Finite Element modeling of Welding processes

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Lecture - 29

Yield Function, Von Mises Yield Surface and Hardening rule

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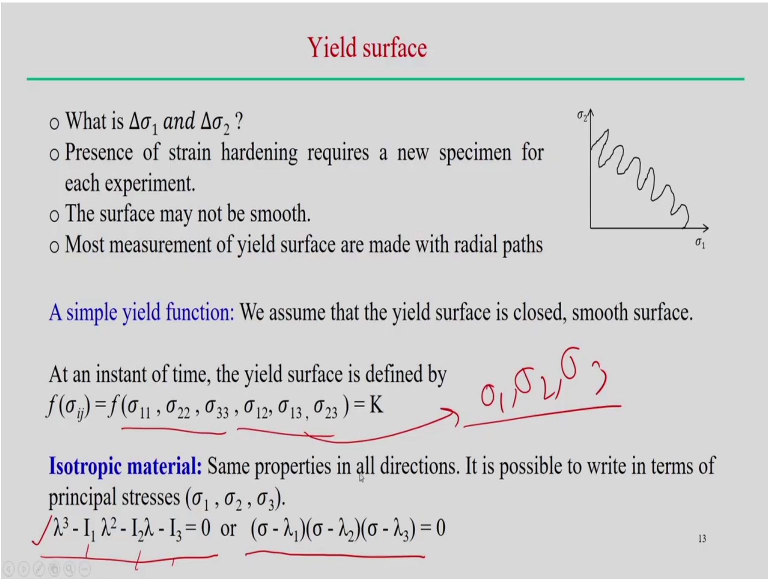

Hello, everybody. We are discussing about the Yield surface in case of 3-dimensional problem. So, once we theoretically assume some the functional form of yield surface and then we can reach we can see that how this functional form can be derived means particular case also in this particular situation.

And, then to do that maybe some assumptions are required to reach certain functional form that will predict the yield surface in case of the 3-dimensional thermo mechanical problem so, in general. So, first we assume that already seen that this functional form is a as a function of the all the individual stress component all the normal stress and the shear stress components. Or explicitly we can say that it can be a function of only those principal stress sigma 1, sigma 2, sigma 3.

Because sigma 1, sigma 2, sigma 3 itself can be represented in the form of a the original or initial state of the stress. To do that we assume this is the functional form. Now, for isotropic material we understand that if we assume the isotropic material the properties are same irrespective of the any direction.

So, it is possible to write in terms of the principal stresses; that means, for isotropic material then this it is a all this directly we can convert the initial state of the stress the original state of the stress in the form of principal stress if we assume the isotropic material.

That means, in terms of the principal stress that this cubic equation to find out the root of the equation or to find out the principal stress components that we can we express this equation like that, that we have already discussed that we can express the from the state of the stress in terms of the principal stress. Also this cubic equation can be represents and the roots of the cubic equation is the represents the principal stress components.

So, now, from this cubic equation we can say it is a in terms of the stress invariant I 1, I 2, I 3 if we put the appropriate value in this case we will be able to reach this thing that sigma minus lambda 1, sigma minus lambda 2 and sigma minus lambda 3 such that this lambda 1, lambda 2, lambda 3 can be the roots of this particular equation. So, in this form.

Now, once we reach this form then it is true; that means, once we representing this yield functional form in the form of a principal stress, it means that we are assuming the isotropic material case.

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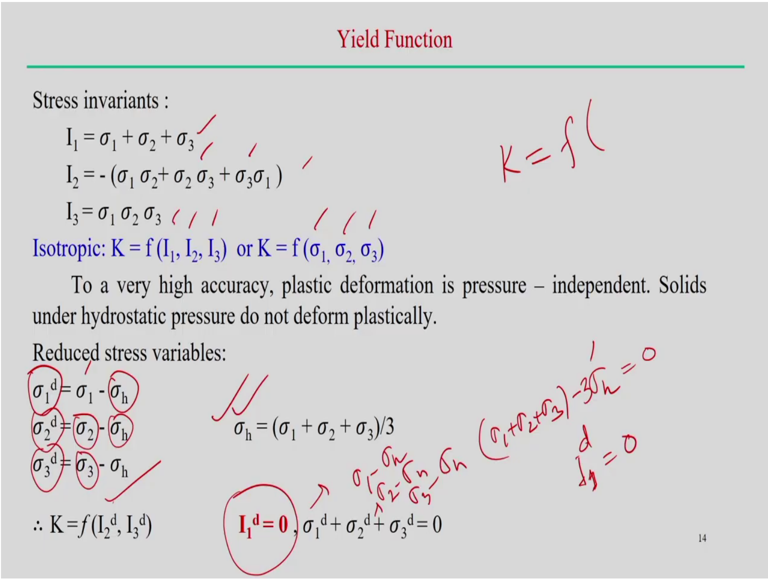

Now, we know the stress invariants can be expressed in terms of the principal stress as well also I 1 sigma 1 plus sigma 2 plus sigma 3. Similarly, I 2 in the form of a sigma 1 sigma 2 sigma 2 sigma 3 and sigma 3 sigma 1 in that form and I 3 equal to sigma 1 into sigma 2 to sigma 3. So, these are the stress invariants in terms of the principal stress components.

Now, this for isotropic material K is the functional from this particular K decide the shape of particular form of a function. So, f this K can be represent this functional as a function of I 1, I 2, I 3 or K can be represent as a function of sigma 1, sigma 2, sigma 3, it means that this functional form can be represent in the form of a either principal stress or in the form of a stress invariance. That stress invariance itself is a function of the principal stress value.

Now, if you want to a very high accuracy in that case plastic deformation is pressure independent that we already explain this thing that any state of the stress can be divided into the hydrostatic stress component and the deviatoric stress component. But, if we assume that plastic deformation is independent of the hydro static stress component then we can reach some conclusion from here also.

So, solids because solids under hydro static pressure do not deform plastically, we normally assume the solid under hydrostatic pressure is basically the elastic deformation is there. So, therefore, once you constitute the plastic deformation, we can reduce the stress variables. For example, we know the hydrostatic stress component, the principal stress component and the actual value of the principal stress minus the sigma i; sigma i is the hydro static stress component.

Similarly, sigma 2 d in the case the deviatoric stress component is the state of actual value of the principal stress minus the hydrostatic stress component. Similarly, for sigma 3 d also we can represent the sigma 3 minus sigma h and we know that hydro static stress component which is simply average of these value sigma 1 plus sigma 2 plus sigma 3 divided by 3.

So, this is the hydro static stress component. Now, if you represents the instead of that stress in invariant I 1, I 2 and I 3 that we can represents this if we assume that plastic deformation is independent of the hydro static stress component, then we can represent this functional form.

The stress invariant of the deviatory component, but if you look into the stress invariant for the deviatory component I 1 d is simply this first stress invariant in the deviatory components that is equal to 0. If you put this value in this case it may be sigma 1 d equal to sigma 1 minus sigma h hydro static component.

Similarly, sigma 2 d sigma 2 minus sigma h and sigma 3 plus sigma 3 minus sigma h; it means that sigma 1 plus sigma 2 plus sigma 3 minus 3 sigma h and we know that sigma h it is a value of this value sigma h equal to sigma 1 plus sigma 2 plus sigma 3 by 3. So, from here we can see that it becomes 0. It means that deviatory first stress invariant, but in terms of deviatory components it becomes 0, so that we can further simplify this functional form.

We can say that this functional form that K an isotropic for isotropic material. And, if we assume that plastic deformation is independent of the hydrostatic stress then K can be represent this functional form can be represent as a function of the two other deviatory component, the I 2 d the deviatoric stress invariant I 2 d in the in form of and I 3 d, it depends on that this.

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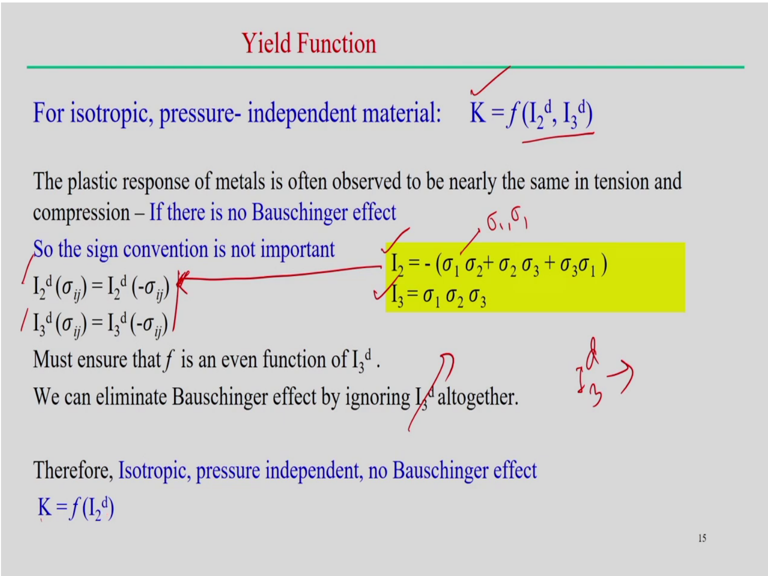

Now, further if we look that for isotropic pressure independent material K can be represent this K can be the any functional form K is equal to function of I 2 d and I 3 d. Now, plastic response of materials of an observe to be nearly the same intension and compression. But a normal engineering material if we consider that deformation behavior that means yield point or tensile and compression more or less almost same.

So, from that point of view we can see if we assume that yield point for tensile and compressive load is same. It means that if we are neglecting the Bauschinger effect; that means, if there is no Bauschinger effect meaning that the yielding happens at the same point irrespective of whether it is tensile load or compressive load.

So, therefore, if we assume no Bauschinger effect the in that mathematically we can say that no Bauschinger would mean that it is having the magnitude is the same irrespective of tensile and compressive load. Mathematically we can see that I 2 d sigma ij that means second or third.

This I 2 d means this are the two both stress invariant should be independent of the sign; that means, I 2 d sigma ij should be equal to I 2 d equal to minus sigma ij; minus sigma ij means just simply changing the magnitude of the stress from tensile to compressive or compressive to tensile.

So, then it should be; it should be same; that means, it then it represent that it is independent of the sign; that means, whether it is plus or minus what we represent the tensile load or tensile stress as a positive and a compressive load as a negative. So, once you do that, then it should satisfy these two equation.

So, therefore, similarly I 3 d sigma I j should be equal to I 3 d minus sigma ij; that means, it is independent of the sign whether it is plus or minus; that means, independent of the whether it is tensile load or compressive load. Now, must ensure that f is an even function of I 3 d we can see that we see we have seen that I 2 d and I 3 d.

Then that means, second stress invariant and third stress invariant we can see that if we change the sign of the sigma 1 and sigma 2 and sigma 3; that means, sigma 1 equal to minus 1 sigma 2 equal to minus sigma 2 and sigma 3 equal to minus sigma 3. In this way if we change the sign, then the expression of the I 2 does not change it remains the same.

It means it is satisfying this functional form, but if we change the sign of the sigma 1, sigma 2 and sigma 3 in that case then this I 3 is basically there is a change of the sign for in case of the I 3. It means that f is not indicates that f is the odd function; that means, it depends on the sign convention.

It means that depends on the sign convention means irrespective; that means, it if we does not consider that means, it does not satisfy this condition of no Bauschinger effect. So, therefore, we can eliminate Bauschinger effect by simply ignoring the I 3 d altogether; that means, we can simply ignoring the I 3 d.

That means, we can eliminate the Bauschinger effect because if we consider the I 3 d; that means, we have to consider the Bauschinger effect. Then even if we consider only the I 2 d, then in that cases there is no Bauschinger effect mathematically we can see this should some even function and that is satisfying.

So, therefore, if we consider the isotropic properties or isotropic material pressure independent; that means, yielding is pressure independent and if there is no Bauschinger effect, then we can say the functional form can be further simplified as the K equal to only function of I 2 d; that means, it is a function of the deviatory component of the second stress invariant.

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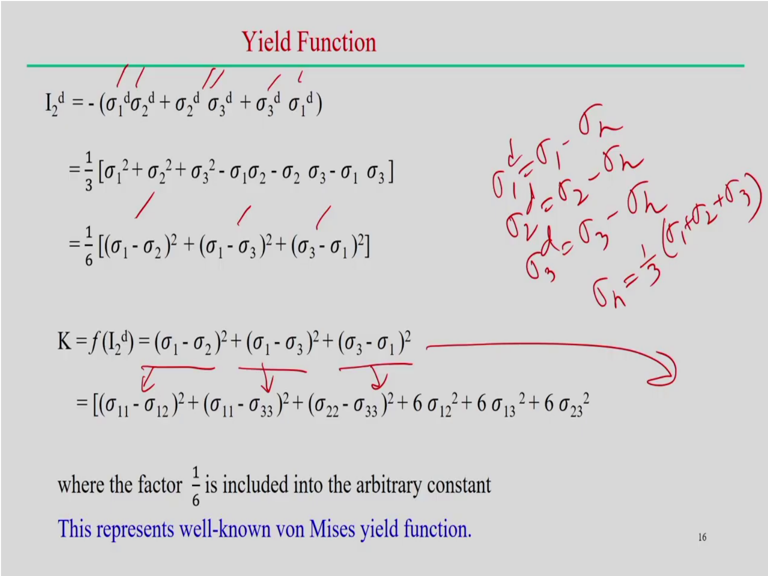

Now, once you get this expression I 2 d we can see the find out the I 2 d. I 2 d is simply minus of sigma 1 sigma 2. This is, but deviatory component sigma 2 sigma 3 deviatory components; sigma 3 sigma deviatory component. Now, if put this value of manipulation of this thing that deviatoric sigma 1 d equal to sigma 1 minus sigma h, similarly, sigma 2 d equal to sigma 2 minus sigma h, so, like that if we put all these value sigma 2 minus sigma h, in this case sigma 3 d equal to sigma 3 minus sigma h.

If we put all these value and replacing the sigma h equal to one-third of sigma 1 plus sigma 2 plus sigma 3. So, in terms of sigma 1, sigma 2, sigma 3 we can represent; that means, in terms of the principal stresses the I 2 d can be expressed in this particular form that 1 by 6 this common to all that sigma 1 minus sigma 2 square sigma 1 minus sigma 3 and sigma 3 minus sigma 1 square.

So, therefore, K can be represent in this form if we ignore this constant term because constant depends on the other depending on the particular condition we will see how constant terms can be vary depending upon the loading condition; that means, whether it is pure shear or biaxial state of the stress or only uniaxial tensile. So, all these cases this constant can be (Refer Time: 11:54).

So, finally, we are getting the functional form particular shape this function, this functional form is that this particular K equal to sigma 1 minus sigma 2 square sigma 1 minus sigma 3 square and sigma 3 minus sigma 1 square. This is the typical functional form irrespective of that constant term can be derived depending upon the loading condition.

Or this can be represent in other way also that in terms of the initial state of the stress; that means, sigma 11, sigma 12 and that means, in these cases this here it is can be represent in the form of a all the normal stress components as well as the shear stress component. Or this functional form can be represent only the principal stress component.

So, therefore, the factor 1 is 6 included into the in terms of the arbitrary constant. So, therefore, this functional form is well known von Mises yield function that we normally use during the simulation also infinite element simulation. We assume that the plastic deformation or equivalent stress that follow in the von Mises yield criteria or von Mises yield functional form, we normally consider.

So, in this case, remember once we reach this particular expression we have to remember the what are the assumptions we consider; first this is true in case of if we consider the isotropic property, isotropic material property these things. And we are assuming there is no Bauschinger effect; that means, the yielding of tensile and compressing is the same value and then it means that there is no Bauschinger effect and this the plastic deformation is independent of the applied pressure.

So, that means, plastic deformation we are considering only the because of the deviatoric stress component not the hydrostatic component. So, therefore, once we with this all assumption we can reach this is the functional form of a von Mises yield function.

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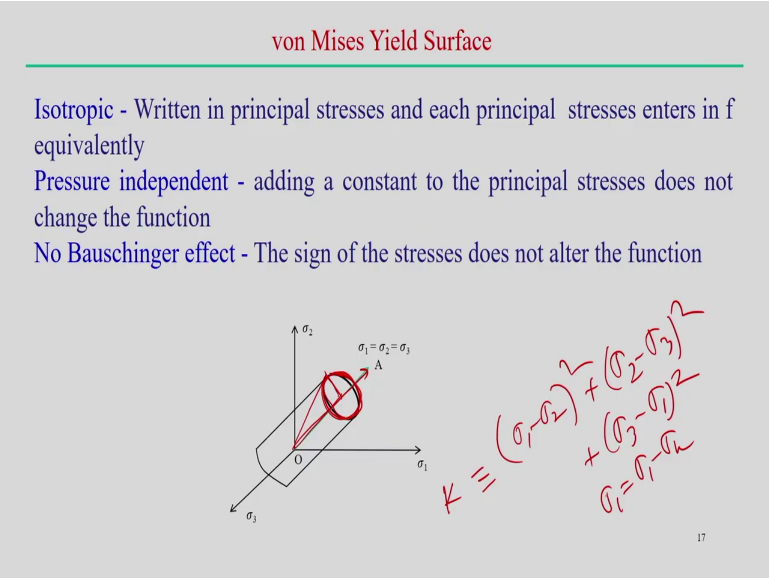

Now, we can look into this thing. So, isotropic we can check back also that whether this condition this expression or particular functional form of the von Mises yield function is satisfying all this criteria or not. So, isotropic that mean written in the principal stresses and the each principal stresses enters equivalently.

If we look that sigma 1 minus sigma 2, this is the expression and sigma 2 minus sigma 3 square plus sigma 3 minus sigma 1 square. So, all this term actually enters equivalently; that means, enters equivalently in this particular expression. So, K equal to for example, this is the functional form.

Pressure independent; that means, if we simply add the pressure adding a constant to the principal stress component does not alter the functional form. So, therefore, we can say that it is also satisfying the pressure independent does not change. For example, if you say that sigma one can be replaced by if sigma 1 or something minus sigma h something like that.

So, similarly sigma 2 sigma 3 also that means, we adding the constant time with the principal stress does not change the functional form. So, therefore, it is satisfying this condition also. No Bauschinger effect – the sign of the stresses does not alter the function. If you change the sign of the stresses then sigma 1 to minus 1 sigma 2 minus sigma 2 and sigma 3 to minus sigma 3, then it does not change the functional form.

Therefore, all these three conditions satisfying for this particular von Mises yield condition and you also graphically represent this for example, this sigma 1 sigma 2 these principal stress axis system.

In this case, we will make one line in such a way that this particular vector makes the equal angle for with respect to all the principal stress axis. And along this line it is representation of the sigma 1 equal to sigma 2 equal to sigma 3. So, this indicates the yield surface and it represents the cylindrical this thing.

So, yield surface and we can see also. So, this is the original state of the stress and we can see that actual state of the stress is the yielding occurs, it is consist of the this part above the this thick deviatoric and the hydrostatic component state of the stress, but this hydrostatic component does not change alter the yield functional form.

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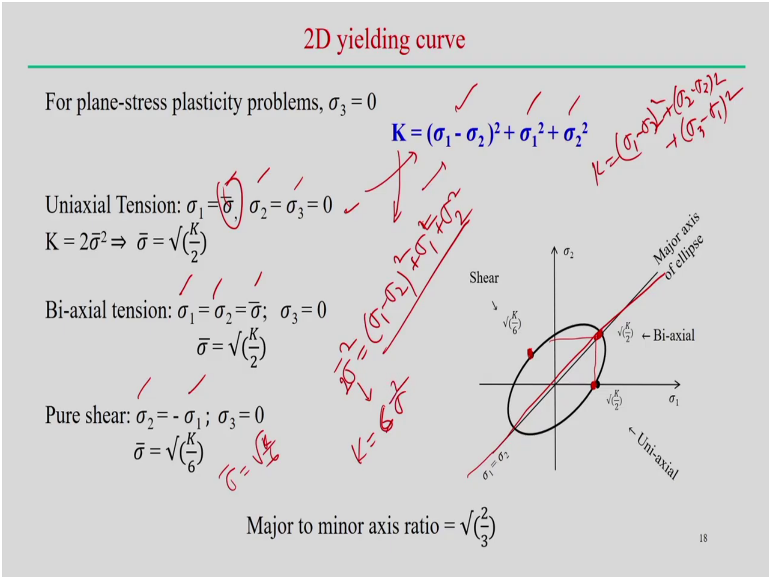

Now, if we consider the 2-dimensional problem 2-dimensional yielding curve for the plane stress plasticity problem sigma 3 equal to 0 in this particular form. That means K equal to say we start with the sigma 1 minus sigma 2 square plus sigma 2 minus sigma 3 square plus sigma 3 minus sigma 1 square.

Now, in this functional form if it is 2-dimensional problem if we assume that plane stress problem sigma 3 equal to 0. Then K can be represented like that sigma 1 minus sigma 2 square sigma 1 square plus sigma 2 square, so, in 2-dimensional problem. Now, in 2-dimensional problem if we see the uniaxial tension testing, if we apply this thing uniaxial tensile testing it indicates that sigma 1 equal to sigma bar.

So, sigma bar is the uniaxial tensile testing indicates maybe we can say that this is the yield point for the uniaxial tensile testing the sigma 1 is the yield stress value of a particular material. So, that means, the principal state is only sigma 1, the yield stress value sigma bar, sigma 2 and sigma 3 equal to 0 in these particular cases.

Now, if we put this value here in this expression then we can reach the K is equal to 2 sigma bar square or other way sigma bar is equal to root over of K by 2. So, we can reach some expression of sigma bar or we can reach some expression of the K also in this case.

Now, if we consider the bi-axial tension, bi-axial tension the principal stress value is the same sigma 1 sigma 2 equal to sigma bar. And sigma 3 equal to 0 if we put then we can reach also the same value sigma bar equal to K root over of K by 2, but in case of the pure shear condition suppose we are making some sample experiment that where we are following the simple shear condition shearing happens.

So, in that cases the principal stress sigma 1 equal to minus sigma 2 or sigma 2 equal to minus sigma 1 and sigma 3 is equal to 0. Put this condition here in this expression then we can reach the sigma bar is the equal to root over of K by 6. This sigma bar indicates the value of the shear yield point value shear yield stress value.

So, it we can see that this particular expression this K equal to this functional form can be represent kind of the ellipse in this case and the on this ellipse that this line indicates that major axis of the ellipse along the major axis of the ellipse is equal to sigma 1 equal to sigma 2 in the along this axis.

Now, we can reach the different point also in this point the in case of the bi-axial; bi-axial the sigma 1 equal to sigma 2 then this is the this value bi axial state of the stress represent the root over of K by 2, but if the uniaxial then only sigma 1 equal to sigma bar in this case then we can reach the root over of K by 2 because this uniaxial that means, stress is along the one direction.

It is the pure shear condition then sigma 1 equal to minus sigma 2 or sigma 2 equal 2 minus of sigma 1. So, pure shear condition it can lie in this particular point and here also root over of K by 6 so that this different cases we can see the major to minor x y ratio is basically root over of 2 by 3.

So, it means that once we know this functional form then we will be able to know that what is the value of the constant in such a way that if you conduct the uniaxial tensile testing in this case the uniaxial tension testing the sigma bar is may be known. Then we can change this sigma bar is known and K is the twice sigma bar square.

So, we can say if we replace in case of the twice sigma bar square equal to sigma 1 minus sigma 2 square plus sigma 1 square plus sigma 2 square. So, in this case this is the functional form, but in this functional form this sigma bar is known quantity and this sigma bar indicates the what the value of the yield point. So, that normal the value of yield point, but with the application of the normal stress value.

So, normal yield point value sigma bar indicates. So, this is the known value for a particular material and we can use this functional form this particular functional form to define the yield surface in case of 2-dimensional problem. Similarly, suppose the experiment can be conducted on the pure shear condition. So, pure shear condition then this constant value can be different or we can say that it K is in this case the K equal to 6 sigma bar square.

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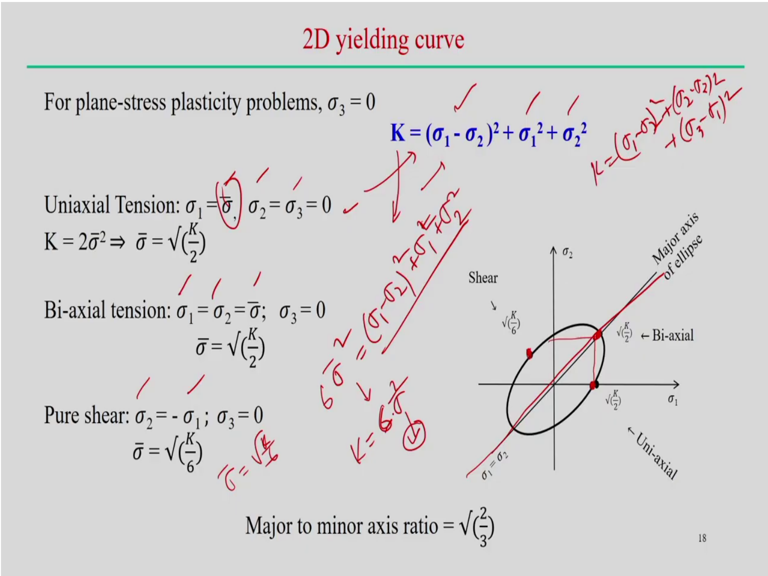

So, in this case if we replace this sigma bar square like that also if we replace this K equal to 6 sigma bar square. So, this expression can also be used the that also predict the yield stress value or yield surface or may be yield curve, but in this case this sigma bar is measured this is the shear yield point value.

The shear yield strength is different from the yield strength which we measure in the application of the normal uniaxial tensile testing. So, that means, this value depending upon the experimental condition this constant term can be defined depending upon the whatever the experimental condition we are following.

Whether we are following the pure shear condition and we measure the only the shear yield point, yield strength or normal yield strength we are if we consider in case of the uniaxial tensile testing. So, depending upon that this constant can also vary.

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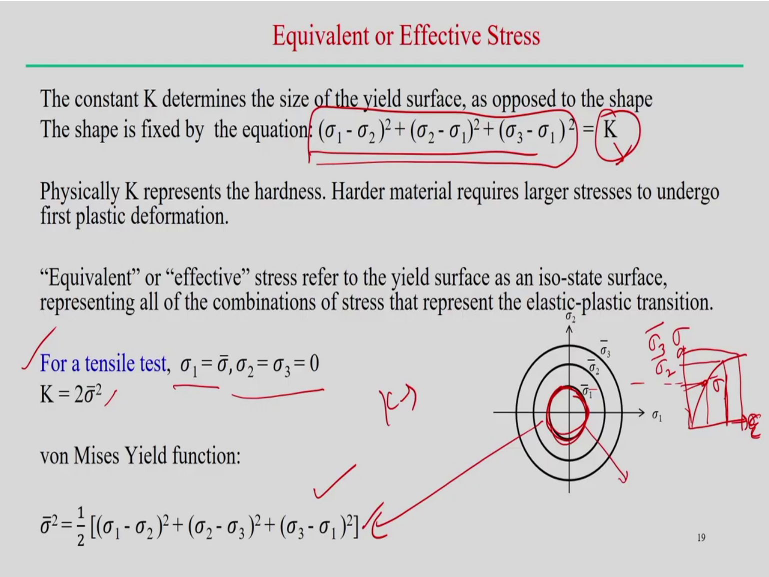

Therefore, this constant term K other way because we represent this is the functional form equal to K and K can be defined depending upon the experimental condition or value of the yield strength value measure and different situation. So, now, the constant K determines the size of the yield surface. It is a size of the yield surface as opposed to the shape.

K actually decide the size, but shape is already decide this by this equation. This equation decide the shape of this particular curve. But size is decided what value we are considering a

K. Now, physically K represent the hardness and harder material requires the larger stresses to undergo the first plastic deformation.

And after the first plastic deformation, then we can consider the strength level depending upon the, this strain hardening parameter of this particular material. So, now this other way also we can represent this functional from that equivalent or effective stress value through the yield surface as an iso-state surfaces.

So, for example, representing all the combination of the stresses that represents only in case of the elasto-plastic transition. For example, this is the one iso-state line. So, in this case it is can be corresponds to the one uniaxial tensile testing, this is the sigma value and this is the strain value. So, suppose this is the yield point. So, then this corresponds to can be the sigma 1 bar.

So, then when it is the yield point first yielding happens at particular point sigma 1 bar and this represent the iso-state surface. And, this surface can be represent in the form of a this von Mises function form. If it is a 2-dimensional we use a 2-dimensional form, if it is 3D we use the 3-dimensional form, but this is the representation of this thing.

For example, the although we are talking about the functional form this is the von Mises yield function for the for example, uniaxial tensile testing the sigma 1 equal to sigma bar, sigma 2 sigma 3 equal to 0, then we reach the K value is equal to sigma bar square and sigma bar square equal to half of this. So, this is the von Mises yield function in case of when the sigma bar is measure in case of the normal yield strength value.

Now, this normal yield strength value and this is the functional form, this is the functional form in this case. So, therefore, this curve is represented by this equation and in this case the first yielding happens sigma bar. Now, for example, after straining the, after strain harden the material become strain harder, the yielding point shift in this particular sigma 2 bar, in this case similarly after strain hardening it can reach the sigma 3 bar.

So, in this case all represent the iso stress surface, iso stress surface that corresponds to the same functional form sigma 1, sigma 2, sigma 3, but this is changing or we can say the K value is actually changing and that takes care of the strain hardening effect and that evolve the yield surface in this particular by looking into the effect of the strain hardening during the deformation process.

So, therefore, in this way we can represents the von Mises yield function and we can predict this plastic deformation following the that plastic deformation such that this either yield curve or yield surface follow the equation of the von Mises yield function.

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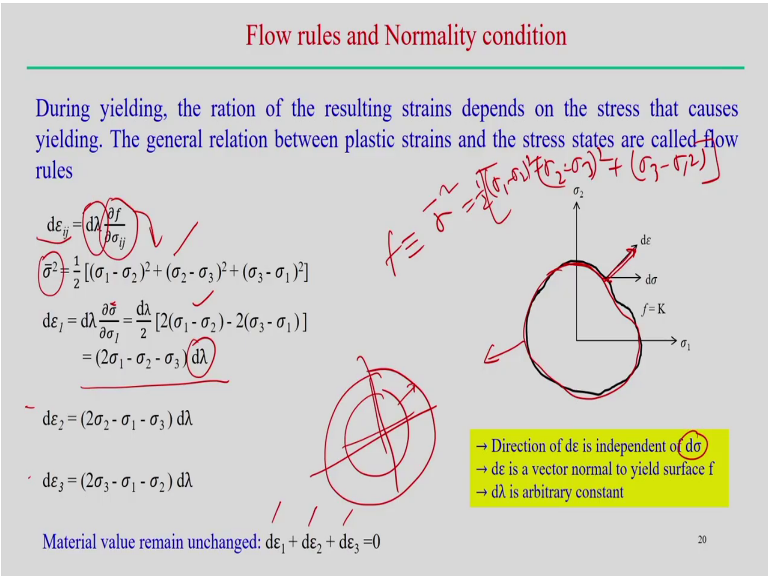

Now, once you predict this is the yield surface by following in maybe von Mises yield function or maybe some other functional form also. But, in general we consider the von Mises yield functional form. If you predict this form, the yield surface it predicts the yield curve it predicts.

Then normality condition or flow rule is important because what way the plastic deformation will sustain during the further deformation process. Sometimes we can use some kind of the flow rule to just represent the flow potential function in this particular. For example, this, what way the yield surface evolve.

So, initially yield point we can define and that may be equivalent to the what is the uniaxial tensile testing data from there where the yielding start that yield strength can be equal to this. But, how what way it evolve from this point to the yield surface evolve from the next point that can be governed, controlled by following some kind of the flow rules or it should maintain the normality condition during this process.

So, therefore, during yielding the ratio of the resulting strains depends on the stress also because strains the resulting strain also depends on the state of the stress that causes yielding. So, the general relation between the plastic strains and the stress states are called the flow rules. So, what way we can represents the plastic strain and the stress how they relate that we can represent in the form of a flow rule.

For example, in this particular case this d increment of the strain incremental strain is d lambda is a constant. We can say sometime it is a plastic multiplier and it depends on the stress. The del f, f basically represent the yield functional form and del f by del sigma ij. So, now, if we assume that this functional form can be represented like that sigma bar square equal to this.

And, then we can say that the increment of the strain d epsilon 1 can be this we can do this simply derivative of this functional form f equal to in this case the functional form equal to sigma bar square and that is equal to in this case half of square sigma 2 minus sigma 3 square plus sigma 3 minus sigma 1 square. And, that is the so, this is a functional form.

Now, from this normality condition if we see the plastic multiplier and then from there you assume this is the functional form and here if we see the del sigma bar by sigma 1 then we can see we can reach this expression. Similarly, and then finally, we can reach this expression twice sigma 1 minus sigma 2 minus sigma 3 into d lambda.

So, this incremental of the strain depends on the principle stress value as well as the actually depends on the plastic multiplier that means d lambda. This is the constant term, this what value depends on this value. Similarly d epsilon 2, d epsilon 3 it is function of the other stress parameter and this ratio.

So, such that during the deformation we can assume the material volume remains unchanged, it means that it maintains the continuity also that means, d epsilon 1 plus d epsilon 2 plus d epsilon 3 should be 0. And, that once we maintain this condition then we can reach the different expression of this thing or sometimes we are interested to know what are the stress state it should maintain for the to avail the some kind of incremental during this deformation.

So, here suppose the incremental strain is d epsilon, this direction of d epsilon is independent of the incremental d sigma; that means which direction the d sigma is moving. So, this incremental of the stress independent of the direction of the stress and d epsilon is a vector normal to the yield surface f.

So, therefore, this if this is the yield surface the d epsilon can be considered as a vector form and which represents the normal to this surface and d lambda is the arbitrary constant. So, that constant has to be decide when you try to analyze all these things. So, therefore, this flow rule or normality this flow rule actually decides what way the yield surface will evolve and for a particular increment of the strain value and what way how this increment of the strain is not it depends on the stress magnitude, but it is independent of the direction of the stress value.

And, then you have to decide what may be the it is also depends on the this plastic multiplied d lambda. So, depending upon the analysis problem d lambda can be decided ok.

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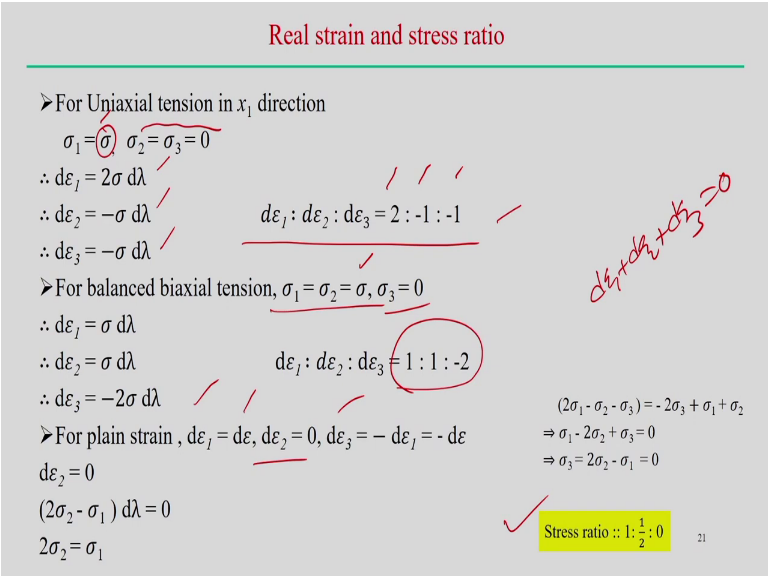

Let us look into this uniaxial tensile testing in x 1 direction. So, here we can see that sigma 1 equal to sigma, sigma 2 and sigma 3 equal to 0 that we know in case of the uniaxial tensile testing. So, therefore, we can express that d epsilon 1, d epsilon 2 and d epsilon 3 in terms of that because the sigma 1 suppose principal stress value equal to sigma in that is the results from the uniaxial tensile testing.

So, therefore, increment of the strain ratio, incremental strain ratio can be predicted this positive minus 1 is to minus 1; that means, 1 can be expand another direction it can contract also. So, for this ratio is applicable in case of the uniaxial tension but, if biaxial tension, then we can see the biaxial condition the sigma 1 sigma 2 equal to sigma bar same, this is the value and sigma 3 equal to 0. So, the ratio can be incremental strain ratio can be different.

Similarly, for plain strain condition and for in this case the d epsilon 1 equal to d epsilon the increment in other 0 and d epsilon 3 equal to minus d epsilon. If we follow and that follow this continuity equation this d epsilon 2 3 using this two equation, then we can reach this ratio the stress ratio can be 1 is to half is to 0. That means, the one direction magnitude of the stress is half double of the other direction. And, then other point that is a stress does not change basically.

So, this way we can make the limit or deformation or restriction of the deformation or strain one particular direction, the principal stress component because these d epsilon 1, d epsilon 2 and d epsilon 3 indicates the principal strain component.

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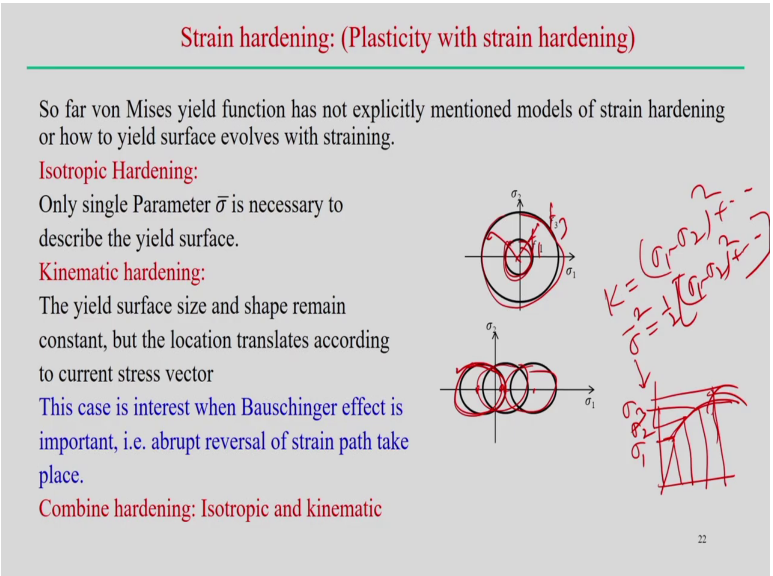

Now, what way we can incorporate the strain hardening or what do we understand by strain hardening in case of this stress model also? So, therefore, von Mises yield function only decide the explicitly mentioned the models of the strain hardening or how to yield evolve during the straining process.

So, von Mises yield functional form decides the shape of the function, but what we can evolve that also can also be here also we can see that already explain this way also because this is the function for f 1 and this is the f 3 for example. So, that means, this is the functional form, but same functional form there is no change in this thing, only the parameter sigma bar is necessary to describe the yield surface.

It means that we represents K as a function of sigma 1 sigma 2 square dot dot. So, now, this K or we can say the K is equal to sigma bar square equal to half of sigma 1 minus sigma 2 square other condition. So, now this sigma bar can be change effect of the strain hardening.

So, it means that this is the first yield point this strain hardening effect means this is the yield point though we can say that it is the sigma 1, now it can be the sigma 2 strain hardening, it can be sigma 3. So, therefore, it will be following the this curve and, but all these analysis of following this curve, but it will be in the incremental mode.

So, that shows that we will be able to evolve this yield surface and taking care of the strain hardening effect ok. So, kinematic hardening this isotropic hardening only the single parameter sigma bar is isotropic hardening means the shape remain the same that is it evolve the same shape. But, magnitude this magnitude can be different, but following the same functional form in this case that is called the isotropic hardening.

So, equivalent way that it is evolving for all the direction sigma 1, sigma 2, sigma 3; now if kinematic hardening means the yield surface size and shape remains size and shape remain the same constant, but the location translates according to the current stress. For example, this is the functional of the yield surface, but this remain the same in size dimension, but it translates the center point from here to here. This is the second one, it translates other point third one.

So, it means there is a yield surface and shape remains the constant, but it translates according to the stress state of the stress. So, this case is very much interesting in case of the when you are considering the Bauschinger effect important or there is a abrupt reversal of the strain path. For example, suddenly there is a reversal of the strain path from tensile to compressive load in that cases may be kinematic modeling is neutral.

But, actually in practically in the welding process problem we normally use the isotropic hardening that is more easy to understand easy to implement also, but in actual practice the we can use the combined hardening model also then isotropic and kinematic hardening.

In these cases model becomes more complicated in the plastic model then we need a lots of parameter to be define. So, this way we can differentiate the in general we can understand that what is the isotropic hardening and what is the kinematic hardening.

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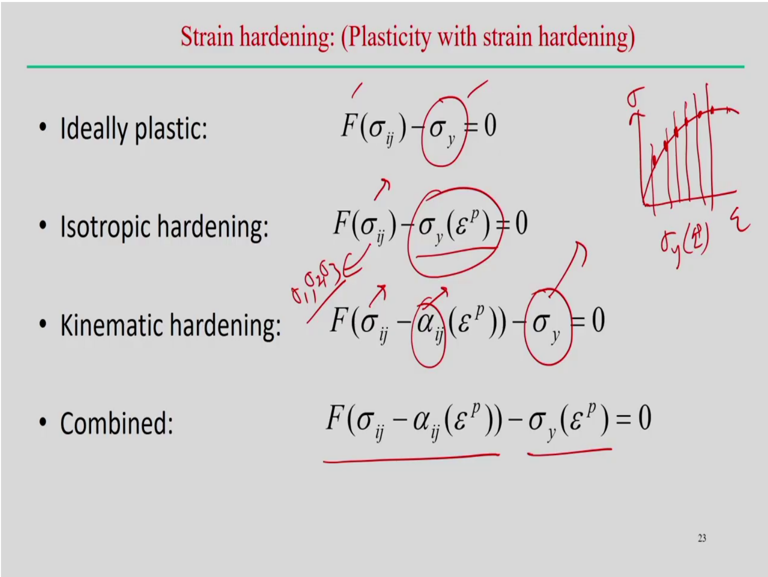

Now, if we look into overall this thing different strain hardening, plasticity with a strain hardening definitely we are considering the elasto plastic material and we are assuming the some strain hardening effect is there, definitely in this particular material most of the engineering materials.

So, if we assume that it is ideally plastic material, then simply this functional is as a sigma ij and sigma one constant yield value for this particular material because ideally plastic value if you remember this is the strain and this is the stress value. So, ideal plastic is the follow the this way also.

So, therefore, this yield point is constant in this case. So, this follow this needs to solve or if you want to find out the law the these cases then ideally plastic we have to follow this kind of expression. Isotropic hardening, we can use the f sigma ij in terms of the state of the stress in particular point and the remaining part this other part is the sigma y; sigma y means, the yield stress value, but that yield stress value is evolving with respect to the plastic strain.

For example, in this case it can be like that this way also, so that yields yield point value is the strain hardening effect. The yield point value is the in incremental value is operating in these cases. So, therefore, the sigma y the yield value is a function of the plastic strain. So, definitely you are talking the deformation happen in the plastic zone only.

So, the different plastic strain value the yield can vary also. So, this way we consider the elasto-plastic material and even if we consider the isotropic hardening. So, that means, we are not changing this expression that it is a function of only sigma 1 set of the state in terms of the principal stress sigma 1, sigma 2, sigma 3, but this continuously changing this parameter this.

But, kinematic hardening if we consider kinematic hardening this we are not there is no change in the yield point value. So, same yield point value, but translates the position the shapes in. So, that translation can be takes care of this thing a sigma ij and alpha ij is that the parameter, what way the translation from happens in these cases and it is itself can be a function of the plastic strain. But this remains constant same value.

So, in this functional form the kinematic hardening effect can be represents and combining these two, so, this is also there and both this yield stress also we want to be combining these two we can get that isotropic and kinematic hardening rule it is possible to apply during the plastic deformation of a particular process.

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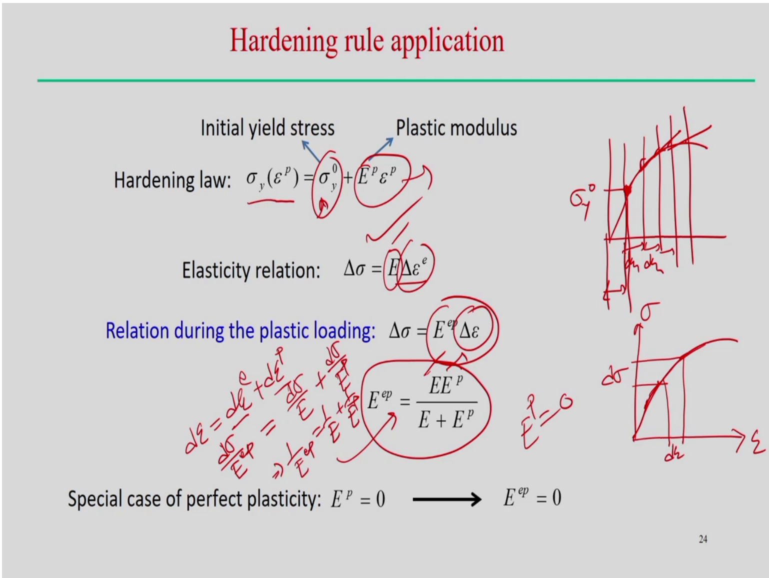

Now, what way we can apply the hardening rule application in this particular this stress analysis of welding process. So, initial yield stress is the value this thing the sigma y is a function of plastic strain, initial yield stress value is this one and that this maybe you can say something like this, this is the initial yield stress value, this is corresponds to sigma y 0.

And, then further as a function of the plastic strain the yield point is basically yield point actually varying. So, that yield point is varying the this is the slope this represents the slope, local slope at this particular point using the slope at this particular point. The slope into the plastic strain value that represent the increment and then it indicates that the incorporating the hardening law then which normally represents the yield point or yield stress value that also evolves with respect to the different strain value.

For example, this is the one incremental strain value d epsilon 1, then this is d epsilon 2 something like that the incremental strain value. So, therefore, with the increment of the strain further straining the yield point also changes and we consider the yield point change as a function of the plastic strain.

And, this is the we can simply add this is the initial value this thing and then gradually incremental value can be represented like that the slope into the plastic strain that indicates this thing. Increment of the stress within the elastic limit can be represent the only the following the Hooke's law E into the increment of the strain.

So, this increment within the elastic limit this is the increment of the strain, but within the elastic limit the increment of the strain can be done at one single step. Such that, this is the increment of the strain this is the flow basically Young's modulus and that indicates the stress value increment of the stress value only single step, which is fallowing in case of the elastic within the elastic limit because within the elastic limit the stress is proportional to the strain.

So, you can use this relation, but relation during the plastic loading then incremental of the stress increment of the stress value is maybe you have to consider the slope or maybe modulus the elasto plastic modulus and the increment of the strain value within this zone. This elasto plastic modulus can be a function of the both this thing the Young's modulus the and the plastic modulus itself. We can see this also for example, this is the particular (Refer Time: 38:39), this is the strain value and this is the stress.

Now, one particular point is the these this this is the increment of the stress value d sigma to let us say and this is the increment of the strain value. So, now, we understand that in a elasto-plastic, the total strain is the consist of the elastic strain and consist of the plastic strain that indicates the total strain any during the deformation. Some part will be the elastic and some part will be the plastic.

Even why it is considering like that? Even if plastic, if it is a plastic zone also if it is a elasto-plastic material we remove the load also some elastic recovery is will also be there. So, therefore, total strain we counting combination of the both elastic strain as well as the plastic strain.

Now, this elastic strain can be represent the same increment of the stress d sigma by Young's modulus this elasto-plastic maybe in these cases E p and then Young if it is the elastic strain we can see the same amount of the stress is correspond to the E d sigma by E. And, in case of the plastic strain this is also d sigma increment of the stress and we can take the modulus of the this local modulo plastic modulus that can be E P.

So, therefore, if we combine this thing 1 by E P ep elasto plastic 1 by E plus 1 by E P. So, from there we can say the elasto-plastic modulus can be calculated by combining the only plastic modulus as well as the elastic modulus both. And, then we can reach from here to this expression. So, therefore, we can see the special case of the perfect plasticity.

So, if it is the perfect plasticity then the plastic strain equal to modulus of plastic strain equal to 0 in that cases then it equal to the E elasto plastic will be 0 in this particular case. So, now, in a elasto-plastic analysis that depending upon that the stress state whether it is within elastic zone or the stress is within the plastic zone, so, therefore, increment of the stress can be calculated the proper modulus in this case.

For example, within the elastic zone then we use the Young's modulus or if it is a; if it is a plastic zone then we consider the elasto-plastic modulus and then we estimate what is the increment of the stress in case of the elasto-plastic material, if we assume the material weaver as a elasto plastic.

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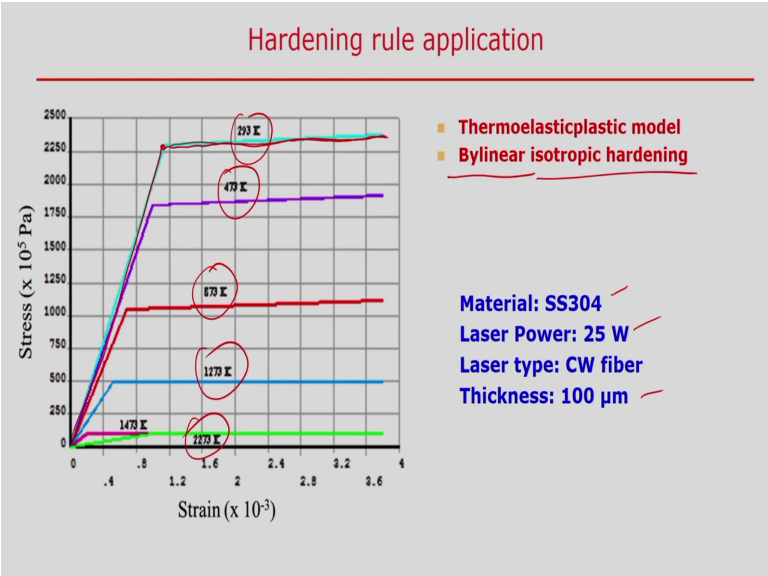

We can see the hardening law application in this case, we can see the thermo elasto plastic model and the it is a bylinear isotropic hardening if we consider in this particular case and this is the one simulation we have done in case of the stainless steel, its laser power 25 Watt and fiber laser and thickness 100 micro meter.

In that cases if we do the stress analysis, then we need to generate this kind of the stress strain diagram which is need as a input to the model because this model we can see the is this model can be this thing as a function of the temperature. For example, at the room temperature this part indicates the elastic and this part indicates the plastic deformation part.

So, elastic part and plastic deformation we can say that bylinear. And, isotropic hardening if we follow isotropic hardening; that means, this hardening effect is there because it is not exactly the horizontal. So, the slope represents some hardening effect is there in this particular material behavior.

So, in this case we consider the bylinear isotropic hardening, then this kind of the data is required to input to the model to represent to define the stress strain diagram in, but this is a simplified way because the plastic deformation we can consider as a linear curve. Then in that cases we can say the bylinear isotropic, bylinear hardening in this case.

And, this we can see that as a function of temperature 293 Kelvin room temperature this is the curve and similarly the lower than this room at higher temperature this value actually changes, the magnitude of the changes. And then intermediate value any other temperature, then is simply interpolation interpolating the data is required from this stress strain diagram.

So, it means that first thing is that plastic part is need to define the this is the yield point and then remaining plastic deformation part plastic pat also be defined, but the plastic curve can be represent in the is a simple linear curve can also be possible. And, that it is easy to implement to define or easy to implement in this particular analysis the stress analysis model.

So, thank you very much.