

**Finite Element modeling of Welding processes**  
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**Lecture - 23**  
**Solid state welding: Friction, FSW and hybrid FSW**

Hello, everybody. Now, we will discuss about the Finite Element Modeling of the Solid state welding process. Solid state welding process normally this friction welding process, friction stir welding process which is extended part of the friction welding process and hybrid FSW process; that means, hybrid friction stir welding process.

In principle the friction and friction stir welding process anyway the frictional heat generation is there. Apart from in FSW Friction Stir Welding process apart from a some frictional heat generation there are some mechanical string action is also involvement in a FSW process. So, we will try to look into how we can do in general the finite element modeling of this particular processes.

So, in this case we will try to discuss only on the FSW process and hybrid FSW process. Because the heat generation term in FSW process is already included the frictional heat generation. So, I am not explicitly describing the modeling approach for a friction welding process. So, anyway that FSW process is may be sufficient to understand the finite element modeling approaches or model development that includes the friction welding process.

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**Governing equations**

This requires the solution of unsteady state heat conduction equation:

$$\frac{\partial}{\partial x} \left( k_x \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k_y \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( k_z \frac{\partial T}{\partial z} \right) + \dot{Q} = \rho c_p \frac{\partial T}{\partial t}$$

T is the temperature and is a function of spatial ( $\xi, y, z$ ) and time ( $t$ ) coordinates. Considering the moving coordinate system ( $\xi = x - Vt$ ), the governing equation can be rewritten as

$$\frac{\partial}{\partial \xi} \left( k_x \frac{\partial T}{\partial \xi} \right) + \frac{\partial}{\partial y} \left( k_y \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( k_z \frac{\partial T}{\partial z} \right) + \dot{Q} = \rho c_p v \frac{\partial T}{\partial \xi}$$

**Initial condition and boundary condition**

<p><b>i. Initial condition</b> <math>T(x, y, z, 0) = T_i</math></p> <p><b>ii. The convection boundary conditions</b> <math>k \frac{\partial T}{\partial n} = h(T - T_\infty)</math></p> <p><b>iii. The radiation boundary conditions</b> <math>k \frac{\partial T}{\partial n} = \epsilon \sigma (T_s^4 - T_\infty^4)</math></p>	<p><b>iv. Symmetric boundary conditions</b> <math>T_{j1} = T_{j2}</math></p> <p><b>v. Tool shoulder/work-piece interface</b> <math>k \frac{\partial T}{\partial n} = q_c</math></p> <p><b>vi. Tool pin/work-piece interface</b> <math>k \frac{\partial T}{\partial n} = q_p</math></p>
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Definitely once we look into that any kind of the modeling heat transfer analysis or maybe stress analysis part. In that cases we need to know what are the governing equation. And accordingly what kind of constitute relation between stress strain, displacement strain all these kind of in relation we are using that should have some idea about all these particular processes.

So, governing equation, if you look into this unsteady state heat conduction equation that we have already familiar with this particular equation here we use the simple heat conduction equation heat generation term is there I think Q dot heat generation term is there rho c p del T, this is a tangent part.

Now, T is the temperature and is a function of the spatial and time coordinates. Basically temperature actually varies with respect to at the space with respect to time also. And,

considering the moving coordinate system it means that this is a tangent equation and from tangent equation to moving.

Because friction stir welding process there is a along with the frictional heat generation, the tool moves one particular direction. So, to look account the movement of the tool, then we can convert into the moving coordinate system. That moving coordination by using relation  $x_i$  equal to  $x$  minus  $Vt$  assuming that the tool is moving along  $x$ -axis with the velocity  $V$ .

So, once we use this relation we can convert into the quasi steady state formulation. So, I can think heat generation and if you look into this particular equation the all terms has been described in terms of this in spatial coordinate system. So, there is no time component involved in this particular equation.

So, once you look into from tangent to some a quasi steady state analysis. This simply converting to the moving coordinate system, then we try to look into different boundary conditions. So, that means, initial condition is required actually specifically if we solve the tangent problem, but or maybe a initial condition can be in quasi steady state analysis.

But, in that cases assuming that at the very initial time  $t$  equal to 0, all particular temperature for the sample that way you can consider this initial condition. Then heat loss from the boundary also simple convective heat loss and radiative heat loss from the boundary.

Then symmetric boundary condition that gradient equal to 0 it means that temperature at the symmetric surface  $T_{i,j}$  equal to 1 equal to  $T_{i,z}$ . And if we make the gradient temperature gradient  $dT$  by  $dx$  that should be 0 at the symmetric boundary condition.

And, tool shoulder workpiece interface there may be we can incorporate some sort of heat flux there. And tool pin or the workpiece interface also you can introduce some sort of the heat flux. So, these heat flux can be incorporated tool shoulder workpiece interface and tool pin and the workpiece interface. Some sort of heat flux can be in it. That can be considered as a heat generation or a heat input to the domain.

So, therefore, along with this boundary interaction and by solving the governing equation we will be able to find out what is the temperature distribution for this particular domain.

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**Heat flux calculation**

There are two processes due to which heat is generated at the interface of tool and work piece due to friction  $Q_f$  and plastic deformation  $Q_p$ .

During the FSW, heat is generated close to the contact surfaces, which can have complex geometries according to the tool geometry.

$$Q_{tot} = Q_f + Q_p$$

- Heat generation from the shoulder
 
$$Q_1 = \frac{2}{3} \pi \sigma_{constr} (R_{shoulder}^3 - R_{probe}^3) (1 + \tan \alpha)$$
- Heat generation from the probe
 
$$Q_2 = \int_0^{2\pi} \int_0^{R_{probe}} \sigma_{constr} R_{probe}^2 d\theta dr = 2\pi \sigma_{constr} R_{probe}^2 H_{probe}$$

$$Q_3 = \int_0^{2\pi} \int_0^{R_{probe}} \sigma_{constr} r^2 d\theta dr = \frac{2}{3} \pi \sigma_{constr} R_{probe}^3$$

The three contributions are combined to get the total heat generation estimate:

$$Q_{tot} = \frac{2}{3} \pi \sigma_{constr} ((R_{shoulder}^3 - R_{probe}^3) (1 + \tan \alpha) + R_{probe}^3 + 3R_{probe}^2 H_{probe})$$

In the case of a flat shoulder, the heat generation expression simplifies to:

$$Q_{tot} = \frac{2}{3} \pi \sigma_{constr} (R_{shoulder}^3 + 3R_{probe}^2 H_{probe})$$

So, let us look in the how what way we can do the heat flux calculation in FSW process in. So, this case there are two process due to which the heat is generated at the interface, one is the interface means if you see this is the tool and then at this one heat generation will be at this a interface between the shoulder and the workpiece.

And, there is interaction of the probe in a FSW tool that probe and the workpiece material. So, there will be some heat generation will be there. So, these two surfaces heat generation will be there and that we can estimate what is the heat flux on the surfaces or you can give the input to this thing.

Now, one is the we assuming the frictional heat and the plastic deformation  $Q_p$ . So, both heat generation will be there. So, due to the frictional heat generation due to the heat generation due to the friction and other thing since is the plasticising the material. So, some sort of mechanical deformation is associated in this particular domain at the interface. So, in that cases due to the plastic deformation some amount of the heat will be generated at that domain.

Now, therefore, heat is generated close to the contact surface definitely between the either interaction at the contact surface between the tool shoulder and the workpiece. Other may be the between in the tool probe and the workpiece.

So, then according to tool geometric  $Q_{total}$  can be these two different components  $Q_1$ ,  $Q_2$ ,  $Q_3$ .  $Q_1$  equal to heat generation from the shoulder assuming that there is a heat generation from the shoulder  $Q_1$ . Heat generation from the probe; so, heat generation from the probe  $Q_2$ ; the surface and the heat generation from the probe at the from the bottom surface also.

Here it be interface this thing. So, here we can assume the heat generation at calculated as  $Q_3$  because that surface is in contact with the workpiece  $v$ . So, therefore, some sort of heat generation we can consider. So, this three now we can estimate by simply the heat generation from the shoulder heat generation from the probe that we have already shown in a heat source modeling approaches also there what we can estimate the total heat generation.

So,  $Q_1$  total heat generation at the this the interface it is basically twice  $\pi \omega r$ . This is the rotational speed  $\tau_{contact}$  the may be what way we can define the shear stress of the contact surface that is called the  $\tau_{contact}$ . And, this is the shoulder and  $1 + \tan \alpha$ ;  $\tan \alpha$  included if there is some angles. If we consider in the tool a surface that it may be not exactly the cylindrical tool in some sort the conical tool is there. And then this term will be counting, but in these cases we can neglect for cylindrical tool.

So, this term is basically 1 or that set  $\alpha$  is equal to 0. So, then it becomes 1. So, we neglect this term so, because this is equivalent to 1. Now, this is the heat generation from the

shoulder surface. We can estimate and this you can see that  $R$  shoulder and probe radius and this is the indicates the probe radius. So, depends on this because only the shoulder workpiece interaction is basically between these two.

So, such that this is the shoulder radius and this is the probe radius. So, this particular zone is basically interacting. So, that is way this  $R$  shoulder cube minus  $R$  probe cube that terms comes into this picture. Now, heat generation from the probe also we can find out the height of the probe equal to  $H$  probe and radius radial distance equal to  $R$  probe or pin radius.

Then we can estimate the this thing total heat generation and  $Q_3$  similarly at the bottom surface this  $\tau$  contact all these things. What three contribution from the three different  $Q_1$ ,  $Q_2$ ,  $Q_3$  this is heat generation at the different contact interfaces. So, therefore, this do not consider this term this is comes into this equal to assume equal to 1.

Now, three combining these two we will be having total heat generation can estimate this thing. Now, in case of the flux shoulder of the heat generation simplifies to do these things. So, basically we can this is equal to 1 the 1 plus  $\tan \alpha$  the total heat generation is this way we can estimate in case of the flat shoulder the heat generation is this amount.

So, that we have already shown the heat generation ok. So, this is the total heat generation for a expression in case of the FSW process.

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### Heat Source Model

The distribution of heat flux over the plate surface due to the tool shoulder was given as:

$$q_{\text{shoulder}} = \frac{3Q_1 r}{2\pi R_{\text{shoulder}}^3}$$

The distribution of heat flux over the pin-plate interface due to tool pin side surface was given as:

$$q_{\text{side}} = \frac{Q}{2\pi R_{\text{probe}} H_{\text{probe}}}$$

The distribution of heat flux over the pin-plate interface due to tool pin bottom surface was given as:

$$q_{\text{bottom}} = \frac{Q}{\pi R_{\text{probe}}^2}$$

Now, once we estimate the heat generation that is a total heat generation over the surface, but maybe we do not have the actual implement in this model you have to incorporate the in the form of a heat flux on the surface. Here you can estimate what is the total heat generation now on this particular surface or at the contact surface.

Now, what is the distribution of this particular flux? We can assume some sort of distribution then you represents the heat flux on the surface. For example, distribution of the heat flux over the plane plate surface due to the tool shoulder is giving this one. We can see that  $3 Q_1 r$ , but twice  $\pi R_{\text{shoulder}}^3$ .

We can assuming this the on this tool shoulder we can assuming this kind of distribution. So, this is the intensity of heat generation is there, total heat generation is there, then a total heat generation is for example at the surface  $Q_1$ . Then in this case this  $Q_1$  is there, but if we

assume this is the distribution that the center it may be the intensity is 0 and gradually increasing its maximum here.

Then this distribution can be represented like these three we estimated this is  $Q_1 r$ ,  $r$  is the radial distance from this point. And twice  $\pi R$  shoulder cube into this  $\pi R$  shoulder cube. I think  $d$  will not be there and only  $r$  will be there ok. So, it means that we can convert from the total heat generation to the in terms of the heat flux by assuming some sort of the distribution of following distribution.

Similarly, heat flux distribution over the pin plate interface due to the tool pin side was given as this one twice  $\pi R$  into  $H$  probe  $Q_2$ . So, we can see also this  $Q_2$  here also  $Q_z$  we can assume some sort of the this kind of distribution or even it is possible you can assume some sort of the uniform distribution. So, when you are assuming the uniform distribution that can be converted in the that in terms of the flux can be converted.

So, that is why once we do different kind of the distribution in this cases  $Q$  probe can be like that this way and this twice  $\pi R$  probe into  $H$ . So, that any kind of distribution we can follow and then accordingly we can represent the heat flux. Similarly, at the bottom surface the plain pin plate. So, at this surface when the interaction we can estimate the what is the  $Q_3$  is there, but we can estimate what is the assuming some sort of uniform flux then  $Q_3$  divided by  $\pi R$  square; so, the radial.

So, that is way so, in actual modeling approach once we calculate the total heat generation on the different surfaces contact surfaces, then it is necessary to convert in terms of the flux. Then that flux can be considered as a input to the model then this model because until and unless you will not be able to convert it to in the form of a in the you cannot estimate the total heat generation.

But, until you are not converting in the form of a flux, then you will not be able to implement in the as a boundary flux may be through the boundary condition. Because if you boundary condition you can see that in the boundary condition there is a option to look into that



basically the implementing the flux over the surface. So, that is why this way we can estimate the flux and we can give the input to the model.

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Example

Friction stir welding (FSW) of 5 mm plate made of 7020-T53 aluminum alloy at 1400 rpm and 40 mm/min.

- All the thermal properties of AA 7020 T53 were considered as a function of temperature.
- 95% of the heat was transferred to the workpiece.
- The heat input is linearly proportional to the distance from the center of the tool.
- The plunging force applied to the plate surface by the tool creates a uniform pressure over the shoulder surface.
- The heat is generated from the workdone by the friction force only.

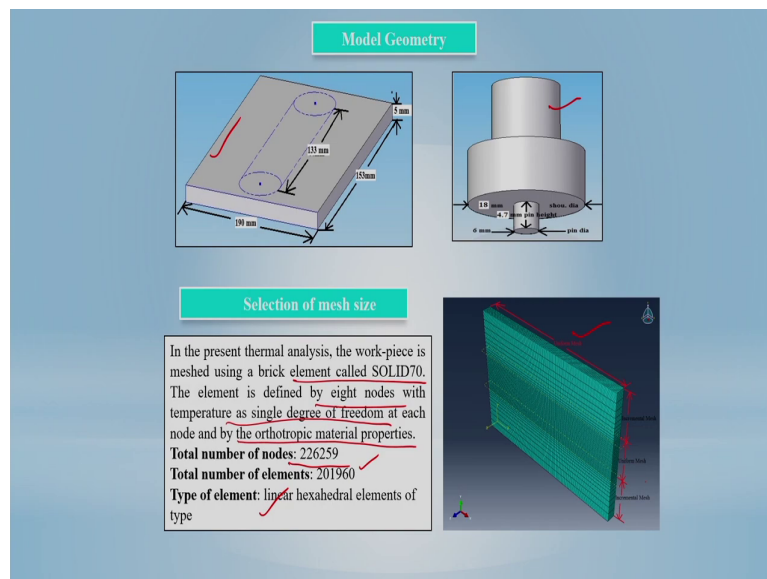
You can take an example to see the result how it works basically. So, friction stir process 5 millimetre plate and 7 in some aluminum alloy consider and the total at 1400 rpm and 40 millimeter per minute.

So, all the thermal properties were considered as a function of temperature. So, in this particular simulation it is necessary to consider the temperature dependent properties. 95 percent of the heat was transferred to the workpiece basically what are the were calculating we can assume that total of 90. So, efficiency may be consider that 0.95 such that 95 percent of the heat or were transferred to the workpiece.

The heat input is linearly proportional to the distance from the center of the tool. So, that means, it is varying linearly from the center to this. We are assuming this in the front distribution. Plunging force applied to the plate surface by the tool creates uniform pressure over the shoulder surface.

So, P times is also there. So, we can assuming that that it creates some sort of the plunging force plate surface to create some sort of the basically uniform pressure that we can assume. So, that accordingly we can decide the distribution of the heat flux can be uniform also. The heat is generated from the workpiece by the friction force only. So, heat generation on the workpiece is considering only on the frictional force.

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Therefore, you can create some sort of suppose this is a model geometry at sample geometry we can create. Then this is the tool dimension if we defined, this is the shoulder on the

shoulder. And this is the tool pin and accordingly the analysis the work piece is meshed using the brick element.

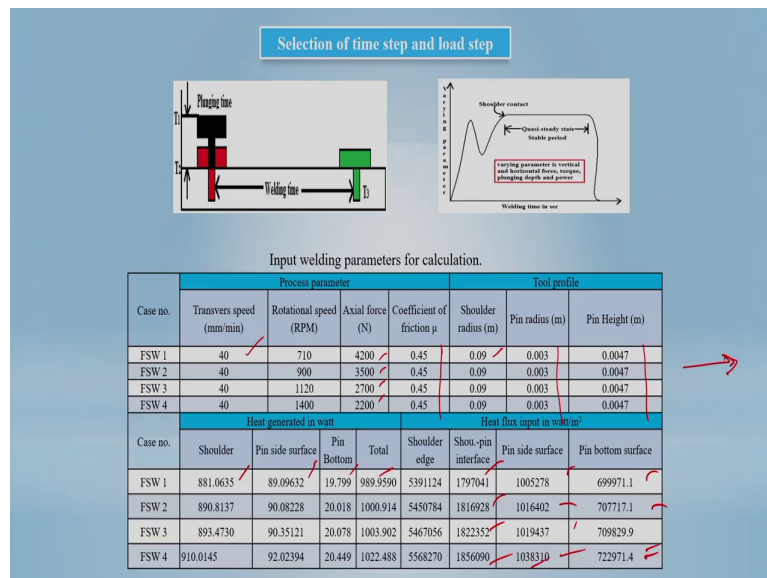
And, then eight noded brick element, temperature is always single degrees of freedom; that means, each and every node doing only for the temperature analysis and we can see by at each node. And by other a orthotropic material properties we can consider all the material properties, but in these cases there is a need to consider only the thermal properties for the thermal analysis.

So, for typical example this is the domain we discretized domain. We can expect this as much as 226,000 total number of nodes and around 201,960 is the number of elements. So, this type of elements linear hexahedral elements of kind of kind of elements we can choose.

So, it means that we can see that in domain and we can see that if you discretize and this is proper way then that as much as possible around 2 lakhs of nodes, elements in the that order is required to handle this thing. It means that, when you try to solve the linear system of the equation the if you follow simple methodology; that means, the there are almost 2 lakhs of equations has to be solved the linear system of the equation.

So, dimension of the matrix may be in general if you assume the square matrix then 2 lakhs by 2 lakhs that matrix has to be solved to get the solution. So, then you can realize the how difficulty to gets or maybe we can understand that what is the computational time required to get particular solution. Then that is why it is necessary to choose some sort of the efficient solver in case of this finite element modeling of any particular problem.

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So, some sort of results we can see also welding parameters see that four different cases at the transfer speed the 40 millimeter per minute; that means, how what is the movement of the tool speed of the movement of the tool. Rotational speed rpm, tool rotational speed the different rpm 722 1400 rpm; axial force you can measure also that axial force is required during this process also.

This there is a variation 4.2 Newton 4.2 kilo Newton to 2.2 kilo Newton. Coefficients friction considered as the same constant 0.45; shoulder radius 0.09 meter; pin radius 0.003 meter pin height. So, all the same pin still same tool actually used, but the different process parameters.

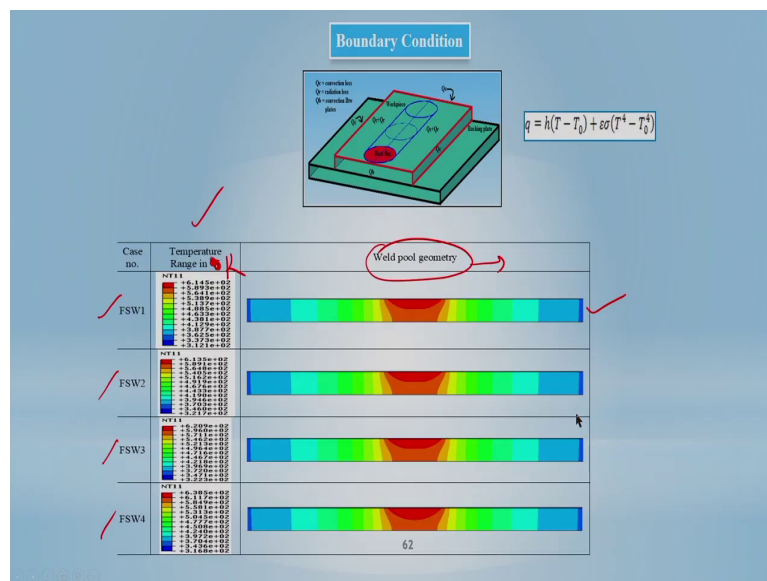
Now, heat generated if we estimate the heat generated and the difference surfaces that we have already discussed what this total amount of the heat generation, we can see the at the

shoulder pin side surface and the pin bottom surface. You can see this is the total heat generation 989 Watt may be.

And then heat flux we can give the we can once we convert it to assuming some sort of distribution. We converted to the in the form of a heat flux then we can get some all these calculation we can do also. And, this can be give input to the analysis to the any kind of the model such that we will be able to get the solution.

So, this is a sample calculation we can do from this total heat generation to how to converted the heat flux. And that this these heat flux can be given to the input to the commercial software.

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So, once you do that boundary with proper boundary condition and solving the governing equation will be able to get the temperature distribution this thing. So, it is a weld pool geometry it is not weld pool actually it is a kind of we can say the nugget zone may be you can see in FSW process.

And, you will get the output that in the form of a temperature distribution something like take the cross section at the different parameter condition process parameter FSW 1, 2, 3, 4 with output.

So, this the you can see the distribution of the temperature the red zone. They signifies the different range of the temperature that is we can see in the bar also it said to be in terms of the Kelvin. So, this way we can solve the problem and in FSW process the same governing equation, but boundary condition is different in this different way you can put the boundary condition and then we can get the temperature distribution as output from this model.

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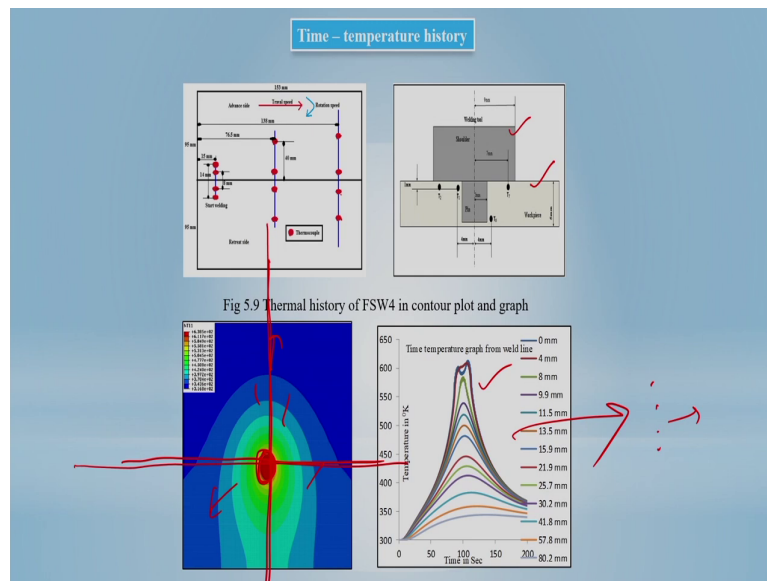


Fig 5.9 Thermal history of FSW4 in contour plot and graph

Similar time temperature history also, even we can extract the time temperature history also. For example, once you do the analysis. So, this is the interaction of the tool and the workpiece material then we can define some characteristic points. So, on this characteristic point say this is that characteristics points normally in practically we put the thermocouple on this particular point.

So, that in during experiment actual experiment at the thermocouple point we will be able to capture the temperature distribution, but when you do the numerical simulation the same point we try to extract the time versus temperature history. If you get the time versus temperature history then the different the position we will be getting the this kind of profile.

So, the at a distance temperature profile from weld line 0, 4, 8 millimeter all these information. Now, once we get the simulation the temperature distribution and then same

thermocouple point we can get the experimental data also and from the experimental data you will be getting actually time temperature profile. And, now compare this experimental data with the numerical simulation to understand how correct your model is.

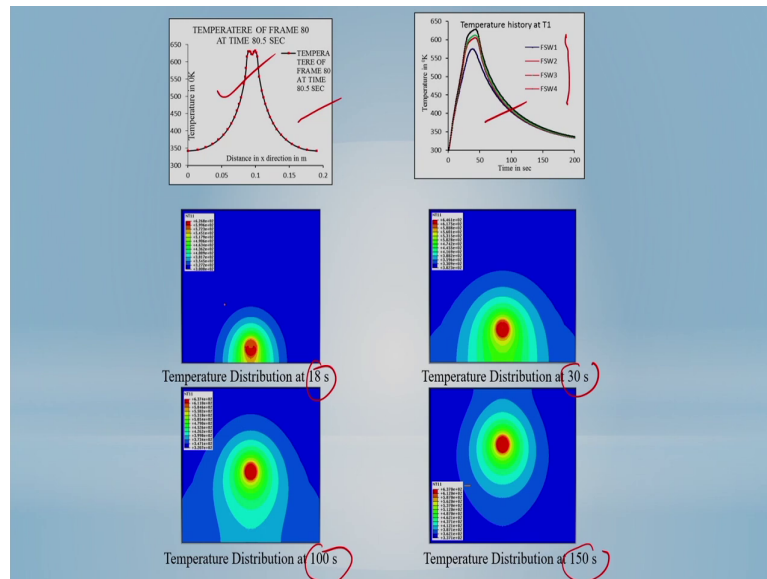
Now, these are the typical output see on the top surface if you see during this process FSW process. We can see the heat generation is the confined with the small zone and be near about the dimension of the tool pin. And, remaining heat affected zone can be like that also depending upon the process parameter and material properties we are using.

And, since if that you can see it is a kind of moving one particular direction so, some sort of a non-symmetric temperature distribution with respect to this plane will be able to get this thing. But, with respect to this it is also it respect to this plane also some very small degree of non-symmetric temperature profile we can expect because since it is moving on one particular direction.

So, the advancing side and retreating side relative velocity and advancing side and retreating side are different. Because of that the temperature distributions should get some sort of non-symmetric profile with respect to this particular axis. And, we are getting that non-symmetric temperature that is obvious with the respect to this plane because it is moving one particular direction.



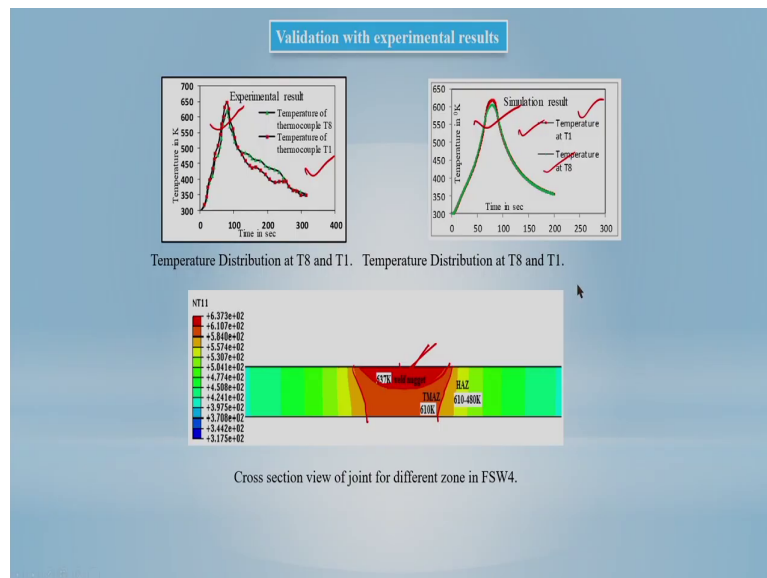
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So, these are the time temperature history also. Even we can see the time temperature at 80.5 second something like that temperature history are all these cases. And this is the distance in x temperature of a frame 8 at 80.5 second. This is the experimental measurement kind of this thing. So, you can compare experimental to the numerical data also.

And, we can get the how the temperature development it is basically starting to the 18 second this is the position, then 30 second it is moves tool from this to this position, then 100 second near about and 150 second this thing. So, that is way numerical simulation it is always possible to capture the temperature distribution at the different time step at different position also.

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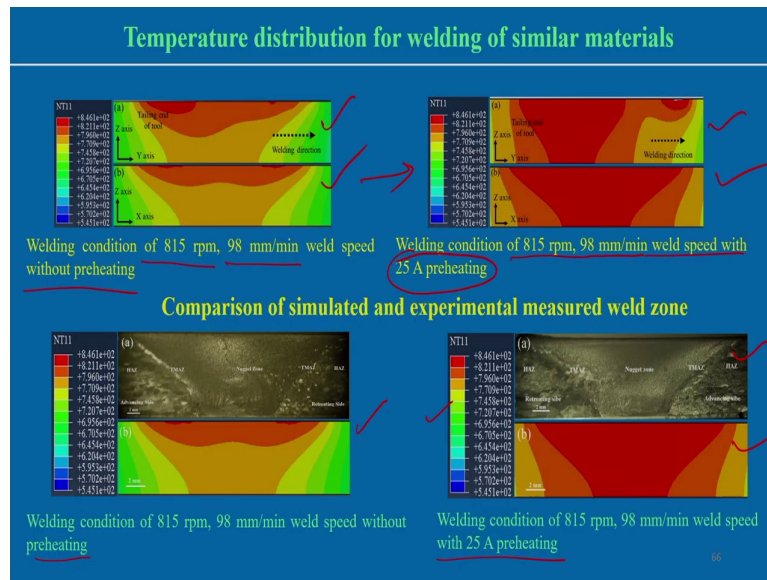
Validation with experimental results: so, here see temperature at T 1 and temperature at T 8 and temperature of thermocouple T 8 temperature of thermocouple T 1. See these are the thermocouple measure term; that means, experimentally measure temperature and this is a simulated result.

So, then we can compare the simulated result means from the model and from the experimental that we can compare. Then we will be able to get the whether this matching with the experimental data or not and finally, cross-sectional view of the joint at the for different zone in FSW4.

We can see that we will nugget zone this is that this isotherm define the well nugget zone. Then thermo mechanically affected zone which is defined characterized by particular isotherm particular temperature and the heat affected zone with a particular range of the

temperature. So, these are the typical output from the finite element base model of FSW process, but this analysis only confined to the thermal analysis.

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Some more results we can see the temperature distribution for welding of the similar materials. Also we can see the welding experiments for 815 rpm this is the weld speed 98 millimeter per minute and without preheating without preheating means we are not heating the sample before run of the experiment. So, we are getting some sort of temperature profile.

Now, if we see that welding condition the same condition 815, 98 this thing weld speed with the same and weld speed 98 millimeter per minute also, but 25 ampere preheating. This is the now there is a change in the temperature profile. Preheating means for example, actually you are starting the welding process FSW welding process are starting at the room temperature may be this thing initial temperature at the room temperature.

Now, before start of the actual welding process FSW process so, some we can apply some sort of the creation of the micro plasma arc to preheat the sample may be with the 25 ampere current to preheat the sample. That means, some sort of temperature rays already there then we starts actually FSW process. So, in that cases the it will be more easier to plasticize the material.

So, therefore, definitely you can expect the distribution of the temperature should be different as compared to the without any kind of the preheating. And that is we can see through the numerical simulation also that there is a some difference in temperature distribution also. Now, we can compare this that welding condition, speed without pre-heating and welding condition.

And, with a particular pre-heating and we can see that with the particular with the when it follows a pre heating may be in this particular material it is more easy to compare between the tool between the material and the simulated result. This way we can find out this material and the simulated result more close with this thing.

It means that from the simulation we can take a that kind of information that it is sometimes there may be need to achieve certain temperature. So, there is a need to some sort of the preheating because simulation we can perform some sort of initial heating initial temperature, some sort of preheating we can model it.

We will show that how preheating can be modeled in this particular case. Then this information can be more easily to get from the numerical model data or may be simulation result. And then accordingly we can design our experiment to cater the different amount of the preheating required for to weld very good weld in case of the FSW process.

But, in these cases the information is restricted to only for the temperature distribution or you can say only for the thermal analysis.

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### Analytical model of strain and strain rate in weld zone

(a) shoulder and pin interface with workpiece and (b) Velocity presentation in r and  $\theta$  direction (top view).

$$\dot{\epsilon}_{rr} = \frac{\partial u_r}{\partial r}$$

$$\dot{\epsilon}_{\theta\theta} = \frac{1}{r} \frac{\partial u_\theta}{\partial \theta} - \frac{\partial u_r}{\partial r}$$

$$\dot{\epsilon}_{zz} = \frac{\partial u_z}{\partial z}$$

$$\dot{\epsilon}_{r\theta} = \frac{1}{2} \left[ \frac{1}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r} \right]$$

$$\dot{\epsilon}_{\theta z} = \frac{1}{2} \left[ \frac{\partial u_\theta}{\partial z} + \frac{1}{r} \frac{\partial u_z}{\partial \theta} \right]$$

$$\dot{\epsilon}_{zr} = \frac{1}{2} \left[ \frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z} \right]$$

$$\dot{\epsilon}_{eff} = \left( \frac{2}{3} \dot{\epsilon}_{ij}^2 \right)^{\frac{1}{2}}$$

The strain values are obtained from the strain-rate using rotational speed (N) as,

$$\dot{\epsilon}_{eff} = \frac{\epsilon_{eff}}{N}$$

$\dot{\epsilon} = \frac{d\epsilon}{dt}$

CDRX phenomenon modeling takes into account with few material constants to predict average grain size in weld zone

$$D_{CDRX} = C_1 \epsilon^k \dot{\epsilon}^l \exp\left(-\frac{Q}{RT}\right)$$

Now, it to some extent it is possible to develop in FSW process to look into the strain and strain rate this thing and to explain the recrystallization phenomena associated with FSW process also. See this cases we can see this is the tool material and the workpiece material and tool means the shoulder is interacting on the surface.

And, this is the pin oh this is the zone domain may be this particular affected join. That zone we can consider as a continuous dynamic recrystallization. Assuming the recrystallization mechanism in case of the aluminum alloy is the CDRX continuous dynamic recrystallization exists in case of the weld friction stir welding of the aluminum alloy.

But, once you try to analyze the recrystallization mechanism and understanding recrystallization phenomena associated with the FSW process. In which case it is also necessary to look into the calculation of the strain and strain rate in this particular case. So,

strain and strain rate the simplified way we can estimate the strain and strain rate that  $\epsilon_{rr}$  radial direction we can see this is the see suppose this is the  $V_T$  the tool of the velocity of the tool and their rotational speed.

And, particular point we can find out this is the at this particular radial distance that component of the velocity is this  $U_P$  as a at a particular angle  $\theta$  with respect to that this is the radial velocity and tangential velocity. Similarly, at a distance this is the radial velocity and this is the; this is the tangential velocity. So that radial velocity tangential velocity each and every point we can get from this modeling approach. So, let us look into this thing how we can estimate the strain rate on the radial direction.

$\epsilon_{rr}$  so,  $\frac{\partial u_r}{\partial r}$  that mean velocity  $u_r$  radial direction in the velocity by  $\frac{\partial}{\partial r}$  that is the representation of the strain rate along the this thing along the radial direction. Similarly, strain rate along the  $\theta$   $\theta$  with the respect to the  $\theta$  at particular angle  $\theta$   $\frac{1}{r} \frac{\partial u_\theta}{\partial \theta}$  minus  $\frac{\partial u_\theta}{\partial r}$ .

So, similarly  $\epsilon_{zz}$  this is the expression for that and  $\epsilon_{zz}$  also  $\frac{\partial u_z}{\partial z}$  by  $\frac{\partial}{\partial z}$ . So, basically  $u_r$   $u_\theta$  and  $u_z$  this  $r$   $\theta$   $z$  coordinate system we need to know at a particular point what are the different velocity components.

And, from there we can find out  $\epsilon_{r\theta}$ ,  $\epsilon_{\theta z}$ ,  $\epsilon_{zr}$  all the expression are standardize this thing. So, from there with this is the strain component strain rate component.

Now, once you get the individual strain rate component what way if you remember when you do the stress analysis while getting the what are the normal stress component it particular position what are the shear stress component. And, from there we can find out what is the effective stress; effective stress means we represent one single component of the stress which is combined effect of the all the different components.

So, similar way what is the effective strain for following the von Mises yield von Mises condition we can find out effective strain  $\frac{2}{3} \sqrt{\epsilon_{ij}^2}$  half that means sticking

into all the strain rate component. Then we represent one single component of the effective strain rate. So, once you get the effective strain rate the strain rate values are obtained from the strain rate using the rotational speed as well also.

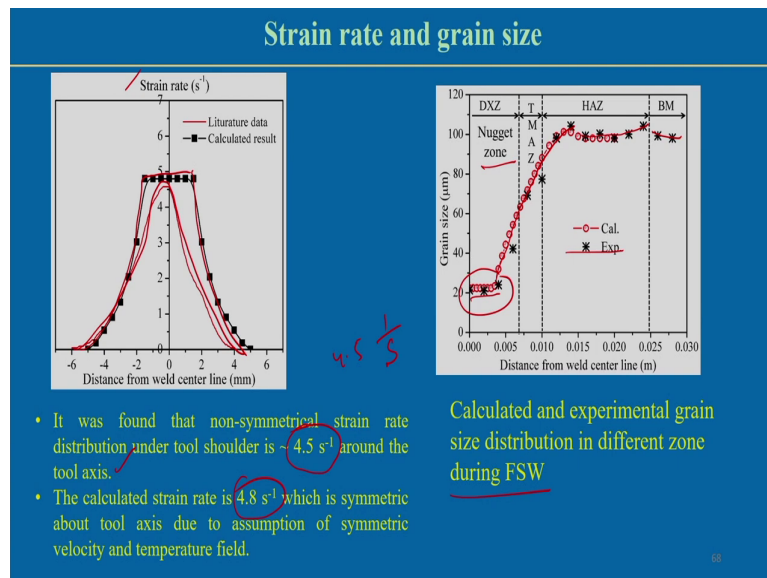
So, then once we get the effective strain rate from effective strain rate we can found divided by  $n$  is the rotational speed we can find out what is the value of the strain. So, strain rate is basically the  $\epsilon$  by  $\Delta t$  change of strain with respect to time that is the strain rate. Now, from the other way also if we have the strain rate value then divided by rotational speed we can estimate what is the effective strain.

Now, CDRX some analytic relation we can follow or some this case assuming the CDRX phenomena modeling takes into account with few material constant to predict the average grain size in the weld zone. So, to  $D$  what is the grain size assuming the continuous dynamic recrystallization mechanism then we can relate this things  $C$  one is the constant material constant we can see the  $\epsilon$ .

That means, strain it  $\epsilon$  dot strain rate and  $D$  maybe the material constant other value and  $Q$  by  $RT$ .  $Q$  by  $RT$   $Q$  may be the activation energy to start the recrystallization process. And, then  $R$  the characteristic gas constant  $T$  is the temperature. So, basically apart from the material constant there is a need to know that actually the this kind of mechanism will always link with the strain and strain rate phenomena basically recrystallization.

So, that information that values are required actual to estimate to predict that what is the recrystallizing the grain size.

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With appropriate value of all these things we can find out a distance from the a weld center we can estimate the strain rate. And this thing strain rate is that literature data, strain rate estimation is there at this particular a position. And, then calculated data a strain rate can be prediction which is close to the experimental value.

So, it has found that non symmetrical strain rate distribution under the tool shoulder is around 4.5 1 by s. That means, second is basically 4.51 by second that is the strain rate around the tool axis. The calculator strain rate is a 4.8 which is symmetric about the tool axis due to the assumption of the symmetric velocity and temperature field.

So, we are getting the symmetric profile for strain rate it should be non-symmetric. But we are assuming that calculator the this thing symmetric velocity and temperature we are



assuming that is why we are getting some kind of the symmetric profile of the calculated data calculated strain rate.

Now, grain size can be exhibited at the different position also calculated and the experimental grain size distribution in different zone during the FSW process also. So, in the nugget zone the grain size is around this and then a calculated value on this and the heat affected zone are near about this value and the experimental value we can see. It is very close to the base metal, this is the grain size.

So, we can see the grain size for the base metal it is very grain size is very large. But in the friction stir welded zone there we can find out that even nugget zone the smallest values of the grain size is a 20 micrometer. But, in the actual base metal it was around the 100 micrometer.

So, definitely some refinement of the grains normally occurs in FSW process and if we assume that it is follow some kind of recrystallization mechanism because this mechanism depends on. So, many factors also, but mainly depends on the type of material. So, which material we are handling?

So, normally aluminium we can assume the continuous dynamic recrystallization happens and based on that that we can even predict the average grain size in this particular case. But, in these cases when you try to create the average grain size, it is also necessary to look into the estimation of the strain and strain rate calculation.

It is very simplified when you can estimate the strain and strain rate calculation that we have already shown in this particular discussion.

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**Findings of strain rate analysis of FSW**

- ✓ Maximum strain rate occurs near the surface of the pin where the maximum velocity gradient exists.
- ✓ The strain rate rapidly decreases away from the tool axis due to rapid reduction in velocity gradient below the shoulder periphery.
- ✓ It is also observed that strain rate in CDRX zone decreases with decrease in rotational speed.
- ✓ The grain refinement decreases as move away from tool axis due to the decrease in effect of temperature and strain rate
- ✓ An increase in temperature leads to an increase in grain size and, on the contrary, an increase in strain rate leads to a decrease in grain size.

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Now, findings for the strain rate analysis FSW process. So, maximum strain rate occurs definitely near to the surface of the pin are the maximum velocity gradient exist. Definitely the strain rate will be maximum near about the pin surface. Strain rate rapidly decreases away from the tool axis due to the rapid reduction velocity gradient below the shoulder.

Actually if you see the strain rate the it is not much extent beyond the tool shoulder a tool pin surface. Because it is very quickly it is a very small strain rate exist is actually very small. We can say that gradient is very high. So, everything the variation of the strain rate from a very high value to the low value is happens over is very small zone near a about the tool a tool axis.

It also observe that the strain rate in CDRX zone decreases with decrease in the rotational speed. So, with respect to the rotational speed the strain rate also decreases. The grain

refinement decreases away as the move from the tool axis because of the decrease with temperature strain rate effect.

So, decrease in the strain and temperature effect away from the tool axis that is why we can may not get that much of grain size also the refined grain size we may not expect because of recrystallization may not happen away from the this tool axis. So, an increase in temperature leads to the increase in grain size and on the contrary an increase in the strain rate leads to the decrease in the grain size that we have already observed in this particular case.

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### Theoretical Model Development

#### Heat Transfer Model

$Q \rightarrow$  heat generation ( $W/m^3$ )

$k \rightarrow$  Thermal conductivity ( $W/m\ K$ )     $C_p \rightarrow$  Specific heat ( $J/kg\ K$ )  
 $\rho \rightarrow$  Density ( $kg/m^3$ )     $V_T \rightarrow$  Welding velocity ( $m/s$ )

#### Transient Heat conduction Equation:

$$\frac{\partial}{\partial x} \left( k_x \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k_y \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( k_z \frac{\partial T}{\partial z} \right) + \dot{Q}$$

$$= \rho C_p \left( \frac{\partial T}{\partial t} - V_T \frac{\partial T}{\partial y} \right)$$

#### Natural boundary condition (convection and radiation heat losses)

P-FSW:  $k \frac{\partial T}{\partial n} = h(T - T_0) + \delta \theta (T_0^4 - T^4) - q_s - q_a$

#### Initial condition

$T(x, y, z, 0) = T_i$

$T \rightarrow$  Temperature at time  $t$   
 $T_0 \rightarrow$  ambient temperature (300 K)

Now, if you look into this hybrid FSW process friction stir welding process we can see that at in this case the conventional FSW process. It is there already the conventional tool is inserting this thing and if you see all the interface are there that is interacting with the workpiece material.

So, this part will be responsible for the heat generation a highlighted part. But, apart from that some external heat source can be used either for preheating or during the process we can heat the sample that is also there. So, this pre-heating sample can be that it can be from a arc, but that arc is the we have to control the arc in such way that it should not melt the substrate material.

So, but it should sharpen the material some sort of raise the temperature of this particular so that it will be easy to plasticize of the material. So, that external heat source is used. So, this kind of a friction stir welding process in general called the hybrid FSW process, but how we can model this hybrid FSW process?

So, we will be using the same kind of the governing equation in this case. And even you can use a moving coordinate system also. And, same sort of natural and boundary condition convection convective and radiative heat loss the similar kind of boundary condition we can use and in general this is the condition.

But, if you see there are two term heat input the  $q_s$  and  $q_a$  – one is the because of the conventional FSW process and that is frictional heat generation for the FSW tool. Other part is the due to the secondary heat source; that means, because of the plasma arc of the heat source will be there. So, we have to estimate both the cases heat flux.

So, in this case already all the terms are defined it. So, it is well defined and we have discussed so many times also.

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**Hybrid FSW**

Total heat generation involved in hybrid friction stir welding process ( $Q_{total}$ ) is equivalent to the input by plasma arc ( $Q_{pre}$ ) and FSW tool ( $Q_{FSW}$ ).

$$Q_{total} = Q_{pre} + Q_{FSW}$$

The heat that is transferred to the workpiece by preheating source is determined by

$$Q_{pre} = \eta IV$$

The heat generated at the interface of tool and work piece due to friction is  $Q_f$  and due to plastic deformation is  $Q_p$ .

The contact state variable can be expressed as

$$dQ_{FSW} = dQ_f + dQ_p$$

$$\delta = \frac{V_{matrix}}{V_{tool}}$$

Coulomb's friction law to describe the shear stress estimates the critical friction stress necessary for a sliding condition:

$$\tau_{contact} = \tau_y = \frac{\sigma_y}{\sqrt{3}}$$

$$\tau_{contact} = \mu p$$

$$Q_{FSW} = \delta * Q_{FSW,sticking} + (1 - \delta) * Q_{FSW,sliding}$$

For a flat shoulder and straight cylindrical tool, the total heat generation is expressed as

$$Q_{FSW} = \frac{2}{3} \pi \omega [\delta \tau_y + (1 - \delta) \mu p] \{ (R_{shoulder}^3 - R_{probe}^3) + R_{probe}^3 + 3R_{probe}^2 H_{probe} \}$$

Thing is that hybrid FSW process also total heat generation involved in the hybrid FSW process is the equivalent to the heat input by plasma arc as well as the FSW tool. Now, if you look into the total heat generation in these cases the due the preheating the plasma arc and due to the FSW tool  $Q_{FSW}$ . Now, if we look into the heat is transfer to the work piece by preheating source is determined by this way  $Q_{preheating}$ ; that means, total we know this is total heat by the arc also.

So, total heat input the efficiency term  $I$  current and voltage, that is the total heat by the preheating source or may be in this case by the arc. Now, total heat generated at the interface by the tool in the workpiece  $Q_f$  and the due to the plastic deformation we can find out total heat is the  $dQ_f$  by  $dQ_p$ . So, this one plus  $dQ$ . This is the frictional heat and this is the plastic deformation.

Now, we can introduce some parameter that is the contact state variable we can say the contact state variable  $\delta$  equal to  $V$  matrix by  $V$  tool. So, that means, contact state variable means the tool is rotating along with the material is also rotating along with the tool. But, there may be some sort of relative velocity between the tool and the material movement along with the tool.

So, to account this relative velocity we can say that some kind of the contact state variable. So, the this can be ratio of the what is the tool velocity what is the matrix of the material  $V$  matrix. Now, in this case we can see coulombs friction law to describe the shear stress estimates the critical frictions stress necessary for a sliding condition. Basically, we can say the total heat generation by the FSW tool can be defined to two component – one is the sliding condition and sliding and sticking condition.

Sliding and sticking condition may be we can say partially sliding and partially sticking during the actual process. So, in that cases when you take the sliding condition and then in that cases the contact; that means, contact shear stress is basically follow the Coulomb's law of friction  $\mu$  into  $p$  coefficient friction and the pressure applied all these thing.

So, that is the this in case of the contact condition is be there if we assume the sliding condition, sliding condition of the friction between the tool and the workpiece. But, if there is a sticking condition sticking condition means this metal is also moving along with the tool. So, the when the sticking condition it should overcome the shear yield stress value then only because of the shear stress it will stick with the workpiece material, the tool material.

Workpiece material will be sticking with the tool material that is called the sticking condition, where sticking condition has to overcome that the shear yield stress value. So, shear yield stress value the  $\tau$  contact can be the  $\tau$  the shear yield stress value, but shear yield stress value with the normal yield stress value  $\sigma_y$  by root 3. This is the  $\sigma_y$  is a normal yield stress value. So, do not confused with the shear yield stress value and the normal yield stress value.

So, we here we can put and accordingly then we can use the total heat generations can be like that delta. They can take the contact state variable delta and that one part than total is for the sticking condition delta into the FSW, the sticking condition and then 1 minus of delta into the sliding condition. So, therefore, if we consider about sticking and the sliding condition then total heat generation by the cylindrical tool can be expressed like that  $3 \delta \tau y 1$  minus  $\delta \mu p$  this is because of the sliding sticking.

And, this is because of the sliding condition and then other part we can look into this thing. So, therefore, the frictional heat generation further modified by looking into the sliding and sticking condition we can estimate this frictional heat generation.

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**Material Model for dissimilar joint**

The dissimilar materials the concept of time varying Functionally Graded Material (FGM)

**Plunging of tool**

**Formation of functionally graded material**

According to the mixture rule FGM material properties such as thermal conductivity, specific heat and mass density can be estimated by using the following relations

$$k = k_1 [1 + (3(k_1 - k_2) v_2) / (3k_1 v_2 + (k_1 + 2k_2)(1 - v_2))] ]$$

$$c = (c_1 \rho_1 v_1 + c_2 \rho_2 v_2) / (\rho_1 v_1 + \rho_2 v_2)$$

$$\rho = \rho_1 v_1 + \rho_2 v_2$$

FGM concept in welding zone of dissimilar material joining.

Now, material model for the dissimilar joint: now, this is another aspect suppose we are joining the two different kind of material. So, for example, aluminium and maybe you can get

the aluminium and copper we can join these two materials also. So, in that cases what way you can define the material properties when you try to do the simulation of in FSW process.

So, dissimilar material the concept of the time varying functionally graded material; functionally graded material concept we can use time varying functionally graded material for example. We can take the concept of the functionally graded material. So, this is a tool interacting with the material; so, this side copper, this side aluminium.

So, then it is interacting all these things and after some time it is basically this zone is filling by the mixture of the aluminium and the copper also. Then this is the pure copper initial and this is pure aluminium and this part will be the mixture of the copper and aluminium. So, we are assuming the properties of the functional graded material. So, like that only you can define the different.

Suppose this is the weld zone. So, this is the weld zone this is the functionally graded material consisting of the both copper and aluminium, but remaining part pure aluminium other part is the pure copper. So, accordingly, we can define the different material properties also, so that formation of the functionally graded material you can see in this way also. This is you just keep on moving on particular direction if you take the cross-section this part is the FGM, functionally graded material.

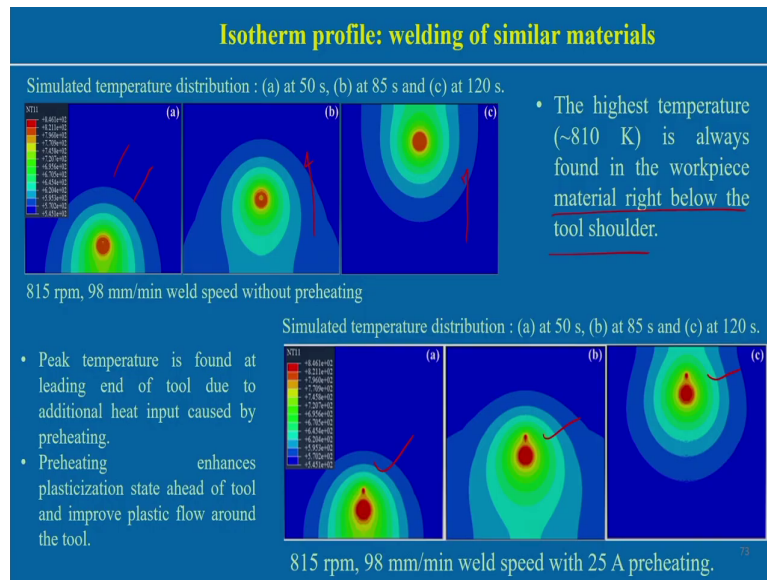
Now, how we can according to the mixture of the mixture rule of the functionally graded material? The material properties has to be defined such that it can takes in the different way the averaging of the both the material. For example, since it is a mixing of copper and aluminium then you can follow some sort of conductivity can be combined of the  $k_1$  and in terms of the  $k_2$  and  $v_1$   $v_2$  basically represent the what is the fraction of the particular material.

Similarly specific heat also we can see, density also we can see  $\rho_1$ ,  $v_1$ ,  $\rho_2$   $v_2$ . So, fraction of particular material of one material and fraction of the other material. So, this way a that functionally graded material FGM material in this particular zone. We can define the



these typical material properties, the combining of both the kind of material properties then we can perform the simulation.

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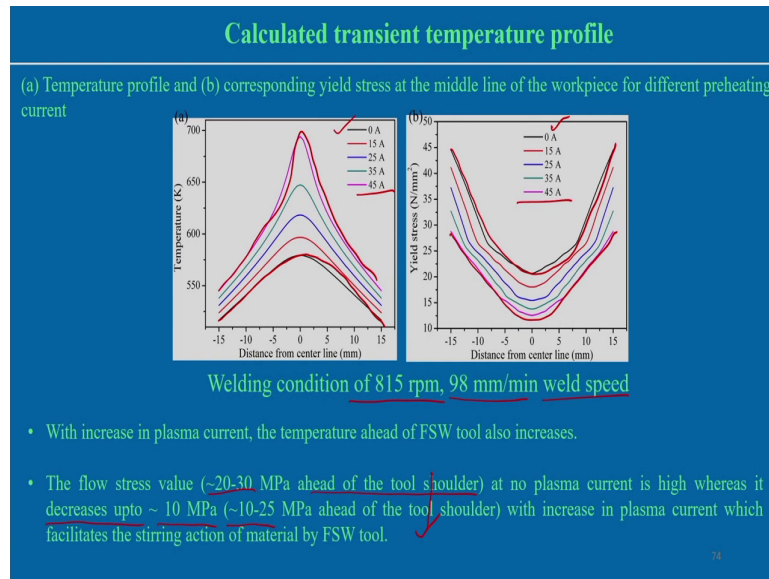


Then, we have observed the isotherm profile welding of the different type of the similar material. We can see different type of the similar material. This we can see that see movement of this material is moving one particular direction and we can see the temperature profile is found of the workpiece material below the tool shoulder. So, that kind of temperature we can getting and without any kind of the preheating.

Now, if we do the simulated profile if you do follow some kind to the preheating we can see there is a difference in the temperature profile. So, this information will be able to do if you develop the model assuming some sort of the preheating or external heat source is using in hybrid FSW process.

And, we can incorporate this thing in temperature analysis and then we can find out there is a difference in the temperature distribution and we can get lots of information from the temperature analysis you will see later on also.

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So, here temperature profile and the corresponding yield stress we can see at the middle line of the workpiece for different preheating see welding condition is this 815 rpm 98 millimeter per second and weld speed. Then we can see that the preheating current 0, 0 amp basically we are not using any kind of the preheating.

So, therefore, the temperature profile is this, but suppose we are using the 45 amp preheating the from the secondary source the temperature profile with this. So, there is a difference

between the between this temperature profile. Even yield stress value also as a function of temperature we can plot it the distant this as a function of temperature.

So, at 0 preheating so, this is the yield stress value this shows the yield stress value. But, if it is a maximum value of the preheating we can get the yield stress value is very lower. So, we can see the yield stress value is lower as compared to the conventional FSW process. So, it means that if we preheating helps to reduce the yield stress value, so then it will be easy for plasticization of the material. So, that kind of the benefit we can get from the hybridization of the FSW process.

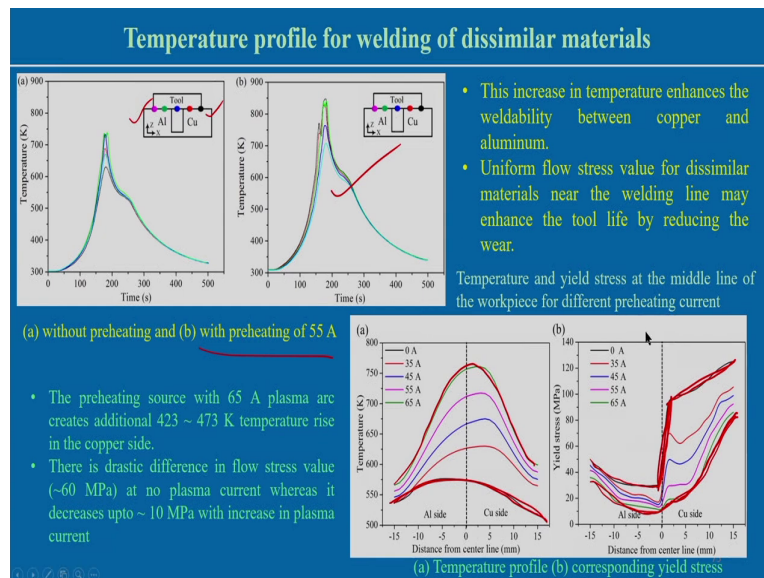
So, here we can see the flow stress value in this case 20 to 30 megapixel ahead of the tool shoulder and at no plasma current is high as high as decrease at no plasma current is high. Whereas, it is decreases up to 10 mega Pascal 10 to 25 mega ahead of the tool shoulder.

So, that some sort of the decrement of the flow stress value because with the application of the preheating we can observe and that kind of information we can get from the numerical simulation also and that numerical simulation of the temperature distribution.

So, once you get the temperature distribution then we can using this particular information of the temperature for a particular position. We can define what is the yield stress value or what is the flow stress value for a particular portion because flow stress value at different temperature it is weld defined property for a particular material.

So, then what is the reduction of the flow stress value or yield stress value that will help to design the process. How much preheating current we should use such that we can make the easier for the plasticization of the material harder material.

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This temperature profile for welding of the dissimilar metals we can see also this enhances the weldability the increasing temperature also. So, here you can see the temperature between aluminium, copper at the different position we can see the measure the temperature also. Without preheating figure a without preheating we can get some temperature profile, but with pre heating 55 amp we are getting this kind of the profile.

Now, what kind of you can conclusion we can get from this thing, preheating? We can see that temperature profile corresponding yield stress value a. So, aluminium side and the copper side this is the without preheating. So, this is the this was the temperature profile. Now, with preheating 65 this was the temperature profile.

Now, what are the corresponding yield stress value? If we see at a 0; that means, without any preheating this is the yield stress value. Because the yield stress value aluminium's definitely

yield stress for aluminium is low as compared to the copper and yield stress value with the copper is this. So, there is a transition at the interface from element to copper side is very high from this amount of the transition value of the yield stress.

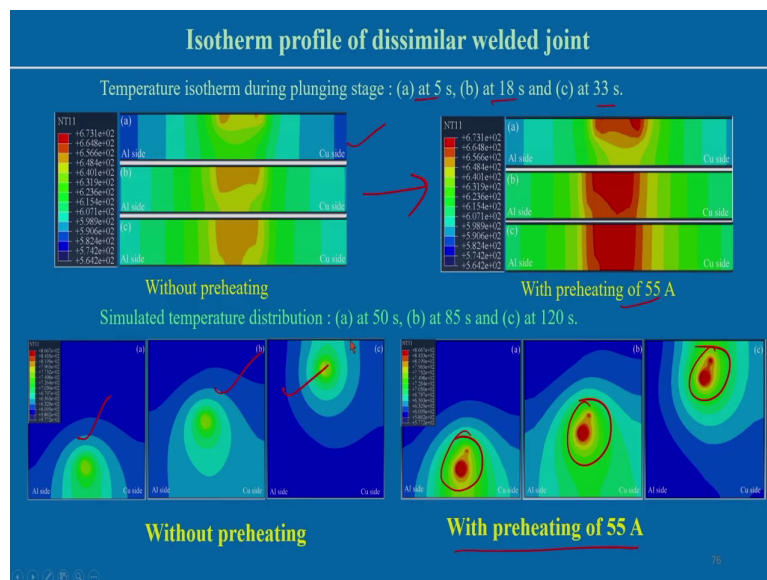
Now, if we apply the pre heating on their particular position or may be preheating either offsetting or exactly at the center point also. Anyway some sort of the secondary heat source we apply in case of dissimilar welding material, then we can see that arc maximum preheating this was the profile of the yield stress value.

So, it means that at the transition state there is not much drastic variation of the yield stress. So, that is why it makes more easier there is not much variation of the yield stress at the transition zone between the one material to another material. So, that is the beneficial effect of using some sort of the secondary heat source in joining of the dissimilar materials.

Even if you know also there is a huge difference in the yield stress or flow stress value between the aluminium and copper. So that utilizing of the preheating actually make easier to plasticization of the material or becomes more successful joint can be made between the two different kind of the metal where there is having huge difference in the kind of the mechanical properties as well as the thermal properties.

So, this kind of information we can get this is results from the simulation also. So, that from results from the simulation this will help to design what amount of the preheating is actually required and even there is a need some sort of the offsetting of the heating source towards the high conducting material or the low conducting material or towards the copper side or towards the aluminium side. That kind of the information also we can get from the numerical simulation.

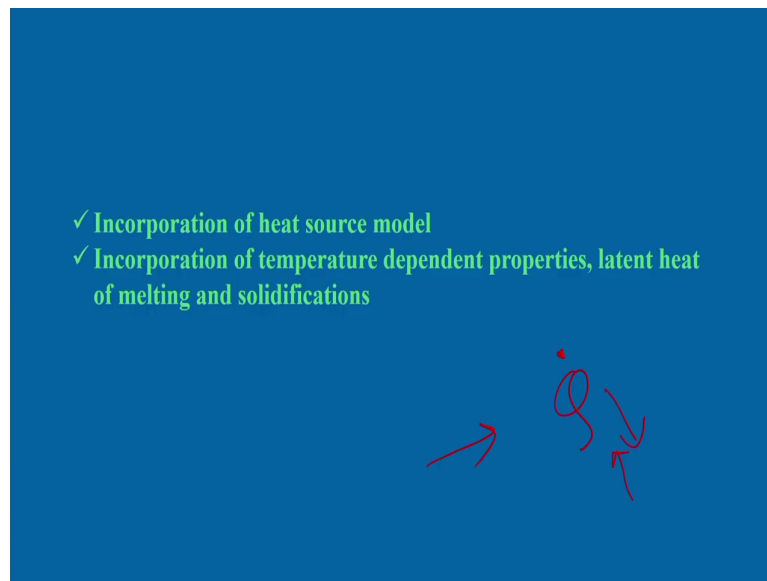
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Some results also we can very get this thing without preheating the development of the temperature profile during the plunging state at the different time step 5 second, 18 second, 33 second. Here also with preheating we can get the different profile see there is difference with a preheating.

Even without preheating and with preheating in case of the dissimilar material, there also we can see that how temperature profile is varies varying we can get as compared to the with pre heating. So, this information is very much helpful to design the process to define these things.

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So, that is all about a related to the how we can do the finite element modeling of the solid state welding process. But basically I have described one the only on the friction stir welding process because this is more widely used and hybrid FSW process and we can understand the what are the difference between the FSW process and hybrid FSW process.

And, also demonstrated through some sort of results that what are they intended to define in case of from a particular problem and that how this finite element the simulation actually helps to get some particular result or to get to design the particular process. So, now we will try to look into that what way we can incorporation of the heat source these two particular topic, incorporation of the heat source model and the incorporation of the temperature dependent properties latent heat of melting and solidification.

So, actually incorporation of the heat source model we have already described the incorporation heat source model. So, basically there we have explained in FSW process that through the once we estimate the total heat generation also we can converted to into in the form of a heat flux assuming some distribution.

Then we can incorporate either through the surface in through the boundary condition; that means, through the surface flux or if there is a need to produce some kind of the volumetric flux. Then we have to find out the distribution of the heat flux over the volume and that can be incorporate over the heat generation term in the basic governing equation.

So, in case of arc welding process we have already there separate model also. What way we can develop the dependent heat source model assuming some sort of the distribution and that this a heat distribution. And, because this  $\dot{Q}$  term if you look into the all the heat source model has been developed the distribution is expressed in such a way that it is expressed per unit volume.

So, therefore, this  $\dot{Q}$  also in the governing equation if you see this is also representing per unit volume. So, that means, this  $\dot{Q}$  is implement through the  $\dot{Q}$  term in the governing equation actual volumetric heat source can be implemented.



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**Incorporation of latent heat of melting and solidifications**

- ✓ In general, melt between two temperature limits – the solidus temperature and liquidus temperature for an alloy system
- ✓ As melting and subsequent solidification occur within the bulk material during the welding process, the latent heat of fusion and solidification is considered in the present work.
- ✓ The main challenge in the incorporation of the latent heat is to account for the sudden release or absorption of heat for a very small change in temperature during solidification or melting
- ✓ The idea of consideration of latent heat through the specific heat term is widely followed in numerical modelling of various problems involving melting and / or solidification.

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But, incorporation of the latent heat of the melting and solidification if we discuss that, normally the incorporation of the melting and solidification normally happens in the alloy system between the solidus and liquidus temperature.

So, what is happening between the solidus and liquidus temperature? Track this particular temperature and then we can we can introduce the latent heat of melting or latent heat of solidification latent fusion and latent heat of solidification can be considered in this particular version. But, definitely there is a challenging task in this particular cases because latent heat is basically there is a sudden release of the phase the heat related to the phase change.

But, this normally happens over in very low range of the temperature and even in case of the pure metal it is one single point temperature there is a transition the release of the heat happens. So, therefore, within the very small time or within in the very in lower range of the

temperature there is sudden release of the energy or absorption of the energy. Depending upon whether it is latent heat of the fusion or there is a latent heat of solidification.

So, therefore, incorporation is a little bit tricky in this particular case, but normally the approach of the latent heat incorporation in a finite element model is through by modifying the specific heat term it will be easy to implement the numerical modeling approach to incorporate the effect of the latent heat during the change of the phase from solid to liquid phase.

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**Incorporation of latent heat of melting and solidifications**

Consider a converged solution set of nodal temperature are obtained at the end of  $p^{\text{th}}$  time step (previous time step). New value of nodal and elemental temperature will now be obtained for the next time step i.e.  $(p+1)^{\text{th}}$  time step (i.e. current time step). During the calculations for the current time step [i.e.  $(p+1)^{\text{th}}$ ], two subsequent iterative steps are considered as  $n^{\text{th}}$  and  $(n+1)^{\text{th}}$  respectively in a sense that the calculations for the  $n^{\text{th}}$  iteration is already over.

$$C_p = C_1 \quad \text{for } T < T_s$$

$$C_p = C_2 \quad \text{for } T > T_l$$

$$C_p = C_n = \frac{L}{(T_l - T_s)} + \frac{(C_1 + C_2)}{2} \quad \text{for } T_s \leq T \leq T_l$$

$\frac{L}{\Delta T}$   
 $\downarrow$   
 $(n+1)^{\text{th}}$

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So, we can see the overall view of all this thing. Now, in a solution that set up the nodal temperature obtained at the end of the particular time step. Now, value of the nodal and elemental temperature will now be obtained for the next time step for example, p plus this thing.

For example, one particular time step  $p$  this thing  $n$ th time step. So, we are getting the convert solution of the particular instance. And now, we are moving  $n$  plus  $1$ th time step in this particular time step what way we can consider the calculation of the particular iteration in calculation of the incorporation of the latent heat.

So, in general you can see the specific heat  $C_p$  equal to  $C_1$  for temperature we can define the temperature flag also here; temperature less than the solidus temperature specific heat is equal to  $C_1$ . Then temperature above liquidus temperature, then  $C_p$  equal to  $C_p$  equal to  $C_2$  some other specific heat.

So, these are the specific heat, but there if within the solidus and liquidus temperature the particular specific heat is basically there average of the specific it at the solidus and liquidus temperature plus latent heat divided by because this is latent heat is divided by  $T_L$  minus  $T_L$ ; that means, the  $L$  divided by some temperature so the range between the solidus and liquidus temperature.

So, that can be artificially added to the specific heat to look into artificially enhancing the specific heat value between the solidus and liquidus temperature. So, that is the approach to incorporate the effect of the latent heat of melting and solidification in a numerical model, but there are different cases can be consider in this case.

So, for example, they just jumping from one particular point say which is out of the solidus and liquidus temperature and this from this say at the  $n$ th step that is the converse solution of the temperature. Now, in the next step the prediction of the temperature can be done in such a way that it can go either this which is even below the solidus temperature or temperature can go between the solidus and liquidus temperature or temperature a solution can be get can go out of the solidus and liquidus temperature. This is the case 1.

Case II – you can start with the say converse solution or particular  $n$ th time step it is may be in between the solidus and liquidus temperature. Now, once you to calculate that temperature for the next step, then it can go directly to the below the solidus temperature it can go more

than but between the less than liquidus temperature or which can be more than that of the liquidus temperature.

Even case III we can start the from between the solidus and liquidus temperature, but it can go on temperature oh sorry outside of more than liquidus temperature. And it can go less than that of the between the less than solidus temperature. This normally happened during the solidification, but one implement this normally happens during the we when we are assuming the heating stage and this may happen during the cooling stage.

So, therefore, it may happens less than that also other temperature. So, looking into the different aspect different possibilities of when one reach to another step. We can decide the different strategy and accordingly, we can develop the algorithm such that we can estimate this thing the possible temperature and from there we can estimate what is the value of the specific.

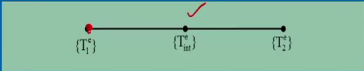
So, say at when you are moving from one particular point to other point either during the either in the solidification; that means, during the solidification during the cooling we can say or during the fusion that means during the heating stage. So, then accordingly different temperature can be assumed this thing and once we look into what temperature it is moving, based on that we can choose the value of the specific heat looking into this a range took this temperature flag of the specific heat.

So, this is a simplified way to incorporate the effect of the latent heat of the melting and solidification in a modeling approach.

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**Incorporation of temperature dependent properties**

This elemental equation can be written in a general form such as  $\{A^e\}\{T\} = \{F^e\}$



An intermediate elemental temperature solution is considered as

$$\{T_m^c\} = \{T_1^c\} + (\{T_2^c\} - \{T_1^c\}) \cdot \lambda_{RF}$$

Nodal temperature solutions at the end of current time-step (or load-step) are recalculated -  $\{T_2^t\}'$

An error norm,  $L_N^t$ , is calculated for all elements within the solution domain as

$$\{L_N^t\} = \frac{\{T_2^t\} - \{T_2^t\}'}{\{T_2^t\}}$$

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Now, incorporation of the temperature dependent properties also in the similar way can be done the other way also. For example, the elemental equation normally we finally, reach this elemental equation  $A T = F$ . So, that means, load and this matrix and  $T$  is the temperature. We solve this equation.

Now, we can solve this equation, but definitely when you try to incorporate the temperature dependent properties, it is necessary to follow some sort of iterative calculation. So, for example, we start with this point  $T_1$  may be one convert solution one particular time step.

Then we can find out that what is maybe the next temperature state that intermediate temperature. We can assume some sort of intermediate temperature, but intermediate

temperature can be like that you know  $T_1$  plus what is the predicting this  $T_2$  temperature minus  $T_1$ ? The difference into some factor we can predict.

So, therefore, nodal temperature solution at the end of the current times step or the load step are recalculated is like that  $T_2^a$ . So, therefore, then once we recalculate the predicated temperature  $T_2^e$  and then we can estimate what is the nodal error in this particular case. So,  $T_2 - T_2^e$  dot by  $T_2$  error.

And, then what is that  $a$  between the  $T_2^e$  dot and  $T_2^e$  is close to each other, then error will be the minimized in this particular case and then on acceptable less than that of particular error, then we accept this is the converse solution during the iterative calculation.

We have already explained that how we can tackle the nonlinearity associated with the temperature calculation we have already discussed. But, this is just a small part that we can incorporate the temperature dependent properties also in iterative calculation during the particular model.

So, with this thank you very much for your kind attention and maybe, I finish this particular module. Now, next class we will try to discuss some sort of the demonstration using a kind of that some kind of using commercial software in specifically focusing on the thermal analysis.

So, thank you very much.