, we have already seen the components of the different stress.

So, then it can be represented in this particular matrix form; f sigma x equal to this E by 1 by nu square and this particular position of this matrix this and this is the strain component. So, here you can say this is equivalent to sigma equal to this is equal to the D matrix, this is the strain and in terms of that this is the column vector that is the column vector, it is the in the 3 by 3 matrix form.

So, that is why this relates between the stress and strain in the single value. Now, we have not included the epsilon z because that is not necessary here. Because epsilon z in this case, that we can keep it separate; but it is a non-zero component, epsilon z or epsilon 3; that means, strain that can be represented once we solve this equation epsilon x and epsilon y. And put this value and then, you can able to find out what is the value of the epsilon z.

Now, the D matrix for the plane stress condition, it is very important the D matrix for the plane stress condition, we can say in this way also that E by 1 by nu square 1 nu 0 nu 1 0 this thing. But you will be getting this expression, if we write the linear set of the equation three equations.

Then, from these three equation, we can convert in the this matrix form; but here it is very important to know what is the D matrix. D is basically representation of the included all the terms in this D matrix is the material properties; so, in this particular form.

So, this D matrix is basically true in case of the plane stress condition; but in other condition, the formation of the of this D matrix can be different also; that means, elements, position of the elements can be different in D matrix in other condition also, that we will see.

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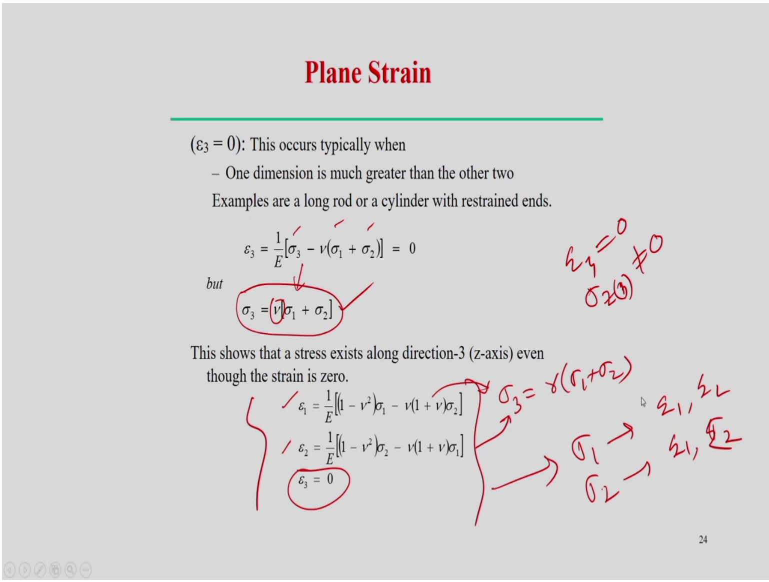

Now, we will try to look into the plane strain condition. Plane strain condition when plane strain condition may be one of the strain components equal to 0. But in this case, if we assume the epsilon 3 equal to 0 or epsilon z equal to 0, that indicates it is a plane strain condition.

But practically, this is typically occurs the one-dimension is much greater than the other two-dimension. For example, long rod or cylinder with restrained end. So, for long rod or cylinder with the restrained end, this indicates the there is a restricted deformation in the third direction.

So, in that case this plane stress condition, we can put the we can consider this is a situation of the plane strain condition. Accordingly, we can use this formula for the plane strain condition. But we will see in plane strain condition what way we can relate between the stress and strain in the form of a matrix.

Now, epsilon 3 equal to 0, then once we put the epsilon 3; we have already seen the epsilon 3 terms of the epsilon sigma 3, sigma 1, sigma 2. So, if it is 0; from here, we can find out the relation between the sigma 3 equal to in terms of the sigma 1 and sigma 2 and only relate with the Poisson's ratio. So, this is the expression for the sigma 3 in case of the plane stress strain condition.

Now, it indicates that although the same thing that epsilon 3 equal to 0, but sigma z it is not necessary or 3, not necessary it becomes non-zero component in this particular cases. In what we have observed in case of the plane stress condition also that epsilon 3 becomes non-zero, but sigma 3 equal to was 0.

So, that is the that was the difference in these two cases. Now, this shows that stress existing along the definitely the z axis even though the strain equal to 0, in this particular direction. So, then from here you can find out what is the value of epsilon 1 and epsilon 2. What is the status of this epsilon 1?

In this case, epsilon 1 equal to we can just manipulate this expression because in this case all these expression, we can put the epsilon 3 equal to 0 and we can get the sigma 3 equal to 2; sigma 3 equal to in terms of sigma 1 and sigma 2, then expression of epsilon y, here we replace the sigma 3 value in terms of the nu into sigma 1 plus sigma 2.

So, all these expression here also with the same relation such that epsilon 1 and epsilon 2 can be represented only in the form of a sigma 1 and sigma 2; similarly, epsilon 2 can be represented in the form of a sigma 1 and sigma 2 such that expression can be manipulated in this particular way.

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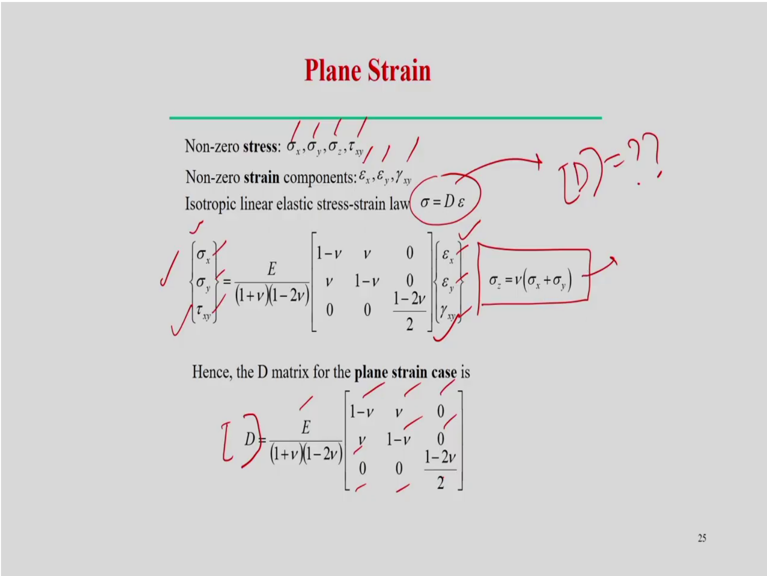

Now, we can see the plane strain condition, the non-zero stress component. So, sigma x is the non-zero strain component; sigma y, sigma z and tau x y, these are the all non-zero stresses.

But non-zero strain components epsilon x, epsilon y and gamma x y; that means, in this case there is a two non-zero strain components and one non-zero shear strain components; and other cases, the non-zero stresses also the three non-zero stress components as well as the once shear stress components becomes non-zero in case of the plane strain situation.

Now, similarly Isotropic linear elastic stress-strain law from the elastic stress strain curve, we can estimate sigma equal to D into epsilon e. So, same way, we can relate between the c, but our objective to what can be the D matrix in case of the plane strain condition, that we can we have already seen that we have already used this expression also in the previous slide.

So, use this expression and this expression from here, we can represent this is the stress components in terms of the strain. That means, we can represent the what is the from this expression the sigma 1 in terms of the epsilon 1, epsilon 2; similarly, sigma 2 in terms of the epsilon 1 and epsilon 2.

So, then if we convert this thing sigma 1 and sigma 2 and from there, we can form the relation between the stress and strain component; this is the three strain component; this is the three stress component and of course, we have not included the sigma z in this particular expression. Sigma z with that, once we estimate from here the sigma x and sigma y; then, we put the value of sigma x and sigma y, then we will be able to find out what is the value of sigma z.

So, this is the three stress components sigma x, sigma y and see it is a column vector and this is the epsilon x, epsilon y and gamma x y, this is the this is also column vector in terms of the strain. So, these are the strain component; these are the stress components and that relation between the by the D matrix.

So, in this particular plane strain condition, the D matrix can be formed in this particular form. So, D equal to E into E by 1 minus 1 plus nu to 1 minus 2 mu and this is the position of the different elements of this particular matrix. So, this is the D matrix for plane strain condition.

Now, we can see that D matrix for the plane strain condition and D matrix for the plane stress condition. In these cases, the formation of the elemental position can be different or different the expression are different in these two cases. So, even it will be different, if we consider the three-dimensional state of the stress having all the six components of the strain and six components of the stress, then formation of the D matrix or particular form of the D matrix will be different also.

So, it depends on the situation. So, now this D matrix is very important to understand because it will further when you do the finite element-based calculation, then we have to look into that all these D matrix at the different situation.

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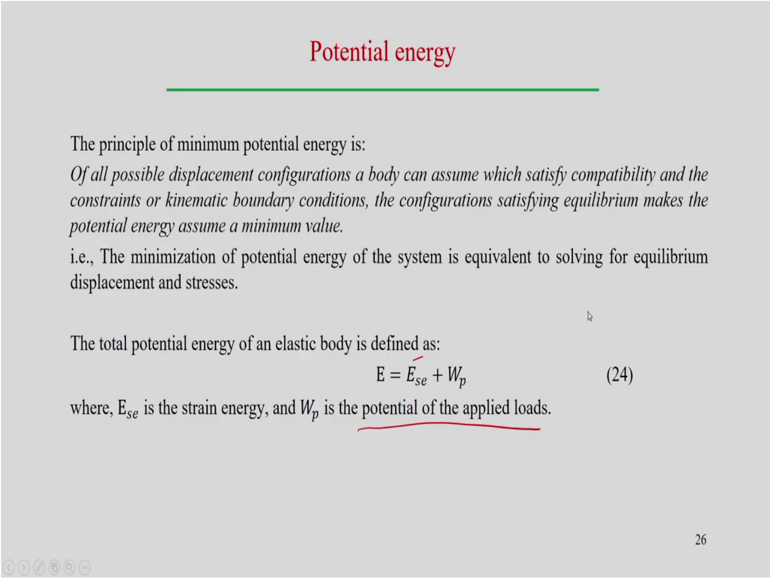

Now, once we look into this finite element formulation of the in case of the stress analysis and assuming that, we will first we will try to look into the elastic stress analysis. Now, once we look into the elastic stress analysis, then it is necessary to understand the minimization of the potential energy to explain the equilibrium condition of a particular system.

It is just its we can simply we can consider that if we apply some kind of the external load on a particular element. So, there will be internal stress will be generated and that. So, that we have considered this thing and then, with the application of the several constant boundary, kinematic boundary conditions in such a way that the system will be in equilibrium, we consider.

In that to satisfy the equilibrium of this particular system, static equilibrium I am talking about this particular system; then, it should follow the minimum potential energy of this particular system. So, that is why minimization of the potential energy, if we follow the formulation for the how we can do the minimization of the potential energy of a particular system, effectively it indicates the system is in equilibrium.

Then, we can relate between the stress and strain and maybe we can estimate what is the displacement field stress field and strain field in this particular finite element problem. Let us look into the principle of minimum potential energy something like that. Of all possible displacement configuration a body can assume which satisfy the compatibility and the constraints whatever constraint is put or kinematic boundary conditions, the configurations satisfying the equilibrium makes the potential energy assume a minimum value.

So, therefore, the system assuming the potential energy assuming the minimum value and effectively indicates the equilibrium condition of this particular system. That but the system may be subjected to internal stress generation will be there, but the system will be subjected to some kind of the external load or some constant may be there or some kinematic boundary condition can also be incorporated in this particular system.

So, therefore, we will see what way we can estimate the potential energy of a system and that such that it will be the equivalent to the solving for the equilibrium displacement and stresses condition of this particular system. So, therefore, it is understand the minimization of the potential energy of a system to establish the formation of the equilibrium; that means, for equilibrium condition equilibrium displacement and stresses exist within this particular body.

Now, total potential energy of an elastic body; definitely, we are assuming this elastic body in this particular in this particular formulation.

So, potential energy of elastic body can be defined in this thing the that is the two components; one is the strain energy of the system of this particular body and associated and W p is the potential of the applied load. So, strain energy generation within this particular system and one component is there, other component is the energy potential for the applied load.

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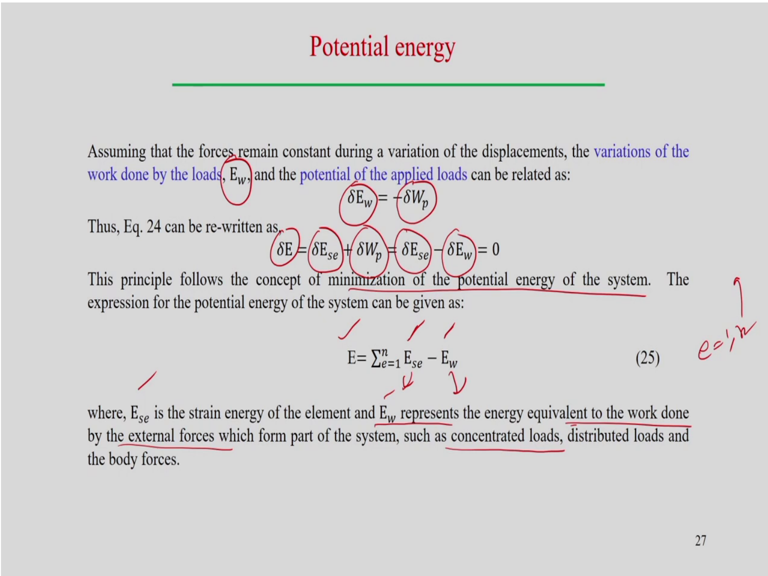

Considering these two components, then we will see that assuming that the force remain constant during the variation of the displacement. So, small displacement variation is there. So, in that cases the variation of the work done by the body or by the application of the loads, then E w that means, E w is the work done by the application of the load E w such that delta E w is the potential of the and the potential of the applied load can be related.

So, delta E w is what is the work done with the small displacement for the application of the external load that is equal to the potential of the applied load delta W p. And then, this is the relation between these two potential of the applied load is equivalent to the work done by this external load and the relation between these two can be this delta E w equal to minus of delta W p.

So, therefore, the previous equation can be written like that, delta this is the strain energy of the system and delta W p is the potency the energy associated with the equivalent to the potential energy applied to the system is equal to minus delta E; E w. So, therefore, delta S is the strain energy and the this comes the work done equivalent the energy associated to the work done by the application of the external for the small displacement.

And this is the delta is the increment of the energy of a particular system. So, therefore, this principle follows the concept of the minimization of the potential energy of the system. Definitely, this equal should be equal to 0 minimization of the potential energy of the system.

So, therefore, the first we have to look into the expression for the potential energy of the system such that potential of the system E equal to the this is the potential energy. That means, strain it is associated with the strain energy and this is the associated with the energy; but because of the application of the external load.

So, therefore, that should be summation over the all elements because E indicates the element number in particular element and E equal to 1 to n; 1 to n means suppose in this particular domain, when you discretize in the there are n number of elements in this particular domain. And we are assuming the summation of this total this potential energy as well as the external work done, potential of the applied load which is can be equivalent to the application of the external load in this particular case.

E s e is the strain energy of the element and the particular element and E w represents the energy equivalent to the work done by the external forces. So, E is the work done by the external forces and it represents the strain energy of the system. And remember, all these cases we are assuming the everything is within the elastic zone; deformation everything the is the following the elastics.

It is known not to go beyond the elastic domain. So, therefore, part of the system such as this E w represents the energy equivalent to the work done by the external forces; that means, external forces because in the particular situation that it may be subjected to some sort of the concentrated load.

In general, I am talking about and it may be subjected to the distributed load also and may be subjected to the body forces also. Body forces means the force distributed over the whole domain or whole body of this particular material. So, this kind of external may also act and then, basically what is the energy equivalent to by this external load in this cases also.

And the same time otherwise you are calculating, what the elastic deformation and from that, what is the elastic strain energy associated with this thing such that system becomes equilibrium, if we minimize the amount of the potential energy of this particular system. But here, I am looking into that we can estimate the potential energy of this particular system. So, there are two components are there in potential energy of this particular system. We will see how we can minimize this thing.

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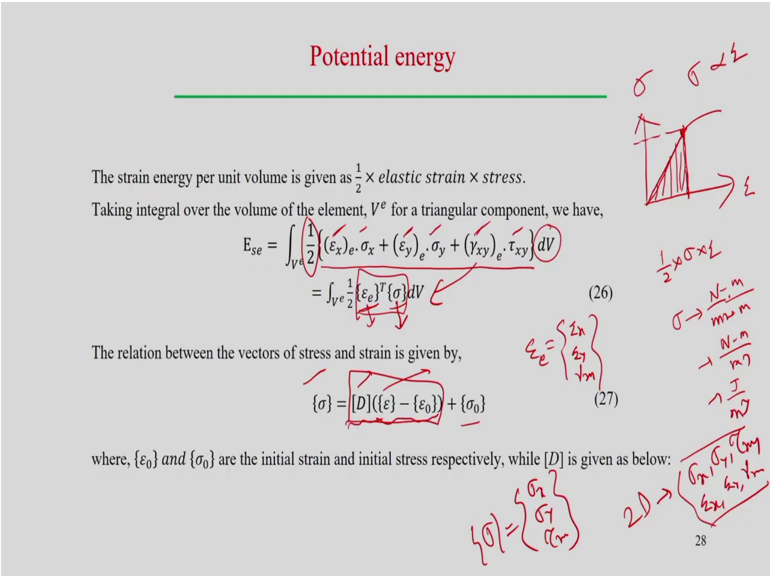

Now, what is the potential energy of this particular system? We know the strain energy. First, we have to know calculate what is the strain energy of this body per unit volume is given by half of the elastic strain into stress strain into stress. So, it is something like that suppose one-dimensional state of the stress is the this is the strain component, this is the stress component and if we look into that, so this is the within the elastic domain. It is the relation between the stress and stress is proportional to the strain.

So, it is a linear relation between the stress and strain within the elastic domain. So, that is mean see this indicates that this is the energy per unit volume. From the stress strain diagram, we can estimate the energy per unit volume is the basically represent this area. So, area means we can see this area of a triangle half of stress into strain.

So, that is when this is the corresponding strain stress value and this is the corresponding strain value. So, from there, this half of this indicates what is the area of this and that area represents what is the strain energy; strain energy per unit volume. Because if we see the units of a stress, you just we can check also unit of a stress say for example, Newton per meter square and strain unit less.

So, Newton per meter square either we can say the Newton meter per into multiplied by meter also. So, it indicates the Newton meter by meter cube or other way, Joule per meter cube. So, it indicates the what is the energy, energy per unit volume. So, basically the half of the elastic strain in the strain that indicates the what is the strain energy associated with this particular body per unit volume.

So, therefore, we take the integral over the volume and this we can for a triangular component that we started with this thing, that we can consider that particular element also triangular element. So, in that particular element, one element we can estimate what is the strain energy. It is simply integration over the volume. So, some half of stress into strain into the d V is the volume, elemental volume.

We integrate over the total volume of the domain, then we can estimate what is the amount of the total strain energy associated with this particular system. But in this case, we are considering the first we are estimating what is the strain energy for particular one element.

So, element can be triangular shape or element can be other shape also; but this is the general formula of this stress into strain. But if you see there are 1, 2, 3 in a two-dimensional state, we can say that in two-dimensional state, we are talking about the two-dimensional analysis.

So, if we look into the two-dimensional analysis which is different from the plane strain and plane stress condition. Two-dimensional analysis, we are assuming the only the epsilon x epsilon y and the this shear stress component; that means, in the two 2D, 2D analysis we have if we 2D analysis, the this is the stress components these are the strain component.

So, do not confuse with the two-dimensional analysis with the plane stress and plane strain condition. Plane stress, plane strain condition that are the two different situations. In particular practical problem, which cases we can consider this is the plane strain condition; accordingly, we can from the D matrix formulation we can do and even it is plane strain condition.

But if it is two-dimensional stress state, then we can say these are the different stress components and this is the three different strain components. Now, two-dimensional stress state, then it is a strain into stress value, strain into stress value, strain into stress. But first two cases, this is the elastic stress and normal stress and normal strain; second case also normal strain, normal strain in other direction and third one is the shear stress and shear strain. So, this indicates total multiplication.

In general, we can say the total stress into strain and half is also there and integration over the volume d V is the elemental volume that indicates at the, that total potential energy of this particular element in a finite element domain. Now, with this particular element, in this case, we can see that strain and stress can be represented in this way also that in the matrix form, such that epsilon for this element is the is something like that epsilon x, epsilon y and gamma x y, this is the strain component.

And stress the sigma x, sigma y and tau x y these are the stress component in the column vector in the form of a column, we can write this thing. So, finally, we can write in this compatibility; accordingly, we can put the transpose of this thing such that this positioning of these things can be multiplied and elements can be.

So, from here to here, we can see the one single strain component in the column vector form, we have written the strain component and there is also stress component. So, epsilon e; e means indicates the for a particular element; so, then stress strain into strain into half into d V over the integration over the volume that indicates the total strain energy of this particular element.

Now, relation between the vectors of the stress and strain is given also in this way also; the stress can be not only this thing, stress can be you know that D matrix. In this case in general, D matrix and then, this is the total strain components. So, this is the one of the strain component and plus sigma 0; sigma 0 is the if in the particular problem there existence of some kind of the initial stress value.

So, then total stress of a particular step, this is the initial stress value plus this we can relate between the stress and strain and that can be the other component. So, then total stress can be sigma.

So, epsilon 0 can and if sigma 0 are the initial strain and initial stress respectively; but in this case, the epsilon the total strain minus epsilon 0, if there exists some kind of the initial strain value. So, then effective stress equal to epsilon minus epsilon 0 that can be this thing, while D is the given below.

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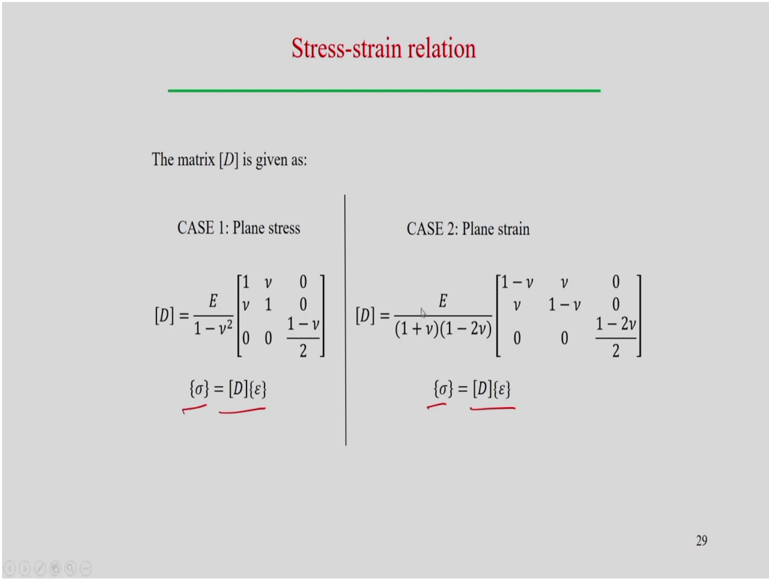

So, D matrix D can be given; the different form also, if we consider only the two-dimensional stress state, if we analyzing the two-dimensional analysis or if we consider the plane stress condition or if we consider the plane strain condition. Accordingly, this we can formulate, the sigma can be written in the form of a D matrix; sigma equal to D into epsilon. So, sigma equal to D into epsilon. So, in general, this is the D matrix.

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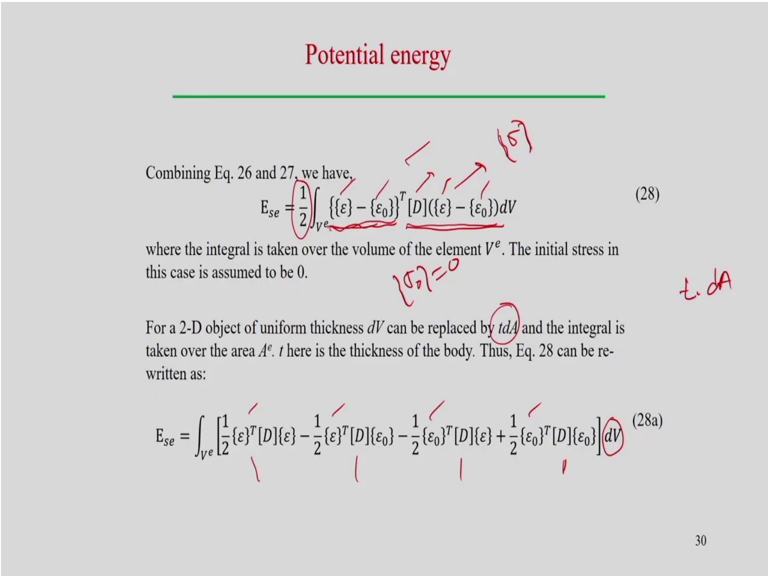

Now, once you look into all these things D matrix, now this energy can be represented like that this, this is the strain component D and the this is also strain component. Because this corresponds to the stress value and this corresponds to the strain value and this corresponds to the stress value.

So, stress simply D matrix, we used to relate between the stress and strain; only that purposes, we are using the D matrix and depending upon the situation, the different forms of the D matrix is possible. Now, this is the strain component and this is the stress component into d V and half, we just get it outside. Now, where the integral is taken over? The volume of the element particular element and the initial stress in this case can be assumed as the 0; that means, in this case is sigma 0 equal to 0.

So, there is no for to simplify the calculation, we are assuming there is no sigma 0. So, then initial stress becomes 0 in this particular case; but there may be some initial strain also, now for a 2-D object of uniform thickness delta V, so this is the expression and now, if we consider two-dimensional object.

So, d V can be represented like that, maybe you can use the constant thickness uniform thickness d V can be replaced t into d A. So, t into d A; t is the uniform thickness and it is uniform, but d A can be the elemental area in this part two-dimensional analysis and the integral is to taken over the elemental area A; t here is the thickness of the body.

So, therefore, this equation can be represented like that can be written something in general from like that we have kept d V also in this case. So, 1, 2, 3 and 4 components are there. So, therefore, 1, 2, 3, 4 components will be getting, if we multiply this thing. Then, this simply this in the matrix form the compatibility of the matrix can be maintained that particular position, we can keep at the transpose where the position of the stress or position of the strain in the in this particular cases.

So, therefore, accordingly, we can put the this transpose matrix T. So, to be very careful that looking into the expression and we can put some that whenever required to come make the compatibility, these things we can put the transpose also. So, therefore, it will be getting the will be getting the 1, 2, 3, 4 components in this particular strain energy component for a particular element.

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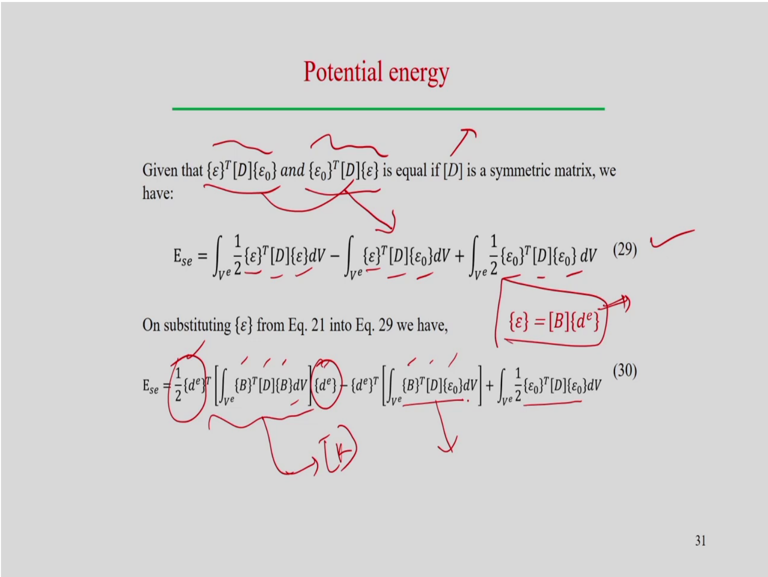

Now, given that this value and this value is equal, this can be equal that is why if the D is the symmetric matrix. If D is the symmetric matrix, so this value and this value can be equal if they are equal, then we can get this value. So, one term we can reduce these things and potential energy can be represent the epsilon D epsilon, epsilon D epsilon 0, epsilon 0 D epsilon 0 that is the way we can estimate this thing in terms of the D matrix.

So, therefore, we can get this expression equation number 29 also from this, looking into the symmetric nature of the D matrix. Now, on substituting epsilon, so therefore, we have already seen for a particular element, the strain can be represent with the displacement field d e, displacement field in the terms of the B matrix.

B is the strain displace; B indicates the strain displacement relation. So, therefore, now we are converting this form of the matrix from strain to, but if we look the formation of this matrix, we have in general the potential energy, we are trying to represent that all the strain components are there.

So, basically the stress value, stress components is replaced in terms of the strain; now, this energy the all the term is basically associated to only on the strain components. Now, we will look into the relation between the strain and displacement. Therefore, if we look into that relation strain and displacement, then it will be possible to replace the strain component in terms of the displacement.

So, the strain displacement relation, we have already established the relation the strain in terms of the B into de. B is the strain displacement matrix. Now, put this value. So, d e is we can see the d e is the basically the value of its not exactly variable.

So, rather the indicates the value of the displacement field in the node point in a particular node. So, therefore, d e we just keep it outside of the integral form and therefore, inside there is B is there B transpose D B into d V and d e is also there and similarly, B transpose D and epsilon 0 is the initial value over the d V.

So, here we can see in this case D just outside and other is the d e is inside, but I think these are the whichever is the variable in these cases D, even D can be also constant. Because the D relates only on the parameter the that properties of this particular elastic properties of the material.

So, therefore, D can be outside of this integral. Now, looking into what else, what we can do? Here also, we can see this relation, here also we can see also in terms of the this matrix. So, now once we get this expression, thus just replacing the strain value in terms of the B and d e.

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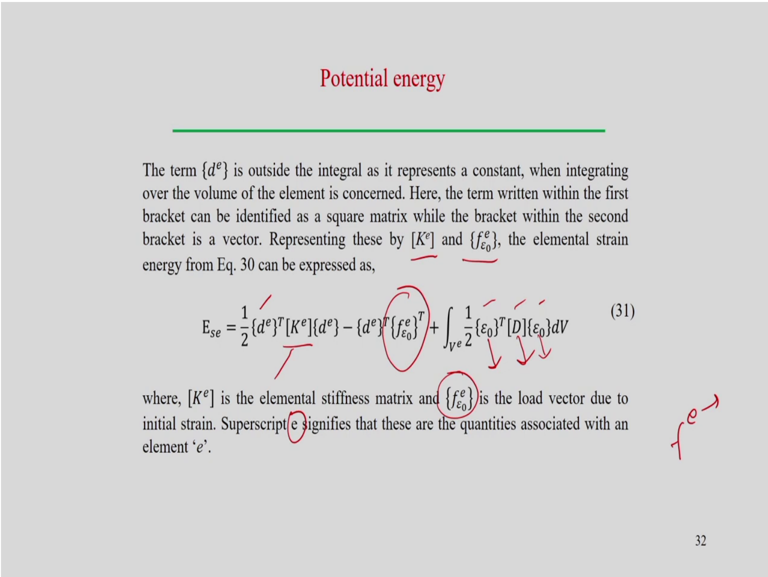

So, finally, we can see that the term d e is outside the integral as it represents a constant assuming this constant. So, therefore, that is why I keep it d e in the outside of the integral. But when integrating over the volume of the element is concerned, definitely over the particular element is concerned. Therefore, d e is kept outside.

Now, the term written within the first bracket can be identified as the square matrix, say we have seen this is identified square matrix; while the bracket within the second considered as a vector. So, therefore, this can be considered as a vector term representing this K e and f 0, the elemental strain energy from equation three can be represented like that.

So, therefore, we can see overall the elemental K, K matrix; then we have seen the we can leave the stiffness matrix also because B transpose D B represents the K matrix. So, from this equation, we can see that K matrix also. So, this represent the I think if we look into the K matrix expression of the K matrix, so this indicates the K matrix or for a particular element, from there we can see that this is the K matrix.

So, K d e is there, half of d e is there, this d e is there and this represents the K matrix, B transpose D B. You see d e K d e. Therefore, similarly the next d e into this one and here you can see that not exactly this represents the B transpose D and this strain components is there; but this initial strain components we can consider as a not actually variable.

So, therefore, this can be considered as a constant. So, accordingly, but B is the variable quantity such that it is representing in terms of this thing; f this thing in terms of the column is terms of the vector also, f e epsilon 0 that indicates the initial strain value. But other part half of the initial strain value if we see D value and this also epsilon 0.

This I think all these cases these are the represent the constant values not varying within the element itself. So, therefore, this can be considered as a constant value. Now, K is the elemental stiffness matrix. So, K is represent the elemental stiffness matrix and this represents the load vector due to the initial strain value.

So, due to the initial strain value, it can be considered as a load vector. So, therefore, subscript e signifies the that is the quantities associated within particular element. So, therefore, the e indicates, the f e, e indicates the one particular element. So, basically you are considering the what is the what is happening is a particular one particular element. So, for that element, we are calculating all these cases. So, therefore, strain energy can be represented in this matrix form, the stiffness matrix, elemental stiffness matrix and in terms of the load vector that is the potential energy represent in this way.

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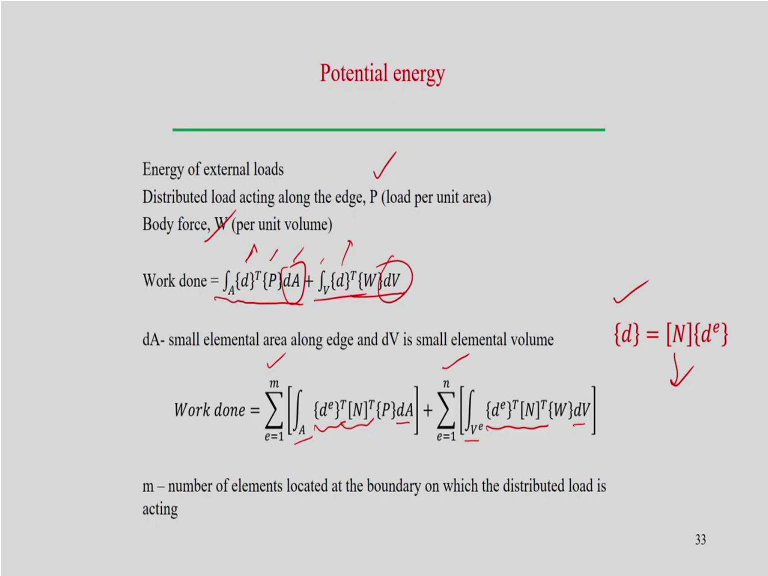

Now, energy for external load, now what happen what we can represent the external load because we have already estimated the one value, what is the strain energy associated, what is the associated with the external load. So, therefore, for example, distributed load along the acting edge P, then load per unit area.

For example, we are assumed this thing and body force equal to W; that means, per unit volume body force is act; but distributed load is acting per unit area and concentrated load is exactly acting exactly one particular node. Therefore, W is the body force per unit volume.

So, therefore, work done by the external load can be estimated like that see d displacement field transpose P into area elemental area d A. P is the distributed load. So, distributed load; that means, integration over the elemental area is basically actually indicates what is the load and over the area. If we see the integrand over the area associated with the distributed load;

but if there is a body force; that means, body force means it is acting per unit volume in that sense.

So, therefore, body force into over the volume if you put the integrand, then d is the displacement field; d is the displacement field. So, therefore, displacement field into this a load or displacement into load that indicates the work done in this particular case.

So, therefore, integration done because its elemental area, we consider the integrand elemental volume we consider. So, therefore, we can put the integrand sign over the volume or integrand over the area. Now, where d is the small elementary area along the edge because the distributed load is basically the load acting, not the whole is only on the along the edge. In the two-dimensional case, it is it must be acting along the edge.

So, what the surface area basically? The distributed load, but the d V is the small elemental volume because body force is acting throughout the volume. So, therefore, accordingly we have to consider and we know that d the displacement field displacement relate to the elemental value of the displacement in terms of the shape function N that matrix.

So, therefore, we convert this in total work done in terms of the d, d e in terms of the elemental and N, this value elemental value shape function and in terms of the displacement value of the element particular point. And then, over the area integration over the volume of this particular element and over the surface area which is involved basically which area is basically involved for acting the surface distributed load over the surface.

So, therefore, area and this it is acting for a particular element, the summation over the all number of elements. So, therefore, suppose there is a m number of element is basically m number of element is associated with the application of the some kind of the boundary load.

That means, this distributed load over the boundary, but not necessary all the element will be acting on the subjected to some sort of the distributed load. So, therefore, this, but all element has to be considered in if there is a this is the volumetric load; that means, body force.

So, therefore, e is the N number of elements over this thing or here m is the number of elements located at the boundary on which distributed load is acting; but small n is the number of elements for the whole domain of the analysis. So, accordingly, we can find out the what is the work done because of the distributed load as well as the kind of body force.

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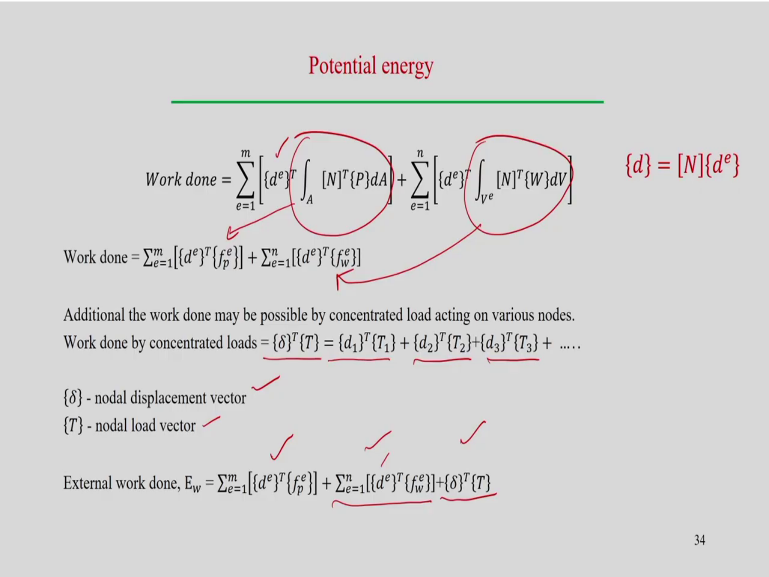

Now, total work done. We can see that it can be expressed the same we have already shown this thing and P and W so, work done can be represented in terms of the because d is the constant term. This integrand say function P dA, it can be represented the f e p and this is also the this can be represented by f w. So, it can be load vector, but load vector on this on the surface and the load vector over the volume.

So, that means, body force basically. This way we can represent these things the total work done and f p e and this is kind of f w e load vector. So, therefore, additional the work done may be possible by application of the some kind of the concentrated load only, acting on the various particular node.

So, when there is a concentrated load, then total work done by the concentrated load can be done like that. So, delta displacement a nodal displacement vector delta for example, in general nodal displacement vector and T is the nodal load vector. So, basically displacement d 1 T 1 basically on the one particular node exactly on the node point, the value is defined.

So, d 1 the displacement on the particular node with the and the concentrated load is acting in this particular node; so, d 1 T 1 similarly d 2 T 2 and d 3 T 3. So, basically T 1, T 2, T 3, we represent the concentrated load which is acting exactly at the node of the different node and definitely, this node point will concentrate load is basically acting only on the surface.

So, definitely it will be defined on the surface, and what particular point basically I can say that particular node point, the concentrated load is acting and d 1, d 2, d 3 can be the overall displacement. So, therefore, total work done because of the distributed load acting on the surface, concentrated load acting on the node point and body force is acting in general in any kind of particular system.

Then, total external work done can be represented in terms of the displacement field. This is the distributed load and this accounts the body force and this accounts the concentrated load. So, therefore, when it is accounting exactly on the concentrated particular node point; so, we are counting the work done simply the what is the load into the what is the displacement on that particular node point.

So, that is why directly delta into T, we can produce there also and here, but it is integration over the it is also required that thus, load vector can be produced, but the formulation can be different as compared to the concentrated load.

Now, we can see the difference this formulation the expression of the external work done; if there is a concentrated load, if there is a distributed load or if there is a some volumetric load is acting in this particular problem. So, then accordingly, we can look into the different expression for that.

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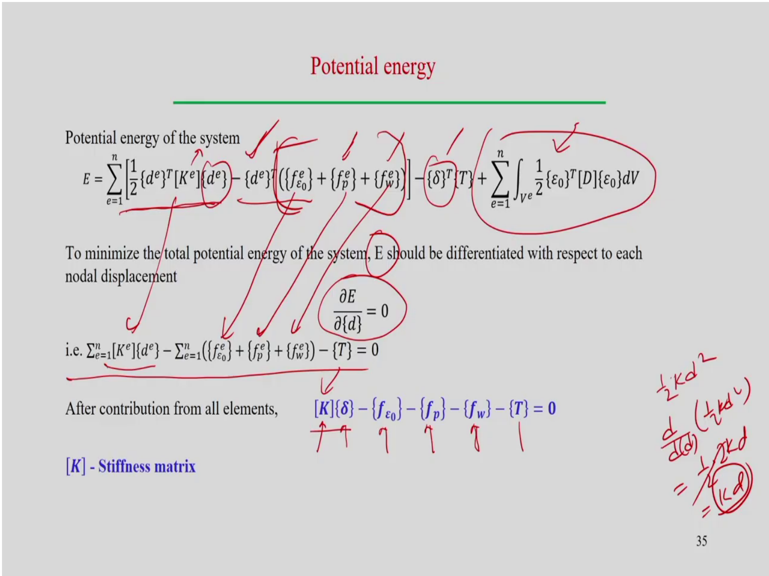

Now, potential energy of the system accounting this thing all these cases that we can see; the potential energy of this system is total system can be represented like that. This is the strain energy, these cases and this is also accounting the strain energy this case; but it is having the initial strain as considered as a load vector.

And then, here the concentrated load this accounts the this distributed load, this account the volumetric load, external load and this indicates the concentrated load and this terminology in that other these the constant terms also. But it is associated with this thing. Strain energy, if fixed strain energy associated with the some initial strain value accounting to this one.

So, once you do this thing. So, total potential energy of the system potential energy of the system this thing. Now, to minimize the potential energy of the system so, this can be differentiated with respect to the displacement nodal displacement value, then we can reach some formulation.

So, therefore, we can see that the minimization of the total potential energy of the system E should be differentiated with respect to each nodal displacement. So, delta E by d d. So, it should be 0. What is what way we can solve any kind of the minimization problem also, with that we find out the derivative with respect to that variable and the then, we make it equal to 0 and then, we find out some condition what is the to minimization of this particular system.

So, then once we do that, to do this thing with respect to del d E; d is the displacement of the nodal displacement field. So, therefore, it is a nodal displacement field we can see. So, it is a basically if we look into in general some K d squares value. So, once we do the half of K d square. So, d by d d of half K d square, we getting something like that half of twice K into d.

So, basically, we are getting the K into d that will be getting. So, then with respect to d e displacement, here we can getting the this term derivative K e into d e. And this will be always only the constant term will be there because we do the first derivative, these things and this there is some displacement field also there it is associated with the d e.

So, this term will directly come and this term will directly come; so, these three term because this was the accounting the displacement field. So, therefore, do the first derivative; but this is the concentrated load delta is the nodal displacement field, also then only the T will be coming this picture.

But this is the constant initial strain. So, therefore, the if we do the derivative, we have count this derivative equal to 0. So, then if we do, thus minimization total potential energy do the derivative this thing. This is the general expression in the matrix form also, you can get by following the minimization of the potential energy; make it equal to 0.

So, therefore, contribution from the all element we can finally reach that K delta this is the contribution from the all element means we are basically estimating for one particular element, making the summation of the condition for the all element. And in general way, we can find out once the contribution from the all element and this is the stiffness matrix K and delta is the displacement field at delta, we have already defined this delta equal to nodal displacement vector.

So, overall nodal displacement vector for the considering for the all nodes point; then, f load vector, it is also load vector. Its load vector concentrated load, distributed load vector, this is the volumetric load vector and this is load vector because of the initial strain value, make it equal to 0. And then, this is the general formulation after contributing from the all elements contribution and K is it is called the stiffness matrix.

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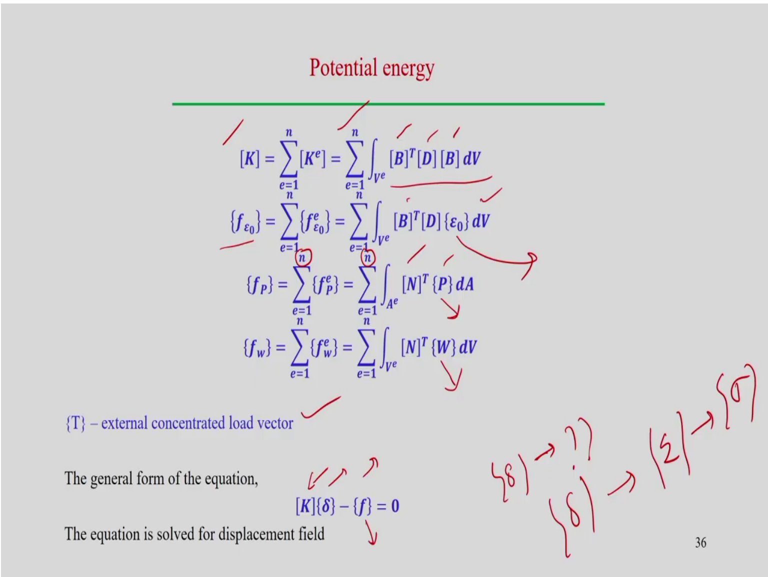

Now, we can see this is the general formula for the minimization potential energy and we can see the K equal to the stiffness matrix; K is the B transpose D B dV over the volume and contribution from the all element f initial for this initial strain component. And here, you can see contributing from the all element B transpose D, but the this will be the initial strain value.

Then, distributed load can be defined, but distributed load can be this can be different from n also, but we are in general we are counting the all element; but not all element may involve for the distributed load. So, may be selective load can be may be applied for as a boundary interaction for the distributed load. So, then accordingly, you have to decide the n value. It is N transpose P per unit area.

It is P should be defined the application per unit area and similarly, this can be defined for the whole domain for all element because it is a volumetric load. So, then in this case W can be defined the per unit volume. So, these are the expression matrix forms, then once you do the integration, one particular element and the sum of these elements this will be able to get the value of the K.

But of course, once we do for the one element, do the calculation, then we form the K matrix for the assembling from the contribution from the all elements. Then finally, we can pull the full stiffness matrix. Now, T indicates the external concentrated load, this can be directly be applied at the particular node value.

So, finally, you can reach the in general expression K into delta equal to f. So, it is a vector form, we can say the column vector K is the stiffness matrix, d is the displacement field. So, therefore, here we can solve for delta from this equation. So, that is the in general form of the stress analysis elastic stress analysis, we can form the K into delta minus f equal to 0.

So, we can go for the solve solution for the delta because delta indicates what is the displacement for the each and every node point. So, once we get the solution for the delta also, then from the delta, we can find out what is the strain value for the whole domain. So, then by looking into the relationship between the strain and displacement, once we get the strain value then we can find out what is the value of the stress.

Because by looking into the relationship between the stress and strain value, we can find out the stress value. So, this is the in general procedure for solving the stress analysis problem, but following the minimum potential energy. Now, we will try to apply this form this expression to look into the other; how we can solve, what we can progress for the elasto plastic analysis in case of the general stress analysis model.

So, thank you very much for your kind attention.