Finite Element modeling of Welding processes

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Lecture - 26
Heat transfer and fluid flow analysis in quasi-steady state

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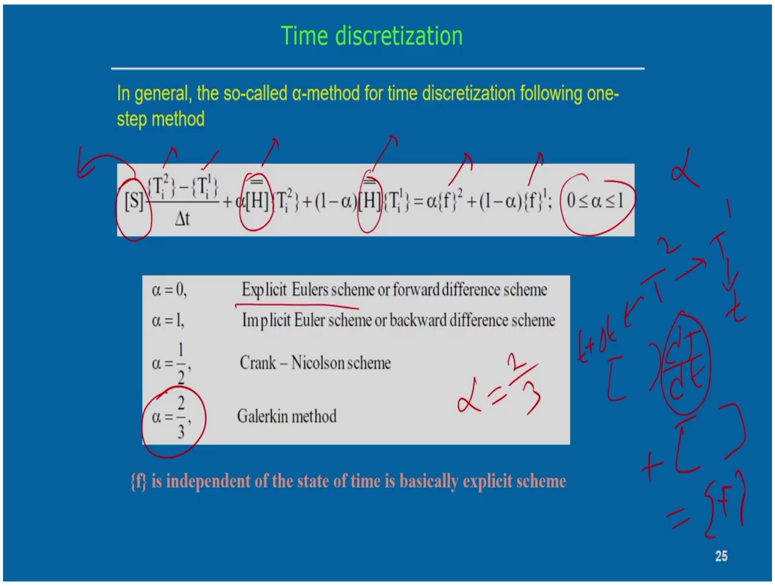

Good afternoon everybody. After discretization of the governing equation in the spatial domain, then we think about the what way we can discretize the similar way in time domain. So, that time discretization is basically in case of any kind of transient problem. So, it is associated with some similar kind of the weighted residue techniques. For example, in this case Galerkin weighted residue technique will be using in the even in the time domain also.

So, if you do that, then any kind of equation even it is applicable in case of conduction based heat transfer model as well; but at the same time this in case of transport phenomena based heat transfer and fluid flow model, it means that even momentum equation also. We can first, we form in the spatial domain, then after that we take the consider the discretization of the, this equation in the time domain.

So, you can see this even heat conduction equation or momentum equation also, in both the cases we can see the governing equation is associated with the first order time derivative. So, in this case similar kind of the time discretization scheme we can use. So, in general, so called alpha method of the time discretization scheme that, which we can show here also; we can see here also in one-step method.

So, it is like that only. Say for example, this is the assemble matrix S and this thing in the form of that S matrix, then H matrix is there and this H matrix is also there and this is the form. So, these are the all the S matrix and H matrix that in general, we can form this equation; maybe in general form of the equation will be always getting like that, we will be getting this three; one is the one matrix is associated with the some this associated with the first order time derivative.

And then, other form of the matrix, then equal to some column vector will be something like that. This is the in general form of the matrix and then, using this matrix, we can look into that what way we can general form in the time discretization came we can express like that.

So, it is like that only. This is a one matrix S and this is another matrix H and that is a one factor that is a alpha. This is called in general alpha method. But other part is that f is the right hand side. It is associated with the mainly the load vector. So, and the load vector in the basically it is a column in the representation is the column vector.

So, then once it is a general form of the equation and this with this alpha is there and this is called alpha method and then, different values of the alpha, then we can take the different scheme in the time domain also. So, for example, in this for if alpha equal to 0, we just simply can the explicits Euler's scheme or forward difference scheme in alpha equal to 0; if alpha equal to 1, the different scheme implicit Euler scheme or backward difference scheme or if alpha equal to 2, then Crank Nicolson scheme.

But in this case, we use the alpha. In these cases a time discretization, we normally use the alpha equal to 2 by 3, that is the corresponds to the Galerkin method. So, in this method or in this time discretization scheme, the f that load vector is basically independent of the time step of the basically it is a explicit scheme.

But at the same time if you follow the Galerkin thing, it is unconditionally stable condition we are following. So, if you simply put alpha equal to 2 by 3, then we will be able to get this thing such that this T 2 is the represents the temperature variable the particular step and it is associated with the T 1.

Then, we can see also that T 2; T 2 represents in terms of the T 1. So, T 1 T 2 is the at time t plus delta t, the temperature distribution and t is basically time at particular time t, what is the temperature? And then that means, previous step, temperature accounting this thing plus rearrange this equation in such a way, then will be able to find out what is the value of the temperature at the time step t plus delta t.

And then, accordingly, we can step once you get the time temperature distribution on particular step. Then, we looking for the what are the temperature distribution for the next step? Like it progresses this thing, but this is the more general scheme, we can follow in time domain that I am trying to showing here, in this particular slide.

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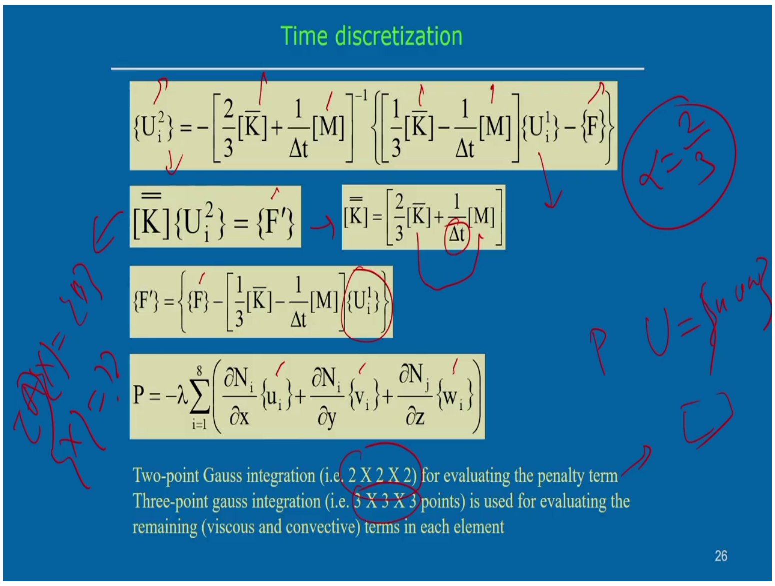

Now, so, similar that I have shown in terms of the temperature, so, we will be using the similar kind of the expression; but in this case same alpha equal to 2 by 3, so Galerkin scheme, we have used. And then, we can predict what is the velocity distribution on particular time step like that U 2; U 2 in the form of the other parameters, maybe the matrix nomenclature are different.

But similar nature of the matrix will be able to will be able to get in this same way that it associated with K M. At associated with K M to square matrix and one is the F is the it is a kind of the column vector and then, U 1 is the what is the velocity of the previous step. So, that is we can estimate what is the value of the velocity distribution also, similarly velocity vector also in the particular time step, that need the information of the what are the values of the previous step.

And by forming these different values of K, M matrix and then from there, we will be able to solve the what is the value of the particular time step. Now, it is finally like that. It can be this thing, this equation can be represented like that also that U K double bar U i 2. Basically, this thing K equal to F.

So, it is like that only the matrix equal to A x equal to B, something like that. So, this is matrix equation, we will be able to solve it. Once you solve it, then we will be getting what is the value of the x. So, it needs some kind of the solver here also. This represent the linear system of the equation. So, with appropriate choice of the solver, finally, we will be able to know what is the value of the x. So, in these cases in the fluid flow problem we will be able to get the value of the velocity component.

So, this is the in general form, but that K double bar that indicates that it is associated with the K single bar and M this thing and delta t is the what are the time step during this process. So, therefore, and F dot; F dot accounts this means if you see the F dot is basically this vector F corresponds to that and it is associated with this thing other. And this F dot, we can calculate it using the value of the velocity of the previous time step.

So, according to Galerkin scheme, we will be able to solve this equation. So, whatever this thing anyway, we are always trying to look into that what are the that form this matrix even in the discretization first in the spatial domain, then discretization in the time domain, then we are making the matrix in the form of a A x equal to B.

Then, we are thinking what way we can solve this value of x. So, then we use the solver. So, different solver, that solver part will coming later on. So, once we get the solution of A, from here the velocity field on a particular time step.

So, from there also then since we are using the penalty finite element method, then pressure can be estimated like that also. Because we remember that pressure term, we linked with the continuity equation expression of this thing by introducing the penalty parameter or penalty term.

So, from there, in this domain the shape function you take the same shape function and then, what we have used for the in the solution of the at the temperature distribution and the fluid flow field. So, similar shape function if we consider, then using the value of the velocity field, so U i; capital U indicates it is a the all the velocity vector. It consists of the that velocity u, v and w. It is a particular node.

So, there are the three component of the velocity will be there along x, y and z direction that is u; small u, small v and small w. So, that indicates the velocity. So, velocity value in the node point and if you want particular element, we will be able to find out the what is the value of the pressure.

Simply what expression we have considered from the link the pressure term with the continuity equation in terms of the penalty parameter, the similar kind of the expression, we can use it by introducing the with the shape function and from there, we can estimate what is the pressure so for particular element.

So, in this way, we can find out that even the first velocity field. From the velocity field, it is possible to estimate what is the pressure distribution also each and every node. Now, once we estimate the pressure that value in a particular element, then we can distribute this equally distribute the each and every node point, if it is a you know brick element. So, which is simple way pressure value we are getting.

We shall make some average value of the each and every node point and then, in that way, we will be able to get the pressure distribution. So, Two-point Gauss integration method: So, basically for the evaluating the penalty term, so to avoid some kind of the solution, maybe the spurious solution during this using the penalty finite element method.

So, it is necessary to follow for the penalty term to this thing for evaluating penalty term, I am talking about that matrix which is penalty term, we have shown that one particular matrix. In that matrix, there is a penalty term, we normally use. So, in that case the whatever integration will be doing that, in that case we will be using the 2 by 2. So, reduce integration method, we should follow.

So, basically that term is the 2 by 2 into 2; that means, in that 8 points, we have to consider the integrate. But, other term that three-point Gauss integration techniques, we can follow. So, 3 into 3 into 3 is used for the evaluating the remaining viscous and convective terms in each of the each of the element.

So, each of the element from the elemental matrix, there when you do the integration about the elemental volume, in that case, so we have to be careful about that for penalty term associated with this particular matrix. The volumetrical, we should do following the reduce integration method. In this case, it is we can the integration points are less as compared to the other elemental term. For example, viscous and convective term in particular to each element.

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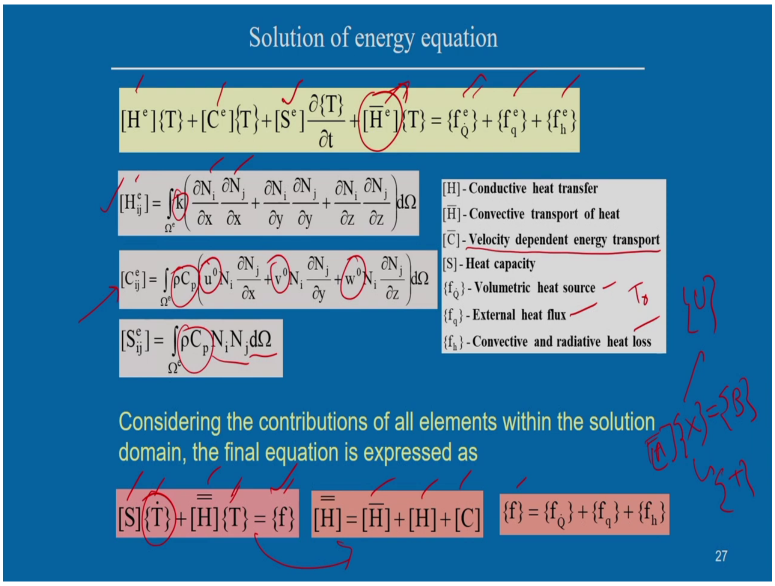

Now, once we solve the velocity field from the Navier Stokes equation and then, in or we can say the from momentum equations and then, once you get the velocity field, then velocity field can be utilized also to some extent.

I have already shown this thing and then, we solve the energy conservation equation to get the temperature field. But, there is a link between this thing because when you solve the energy conservation equation, then there is a need of information of the velocity field is also there.

So, certain term in this case is associated with the velocity field and that we can estimate this velocity field. For example, on this term C, then this elemental matrix we can see. The elemental matrix H associated with the C; H is called conductive heat transfer. Basically, that conductive heat transfer, we can see the expression of the H also that k thermal conductivity is there and remaining is the shape function that differentiation of the shape function delta N by del x; similarly, delta N delta y.

So, this is the form of the elemental matrix. Now, if we do the integration here, we can follow the simple in a three-dimensional state, we use a 3 into 3 into integration point; 3 into 3 into 3, so total 27 integration points we can use to get this term, the elemental term H. Similarly, C term; C term indicates that velocity dependent energy transport.

So, if you see the velocity dependent energy transport is associated because this velocity field, we normally count from the velocity field what we have estimated. So, that is why there is a need of this convective term also to incorporate in the energy equation to know about the, this velocity field also.

From there, we can estimate the C matrix; C elemental matrix and it is associated with not only this velocity dependent energy transport, but it is a rho C p, then specific heat and density of this particular material is also required here. Now, S, S corresponds to see this S corresponds to the heat capacity.

So, heat capacity one is rho C p and then, shape function and over this elemental volume and we can do the what the this is the all this cases, we are doing the volumetric equation, because this depend on the domain also within the element. Now, once we estimate this H, C, S term, then H bar also, H bar accounts I think this term is the basically H bar, we have the convective transport of heat.

That means, H bar into T so that I am showing this term; but H bar is the convective transport of the heat you know we considered here because this energy equation is solved with the combining the boundary condition also. So, from the boundary condition, we can see that convective and radiative heat loss from the surface. So, H bar accounts that part, the convective transport of the heat.

So, then this is the this account merely from that convection and radiation that part, it comes is elemental matrix. Then, it should be equal to the that this is the load vector; that means, volumetric heat source. If we use the volumetric heat source term, then we have this matrix accounts the volumetric heat source and if you use the some kind of the external heat source on the surface.

That means, if you consider only on the surface flux, then f q can takes care of that surface quantity. And f h is the convective and radiative heat loss, in the sense that this f h is accounts there is a some reference temperature the T 0. So, that means, ambient the outside environment temperature. So, that term is comes that is f h.

So, that will account all this right hand side, it is say basically represent the column vector; all these terms in the cases and this H, C that elemental matrix is there, it represents the square matrix. So, in this case, we can considering the contribution from the all elements. So, this is the elemental form. Now, once we calculate the for one particular element, the contribution from the each element, then we assemble this contribution from the all the elements in the domain.

So, then we will call the one by one element and that keep on assembling the in the global matrix form such that finally, we will be able to get the expression is like that only S T dot H bar T in general form and f. So, that similar kind of form of the equation, we got in the what we discussed in the just in case of the fluid flow analysis also; the similar form.

So, that means, one is associated with the some temperature derivative with temperature derivative with respect to time; the time derivative of temperature and this is only temperature and right side is the basically load vector. So, in this case, we can linearize the system; we can make the system of the equation in this way also.

And such that H double bar equal to account all this similar kind of the matrix here and f counts this is the basically the similar kind of column vector and H is basically the square matrix. So, then once we get this kind of the form, then we think about we need to go for the similar kind of the analysis; that means, that just I discussed, then we have to follow the discretization in the time domain.

So, similar kind of the exercise, we can do, we can form that what way we have did in these cases also. Then, I will get the similar kind of expression, but only thing is that instead of U, we will be getting the in the I think it is better to represent this equation here also. So, here we can this time domain; only thing is that this alpha will be 2 by 3 in this particular case and then, we solve for the temperature in the after discretizing the time domain, then we solve the we solve for the temperature.

So, after discretization of the time domain, then it normally forms in this particular form. So, in this is the form. So, this that means, A X equal to B, the linearized form of the equation is we finally we are getting the A X equal to B in that form also. Here also, we will be getting finally the A X after discretizing the time domain. So, A X equal to B in that format we will be getting.

Then, we will be solving for X. So, X here in this case X will be the temperature distribution. So, temperature value in the whole domain; that means, temperature value each and every node point and previous cases, we solved for the similar kind of expression, but that was we are getting the velocity vector. So, velocity vector means all the three components of the velocity each and every node point; but finally, we are reaching in the final form of the equation something in this format.

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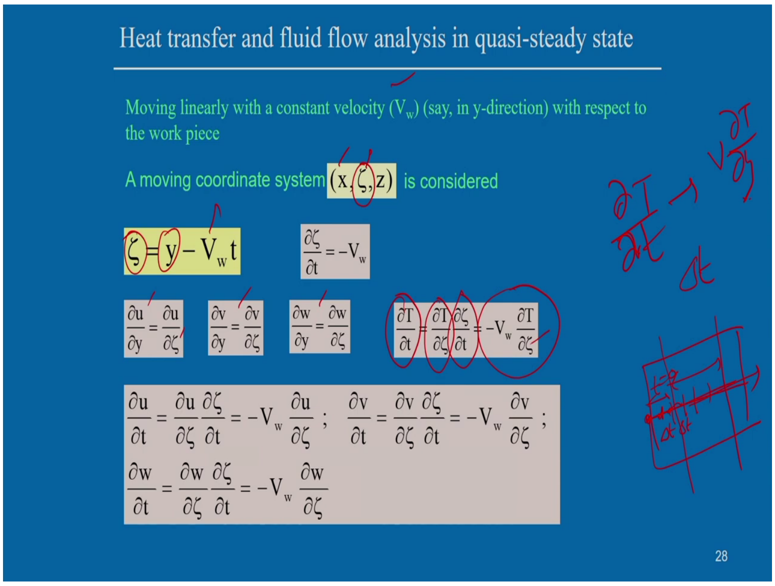

Now, talking about the heat transfer and the fluid flow analysis, in case of the quasi-steady state analysis; so, quasi-steady state means so far we have discussed in the transient problem. So, transient problem, we need to discretize the domain for the for spatial domain as well as the time domain also and at the same time, once we solve the transient problem also, if it is stationary heat source, then also we can solve for the transient problem for a stationary heat source.

But, it is the moving heat source also, we can solve the transient problem; but each and every time step, there is a movement of the heat source. So, movement of the heat source can follow with a particular velocity and then, means it each and every point with the over the increment of the time delta t, we need to solve this transient equation and then, we will we are getting the temperature distribution.

For example, suppose in the domain, the is domain the heat source is moving along this line. So, we start from here. So, the time t equal to 0 and then once the velocity; that means we discretize in the small time step delta t this thing. So, next step the this heat source moves from here to here.

So, we are solving this transient equation for this particular point, then we are solving the against the same equation in the next point, solving the same equation in the next point; that mean, computational time can be more, if we continuously solve the one the discretize in the simply just replacing the heat source one particular point and solving the same equation. So, this kind of the situation in the takes much more that computation time is very much high in this particular situation.

Now, if we consider the quasi-steady states in the in welding problem, it is like that only. Suppose, the heat source is moving in an particular direction; but that means, if we after initial period of the time that is a very initial starting time, suppose, we start from this point. So, very initial time, it is in the transient state. But in between, the space in between it is the whatever point we consider, we can it can reach the similar kind of the temperature profile.

Then, this kind of situation, we can say that it is the quasi-steady state problem. It indicates that within that domain a particular length that if we solve the energy equation not only not in the several discretized time step rather if we single time, if we solve it; then, we will be able to predict that this is the particular profile in a quasi-steady state analysis.

It means it is simply that the solution is independent of the time. So, that is possible in case of the welding process. So, it one from one side, it actually reduce the computational time as compared to the pure transient problem and transient problem; but at the same time, it is a moving heat source problem.

But in that case, we can reach from the transient heat conduction equation to the steady state equation from this particular way. For example, suppose heat source is moving constant velocity V w and particular direction y-direction, so with respect to the work piece domain. So, therefore, we define the moving coordinate system, the x because the zeta is basically it is a replacement of the y axis because it is a moving coordinate system.

Zeta is considered and such that moving and the stationary coordinate system with the fixed coordinate system can be linked like that only. So, zeta equal to y minus V w into t. So, V w t indicates that distance and the particular time t. So, this link the this is the moving coordinate system and y is the fixed coordinate system and the suppose welding velocity uniform velocity V w is moving and t is the time variable in this case.

So, therefore, this equation you can say d delta zeta by delta t equal to simply minus V w. So, now, similar way that even in case of velocity component also, we can say the del u by del y is the same velocity by del in terms of y. We just simply replace y with the fixed coordinate system to the moving coordinate system zeta, so, same thing for w also and V also.

Now, del T by del t; that means, this term we see the transient term. If you see the transient heat conduction equation, the transient term del T by del t as a small t; that means, this with respect to time. So, it consists the del T by del zeta and then, otherwise we can see this thing that such that we will be able to get that time variable, we can replace with the spatial variable. See zeta is the in this case, the we represent this in the form of a moving coordinate system; so, del t.

So that means, we can it is possible to eliminate the f transient problem to the quasi-steady state just by everything governing equation, we explain in the form of a spatial domain only. So, in this all this transformation, we can consider similar way; then, we can reach in the spatial domain such that the governing equation can be solved in such a way that it will replace the right hand side the this right hand side T by del t.

It is in the form of the velocity and del T by del zeta; that means, this is the moving coordinate system. So, in that form, it will the equation actually form.

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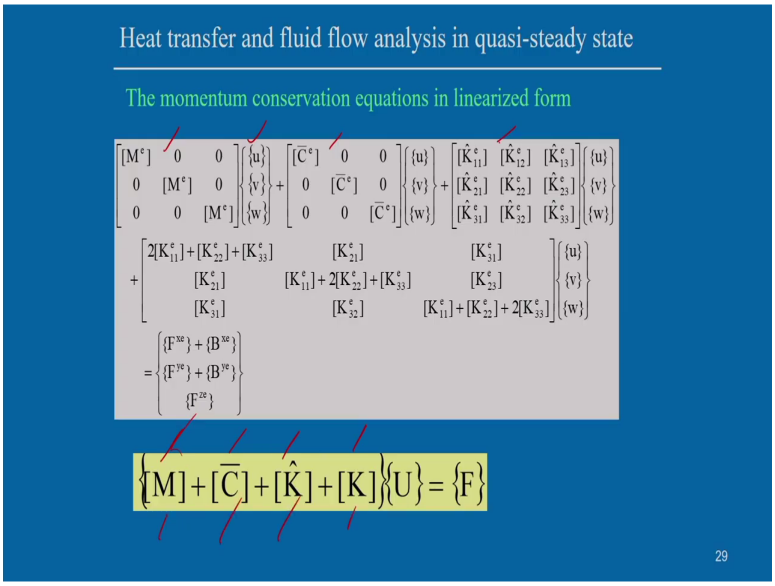

So, once we get this thing, then we need to consider the form of the equation in the spatial domain also. So, discretization of the spatial domain is sufficient to get the solution of this particular equation. So, then if you consider the momentum conservation equation in the linearized form, we can express this in this format.

Say it has similar kind of M; M term will be there. The, this is the velocity component and then, C is the C term is there also a velocity and if you see the K the similar kind of the expression what we are getting in case of the transient problem also then, the similar expression.

But in this case, there is no time component here and we can get that the matrix can be formed, if we follow this similar analogy, what we consider the element and we follow the similar discretization strategy, the Galerkin weighted residue technique and three-dimensional linear brick element if you consider.

Then we will be getting this form of the equation and matrix. Elemental matrix can be form accordingly and after the formation of the elemental matrix, if we do the assemble; then, we will be getting the what the whole domain, the matrix; global matrix. But all the global matrix, the it will be the same dimension or this thing. We can simply merge it and then, it is possible to solve also.

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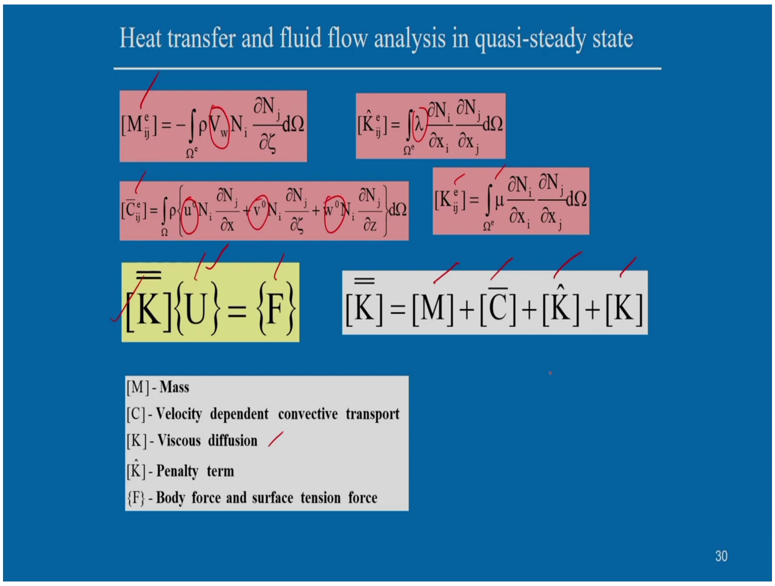

For example, here you can see that what are the expression of the different elemental matrix. So, M is the basically mass matrix then if you see there is a slight change in the formation of the M matrix, M mass matrix as compared to the transient state also. Transient it was different there, but here the velocity components introduced in this case.

So, this takes part, it is it expressed in the moving coordinate system. Similarly, K; consider the penalty parameter, K is the basically I think penalty term, we can use this and the other K indicates that the viscous diffusion also, the similar viscosity associated with the viscosity and all we discretize in the domain the of the over the elemental volume.

Now, C can also takes care of this viscous dependent convective transport; that means, it is associated with the velocity; this u 0, v 0, we have already shown in this case that in a fluid flow problem. So, we can linearize the equation because there is a two velocity term also. So, we can linearize the equation. So, therefore, this finally, once we merge all these matrix form, then we will be getting this is the final form of the equation. A X equal to B, the similar form equation we are getting.

So, ultimately, we are aiming the discretizing the spatial domain if or even after spatial domain, we try to reach the expression in the form of a A X equal to B in that form matrix form. This indicates simply linear system of the equation; where, K is the square matrix is the column; V is the F also column matrix column vector. So, then K accounts all these similar kinds of the matrix, similar dimension of all the other.

So, mass may be convective, diffusion, penalty and the viscous diffusion convective transport. All kind of matrix from its accounts and finally, F is the body force. Then, we solve this equation then we will be able to get the what is the velocity distribution in this fluid flow problem.

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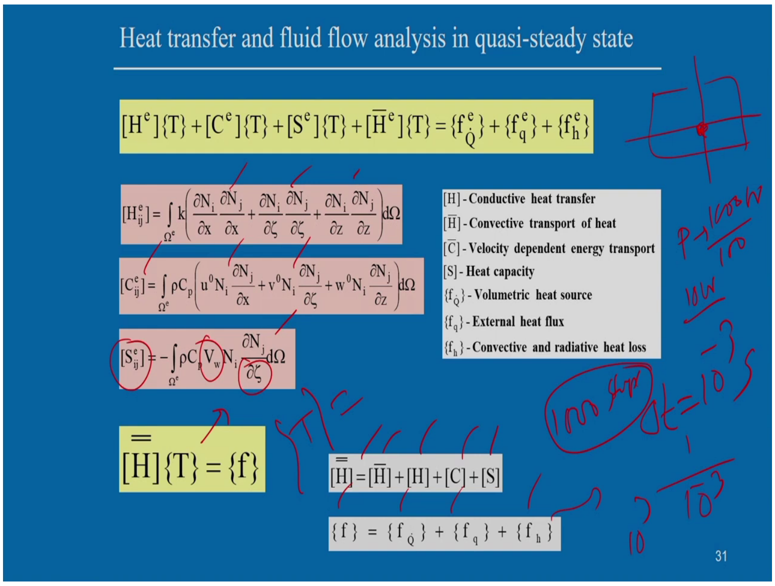

Similarly, in case of the energy transport also, we use the similar kind of the expression; but in these case is that see there is a difference also. Here the S term, the elemental matrix in case of the energy transport equation that it is accounting this velocity term. This velocity is actually this V w is the constant term. This velocity is the simply moving heat source.

This is the velocity of the moving heat source. So, this velocity is different from the velocity field in case of the what we are getting in case of the fluid flow analysis. So, this expression, see it is associated with the moving coordinate system and then the similar kind of the expression we are getting.

See that is the C is the basically is the velocity dependent energy transport and this thing specific heat and we can solve this equation and finally, we will be similarly, we will getting this kind of expression H T equal to f.

So, that means, H bar equal to accounts all these elemental matrix form and f accounts all these column vector. So, associated with the volumetric heat if we consider volumetric, otherwise it will be it will be 0 all the entry with this particular column vector. If we consider the surface flux, then it will account this thing and similarly, if we the heat definitely if heat loss is there.

So, it is accounting the heat loss by the it say comes the heat loss because of the some reference temperature, we are considering, from that point this f h term which comes into the picture. So, therefore, we solve this equation and finally, we will be getting the what is the temperature distribution.

So, in this cases there is no need of to going into the discretization in the time domain; but in this case the solution can be done in the different because in transient problem basically, we are getting the temperature distribution or velocity field each and every time step.

So, at the end of the time step, we are getting. So, normally the time step is very small. So, in our particular maybe we can use a 10 to the power minus 3 second maybe time step typically in for example, in case of a GTA, simple gas tungsten's arc welding or laser welding problem. This is the typical step of the time. So, this is the order of time step.

So, that means every if it is run for the 1 second the welding problem, so that means, 1 second means the how many steps we have to see; 1 by 10 to the power minus 3 second. So, 10 to the power 3; that means there may be the 1000 steps.

So, 1000 steps need to solve the every time. Every time step gap we have to solve the this both the equation, then we will be getting the temperature distribution or temperature distribution as well as the velocity field and we are keep on updating this velocity field this.

But the transient problem, the number of time steps involved is more. So, that means, 1000 steps has to be solved basically; 1000 times basically you have to solve this equation to get solve for a problem associated with suppose the heat source moves from one point to another for total 1 second.

So, that is a difficulty or maybe this in transient problem; but rather if we follow the quasi steady state problem, we can pick up on the same domain also we can consider. But in this case, maybe we can solve assuming the heat source at the some intermediate point at the center point and then, we solve this equation and at the single step we will be getting the temperature distribution.

But single step in the sense means it is not necessary to though the different step, the different position of the heat source we have to solve. We can assume at the one position of the heat source is there and we solve this equation, then we will be getting the temperature and velocity field.

But to do that in these cases, we also follow some steps also in the sense that the load cannot be applied the total load, for example the power or in case of laser power or maybe in case of the arc welding process, the total energy supplied to the domain that cannot be done in the single step.

So, that power can be ramping of the power is required in these cases. So, for example, suppose there are power laser power is 1000 Watt. So, in this case for example, we can divide this 1000 Watt into the 100 steps or 10 steps for example. So, each step so if it is assume that is a 100 steps means 1 step, there is a 10 Watt.

So, we start the solution assuming that same position of the heat source is fixed, then we start the applying the load a 10 Watt; then next step, we apply the load 20 Watt; next step, we apply the load 30 Watt. So, that is why ramping of the power is required to get the final solution of this equation.

But definitely in this case, the steps requirement is less as compared to the transient problem. So, once we do the similar kind of the analysis, similar kind of the solution procedure also we can follow both in the heat transfer and the fluid flow analysis and at the same thing also we can done in case of simply, if we do the conduction based heat transfer analysis as well.

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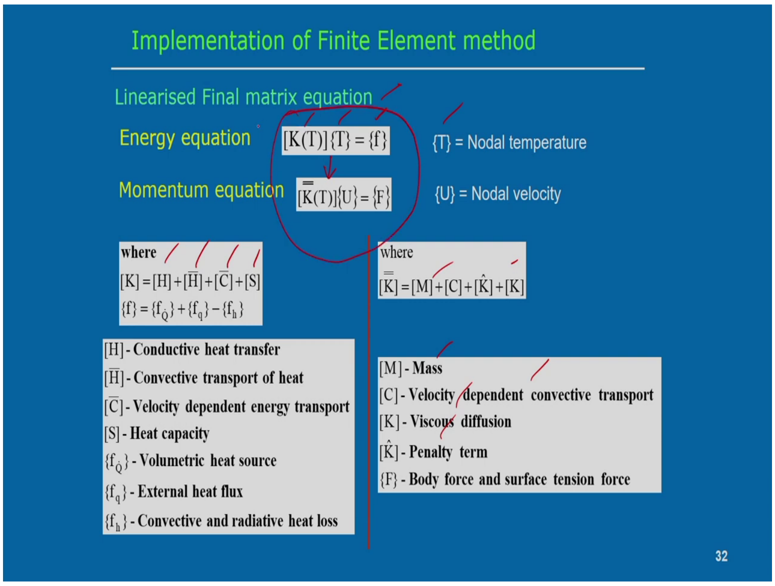

So, there also we have very clearly way that implementation in the finite element method. But basically always we try to reach this linearized equation, final matrix equation. So, in case of transient problem it can be like that; energy equation can be like this. K T equal to f. So, K is the accounts all the similar kind of the matrix. So, square matrix in this case it forms; finally, accounts the contribution for the all the elements in the different - different form.

So, in this case, different - different form means here you can see H, H bar C bar and S that we have already explained the conductive convective heat transport, velocity dependent energy transport and the heat capacity. All take this thing in the form of the same equation, then T, we solving for the nodal temperature T.

And each and every node point will be getting the temperature distribution or temperature value and momentum equation also, we can the similar kind of the expression; the same form of the matrix equation. So, my point is that we can use the similar kind of the solver in both the cases; that means, same kind of the solver can be used because in the both the cases the nature of the equations were same.

So, linearized form of the final matrix equation and here also in momentum equation, the expression of the matrix are different. In these cases, the mass, velocity dependent, viscous diffusion, penalty term and the body force are all these dependence. But finally, we will be reaching this or the same of the similar kind of the equation will be getting and will be solving for the either temperature distribution the, in case of the heat transfer problem.

Only heat transfer problem or both temperature distribution as well as the velocity distribution in case of the transpose phenomenon heat transfer and fluid flow model.

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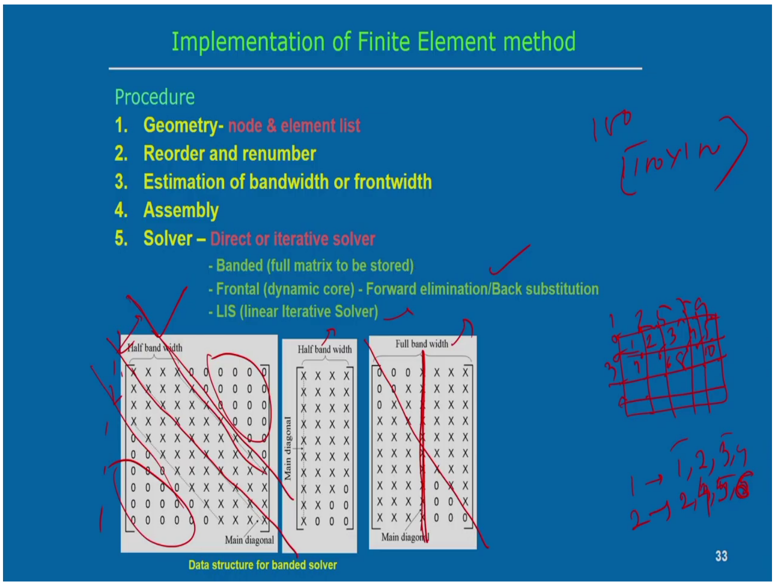

Now, in general implementation of finite element method means in solving the this either this only heat conduction problem or combined heat transfer and the fluid flow problem also the, in general, this is a general procedure. 1st, we have to create the geometry: Basically, the we have to decide the domain, solution domain. So, once we clear the solution domain, then we have to discretize the domain.

So, discretize the domain can be done the different discretization scheme depending upon the this thing that even that is meshing scheme also there. So, in this case, we can use the simple the brick elements also because it accounting is easy in case of the if we assume the in three-dimensional problem in the linear brick element, if we consider.

Then, we will be getting the others total domain in the discretize form. But in the discretize form, it is also necessary to understand there is a node point. So, this point indicates the node value, node there is a some numbering of the nodes is required for in discretize space also and at the same time, numbering of the element is also required.

Now, once we numbering of the element and numbering of the node, then what we can link between these two; then, the list is the node and elemental list that mean list of the element and what are the node is connected each and every element, that list is required from the elemental matrix also.

So, therefore, that is very needed in a any kind of the standard finite element problem. So, that means, first with these things node and element list to this thing. For example, we create the element number something 1, 2, 3, 4, 5 and then, node number can be 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

So, for example, in this case the one element, element number 1 is connected to the 4 nodes; 1, 2, 3, 4. So, that element number 2 is connected in the nodes 2, 4 5 and 6. So, that is way, so each element number which are nodes are connected to which element number that complete list is required to form the elemental matrix.

So, once it is done, the next part is the reorder and renumber. It is very important because this is the we connected that element number 1 is connected to the node number 1, 2, 3, 4. Then, element number 2, it is connected to 1, 2, 3, 4, 5, 3, 4. So, 2, 4; basically 2, 4 and 5, 6, this is the for element 2.

So, therefore, this is the this sequence we are following 1, 2, 3, 4. But in this case, reorder renumber means, the this finally, we are when we assemble the global matrix and they basically global matrix in form and we count the what is the node number. So, for example, if the total node number is 100, so, if we assemble this thing, then we will be getting the dimension of the matrix 100 by 100.

So, it is basically following the what are the total node numbers in this particular domain. Now, if the node numbers is in the numbering in such a way that the matrix can be formed such a way that we will get in lots of non-zero elements within this matrix also. And some lots of zero-element that there is a non non-zero element also will be there at the same time the there will not be anything it zero-elements.

That means, the entry of the particular position will be the 0 also; that means, it depends on the what way the node is connected, one element is connected to with which nodes and how one nodes is connected to accordingly one nodes connected to other nodes also to form this equation.

Therefore, it is necessary to arranging of these nodes in such a way that we can make the non-zero entry in the scatter in the very small domain maybe, so, that here you can see that also, this figure also it is very easy to understand. So, reorder reordering of the nodes is required in such a way that it can minimize the bandwidth.

Bandwidth means so basically it can accumulate it can accumulate the non-zero entry towards the scatter the near about the diagonal; but away from the diagonal this becomes zero-elements basically. This is no use of that thing this zero-element. So, that kind of it depends on the what is the nature of the typical nature of the matrix.

So, how the non-zero and zero elements will be arranging within this matrix? It depends on what way we are considering the some reordering scheme of the elements. So, that means, it simply depends on the numbering of the nodes and numbering of the elements. So, that is why once we get create the geometry, we get the elemental list and node list numbering of the nodes is over.

Then next step can be reordering this things such that the non-zero entry can be scattered and non-zero entry can be gathered near about the diagonal of this particular matrix. So, it will help to develop some kind the solution procedure or computational time. It saves the computational time of the solution of the linear system of the equation.

Renumber is required for examples the once a reordering of the elements nodes are done, then renumber will help this thing in the sense that it is renumber in the sense that it is been following the top one particular element, the one particular sequence will be following the top of the surface and bottom of the surface and other sides of the surface.

Same sequence; so, 1, 2, 3, 4, the same sequence positions should be maintained for each and every element, such that it will easy to implement the boundary condition also. So, that is why renumbering and reorder first and the renumbering of the each end every element and nodes are also required. Then, once it is done, so then, estimation of the bandwidth or front width, if we follow the some kind of the frontal solver, then either bandwidth or front width can be estimated.

And such that, it can be form this kind of the equation; that means, up to which the maximum position the some non-zero entry is there, that indicates the bandwidth. For example, this is the diagonal matrix and this is the actual matrix after assembly for contribution from the all elements. Then, it is counting the node number x and this thing, row and column will be simply the counting of the node numbers.

And then, if you consider the half because if it is symmetric matrix, then we consider only half of this entry, we can store only this non-zero entry also in this particular format. So, that is called the this called the this is the band, the this width and this half it is called the half bandwidth, such that it is simply storage space can be optimize this thing storage space to get the solution of the equation.

So, even it is so non-symmetric matrix, then we can solve we can store in the form of a full bandwidth also, that is also possible. So, in this case if you see this is the main diagonal, this is the main diagonal because the diagonal matrix should be non-zero entry definitely. So, with respect to this main diagonal, we can store the half part this thing; but this is the actual matrix and this is the storage of the half bandwidth and this is the storage of the full bandwidth.

And this half bandwidth is equal to if the problem is a symmetric or matrix form is the symmetric form, then half bandwidth storage of the half bandwidth is required up to that and if it is not symmetric, then storage of the full bandwidth is required. So, it simply reduce the amount of the storage. So, therefore, once is the assemble then we stored in the particular format, then we think about the solver.

Now, solver can be both direct solver or iterative solver can both can be used and there are so many in standards commercial software, there are several options are there also solver while choosing of the solver. The solver depending sometimes if iterative solver, then it depends on the kind of the problem; what is the nature of the matrix; whether any preconditioning of the matrix is required or not.

So, see if the iterative solver depends all kind of these factors. But if we consider the direct solver, direct solver will be always getting some kind of the exact that means solution. For example, you can in direct solver; we can simply follow the Gauss elimination method; that means, forward elimination and back substitution.

This way we can forward elimination and back substitution, you can follow simply that follow, then Gauss elimination method we can follow that things and we can solve this matrix also.

But in case of the iterative solver, then [FL] linear iterative solver, there are several solvers are also available, we can use the solver. But in this cases, we can start with the some approximate. So, therefore, it depends on the number of iteration is required to reach the particular solution; but in case of the direct solver, in one step, we will be getting the solution of this from this particular matrix.

So, therefore, there is a so many options are there in by choosing the solver depending it depends on the, whether computational time definitely. So, that is very important. So, but we will discuss simply the frontal solver in this case particular case because this frontal solver is maybe useful in this particular context to this thing solve this equation.

So, that means, once we in general assemble this matrix, we get this complete matrix contribution from the all the elements in the total domain. Then, after that we solve it; after solution, we are getting the temperature distribution, velocity distribution each and every node point in a finite element based problem.

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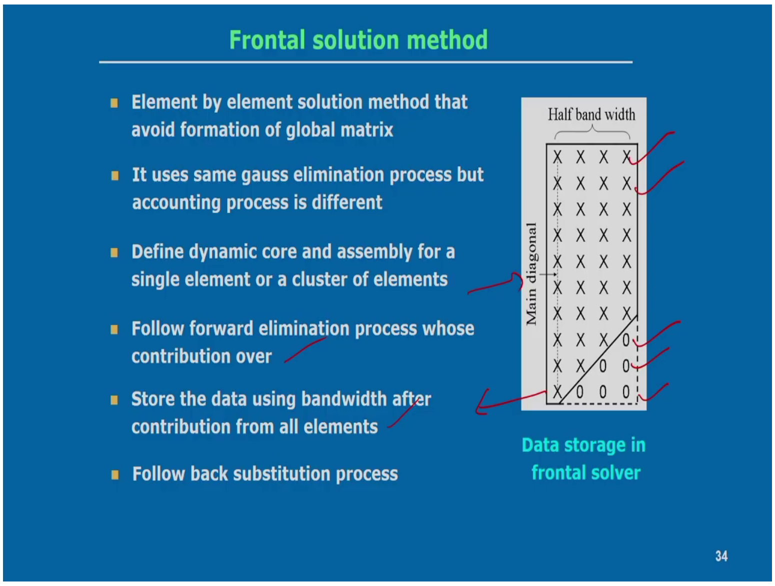

Frontal solver, but in particular to the frontal solver, it is a basically element by element solution because in finite element-based problem what we normally do that in general, once we assemble this matrix. So, we take the contribution from all the element and then, assemble; then, we will getting the global matrix form.

Then, there is a requirement of the heat space in the, if the if number of nodes or number of elements are very high in a particular finite element-based problem. So, that means, until and unless, you are not assemble all this thing, after assembling from contribution from the all the element, then you form the global matrix, then after that you are trying to solve it.

So, then it is the computational time is very much involved in case of finite element-based problem. But if you consider the Frontal solver method, it is also possible to develop some element-by-element solution method and of course, the frontal solver this is a very old concept.

We can use this, but till we can use it. And since element by element solution method, but then not necessary to form the avoid the formation of the complete global matrix so that we can avoid these things. Is uses the same kind of the Gauss elimination process? But accounting process is different this thing.

In this case, we can define the dynamic core and the assemble for the not a single element or cluster of elements. Then it can form the once with the within the cluster of elements or single element contribution from the each node is over, then we go for the solution.

Solution in the sense, we can not complete solution that we can follow the forward elimination process and once you follow the forward elimination process and in a particular file, we can keep accounting in the half bandwidth or full bandwidth form also. So, for example, in symmetric problem after forward elimination we keep on accounting the half bandwidth form, so, in this particular form.

So, then once for each and every cluster of the element, so, once the forward elimination process contribution over, then we store it in this particular form. Now, store the data using the bandwidth after contribution from the all the elements and then, once or contribution from the all the elements is over and this can be stored in a particular not in the dynamic in the variable rather we can say it can be stored in a file also. So, in that particular file, we store it.

Once the forward elimination over for a contribution from all the elements, then we can finally will be getting this kind of the expression this, in this form. Then, we follow the back substitution [FL] once you follow the back substitution, then we you start solving from the lower side, you get the solution of this equation first; then next equation, then next, like that will be getting the value of the all equation. And then, we represent the laser in this way.

So, this is the typical frontal solution method and in particular cases, it is possible to follow this frontal solution method to get the solution. Of course, nowadays in standard font this thing in a standard commercial software also there are so many iterative solver are also available. We can show some results also by following this kind of solution from the finite element based heat transfer fluid flow analysis.

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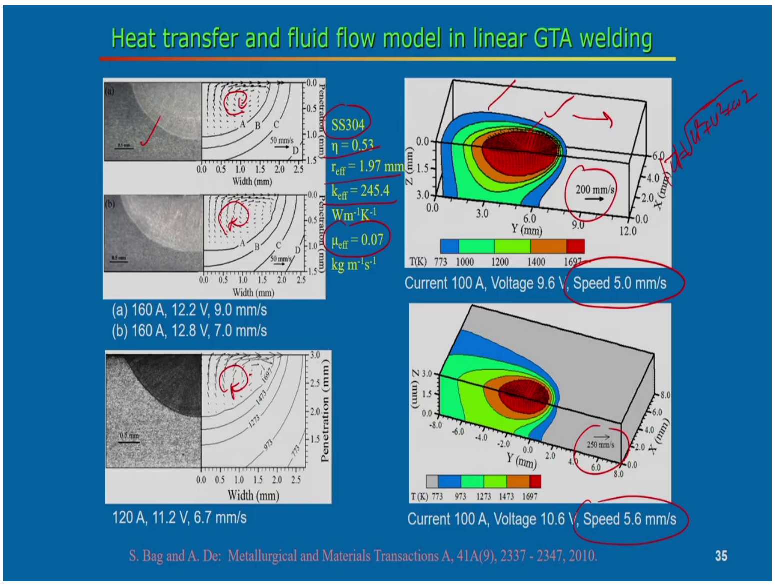

So, these cases we have done this analysis by developing our own (Refer Time: 44:02), we have not using in this particular case; no kind of we have not used any kind of the commercial software. We can first do the formulation and then develop the code also for heat transfer and fluid flow analysis.

So, we can show some results also by using this thing. For example, this is the material for SS304 stainless steel and with this value, different value of the efficiency, effective radius on the arc and the all other parameters. So, we use the effective conductivity of the material and effective viscosity of the material and then, we get this kind of the solution. For example, this is the domain.

And of course, we can see the since this is a quasi-steady state, I think this is a quasi-steady state problem; that means, the heat transfer problem that means, speed is 5 millimeter per second. That means, in this case the voltage heat source moves 5 millimeter per second. So, we follow the we assuming the quasi-steady state analysis.

Then, one step we have we are getting the solution of the temperature distribution as well as the velocity field that we can see also here. So, it this is the quasi-steady state problem. So, with all the parameters we will be able to predict the velocity field, if this indicates the black color inside the red zone is basically indicates the molten zone and within the red zone, we are indicating the arrow indicates the velocity field.

So, velocity vector: So, each and every arrow is basically the velocity vector means that it represents the what is the components u square plus v square plus w square. This is the magnitude of the velocity on particular node point. So, this way, we can get the output this thing the temperature distribution as well as the velocity field also. Same thing we have done in the different velocity.

We can see the maximum magnitude of the velocity is at 200 millimeter per second. Here also, 250 millimeter per second. But here, in this case, the speed are different or parameters are different also in this case. So, therefore, we can see also that the here it is the particular section, we have considered and then, we compared with the weld with an penetration, the macro graph, the actually experimental data also.

So, here the if you see velocity pattern is follow in this way. Here also you can see the velocity pattern in this way; that means, in this case we have not used any kind of the surface active elements.

So, then we are getting the clockwise direction, velocity field is clockwise direction with respect to this is the center point between this thing. So, this way, this is the way to get the velocity field and we can compare the results also with respect to the experimental data in case of the fluid flow problem.

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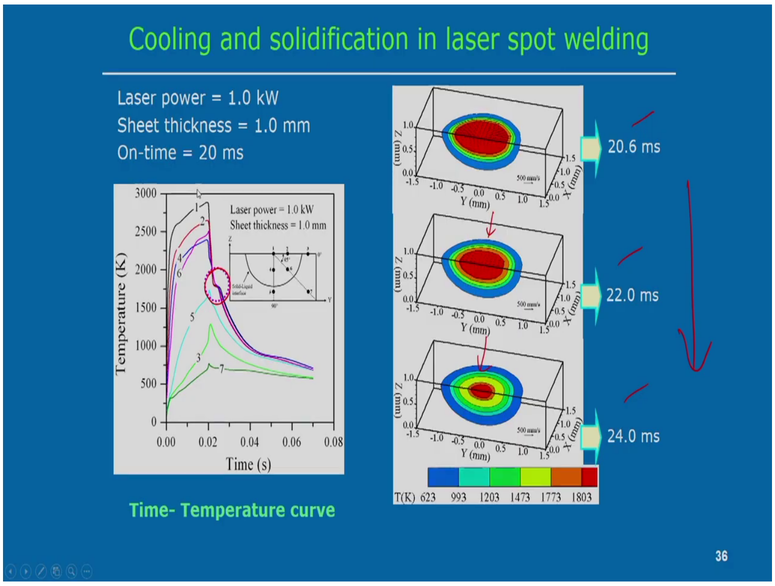

So, apart from this thing this, temperature distribution is also useful because other way also this lots of information can be get that means, cooling and the solidification behavior in case of the laser spot welding process can be always predict. See this temperature distribution, we are getting the particular position.

That means, different zone in a welded structure and this simply, we can extracting the time temperature curve in case of, but in these cases, the this time temperature curve, we are getting in case of the spot welding process. So, this is one example, the spot welding process also; that means it is a stationary heat source.

In this case also, we can predict the velocity field that; but it is a transient problem. And that and it is particular position then, heat is we have applying the heat flux, then if we see this is the at 20.6 millisecond, this is the temperature and velocity field also 22 millisecond and 24 millisecond getting this thing.

That means, after the up to one time is 20 millisecond means that laser on time up to 20 millisecond then we stop the heat source. Then, once you stop the heat source, they start cooling the particular zone. Then once we can capture the different position, different time, what is the velocity and temperature field. It is gradually the shrinking, weld pool is sinking because cooling phase is following this particular time different time steps.

So, this way, you can get the this field also; but at the same time, you can see that also that time temperature profile. Here, you can see there is a some there is a change of the slope in the during the cooling phase. So, initially, it was the heating phase, there is a temperature increment; heating phase reaching the peak temperature and then, gradually you are following the cooling step also.

During the cooling step, we can say there is a change of the slope. This indicates there is a phase change happens in this particular at this particular point. So, that is why phase change means the liquid phase to solid phase. So, since the during the phase change is associated some amount of the we are considering the latent heat of latent heat of phase transformation, that is why there is a delayed in this risk in the there is a not smooth continuous change in the temperature.

There is a delay and some kind of the slope or bump, we can observe. That indicates that we in this particular simulation, we are considering the effect of the latent heat in this particular problem.

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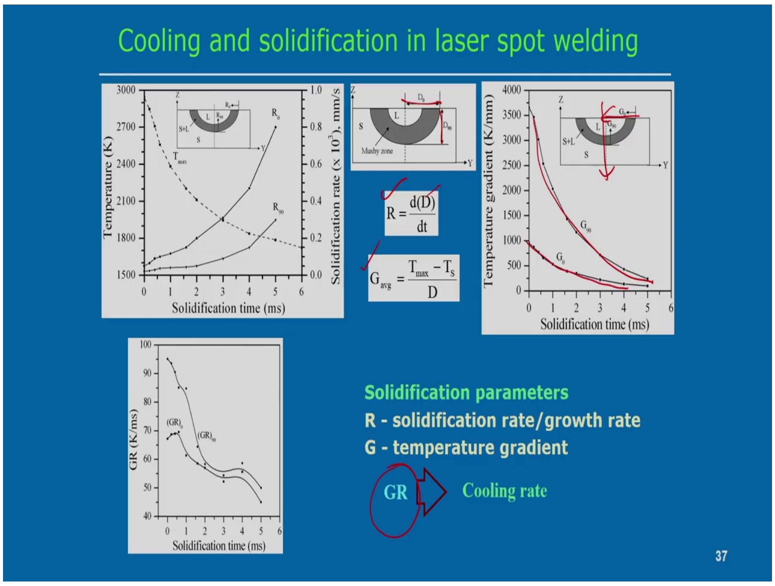

Now, once we estimate this time temperature profile, even the we capture the different times temperature profile, it is possible to estimate the, what is the solidification parameters. Then, R and D are normally called the R and G solidification parameter. So, R is the represent the solidification rate or growth rate and G is the temperature gradient; these are the two important parameter in associated to predict the solidification behavior of a weld pool also.

So, therefore, point is that once you develop the model, from that model, we can extract the data in the form of a time temperature profile. Even from the time temperature profile, we can extract the, this kind of the phenomena, what way there is a change of the R and what way there is a change of the G also. Because finally, G and R is to be G into R is representing the cooling rate and G by R, this we are this is the variation the G see in the particular direction, this is the variation of the G also is 0.

So, G 0 means in this particular direction, what is the value of the G and in this particular direction, what is the value of the G. So, these two cases we can see the temperature gradient, different nature depending along the this direction one on the width direction and along the depth direction, the temperature gradient also different, G R also different. At the same time, R is a solidification rate also different and the, because it indicates it represents d D by dt.

So, D is the basically diameter of the weld pool. So, D 0 is the this direction so, how D changing with respect to time that indicates the solidification rate and growth rate. So, therefore, this G and R parameters is defines the mode of the solidification or the cooling rate decides the that structure of the solidified structure; (Refer Time: 50:22) the fine structure or core structure.

So, that kind of information will be getting from this thing and we will be able to predict the, we can link the micro structural phenomena or solidification behavior in case of the welding problem.

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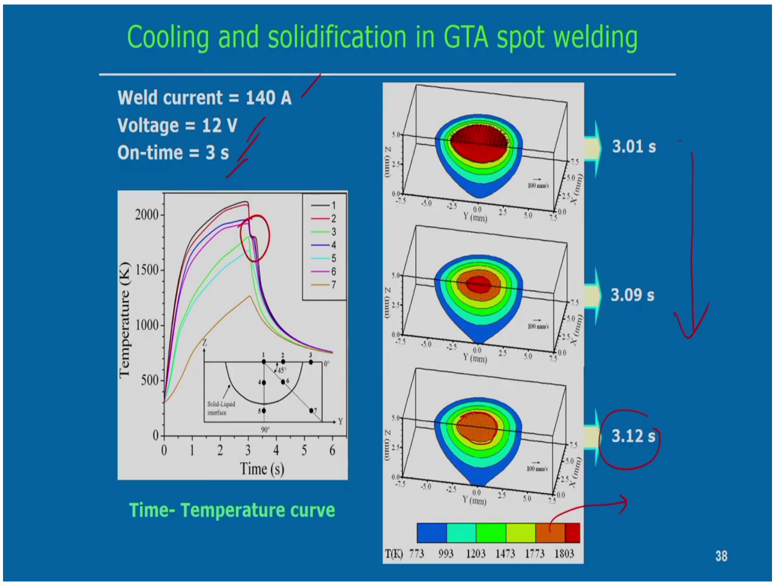

So, here also we can see that the cooling of the solidification in GTA spot welding also. In this case, we can see that welding current this one this three different cases and we can see that in the GTA spot welding, so this is the stop the applying the flux basically arc at the 3 second. So, on-time 3 second, then we stop it, then we are following the how it is cooling.

So, then we can see this is the, this indicates the last point that 3.12 second is a mushy zone. Basically, this indicates that this zone is very high the between the solid this thing the solidus temperature and liquidus temperature. So, actually this mushy zone is very much response a very much useful to understand the kind of the solidification solidified structure.

That means, whether it is very kind of equiaxial kind those structure or other kind of the structure is possible to get, that information we can get from this thing mushy zone. So, that we are getting simply predict the simple heat transfer or fluid flow analysis, that is the output from this heat transfer model.

So, same kind of temperature change we can observe even at the between the solidus and liquidus temperature because or maybe we can say temperature at this, where the phase transformation happens. So, same kind of the prediction is possible from this curve also.

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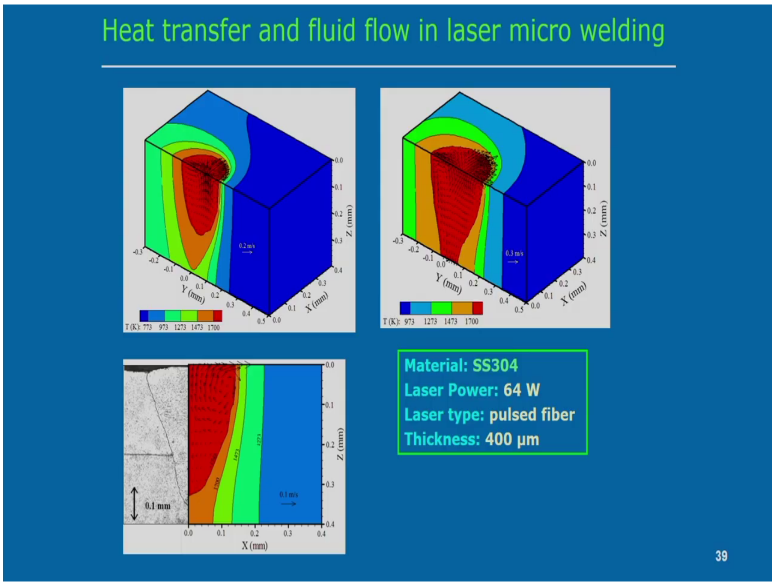

Even we can see even heat transfer and fluid flow in the laser micro welding process also; here, we can see that the different kind of the temperature distribution or velocity field also able to predict. In this case the material is SS304 stainless steel; laser power 64 Watt and the laser type is the pulsed fiber; laser thickness equal to 400 micrometer.

So, there also we can predict the temperature and the fluid flow phenomena also even in the laser welding process. So, there you can see that high depth of penetration is possible to predict, if we consider the metal flow also within this structure itself.

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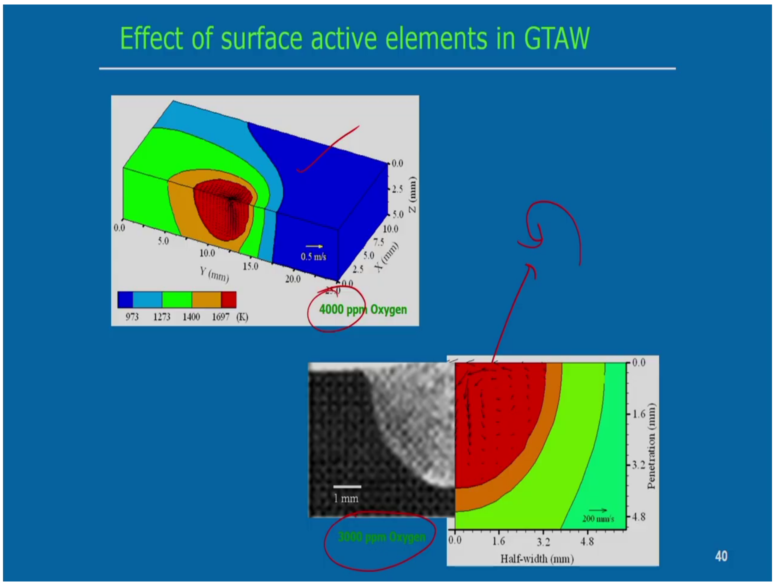

Now, here you can see the effect of the surface active elements in GTAW process that we started with the flow analysis in case of presence of the surface active element. So, in this case, we can do the similar kind of the analysis and we can see that in case of the it is a GTAW process and this material is having 4000 ppm oxygen and another case is 3000 ppm oxygen available there.

So, you can see the material flow pattern is the opposite direction. This z zone is basically indicates the z zone indicates the fusion zone and the material flow pattern is the opposite direction; that means, this indicates the material flow pattern because in these cases, we have considered the effect of the surface active elements.

Then, we can predict the material flow behavior and in this case, this kind of prediction is only possible if we consider the fluid flow phenomena; otherwise, it is if we consider only the conduction based model, then it is very difficult to predict this kind of the phenomena. That means effect of the surface active elements. So, it means that even the simulation will can help to predict the material flow behavior as well as the in presence of the surface active element.

So, in this case, the oxygen can be act as a surface active elements; but 4000 ppm oxygen, this oxygen normally mixed with the shielding gas and then, that act as a surface active element, then it completely change the weld pool behavior; simply presence of the surface active element. So, this kind of the phenomena, to explain this kind of the phenomena simply the effect of the surface active elements, it is necessary to consider the heat transfer and fluid flow model in case of the welding process.

So, that is all today. Thank you very much for your kind attention. Next class, we will discuss the remaining parts of this module. Basically, the we will discuss some sort of discussion of the interface tracking method or may be free surface modeling associated with the welding process.

So, thank you very much.