Finite Element Modeling of Welding Processes
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Module - 08
FE model of non-Fourier heat conduction
Lecture - 35
Ultra-short pulse laser welding

Hello everybody, I hope you have already learned the basics of finite element methods that we have discussed, and that how to develop the finite element model in case of simply transfer. That means, then fluid flow and stress analysis model. Now, we will try to look into the different font that what way we can do the finite element model in case of non-Fourier heat conduction.

Actually this non-Fourier heat conduction is comes under this category when you try to analyze and specific to welding process that it involves the ultra-short pulse laser processing. So, ultra-short pulse laser processing we can explain the heat transfer mechanism assuming that it is not following the Fourier exactly Fourier heat conduction equation whether it is following the non-Fourier heat conduction.

First we try to derive that what way we can done the formulation in case of the non-Fourier heat conduction model, then we try to look into the how finite element model can be developed using this particular equation.

And definitely at the same time, we will try to differentiate between simple heat Fourier heat conduction model and non-Fourier heat conduction model, what are the difference in terms of the computational aspect in terms of the formulation, all this aspect we try to look in this particular module.

So, first we have to understand what is ultra-short pulse laser welding, and then we will try to look into how we can model this ultra-short pulse laser welding. And then not only ultra-short pulse laser welding, the apart from that we will try to look in the heating of the nano film and by using the application of the ultra-short pulse laser.

At the same time, here we will try to look into the lattice distortion that the simple formulation assuming the simple elastic deformation and it is possible to develop the lattice distortion model in the front of or may be in particular to ultra-short pulse laser processing.

So, these three is the main aspect I will try to cover in this particular module. First try to look into what is the difference between the pulse and continuous power laser that we have already discussed. But here the prospect is to understand that basically why there is a need of ultra-short pulse is a particular material processing.

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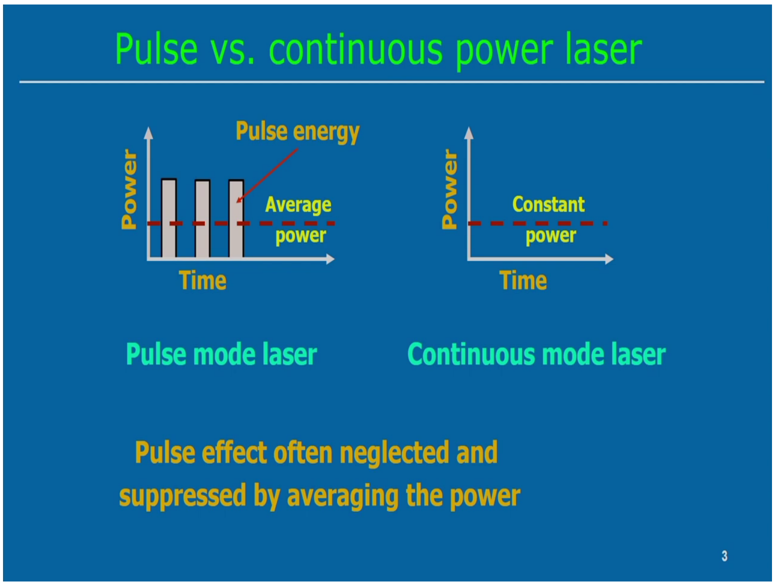

So, it is obvious that in case of pulse laser processing, the application of the pulse energy is normally happens over the duration of the pulse that is normally called the pulse on time. And there is a variable that depending upon the frequency or pulse repetition rate the per unit cycle, the what is the number of per unit time. What is the number of pulses can be generated from the laser source.

And then it seems like that; that interaction of the energy supplied by the laser that is the interpretation of the pulse laser source. So, graphically you can represent these things that power – y-axis, and x-axis represent with respect to time. So, at a over a short period of time, total pulse energy is supplied. So, that area that indicates the what is the amount of the pulse energy in case of pulse laser process.

Now, if you know all the pulse parameters, that means, the what is the peak power, what is the pulse duration, cycle time means if you know the frequency, then it is possible to define what is the average power, average power in case of the pulse laser. In sometimes this we can do the analysis by neglecting the effect of the pulse rather by simply calculating what is the average power.

And based on the average power, we try to analyze the different aspect of heat transfer in other mechanism also. But definitely the, if we consider the actual pulse and the shape of the pulse that is most important and more precisely you can develop the numerical model or you can say the heat transfer model or other kind of other phenomenological model using considering the actual pulse energy supplied from the laser.

Now, if you compare with respect to the continuous mode, laser continuous mode laser there is a continuous supply of the power without any interruption. So that means, in this case with respect to time, the continuous laser energy is falling on the substrate material. And then yeah either we can do the heating or we can do the welding process by continuous mode of the laser welding process. So, these are the basic differences between the pulse mode and the continuous mode of the laser.

Now, I have already mentioned that is that pulse effect of a neglected and the suppressed by averaging the power, but definitely that will just more simplify in some calculation. But if you look if you consider the actual pulse, the energy supply and that variation of the energy supply with respect to time. Then interpretation of this pulse energy in the material processing is more accurate for the development of a any kind of the model.

Now, come to this point the ultra-short pulses, how it is different from the conventional laser sources what we know. So, you can divide this pulse laser into basically three different category with respect to the duration of the pulse. One is the long pulse laser. In these cases, pulse duration is normally up to the nanosecond or greater than that.

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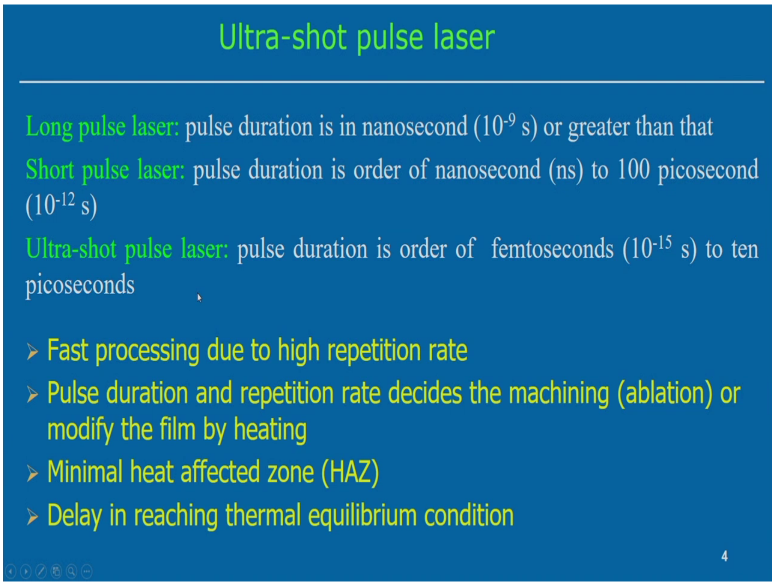

So, it means that the millisecond pulse and microsecond if the order of microsecond and order of nano second pulse, it comes under the category of the long pulse laser. And then short pulse laser; the pulse duration is order of the nanosecond to 100 picosecond; 1 picosecond is 10 to the power minus 12 seconds. So, within that duration, we normally consider as a these are the called short pulse laser.

But ultra-short pulse laser if the pulse duration is order of femtosecond to the 10 picosecond approximately with that range we can say this is the ultra-short pulse laser. So, mechanism interaction of the pulse laser energy with the material are completely different as compared to the long pulse and with respect to the long pulse laser.

So, therefore, we have discussed that in laser welding process, mostly we associated with the either millisecond pulse and the microsecond pulse. And based on that, we have analyze by considering the Fourier heat conduction model, we are analyze the heat transfer mechanism.

But in this case if you look into this ultra-short pulse laser, the pulse duration is too short, then mechanism of the heat transfer or mechanism of the heat generation can be completely different what we normally assume in case of the Fourier heat conduction model. So, we will see that.

But the ultra-short pulse laser is actually very fast processing due to the high repetition rate. Pulse frequency is normally very high in ultra-short pulse laser that is why it is a very fast process. And pulse duration and repetition rate decides the actually the machining or ablation or the modify the film by heating, that means, whether these parameters in ultra-short pulse laser is actually normally used in case of the material ablation process or maybe you can say the machining processes.

It is normally mostly useful this ultra-short pulse laser because the diffusion of the heat from the ultra-short pulse laser is very less as compared to the conventional laser. And that is why if the diffusional heat transfer we normally explain the Fourier heat conduction model, but if the if it is non-diffusional kind of the transfer that means, it then in that cases probably we can utilize the concept of the non-Fourier heat conduction and then we can analyze the different phenomena.

Now, pulse duration and repetition rate is also other parameter that actually decides whether that particular ultra-short pulse source can be used in case of the material ablation or the just finishing process or it can be used some welding or heating purposes.

A minimal heat affected zone that is the one even if you consider the long pulse laser, that in that cases also heat affected zone is very minimum, because it is very small as compared to the even arc welding process. But even if you follow the ultra-short pulse laser, in this case the heat affected zone is minimal, that means, very small heat affected zone is there in case of the ultra-short pulse laser. And that is the advantage in case of the as compared to the other laser sources.

But the mechanism is completely different in this cases delay in reaching the thermal equilibrium condition. That means, once we apply the laser flux on the substrate materials there are some time delay to reach the equilibrium condition. That means, some time delay is required to develop the temperature gradient within the substrate material, so that it accounts that kind of the delays.

But if you consider the only Fourier heat conduction model, then not necessary to account we assume that if with the application of the heat flux, there is a instantaneous development of the temperature gradient within the body, so that means, without any time delay.

But when you try to analyze in case of the ultra-short pulse laser, the pulse duration is such small as compared to the interaction time or the laser and material then there must be some delay to transfer the to develop the temperature gradient within this body. So, that is the difference in with respect to the conventional laser sources.

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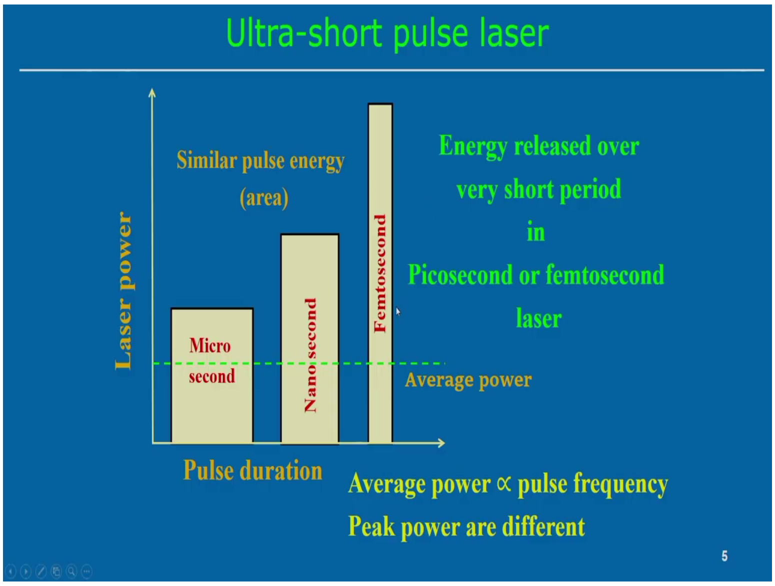

Now, similarly we can explain these things. Even if there is a average power is the same, if we assume in these cases and then if there is a due decrement of the pulse on time. So, laser power with respect to pulse duration or time, in this cases, microsecond laser. That means, if pulse duration is microsecond, in this case, the average power can be the same. But peak power relatively low, and then duration of the pulse relatively better because already I mention this is the microsecond laser.

Similarly, nanosecond means duration becomes narrower. In this case, the peak power will be much more to maintain the same average power. Similarly, if it is femtosecond, and the if femtosecond, the pulse duration is too short. It means that femtosecond pulse laser within the very short span of time there is a high amount of the energy is particular amount of the energy is supplied to the substrate material, but duration is very small.

And then it means that energy released over very short period of the time in picosecond or femtosecond laser, and that is the characteristics of the femtosecond laser or ultra-short pulse laser. In this case, of course, average power also depends on the pulse frequency. But all these cases in the in cases specifically I am talking about the ultra-short pulse since the pulse duration is very small.

So, for a moderate average power, then peak power should be very high in case of the ultra-short pulse laser. And that is also obvious if you look into this graphically the difference of the peak power with a micro second laser, nano second laser, and in case of the femtosecond laser.

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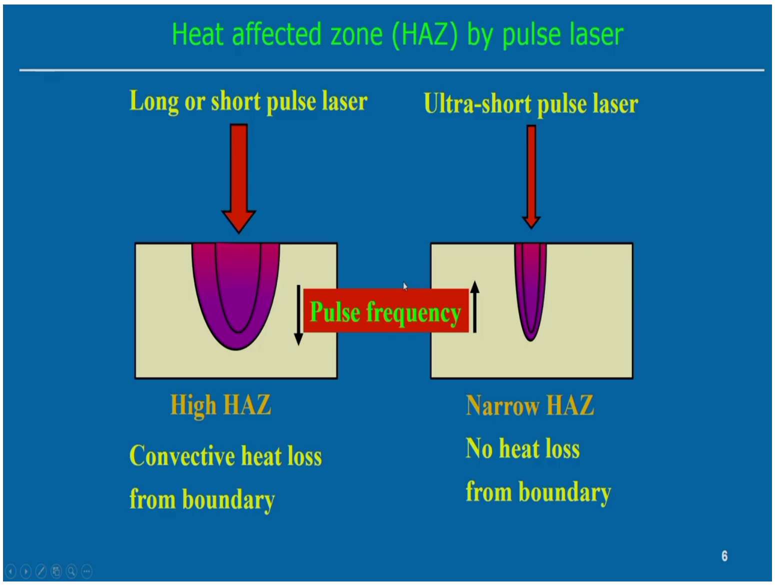

Now, if you look into that what are that heat affected zone is associated with some kind of the laser. And if you compare this thing long or short pulse laser, in this case, the heat affected zone is considerable or that as compared to the short pulse ultra-short pulse laser. The heat affected zone is relatively high in case of the long or short pulse laser.

And molten volume can be created by can be high in these cases, but at the same time from the boundary there may be the convective heat loss. That means, this diffuse heat is conducted away to the boundary, and then from the boundary there is a heat loss will be there. And we normally once we try to solve this kind of the laser welding, we have already observed in the solving the laser welding problem there we use the Fourier heat conduction model.

Fourier heat conduction model, in this cases so heat will be conducted to the boundary, that means, there are trying to conducted I to the boundary is the steady state condition, and from the boundary there is a loss. And the we account always the loss from the boundary and that is a typical characteristic in the perspective of the model development in case of the long or short pulse laser.

But, if you consider the ultra-short pulse laser, in this case, you can find out that heat affected zone and the I think molten zone or ablated zone is very small. Exactly at which part the what is the shape of the laser energy is focused only that part material can be removed, or even some diffusion very small diffusion may also happen. But that is the diffused zone can be very small, that means, heat affected zone is very small.

And that narrow heat affected zone and almost no heat loss, that means, the heat may not conducted away to the boundary. So, that is why most of the cases in case of analyzing with the ultra-short pulse laser, we do not account the any kind of the boundary conditions may be because there is no heat loss from the boundary. So, that are the typical characteristics of the ultra-short pulse laser.

And of course, in case of long or pulse short pulse laser, the pulse frequency is normally very low. And in other cases the ultra-short pulse laser is normally is the pulse frequency is very high or we can say the pulse repetition rate that mean per unit time there is a so many pulses can be overlapping on the surface.

And then it can affect the accumulation of the temperature or some other it can decreases the temperature also that we will see with some example also. But before that we can see that what are the different process parameter is associated with the ultra-short pulse laser.

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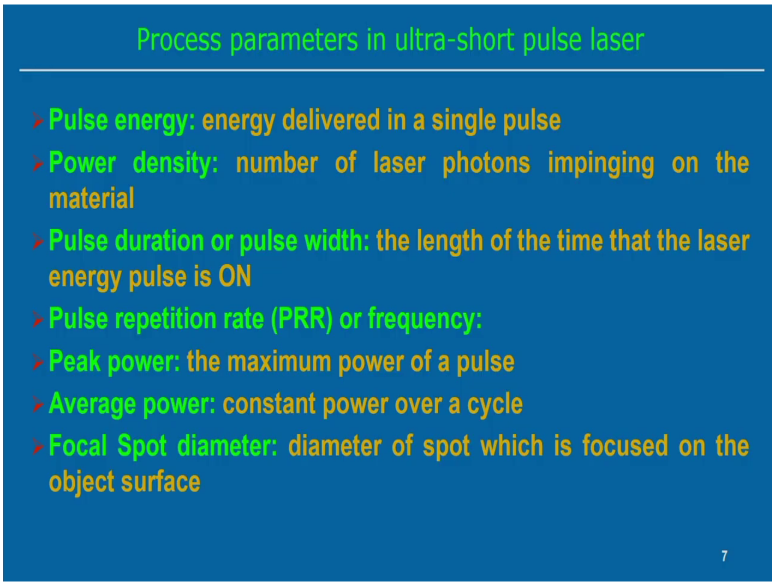

One process parameter that is called pulse energy. So, it is basically energy delivered in a single pulse that importance I have already mention that the graphically we can represent the what is the area on the we are showing in this time versus power in this graph.

The area represent the pulse energy in a single pulse. Then power density is the number of laser photons impinging on the material in other way power density simply estimated the what is the laser power we can estimate and then divided by the what is the cross sectional area over which the power is focused that represent the power density.

Now, pulse duration or pulse width or that is represent the what are the length of the time over which the laser energy is supplied that indicates the what is the pulse duration. And that means, when energy pulse energy that laser energy pulse is on. Other part is the pulse repetition rate or frequency that is called the that means per cycle time.

What that means per unit time what is the number of pulses of a number of times of the energy is supplied to the substrate material that indicates the pulse repetition rate or frequency we can use the that means, a frequency we can induce the cycle hertz basically cycles per second. Then per second how many cycles are there, one cycle consist of the pulse on time and remaining time is the pulse off time.

Then peak power is the another parameter, the maximum power we have already given that y-axis what is the amount of the power reach up to this that is called the peak power and the of a laser of a pulse. Then average power which is simply averaging over a one cycle constant power over a cycle that represents the average power. So, that means, it is obvious in pulse laser the cycle average power will always be less than that of the peak power.

Finally, the spot diameters, spot diameters means that when you are using the laser source what is the area over which laser is focused that effective area on the work piece surface that indicates the lasers for diameter and which is basically used to calculate the power density as well also. That means, power and divided by the area which is the area over the spot of a particular laser on the surface.

Now, physical aspects of the ultra-short pulse laser we can explain in that way that there are the two possibilities the low pulse repetition rate, and other is a very high pulse repetition rate. But which cases it is favourable to produce the heating purposes or which cases you can use the ablation purposes?

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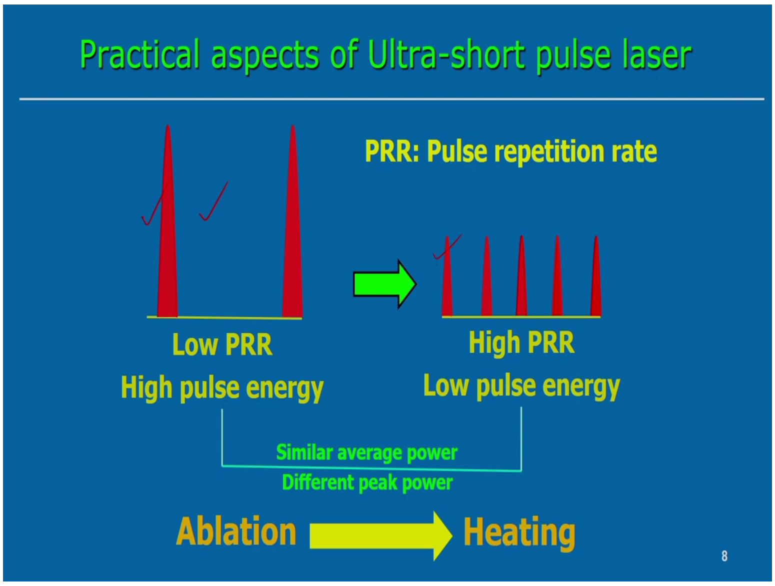

So, if you look into the first figure, here you can see that low pulse energy, low pulse repetition rate. So, low pulse repetition rate that normally peak power is very high and may be energy can be different. Peak power at as peak power is very high, but the gap between the application of the one pulse and the application of the second pulse is much more. It means that pulse repetition rate is low.

If pulse repetition rate is very low, in this case, that relatively high pulse energy is supplied in within one pulse. And if you compare that low high pulse repetition rate but low pulse energy, but pulse in the second cases the pulse energy is low because the area per this area represent the pulse energy; here the area represents the pulse energy.

So, pulse energy as well as peak power is low, in this case, but pulse repetition rate is very high, that means, this is the gap between the application of the two pulses is small in this cases. So, that means, whatever per unit time the number of pulses apply is less in one cases, and second cases is the per unit time the is the there are several pulses can be apply per unit time.

So, therefore, although there are so low pulse energy, but in these two aspects. Although we are having the same average power because the same average power, but in these two cases the different peak power is there. So, one cases, if we shift from the very low pulse repetition rate although thus similar average power to high pulse repetition rate with the same average power, then the applicability shifted from the material ablation to the heating purposes.

We can see that what way the it comes the from material ablation to the heating purposes from the next slide.

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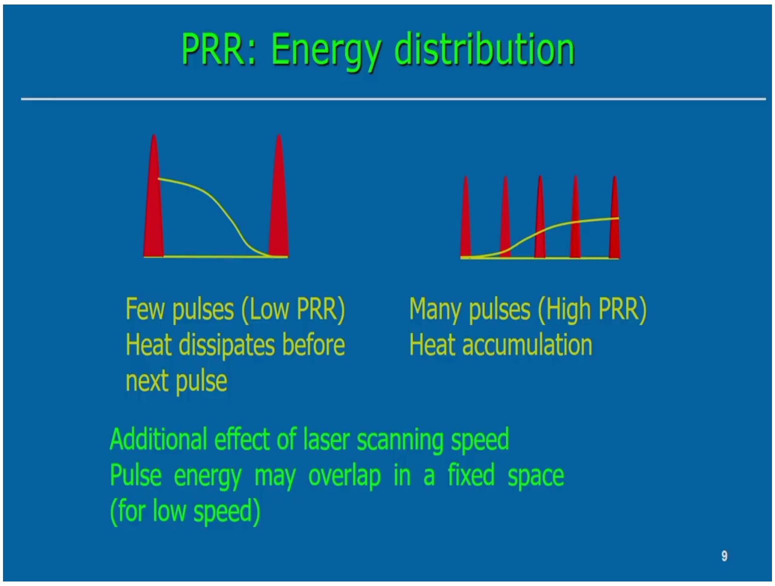

So, here first cases the very few pulses, that means, pulse repetition rate the gap between these two pulse is very high. So, what are the heat accounting that with application of the first pulse. Then there may be some heat disappears before reaching the next pulse.

So, from that sense, before reaching the next pulse, the heat disappears and that is why once the pulse energy high pulse energy is applied one for a single pulse. Then immediately it try to remove the material by vaporization from this thing which is normally called the ablation mechanism.

The, it means that through ablation mechanism its remove the material. And then once removed the material, then heat diffuse away whatever then reach to the low temperature before application of the next pulse. But since over a high pulse repetition rate and over a short period of time, there is several pulse energy is applied energy is laser energy is applied in the form of a pulses then there may be some heat accumulation gradually.

So, that is why once there is a heat accumulation gradually, then the second cases since pulse energy is low. So that amount of the pulse energy is may not sufficient to start the ablation process; or even if it is there, the ablation may be confined with a very small zone. So, that is why the high pulse repetition rate and the low pulse energy is basically suitable in case of the heating of the substrate material.

We can say that maybe this high many pulse the high pulse repetition rate may be suitable in case of the welding process as well, but not the low pulse repetition rate. So, low pulse repetition rate is mostly used in case of the for the machining purposes or material ablation purposes.

But of course, additional effect of there is always that that is a laser scanning speed and pulse energy which may overlap in a fixed space. And that means, the depending upon the because pulse is applying with respect to because whatever we are defining the pulse application of the pulse that is we application of the pulse and the gap between these two pulses everything is defined over the time scale, that means, with respect to time you are defining all these things.

But at the same time laser can be used with the some scanning speed that means, laser is moving with a particular direction. So, therefore, there may be possibility of the pulse overlapping is there depending upon the what is the scanning speed. So, at the same time, apart from this, pulse repetition rate there is a role of the additional effect that is called the laser scanning speed to decides the mode of the energy distribution.

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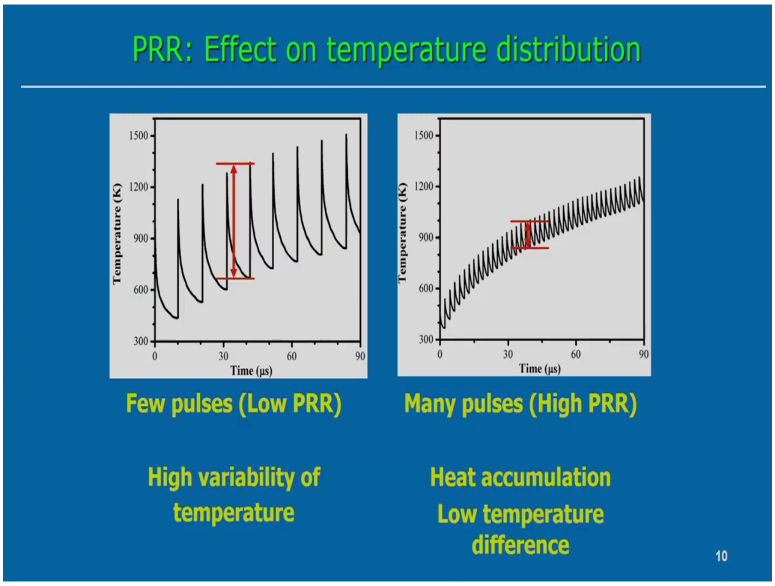

That means whether it is it is suitable for the ablation purpose or whether it is suitable in case of the heating or welding purposes. We can see some example also. The temperature simulation, we can see the few pulses, that means, low pulse repetition rate we can see that the, but single pulse there is a application of the that is it reached very high temperature. Probably the pulse energy is very high.

Then with a application of the single pulse the energy reach to the particular point, that means, temperature rise is very high in this particular case. But at the same time the it reach to the very low temperature before application of the next pulse. So, that is obvious from the first figure. And we can see that wide variability is there.

Very high variability of temperature that means, it reach to the very high peak temperature which may be suitable in case of the material ablation process. Because it reach to the with the short period of time at this vaporization temperature, so that energy is supplied. It is the vaporization temperature and immediately very quickly it reached come back to the very low temperature before application of the next pulse, and that kind of characteristics we can observe in the first graph.

Similarly, if I we can see that the second one many pulses, that means, very high pulse repetition rate and you can see the variable if the temperature variability is very low in this case, second case. And then gradually there is a accumulation of temperature and gradually it reach to the average temperature is particularly very high in the second case it is and that means, we need a large number of, that means, over a considerable time with the application of the high pulse repetition rate.

And with the average temperature can be very high and that it takes much more time to reach a particular temperature. And the because in the second case, there is a accumulation of temperature happens because of the high pulse repetition rate. Although low temperature difference and temperature magnitude can be low in with respect to time as compared to the high variable low pulse repetition rate.

But in case of the high pulse repetition rate, the low temperature difference is very small between these two. Because the gap between these two is small also, and at the same times heat accumulation occurs. And then second case, it is most suitable in case of the heating purposes or maybe this can be used in case of the welding purposes. So, that kind of the inferences we can derive just to look into that simply the effect of the pulse repetition rate.

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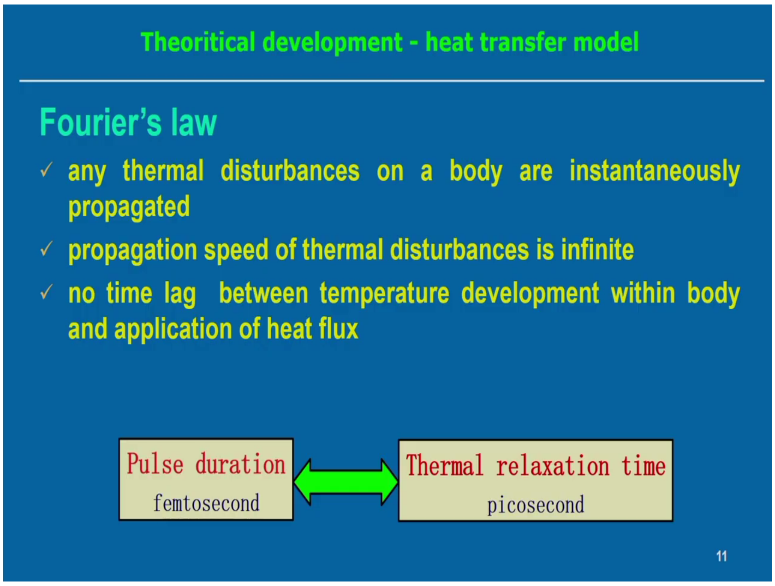

Now, what way we can analyze the heat transfer in using the non-Fourier heat conduction model? But before doing that to develop the theoretical model we have to understand the Fourier's law, and what are the assumptions in case of the Fourier's law of heat conduction.

So, any thermal disturbance on a body are instantaneously propagated, that means, with the disturbance means once there is a supply of the pulse energy to the substrate material to the works material, there is instantaneously, that means, immediately there is a development of the temperature within this body. So, there is no lag of the time with the supply the energy and the development of the temperature within the body.

So, that is the assumptions was the Fourier's law. We are assuming we are not accounting any delay between these two. And second thing is that propagation of the speed of thermal disturbance is infinite. So, if we assume the propagation of the thermal disturbance, thermal wave is propagated within this medium, in these cases we are assuming the it is the speed is infinite. So, that were the assumptions in case of Fourier's law.

And third, no time lag between the temperature development within the body and application of the heat flux. In the sense, we analyze these things with the solution domain and we are assuming the through the boundary there is a interaction of the heat, that means, heat flux supplied through the boundary.

So, it means that in these cases the once the heat flux supplied through the boundary, then there is a immediately there is a development instantaneously there is a development of the temperature; no time lag between the application of the heat flux and the development of the temperature within the body. So, these were the assumptions when you associated with Fourier's law of heat conduction, but because this Fourier's law of heat conduction is applicable actually when the pulse duration is very long.

For example, may be you can apply up to the nano second pulse duration in this case, it is possible to apply the with the assumptions that there is no time delay to application of the pulse energy to the work substrate material. With this assumption, we assume that it is following the Fourier law of heat conduction, then accordingly we can estimate the temperature distribution in substrate material or maybe in case of the laser heating process or laser welding process.

But if pulse duration is too short, that means, for example, femtosecond pulse duration or maybe ultra-short pulse laser processing, pulse duration is too short, then it may not reach the instantaneously to reach the equilibrium with the that means, it means the with the application of the flux it is not immediately develop the temperature within the body. So, there may be some delay within these thing. And this delay happens that accounts in the this terminology the thermal relaxation time.

So, some time is required since in these cases to develop the temperature gradient within this body. So, that time can be represented as a thermal relaxation time. And then non-Fourier heat conduction model is simply developed just by modifying the Fourier heat conduction modeling accounting the sum these two terminology, that means, thermal relaxation times.

And this thermal relaxation time is comes into the picture once the pulse duration is comparable with the thermal relaxation time. Because pulse duration is comparable in the sense the pulse duration is very small, then only thermal relaxation time has to be accounted. Otherwise, pulse duration is very high then it is not necessary to account this thermal relaxation time.

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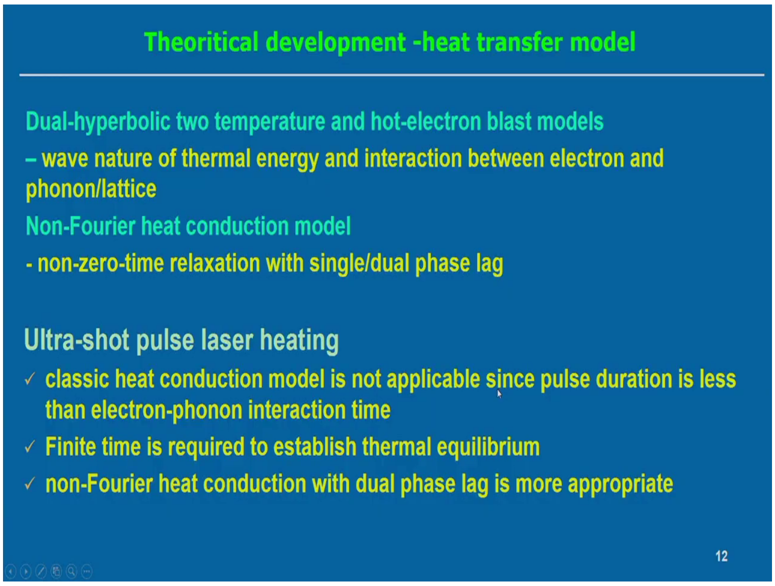

So, here you can see that most of the theoretical or heat transfer model specifically associated with the ultra-short pulse laser is either dual-hyperbolic two temperature model and hot electron blast model is based on these two models normally used, and these are all analytical models.

In this case, it is it was assumed that wave nature of the thermal energy. And this accounting the interaction between the electron and photon or lattice, that means, actually there in this cases, it is a there is two medium the electron and the lattice. So, what way the electron on lattice or electron on phonon coupling is there or interaction is there, based on that normally this is called the dual-hyperbolic two temperature model has been developed.

But the ultra-short pulse laser can also be explained by using the simply modifying the Fourier heat conduction model, and that is the objective in this particular module. So, here non-Fourier heat conduction model is the basically non-zero-time relaxation with the single or dual phase lag.

That means, we can account either single phase lag, that means, single time relaxation, or we can consider the dual phase lag; that means, with the two time lag has to be can be accounted for the development of the non-Fourier heat conduction model.

Now, you will see what way we can develop the non-Fourier heat conduction model from the Fourier heat conduction model. So, therefore, non-Fourier heat conduction model is practically used for the analysis of the ultra-short pulse laser heating. In these cases, the classic heat conduction model is not applicable since the pulse duration is less than that of the electron phonon interaction time.

Actually, if you represent the short pulse duration the heat transfer mechanism, the electron phonon interaction time the pulse duration is even more smaller than that of electron phonon interaction time that is why the classic heat conduction model is not applicable in this particular situation.

Then finite time, that means, finite time is required to establish the thermal equilibrium. From that sense; that means, once the energy is supplied to the substrate material, some finite time is required to reach the equilibrium with respect to reach the or to establish the thermal equilibrium.

So, that time gap is normally accounted in the form of a relaxation time. So, therefore, non-Fourier heat conduction with dual phase lag model is more appropriate, and we will try to I will try to explain the mainly the not the single phase lag model we will try to explain the dual phase lag model.

But it is possible to come back from dual phase lag model to single phase lag model, that means, only accounting the two different relaxation times or in single phase lag model we can account only one relaxation time. We will see the dual phase model, and then it will be easy to understand then or may be derive from the dual phase lag model to the single phase lag model.

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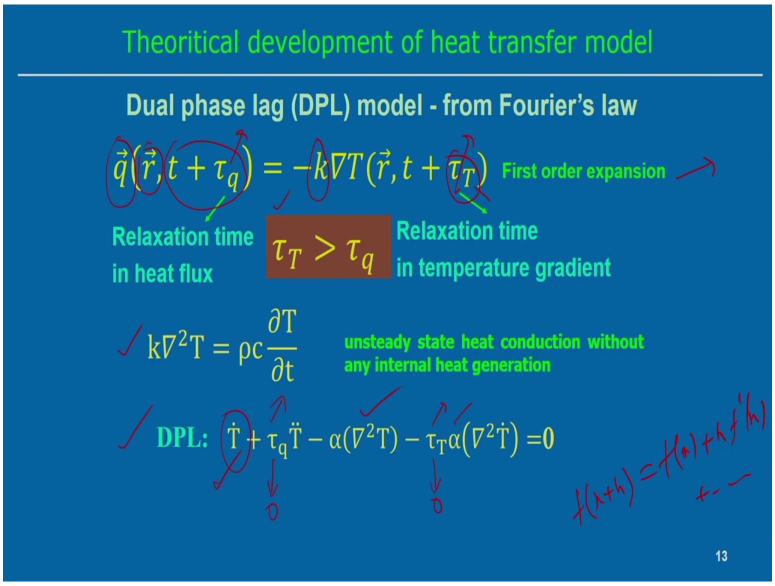

Now, we will can look into this theoretical model of the heat transfer. We start with the dual phase lag model from the Fourier's law. From Fourier's law, we know this is the Fourier's law can be represented like that the heat flux representing these things; and in the steady state we can see the k into delta T.

So, these are the this is a representation of the q equal to minus k delta T, that means, delta T represent the temperature gradient. So, if it is one direction I say that q equal to minus k into delta T by del x, so that is the representation we know in the steady state this is the Fourier heat conduction model.

Now, we if we account the time relaxation factor, in these cases, we can simply this is the spatial space spatial with respect to the spatial coordinate and account the both the time part also. That means, it is as a function of time as well as the space, but this time we just introduce t plus this another tau q that indicates the relaxation time in the heat flux.

That means, with the application of the heat flux there is a sometime is required in this cases, for the development of the temperature within this body, and one of this is accounting in the term of the relaxation time in heat flux. Then similar kind of; that means, that is why the relaxation time in heat flux is accounted the in the terminology the heat flux in that part.

Now, relaxation time in temperature gradient we are using that other relaxation time with the temperature gradient the tau T. So, these two relaxation time, we have we can introduce thus in a Fourier heat conduction model, and we can modify the Fourier heat conduction model so to account the these two relaxation time.

So, one is associated with the heat flux another is associated with the temperature gradient and that is why two relaxation time is introduced in the Fourier heat conduction model. And this is called the dual phase lag model and that comes from the Fourier heat conduction law. So, we see how further process we can reach the final equation.

So, if you do the first order expansion this equation first order expansion that Taylor series expansion may be you can look into the f of x plus h equal to f x plus h f dot x, so that is why we can use the Taylor series expansion. And we neglecting the higher order term if we consider on the first order term it can reach this kind of we can reach some relation or that we can derive this particular non-Fourier heat conduction model or you can say rather than dual phase lag model.

But in this case, we use this equation in the relaxation time is normally temperature gradient, relaxation time is more as compared to the relaxation time in the heat flux that we in the sense because first the equilibrium established for the in the relaxation time in the heat flux. So, with the application there is a delay in the application of the heat flux. So, heat flux goes to the substrate material.

So, that delay once it established there, then temperature gradient reaching the development in the temperature gradient with this body that will account to these thing. So, then after that it reach the equilibrium after this time. So, that is why the relaxation time for the development the temperature gradient is more than the relaxation time associated with the heat flux. Now, this is the one equation.

Then second equation we can use the unsteady state heat conduction equation without any internal heat generation. But of course the same thing can be derived if you consider the internal heat generation terminology as well. So, in this case, if we consider first cases, we consider the without any internal heat generation this we know, the this is the unsteady state heat conduction equation this equation and it is also useful here.

So, using these two equations maybe we can reach this kind of expression. So, T dot means is basically time derivative of temperature, T double dot means second order time derivative of temperature. These two relaxation times are there associated with this alpha thermal diffusivity these are the associated with the two, two relaxation time. And this can be the expression of the dual phase lag model.

Now, in this dual phase lag model, if we put the relaxation time 0, and other cases if we put the relaxation time 0. Then this is the simple the other this expression and this expression is equal to 0, that indicates that it is the representation of the normal Fourier heat conduction model. In that cases, that means, simply in Fourier heat conduction model, we do not account the any kind of the relaxation time.

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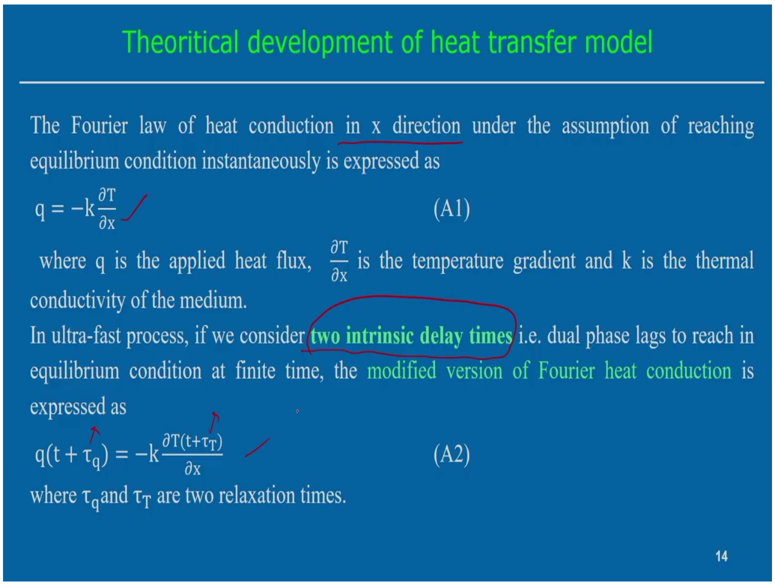

Now, you will see expression what way we can express the theoretical development of the heat transfer model using this non-Fourier heat conduction model. Now, Fourier heat conduction model we already explained this thing the assumption instantaneously development, that means, q equal to first we start with the we try to express in the one dimensional problem. We assume the heat conduction in only x-direction.

And then this is the first equation q equal to minus k into dT by dx that indicates the q is the applied flux and dT by dx is temperature gradient, and k is the thermal conductivity of the medium. Now, we have already accounted this expression that in the ultrafast process or ultrafast process means ultra laser processing two intrinsic delay times we can account.

And that to delay time can be accounted here also that we have already explained that tau q into tau T that two delay time was accounting the delay in establishment of the heat flux and delay in establishment of the temperature gradient within this body, its accounts. And we reach the equation a two and that were here which is associated with the two relaxation times in this case.

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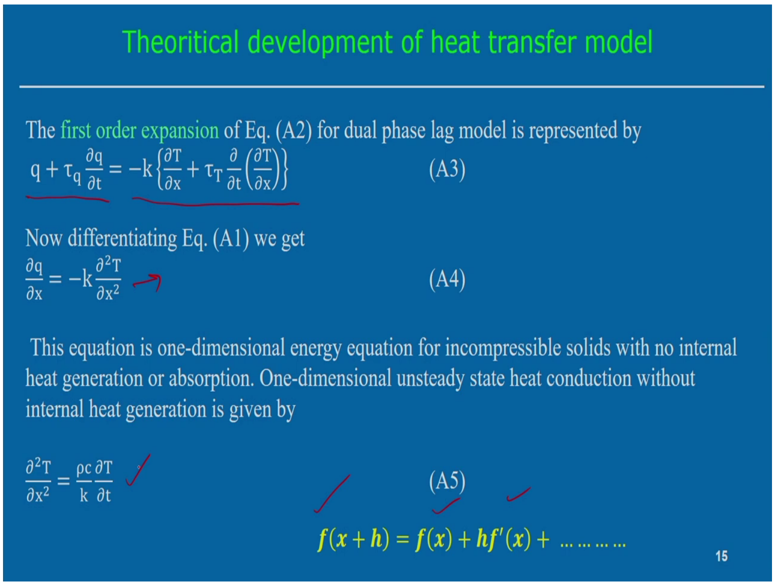

Now, if you do the first order expression of the equation A 2, then dual phase lag mode is represented these things that means, we neglect the higher order term. If we this is Taylor series expansion, if we look into that if we neglect the higher order term, that means, account on the first two terms, then we can reach this expression q equal to tau q del q by del t. Similarly, the right hand side also we can reach this expression A 3.

Now, differencing equation A 1, that means, the steady state q equal to k minus k into d t by d x that differentiate these things we can reach this expression. And this expression is one-dimensional energy equation for the incompressible solid and definitely it is associated without any heat generation term.

So, therefore, one dimension unsteady state heat conduction, that means, as a function of time the heat conduction equation if we consider what variable heat condition equation with respect to the unsteady state; that means, as a function of time we can see this is the this is the one-dimensional unsteady heat conduction equation. That we know that already explained these thing that we can use this equation also unsteady state heat conduction equation.

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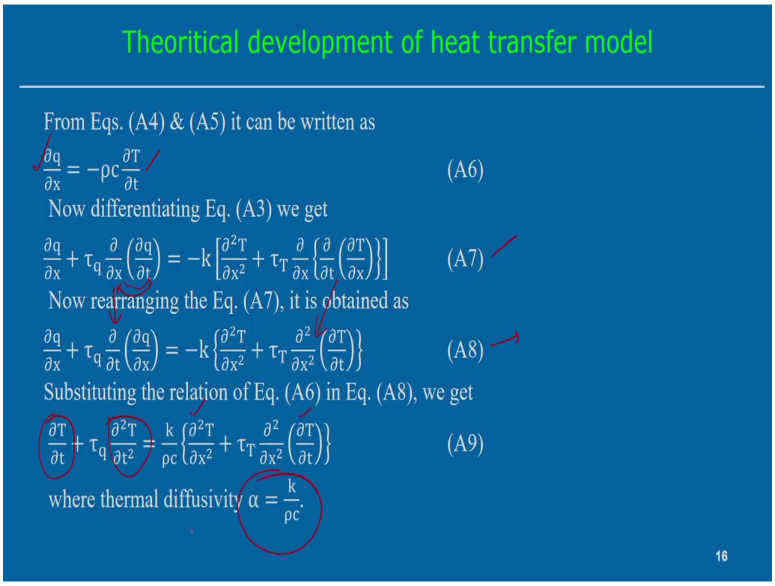

Now, using this expression A 4 and A 5, we can reach the A 4 and A 5. This A 4 equation and A 5 equation, if you use this, we can reach this expression that in terms of the heat flux see and in terms of the temperature and this is a tangent this temperature as a function of time.

Now, differentiating the equation A 3, differentiating equation A 3 means this equation this is we called from the Fourier heat conduction model by accounting this time effect here. Now, differentiating this equation, we can get this expression – expression A 7. Now, rearranging the terminology we can get this kind of expression. That means, rearranging means this del x that t here and x here with this kind of rearranging we can reach this expression from here.

Similarly, here also we can reach this expression simply rearranging second order spatial derivative. And then we can express this also in this form that the relaxation time this is first terminology, then second order temperature derivative, this is also the spatial and the temporal derivative single order. Then where alpha equal to k by rho c p, we know that alpha represent the thermal diffusivity of this particular medium that, k is the conductivity, and rho is the density of specific heat.

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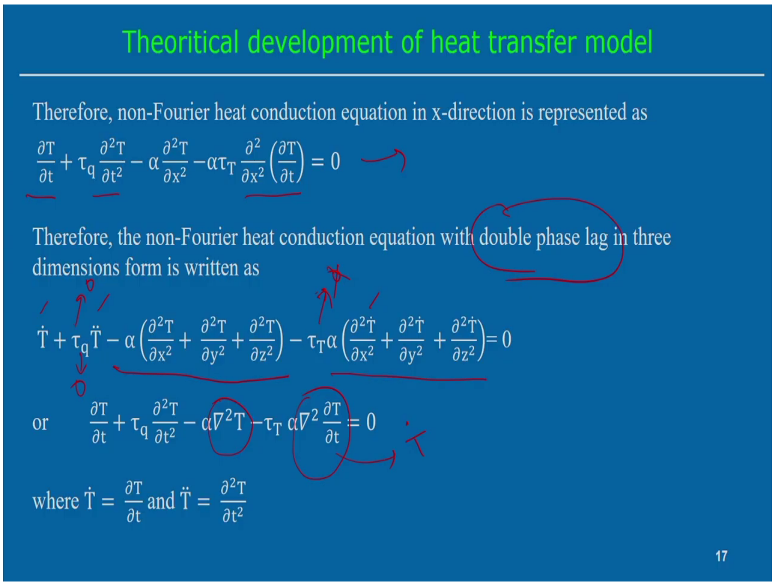

So, the thermal diffusivity in that term we can represent this equation. And therefore, non-Fourier heat conduction equation in particular direction, that means, non-Fourier heat conduction in x direction is represented as this one. So, we can see that as a function of time, this is the expression for the non-Fourier heat conduction model. But in this cases, we are not using any kind of the internal heat generation term.

Therefore, non-Fourier heat conduction can be represented in this way for three-dimensional case. This I have shown in the in case of the one-dimensional case. But if you follow the similar kind of calculation similar analogy if you follow, then we can reach the three-dimensional equation.

So, therefore, non-Fourier heat conduction equation with the double phase lag, that means, dual phase lag in three dimension form can be represented like this is the T dot represents that the time derivative, here T double dot second order time derivative. And then similarly the spatial derivative is there.

And then here also accounting the del 2 y is T dot represent the first order time derivative, and the spatial derivative is also, second order spatial derivative is also there. So, in this case, we can see this is the one relaxation time and it is account the another relaxation time both the relaxation times are there. Now, if we put simply these two relaxation time equal to 0, then we come back to the Fourier heat conduction model.

So, this is equal to 0. And this is equal to 0. Then we come back to the Fourier heat conduction model. Now, if we put the only this term 0, but it is this second this is non-zero, then that is the expression for the single phase lag single phase lag Fourier non-Fourier heat conduction model. Now, this can be expressed in the different way that is that means, three dimensional form that is del to the temperature gradient.

And then it is also represent the gradient but here del t by that means, if that time derivative is there T dot when it account to the time derivative from. Now, we reach that equation, but if you clearly observed that expression of this equation, we can see that it is associated with the second order time derivative terms are there, spatial derivative second order is there.

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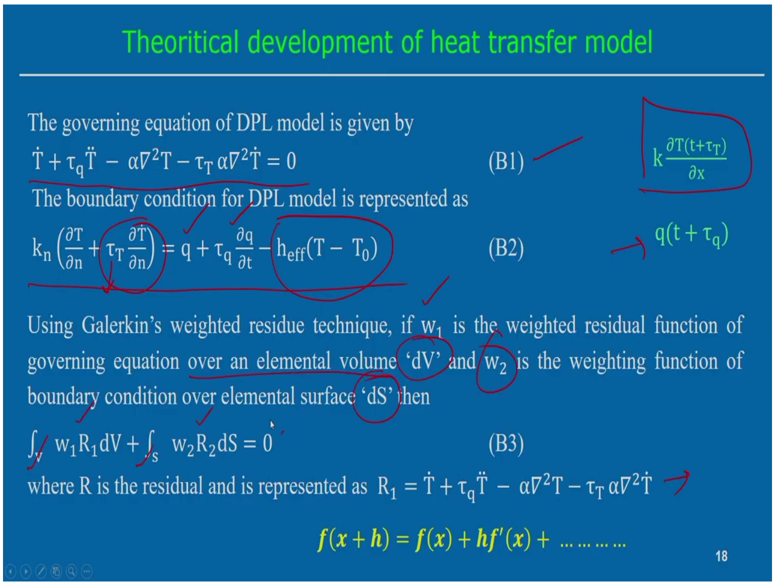

But at the same time second order time derivative is also there which is different from the in case of the Fourier heat conduction model. And we can see the difference between the Fourier heat conduction non-Fourier heat conduction model. But before that we can look into that different finite element model of this particular. So, once you reach the dual phase non-Fourier heat conduction model, and we can look into that.

So, what way we can apply the finite element method in this particular equation. So, first equation B 1 represent that this is the governing equation for DPL model. And we can account this expression of the these things. And for the simplicity output generalize this expression, we assume some boundary condition.

But in this cases, boundary condition, we can introduce the boundary condition assuming that what is interaction happening at the boundary in the sense that temperature gradient, we can some delay will be there development of the temperature gradient at the exactly at the boundary. So, we can account some delay. So, that same delay what we develop the temperature gradient within this body the same delay, that means, the time delay we can incorporate in the boundary conditions.

And how these two terms is coming? Because if we expand these things in the Taylor series model, we can find out this express. Similarly, if we expand this first order expansion, then we can reach these two term will be there in this case, that q and plus these two term will be there.

And this we are assuming the heat loss from the boundary by convection. So, this way we can formulate the boundary condition, but it is not most of the non-Fourier heat conduction analysis model, we normally do not count the boundary condition. But in these cases, we can incorporate the boundary condition for the generalized form of the equation.

Now, once we look into the governing equation, we decide this is the governing equation, and this is the boundary condition, then we can discretize using the finite element method. So, we know that Galerkin's weighted residue technique, we know already this technique.

And here w 1 is the weighted residual function of the governing equation over an particular element volume elemental volume dV. And w 2 is the other weighting function which is accounting the boundary condition, and that boundary condition is accounting on the surface, that means, over the surface area dS.

Now, this integration over this thing keeping the weightage function and if you follow the Galerkin weighted residue technique. Then we can express this R residuals R 1 and R 2 can be it here R 1 the residual is represented is the simple this expression which comes from the governing equation, similarly R 2 comes from the boundary condition.

Then and the residuals putting some weightage and summation of this integrant over the because governing equation is satisfied over the volume that is some elemental volume we can count. It is integrand over the elemental volume. And other cases the boundary condition is satisfied should be satisfied on the boundary, that means, over the surface that is over the surface we can account with the weightage along with the residual that should be 0, and this is the principle of the Galerkin weighted residue technique.

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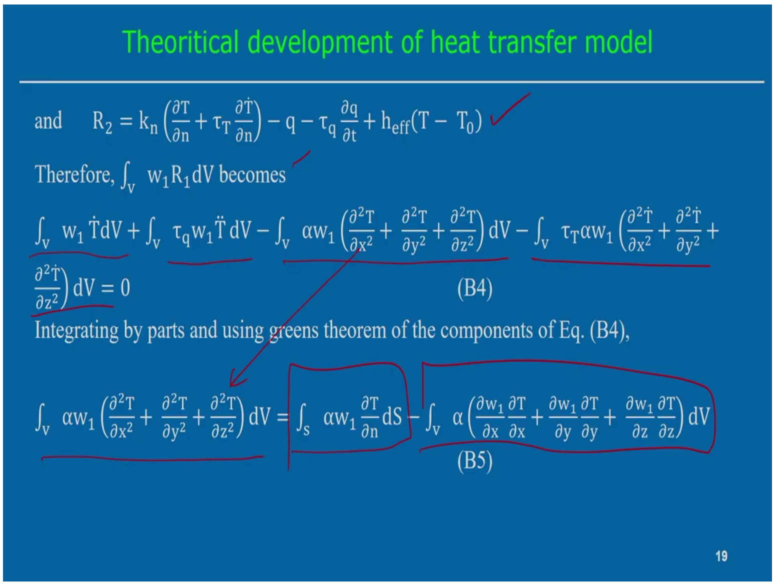

So, once we put this expression and then accordingly R 1 and R 2 is already defined. If R 1 is comes from the governing equation, R 2 from the boundary interaction or boundary condition, and such that here we can represent the R 2 also. Therefore, we can expand the w 1 R 1 dV it comes this thing that w 1 into this term and this other term and all w 1 and the over the volume.

All this particular this first term we are we can expand the first term weighted residue technique. But alpha we can do these things now this is the using the Greens theorem of the components because that is that part second order spatial derivative is there. So, in this cases, if we apply the Greens theorem, from here we can express these things over the surface this part alpha w 1, and other part can be over the volume this term can be expressed.

So, here the same way what we have done in case of the simple conduction heat conduction model what way we can did the discretization using the finite element method. We are following the similar methodology here, but here you can see the details expression of this thing.

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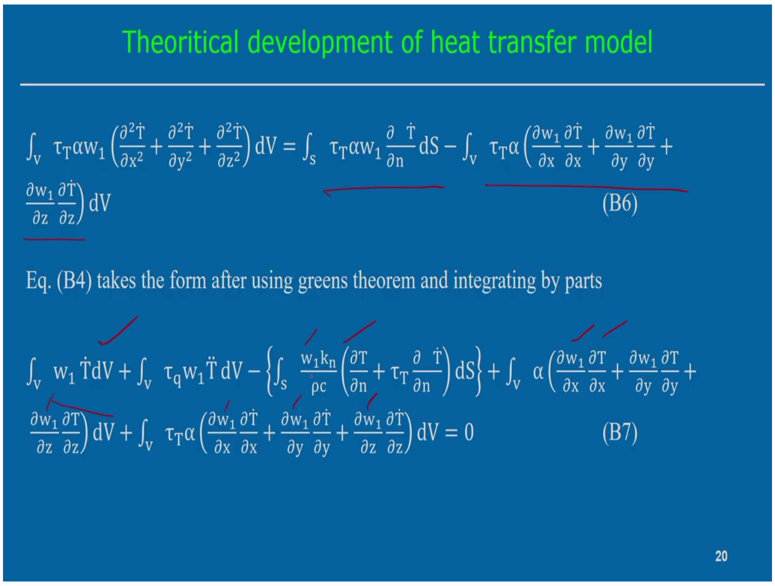

So, once we apply the greens theorem these things and then we put this we can reach this expression and what the volume in the surface. But takes the half form after using the greens theorem and the integrating by parts also, if you do the integrating by parts, then we can reach this kind of expression. So, here we can see that all this terminology that I think the equation B 4, B 4 greens theorem and integration by parts this equation, equation B 4 is here.

So, here from here also from this B 4 from equation this B 4 we follow the integration by parts. And as the after using the greens theorem then we can get the so many here the so many terms you can get which is associate you we express all this term in the integrand form is the weightage function w 1. If you see that weightage function w 1 is incorporated here, so w 1 is associated with only the this term which is comes from the residual actually comes from the boundary governing equation.

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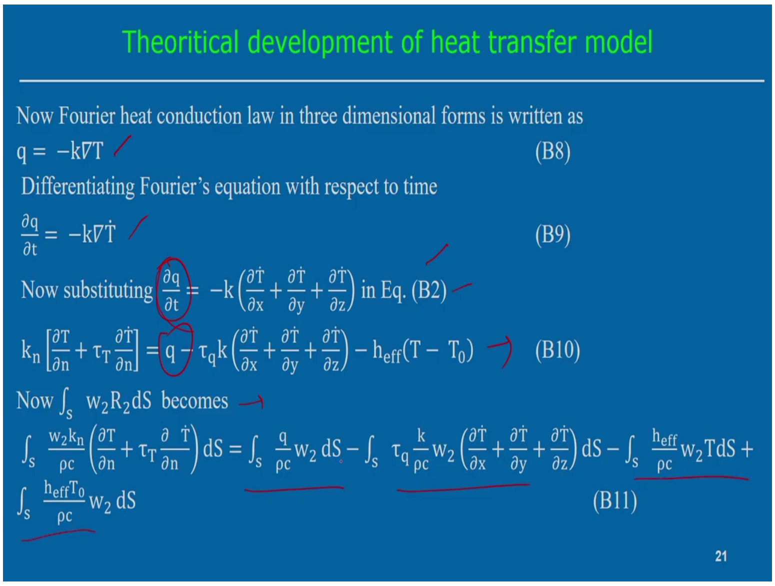

Now, Fourier heat conduction law in three-dimensional form can be written as that we have already explained these thing. Similarly, differentiating this we can Fourier with respect to time. Now, substituting this equation del q by del t in the equation B 2. And we can reach this expression because this del q by del t with differentiating Fourier with respect to time that already there.

And from in B 2 equation B 2, we can put this a B 2, then we can reach this expression, that means, we modifying the boundary condition that we try to bring this boundary replacing the term the heat flux term in the boundary condition. And or first order derivative of the heat flux term the q; del q by del t, basically that term we replace in the from the boundary condition expression in terms of the other parameter.

So, then now if you do the integration of the second term that in the Galerkin weighted residual technique, when we apply the second term w 2 R 2 dS when we apply these things then we can reach this kind of expression that over the surface. All these component if integration if necessary if I do the integration by parts also, then we reach this expression that is defined over the surface.

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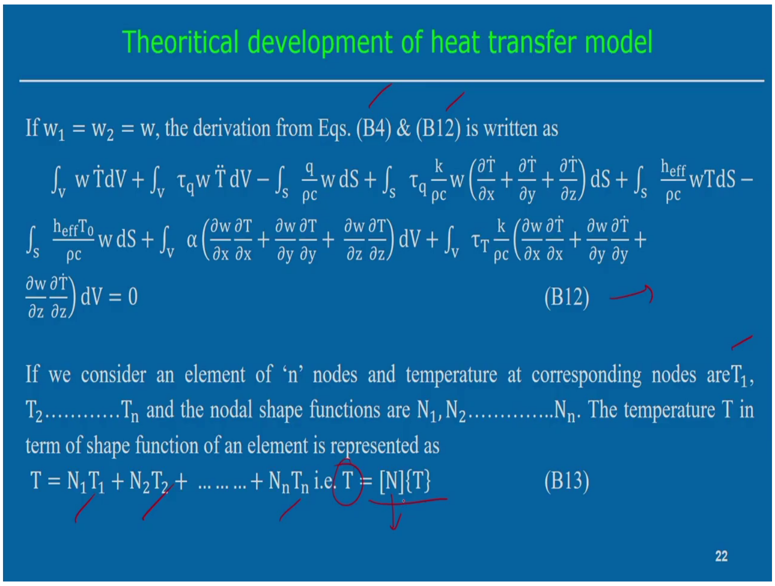

Now, using these two expression, now if you put w 1 equal to w 2 equal to w. So, that the same weightage putting then derivation from the equation B 4 and B 12, we can find out for the simplify the replacing w and w 2 by the same weightage function. We can reach the equation B 12 or the explaining all terminologies are there in this cases.

Now, if we consider the element of n nodes and temperature at corresponding nodes are particular T 1, T 2 in the particular indicates the temperature in particular node and the nodal shape function can be represented as N 1, N 2 up to N n. Therefore, the temperature T variable can be represent that N 1 T 1 N 2 T 2 and N n T n, that means, it can be represent temperature can be represented by this way that represents the shape function and temperature.

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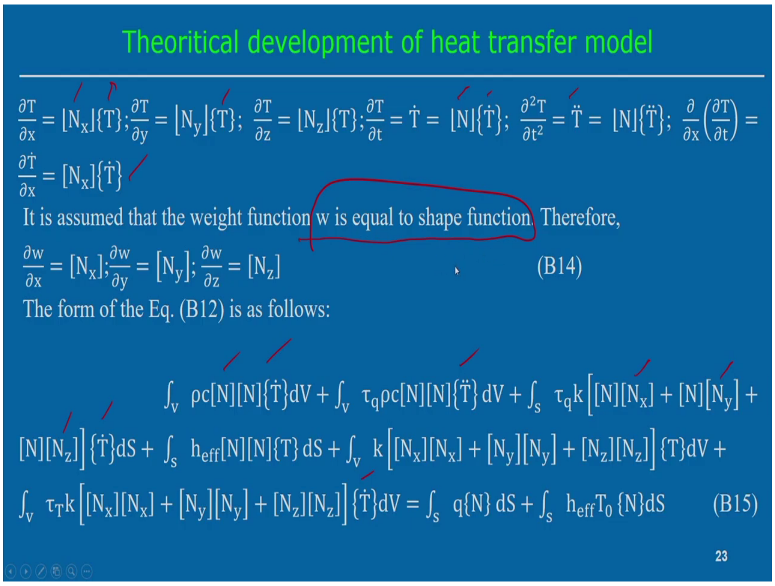

Now, del t by del x is the temperature variable with respect to x, we can represent the derivation with respect to the shape function N. So, that we represent del t by del x equal to N x, and T is the temperature that is the nodal temperature in this case. Similarly, del T by del y N y T, and del T by del z, similar way we can do these things. And we represent del T by del T is the basically shape function N and the T dot.

And T dot it represent the column vector in this case, and N in the form of a matrix we can represents these thing. And a similar way T; T double dot represent the N T double dot, that means, the it is not associated with spatial derivative, but it is a temporal derivative we account the temporal derivative separately. And then this terminology we can use, and then it is assumed that the weightage function w is equal to the shape function.

Now, if we assume that weight weightage function is equal to the shape function, then del w by del x is simply represented, that means, shape function, that means, it should be n del w by del x is equal to N x. Similarly del w by del y is represent the N y, del w by del z equal to N z. Such that B 4 equation can be expressed in terms of the shape function N, if you see n in terms of the shape function N and here also N x, N y, N z are also accounted here.

And we had only spatial derivative here you can show, and we keep the as well as the temporal derivative is also there, T dot is terminology is there. And of course, T double dot term will be there, also here also T dot, T double dot term is there. Now, this is the after spatial the discretization of the spatial domain, we can represent these are the expression. Of course, it is associated with the temporal derivative terminology also that we have already explained these things.

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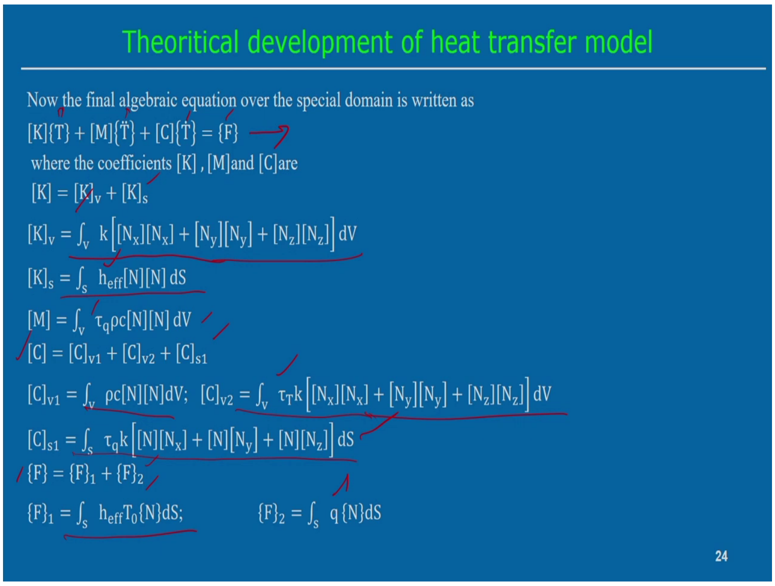

Now, the final algebraic equation over the spatial domain we represents like this in the most compact form of this final expression such that K is associated only the temperature T column vector and M is associated with the T double dot. That means, temporal derivative of the temperature and second order derivative and the first order derivative of the temporal.

So, these are the final form of the equation and in the F represent the load vector basically represent the load can be calculated there. That means, in this cases, may be heat input to the substrate material that can be represented in the form of F.

So, therefore, this coefficients this expression is the second order time derivative in this cases, that we can this we can get from this expression. And here we can find out that K can be represented there are two term one the volumetric term and surface term is there such that this volumetric term is this expression over the elemental volume dV, and K s over the surface area dS.

And that in this cases, account h effective, that means, heat transfer coefficients is accounting this that is definitely heat transfer coefficient is defined over the surface. So, K s represent the surface component. And then similarly, M the volumetric component which is accounting the one relaxation time rho c, and then simply to shape function and the dV.

Similarly, C can be represented there are two volumetric terminology is there v 1, v 2 and one surface component is there. So that is why C v 1 represented like this C v 2 here in this cases, represented like this. So, here accounting one of the relaxation time; C s 1 you know surface it represent the one of the relaxation time.

If you see the surface relax if the surface terminology in this case, the relaxation time is accounting. Because there may be the tau q can be represented like that what is the time delay to develop the heat flux within this body also. This we can express these things.

And finally, F can be represented in the form of a F 1 and F 2 both components are there. F 1 is associated with a h effective and T series the reference temperature over which we can estimate the heat transfer heat loss from this basically T 0 the ambient outside temperature, so that accounting these thing.

And F 2 is account with the what is the heat flux is applied to the substrate material. That means, it basically F accounts the what is the input, that means, what is the boundary through the boundary interaction, what is the heat flux applied to this substrate material. So, this way we can represent this equation in a that is called the final object equation on the spatial domain discretization and process, and that is associated with the dual phase lag model.

So, once we form the different elemental matrix, then we will try to look into this what are the overall nature of the equation which you are supposed to solve this thing. So, if we can get the linear system of equation as following the Galerkin weighted residue technique, we can get this equation that K T M T double dot C T dot equal to F.

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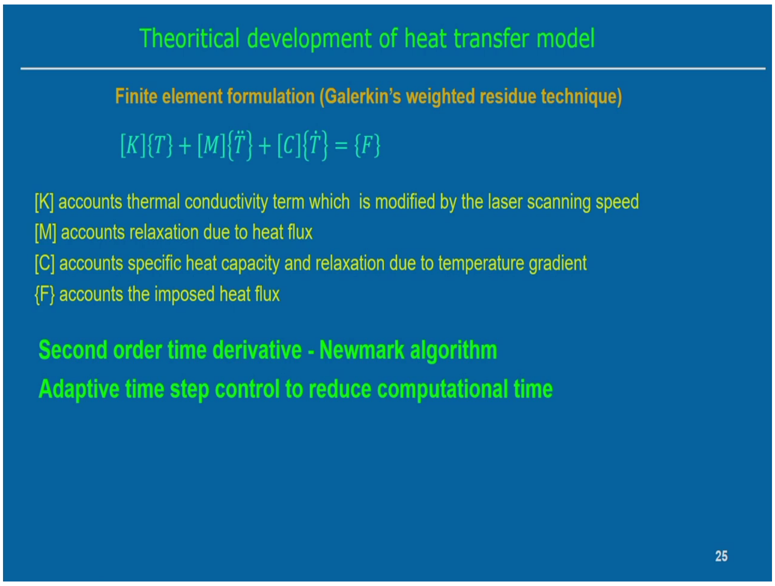

Actually, K accounts the thermal conductivity that we have already explained the K in the different components of the K which is modified by the lasers scanning speed. Actually the laser scanning speed is included in this term K. Then M accounts the relaxation due to the heat flux that way we can finding out the M is this that quantity. And we have already shown that what are the expression for the M which it consider the different components.

Then C accounts the specific heat capacity and the relaxation due to the temperature gradient. So, M in this terminology relaxation due to the heat flux term is there, and in C the relaxation due to temperature gradient as well as the specific heat accounts the C term. And then F accounts the imposed heat flux that is the that is expressed in terms in the form of a column vector we represent F. And otherwise K, M, C, these are the kind of square matrix.

And then, but in this case, the it is a spatial derivative, but as well as it shows that there is a it also associated some second order time derivatives is also there. Now, once we form in this spatial discretization of this domain of interest, and then after that there is a it is also necessary to discretize in the time domain also. Then we will get the linear system of the equation in the form of A x equal to B.

And if you solve this equation, then you will be able to know the unknown variable here, in this case, it is the temperature. So, now it is quite obvious the second order time derivative which is completely different from the heat conduction Fourier heat conduction equation that we normally solve in case of the welding processes only heat conduction equation.

But non-Fourier heat conduction in these cases, the second order time derivative comes into the picture. Then we can follow the Newmark algorithm to solve this in the time domain basically to look into that what way we can find a different expression in the time domain that if you follow the Newmark algorithm.

But of course, we observed in this case, that ultra-short pulse laser, and since we are want to capture the effect of the particular pulse and pulse duration is very small say in the femtosecond order of the femtosecond. Therefore, the time step should be less than that of the pulse duration when you try to implement in this particular new algorithm.

But once we deal with the once call with the time step which is less than that of the femtosecond, then the computational time will be huge, in this case. Therefore, it is necessary to look into that adaptive time step control to reduce the computational time. And mostly this adaptive time steps maybe you can look into in the pulse during the pulse off period. Because pulse duration is very small it is with the range of the femtosecond and maybe pulse off time may be much more high.

So, during the pulse off time, we can increase the time step. And by following certain algorithm, or some by finding the suitable adaptive time step algorithm such that computational time can be reduced.

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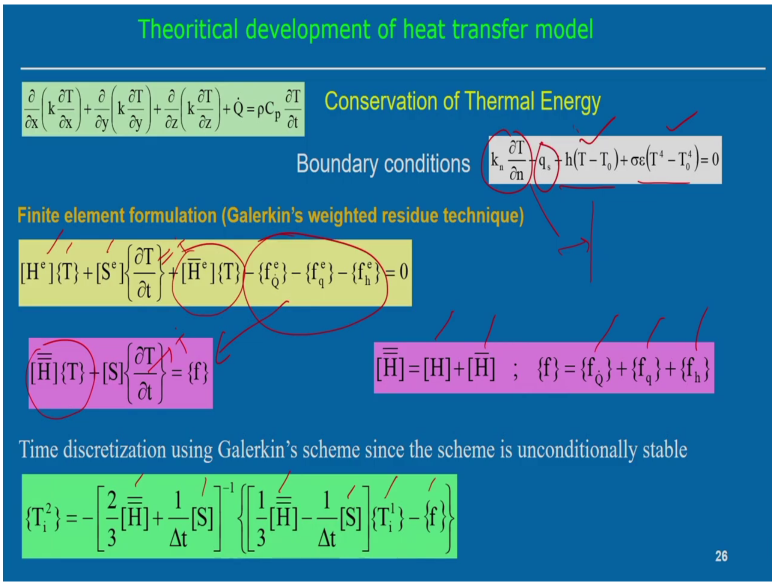

Now, if you want to compare how this discretization form of this equation or this linear system of the equation is different from the Fourier heat conduction model, we can compare one by one. For example in Fourier heat conduction model, this was the expression that k del x d by del x k into d T by del x plus other term into y and z, and the Q dot in the internal heat generation term and rho C p delta T by T.

So, that means, it is a transient heat conduction equation that is Fourier heat conduction equation we solve. And we also say the conservation of energy equation that is solved in the domain along with the appropriate boundary condition. And see the boundary condition is also different from the what way we implement in the boundary condition in case of non-Fourier heat conduction model.

So, here there is no kind of time relaxation time is not associated with any of the term which is simply the what is heat conducted away to the surface of the boundary. And then at the same rate, the heat will be heat loss in the in terms of the radiation and convection. And then mathematical form we can write this equation.

And of course, in the solution domain q x represent the interaction with the boundary, that means, flux applied to the boundary. So, heat input to the boundary and remaining others are the heat loss by convection and radiation in the represented by these two terms, these first term and second term. This is the heat loss by convection and heat loss by radiation we can include.

And q s represents the heat flux on to substrate that is input to the substrate that is negative. And k del T by del n is the at the boundary what is the heat conducted at the boundary such that it make the balance the heat loss from the boundary. So, this is the mathematical form of boundary equation in case of the Fourier heat conduction model. There is no need to account any kind of the time relaxation as compared to the non-Fourier heat conduction model.

Now, if we apply the Galerkin weighted residue technique the same finite element formulation with a similar methodology if you put, this thing we can achieve the equation is something like this is the in the elemental form the H term is there, the temperature, S term, but here the its correspond to the T dot because it is a time first order time derivative.

This is the I think H the other term we have seen already in Fourier heat conduction model that expression of the different elemental matrix. These three terms represents the basically the load vector which is equal to the, so which is equal to the F in this case. And then finally, we reach this equation in the form of a H T which is and S, and S we can see it is correspond to delta T dot.

So, H T dot equal to f. So, this is the general expression for the final we can reach this equation, but in this case, time derivative this on the first order time derivative. Even if we use the time discretization using the Galerkin's scheme also and then in for in case the scheme is following the unconditional stable. Then we can reach that we can in the time domain we can get the T 2 means at time step 2 that is the temperature will be in terms of the other parameters.

Say in the form of a T 1 F load vector and H and S terms are also there, and H double dot is basically combination of both H S, H and H bar. And f consist of the all the components if exist there. So, this is the general form of the Fourier heat conduction model and it is very much obvious that how this expression is different from the non-Fourier heat conduction model. Because non-Fourier heat conduction model the equation was the something like this which is different from the Fourier heat conduction model.

So, therefore, solution discretization is the in spatial domain, it is a it is a follow the similar kind of methodology, but discretization the time domain are different in these two phases.

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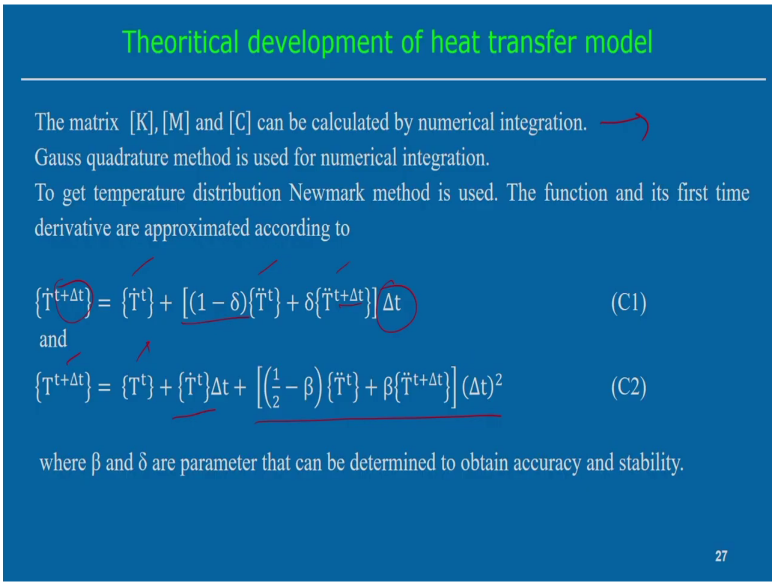

Now, we will look into the what way we can discretize in the time domain according to Newmark's algorithm. We can see the time domain the matrix K, M and C can be calculated by the numerical integration. It is a spatial domain that we have we are following the same thing what when you try to describe the Fourier heat conduction model. Now, Gauss quadrature method is used for the numerical integration.

We normally use the Gauss quadrature method in the numerical integration on the spatial domain. Now, to get the temperature distribution Newmark method is used one particular step. The function and its first time derivative are approximated according to these things. That means, suppose you want to estimate what is the temperature at time t plus time step t plus delta t.

So, at time t plus delta t t dot can be expressed like that it is the T dot T at the what is the value of at time step T plus some 1 minus delta into expression is the T double dot T and delta T double dot T t plus delta t, and this is the time step. It means that this is the way we can approximated according to Newmark's algorithm method. We can approximate what is the T dot value that means, first order time derivative at time step T plus delta t. This is the expression which is expressed in the form of equation C 1.

Similarly, what is the expression T temperature at time T plus delta t, that means, the time step T plus delta t temperature can be represented in other form temperature. If the previous time step at the time step T, and then T dot first order time derivative and this thing at time step T, and we can express this thing in the form of a other because here T double dot in terms of the delta t.

This expression actually comes but following the Newmark's method. These things where beta and delta are the two parameters that can be determined to obtain the accuracy and stability of this particular solution in the time domain. So, these are the expression for the T plus delta t the different the what is the value of temperature, and what is the value of first order time derivative of temperature in the form of a other parameters.

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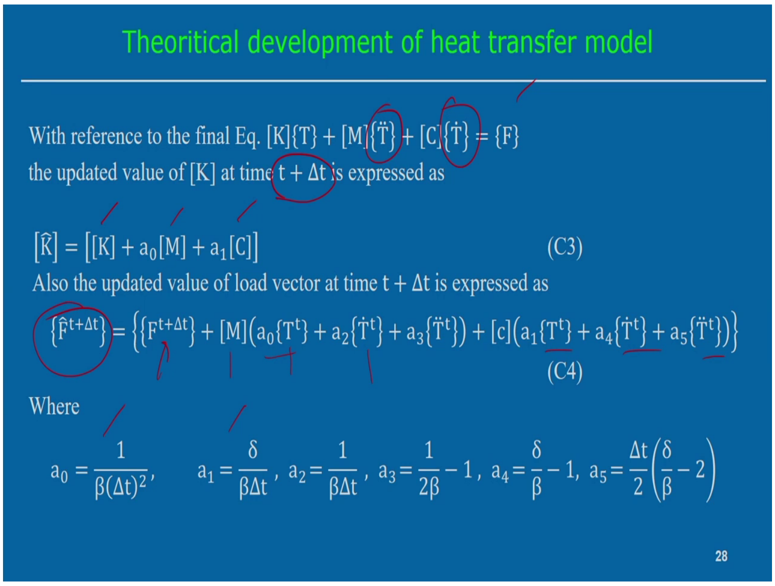

Now, once with reference to the final equation, if you look into these are the form of the final equation, this is the second order time derivative is there and first order time derivative is there, F represents the load vector, and K is the usual meaning of that we already explained this thing. Now, updated value of the K at the time t plus delta t.

So, at the next time step t plus delta t, the updated value of K can be represented like that K what is the value was the K a 0 M a 1 C. Such that a 0 is related to some time step and the stability parameter of beta. And a 0 and a 1 is corresponds to the delta by beta delta t, that means, it is also associated the what is the value of the stability parameter delta and the time step.

So, now, the updated value of the load vector at time t plus delta t is also expressed as that means, at updated value of load vector at the time step t plus delta t which is a which need the information of the previous term. So, what is the value of F at exactly that time step and the other values M, and this thing temperature at the previous time step temperature first T dot at the previous time step. Similar T double dot at the previous time steps, all these values are required, then we can estimate what is the value of F t plus delta t and this way we can get the expression.

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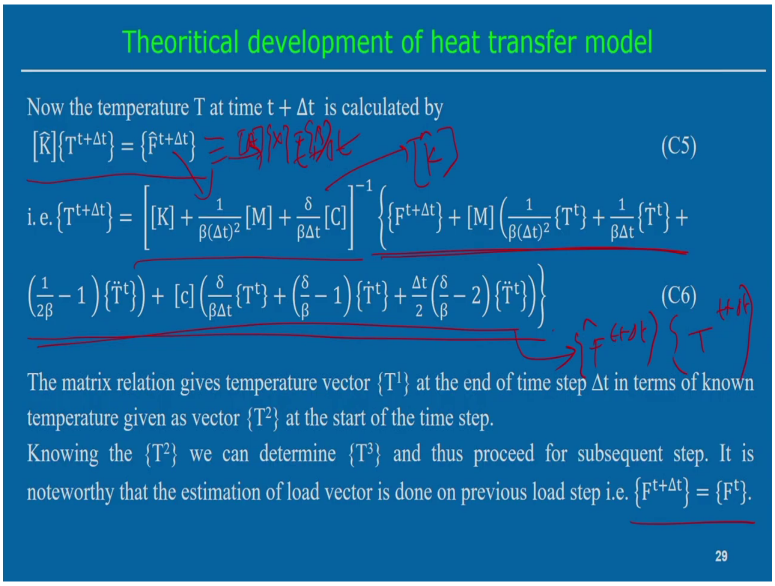

And once you get estimate this value, now the temperature T at time t plus delta t is calculated as we just putting these value we can see. Finally, we are expressing these at the time at the time step t plus delta t. And this expresses that in the form of a; A x equal to B, it is equivalent to if you look into this expression, it is a A x equal to B.

So, now, it is a linear system of equation is represented like that and if you solve this equation A x equal to B. Then we will be able to solve what is the value of temperature at the time t plus delta t. So, this is the expression for that, so that means, or you can do we can see from here this expression T at time t plus delta t the simply inverse of this matrix can be represented like that K, this term inverse and this F, F t plus delta t this term represent the F t plus delta t, and this term represent the K cap.

So, this is K cap and this total value this represents the F cap t plus delta t. Now, once we get this expression, then we solve it from the solver. Then we can we will be able to finding what is the value of temperature we can find out the is the output will be the T at t plus delta t. So, this is the output. That means, each and every node point will be getting what is the value of temperature at time step t plus delta t.

So, therefore, the matrix relation gives the temperature vector T 1 at the end of a one particular time step delta T in terms of the known temperature given as the as a vector T two at the start of the each time step. So, this way we can estimate the value of the, at each and every time step, we can value we can estimate the value of the temperature and for each within the solution domain and particularly in the discrete point of the node.

So, therefore, knowing the T 2, we can determine the T 3 similar way and thus proceeds subsequent step. But if we observe these thing in these cases the we assume that load vector F t plus delta t, that means, what is the value of load vectors at time t plus delta t equal to load vector at time t.

That means, this case is the it is the same value otherwise you will not be able to solve this expression. So, this assumptions is there. So, that means, previous time steps what is the load vector and the next time step what is the load vector and that at the same. So, this way we can estimate the value of temperature.

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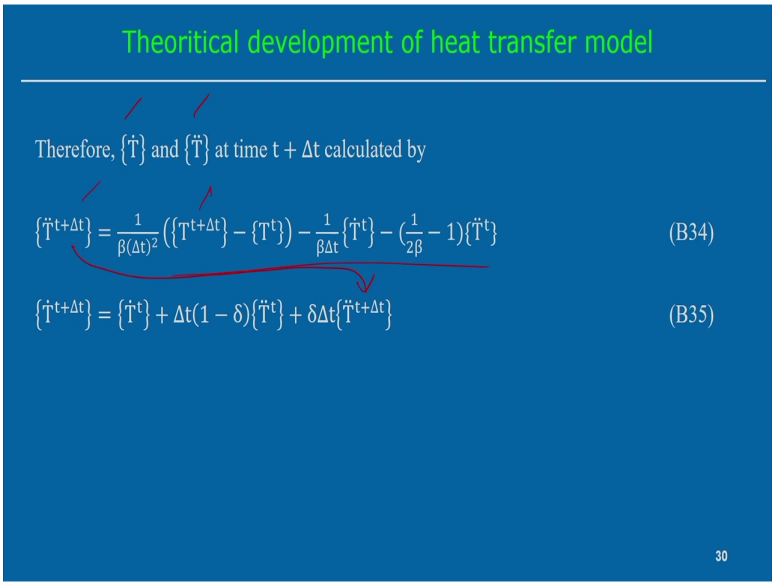

Now, once you get these things estimate the temperature then that information is also required in this particular case, that means, T dot value is required, at the same time T double dot at the t plus delta t can be estimated like that expression that T dot in the form of a in the if T dot at t plus delta t if you see that what is the time temperature the current time step t plus delta t, and all the information and equate the previous time step. So, previous time step, putting these value we can get the T T double dot t plus delta t.

Similarly, T dot t plus delta t we can get. Once we estimate T double dot t plus delta t is the current time step, then from here if you put this value, then we can estimate the T dot at t plus delta t time by taking the considering the information or may be the value as the previous time step.

So, this way the sequentially you have to solve this one by one equation, you have to arrange the equation in such a way that information from the if we estimate, for example, in this case, if we estimate the current time step what is the value of T double dot and that information is required to estimate the value of single T dot in the current time step.

So, that kind of information is required such that sequentially we can finding we can estimate the different values of the variables and in the equation we can solve this equation. So, thank you very much for your kind attention. This is the first part of the non-Fourier heat conduction model. I have shown the theoretical part of this a non-Fourier heat conduction model using the finite element model.

Now, we will see the some application of for the solving the different kind of the problem using the non-Fourier heat conduction model.