Finite Element modeling of Welding processes
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Module - 06
FE- based elastic–plastic stress model of welding processes
Lecture - 28
Principle stress, Hydrostatic and Deviatoric Components of Stress

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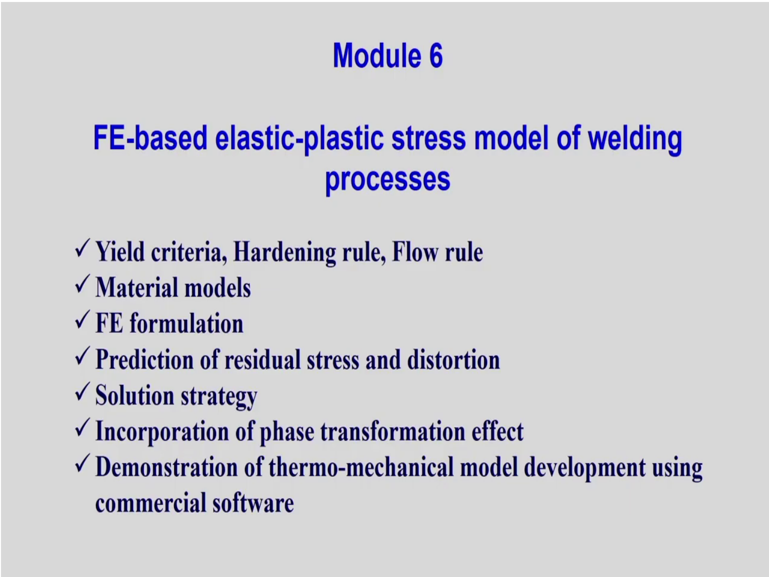

Hello everybody, now we will discuss in module 6 in particular in this particular module that, how finite element model can be developed by following the elasto-plastic analysis in case of the welding process.

So, basically from this module, we will be able to learn how we can predict the residual stress, which is associated with the welding process, distribution or combining the temperature and the residual stress distribution, stress distribution associated with the welding process.

So, this module has been first divided into the different subsection; first we will try to understand that, what are the plasticity model; to understand the plasticity model, then we need to know what are the yield criteria, how we can decide the yield criteria, what way we can decide the hardening rule, how the flow rule or hardening rule can be applicable in case of the finite element based stress analysis model.

Then we will be looking into the materials model also; what way we can define the materials model or what we understand by the materials model. Then finite element formulation of the stress analysis model also and some part we have already covered in the very into this thing and the basics of the finite element there also.

So, in this case, I will continue from the after assuming that, you have basic preliminary discussion is already done in the module 1. So, then finally, prediction of the residual stress show some, I will try to show some results also; it will be able to see that how, what way the distribution of the stress which can be, nature of the distribution of the stress field associated with welding process.

Then solution strategy, what are the different coupling mechanism we can apply to in case of the stress analysis model. And then finally, the incorporation of the phase transformation effect; basically the we instead of simply doing the thermo mechanical model, in case of the welding process, we will I will try to show you that, what way we can incorporate the phase transformation effect to improve the prediction accuracy of the residual stress from a stress analysis model.

So, finally, some demonstration of the stress analysis model using the commercial software will, I will show in this particular module. Before start of this thing, we need to understand what is the our three-dimensional state of the stress and then how this different equivalent or single component of the stress can be derived from the say three-dimensional state of the stress.

Actually, when we talking about the thermo mechanical analysis in case of the welding process. So we normally solve the static equilibrium equation. Static equilibrium equation it means that, suppose the one domain is there, solution domain is there and the, with the application of the some kind of the external load; some internal stress will be generated.

So, with the application of the load or even the existing of the body load, inside the body also and the corresponding generation the induced stress also, that should be balance and that is called the static equilibrium equation. So, basically we will try to look into that and from that point of view we will show that, what we can do the stiffness matrix in case of the stress problem.

And what after that the standard procedure to show the solution of this stiffness matrix will get the distribution field. And then from the distribution field or distortion field each and every node point, from there what way we can estimate the stress and strain during this particular welding process.

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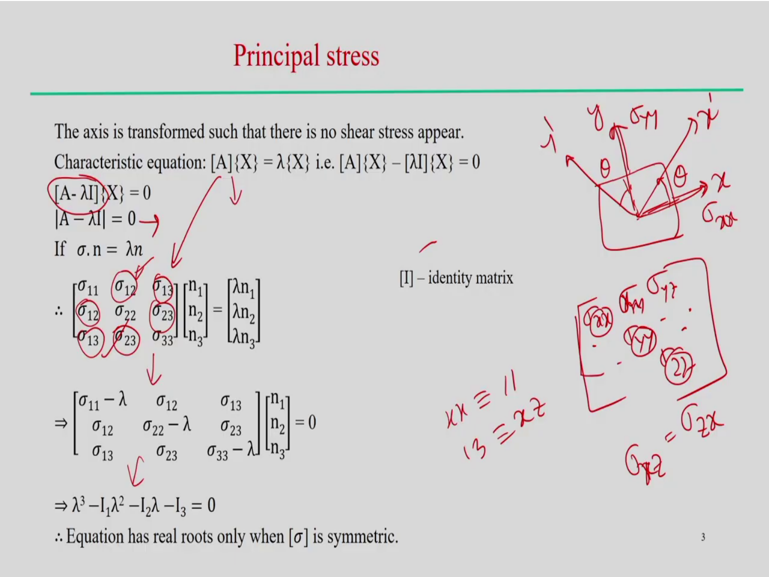

But before that, we need to understand the state of the stress in a particular element. So, suppose there are three dimensions body is there; three dimensional body we can represents this, for example, this is the x, this is the y direction. So, basically we can represent this sigma x, xx, sigma yy; these are the normal stresses acting in this particular element.

So, this is I am showing 2 D element for your understanding this thing. But we need to identify what are the principal stress in this particular situations; principal stress if we look into the transformation of the stress from one axis to another axis. So, suppose there is another axis. So, it is certain angle theta and suppose this is the x dot and similarly it can be y dot also, this is the same angle theta.

So, if we rotate this x dot and y dot this particular system, such that in particular angle, particular direction, there may be disappearance of the shear stress; but that we are then with respect to that x dot, suppose in these cases with respect to the x dot axis, we define the state of the stress.

So, that state of the stress can be considered as a principal state of the stress; but before doing that, we see that what are the different components in a stress analysis sigma yy, sigma xx, sigma yy, and sigma zz, sigma xy and sigma xy and sigma yz so, like that, there are; if you say stress tensor, in three dimensional case that any particular point.

So, particular element, we can represent this element having the 9 component of the stresses. So, 9 component of the stresses, out of that 3 are the normal stress components and 6 of the shear stress components. Now, if it is a symmetric problem, symmetric matrix; then we can say the total 6 components will be able to know.

So, in this cases, sigma if we say sigma yz equal to sigma zx; so basically when we are talking about sigma xx, sigma yy or sigma zz, this indicates these are the normal stress components and remaining yz, zx and that represents the shear stress components. But when you when we talking about the sigma xz equal to sigma zx; it means that this is the, if you follow the similar kind of the situation, then for the other shear stress components also this indicate the symmetric.

So, in that cases symmetric matrix, then it is having 6 components; 3 normal stress components and 3 shear stress components and that is the representation of the state of the stress in a particular element. Now, we will dealing with this the 6 components of the stresses. Now, before doing that, what way we can estimate the; first point is that, how we can estimate the principal stress in this particular situation?

So, we can already explain that this is the initial state of the stress the x and y axis it is acting and, but we just try to rotate, we are trying to finding out the another axis systems; such that along the (Refer Time: 06:58) principal stress will be defining. So, in principle, the axis is transforms such that, there is no shear stress appear; in this particular situation, this represents the state of the principal stress components.

But mathematically we can say that, suppose this is the equation, the A is the that particular this state of the stress in this case and A into X is the variable here and equal to lambda X. So, we can use this say A X equal to lambda X or A X minus lambda I X, I is the identity matrix and such that A minus lambda I this is the matrix and X equal to 0, this indicates the state of the principal stress.

If we solve this equation and once we solve this equation or we can say that A minus lambda I equal to 0, the determinant of A minus lambda I equal to 0; then if we solve this equation, then we will be able to find out what is the values of the principal stress components. And similarly the principal stress direction; but here I am saying the only the principal stress components.

So, this once we get the characteristic equation from the three dimensional, this is the initial say; I can say this is the initial state of the matrix, instead of here xx, which is equivalent to 11 here also this notation. And similarly, 13 is equivalent to xz like that. So, it means that, the what way we represent the sigma 11, sigma 22 and sigma 33 in this particular case; that is representing the normal stress components and remaining are the shear stress component.

So, once we form the characteristic equation, then if we solve this characteristic equation A minus lambda equal to 0; we can say A minus lambda, lambda I is the identity matrix. From there, we will be able to get this kind of equation. So, determinant equal to 0; this if we solve the equation, then we will be getting that one cubic equation.

So, then cubic equation means, the root of this equation; once we get the cubic equation say, lambda cube minus I 1 lambda square minus I 2 lambda minus I 3. So, this I 1, I 2, and I 3 represent the stress invariance and this cubic equation; if we find out the roots of this particular equation, that roots represents the values of the principal stress.

Now, equation has the real root, it is, if we having the real root, if sigma is symmetric; if sigma is symmetric means, this matrix is symmetric matrix. Here also see symmetric matrix means, this is equal to this sigma 12; here also sigma 13 equal to sigma 13 and sigma 23 equal to sigma 23 in these cases, they are same.

So, in this particular entry that, this matrix is symmetric in nature so, once this is symmetric; if we solve this cubic equation, we will be getting the 3 roots. So, all are real roots; that values of the real roots indicates the principal stress components in this particular state of the stress, in this particular matrix. So, these are the way to find out the principal stress components.

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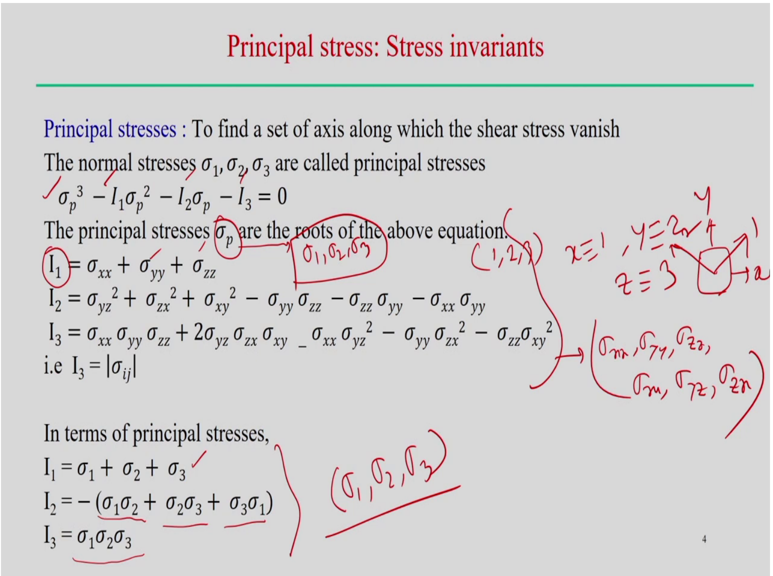

Now, here also to find out the set of the axis along which the shear stress vanish; therefore that indicates the normal stresses sigma 1, sigma 2 are the normal stresses along this direction and which direction shear stress vanish that, indicates the principal stress value.

It is a simply the equation sigma p cube the same as the what we discussed in the last slide also; then I 1, I 2 and I 3 are the stress invariance and then such that, the solution of this principal sigma p, solution of the equation this represents the roots of the equation represents the principal stress.

For example, root of the equation of this is sigma 1, sigma 2 and sigma 3. So, these three indicates the principal stress in this particular case. Now, this stress and invariants can be represent in the different way also; that is very important term, the stress and invariant definitely it should have the components of the other stress components.

So, I 1 can be represent that, sigma xx plus sigma yy plus sigma zz. So, this can be done simply if we find out the determinant of this equation. So, if we find out the determinant of this and the looking into the expand this value determinant; then we will be able to know that, lambda square is associated on I 1, I 2 so, I 1, I 2 and I 3 in terms of the all the stress components.

So, sigma 11, sigma 12, sigma 13 like that. So, if we do that, then I 1, the stress invariant I 1 can be like that sigma in terms of sigma xx sigma yy and sigma zz; that was the initial state of the stress. And I 2 also in terms of the all the initial state of the stress sigma yz, sigma zx, sigma xy like that also and the I 3 also in the similar way.

But it is also possible to represent the 1, 2, 3 also; just like x equal to equivalent to 1 and y equivalent to 2 and z equivalent to 3. So, it is possible sometimes we represent this in terms of the other axis 1, 2, 3 in that way also. So, we can simply replace yy, it should be replaced by 22 like that. Now, I 3 can be represent in terms of the all the stress component; but I 3 can also be represent in the simply the determinant of sigma I z, that means the total state of the stress, then I 1, I 2, I 3 can be represent in this way also.

But this is the initial state of the stress. But once we getting the, if we know the what are the principal stress and the particular state of the stress from here; then this stress invariants can also be represent in terms of the principal stress components. For example, in this case if we this in terms of the principal stress, the I 1 can be sigma 1 plus sigma 2 plus sigma 3; similarly I 2 can be sigma 1 2, sigma 2 3, sigma 3 1 and I 3 equal to sigma 1, sigma 2, sigma 3.

So, these all these stress invariants are represented in the form of a principal stress and in this case, the all the stress invariants are represented in the form of the initial state of the stress. So, advantage of these things in this case; if you see all the, in this case the these stress invariant is represented as a in terms of sigma 1, sigma 2 and sigma 3 these three principal stress components.

But in this case, all there are 6 stress components; sigma xx, sigma yy, sigma zz, in this case sigma xy, sigma yz, sigma zx. So, basically these stress components are represented all the in terms of the 6 stress components; but this is the initial state of the stress.

What we define the this is x, this is y, but once the stress (Refer Time: 13:23), suppose this is 1, this is 2; then this transformation, that means once we find out the principal stress, then in terms of the principal stress, we represent the stress invariants.

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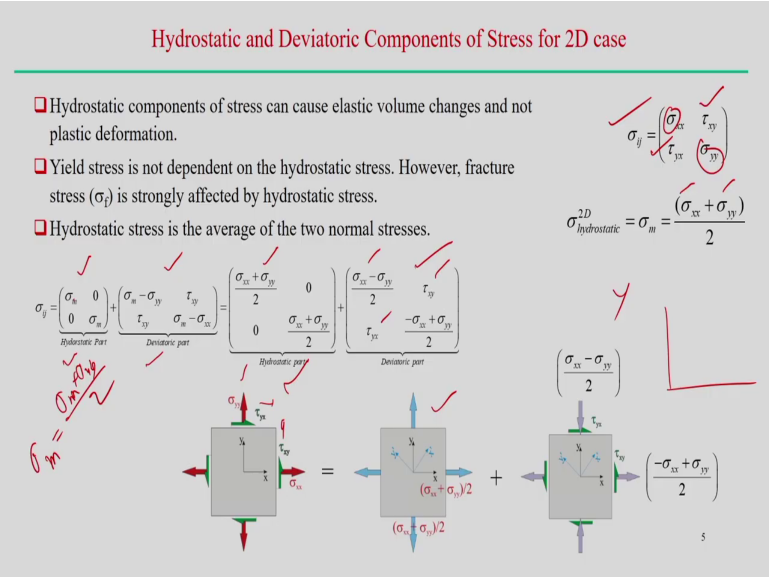

Now, any state of the stress can be represented in the two form; one is the hydrostatic stress component and the deviatoric stress component. The simple way, suppose this is the state of the stress; suppose this is the 2 dimensional state of the stress, this is the normal stress, this is normal stress, and this is shear stress, tau xy equal to tau yx in this case.

Then hydrostatic stress can be represented like that, average of the normal stress components sigma xx sigma yy by 2. And such that, this total stress, stress state any stress state is represented in the two parts; one is the hydrostatic, another is the deviatoric part. So, if we see, if you merge these two; hydrostatic part and deviatoric component it merges two, you will be getting the initial what was the original state of the stress.

That is why it means that, any state of the stress we can divide into these two components. But sigma m is the simply here, the sigma m is the mean stress value; if you see the this sigma and sigma m the normal stress exist in these cases, there is no shear stress components and this normal stress are also equal.

So, it is correspond to sigma xx sigma yy by 2 and remaining is the deviatoric part. So, in principle, our understanding this hydrostatic stress component of the stress that can cause the elastic volume changes; that means it is do not change the shape changes and not plastic deformation. The basically hydrostatic stress component do not takes part the plastic deformation, rather elastically it deform the material.

For we can say like that, in this way also, I represent these things or it is can be represented as compare analogy to the pressure term also. Because if a round ball, spherical ball if we put the under the pressure, water pressure suddenly; then under the water pressure is acting all the direction, if we assume the x y and z direction, all the directions pressure is the same.

So, even it is all the directions pressure is the same; then round ball becomes round, even there is a small change in the volume, but it becomes round. So, there is no change in the shape, only change in the size. So, that happens because of the presence of the hydrostatic stress component or which is analogous to the only the pressure component.

Since in these cases the pressure is acting all x, y and z direction is the same; the same concept we can use the hydrostatic component state, component of the stress is acting equally in the all direction. So, that is why it do not take on the plastic deformation, rather I can say it only takes part in the elastic volume changes or size of the particular component.

Now, yield stress is not dependent on the hydrostatic, that is the one of the very much understanding in case of the plasticity; because the yield stress is not depend on the hydrostatic stress, I can say the yield stress is basically is a function of the deviatoric stress component not the hydrostatic stress component. Because it is not the, does not take part the yielding, plastically deform in plastic deformation process.

But however, fracture stress is strongly affected by the hydrostatic stress. So, in this case we assume the yielding; yielding means plastic deformation. So, during the plastic deformation means, once we cross the elastic value; then plastically the material deform plastically with the application of the load.

So, once we cross the yield value. So, even it is plastic deformation is there; but the in these cases the state of the stress, the hydrostatic stress component does not take parts in the yielding of a particular material or I can say the plastic deformation of a particular component.

But hydrostatic stress component is the average of the two normal stress component that we have already shown that, hydrostatic stress component can be in represented as the average of the two normal stress component. So, this all way also, this is the hydrostatic part and this is this represents the deviatoric part. So, graphically as we can represent these things. So, suppose sigma xx is acting and the shear stress is acting in this direction and the here the shear stress is acting and sigma yy is acting in this particular.

So, it is have two components, we can divide the state in the two different components; this represent the all the direction with the same state of the stress, that is the hydrostatic component. And if we see, this is the state of the stress represent the deviatoric part; but in this case the deviatoric part is the shear stress is there component, at the same time the some normal stress component are also there, so in the deviatoric component.

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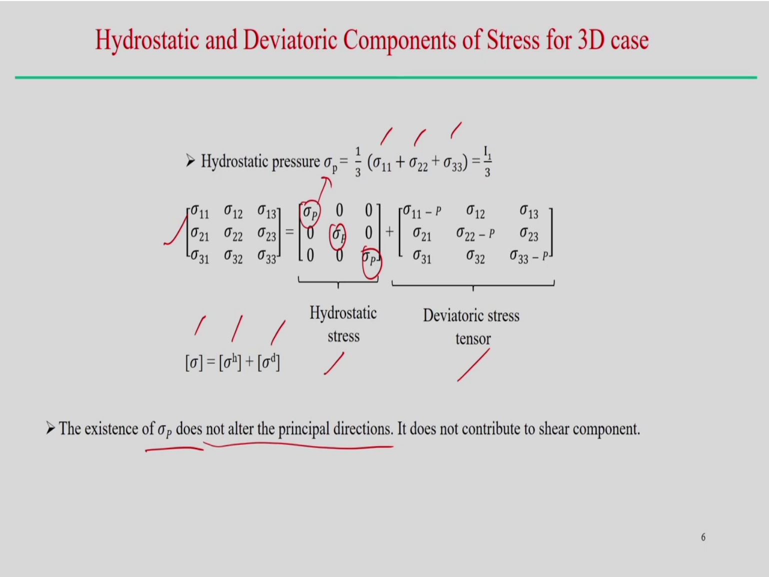

So, in 3 dimensional state of the stress also, we can represent the similar way; the hydrostatic pressure can be represent or hydrostatic stress can be represents like that, the one third of this three component sigma 11, sigma 22, and sigma 33 and this is equivalent to I 1 by 3.

So, first stress invariant by 3. Similarly it can also be represent the any state of the stress sigma 11, sigma 22, and sigma 33; it is consist of the hydrostatic stress and the deviatoric stress tensor.

In this case, the hydrostatic stress sigma p is acting; so such that sigma p is the, this is the value of the sigma p in terms of the normal stress, they are simply average of the all the normal stress component, this is the sigma p. And if we see in this there is no shear stress, only the normal stress is acting; but all normal stress are same for all three direction.

But, deviatoric stress component is different in other way also; it means that remaining part is the deviatoric stress component. So, in this case the sigma can be represent in the hydrostatic component and the deviatoric stress component; this the and the total at the state of the stress can be represent these things. And it is also another important conclusion that, sigma p; this hydrostatic stress component does not alter the principal direction.

So, that means principal stress or principal stress axis in particular direction; but presence of the existent of the sigma p, that means hydrostatic stress does not alter the principal direction remains the same. Even if you find out what is the principal stress, if we consider only the deviatoric stress component; but deviatoric stress component principal direction remains the same in this particular situation, and it does not contribute to the shear component.

So, this hydrostatic does not helps to for the shearing of the particular situation; that is why already explained this thing that hydrostatic component in such a way that, it is takes the elastic deformation without changing the shape. But when the shear is acting, then there must be change of the shape on a particular object. So, that is why, hydrostatic does not change the shape; it means that, it is does not contribute to the shear component, only the deviatoric stress component is contribute to the shear component.

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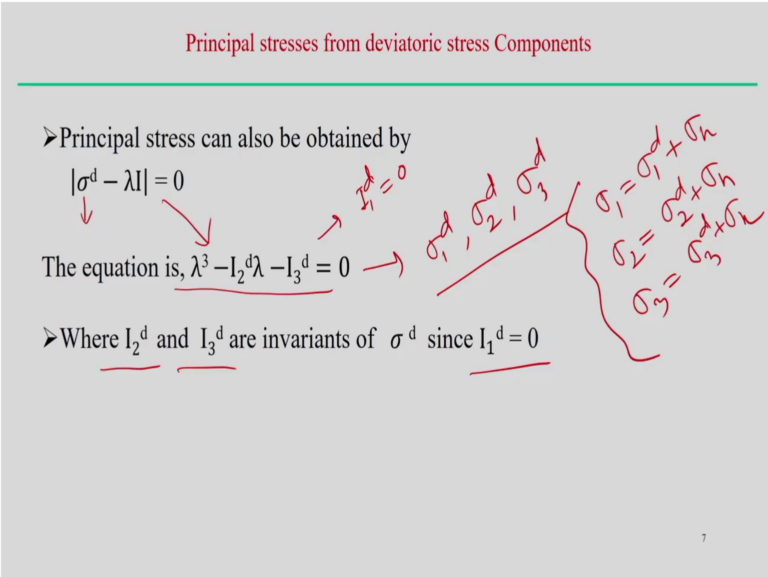

Principal stress can also be obtained, for example, this is the another methodology also principal stress can also be obtained only if you know the hydrostatic, deviatoric stress component also. So, similar characteristic equation, but in these cases the sigma d represents the deviatoric stress components.

That means the total stress minus the hydrostatic stress that, indicates the deviatoric stress components minus lambda I, the characteristic equation we can find out and this equation is transformed to this in this particular form. See in this case that, this nature of the cubic equation are different; because there is in this case I 1 d, deviatoric stress component equal to 0.

If you follow this thing, only two stress invariants are there; I 2, but deviatoric component only and I 3, that is also deviatoric component, but there is no I 1 d. Since I 1 d equal to 0 in this particular case; this I 1 d is equal to 0, then we can more simplify this equation.

And then if we solve the equation, then we will be getting the three roots of the equation; that three roots represents the, for example, we are getting the sigma 1 d. But, in terms of the hydrostatic sigma 2 d and sigma 3 d, these three real; if we solve this equation, we will be getting these three roots of this equation.

So, that represents the stress component, the principal stress components, but in the form of a deviatoric part; but actual principal stress can be represented like the sigma 1 d plus this hydrostatic component as sigma h, similarly sigma 2 the actual principal stress, d plus hydrostatic component. Similarly, sigma 3, the principal stress; sigma 3 d plus hydrostatic component.

So, this way we can find out the three values of the principal stress component, just by solving the characteristic equation from the deviatoric stress component part. So, these are the ways to find out the principal stress; I hope there is some understanding on the principal stress value and how it is related to the initial state of the stress, as a function of the initial state of the stress.

Was most important here is the two things are important in this particular case; one is that what way the any state of the stress, what way we can divide into two component, the hydrostatic stress and the deviatoric stress component, that is one part.

And second part is that, this stress invariant; this stress invariant is simply can be represented in the form of a only the original state of the stress or the stress invariants in the can be represent in the form of a principal stress components, both way it is possible. Now, we can do the further the analysis of this particular state of the stress.

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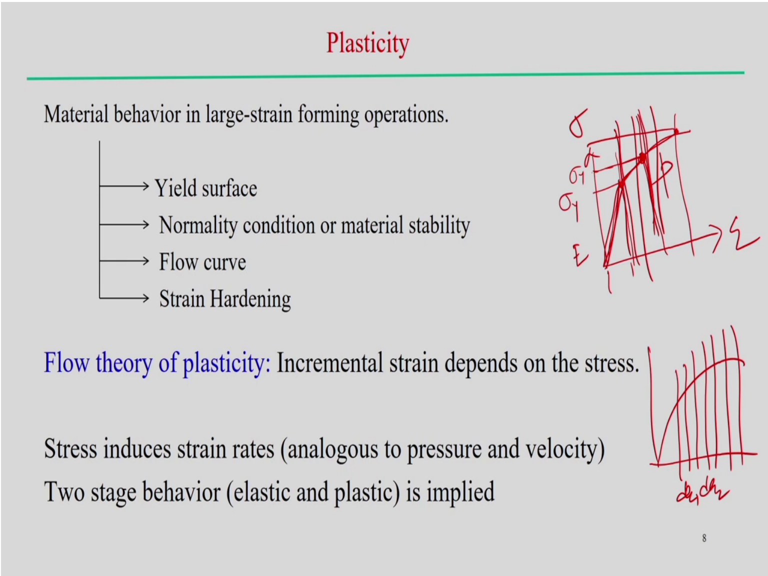

Now, come to this point the plasticity; what way we can analyze the plasticity? Plasticity, what do you mean by plasticity? Even if you observe in the uniaxial stress strain diagram also; you can find out thus this is the typical pattern of this thing and thus yielding happens at this particular point, the say up to this thing.

This is a stress is proportional, this is the strain axis, this is the stress axis; stress is proportional to the strain and this is the this is the constant slope that represent the Young's modulus and then we represent this is the up to this that, within this below the sigma yield value, that is the stress is proportional to; that means, it follow the Hooke's law of elasticity that mean.

But beyond that sigma y, then stress is not proportional to the strain; I think the relation between the strain and sigma is some kind of a non-linear relation. So, this represent the plastic deformation, once we cross the yield point value for a particular material. So, different material having different this particular stress strain diagram; if we that we will be able to know, if we do the tensile testing using the uniaxial tensile testing machine.

If we follow this uniaxial tensile testing, we will be getting the different curve. And from that curve we can easily say, this is the yield point value and beyond that yield point value, the stress is proportional to the strain; below the yield point value, the elastic zone and beyond the yield point value that is the plastic zone.

So, that point is that, in the most of the material, we handle; that is having the not only the may not be the perfectly plastic or perfectly elastic material, rather the material behaves will be elastoplastically, so some part elastic and then plastic. So, now we will see, what way we can handle this elastoplastic material.

So, to do that, we have to understand this plasticity theory also. Now, this material behavior in the large strain forming operation also, even other cases also; the this most important part is the to analyze the plastic deformation is the yield surface. So, basically once we are talking about the uniaxial tensile testing, then we can define the yield point.

But, in practical problem, when the material deform three dimensional state of the stress exist; but material deform plastically in these cases, the yield cannot be a yielding may not happen one particular point or rather it is necessary to define the yield surface. Second point is that, normality condition of the material stability. Even yielding happens in this particular point, then when it is just enter the plastic deformation domain.

In this case, the whether the continuity of this plastic deformation and following the plastic deformation, how what way the continuity condition can be maintained beyond the yield point, in this yield point that can be represented by the normality condition or the material stability. So, in these cases, it is necessary to define the to consider the flow curve.

So, plastic flow of the material or plastic deformation of the material beyond the yield point, that is the flow curve has to be defined has to be known in this particular during the plastic deformation of particular material. And there is another point is the strain hardening. Strain hardening means once it depends on a particular material also, some material having the strain hardening effect is very high and certain material their strain hardening effect is may be very low.

So, strain hardening means, if we once we deform the material beyond the yield point; if we see the beyond this yield point, the material become harder. So, it means that, the further straining the material become harder. So, in this case the this can be the yield point corresponds to this particular strain.

And gradually this can be the yield point particular corresponding to the strain; that means, if we remove the load beyond the yielding point, then if we reload. Once we reach this particular point and remove the load; then some elastic requirement will be there, but elastically deformed. Now, if we again appear the load, then yield point can be different. So, yield point can be start from this point.

So, in that way the; it means that material is having the strain hardening effect, that means the strain level is always increases with a further straining of a particular material. So, that comes into the strain hardening effect. So, and definitely the strain hardening effect can be represented, just it depends on the particular material; but or uniaxial tensile stress strain diagram, we can easily predict the strain hardening behavior.

Now, flow theory of the of plasticity indicates that incremental strain depends on the stress; basically when you do the plasticity analysis, this is one approach that we cannot predict the value of the plastic the state of the stress just a single shot. It means that we need to analyze the in the incremental mode; incremental mode means, when we enter the plastic deformation zone.

Since it is the plastic deformation curve is non-linear in sense, we can analyze the theory can be based on like that small-small elements we can consider. For example, suppose this is the curve; so then at the plastic zone, we can small small division small elemental, so maybe d epsilon 1, d epsilon 2.

So, in this case we can do the analysis for the incremental mode. So, what is happening or the incremental of the small strain and then what maybe the correspondent requirement of the stress value. Then we consider the next increment just updating for the stress value and strain value.

Accordingly, this incremental mode, we normally decides the flow of the plasticity; because this incremental mode strain depends on the stress value. Now, other points is that stress induces strain rates also; we were doing the deformation, it may be associated not only on the strain; maybe it is associated with the strain rate, which is analogous to the pressure and velocity.

It means that, stress is analogous to the pressure and strain rate is analogous to the velocity in case of the other fluid mechanics problem also. That is why whole problem is decided when we talking about the only, we normally use this material behavior as elastoplastic. So, therefore, two stage of behavior, elastic and plastic is normally used for this analysis. So, in these cases it is very important to look into that what is the elastic recovery during this process.

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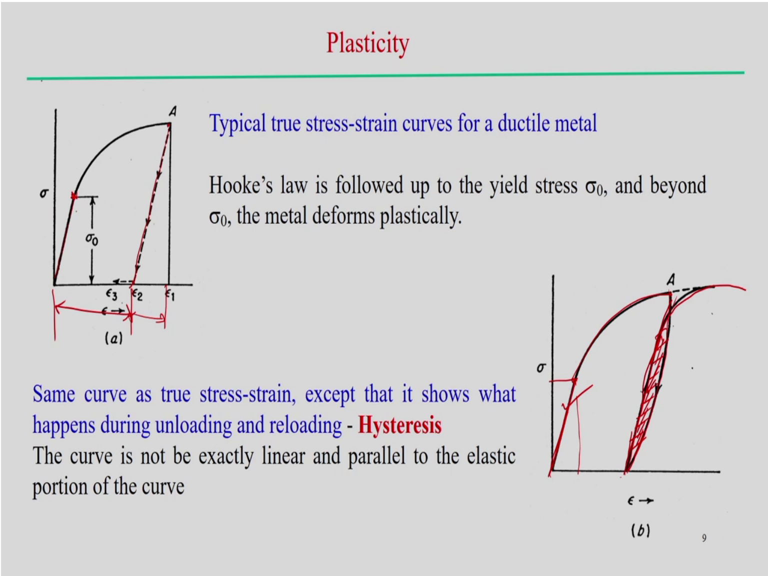

So, typical true stress strain curve for a ductile material, you can see the; this is graph shows that, for a ductile material that, Hooke's law is followed up to the yield stress value. So, this is the up to the yield stress and stress is proportional to the strain and this slope on this curve represent the Young's modulus and the beyond sigma 0, the material deforms plastically.

So, then in these cases, once the beyond the yield point; then some plasticity theory has to be look into that. In this case also we can see the another point, see this is the plastically deform this material and at this point. If you remove the load at this point, so it will come back to the will be the in such way that, this is the this line should be parallel to this line elastic.

And then this part can be say that, it is the kind of the this elastic recovery during the deformation process. Since it is elastoplastic material, so some sort of elastic recovery must be there when you remove the load in a particular position. So, in this case, this is the plastic recovery and this is the permanent deformation within this material.

So, that is representation of the permanent deformation, this part represent the elastic recovery strain. And similarly, the same curve as the true stress strain curve, but except that it shows what happens during the loading and reloading conditions. So, some this curve represent something like that, suppose this is the initially yielding point and then this load up to this point, remove the load; then in practical it will be following this path.

But, once we reach this point, then you reload the component; the we application reloading, then it follow some other part and then its further deformation happens. So, therefore, it is may not exactly follow this particular path and that is called the this zone is basically represent the hysteresis for a particular material. So, this curve is not exactly linear and parallel to the elastic portion of the curve. So, basically this curve is not exactly linear, but it try to follow parallel to this.

So, this always try to follow the parallel to this curve, this curve, the elastic and this zone is practically it is called this the material depending upon the material property. So, if material is having, it can be link with the micro structural aspect also, if material having the is this the hysteresis part is very large, area is very large; then we can say the material having very good damping properties, if this area is basically very small, they are not having very good damping properties normally we can say.

So, anyway, so this is the real phenomena; but in practical problem, we try to or theoretically we can say it is follow some kind of the linear curve and based on that we can say calculate what is the elastic recovery in elastoplastic material. And based on that, we can define the different plasticity theory we can apply for the analysis.

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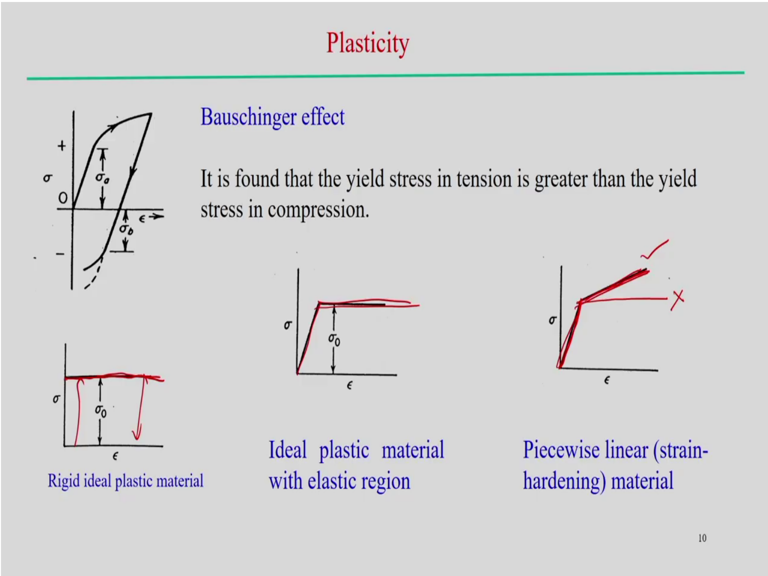

Now, there is another point, the in plasticity that is called the Bauschinger effect. Bauschinger effect which is that, if we observe, if we experimentally do some kind of the stress strain of the particular material that the yield stress in tension; that means when you applying the uniaxial tensile testing, the what is the yield stress value, that may not be the same of the yield stress in case of the if we apply the compressive load.

So, that means if the yield point value in tensile load and the yield point value in compressive load are different; but in general the yield point in tensor is greater than the yield point in case of the compressive load. In that case, we can say the material is having some kind of the Bauschinger effect.

So, that is the it is Bauschinger effect means, we simply understand the yield point value for the tensile and yield point value of the compressive are different. And this is the other part in case of rigid ideal plastic material; that means if you start the deformation, initial that the strain zero, it start deform plastically.

So, it start from this point and there is no strain hardening effect; means it is this is the; this is the line parallel to the axis. So, it means the rigid ideal plastic material. So, it is having the yield point, this is the value, this is the value or we can see some is having some flow stress value also.

Now, ideal plastic material with elastic zone. So, is the elastic initially elastic deformation, but ideal plastic; ideal plastic means there is no strain hardening effect, it is parallel to this, this part is parallel to the axis, horizontal axis. So, in that cases we can say, ideal plastic material with elastic region.

But in these cases piecewise linear; linear material means strain hardening effect is there. So, if stress strain diagram is something like that, this is the slope is there; that means elastically deform, but this is the yield point, then this plastic deformation not exactly the parallel to this axis, x horizontal axis, but rather it is continuously increasing. It means that this material having the plastic, it is having some strain hardening effect.

So, this kind of the, this is typical representation of the elastoplastic material most of the engineering material follow, some sort of strain hardening effect is there. It means that if it is exactly the same slope, that in these cases there is no strain hardening effect, but in this case there is some strain hardening effect.

So, that is why we use the in ideally and this kind of the curve to analyze the elastoplastic behavior during the in particular material and that what we can implement in case of the finite element model of the stress analysis. But in this case we can say bilinear means, one linear part represents this elastic part and other linear part represent the plastic part.

So, we can say the bilinear strain hardening effect in this particular case, this particular material follow the bilinear curve; that simply representation of the stress strain curve or for a elasto plastic material is the two linear components, one linear part is represent the elastic part, another linear part represent the plastic part.

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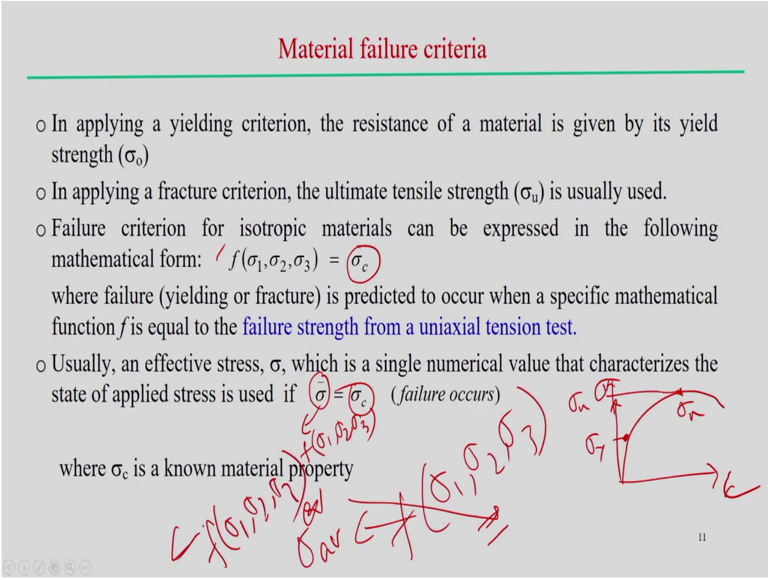

Now, material failure criteria; what we can decide that, material failure criteria that is also necessary to understand. So, first thing is then in applying an yielding criteria, yielding happens this thing, the resistance of a material is given by the yield strength. Basically, what way we can decide the yielding criteria?

So, we can decide the yielding criteria means, once it over's overcomes the elastic part; then we can say the yielding starts and it enters the plastic deformation zone. So, that is simply yielding criteria we can decide, the simply yield point or a particular material. Now, in applying the fracture criteria, that is the yielding criteria; but fracture criteria we normally use the ultimate tensile strength of you normally use in the stress strain diagram.

So, it is something like that. So, you can say this is the sigma ultimate tensile strength and this is the sigma y value. And this correspond to the ultimate tensile strength. So, fracture criteria we normally follow that, decide the any kind of fracture criteria with this we can have this value of the ultimate tensile strength; but yielding can be defined by the yield strength value for a particular material.

Now, failure criteria for isotropic material can be expressed in the following mathematical form. So, like that only the failure criteria or even yielding criteria, we can follow in this particular way also, f equal to sigma c for the failure; that means, yielding or fracture depending upon the is predicted to occur when specific mathematical function f is equal to the failure strength from a uniaxial tension testing, it is simply like that.

Suppose this is the sigma c value. So, what kind of the value we can predict? So, we can do simple tensile testing of a uniaxial testing, particular material sample; we prepare the sample, do the uniaxial tensile testing, we will be getting the single value of the yield point of a particular material. And same thing we can get the single value of the; if we decide the failure criteria happens at the ultimate tensile strength, so, we will be getting the single value.

But, this value, the single point value in case of the uniaxial tensile testing. But in actual manufacture component or maybe in metal forming process or even in welding process also; we when the deform the material or deformation of the material is associate, not the uniaxial condition, the deformation of the material, it is always associated some kind, it is associated with a three dimensional state of the stress.

So, therefore, three dimensional state of the stress, there are so many components of the stress value and we normally converted this stress state in the form of a principal stress component. So, once we convert the actual state of the stress which is represented by the 6 components of the stress and that converted to the state of the principal stress components.

In the principal stress components, we are having only the three state of the stress; that means only three values sigma 1, sigma 2 and sigma 3 three components of the stress. So now, then these three values what way we can compare the single value of the uniaxial tensile testing value.

So, therefore, we can follow some functional form, such this functional form link all this as a function of sigma 1, sigma 2, sigma 3 value. And then that when we compare this functional value in such a way and sometimes we represent the single component which is a function of or which can be represented in the form of a all this value of the sigma 1, sigma 2, sigma 3.

So, therefore, to do that, we need some functional form of sigma 1, sigma 2 and sigma 3, such that it will be it represents theoretically as a single equivalent value, average value or equivalent value. And that is compared with the value of the experimentally measured value, which experimentally we can measure, the simple experimental measurement we have only the single value of a simply uniaxial tensile testing value.

Now, if we compare these things, then we decide the different failure criteria or yielding criteria we can decides. Now, point is that, what way we can decide this functional form and how this different functional form as a function of three principal stress is linked with this thing, based on that we can derive also that is in case of we know that is well known the Von Mise's yield condition.

We normally use in the this stress analysis model, that we will see. Usually an effective stress sigma which is a single numerical value, that characterizes the state of the applied stress which is this single value, which is can be is a functional from sigma 1, sigma 2 and sigma 3 and that is compared with the sigma c value. Then we decide the whether it is failure occurs or whether it is yielding occurs or not.

And sigma c is the known material property that can be evaluated simple the uniaxial tensile testing or simply compressive testing we can find out the values of the sigma c. Now, theoretically what way we can predict this f value that, this functional from sigma 1, sigma 2 and sigma 3 and that we will see in this particular module also.

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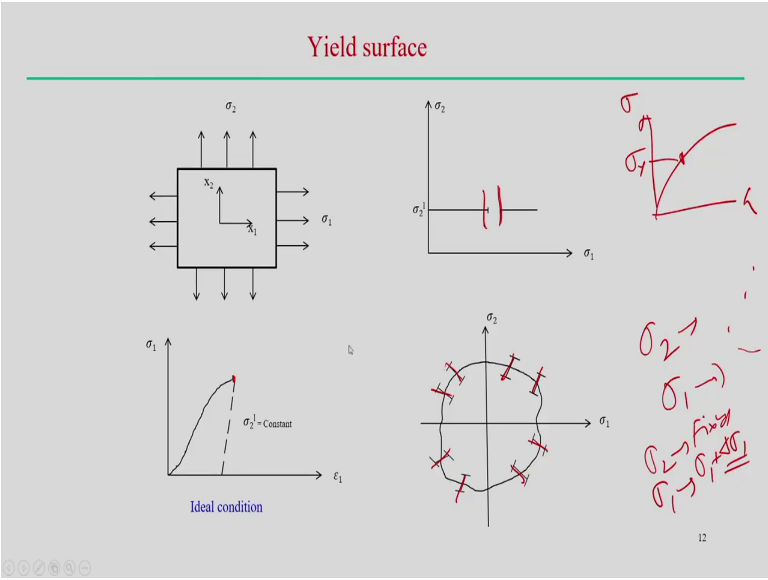

Before doing that, the other important point is that, normally when we are in case of actual state of the stress is associated with the 6 components or 9 components I can say. The 9 components of the stress it is the yielding can be represented not only in the single point; because single point yielding we represent in a case of uniaxial tensile testing.

But, if it is a multi-axial, that mean all the 3 dimensional state of the stress is acting; then it can be represents as a over the surface. So, that means it is necessary to predict the yield surface; but what we can do? Define the yield surface; for example, uni-axial tensile testing we just simply find out this is the point of yielding, this is the yield point value, this is single point value; because in these cases the load is acting in one direction.

Now, if this loading acting the bi-direction say, for example, sigma 1 is acting one direction, sigma 2 is acting other direction. Then what way we can experimentally the define the different values of this yielding happens on particular point? In this case what way we can do these things, for example that, we keep one single value of the sigma 1.

So, sigma 1 can be keep one single value of the sigma 1 or we can keep the sigma 2 fixed; sigma 2 as a fixed value, then we can vary the sigma 1, different values of the sigma 1 for the different different sample. Then we can find out the location of this, this is the state of the stress and this small variation, but depends on this thing.

For example, sigma 2 equal to fixed and sigma 1 can vary; but it depends on the what should be the sigma 1 increment for the next value. So, that means the stress, the yielding can occur over a particular zone domain; because what way we can consider the sigma 1, what is the next value say sigma 1 plus delta sigma 1.

Now, what is the delta sigma 1 value? What is very large or big small accordingly, the this yielding may happen; not exactly one single point or we can say the over a range of the point, now depending upon what is the value of the delta sigma 1. So, in this case keeping the value of sigma 2 fixed and then lots of experiments are required to by the varying the value of sigma 1 for the different different sample.

Same thing now, keep on another level of the sigma 2 value; then we are keep on varying the sigma 1 in the different way also. So, such that if we follow this kind of the. So, lots of experiment has to be done. So, in this case, huge number of experiments; but in principle we can represents that over this two dimensional stress state if we assume the sigma 1 and sigma 2, then yielding may happen over a range particular this thing.

And then and this is the case of the one particular value that, where sigma 2 remains the constant, we can vary the sigma 1 also; then we getting the this is the yield point ideal condition. But this way also, even if we consider three dimensional; then other variable is also there. So, it becomes sigma 3 also comes into the picture.

So, keeping sigma 3 and sigma 2 fixed, keep on varying the sigma 1; similarly sigma 3 with the varying of the. So, lots of combination of the experiments are required to evaluate the yield surface. So, in that sense the huge experimental data are required in this particular case. So, that is the not as good approach, rather if we follow, only the uniaxial tensile testing, we can predict the yield value.

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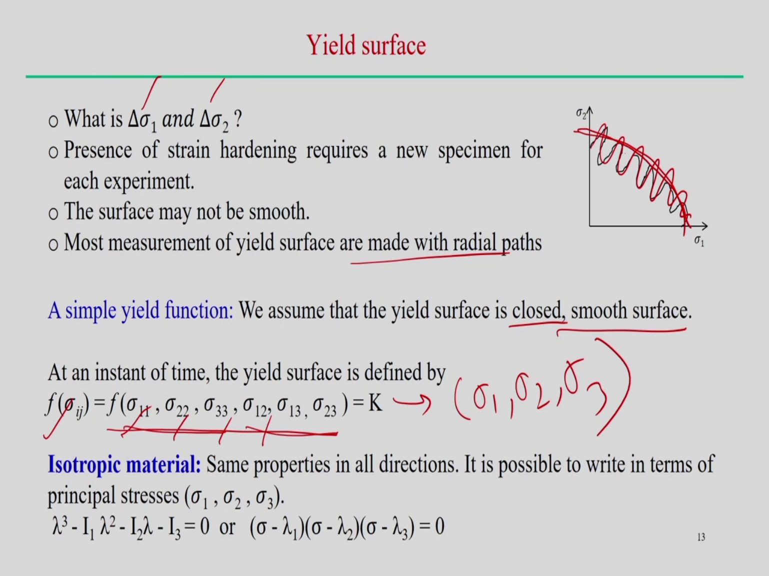

But, in this case, from the analogy of these things that we can assume that, defining some theory; such that, the yield surface can be a close surface and we can predict the different theory and then from this theory we can represent this. The theory means in the sense that, it is we can define some functional form of the yield functional form and then that is a function of the all the principal stress component sigma 1, sigma 2, sigma 3.

And then we can compare with the respect to the experimental value which can be evaluated or the single tensile testing value. Now, these cases it is very important; when you do this kind of the experiments also, the what maybe the delta sigma 1 and increment of these value delta sigma 2 for the next to the experiment.

So, presence of the strain hardening requires a new specimen for each experiment as the same size the surface may not be more smooth also; surface normally the there is a value zone this kind of this thing. Most measurement of the yield surface are made with a radial paths.

So, therefore, even this surface cannot be moved in these cases; we can assume the theoretically, simple the functional from a particular yield function and we can assume it is smooth surface.

We assume that the yielding surface is a closed smooth surface, such that it can be represent by this particular equation yield surface is defined something like that; f this functional form is a function of the all the stress component the six component of the stress, the three normal stress, three shear stress equal to K.

Or other way, this K or other way also it is simply represent not all the shear state; because this state of the stress, the 6 state of the stress the normal stress and shear stress that can be linked with the principal stress value. So, therefore, it is possible to represent this K as a function of only the sigma 1, sigma 2 and sigma 3 value.

So, this way we can represents this thing, this functional form of the yield surface and then we can apply the; we can develop some plasticity theory. And then we can apply the analysis in this particular case. Otherwise if we want to evaluate all these, if we do not fill this functional form; it is almost impossible to conduct all the huge number of experiment simply to define the yield surface.

So, thank you very much. Next class I will discuss about this other part also; for example, what way we can develop the this yield functional form and specifically I will try to discuss the Von Mises yield functional form in this particular case.

Thank you.