Finite Element Modeling of Welding Processes
Prof. Swarup Bag
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Lecture - 37
Theoretical development - Stress analysis model

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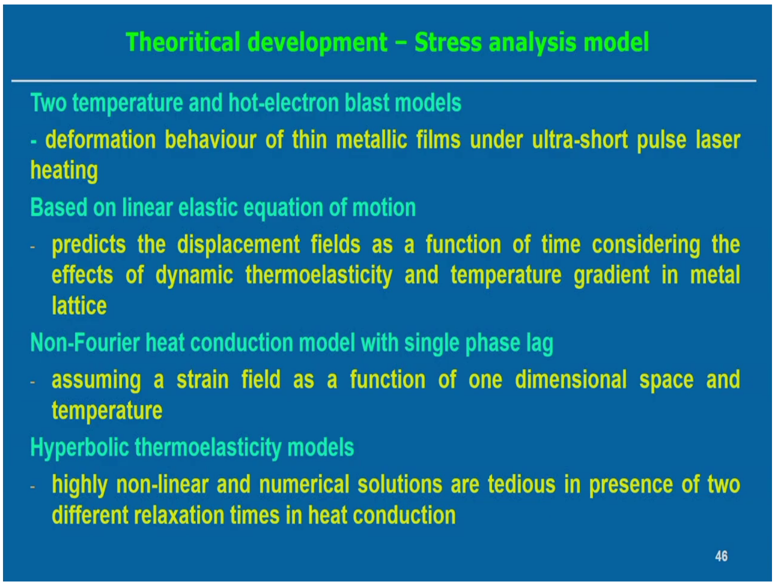

So, after discussing a Non-Fourier heat conduction model and that is specifically applicable in case of welding, heating processes. Now, we will try to discuss the Stress analysis model and that is associated with ultra short pulse laser heating process. So, we will see that in this cases stress analysis model, how the stress analysis model is actually different from the conventional laser welding process.

So, in this case we know that two temperature and the hot electron blast model, that deformation behavior of the thin metallic film can be analyzed and that is under short pulse laser heating process is basically analyze this thing assuming that this thermal analysis is from the two temperature and hot electron blast models. But, in this case the actual equation will be getting the based on the non-linear elastic equation of motion.

So, that actually predict the displacement field and that is a function of the time considering the effects of the dynamic nature of the earth, we can call that dynamic thermoelasticity and temperature gradient in metal lattice. So, in this case if we look into this two temperature model and the hot blast electron blast model, in the literature we can find out that they have analyzed the dynamic thermoelasticity and temperature gradient the analysis can be done.

But of course, in this case also the displacement field can be represented as a function of time. But, if you look into even non Fourier heat conduction model with single phase lag also in this case assuming the strain field is a function of the one dimensional space and temperature. And then once we assuming the strain field and from the strain field we can estimate the static equation and then static equilibrium equations and from that point of view we can make the in the equation in the form of a displacement.

And then if you analyze the displacement and then after analyzing the getting the displacement field or we can say the lattice displacement field then we can predict the strain field and from the strain field it is possible to predict the stress field. This is the usual procedure for the stress analysis model.

But, in this particular case if we consider the hyperbolic thermoelasticity model in this case it is very much non-linear and the numerical solution is actually tedious in presence of the two different relaxation times in the heat conduction equation.

So, in that case if you introduce this, if we solve this hyperbolic thermoelasticity model if you follow this particular model then equation becomes very tedious and it is complex and may be computational part may be an issue in this particular process because we account the two relaxation times in the heat conduction equation.

So, we will try to find out the stress analysis model, but simply what we have learned in the simple elastic analysis or simple elastoplastic analysis, the similar kind of the analysis can also be applicable.

But of course, in this case this application of the, in ultra short pulse laser application of the laser flux is so rapid that in these cases probably we can use only the thermoelastic model may be sufficient to explain the different distortion field. And that, in these cases we have analyzed the simple thermoelastic model, but with so many assumptions.

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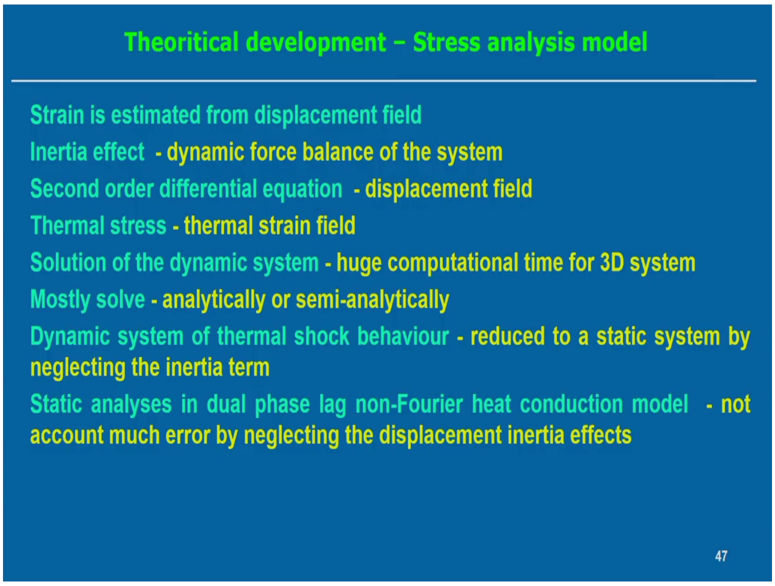

Strain field is estimated from the displacement field, that we have already seen when we have analyzed the what way we can develop the stress analysis model. Because in general in stress analysis model that first we take the input other from the thermal analysis or maybe if there is any concentrator or distributed load as a boundary condition.

So, from there we can find out the first we solve the this displacement field to solve for displacement field. So, first we estimate the displacement field from the displacement field we can estimate the strain field and from the strain we can correlate with converted in terms of the stress field that is the usual process.

But, if you consider the thermoelasticity, the dynamic thermoelasticity model in this case there is a inertia effect so; that means, dynamic force balance of the system is required to consider the effect of the inertia in this particular system, specifically to the ultra short pulse laser heating system or ultra short pulse laser welding system.

And it is associated with the second order differential equation and then we need to solve this second order differential equation then we will be getting output as a displacement field. And definitely apart from that thermal stress is associated with the thermal strain and this thermal strain normally we assume the elastic strain and then once we estimate the thermal strain from the thermal expansion coefficients the and the Young's modulus from there, we can finding up we can find out what is the value of the thermal stress.

And then the solution of the dynamic system, if we consider a second order differential equation in the form of a displacement field then if we solve this equation differential equation we will be getting the displacement field. But, this dynamic equation may consume the huge computational time are specifically for the 3 D system. Therefore, most of the problem associated with the ultra short pulse laser heating process is either these are the analytically or semi analytical approaches.

But, numerical approaches or numerical solution of this kind of equation is very much computationally expensive. At the same time dynamic system of the thermal shock behaviour, if we consider the dynamic in this dynamic system and if you consider thermal shock because it is very rapid process the application of the heat flux and development of this thing some time relaxation is there.

So, then it may induce the thermal shock that at the same time because there is a pulse heating is there and when one material is subjected to the pulse heating there is a continuous change in the temperature or variation of the temperature and wide variation in temperature may induce some amount of the thermal shock behavior associated with this particular system.

So, therefore, once we consider thermal shock behavior with the application of the ultra short it is actually converted reduce it actually converted to the simple static system to dynamic system. But, if it is possible to convert this thing if we neglect the inertia part and then if we simply represent this system as a static system, then we can then it will be we can simplify the modeling approach of the stress analysis associated with the ultra short pulse laser welding.

So, therefore, in this particular discussion we can look into the static analysis in the dual phase lag non Fourier heat conduction model and therefore, in this case not account much error by neglecting the displacement inertia effect. If we effect if we neglect the inertia effect there may not be that much of error in calculating the displacement field. Anyway, we simplified the model in case of ultra short pulse laser heating model as a simple static system.

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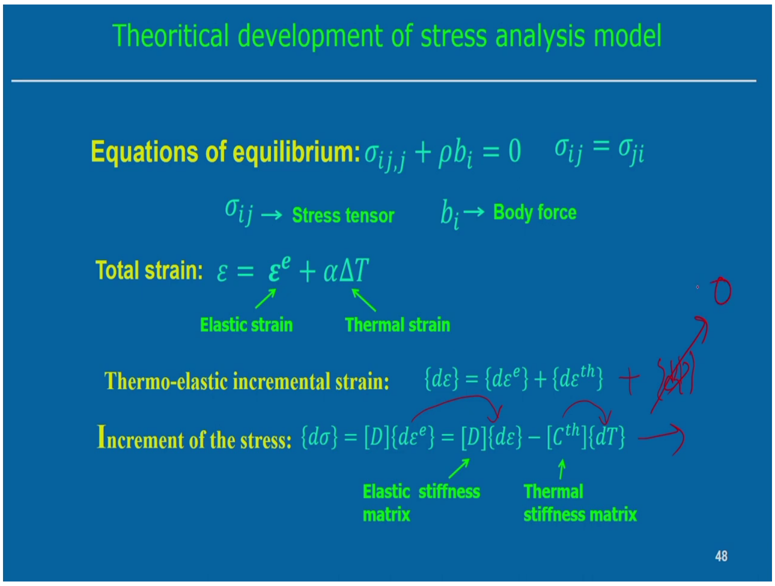

Then we will see that, we know as a simple static system the equilibrium of equations can be represented like this. That what is the internal stress internal, stress generation in the body sigma i j and then external forces or in terms of the body forces that make the balance of the system equal to 0.

We can see the rho b i, b i represent the body forces and sigma i j is the stress tensor, but in these cases we consider the symmetric stress tensor. Now, these are the system of equilibrium equation. Now, total strain we estimate you know that there is either elastic model, thermoelastic model or thermo elastoplastic model it is necessary to estimate what is the total strain component, which component we have to consider in this case.

Since, we use only the thermoelastic model, therefore we consider the elastic strain if the total strain consists of the elastic strain plus the thermal strain. And thermal strain depend on the temperature variation and thermal strain the temperature variation as well as the temperature thermal expansion coefficients of the particular material. So, once we, if we consider both elastic strain and thermal strain, then total thermoelastic incremental strain can be represented d epsilon equal to d epsilon e elastic part and the d epsilon th.

So, that is the thermal part thermal strain component. Now, increment of the stress is simply d sigma equal to D is the this matrix associated with the all the material properties and into d epsilon e; that means, elastic component; that means, we use the thermo elastic analysis therefore, the stress we relate the strain as being the elastic component only.

So, we are not considering the plasticity part in this particular process. Then elastic stiffness matrix d elastic it should consist of the total strain minus of this is the thermal stiffness matrix and the temperature difference dT. So, this will induce the total incremental stress. So, therefore, total incremental stress can be represent the total strain minus the only the thermal strain component.

So, if we find out in these are the relation and we have observed in the in simple stress analysis model or elastoplastic analysis model, it was necessary to incorporate the other plastic strain component also, but in this cases we are neglecting this plastic strain part.

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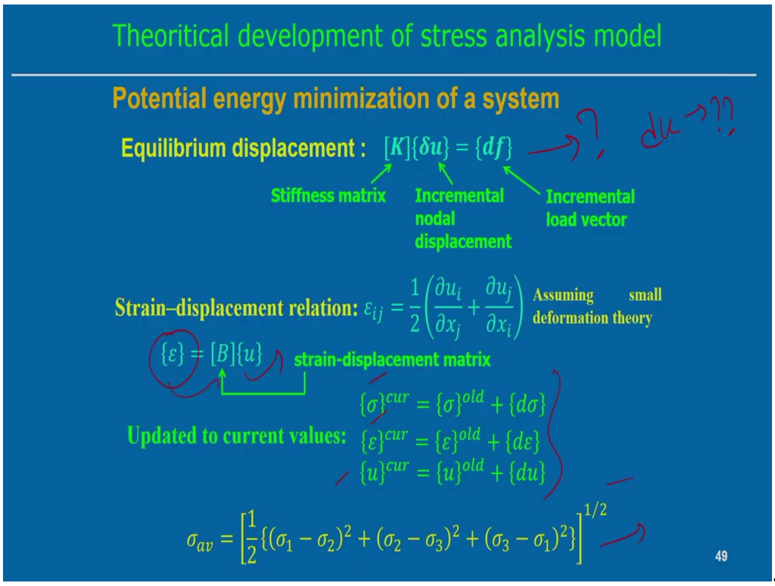

Now, if in a static system if you follow the potential energy minimization problem, then we have already shown in the module 2, then during the stress analysis model the what is the potential energy minimization of a system and what way we can reach this kind of equation that equilibrium displacement.

So, K is the stiffness matrix and delta u is the incremental nodal displacement equal to d f, d is the incremental load vector. So, in this case the incremental load vector we can see the stiffness matrix and incremental nodal displacement term equal to the incremental load vector. So, this the equation of the ax equal to b in the matrix form.

So, if we solve this particular matrix ax equal to b, we will be getting the x value; that means, what is the incremental value du will be the output in this case. So, that means, displacement field will be getting each and every node point. But, remember in this particular case when you are looking into only the thermo elastic analysis.

So, in this case maybe if we assume the strain is the stress is proportional to the strain up to the yield point. So, that is follow the linear curve. So, in this case incremental can be very high, even incremental can be in single step incremental can be considered. Because in this case the elastic the in within the up to the yield point the stress versus strain follow the linear path.

So, which is different from the plasticity model, because plasticity model if you look into the elastoplastic; that means, in if we look into the simple stress strain curve of universal tensile stress in specimen in this case that when it cross the yield point then it becomes, the curve becomes non-linear.

So, therefore, in that cases it is necessary to make the analysis for the by dividing the total small small strain component and we analyzing this and keep on updating the stress or strain value. But, in is if path is linear, then large amount of the increment can be considered as a single step.

So, that is the difference from the simple elastic analysis as compared to the elasto plastic analysis. Now, once we look into the equilibrium displacement in this case, but we need all this relation then what way we can correlate we can relate between the strain and displacement.

So, if we assume the small deformations theory may be rotational degrees of freedom other component can be neglected or may be second order differentiation of the displacement, if we neglect this part then we can use the this simple relation between the strain versus displacement. So, that has to be decide first what is the relation between the strain and displacement, will be using this particular equation.

So, in the matrix form it can be represented this thing epsilon equal to B into, B is the strain displacement matrix that we have already explained in the stress analysis model. So, this is the relation between the stress strain and the displacement field. Now, we solve for this equation because finally, we will be solving this equation. In this case if we solve this equation then we will be getting what is the value of the incremental or displacement value. So, once we get the displacement field.

So, we can estimate what is the value of the strain value. So, once we get the strain value then from the strain value, soon we can estimate the stress value. So, this way we can find out the what is the value of the it means that first we are calculating the displacement from the matrix equation, we solve it for the displacement field, we will be getting the displacement field each and every node point.

Then we convert it to this displacement to the strain value. So, once you are getting the strain value then this strain can be converted to the stress value. So, that is why we will be getting the displacement field, strain field and the stress field for each and in a particular domain. And once you get the in incremental form also current strain value and current displacement can be approximated and that can be updated with compute the what are happening the previous step and then we will be getting this solution.

But, it is also sometime necessary to estimate what is the value of the effective stress equilibrium or effective stress value, the effective stress value which is a component of the three different stress component. In this case if we assume the sigma 1, sigma 2 sigma 3, the three different components. Then we can finding out the effective value in terms of the it means that at any state we will be getting the three dimensional stress value.

So, say in this case maybe within the elastic limit it can be say sigma x sigma y and sigma z because or this sigma x sigma y and sigma z represent the principal stress values. Now, once you get the principal stress value then the single value can be represented using this equation. So, it means that the three dimensional state of the stress can be represented on a single value that is the equivalent stress value.

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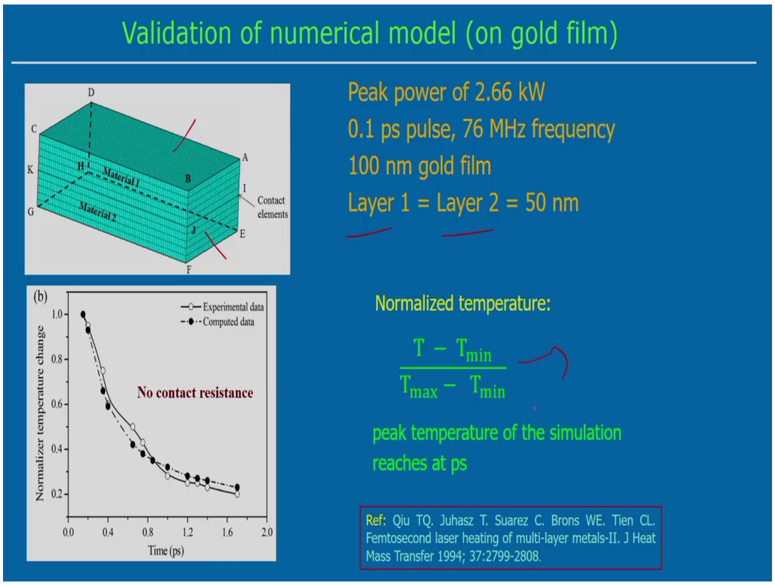

So, we have done some analysis; that means, validation of this model stress analysis model and what way we have done some thermal analysis on the of a gold film and that does with the similar material we can analyze the stress and strain field also as well. But, in this case we have considered the multi layer; that means, there are two layer of the gold film, the layer 1 and layer 2 both are 50 nanometer thickness so; that means, total domain is the thickness of the domain will be the 100 nanometer gold film.

So, in this case peak power of the ultra short pulse laser is used peak power is 2.66 kilowatt and pulse duration is 0.1 picosecond is the pulse duration and 76 megahertz is the frequency. So, using these cases we have analyzed the stress, we have analyzed the stresses in this particular thin film of the gold.

Now, this in this case if you look into the domain we have created this domain, the layer 1 and layer 2, but in between we have created so one single layer of the contact elements. So, this contact elements takes care only the thermal contact resistance between these two layer. So, we can validate the we can do the similar heat transfer analysis what we have done using the non Fourier heat conduction model.

So, if we can do the heat transfer analysis, then we will be getting the temperature distribution and we can validate the model by the known experimental data also. So, we can validate this experimental data is available actually the 100 nanometer thickness. So, if we assume the no contact resistance, the continuous layer of the 100 nanometer gold film and for that purposes we have done some thermal analysis using non Fourier heat conduction, then we can find out that this is the temperature distribution. And from and we can see this is the normalized temperature we can find out and then we compare with the experimental data.

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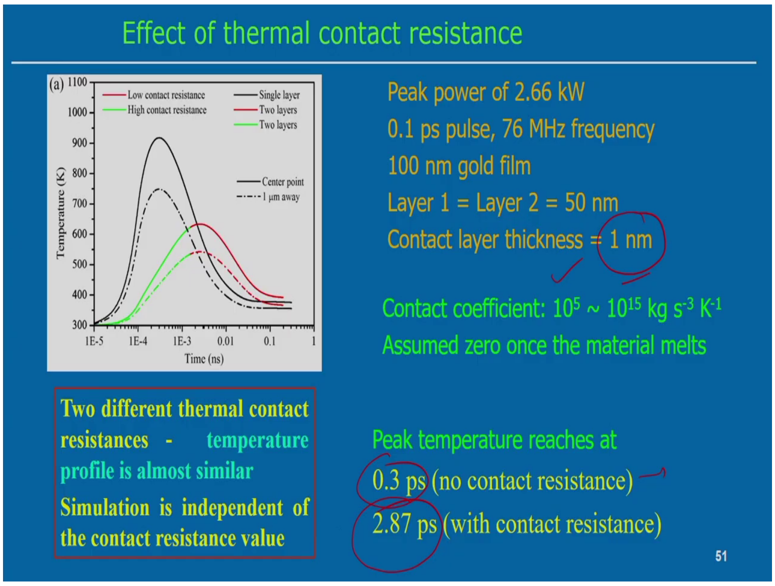

So, similar kind of the graph we have analyzed in this case also, that in this case contact layer is actually 1 nanometer, only the 1 nanometer the contact length the; that means, contact layer thickness 1 nanometer is used in this particular problem and we have analyzed this thing what is the effect of the contact resistance. So, contact coefficients is actually varied from 10 to the 5 to 10 to the power 15 kg per second cube per Kelvin and assume the zero once the material melts.

So, once the material melts. So, contact resistance becomes zero, but before melting we assume that this two different there is wide variability in the contact resistance and we are we are getting this kind of the profile temperature profile so with respect to time. So, you can see that the that single layer and the two layer, single layer means single layer 100 nanometer and the two layers of the 50, 50 nanometer with some contact resistance.

So, at the center point and the 1 millimeter 1 micrometer away, from the center point these two cases we can plot the time temperature profile we can see there is a temperature difference is there. So, it is at the center point as well as the 1micrometer away from the center point, these two points there is a temperature difference at there, the same time that single layer and the two layer.

So, therefore, two different thermal contact resistance we can find out that even if I use the thermal contact resistant there is a wide variation thermal contact resistance. But, temperature profile is almost similar, it means that we I can optimize the contact coefficients in such a way that simulation is independent of the contact resistance value. So, that is why that we have shown that independent of the contact resistance.

Then we can analyze, further analyze this temperature this we accept this temperature distribution and further use this temperature profile for the analysis of the stress, stress generation in this particular case. So, in this case the low resistance and the high resistance this particular see and the single layer and the two layer there is a temperature profile are completely different, that is obvious in this case. But, peak temperature reaches at 0.3 picosecond although the pulse duration was 0.1 picosecond.

And in this case no contact resistance, but in this case of the with the contact resistance there is a delay to reach the peak temperature, that is 2.87 picosecond so; that means, if you use the contact resistance the contact; that means, there is a multi layer film. In this case the thermal inertia effect is more active because it takes much more time to reach the peak temperature.

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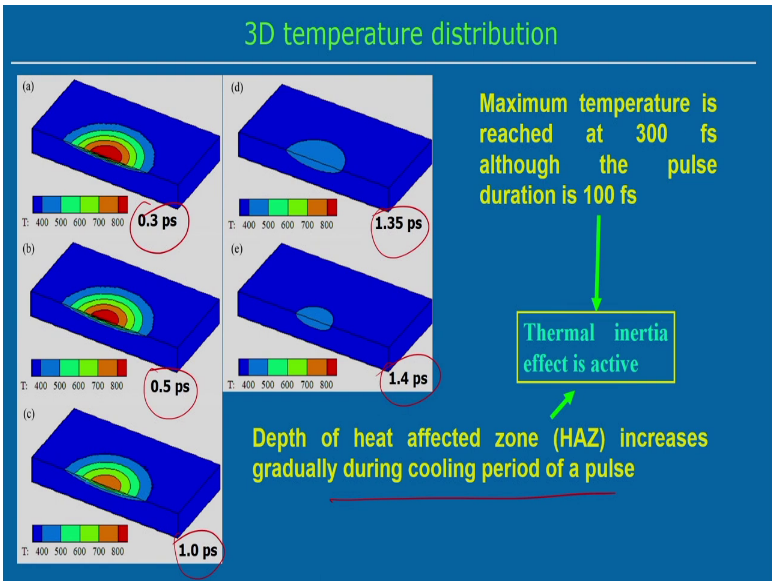

See, we have done some simulation that for these cases, the temperature profile you have tried to plot it the temperature profile and we can see that at 0.3 picosecond; that means, at this point the it reaches the maximum temperature that means peak temperature of the system.

And then, gradually decreasing 0.5 picosecond, 1 picosecond the temperature profile we can observe 1.35 and then 1.4 picosecond, it means that maximum temperature is this at 300 femtosecond or 0.3 picoseconds, although the pulse duration was only 100 femtosecond.

So, this it means that thermal inertia effect is active in this particular that we have already discussed and when the in non Fourier heat flow conduction model also. But, depth of the heat affected zone increases gradually during the cooling period of a pulse, particular pulse if you look into the depth of the heat affected zone if we consider and that actually gradually increasing even depth during the cooling phase of a particular pulse. So, that is the typical characteristics of the ultra short pulse laser process.

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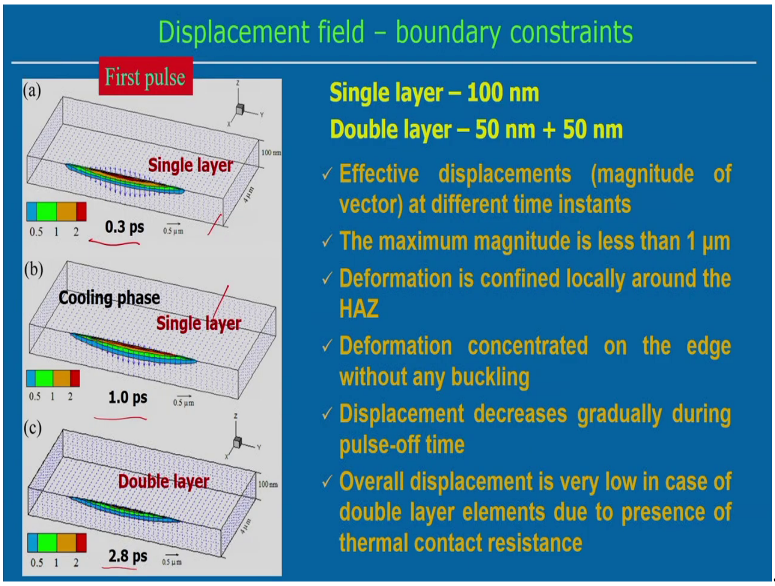

Now, we have analyzed the distortion displacement field first and then we have plotted the displacement field and the using some boundary constraints. So, at the we put some boundary condition, the restrict the boundary through this particular deformation to restrict the deformation of the boundary. Then we have done simulation for single layer and double layer both and double layer in 50 nanometer plus 50 nanometer and single layer 100 nanometer.

These two cases with boundary constraints and then first pulse with application of the first pulse we can see this the shows the figure shows the single at the 0.3 picosecond what is the distortion field or displacement field. So, effective displacement may be magnitude of the at different time instants, [FL] magnitude of the vector.

So, effective displacement though; that means, effective displacement means root over of u square plus v square plus w square; where the u, v, w represent the displacement along the x y and z direction. So, the magnitude if you represents the this first figure represent the effective displacement. The maximum magnitude is less than that of the 1 micrometer, we can see observe that maximum magnitude is less than 1 micrometer.

So, with the constraint value if the single layer, single layer and the third one is the double layer. In this case, that is 0.3 picosecond and 1 picosecond and 2.8 picosecond. So, in this case, deformation is confined locally around the heat affected zone, that is obvious, the this deformation is actually confined mainly the heat affected zone not distributed to other and we can see there is no buckling kind of things are not there.

And but, deformation concentrated on the edge without any buckling. So, at the edge without any buckling it is concentrated at the edge. But, displacement decreases gradually during the pulse off period; that means, up to the 0.3 second the up it reaches the maximum point, then that displacement decreases gradually during the pulse off time means the once pulse is off, at that period then the displacement field actually gradually decreases. So, overall displacement is very low in case of the double layer elements.

So, we can see as compared to the single layer, the double layer element overall displacement field is very low because presence of the thermal contact resistance at the interface. That means, the thermal contact resistance reduces the temperature difference spatial temperature differences because we estimate the thermal strain is the alpha delta t. So, the delta t is small in these particular cases when we are using any kind of the contact resistance or multi layer film.

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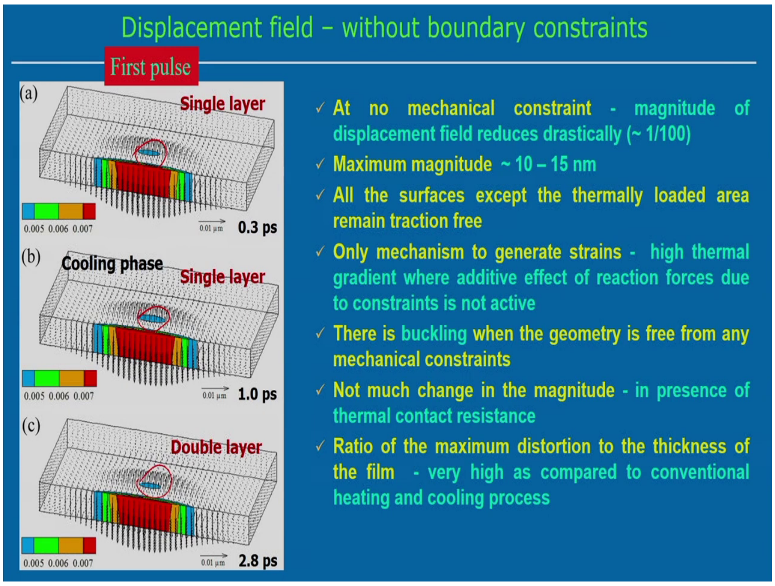

So, similar kind of analysis also done, but without any boundary constraints. So; that means, the thin layer or film is free to move. So, if you consider if you look into that in a single layer as well as the double layer and the similar time we can see that at no mechanical constraint a magnitude of the displacement is reduced and drastically. It is almost 1 by 100 times less as compared to the with the boundary constraint.

So, maximum magnitude is only 10 to 15 nanometer only, but all the surfaces except the thermally loaded area remain the traction free. If you look into only the displacement, we can observe only there near about the thermal field, but thermally loaded area which remains the traction free. But, only mechanism to generate the strain because high temperature gradient where additive effect of the reaction forces due to the constraint is not active. It means that since we are not applying any kind of the constraint.

So, there is no kind of the additive forces available in this cases and that is why it is free to move. So, it clears very less magnitude of the displacement field. There is buckling also we can observe in the similar we can this field, if you see this part it shows there is a some small amount of the buckling is there when geometry is free from any mechanical constraints.

That means, if there is a mechanical constraint we observe there is no buckling, but it is free from mechanical constraint we can observe some buckling is there. But, not much change in the magnitude, magnitude not much changes, in presence of the thermal contact resistance.

We can see the presence of the thermal contact resistant does not change the magnitude much. But, ratio of the maximum displacement distortion to the thickness of the film, when you compare the what is the maximum displacement field, in this particular cases the ratio of the maximum displacement field with the thickness is very high as compared to the conventional heating and the cooling process.

In this particular, the thin film they may at the localized position it may the ratio can be very high; that means, it means that what is the maximum value of the displacement field, that is very much high as compared to the thickness of the particular layer.

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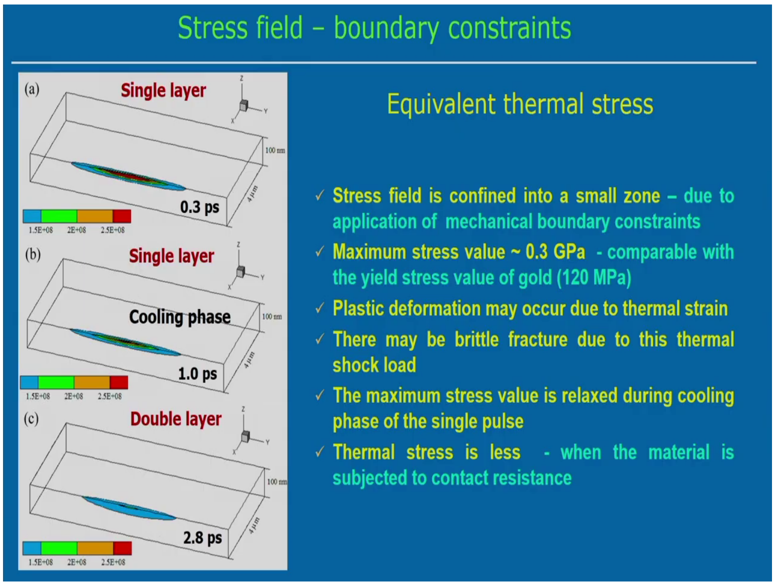

Similarly, stress field also with the boundary constraint we can plot the stress field. We can observe the different both the single layer as well as the double layer, we can find out that equivalent thermal stress represented in this case the distribution of that also. Equivalent thermal stress means the three dimensional stresses we converting in the representing in some on the single value of the stress.

So, therefore, stress field is also confined in the very small zone and due to the application of the mechanical boundary condition we can see the, it is confined to the very small zone. And the maximum stress value is 0.3 Giga Pascal and which is comparable with as compared to the yield stress value of the gold, because yield stress value of the gold is 120 Mega Pascal.

So, in that case the maximum stress value 0.3 Giga Pascal is comparable with respect to yield stress. It means that plastic deformation may occur due to the thermal strain, but we have not analyzed the plasticity in these particular cases. So, there may be the brittle fracture due to this thermal shock load because in this particular case the application of the load is there is a temperature variation is there.

Because this is a pulse we can see that during off pulse there is a decrement of the this cool down the temperature from the maximum temperature. Therefore, this kind of nature of the temperature can induce some amount of the thermal shock load. So, therefore, the maximum stress value is relaxed during the cooling phase of the single pulse because of the presence of the thermal shock and that thermal shock can induce some thermal shock may induce the brittle fracture.

But, thermal stress is less when the metal is subjected to the contact resistance. So, we can see when thermal stress is less, if you see as compared to the other cases; that means, single layer then due to the contact resistance thermal stress value is actually less in this particular case.

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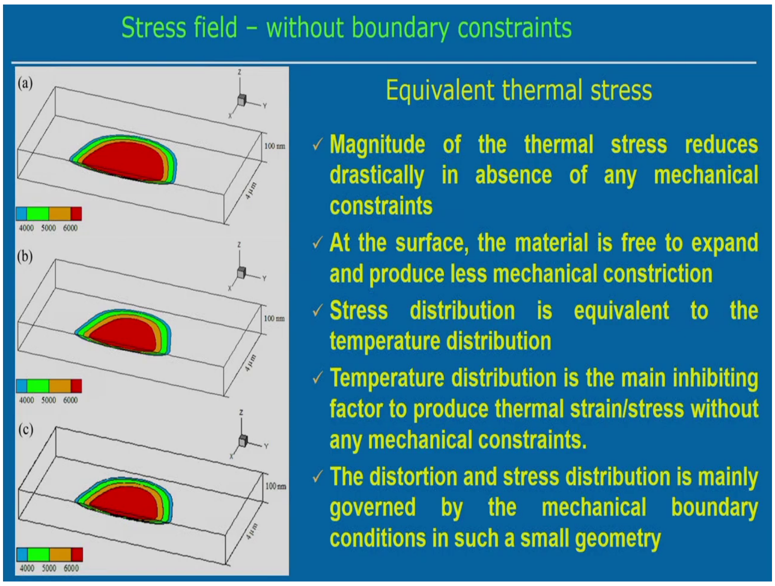

Similar analysis we can do equivalent thermal states, but without any boundary constraint. But, if you look into, if you do not put any kind of the boundary constraint the magnitude of the thermal stress is mainly confined to the zone of the heat affected zone in this case.

So, but magnitude of the thermal stress is reduced drastically as compared to the with the constraint, but surface metal is free at the surface the metal is free to expand and produce less mechanical constraint because in this case the surface is free. So, in there is no constraint there it is free to expand that is why the generation of the stress in these particular cases thermal stress is very less, that almost stress distribution stress distribution is equivalent to the thermal temperature distribution.

Basically, what way? Which zone? There is a temperature distribution at the same zone. We can see the distribution of the stresses all the magnitude is very less in this particular case.

Therefore, temperature distribution is the main inhibiting factor to produce the thermal strain or stress without any mechanical constraint.

It means that since we are not applying the any kind of the mechanical constraint, then temperature distribution is mainly the temperature difference is mainly the responsible to induce the thermal strain in this particular case. But, when you put the boundary constraint, the constraint can induce some amount of the strain value also.

So, therefore, the distribution distortion in the stress distribution is mainly governed by the mechanical boundary condition in such a small geometry. It means that it is very important to precisely define the boundary constraint because the maximum amount of the strain or stress can be generated because of the boundary constraints.

But, if there is no boundary constraint then magnitude of the stress or strain is very less. It means that the in the thin film structure it is very important the boundary constraint is actually induce the maximum amount of the strain or stress as compared to the only thermal stress or thermal strain. In summary we can say associated with the non Fourier heat conduction model and the as well as the stress analysis model.

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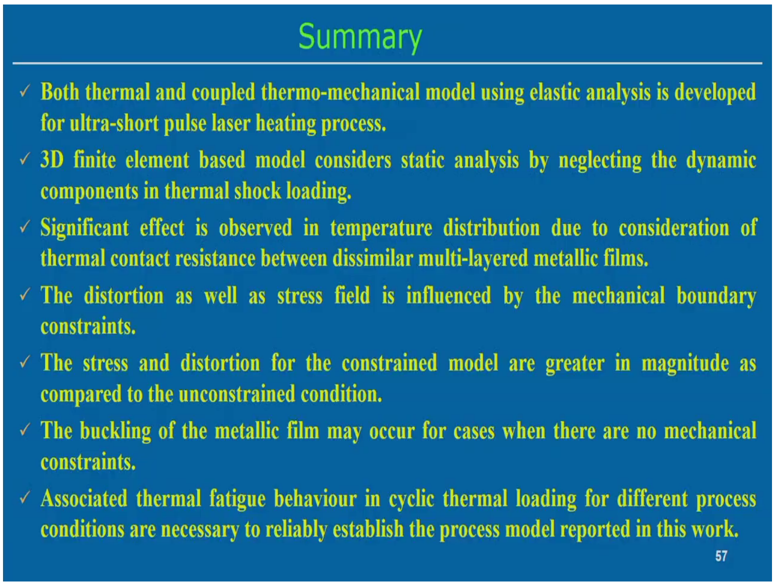

We can see the both thermal and the coupled thermo mechanical model using the elastic analysis is performed or maybe in these cases we have explained in the light of the by using the ultra short pulse laser heating process. So, 3 D finite element based model is possible to develop using the non Fourier heat conduction model and we have shown that dual phase lag model can be possible to develop.

And but, from the temperature distribution it is obvious that when it is the any component even it is a very thin component subjected to the ultra short pulse laser then it is basically associated some kind of the thermal shock loading. So, thermal shock loading it can may create the thermal fatigue in the structure. So, that stat is different aspects. Therefore, significant effect is observed the in the temperature distribution due to the consideration of the thermal contact resistance.

We have seen that if we consider thermal contact resistance in the multi layer field the temperature distribution there is a variation of the temperature distribution. Distortion as well as the stress field is influenced mainly by the mechanical constraint that we can observe also.

Only the thermal strain can induce not much that much of the stress as value. The stress and distortion for the constrained model are greater in magnitude as compared to the unconstrained conditions, that is obvious because when you put the some constraint in the boundary constraint, the magnitude of the distortion and the strain or strain field is much more as compared to the without any boundary constraint. Therefore, associated thermal fatigue behaviour of the cyclic loading.

In this case, the cycle loading means the temperature variation the follow some kind of authorities or cyclic loading or follow kind of pattern and for the different conditions are necessary to reliably establish the process model reported in this particular module. So, far we have discussed, it means that it is also necessary to consider the thermal fatigue or that can be another direction of the study associated with the non Fourier heat conduction.

So, thank you very much for your kind attention and here the this is the end of the module 8 of this particular course Finite Element modeling of the Welding process. Next time we will start in the last module that is the module 8 which is associated with the additive manufacturing process, so.

Thank you very much.