Finite Element modeling of Welding processes
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Lecture - 17
Heat source models in welding-I

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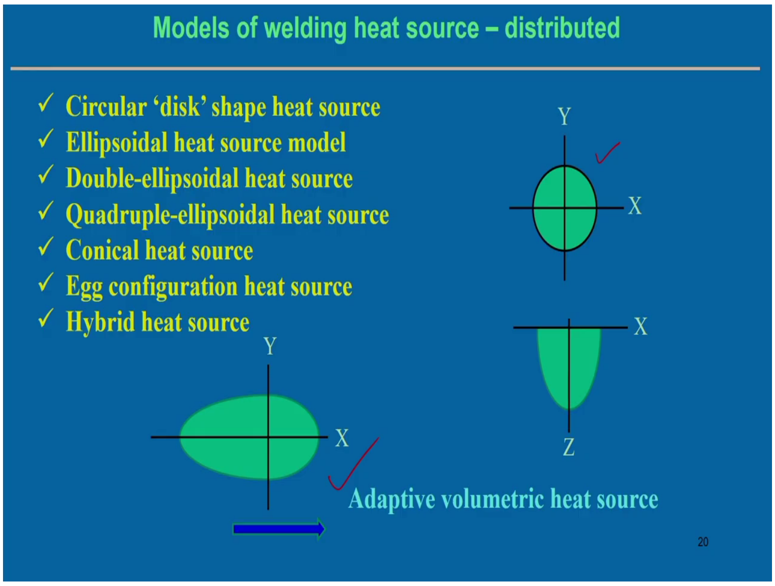

Hello everybody today we will discuss the models of the welding heat source, last class we have discussed the analytical solution of the different heat source are associated with the welding process, but in analytical solution we assume that heat source can be represented like a point heat source kind of line heat source and based on that with other assumptions.

We can get some kind of the analytical solution of the temperature distribution in a welding process. But in actual practice the representation of the heat source in the form of a point or line heat source that may not be the feasible and because if you look into simple arc welding process also or even other welding process.

That energy is not distributed exactly focus concentrated on one particular point. Rather it is always in the distributed source and accordingly we have to look into the different aspect of this heat source model. First thing is that what is the distribution follow what kind of mathematical equation can be represented using the distribution of particular arc welding process or any other welding processes.

And second point is that the distribution happens over what kind of the regular geometry. So, that has to be decided and based on that different others heat source models has been developed. So, in this case we will try to look into the distributed heat source, but there are two kind of the distribution heat source maybe first is the circular disk shape heat source. That means, distribution we are assuming over the surface our distribution may happens over the particular well defined volume.

So, that volume has to be predefined for example, most of the cases we use some kind of the ellipsoidal ellipse shape of the geometric volume can be ellipsoidal shape or maybe other. In these cases, we will show that kind of oval shape or maybe a configuration heat source model.

And sometimes according to the welding process the there is a possibility of looking into the heat source model development that is not only the single kind of the heat source model then hybrid heat source model can also be possible to develop.

But before doing that we will try to look into that, how the mathematical equation of a surface heat flux will gradually develop this knowledge or this thing understanding that is first surface heat flux. And then we will look into the volumetric heat flux or maybe in volumetric heat source model in this particular context. So, here we can see the circular shape heat source model.

The first if you see circular shape heat source model it is over the surface the, it is the over the circle the heat flux is distributed over the circle, that is one kind of the heat source. Or it can be ellipsoidal heat source model ellipsoidal heat source model maybe look into that in case of stationary welding process normally we use the ellipsoidal heat source model this is a well known heat source model, but it is over the volume.

So, distribution happens over the volume, but what are the heat flux density distributes that we normally follow the Gaussian distribution, but it is not necessarily every time we have to follow the Gaussian distribution of the heat flux intensity, the distribution may follow in other equation also.

We will show if distribution changes then what way the volumetric heat source model actually changes. Now apart from that double ellipsoidal heat source model also we need to consider. Because in this case double ellipsoidal means simply merging of the two ellipsoidals and making a double ellipsoidal heat source model which is more practically significant when we are using any kind of the moving heat source problem.

So, that means, heat source is moving one particular direction normally in a welding process though it is not always the stationary heat source, but sometimes the source can move one particular direction. So, in that case simply ellipsoidal heat source model not sufficient in that cases we need to merge two different ellipsoidals. Such that non symmetric energy distribution along the because of the welding velocity can be incorporated.

So, in that sense that double ellipsoidal heat source model has been developed. Then quadruple ellipsoidal heat source model also we will look. In this case the non-symmetric energy distribution not only once because of the welding velocity, but apart from the welding velocity if we consider the non symmetric energy distribution due to the different material properties in case of dissimilar materials.

In these cases, the double ellipsoidal heat source model can be extended to the quadruple ellipsoidal heat source model or I can say that quadruple ellipsoidal or double ellipsoidal model is basically simple one specific derivative of ellipsoidal heat source model. So, in that case we will try to understand first how the ellipsoidal heat source model can be developed.

So, apart from that there are some conical heat source model, egg configuration heat source model and hybrid heat source model can also be developed and we will discuss a little bit about the other kind of heat source models. But apart from that there is another heat source model that is called the adaptive heat source model.

Adaptive heat source model, we have some specific computational advantage in heat source model, or it overcomes certain inheritant difficulties, in case or in the in terms of the parameters of a welding heat source.

So, that can be overcome by using the adaptive volumetric heat source model. We will discuss also the adaptive volumetric heat source model is particularly important. And maybe that may be significant more if we try to develop some kind of the finite element where heat transfer analysis of your own.

In that case of your own code then it is easy to implement adaptive volumetric heat source for them, rather than using a as compared to the kind of commercials finite element software. In that cases it is little bit difficult to implement the adaptive volumetric heat source model. Anyway, we will discuss all these kinds of the source model.

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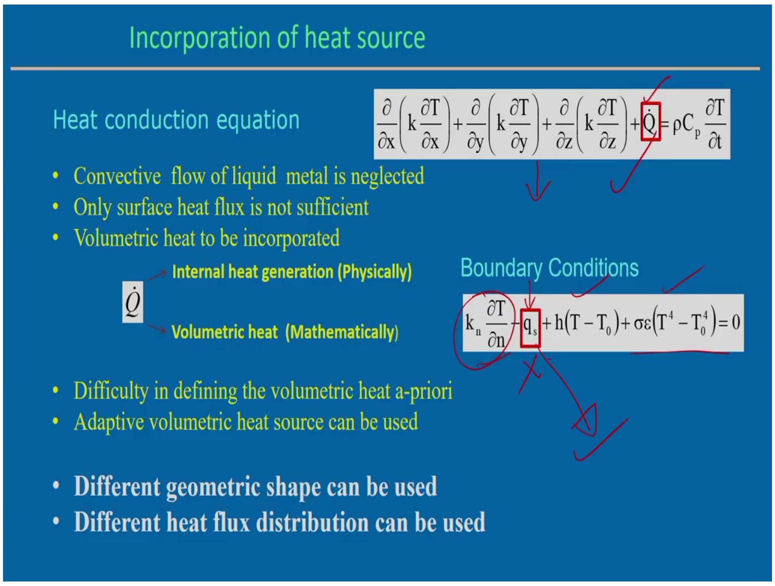

So, first start with this different heat sources model, but what way we can incorporate this heat flux or distribution of the heat flux heat intensity into the model into the heat transfer model. So, this heat conduction equation we know this is the 3 dimensional heat conduction equation, we use that this term the highlighted part is that Q dot that is called the.

According to this heat conduction 3 dimensional heat conduction equation this is called the internal heat generation term, but mathematically if we want to implement the volumetric heat source in the in particular welding process. Then we have to incorporate this volumetric heat flux through this internal heat generation term.

And ultimately we try to look into the internal heat generation term in the sense that we will try to estimate what is the volumetric heat distribution heat intensity or volumetric heat flux that we will try to estimate and then after estimating this thing we incorporate this through this Q dot. So, what actually the convective flow of the liquid metal is neglected that is true because when we analyze the any heat transfer problem we, if you use only the heat conduction equation then basically we are neglecting the material flow within the small weld pool.

In that case since we are neglecting the material flow in a small weld pool then surface heat flux if we assume sort of the surface roughness is not sufficient because that may not be able to give you the correct estimation of the heat input to the particular welding process. So, definitely in that case it is necessary to incorporate the effect of the volumetric heat flux.

So, basically, we have to represents in that when there is a finite volume of the molten pool exists and material flow is significant in that case it is necessary to incorporate the volume at heat flux. So, surface heat flux is not sufficient in that case.

Now Q dot term definitely this is the internal heat generation physically it is called the internal heat generation term, but mathematically it is called the volumetric heat and specifically it is in that sense that in welding problem we just incorporate this volumetric heat through this Q dot term.

But difficulty in defining the volumetric heat a priori. That means, the main difficulty is that suppose when you define the particular volumetric heat source term in welding process. It is necessary to pre define what may be the geometric parameter of this particular volumetric heat flux. So, for example, if we assume the it is a ellipsoidal heat source model, but in that case is ellipsoidal heat source model is having geometric parameters.

So, a semi axis length of all semi axis length of this ellipsoidal is necessary to define to define the volumetric heat flux. So, that is one of the limitation although we can represent the different heat source model, but that is the limitation. So, before start of the simulation we need to define these the geometric parameters of this particular volumetric heat source.

So, to overcome that difficulty then adaptive volumetric heat source can be used in this case. But here in the boundary condition because this is the governing equation the heat conduction equation, but it is we have to supply some kind of the boundary interaction. So, boundary condition the mathematical form can be represented like this the all the terms are a convective heat loss and radiation from the surface that is the account the heat loss radiation heat loss and q is the heat flux that is the applied to the domain.

And then this is the heat conducted array exactly the heat conduction what is the value of each conductor exactly on the boundary. So that make it balanced in the boundary condition in the. So, this is the typically represent the boundary interaction in this particular mathematical equation but see there is a term q s q s is the surface heat flux.

So, once if we incorporated the volumetric heat flux normally in arc welding process we know we if we assume the volumetric heat source term exists in this particular arc welding process, then we incorporate these volumetric heat flux through the Q dot term. So, in that case it is not necessary to incorporate the heat flux to the surface heat flux term; that means, through the qs.

So, then in that case we do not consider qs, but in certain particular problem maybe its not necessary to incorporate the volume at heat flux in that case maybe it is possible to incorporate the heat flux and the supply heat flux through the surface heat flux term.

So, there is a; there is a flexible enough in this modeling approach that a both way it is possible to incorporate the heat flux. Either through the Q dot term the volumetric heat generation term or heat flux can be incorporated through the surface boundary surface flux.

So, both way we the heat flux can be incorporated in the model and even certain cases also we have already given an example then a resistance hot welding process it is also necessary to incorporate the heat flux from both way. So, that means, some in some part depending upon the problem.

So, some part it is also necessary to volumetric heat generation term then volumetric heat flux we incorporate through the Q dot term and in the certain part it is necessary to incorporate that surface heat flux an surface heat flux can be incorporate through this term. So, that is why it is very important to understand that what way we can incorporate the volumetric heat flux or surface flux in a finite element heat conduction base heat transfer model.

Basically heat in these cases we are not considering the material flow only the heat we are solving the heat conduction equation to get the temperature distribution, but overall if we look into the volumetric heat flux term in these cases different geometric shape can be assumed. Different geometric shape can be assumed means, its not necessarily always we have to incorporate the ellipsoidal shape or w ellipsoidal cell maybe some other kind of geometric shapes can also be considered.

We will show that even if we consider the a configuration heat source model also, that is the geometric shape is different from the ellipsoidal or double ellipsoidal form. And apart from that not only that geometric set, the second part is that different heat flux distribution can be used also. So, most of the cases we are assuming that Gaussian distribution of the heat flux.

So, Gaussian distribution did not normally follow the exponentially decaying the heat flux from the maximum intensity at the center to gradually decreasing towards the boundary, but it is not necessary some it is a heat flux density distribution can be a different way also a different equation we can use also, that we will show.

So, it means that once you try to understand the what is the volumetric heat source model, that can be completely defined by looking into the two aspects. One is the what geometric shape you are considering and second (Refer Time: 12:56) is the what distribution you are considering both are important to understand to develop our volumetric heat source model.

Now we will look into the most widely used one by one the first start with the surface heat flux then we will try to look into the other volumetric heat flux model.

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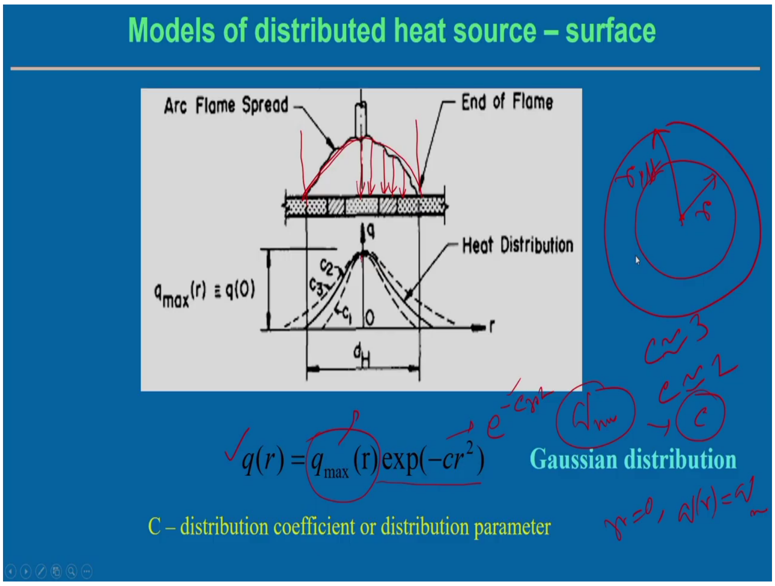

So, models of the distributed heat flux on the surface and we can see that what we can incorporate this thing. Just let us look into the surface heat flux only. So, this figure indicates that this is the typical arc the shape of the arc. If you look into the what are the typical shape of the arc that the intensity and the gap is there is the arc at the intensity this is the maximum and the intensity is gradually decreasing both the sides in this form, this is the typical shape of arc.

So, this is the flame is the arc flame spread in that way that is from this forging to the gradually decreasing to these things, but we can mathematically represent in the form of intensity heat flux intensity. So, that means, heat flux intensity means in the sense that its a per unit area that it is the at the center point it is become maximum intensity is the maximum and gradually if you see the towards the boundary it is gradually decreasing.

So, this kind of nature mathematically can represents the heat distribution is that in this way that particular equation the q r it is a radial distance. So, assuming that the heat flux is false on a circular shape over a circle. So, its like that only the arc can be represented as a circle and this is the center point is the maximum intensity and the boundary is a towards the boundary it is the minimum intensity gradually decreasing from center to these things.

So, this mathematically can be represented by the equation q r equal to q max is a function of r an exponentially decaying exponential e to the power minus c into r squared r is the radial distance. So, r from the center to this is the radial distance. Now, as the r is the variable here. So, c is the; c is the call the distribution coefficient distribution parameter. So, c is the constant term some distribution coefficient or distribution parameter. It means that indicates c actually indicates that whether it is very stiff can be distribution can be very stiff weld can represents or it can be the shallow kind of these things.

So, (Refer Time: 15:17) distribution. So, it is a spread of the arc is better represented by this term the distribution coefficient c parameter I can give an example also. For example, c equal to 3 if you use the c equal to 3 value this is normally in the arc welding process, but this if c equal to 2 or lower side maybe in case of 2 or less than that can be used in case of the laser welding process. Because if you compare the intensity of the laser and the arc, that means, over the surface over his laser falls there is a that the spread should be very stiff in case of the laser welding process.

But the spread of the arc is not that much of stiff as compared to the laser in case of the arc welding process. That is why the c can with 3 approximately in case of arc welding process or c can be lower side equal to or less than that in case of arc laser welding process.

So, that is where we can differentiate depending upon the spread of the arc, but that has to be decided that what can be the c value or it is possible to decide also in terms of other parameters we will show, how c can be evaluated or c can be decided. Now, q max indicates the maximum intensity at the center point and exactly at the center point q is the max. So, basically the r equal to 0 at r equal to 0 what is the value of the maximum intensity and that is this is the condition.

So, r equal to 0 q equal to q max. So, if you see r equal to 0 then q r equal to q max. So, then maximum value. So, then from the maximum value to gradual decreasing as a function of r. So, this way you can express these things. Now, there is other part this is the c this arc. So, arc this part is called the effective what is the arc spread for example, arc spread up to this point.

So, then it is called an effective arc radius. So, we can see up to that point we can count that heat intensity distribution beyond that we may not count even there is a small amount of the distribution may be there, but what is the effective radius of the arc in case of the arc welding process this has to be defined in this case.

So, this way we can represent that distribution of the arc in the particular equation. So, that means, q max then this is the maximum value and gradually decreasing the exponential way towards the boundary and this is this kind of distribution is normally called a Gaussian distribution.

So, in this case Gaussian distribution we do not know what is the value of the q max. If we know that the q max even also we do know what is the distribution parameter. If these two parameters are known us then we can completely define the heat flux intensity distribution in this particular model.

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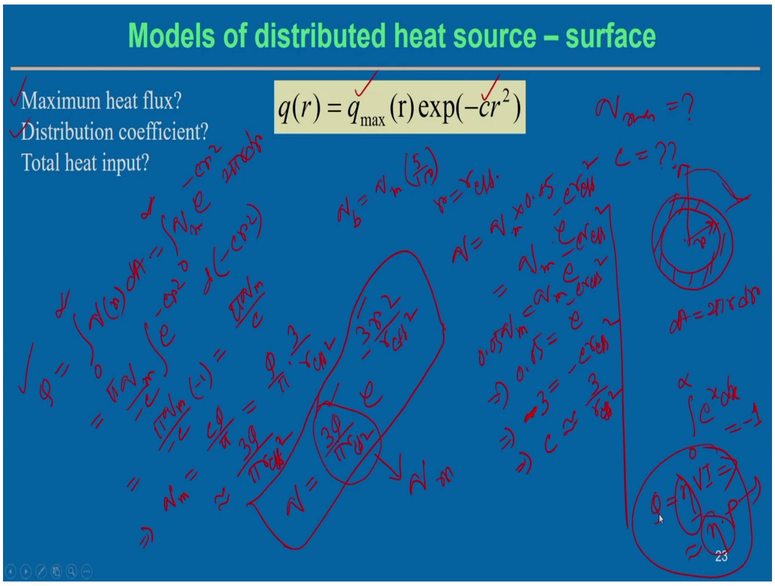

So, that we have to look that what we can estimate the heat flux density distribution. So, in this case we can see that q r can be represented to the q max are exponential seen to r. So, these two things are there what is the maximum value of the heat flux. So, what is the value of q max that has to be estimated and what is the distribution coefficients value that has to be estimated we can estimate this thing and this curve.

So, let us look into that part we can estimate this two things first is the. So, q equal to q max. So, if we assume that total heat input to this domain q because this heat flux intensity it indicates the per unit area. So, according to the Gaussian distribution, it actually see we can see the q r and if we take suppose this is the circle part as a radial distance r, we take an element through this area elemental area d a d a equal to twice by r d r this is the elemental area.

So, now, if we integrate over this d a and that integration 0 to infinity this is not finite because if you look into the Gaussian curve also according to nature curves this is the asymptotically converts on the x axis or as the radial distance r. So, then we can extend 0 to infinity. So, that integrates up into get over the area. So, over the elemental area intensity falls then if we integrate over this elemental area then we will be getting that total heat input to particular this particular surface.

So, that take element it is like that only. So, it can be 0 to infinity. So, qr equals q r here to the q max we can see that q max into e to the power minus c r squared d a d a equal to twice pi r d r. So, that is where we can estimate this integration q m integration e to the power minus c r squared, we can say d equal d of minus c r square. Then it can be pi outside and 2 is there divided by minus c, so making the balance.

So, pi q m maximum by minus c and this is the integration. Now we know this integral form 0 to infinity e to the power x d x equal to minus 1 escapes. Now this integrates that equal to minus 1. That means, it can be pi q m by c or we can see that. From here we can see that q m maximum intensity equal to c into q by pi.

So, this is the maximum intensity value what we this is c is; c the distribution coefficient is unknown, but other part also I can say that q also q can be estimated like that. So, then q is the total heat input. So, q can be like that v voltage into amp for example, in arc welding process the input voltage and input current we can measure the input voltage and input current also.

So, volt into ampere current that indicates the power or maybe in this case is the energy input power unit time. So, in this case if we multiply the efficiency term because one of the creating the arc what is the input power. That may not be transferred to the workpiece surface because we here we are called calculating the this volumetric or surface it will whatever based on that, what is the amount of the energy transferred to the workpiece?

So, then in within the workpiece we are assuming some volumetric heat flux them accordingly we define the geometric set either the surface of volumetric and then we are integrating over this thing to estimate the what is the total energy input because of this particular geometry or that hold by this particular geometric say whether volumetric or surface.

So, that you know amount of energy q can be estimated that suppose it is the efficiency. So, if you incorporate efficient ofthis is the effective energy input to this exactly to this surface. So, that q will be like that only. So, then assuming that then q is known then is the c is the unknown in this case, but c is the distribution parameter.

If we assume that distribution parameter is something like that. Say over at the boundary because it is at the center it is a maximum and gradually decreasing to the boundary. So, it is the exactly the at the boundary q m intensity at the boundary is basically maximum the 5 percent of the maximum intensity with this assumption.

So; that means, 5 percent of the maximum intensity at the boundary, that means, at the boundary r equal to r effective at the boundary. So, this boundary. So, we can this the intensity because intensity we have assuming that q r. So, q at the boundary value equal to q m into 0.05.

So, that is equal to q max q m here maximum intensity into e to the power minus c into say r effective square. So, we say that at the boundary only the 5 percent of the energy falls exactly at the boundary and at the boundary effective value of the radius equal to r e f f. So, put this condition then we can say that 0.05 q m equal to q m into a to the power minus c into r effective square.

So, from here we can say that 0.05 equal to e to the power minus c r effective square. So, logarithm both side, so l n 0.05 is around minus 3 approximately minus 3 equal to minus c into r effective square. So, from here we can see that c is basically approximately c equal to 3 by r effective square. So, q into q by pi c equal to 3 by r effective square it means that it is approximately 3 q q by pi r effective square intensity.

Now, what we can estimate the distribution parameters? So, q equal to so we can see q equal to q max, q max equal to 3 q by pi r effective square. And e to the power minus cr square c equal to the r square divided by c c equal to 3 by r effective square. So, now, so this is the expression now q is the intensity heat flux intensity at the center and a q max and then in this cases we can estimate the q max equal to this is the value of the q max.

Now, distribution parameters we can see the q equal to q max into e to the power minus c r a square we can estimate that from the boundary condition. In the sense that if you put the boundary interaction that exactly at the boundary only 5 percent energy falls on the boundary from that condition, we can find out what is the value of the distribution coefficient c.

Of course, the similar kind of the value it will be about estimate that within this zone. That means, within the effective radius r the 95 percent is energy within this effective radius r and remaining 5 percent is outside of the boundary of the arc. So, in this cases because that distribution parameter is fusion in such a way components will because it is not you will not get the definite value exactly on the boundary.

So, in that cases if we make the energy balance. That means, 95 percent of the energy inside the arc make the similar kind of the calculation we will be able to get the similar value of the distribution coefficient. So, this boundary condition is required to finding find out the value of the distribution coefficient c equal to this thing the distribution coefficient.

And if you know the distribution coefficients and there also you can find out what is the value of the maximum intensity at the center q m interest with the q. And finally, we can reach this is the distribution equation of the distribution in case of the circular kind of the disk heat source. So, that is the you see that q m is in this case is intensity that q should be known and we can estimate what is the q value q is the total energy input to this domain and that energy input can be estimated from this thing.

I have shown that we know the efficiency term thermal efficiency then in case of the arc welding process or in case of laser welding process this expression can be the efficiency term into the power because directly the laser welding process we directly estimate what is the power input. So, input power can be estimated if you multiply the efficiency time then effective power is basically a goes into the workpiece.

So, that effective power here we count as a value of the q and that effective power in terms of that effective power we can estimate the distribution of the heat flux distribution over the surface. So, that c 3 is the distribution coefficients here and 3 is the distribution coefficient. So, actually the distribution coefficient can be different value if we assume here we are assuming that only 5 percent energy is false on the boundary and based on that we are getting the distribution coefficient value of the 3.

But if we assume the assumptions can be different then c value can be different. So, depending upon the welding process itself or I can say that other than the experience in the practical of different heat intensity associated with the different welding process. This is possible to find out the different value of the c or distribution coefficient value we will show the different calculation also based on this basic testing.

So, but in this cases we first asked we have to find out what is the value of the maximum heat flux that we can estimate, second what is the distribution coefficient the value of the distribution coefficient, that means, c in this case c in this case and a maximum heat flux in this case. And then total heat input total heat input we can show through the integration of the total heat input of what the surface.

Similarly, if you want to estimate the what is the total heat input in case of the volumetric heat, if you know that know the expression of the volumetric heat flux density distribution.

So, from there if we integrate over the volume then you will be able to find out what is the value of the total heat input to this substitute and that total heat inputs that should be must be equal to that can be linked with this other process parameters in a welding process just I explained that total heat input q here in this cases.

If you know the volt ampere input voltage and ampere in a welding process and then multiply by the efficiency term then you will be getting the total heat input or in case of the laser if you know the laser power use in this experiment. If you multiply the efficiency and then we will be getting there basically thermal efficiency then we will be able to know what is the value of the total heat input to the substitute metal and that is equal to that equal to q.

So, this way we can understand that heat flux density is the same principle the different heat source model can be developed we will show one by one. So, this thing we have already explained this thing that models of distributed heat source in case of the surface.

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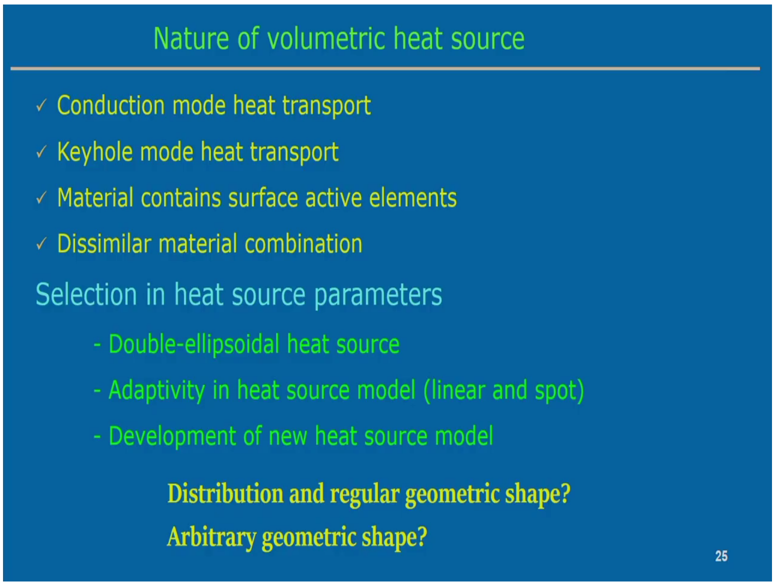

Now we will try to explain the nature of volumetric heat source; that means, we will look into the volumetric heat source. So, what kind of the nature of the volumetric heat source is decided by the actual practical problem or in pertaining to the welding processes. First is that whether conduction mode heat transfer, definitely we will try to focus on too much of this volumetric heat source model in the case when we are looking into the conduction mode heat transport problem.

Second is that keyhole mode it is possible to develop also that volumetric heat transport can be in corporate in case of the keyhole mode heat transport problem, but this problem has to be treated in a different way. It is also possible to develop because keyhole model heat transfer model in this case is the keyhole formation or shape of the keyhole it is not exactly a very smooth that can follows the kind of the regular geometry shape like ellipsoidal, double ellipsoidal kind of conical shape.

But it may not follow these things. So, most of the cases keyhole shape or volume of the keyhole can we assume that adequate arbitrary volume. So, in this case if it is it follow kind of the arbitrary volume then what we can develop the heat source volumetric heat source model in when there is a generic the keyword that. We will discuss in this one of the particular in this module also later on.

Third part is the material content surface active elements definitely suppose metal is having surface active elements in this case is that possible to explain this effect of the surface activity element just looking into the conduction. Conduction based heat transport problem. So, I think in that cases I say that it is no because presence of the surface active elements alter the nature of the metal flow pattern.

So, in that sense I can say that presence of the surface active element is more influence on the material flow behavior. So, in that cases it is necessary to look into the material flow. That means, we have to look into the fluid flow phenomena in a welding process in that cases not necessary explicitly to develop some kind of the heat transport model in this particular case fourth case maybe dissimilar material combination.

So, in this cases we are discussing this thing, but if there is a two different material if you want to join. So, two different material the thermal diffusivity of the two different metal may be different.

So, in these cases the shape of the volumetric shape of the heat source may not be the symmetrical as compared to the similar kind of the material when you are joining or when you are developing some kind of the source model. So, definitely dissimilar material model how we can handle or develop the heat source model.

In case of the dissimilar material we will show also that of course, we can say that in double ellipsoidal heat source model although we are taking the non symmetry energy distribution, but double ellipsoidal heat source model we are taking the non-symmetry energy distribution just looking into the because of the welding velocity, but dissimilar metal welding that there is another non symmetry.

That means, welding velocity these accounts on kind of the non symmetry energy distribution at the same time two different materials also account some amount of the non symmetry energy distribution.

So, both of this non-symmetric energy distribution we can takes care of the different way simply extending the double ellipsoidal heat source model to the quadruple ellipsoidal heat source model, but apart from this thing there is one issue that what we can estimate the selection in heat source parameters.

So, based on that different heat source model has been developed and people are working to finding out what is the different way to count or to account the different heat source parameters. I am talking about the heat source parameter means suppose we define the volumetric heat source what do we can take the different size the semi axis length in case of the ellipsoidal or double ellipsoidal heat source models.

That is the one particular issue associated with the heat source model that we already explained this thing double ellipsoidal heat source model water you can account the heat source parameters and that induce some amount of the uncertainty in the heat source model development.

So, to overcome these things it is possible to certain extent to some extent takes care of by developing the adaptivity in the heat source model both in the linear modeling process we will discuss the linear spot welding process the adaptive heat source model.

But finally, it comes to this one that even. So, within these issues associated with the selection of the heat source parameters that drives to develop some kind of the new heat source model says that the model parameters can be reduced in this new heat source model. We will show that if you in a particular development of the new heat source model in this case.

The a configuration in heat source model there is in need of the we can reduce the one parameter needed as compared to the double ellipsoidal heat source model, but finally, should it follow the distribution over the regular geometric set I say no not necessary all as we have to fill regular geometry even if you look into the arbitrary shape of the volumetric, it is possible to develop the volumetric heat source as in the arbitrary shape that will.

So, and in particular to the keyhole mode laser welding process. So, in distributed volumetric heat source model we will start with the ellipsoidal heat source model.

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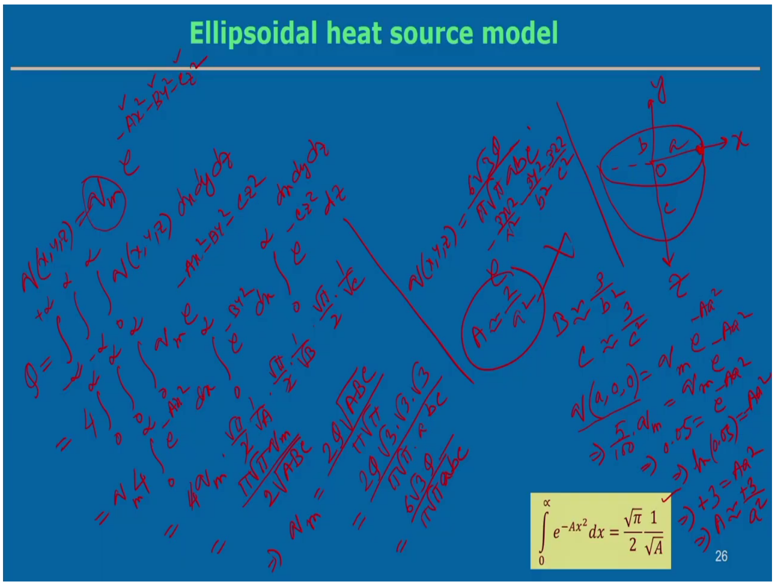

So, ellipsoidal heat source model we can see that in case of the ellipsoidal heat source model that we normally using gives the spot welding process we can see an ellipse kind of this thing as being that these are the 3 different axis. So, semi major axis a y axis b and z it is the length semi axis length equal to c.

So, abc is the axis length semi axis length in case of the ellipsoidal heat source model, so it is a volumetric heat source model. Now as in the heat flux in disability it can be represented the volumetric heat flux intensity the q x y z, e to the power minus A x square minus B y square minus C z square. Suppose this is the distribution of the heat flux we assumed the Gaussian distribution the qm is the maximum intensity at the center point o and A B C capital A B C are the distribution parameters.

So, capital A B C though there are two things what we can estimate the maximum flux and what we can estimate the A B and this distribution a xyz are the variable in this case. Now first we can look into that what we can estimate the maximum heat flux intensity.

So, same kind of this thing what is the total heat input over this volume, over this we have defined the geometric shape as the ellipsoidal says. Now about the geometrics of water we can estimate the total heat input assuming this is the volume.

So, we can say that total heat input in this case Q equal to. So, integration over say if you see the extent x axis in this case minus infinity to plus infinity because y axis also, we can see minus infinity to plus infinity and z axis we can say 0 to infinity and q x y z. We can say the element d x d y d z over the volume the elemental volume dx d y and dz now we can see the symmetric kind of 0 to, we can convert all 0 to infinity.

0 to infinity I want the x and y axis and z axis also 0 to infinity and see this 4 term will be there to symmetric minus infinity to plus infinity is equal to 2 into 0 to infinity because of the symmetric nature. Now q x y equal to q m e to the power minus Ax square minus v By square minus Cz square its a dx dy and dz.

So, we have to do this integration now first thing is that what we can estimate the distribution parameter there are two things are there what we can estimate the distribution parameter and second thing is that intensity we can find out. So, let us look into do this thing.

So, 0 to infinity 4, I say that q m is the keep outside q is a constant in this case at the center for the intensity then e to the power minus A x square dx we just separate intrigant 0 to infinity it will e to the power minus B y square dy 0 to infinity e to the power minus C z square dz.

Now, 4 q m, so integration if we look into this definite integral 0 to infinity minus e to the power Ax square dx equal to root over of pi by 2 into 1 by root A. So, apply this formula here also. So, it indicates that root over of pi by 2 1 by root A.

Similarly for y root over of pi by 2 into 1 by root B root over of pi by 2 into 1 by root C this indicates the total equal to 4 sorry 2 2 4 balance pi root pi q m by twice the root over of A B C. So, that is equal to q or I can say that q m equal to the maximum intensity twice q root over of A B C and by pi root pi.

In this case we have considered that 3 different distribution parameter that is flexibility is there because we are assuming the distribution parameter along x axis is different a distribution parameter A along y axis it is B and along z axis it is C.

So, we are considering the distribution parameters can be incorporate in the different way not exactly we are assuming the same distribution parameter for along x y and z axis. So, that kind of flexibility is there, but point is that what we can estimate the distribution parameter A, B and C the different ways.

So, simply assuming the same analogy that the maximum intensity falls exactly is a 5 percent on the boundary. So, intensity on the heat flux intensity on the boundary indicates the it is a max the 5 percent of the maximum intensity. With this condition we can find out what is the value of the A B and C, let us look into the what are the value of A B C in this case.

So, we can consider one case for example, that q equal to say at this boundary a. So, q equal to x equal to a y equal to 0 z equal to 0 then in this case q m e to the power minus A a square and e to the power other term will be the for y and z component it will be to the e to the power 0 1 e to the power 0 equal to this thing. Now this value equal to q at the this at this boundary value that heat flux intensity of the boundary along the x axis.

So, that value is the 5 percent of the. So, 5 divided by 100 5 percent of the maximum intensity equal to q m that is equal to q m into e to the power minus A x square. So, from here we can see the 5 by 0.05 equal to e to the power minus A x square. So, we can take the logarithm ln 0.05 equal to minus A x square. So, it is approximately minus 3 equal to A x square.

So, we can say that A is basically approximately A equal to minus sign here also minus sign this will be plus, so plus 3 by a square. So, A equal to it is clear 3 by a square similar analogy we applied the earlier also when you try to estimate the surface heat flux intensity the same analogy we can apply also.

Then if we put the similar condition for the y axis and similar condition for the z axis then we can reach this kind of expression that B is approximately 3 by b square and C is approximately 3 by c square. So, it means that this distribution parameter is actually vary depending upon the value of the different saving axis length.

So, if A, B, C are same equal then we can see that if A, B, C are equal then distribution parameters are same because that we also observe in case of the circulars kind of the heat source model in that cases the A was equal to B A equal to B. So, that then it is a single distribution parameter 3.

But this B depends on this value of the what is the semi axis length basically from the centre to the what is the boundary the distance depending on that the A, B, C distribution parameter can vary also. So, we can estimate what is the maximum intensity value in this cases putting the value twice q root over of A, B, C. A, B, C means root over of 3 3 3 will be coming by pi root pi and here it is a b and c.

So, finally, it will be coming 3 6 root 3 q by pi root pi a b c. So, this is the maximum intensity exactly at the center point in case of the ellipsoidal heat source model. So, that we can estimate these things.

Now, a b c is the distribution parameter in this case. So, what it changes the distribution equation if we see that the heat q equal to x y z equal to q m 6 root 3 q by pi root pi root pi A, B, C into e to the power minus 3 x square by a square minus 3 y square by b square minus 3 z square by c square.

So, now, we can get the complete equation of the volumetric heat flux density distribution in case of the ellipsoidal heat source model. So, that indicate the 6 root 3 q pi root pi a b c into e to the power minus 3 x square by a square minus 3 y square by b square minus 3 z square by c square.

So, remember if we change the boundary condition actually the this thing. So, this distribution parameter can be different in these cases the heat flux this distribution can be different also depending upon the z, what we are putting the boundary interaction; that means, what in this case we assume the 5 percent of the intensity falls auxiliary on the boundary which change this intensity with the less than that or more than that then this distribution parameter will change and the equation will be different also.

So, depending upon the situation or problem we can make the different this thing may be. For example, when you use the double ellipsoidal heat model it is better to use this thing this expression the intrinsic flux q equal to 6 root 3 q by pi root pi ABC into e to the power minus 3 x square by a square, but if you change it.

For example, if you change it then you find out the distribution parameter not exactly we can find out the distribution parameter is maybe 2 by a square something like that a distribution parameter. So, then if you see the if you do the similar calculation also then your expression will be completely different if you change the distribution parameter.

So, that is be careful into that in this case that, if you try to understand that what a the distribution varies and what we can change the parameter also depending upon the problem we can also do that this thing. So, this way we can estimate the ellipsoidal heat source model.

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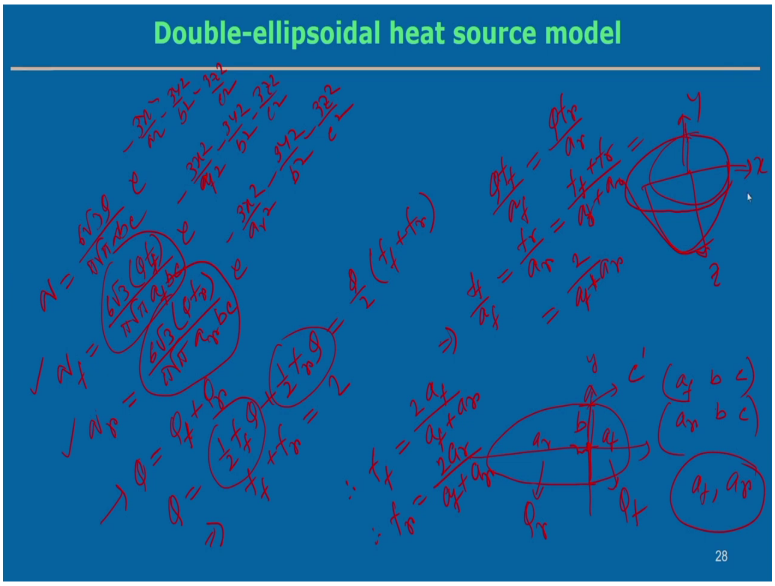

Now we can see what we can estimate the double ellipsoidal heat model. So, a ellipsoidal heat source model how what we can be extend to double ellipsoidal heat source model. So, now, that we see that we use some ellipso having the axis line a, b and c, but in case of ellipsoidal model is more suitable in case of the stationary heat source model, but suppose there is a heat source is moving on particular direction for example, this is the x axis and this is y axis and z axis.

So, now heat source is moving along x axis. So, definitely the energy distribution in the front part energy distribution from the rear part will be completely different. So, to account these things simply merging of the two parts of the ellipsoidal. So, that means, front part of the one ellipse and the rear part is a part of the another ellipse can be merged together then it is possible to develop the double ellipsoidal heat source model.

So, it is possible to extend the double ellipsoidal heat source model from the calculation of the ellipsoidal heat source model that we will see. So, suppose a f b c is the part of the one ellipse a r b and c may be other ellipse. So, it is like that only with respect to y z plane and since if heats source moves in the arc is moving on x direction.

Then we have to configure the two ellipse may be part of the one ellipse this is the one major semi axis length other cases semi axis length or this one and its merged together. And at the same time this part of the one ellipse and this is the x axis this is y axis, and this is a part of the another ellipse.

So, that is the so this is called a f, and this is a r and b is the other axis and c is the depth. So, in this cases c is the along the z axis. So, in these cases there is a variation of the ellipse in the this a f and a r two part of the two different ellipse, but b should be the same for both the ellipse.

So, when you merging together and of course, it should follow the C 1 continuity C 1 continuity means at the center point the slope should be same when you was merging this two slip and the dimension is the same b for both the ellipse. So, that it will try to maintain the C 1 continuity and makes the double ellipsoidal heat source model.

So, definitely in this case we have seen that q we can estimate this thing double ellipsoidal heat source model and we can see that q equal to heat flux equal to 6 to 3 q by pi root pi A, B, C this is the maximum intensity and e to the power minus 3 x square by a square minus 3 y square by b square minus 3 z square by c square this is a expression or intensity in case of the ellipsoidal heat source model.

But double ellipsoidal there is two ellipse. So, now, we assume this we can change it in the two parts say for the front part q f can be represent to the 6 root 3 and q can be there is say q f f power by b square 3 z square by c square, similarly rear part also we can represent like that 6 root 3.

So, q f r a r b c. So, e to the power minus 3 x square by a r square. So, this is the heat flux intensity of the front part and heat flux intensity of the rear part, but total heat input what is a total heat input we can estimate that can be divide into these two parts maybe not exactly the.

The front part what is the total heat input q and for the rear part total heat into q 1 and q 2, but we can make it as a fraction maybe f if we have introduced the f f and f r, f f is the fractional heat deposit in the front part because of the non-symmetric energy distribution and f r the fractional heat deposit in the rear part and of course, for the form part this is a part of the one ellipse. So, we use the a f term and rear part we use the a r term.

But it is a this instead of taking q total heat input equal to that what we have seen this thing in this case we use that q f f f r such that we can divide the total heat input into Q equal to Q 1 Q f plus Q r total heat input and if you balance these things do the integration similar way we can find out that total Q can be represented like that. That Q equal to half of f f into Q and plus half of f r into Q the total heat input.

So, 50 percent of that and then accordingly that we introduce the fractional heat front end care such that it can be Q by 2 into f f plus f r even if we do the integration also we can show the similar kind of equation f f plus f r equal to 2 in this case.

Now, definitely double ellipsoidal at the center point the maximum intensity should be the same even if we consider the front part may be one ellipsoidal and even if you consider rear part. Such that intensities will be the same at the center point it means that this maximum intensity this maximum intensity is this.

So, if that should be same then I can say the Q into f f by a f equal to Q into f r by a r it means that this f f by f f by a f equal to f r by a r and that can be represented f f plus f r divided by a f plus ar. Now from the energy balance also if you estimate even if you this q equal to q plus q r front and we have divide that energy into these two part this thing and the these two parts ar the f a plus f r equal to 2.

Even if you integrate over if you do the integration a over what the first part say this is the Q 1 Q r and this comes to the Q f. So, that means, total energy for the front part rare part if you follow this calculation similar wave integration over the domain we will be able to reach this kind of the expression f a plus f r equal to 2.

Now, f f plus f that can be written can be that f a plus f r divided by a plus a r. Such that we can see that equal to f f plus a r equal to this equal to twice and a f plus a r it means that. So, fractional of heat deposited can be estimated f f equal to I can say that a f twice f by f plus a r and fraction deposits to the rare part equal to twice a r divided by a f plus a r it means that.

So, as compared to the ellipsoidal single ellipsoidal heat source model if you look into the double ellipsoidal heat source model then we can see that 2 this term is comes into the picture, what is the fractional heat deposited in the front? And what is a fractional heat deposited at the rear part?

So, that we can estimate in terms of the other parameters. So, a f plus a f by a r twice or f are equal to twice b plus f by a r, that it means that a f and a r should be explicitly defined then we can estimate, what is the value of the fractional heat deposit of the front part? And what is the amount of the fractional deposit on the rear part?

Apart from the because this f f and a r is independent of the other two parameters in the independent of the b and c parameters. And why you have considered the f f and f r comes into the picture because we want to takes care of the non uniform energy distribution at the front part and rear part that is why we merge two different ellipsoidal and making the maintaining the C 1 quantity we can develop the double ellipsoidal heat source model.

So, this is the one of the issue that what is the value of the a f and a r should be considered in the double ellipsoidal heat source model that is the one particular issue and that enhance one need the that parameter is not explicitly defined until and unless it is a f and a r can be defined based on the experience or.

Directly the a f and a r parameter can be linked with the welding velocity and we can estimate the value of the a f and a r as a function of the welding velocity if we put it then we will be able to estimate what is the value of the fractional heat deposit in the front and rear part, but remaining the maximum remaining the intensity will we the same value, what we can estimate the ellipsoidal heat source model.

And, but this these things are extra in these cases to estimate the a f and a r value in case of the double ellipsoidal heat source model.

So, that is all today next class we will discuss the remaining that apart from the double ellipsoidal heat source model what are the other kind of the heat source model or, how we can extend the double ellipsoidal model to the quadruple heat source model with a similar analogy by introducing the fractional heat deposit in the front and rear part.

So, thank you very much for your kind attention.