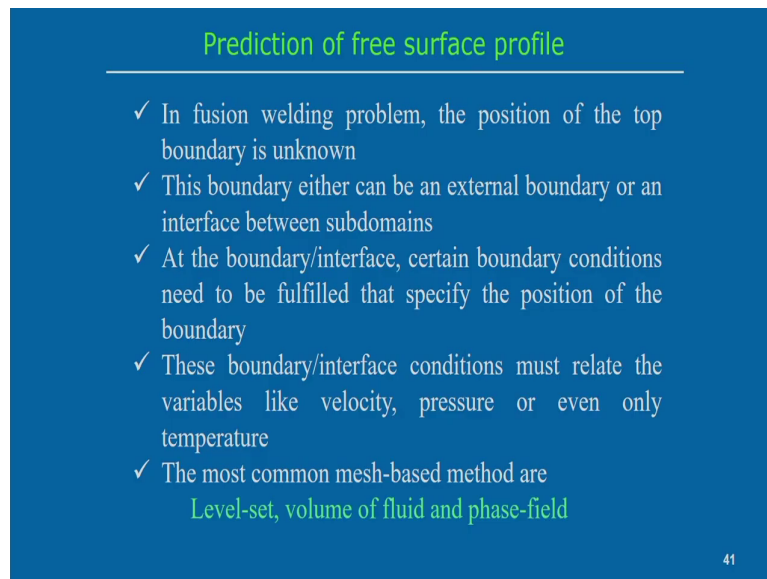


Finite Element modeling of Welding processes
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Lecture - 27
Prediction of free surface profile

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Prediction of free surface profile

- ✓ In fusion welding problem, the position of the top boundary is unknown
- ✓ This boundary either can be an external boundary or an interface between subdomains
- ✓ At the boundary/interface, certain boundary conditions need to be fulfilled that specify the position of the boundary
- ✓ These boundary/interface conditions must relate the variables like velocity, pressure or even only temperature
- ✓ The most common mesh-based methods are
Level-set, volume of fluid and phase-field

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Hello everybody. Today, we will discuss about the Prediction of the free surface profile associated with the welding process. So, if you observed in any kind of the fusion welding process that after welding the deformed surface specifically I am talking about the top surface profile which may not be exactly flat.

So, what we can model this surface basically curved surface exist in a particular associated with the welding fusion welding process. So, when we will discuss about the modeling approaches on the free surface profile in case of the fusion welding process. So, definitely, I

put in the mathematical sense the profile is normally unknown or position of the top surface boundary is specifically unknown in case of the welding process.

So, then boundary can be represent in the mathematical sense either in the form of a external boundary or say interface between the two different domains, that way boundary can be represented. But, what way we can predict the boundary that is the point in that is a discussion in this particular module.

So, boundary interface certain boundary condition need to be fulfilled definitely. So, we have to impose the boundary conditions, definitely in terms of the variable like temperature, velocity, in that critical even in terms of pressure also, that can be defined that you have to link the boundary conditions in terms of the all these variable parameters.

But there are several methodologies we normally follow in case of the fusion welding process to predict the boundary profile. One of them is that level set method can be used a volume of volume of fluid at the phase field. These are the most commonly used the methodologies associated with a free surface profile prediction in case of the welding process.

But definitely, these methodologies having certain advantage, disadvantage, and applicable in a particular situation that approaches only will be describing in this particular thing.

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Prediction of free surface profile

Interface Capturing

- Based on Eulerian description and define the interface implicitly on a fixed mesh
- The scalar field is defined that is used to identify two phases as well as interface
- The discontinuous scalar field may be expressed by Heaviside function or signed-distance function

$$\frac{\partial \phi}{\partial t} + \mathbf{u} \cdot \nabla \phi = 0$$

ϕ – scalar field
 \mathbf{u} – fluid velocity

Particle methods, volume-of-fluid methods, level-set-method, phase-field method (share some features)

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Now, before doing that actual process of all this different methodologies, for example, level set methodologies, before that prediction of free surface profile can be categorize into this two different way. One is the, we can say that it interface capturing. So, interface capturing is the method it is basically based on the Eulerian description and that is defined the surface over a fixed mesh.

Fixed means suppose your domain is fixed and the domain is, solution domain is fixed discrete in that which is having so many elements on the discretized space. Now, assuming this mesh is fixed then what way we are simply tracking what are the boundary or what is the interface between the two different phases or maybe we can the on the free surface also that we can simply capturing over a fixed mesh that is a one way.

So, in this case, the one of the scalar field is defined to identify the two phases as well as the interface also, that way the necessary. And this discontinuous scalar field may be express in the form of a Heaviside function or sine distance function can be expressed this discontinuous scalar field. So, definitely we need to define on the scalar field that scalar field explicitly able to capture the interface capture or the free boundary capture is possible.

So, this if you look into this expression also that $\Delta \phi$ by Δt . So, ϕ is a that scalar field and u is the velocity field. So, in these cases maybe in we need to know the what is a fluid velocity at the boundary and it is a gradient of ϕ equal to 0. So, this is a dot product between these two quantity, and it should, this is the equation such that we will be able to interface tracking and in the by defining ones particular scalar field ϕ .

So, of course, this scalar field ϕ although we are defining this free surface, but it is link with the fluid velocity. And this by solving this equation, we will be able to find out what is the particular interface free surface profile or the interface, in case of the fluid flow problem associated with the welding process.

But in this case, we need to solve this equation, this equation over the same domain the same solution domain where we are basically analyzing the other field variables there. For example, in a on a fixed domain particularly we are analyzing, what is the temperature distribution, what is the velocity distribution by looking into the governing equation and boundary condition we are getting and solving for this thing.

At the same time, this can be linked that this free surface profile also can also be predicted, but it is necessary to solve this equation in this particular situation, such that this equation is defined in such a way that it is in the scalar field is defined explicitly what is the interface or free surface boundary, free surface profile, but it is has to be link with the variable like velocity field in this particular case.

So, following this scalar field, solving this governing equation there are several methods one is the particle methods, volume of fluid methods, and level set method and all are in the we

can say the interface capturing method and phase field method also that is share some features not exactly the interface capturing is it can follow the other interface tracking also.

So, these are the name of the methods; volume of fluid, level set method, particle methods and the phase field method. These are the common methods we normally use for the interface capturing or the free surface profile in case of the welding problem. Now, this is talking about the interface capturing that can be defined over the fixed domain and we are explicitly defining what is the nature of the free surface profile or the interface.

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Prediction of free surface profile

Interface Tracking
- Track the position of the mesh nodes x in a Lagrangian fashion

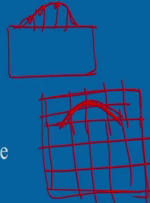
$$\frac{\partial x}{\partial t} = u(x, t)$$

u – fluid velocity in the domain

More accurate and computationally efficient approximation

Imposition of boundary conditions at the interface is easy as the nodes lie on the boundary

In large deformation problem, re-meshing strategy is required
Projection of field variables from old to new mesh is computationally costly



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The other way is the interface tracking problem, but in the interface tracking problem it follow the mesh follow the Lagrangian approach, in this cases Lagrangian fashion. Such that we track the position of the free surface or track the position of the boundary in the exactly mapping with the mesh.

Actually, that it means that there is a we if we consider there is a distortion of the mesh during this solution, that is, in principle that is a Lagrangian approach basically we are tracking we are following the each and individual mesh in this particular problem. So, that follow this equation Δx by Δt and the u is the fluid velocity domain in this fluid velocity field which is a function of the x is the vector in these cases, so and the as a function of t also time.

So, therefore, in this case the mesh is moving important than the, in other way you can say this interface tracking method is basically more accurate and computationally efficient approximation because in this case it is easy to implement the boundary condition because the imposition of boundary condition as the interface is easy because as the nodes lie on the boundary.

So, if there is a deformation process. So, for example, this is the initial position and then after a welding process or during welding process the surface can be this or deform like this way also. So, it is imposition that we can this is confirmed to the mesh on this particular the free surface boundary, and then it will be easy to implement the boundary condition by since tracking the free surface along with the movement of the nodes in this particular situation.

So, therefore, if we compare with respect to the other cases, for example, the interface capturing method. In interface capturing method, suppose this is the domain we can take some extra domain also, and suppose this is the interface in particular or this is a free surface profile during the welding process which is not exactly the flat.

So, in these cases, over we can say this is the domain over this domain we can explicitly defining this profile. With that this profile and this is profile is not exactly mapping, this is the independent, the mesh size is fixed, over the fixed mesh size we are basically capturing the interface or free surface boundary.

But that is not the case in case of the interface tracking method, interface tracking method the mesh is not fixed in these cases. The meshed can distorts and basically that is why talking

about the interface tracking method we normally follow the Lagrangian fashion. Lagrangian fashion basically we are following what is happening to a particular node, how it moves, what is the velocity it moves. So, that we can look into this all these aspects.

So, therefore, in the second one interface tracking although it is more accurate process are at the same time the condition at the interface is easy and as the nodes lie on the boundary. But difficulties is the large deformation when the it is definitely mostly the this in case of large deformation problem it is necessary to construct the re-meshing; that means, meshed distortion is very large.

Then, it is necessary to re-mesh the basically redefining the meshing in such a way strategy is required. Such that projection of the field variables from the old; that means, what was the initial value after remeshing then distribution of the all the or projection of the all field variables to this new mesh is difficult or may be computationally more costly in case in this particular approach.

So, therefore, we can easily distinguish the between these two approaches, one is the interface tracking method and interface capturing method. So, basically, we will be discussing the interface capturing method because in these cases we are not considering the distortion of the mesh, we externally tracking what is the; what is the free surface boundary or in the interface also, simply by solving this value.

But there is a different way how, what way we can consider this scalar field variable and what are the different functional form we can consider. Accordingly, the different methods has been developed one is the particular volume of fluid method, level set method, and the phase field method.

So, definitely we are not discussing the particle method, it is very old basically it is a it started actually interface tracking method in the particle methods, rather nowadays this has been more accurate or volume of fluid methods. So, level set method and phase field method, these are more generally used in these cases, so we will be discussing only these three methods.

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Prediction of free surface profile

Volume of Fluid Method

- A global field is used where the volume-fraction field for each element is defined by $0 \leq F \leq 1$

The characteristic scalar field is also defined by $\phi = \mathcal{H}(x, t)$, where \mathcal{H} is the Heaviside function

Therefore, the volume fraction for each element is calculated as

$$F(\Omega, t) = \frac{1}{|\Omega|} \int_{\Omega} \mathcal{H}(x, t) d\Omega$$

The advancement of the interface in time is governed by

$$\frac{\partial F}{\partial t} + \nabla \cdot \mathbf{F} = 0$$

The volume fraction function is connected with the mass conservation equation due to its discontinuous nature

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Volume of fluid method in this case the interface tracking or free surface profile in this case is global field variable is defined, but the volume fraction field is defined for each element volume fraction field which is defined as a function F. So, definitely F should be between 0 and 1. So, 1 means the completely filled with the liquid fluid and 0 is not the filled with a it is a solid state.

So, therefore, in between partially field, so F should be between 0 and 1 value. So, by simply this concept we can define this global field. But this scalar field this fraction volume fraction field this particular scalar field can also be defined as a characteristic scalar field can also be defined by the Heaviside function, phi can be a function of this thing where H is the Heaviside function.

So, therefore, the volume of the fraction for each element, for particular element each element the volume fraction can be estimated like that in the this Heaviside function the over the elemental volume. This is the particular divided by the total magnitude of this volume for this particular element. And this indicates the fractional value for each element F this thing.

Now, the advancement of this interface at because in these cases it is necessary to for the advancement the interface because interface we are defining in such a way that is it is based on the fixed mesh. We are not moving the fixed mesh. But it is not conforming of the mesh is required with this interface, rather external you are tracking the interface independent of the what way the mesh distorting happening or not.

So, practically this method is best suitable if you define the interface advancement of the interface independent of the moving of the mesh, or other way we can say it can be depend over the fixed mesh. So, therefore, the advancement of the interface with respect to time can be governed by this particular equation the similar kind of the equation ΔF by Δt , F is this field variable in this cases.

And F if it is a vector quantity then Δ this is a gradient of this the dot product of this F and this gradient we can find out, solve this equation. Then, we will be able to find out the this by applying the volume of fluid method to track the boundary. So, therefore, volume fraction is connected with the mass conservation equation due to its discontinuous nature.

So, they are definitely it is volume fraction when calculating this is a not a continuous function because of the discontinues nature. This volume fraction can be linked with the mass continuity conservation equation.

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Prediction of free surface profile

Level set Method

- Overcome the difficulties associated with a discontinuous function by using a smooth level-set function

However, it is necessary to solve the level-set transport equation to update the evolution of interface over time scale. In level set theory the interface motion is governed by the conservation equation.

$$\frac{\partial \phi}{\partial t} + \vec{v} \cdot \nabla \phi = 0$$

where, $\phi(\vec{x}, t)$ is the level set field and \vec{v} is the vector velocity field that advects the level set field. $\phi(\vec{x}, t)$ is the sign-distance function which stores the shortest distance to the interface.

This level set field is discretized by the standard finite element basis function.

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If you look into the level set method also this is to overcome the difficulties of the associated with the discontinuous function. Instead of discontinuous function, in the level set method we can define a very smooth level set function. But in this case, it is necessary to solve the level set transport equation.

So, that is required the level set transport equation necessary to solve over the same discretized domain, over the same discretized because that is the fixed domain. So, level set transport equation to update the evolution of the interface over the time scale. So, that means, to update the evolution of the interface over the time it is necessary to solve this transport equation.

So, therefore, in level set theory the interface motion is governed by the conservation equation. This conservation equation we can use the similar kind of the equation. This is the

ϕ the scalar field here and, but this is the a smooth level, the scalar field can be smooth level set function in this particular method and v is the velocity field that advects the level set. This is the velocity field the advects the level set from one time step to the next time step.

So, therefore, and $\phi(x, t)$ is the level set field and say v is the velocity over which advects the this field. So, therefore, if we solve this equation over the fixed domain and then we will be able to track the what is the interface or free surface boundary in this particular case, but mostly this scalar field can be represent with the sign distance function we normally used.

And that actually store the shortest distance from that to the interface, distance through the interface it stores all these data. And then, once we get this governing equation can be solved, but discretized by the standard finite element basis function. So, this can be the similar way we can discretize the domain and we the same kind of the shape function we can use here also, linear shape function.

And this, same equation transport equation we can solve such that the particular value this ϕ , ϕ distribution the variable, the ϕ or distribution of the ϕ each and every node point, it will define the what is the value of the interface during this process, during the level set method.

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Prediction of free surface profile

Phase Field Method

- Works with diffuse interface – transition layer between phases has a finite size
- There is no tracking mechanism, but phase state is included in the governing equations
- The interphase is associated with a smooth and localized phase-field variable ϕ
- If two phases, ϕ is scalar quantity and if more than two phases, ϕ is vector
- ϕ is assigned distinct values for different phases (i.e. -1 or 1) and the interface is defined by a value of 0

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A phase field method is one of the methods, in this case it depends on the diffuse interface in principle and that means, transition layer between the phase has a finite size. So, it uses a transition layer that is smooth transition from one phase to another phase. So, in these cases, there is no explicitly you are not exactly tracking mechanism, but phase state is defined or included in the governing equation.

So, therefore, in this case also it is necessary to solve this governing equation or the phase can be used in the governing equation included in the governing equation you can solve the particular domain. Then, we can define this phase field variable and that based on the variable what a definition, for example, the phase field can be defined say we assume the single phase.

So, then it is a 1, maybe we can define 1 value indicates particular phase and when it is reached to the 0 value. So, then we are tracking the 0 value, so that indicates the interface or

boundary. So, therefore, it is also possible the more number of phases can also be present. So, it interface is associated with the smooth and localized phase field variable.

So, this is the ϕ this scalar variable can be represented as a small localized phase field variable. If the two phases if we consider then ϕ is the scalar quantity. If two phases then; if more than two phases; that means, 0, 1 something like that then ϕ can be represented scalar quantity more than 2 phases ϕ can be vector quantity. But ϕ is assigned distincts values for the different phases.

For example, ϕ can be minus 1 or plus 1 and the interface can be defined as a value of the 0. So, therefore, in this cases, so there is a there are several scalar variable can be, several functional form of the scalar variable. So, that scalar variable can be included in the governing equation or this can be explicitly solve the defining the governing equation on depending upon the particular problem.

Then, we can track the interface or the free surface boundary by using the phase field method. For example, phase field method it is also very common method to use in case of that if we want to know dendritic structure during the solidification process. There is a extensive use of the phase field method there to predict the dendritic structure in case of the solidification process.

So, therefore, apart from all other application the same phase field method can also be used or the free surface tracking or free surface profile in case of the welding process. So, thank you very much for your kind attention. And now, we will discuss the next module.

Thank you.