Finite Element modeling of Welding processes
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Module - 02
Fundamentals of finite element (FE) method
Lecture - 08
Elastic stress analysis - I

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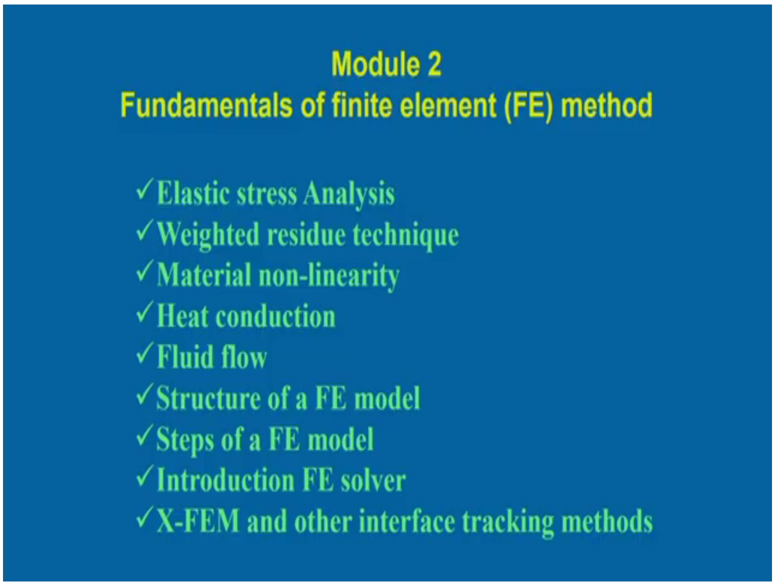

Hello everybody. Now, we will discuss about the application of the finite element method in modeling of the different aspects of fusion welding processes. So, we have seen that the fusion welding processes there are several aspects; that means, heat transfer phenomenon, material flow, stress analysis, distortion field all these are significant to analyze associated with the fusion welding processes.

Now, we will try to look into that how we can develop a model, but to do that we need to understand the basic aspect of the finite element method. So, in this case the discussion on this finite element method is not intended to exclusively the specific to the theoretical aspect of the finite element method.

Rather I will focus on most on the application, but very basic understanding of the FEM method and what we can apply this FEM method for the development of the model that is the main objective in this particular module.

So, we will start with that in finite element method fundamentals of finite element method. Actually we start with some very basic elements and we will gradually so, how we can develop the model and the elemental level and then what we can do the assembly and this then particular structure in a finite element base model.

So, to do that, we start with elastic stress analysis then basically to understanding of the stress analysis model first and then what we can apply the weighted residue technique in any kind of the finite element technique.

And then how to incorporate the material non-linearity that we will see and what brings the material non-linearity in any kind of the welding problem then gradually we will show how to develop the heat conduction model then fluid flow analysis how it can be done using finite element method.

And then to do that what may be the overall structure of a finite element method that will show and then the what are the different steps to solve this particular equation to get the solution of a particular problem associated to the welding process that will be done.

And then definitely one of the important part is the solver; because, in any finite element waste model, the main time consuming part is the solution methodology. If we choose the proper solution methodology or specifically if we choose the part good solver, then we can reduce the computational time also. Or of course, there are different types of the solver having advantage and disadvantage in maybe in certain problem this particular solver is more applicable.

So, that we will discuss in the for the in this particular part when will be looking into the different solver associated with any kind of the finite element waste program. Then some deviation from FEM; that means, extended finite element method will see that what is the application of the extended finite element method as well as the other interface stacking methods is associated with the welding process, because it is quite possible in welding process that it is necessary to predict the free surface profile or solid liquid interface.

So, that kind of phenomena is it is necessary to understand the integral part of the finite elements model that how we can develop some kind of the interface tracking method. So, little bit discussion of that particular interface tracking method will be done at the end of this particular module.

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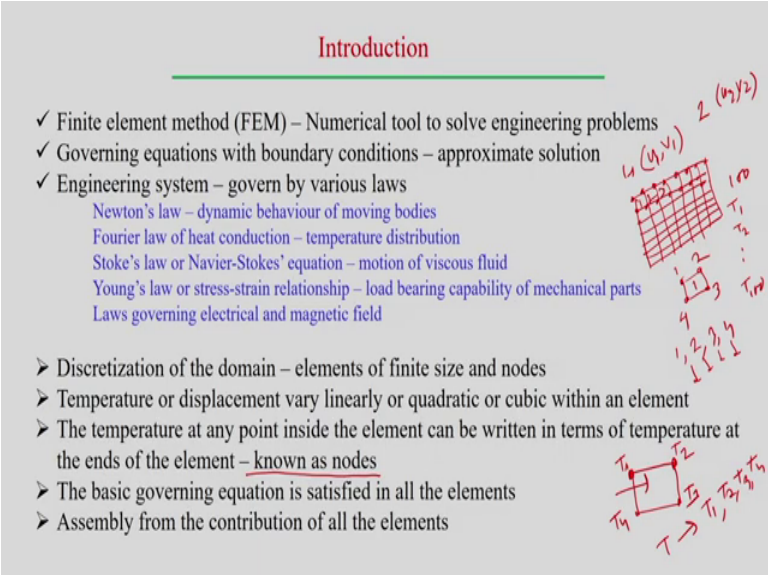

So, start with that this stress analysis, but before that when there is a need to look into the finite element method because finite element method we normally consider as a mathematical tool or as a particular tool to solve the engineering problem. So, that is the objective.

Now, in which cases we can need when there is a need of following this finite element method. First thing is that we know that any kind of the governing principle of governing law is represented by some normally partial differential equation or differential equation and along with some boundary condition also.

So, when this partial differential equation there is a there does not exist any kind of the analytical solution of this partial differential equation because of the complexity of the problem or several physical phenomena is driving this particular problem. Then it is not always possible to solve this particular equation which represents the physical problem.

So, in that case, there is a need to solve kind of the approximate solution. So, if we want to go for approximate solution of a partial differential equation, then in that case we can add up the finite element base method. So, it is a basically finite element, the solution is the not exact solution rather I can say the approximate solution we can get.

Exact solution will be getting for a differential equation if in that cases there exists some analytical solution available in this particular problem; that is represented by some kind of the differential equation. So, therefore, governing equation along with the boundary condition is solved using the finite element method then we get the approximate solution of this particular physical problem.

Now, we know the engineering system they are governs by the various laws. Basically if we see the Newton's law, the dynamic behaviour of a particular moving body if we want to analyze this thing, then we need to apply the Newton's law. Then Fourier's law of heat conduction also if we want to understand that temperature distribution in a particular body or to get some idea about the temperature development there variation then we need to follow the Fourier's law of heat conduction; that is a basic governing equation.

Now, suppose we want to know what is the metal flow field or viscous motion of the viscous fluid then it is not know the we need to follow the governing equation as a stokes law or Navier Stokes equation and that if we solve this equation, then we can get the output as a we can able to analyze the motion of the viscous fluid flow.

For example, even if we want to know the stress distribution or residual stress generation in a particular body then we need to know what is the Young's laws or stress tests stress strain relationship on how to apply the plasticity law. All this kind of the theoretical knowledge is required to understand table of the model associated with the stress analysis or displacement on deflection in a particular body also.

Even laws governing the electromagnetic or electric or magnetic field also. So, certain that represent with some equations also and that solving that particular equation with the appropriate boundary condition and that to solve this particular equation we use the tool as a finite element base finite element method. So, it is very important or significant in case of the problem solving cases in engineering problem solving I am talking about. So, it is very useful this to understand this particular tool.

Now, what are the basic steps in the finite element method? So, we start with these things for example, we want to know any kind of any kind of physical process for example, even talking about the welding process also. So, welding process we are interested to know that what is the temperature distribution or what is the material flow field when the application of the heat on a particular domain.

Now, to understand these things we have to fix the domain first. So, we have to fix the domain through within that domain we will be able to solve this particular governing equation along with the boundary conditions.

Now once we select the domain and then first step for the application of the finite element method the discretization of the domain and creation of the small small elements. And within that element such that when we choose some governing equation and that should satisfy this part within that particular domain or maybe I can say that if satisfy the governing equation is a particular node point.

Say for example, we start with suppose this two-dimensional domain it is a this is the solution domain. So, within this domain we want to solve the heat conduction equation Fourier law of heat conduction. It is well known that the governing equation is well defined and according to the problem, we can define the boundary conditions say different manufacturing process or different manufacturing processes having the different kind of the boundary condition may have.

So, look into this boundary condition then we can solve the problem, but before that when you apply the finite element base finite element method or this particular domain say discretization of the domain is required. So, small small element we can create and the solution accuracy that depends accuracy of the solution output depends on the what is the size of the element that vary matters.

So, as small as possible we can create this domain. And of course, other side also if you create two very small element also a number of element then in that cases number of elements for a fixed domain increases; so, that when the number of element increases the in general the competition time will also increase. So, it is can cannot be compromised the quality of the solution and the computational time to get a particular to solve a particular governing equation along with the boundary condition.

So, once we do that, we can get this is the element. So, element 1, 2, 3, we can put the number. So this, these are the particular element numbering we can put, we have to keep track also that this is the element number. So, now, one element is associated with the node point. So, this node points has to be count. So, for example, element 1 is associated to the 1 2 3 4 this is the node number.

So, in this case it is more important this element of finite size along with the associated node and keep on accounting this elements how many number of elements in a particular domain that all information is required. Apart from that one secured element geometric domain. So, then the coordinate of the each and every node.

For example, this node point 1 2 3 4. These are the node each and every node point what is the coordinates of this particular node point; that information is also necessary to develop the finite element waste model.

So, that is the discretization of the domain is very important and this case is, but it is not necessary we have to discretize the domain exactly it is a very symmetric kind of way, but we can create some triangular elements also or other kinds of the elements or sometimes in the commercial software this available automatic machine option also available.

So, depending upon the problem and which cases the gradient exists maybe variation is very steep. In that cases we can use the fine mess also; that means, the element size can be very small also and in certain position the change is not gradient of the variable is not much. In that cases we can use the course means the bigger size of this thing. So, all this you will get all information is important to understand the finite element based model development.

Now, once you look into basic elements of the this thing. Now within particular element the what are the variable? So, variable means for example, in the thermal analysis temperature is a variable in this cases or in case of the stress analysis model the displacement can be the variable.

So, in that case temperature or displacement can vary within a particular element either linearly or quadratic or cubic. So, once the temperature or displacement vary linearly. So, this point it is necessary to get the information what is the value of temperature say T 1 at this particular point and what is the value of temperature T 2.

We not necessarily to know it is varying linearly. So, not necessarily go the information in between, but if it is quadratic maybe one quadratic equation if you follow that another node can be there also.

So, then the values T 1, T 2 and T 3 maybe it is necessary to define if the variable temperature displacement vary quadratically within this element. So, that according to this problem we can vary linearly it quadratic or cubic within an element can be considered. The temperature at any point inside the element can be written in terms of the temperature at the ends of the element which is known as the nodes. So, at the end of the elements which is known as the nodes.

So; that means, temperature can be written in terms of the node point. For example, if it is a linear variation of the variable within this thing. So, T 1 to 2 suppose T 3 and T 4. So, then temperature can be represent in T 1 T 2 T 3. So, in the by using the value of the at the different node point. So, that temperature variable can be represented for a particular element also T as a function of T 1 to 2 T 3 and T 4.

Now, such that basic governing equation is satisfied in all the elements also. So, if we over the domain we discretize these things. So, we have to discretize in such a way that the governing equation should satisfy for all the elements. Now, once is done governing equation satisfied for all the elements or other way we can see what is happening one particular element, then assembly from contribution from all the elements can be done also. Then once we assembly from all elements is done, then we can get 1 linear system of the equation.

So, then we solve the linear system of the equation to get the output. What is the output we can get what is the value of the temperature of this thing. For example, in this particular domain we can get the what is the value temperature for each and every node point. So, then if there are say for example, 100 node points you will be getting the T 1 T 2 dot dot dot T 100.

So, that values will be getting as a output. Such that we will be able to know what is the distribution of the temperature and that distribution of temperature is exactly defined each and every node point.

So, this is the general procedure to develop a finite element model or we start with this finite element model assuming some kind of the temperature distribution. So, then same thing may also happen may be similar vary if we change the variable in for example, if we consider not only temperature suppose we want to do the stress analysis.

So, then in that cases the will be getting the displacement field each and every node point. But displacement field each and every node point it can be off if it is a two-dimensional problem.

So, we will be getting the two displacement field u 1 is node number 1 u 1 v 1 there is a displacement in 2 direction and similarly u at node number 1. So, node number 2 will be getting the displacement value u 2 and v 2. So, this way we can get the if it is 3 dimensional problem then each and every node point will be getting the variable as output in the case of stress analysis model the u 1 v 1 w 1.

So, this the displacement components along x y and z direction. Same thing if we follow the flow analysis the fluid flow analysis problem in that cases we will be getting the each and every node point what is the value of the different velocity components. So, in that cases maybe u 1 u 2 u 3 or u 1 v 1 w 1 that kind of the 3 different components will be getting each and every node point.

So, depending upon the dimensionality of the problem we can get the analysis, but in case of temperature analysis, thermal analysis will be temperature the scalar quantity we will be getting the temperature distribution; that means, we will be getting the temperature value each and every node point the single value of temperature.

So, that is way or also sometimes the we can combine the thermo mechanical model; that means, we can get the each and every node point not only that temperature if you follow the thermo mechanical model, we will be getting the output as a temperature as well as the displacement fluid. Both we can get each and every node point. So, that is the overall idea or overall procedure for the development of the finite element method.

But now we will focus on the what we can create the we will focus what happens when a particular element. So, that, we will focus on the element.

So, once contribution from each element can be done then after that we can assembly for each and every element then after assembling we solve this whole domain, solve the problem for the whole domain and after will getting the output as a variable as it can be temperature and temperature for all each and every nodes.

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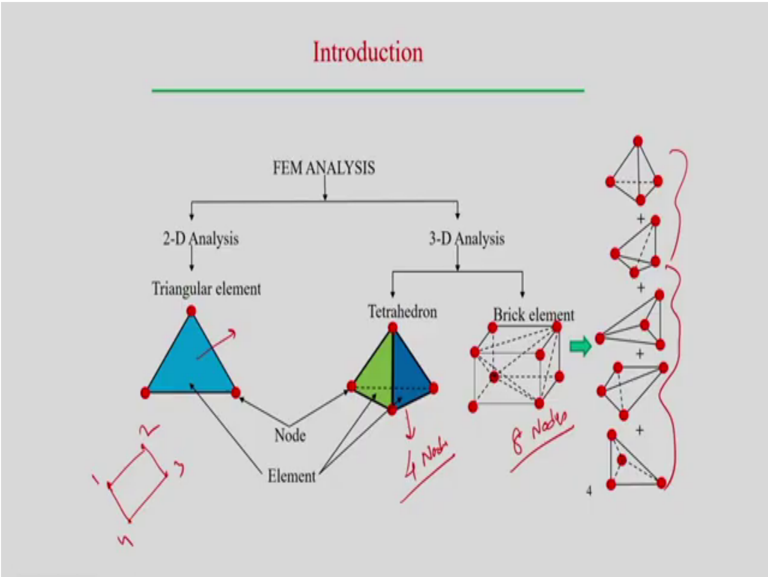

Now, look into this the FEM analysis finite element method maybe we look into this if we did two-dimensional analysis and 3 D analysis also. two-dimensional analysis for the simplicit we assume the triangular element. So, this element is defined in such a way that triangular elements. So, this is the element shape of the element and then there are 3 nodes in it in this particular element.

And then if it is 3 dimensional analysis, then we can start with the tetrahedron element or we can start with a brick element. So, in case of the tetrahedron element we it is having 1 2 3 4, 4 nodes there are 4 nodes tetrahedron, but it is an if it is brick element there are 8 nodes. Or other you can see this brick element can consist of the so many tetrahedron.

So, such that will be getting the this thing 8 nodes. So; that means, we can discretize the domain the by choosing the different types of the elements. So, if 3 dimensional domain we can choose the tetrahedron element we can use the brick element also and even if it is a two-dimensional analysis we can choose the triangular element or we can choose some other maybe in this cases we can use this type of element also kind of rectangular shape element this kind of elements.

So, there are 4 nodes 1 2 3 4. So, depending upon the problem we can choose either these different types of the elements and each element is associated with the different nodes. Now then in this case now will see that what we pick up only the triangular element for two-dimensional analysis, then we will see how the different equations or different relations can be formed in associated with the stress analysis problem.

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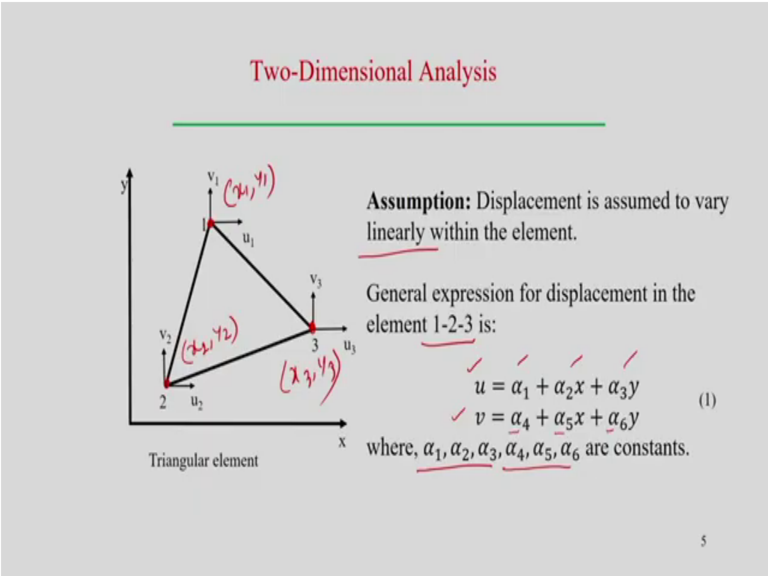

So, let us consider the two-dimensional analysis and using the triangular element. Having this triangular element the 3 node points are there and this variable displacement is assumed to vary actually linearly within this element.

So, the variation of the particular variable to in this case is displacement it is varying within the element a linearly. So, that is very important linearly. So, once you assuming the linearly variation of the variable. So, the general expression for displacement in the element 1, 2, 3; so, this is element 1, this is node 2 and this is node 3.

So, in this particular element 1 2 3 say is form particular elements. So, at the point at the node point 1 2 and 3 node point the displacement field is defined that u 1 is that; that means, displacement along x axis and v 1 displacement along y axis. Similarly, at point node 2 displacement along x axis u 2 displacement along y axis it is v 2 and similarly for node 3.

So, this way, but general expression for displacement field so, you assumed u as a variable it considered as a displacement field and along x axis. And v is also variable the displacement field along y axis in general. So, now, then since it is varying linearly we can assume u as the expression of alpha 1 alpha 2 into x and alpha 3 into y. So, x and y also variable in this particular case. So, if this is x axis this is y axis.

So, similarly v can be alpha 4 alpha 5 and alpha 6 into y. Now, in this case alpha 1 alpha 2 and alphas there are 1 2 3 6 are constants value. Now, if you want to evaluate this value all these 6 constant value; the alpha 1 alpha 1 alpha 2 to alpha 6.

Then with a known quantity the known variable maybe we can see the geometric value, because the coordinates of 1 2 3 is that is well defined. So, coordinate of 1 node 1 its say x 1 y 1 that is the defined value within this domain and that is known quantity maybe in within this domain.

Similarly, point 2 also, the x 2 y 2 say for example, node 1 the x 1 y 1 is the coordinate y it is also x 2 y 2 is the coordinate value and x 3 into y 3. So, with respect to the reference axis this x 1, y 1, x 2 y 2; that means, coordinate of these 3 points are well defined. Now, the we can evaluate these value alpha 1 alpha 2 and alpha 3 and similarly alpha 4 alpha 5 and alpha 6 that in terms of the x 1 y 1, x 2 y 2, x 3 y 3. We will see how it can be done.

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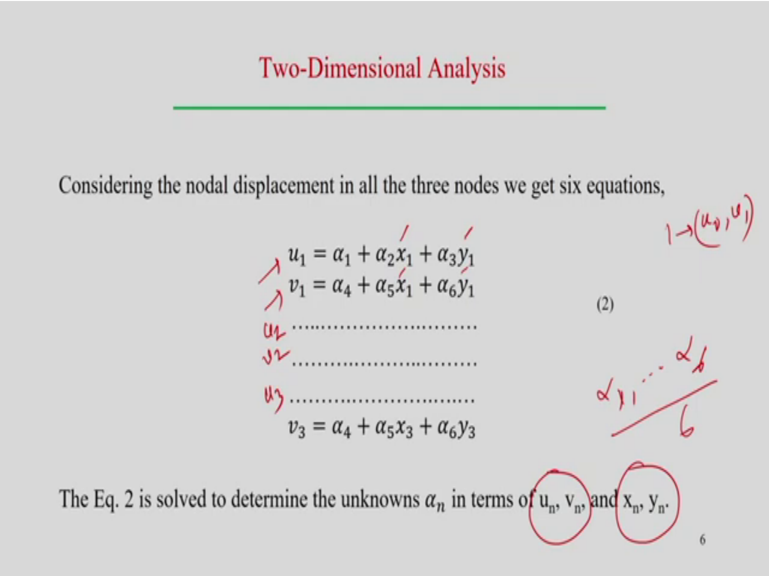

Considering the nodal displacement in all the 3 nodes we get assuming that nodal displacement in node 1 is the u 1 and v 1 so, but u is the variable in this cases displacement u and v are the variables in displacement fields. So, we put this thing u 1 equal to. So, u 1 equal to alpha 1 plus alpha 2 x 1 plus alpha 3 y 1.

So; that means, in sums of x 1 y 1 given similarly v 1 this can be x 1 y 1 also, but constants are different in these cases. So, similarly if we follow all this u 1 v 1 u 2 v 2; that means, all 3 node points then you will be getting to the 6 equations ok. So, u 2 v 2 and u 3 v 3; so, u 1 v 1, u 2 v 2 and u 3 v 3. So, total 6 equation will be getting.

So, now, we have the constant term which is alpha 1 to alpha 6 there are also 6 constant term. So, if we solve this equation then we will be able to know what is the we can evaluate the value of the alpha 1 to alpha 6 in terms of the known quantity u 1 v 1 or in terms of the x 1 y 1 all these are these things.

Anyway there are 6 equation and 6 unknowns also. So, and all this they are linear related. So, therefore, by solving this linear system of equation we will be able to know what is the value of the all constant value. Now, equation 2 is solved to determine the unknown alpha N in the values of a in terms of the u n, v n and in terms of the x n, y n.

So, n is the basically x n is depends on the what is the number of nodes in this particular element. So, in this case there are 3 nodes.

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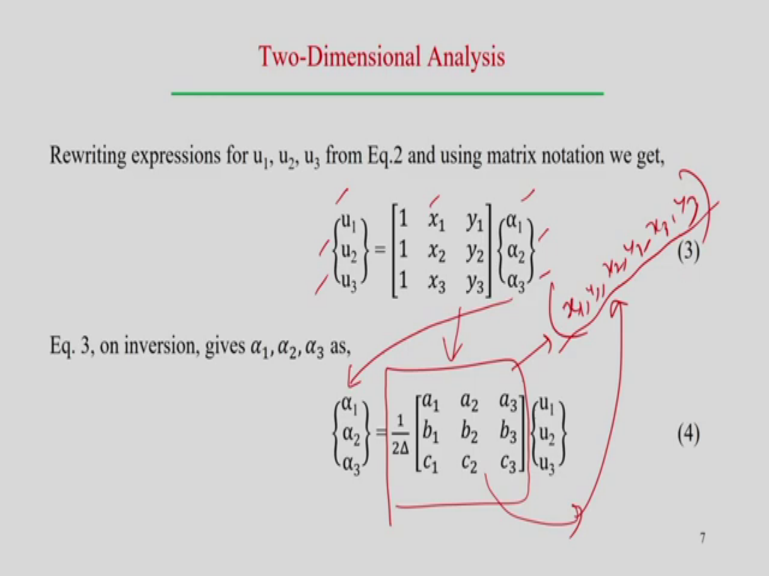

Now, u 1 rewriting this if you pick up the only the components of the u 1 3 equations in terms of the u 1. In the matrix form we can write this expression using the equation number 2. So, only in terms of u 1, u 2 and u 3, u 1, u 2, u 3 if you see x 1 x 1 y and x 2 y 2, x 3 y 3 in terms of that and its alpha 1, alpha 2, alpha 3. Just similar expression of that representation of this system of the equation in the matrix form.

Now, once you know that u 1 u 2 u 3 in this matrix form. Now, if we follow the inversion of this matrix, then we can evaluate what is the value of alpha 1 alpha 2 alpha 3 that should be in terms of the x 1 y 1 and u 1 u 2 u 3 or maybe we can say that if we do this transformation maybe alpha 1 alpha 2 alpha 3 simply in terms of the known quantities in terms of the coordinate x 1 y 1 x 2 y 3, x 3 y 3.

So, in that form will be getting this value of alpha 1 alpha 2 and alpha 3 this constant terms.

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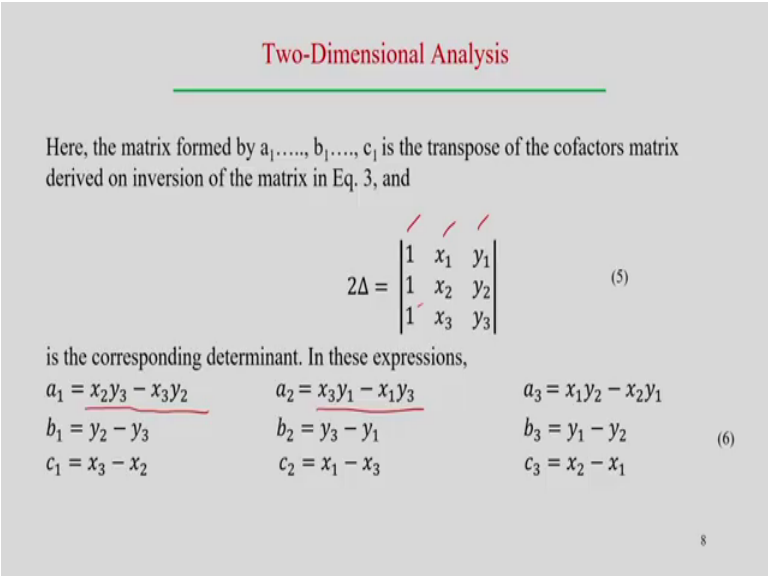

Now, we see that how it can be done. So, here the matrix formed by the a 1 b 1 c 1 we have written this a 1 b 1 c 1 actually in terms of this thing, but is this a 1 a 2 a 3 b 1 b 2 and b 3 c 1 c 2 and c 3 all in terms of as a function of in terms of the known quantities. Basically coordinate of these 3 node points; in terms of that we can write.

So, therefore, this matrix formed by a 1 b 1 c 1 is the transpose of the cofactors matrix derived by the inversing of this on the previous equation that we have seen and, but in this cases 2 delta represent the determinant of this expression. So, 1 x 1 y 1, 1 x 2 y 2 and 1 x 3 y 3 it is basically a representation 2 delta.

Now, a 1 can be a 1 is the in terms of x 2 y 3. So, that expression can be done we can see also a 1 a 2, b 1 b 2 in terms of x y x 1 y 1 z 1 I think like that a 3 also. So, all a 1 a 2 are related to this in terms of the coordinate value of this particular node point.

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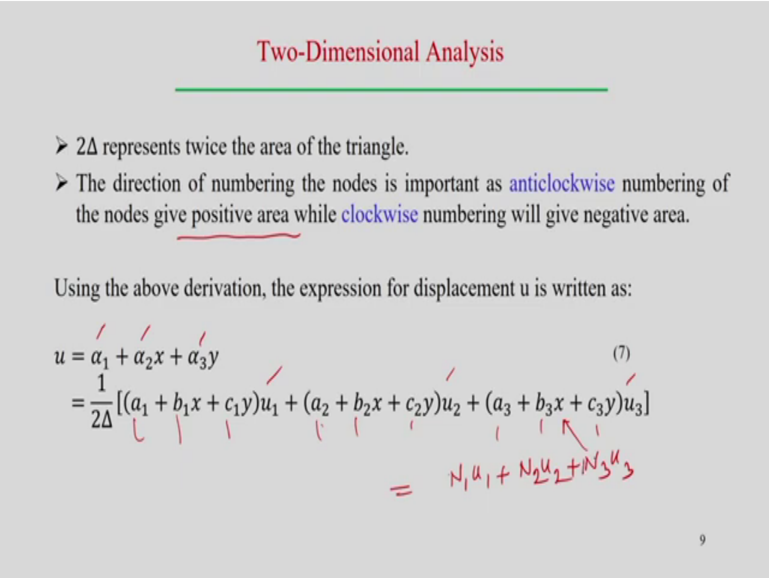

Now, in this case 2 delta represents twice the area of the triangle that we have seen this direction of the numbering of the nodes is important also. So, it says we should follow the anti clockwise direction numbering of the nodes given positive area and then positive area while the clockwise numbering will give the negative area. So, we have to be careful to choose that choosing the numbering of this thing node we should follow some anti clockwise direction.

Now, using the above derivation the expression for the displacement u is written in the form of a. So, therefore, we have seen the u in terms of alpha 1 alpha 2 and alpha 3 and x and y is the variable and u is also variable. So, now, if we represent if we replace the value of the alpha 1 alpha 2 alpha 3 we have that we have already derived in the form of a by inversing in that in terms of the x 1 in terms of a 1 a 2 or other way in terms of x 1 y 1 to x 3 y 3. So, using that known quantity.

Now, if we rearrange these things then we will be able to see that u can be represented by putting the value of this 3 constant alpha 1 alpha 2 and alpha 3. So, here we can see 1 by 2 delta will come here and we see a 1 in terms of b 1 c 1, a 2 b 2 c 2, a 3 b 3 c 3 and here u 1 u 2 and u 3 are there also. But some little bit of manipulation is required or some calculation is also required to rearranging of this variable we can put we can get this kind of expression.

So, u in terms of the alpha 1 alpha 2 alpha 3 just replace alpha 1 alpha 2 alpha 3 in terms of the other known quantity.

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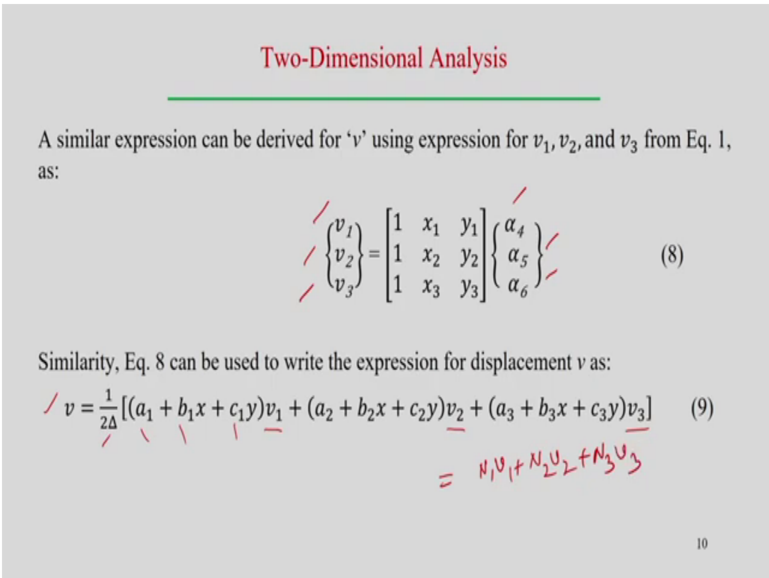

Now, a similar expression can be derived also for v also using the expression v 1 v 2 and v 3 and from equation 1 also. So, then the same exercise we can do. So, v 1 in terms of this thing that if you remember that alpha 1; v can be represented that the alpha 4 alpha 5 and alpha 6 in that constant also and such that v is related to the v 1 v 2 and v 3. So, same kind of expression can also be written.

So, then v 1 v 2 and v 3 can be in the matrix form from the linear system of the equation we can find out in terms of x 1 y 1, x 2 y 2, x 3 y 3 and as well as the an alpha 1 alpha 5 and alpha 6. Similar way if we find out the value of the alpha 4 alpha 5 alpha 6 these are the constants value then similar exercise we can pump out. So, what we can derive the values of the u 1 u 2 and u 3.

Same way we can find out the alpha 5 alpha 4 5 and 6 and we can put this expression v v can be represented this in terms of the even a 1 a 2 and the this 2 delta term a 1 b 1 c 1 in terms of v 1 v 2 v 3 also and remainings are x for the variable. So, from here we can represent u in terms of the u 1 u 2 and u 3 and v in terms of the v 1 v 2 v 3 as well as the variable x and y.

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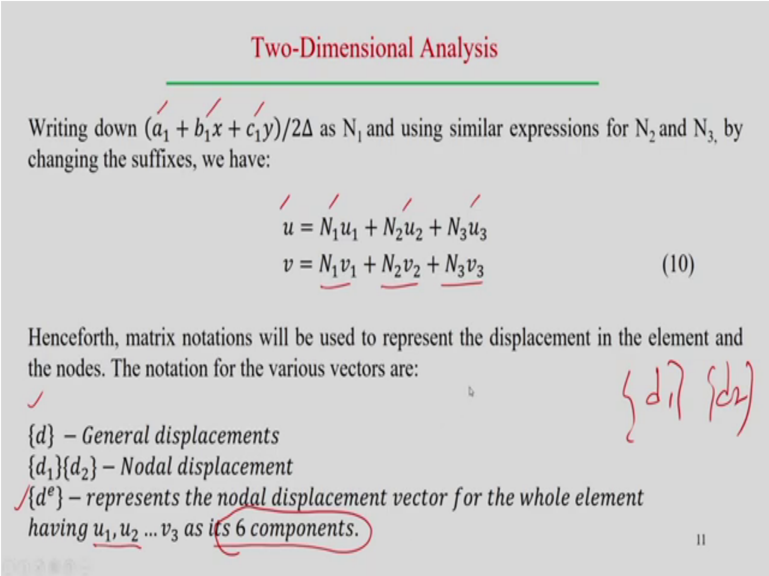

Now, in this expression if you see the v a 1 v 1 x plus u and y into v 1. Now, other way you can see if we assume that a 1 b 1 x plus c 1 y by 2 delta is represented as N 1. Say expression the N 1 is that so, if you follow this expression we can see that N 1; u can be represented N 1 u 1, N 2 u 2 and N 3 u 3.

So, like that also we can see that it is a basically we can see N 1 u 1 plus N 2 u 2 plus N 3 u 3. We can see that for example, N 3 u 3. N 3 u 3 is the in terms of the a 3 b 3 and c 3 and variable x and y that can be represented as a function N 3 and u 3 is there.

So, similarly for v also we can write that N 1 v 1 plus N 2 v 2 plus N 3 v 3 in this way we can write this expression that v also. So, these two variables. So, finally, similar expression for N 2 and N 3 assuming this thing. So, changing the suffixes we can find out u is basically that N 1 u 1, N 2 u 2 and N 3 u 3. So, u 1 u 2 enter the displacement field associate along the x axis also at the different node point this u 1 u 2 and u 3 is defined at the node 1, 2 and 3 the displacement along x axis.

Similarly, v in general v is the variable the displacement we can see along y axis that consists of the N 1 v 1. So, v 1 is defined at the node 1 v 2 is defined node 2 and v 3 is defined and node 3. So, this way in general way you can present we can write this equation and the different functional form.

Now, henceforth matrix notation will be used to represent the displacement in the element and the nodes. We can use instead of this thing we can use some matrix notation to represent the displacement field also. The notation for the various vector quantity we can see d can be (Refer Time: 30:11) as a general displacement.

General displacement, we can assume d can be the variable. So, in it is a within the d can vary is a what are the x and y is variable quantity within this element. Similarly d can be the general displacement field, but once we define d 1; the d 1 represent the displacement field with the nodal displacement.

So, basically when denoting this 1 suffix 1 that represents the displacement field particular node point. And similarly once we d e we represent this thing third time also d e represent the nodal displacement, but vector for particular element. So, it is a particular element and d is a general displacement and once we d 1 d 2 it depends the displacement in the particular node point and d e represent the overall the nodal displacement vector in a particular whole element.

So, therefore, d consists of the maybe in terms of the other nodal quantity also displacement we will see. So, definitely here we can see the d e represent the nodal displacement vector, but it is having the whole element components; that means, u 1 u 2 v 1 v 2 and u 1 u 2 u 3 and v 1 v 2 v 3; that means, total 6 components consist is having there in the nodal elemental displacement vector.

So, these are we use the notation, then it will be easy to understand that how this all this functional form how we can relate the 1 variable to another variable.

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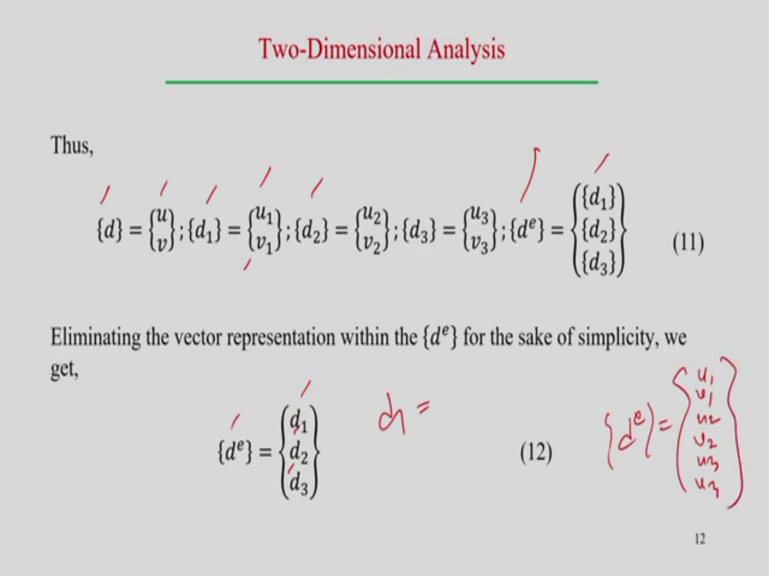

Therefore, d is the general displacement we see it is the variable its consist of the u and v is the in this case we can see it is the column vector d 1 is the nodal displacement value. So, nodal displacement it is consist of the u 1 v 1 because this particular nodal displacement both x and y components are there.

So, d 1 represent the u 1 v 1 similarly d 2 d 2 represent the u 2 v 2, d 3 also u 3 v 3 and finally, d e. So, total elemental displacement and then is consists of the displacement of all the 3 nodes d 1 d 2 and d 3. So, this d e represents that all the this thing.

Now, eliminating the vector representation within the d e also for the sake of simplicity we can see that d e simply element d 1 d 2 and d 3 compressor are there. Definitely d 1 again it is a consists of the u 1 v 1 and d 2 u 2 v 2 and d 3 u 3 v 3. So, sometimes we can write also d e can we say u 1 v 1 u 2 v 2 and u 3 v 3. So, this way also we can right. So, these are the represent the total elemental displacement.

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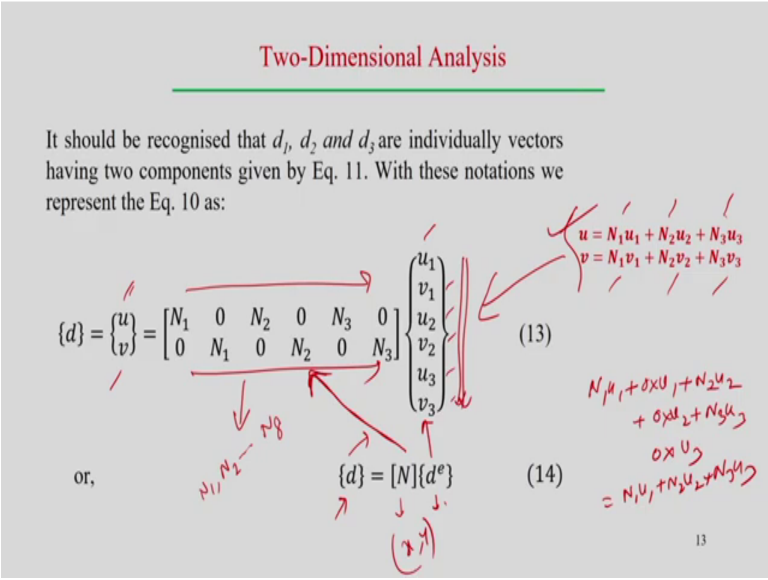

Then it should be recognized that d 1 d 2 and d 3 are individual individually vectors having the two components that is already defined. So, d 1 represent itself the two components are there u 1 and v 1 given by the equation 11.

So, therefore, with this notation the equation 10 can be represented as the d can be represented that d is the u into v. u v is the variable and we have already seen that u can be represented that u equal to N 1 u 1, N 2 u 2, N 3 u 3 that we have already seen. Similarly b is represented by N 1 v 1, N 2 v 2 and N 3 v 3.

So, therefore, u and v that can be in the matrix notation we can write this way also that N 1 0, N 2 0, N 3 0 and this is the matrix N 1 in the we separate out this variable in one set it we can keep in the matrix form that the variable N other set with the variable the displacement. So, then u 1 v 1 it is a completely v 1 u 2 v 2 v 3 v 3 and this way we can write this expression also.

Now, you can cross check also we can multiply this matrix notation then we can see that N 1 u 1 plus 0 into v 1 plus N 2 u 2 plus 0 into v 2 plus N 3 u 3 0 into v 3. So, finally, it is coming N 1 u 1 plus N 2 u 2 plus N 3 u 3.

Similarly for v also; so, similarly this and this if you multiply we will be getting this expression. So, we can validate this the u equal to N 1 u 1 simply the set of the equation we have written in the different form and to separate out these 2 matrix; so, such that we can write in the matrix from this equation. So, in general we can see more compact from we can write the d is equal to N; N is this matrix and d is the elemental displacement vector.

So, N is the in the form of a matrix and d is the column vector in the form of a column vector we can write. So, basically displacement field we can see the we assume d is the variable quantity displacement field equal to N into d e. And d e is defined the d e is actually defined the displacement of each and every node point, but having both the components the x components as well as the y components.

And if you see the N itself is a function of although we are taking the N 1 N 2 N 3, but N itself is a function that we have seen also with its a x y, N can vary with respect to x and y, but d is fixed. d is only the defining the node point. And here also in that sense d can be also variable which is combining the effect of the this the cip N as well as this function N as well as the displacement d.

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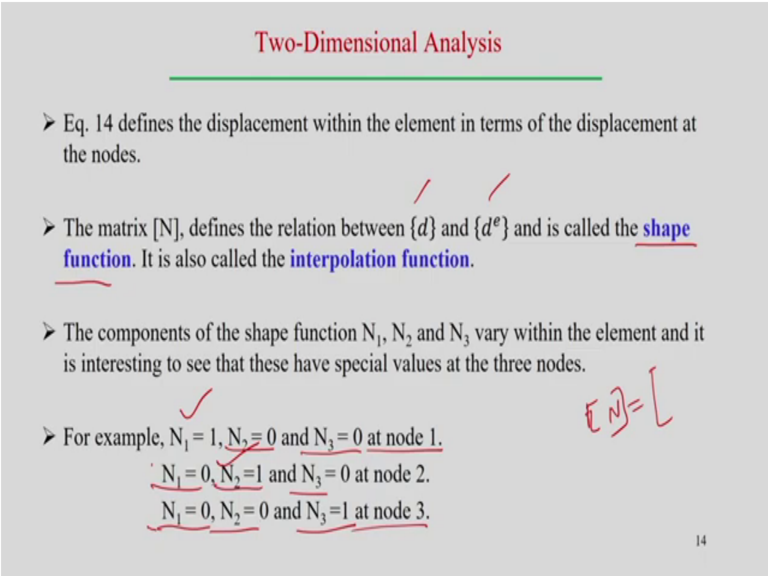

Now, therefore, equation 14 defines the displacement within an element in terms of the displacement at the node. So, definitely we can see that equation in the displacement field, but it is a this displacement field is connected with the displacement each and every node point. We connected through the functional form of N. So, therefore, elements of the displacement at the nodes.

Now, in this case the matrix N which is defines the relation. So, matrix N is actually defined the relation between the d and d e. So, d is the general displacement and e is the elemental displacement which is defined in the each and every node point. Therefore, N is called also the shape function. So, N is called a shape function and it is also called the interpolation function.

So, we will see we will look into this that how the shape function is defined. So, shape function is basically link is the general displacement within a particular element with the associated with the nodal displacement; that explicitly defined each and every node point.

So, now, shape function having these components of N 1 N 2 and N 3 and that can vary within the element also and it is interesting to see that these have this particular special value each and every 3 nodes also.

It means that the shape function N is that we have seen the shape function N for a triangular element how what may be the shape. For a shape function N for triangle elements it follow this particular relation also that we can see this N is defined in this particular situation.

But this arrangement the shape this arrangement of the N 1 N 2 and N 3 or maybe it can the other components also N 4 may also come if there is a change in the shape of the element. So, if we consider the some other kind of the element for example, if we consider the brick element.

So, if we consider the brick element then in that case the N can be N 1 N 2 and up to N 8 and they follow certain arrangement depending upon the how many nodes are there in a particular element.

And then accordingly we can arrange this thing. So, N is basically called the function and which is defined as a function of the depending upon how many nodes are there based on that we can define the shape function N. But if you look into the special properties of the shape function we can see also in the particular case.

N 1 equal to 1 at node point 1 at node 1, but other cases the N 2 equal to 0 and N 3 equal to 0 and node point, but only N 1 equal to 1 in the particular node. Even, similarly if we consider the node point 2 also then in that cases N 2 equal to 1, but other N 2 equal to 1, but other N 1 and N 3 becomes 0.

Similarly, at node point 3 we can see that N 3 equal to one, but N 1 equal to 0 and N 2 equal to 0 in the other node points. So, these are the particular properties we can observe, but shape function.

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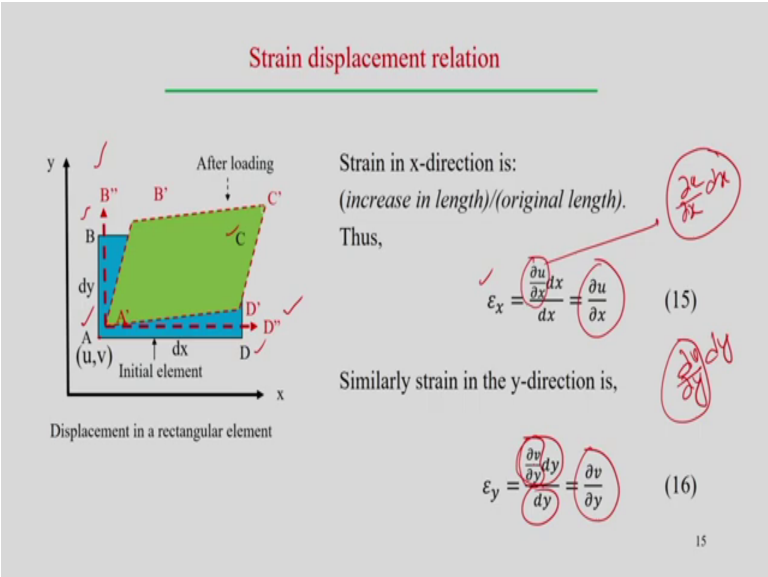

Now, you can see for the stress analysis; once you understand that how the shape function form in a particular element, then we will try to look into that how in a stress analysis it is also necessary to relate between the stress and strain displacement.

Because this relation is equal to establish the formulation of the shape function and the their solution. So, we will say basic things of the how strain displacement relation assuming the they just small deformation theory also in that cases we can neglect the rotation of this particular in this particular element.

So, only small displacements are there if there is a small displacement and we can neglect the higher order differentiation higher order term in their particular Taylor series expansion, then we can establish the relation between the strain and displacement and that is very important in finite element to understand the how this formulation normally happened.

Now, say we consider 1 particular element and so, initial size of the element rectangular element we can see that element having the size is the dx and dy that along the x axis the elemental length is dx, along the y axis the elemental length is dy. Now small deformation happened. So, in these cases the deformation or with the application of the load after loading condition it deforms to some other shape, but neglecting the rotation.

So, deform to some other shape we take these are the reference say initially the this B B A D and C points are there. Now, it deformed to A dot B dot C dot and D dot in this particular form. And we take the difference along the x axis is the D double dot, along y axis it is B double dot.

And then we can estimate what is the strain with this deformation we can get formed. So, definitely we look into the what is the strain along the x axis along the x direction. We know the strain can be defined what is the increase in the length with respect to the original length. So, that is the normal definition of the length.

Now there is a deformation happens in the; it is a two-dimensional deformation happens in the both x and y axis also. So, in this case the strain can be defined epsilon x then we normal strain along x axis we define in such a way that del u by del x is the variable. So, point A is the thing the displacement field is the defined as the general variable u and v. So, it deform the following this displacement field that u and v. So, u and v may have some functional form also.

Now, u and v is general displacement field in general. Now, epsilon x equal to what is happening that along the x axis what is the deformation along the x axis? So, that can be defined this del u by delta x del u by del x we can see it is a simply gradient basically change of u with respect to change in the x that is defined del u by del x, this functional form.

Now, this differentiation and, but total length or total initial length along x axis is dx. So, once you multiply by the this into dx that depends on the increment along the x axis. So, this represent the increment along x axis and dx over the original length about the origin in a dx also. So, in this case we are getting the final epsilon x equal to del u by del x. So, then we simply as the strain along x axis is the representation of the x axis equal to the differential form del u by x del x u is the here the displacement simply along the x axis.

Now, similarly strain can be y axis can also be y direction can also be estimated that y direction the functional form is the variation is the v v functional fold. So, del v by del y as the change of del v along the y axis. So, change displacement basically this deformation happens along the y axis del v by del y. So, del v is the basically displacement, but with this per unit elemental length.

So, that it represent the del v by del y; so, the integers per unit element. So, done what is the total deformation along y axis when you multiply by the dy. So, this is per unit and then unit length I can say and then we multiply the elemental dy that indicates the total deformation along this y axis.

So, this deformation that means, we can see the increase in length divided by original length. So, that is the increment in length with respect to the original length that this represents the epsilon y. So, then epsilon y the strain along y axis is the del v by del y. So, this way you can estimate strain x and strain y.

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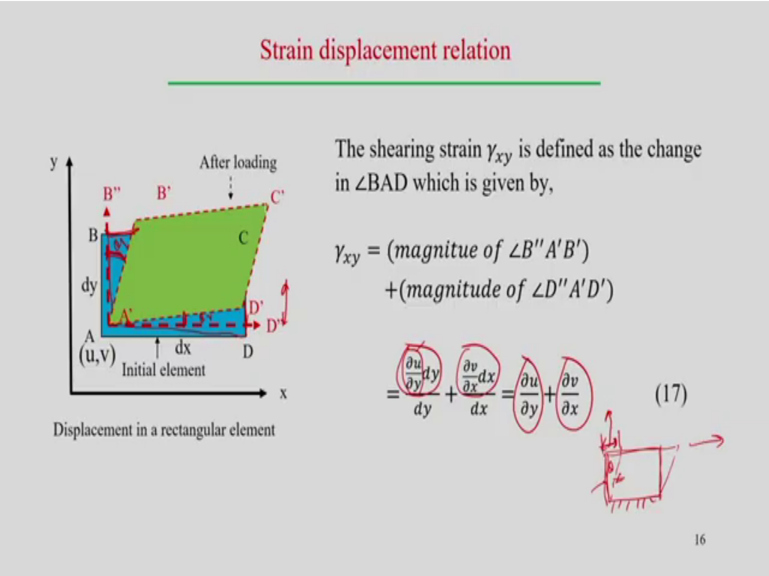

But how what are you can define the shear strain also, because it is associated with the normal strain epsilon x and epsilon y normal strain along x and y axis. Now, shear strain can be defined in mathematically that the change the what is the angular change this thing. So, if you remember once and in the basic element the shear deformation is normally the shear strain it say with respect to this base if that was the original length and there is application of the shear along this direction.

So, it deforms and its deforms angle is theta. So, this basically this way we can define the strain, the shear strain is basically representation of this the angular deformation there is basically the theta. So, theta can be estimated that this distance divided by this distance.

So, accordingly we can estimate what is the value of the theta or tan theta because theta is very small in this case is that represents the shear strain in case of, but when there is a deformation happens with respect to both x axis and y axis also. So, then total shear strain represents the what is this angular from this angle as well as the this angle 1 and theta 2. So, total deformation.

So, therefore, shearing strain is defined as the change in the b a d which is given by gamma magnitude of the change of the angle. So, initial angle was a BAD and then I was changed to B dot A dot and D dot. So, therefore, it consists of the with respect 2 different angles. So, one is the magnitude of the B double dot A dot and B dot that is a one this angle and second one is the other angle.

So, then this total represent a what is the amount of the shear strain during this deformation. Now, in terms of the displacement field is already there u and v displacement field. So, what is this angle we can represent these things and first we estimate this angle B double dot A dot and B dot, what is this angle?

This angle we can estimate the del u by del y and in this case the deformation along the deformation with respect to per unit and then d y is the that represents the del u by del y into dy represents that this value and with respect to dy the ratio this value and this d y this is the total deformation of this part and then what is this value. So, that is the with respect to dy. So, then he represents that that one part other part the magnitude of this angle.

And of course, there is a small deformation is there are magnitude of D double dot D double dot at this angle this angle represent what is happening the along this direction what is the deformation with respect to this value that represent the shear strain. So, that del v by del x into dx that is a total this value increment in this case equivalent to this value also divided by dx that represents that del u by del y plus del v by del x.

So, then shear strain is consist of this two common del u by del y plus del v by del x and the shear strain of the xy plane that gamma x y. So, that represent the shear strain in case of the that shear strain this is the relation between the strain and displacement field; therefore, the strain vector can be represented like that.

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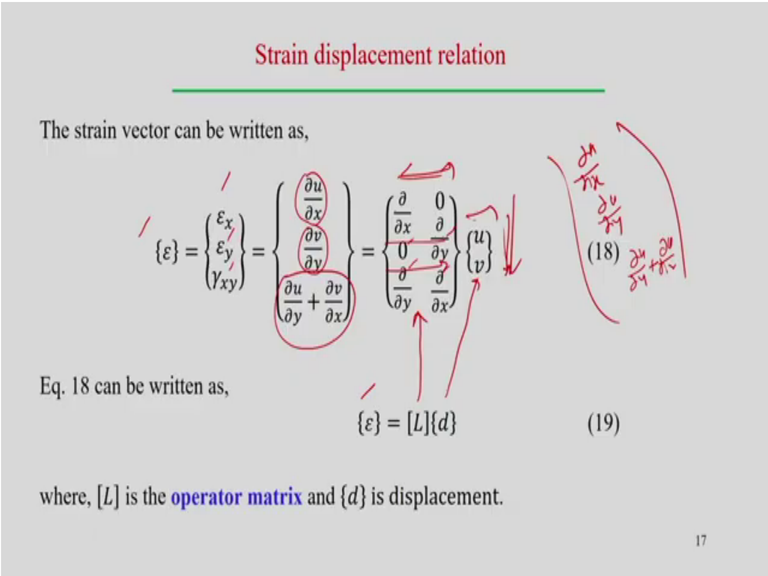

So, we know in a strain vector if you assume the two-dimensional case, the strain consists of the epsilon x epsilon y and gamma x y. If you look into this then strain component can we represented the epsilon x equal to del u by del y del x del v by del y and that this is a shear strain component.

Now, we can for the represent using in terms of operator also that del by del x 0 del by del x then 0 0 del by del y and del by del y into del by del x into u v. Now, you can cross multiplication this thing if you matrix in the matrix form also we can write this here you can see that this one and this one we can see the del u by del x and other components equal to 0.

Similarly, this to this we can see the del v by del y similarly this direction and this direction you can see in this case del u by del y plus del v by del x. So, that is in the in terms of the operator we can find out the strain displacement relation. So, finally, it can be written the strain components and this is the operator L and this is the this u v is the general displacement field.

So, now L is the operator matrix and d is the displacement. So, this way we can relate between the strain and displacement field. So, strain and displacement field which is linked with the operator matrix L; so, L matrix.

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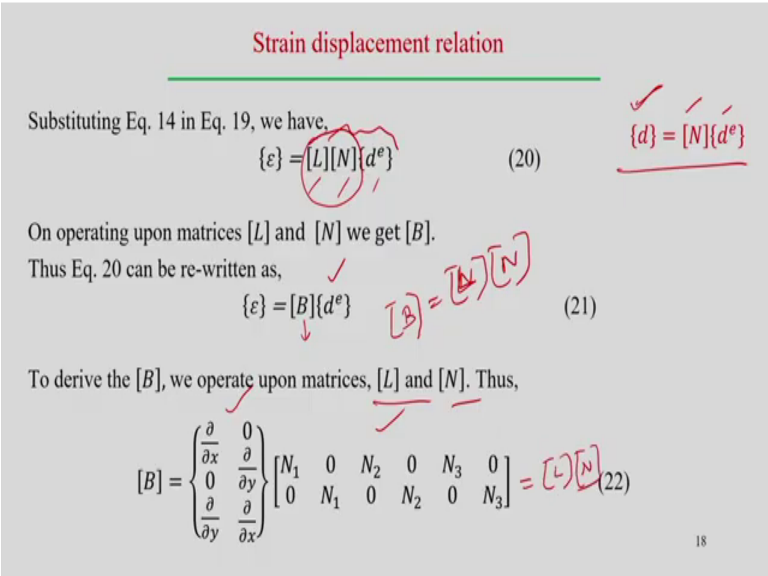

Now, this L matrix now substitute equation 14 into the equation 19 we can get this thing equation 14 is the general displacement. If you remember displacement field in terms of the shape function also the displacement field d is the general element this is the shape function and this is the displacement for each and every node point.

So, with using this relation we can see that replaced this d e in using this relation we can find out epsilon equal to the in terms of operator N shape function and the d e elemental displacement function elemental displacement. So, therefore, on operator upon matrix L and N we can get the B. So, B is basic equation 20 can be written also that L into N that can be represented is the B. B is basically sorry L and N.

So, therefore, a strain is basically B into d e. Now, the to derive the B also we have upon the matrix L and N N. So, this is the l matrix and this is the N matrix such that it becomes that L into N. So, therefore, we can represent the strain in terms of the B matrix as well as the other operating matrix.

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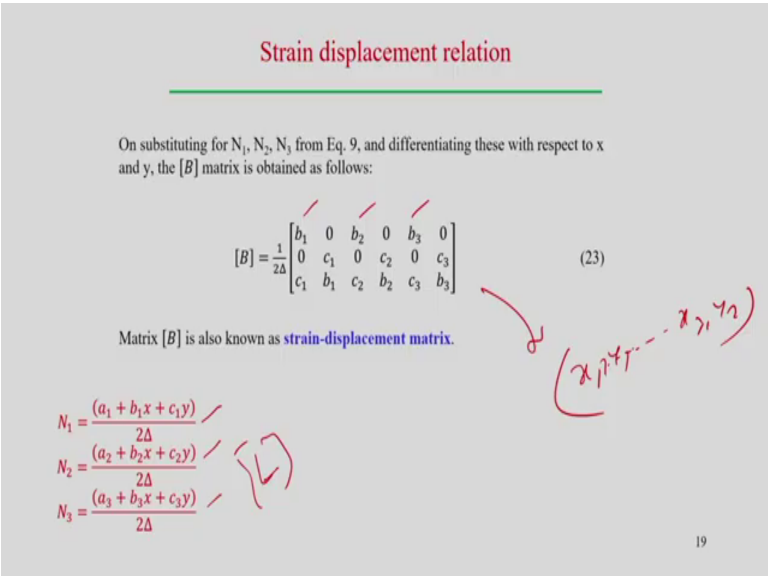

So, now on substituting N 1 N 2 from the equation 9 also and differentiating this with respect to x the using the operator L matrix B matrix can be represented in terms of the in terms of the b 1 if you see b 1, b 2, b 3 and which is basically that x 1, y 1, x 3, y 3. So, basically we can link this B matrix with a known the geometric parameters x 1 y 1 in terms of this thing. So, B is basically called the strain displacement matrix.

And we have already seen the N 1 is this functional form N 2 and N 3, but here L; the operator matrix is basically eliminate this variable. So, operator matrix the faster differentiation with respect to either x or y; so, in that case it is eliminated that the variable x and y.

So, we finally, reach the value in the form of a some known quantity in the as a function of the geometric; that means, coordinate of this particular node point. So, that is why B is called the strain displacement matrix.

So, this way we can relate the different elemental matrix and different symmetric also. And, such that we can in this case particularly it is very important to understand how the different metrics are used and what are the known quantity what is the unknown quantity.

So, this expression is basically useful the B matrix is basically here the if you see the b matrix is basically relating between the strain displacement matrix. So, it relate between the strain and displacement component. So, that is why it is called. So, the strain is a B matrix and the d accounts the what is the displacement each and every note point and B accounts the in this particular case when there is a linear variation of this particular element the linear variation of the displacement field also.

B is basically the constant in form of a or the known quantity; that means, it is basically the coordinates of the each and every node points. So, that is very important to understand and it is very useful to relate the between matrix in the this formulation of the stress analysis part. So, that is all today and thank you very much. Next class we will discuss the other part.

Thank you.