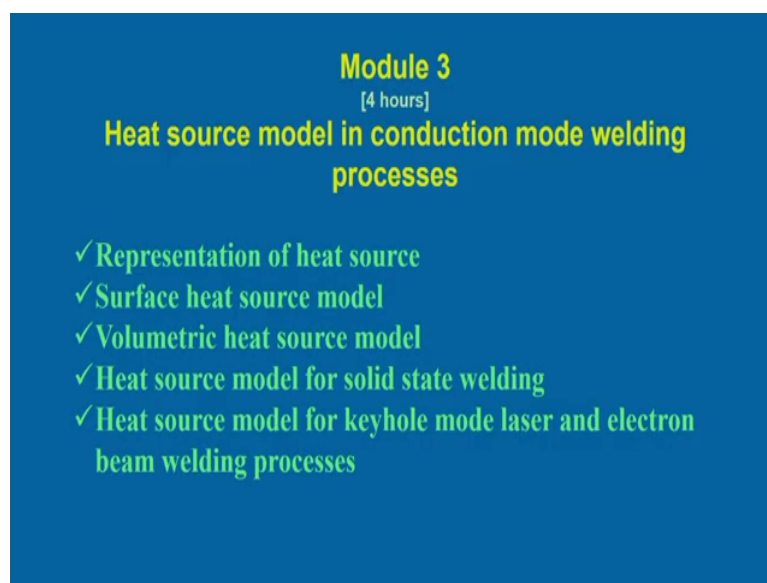


Finite Element modeling of Welding processes
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Module – 03
Heat source model in conduction mode welding
Lecture - 16
Introduction to heat source model

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Module 3
[4 hours]
Heat source model in conduction mode welding processes

- ✓ Representation of heat source
- ✓ Surface heat source model
- ✓ Volumetric heat source model
- ✓ Heat source model for solid state welding
- ✓ Heat source model for keyhole mode laser and electron beam welding processes

Hello everybody. Today we will discuss the 3rd module and that is related to the heat source model in conduction mode welding processes. Actually the one of the critical aspects associated with the mathematical modeling of the welding processes that the, what way we can represent the heat source? It is the simply heat source model means we understand that simply representation of the different sources of the heat.

Can it can be different for example, in case of arc welding it can be different from laser welding processes even electron beam welding process. Even in friction stir welding process all these cases it is necessary to define a correct representation of the source of the heat and in general that is called the heat source model associated with a welding process.

So, we will try to cover in this particular module is the one is first is the representation of the heat source what way mathematically what we can represent the different heat sources. Then second surface heat source model means what way the it is interacting heat source is interacting assuming the interacting only on the surface. And some cases the heat source is interacting not only on the surface.

Its a the amount of the heat energy is penetrate into the depth of the weld pool or maybe substrate material. So, that in that case it is necessary to represent this source of the heat as the volumetric heat source term. So, definitely we will try to look into all these different formation of the heat sources their mathematical equations and finally, what way we can integrate this heat source model in the modeling of the welding process.

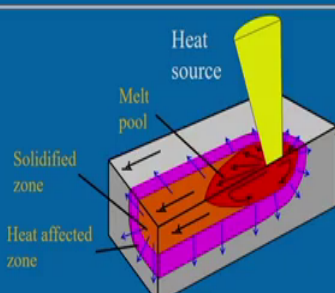
Apart from that we will discuss the heat source model for the solid state welding process. For example, in case of the friction stir welding process what way we can estimate the heat flux and then how we can implement in any finite element based numerical model? Then heat source model for the keyhole.

So, actually the keyhole formation of the keyhole mechanism this or maybe I say the presentation of the source of the heat or interaction with the laser with respect to substrate material is somehow different as compared to the conduction mode welding processes.

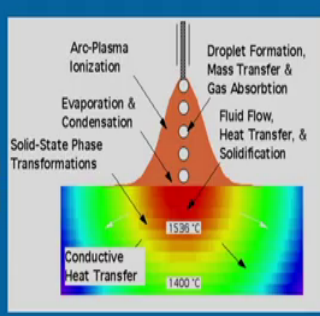
So, we will look into that aspect also that what way we can develop the heat source in case of the keyhole model or what way we can represent the heat source that in case of the keyhole modeling process as well as the electron beam welding process. So, in the light of the different heat source representation this module has been prepared.

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Introduction: Physical aspects of fusion welding process



- ✓ Heat flux from heat source
- ✓ Localized melting and solidification
- ✓ Flow of molten material



- ✓ Differential thermal expansion and contraction leads to distortion and residual stress
- ✓ Structural changes after solidification

If we look into the physical aspect of the welding processes that it comes that it is like that only, that there is substrate material is there. And from the source it can be there is a creation of the arc in case of arc welding process or it can be laser also the laser energy imports on this surface of the substrate material, but this energy can penetrate up to the particular depth, but it means that the laser is interact with the substrate material or the work piece material.

Then it creates the small weld pool; small weld pool which is the field why the molten material and remaining part it creates some kind of the heat affected zone with that heat affected zone with the temperature of the heat affected zone is below the melting point temperature.

So, there is it is not associated any kind of the molten state, but it can go through some kind of the solid state transformation phase transformation. Actually we defined the heat affected

zone just looking into the microstructure whether this particular structure is affected with the application of the heat energy or not. Based on that we decide what is the particular amount of the heat affected zone.

So, it is like that if there is a moving heat source only there is a continuous moving up the heat source one particular direction and accordingly the melting and subsequently solidification of the backward part may happen and then it creates the solidified structure. So, definitely the solidified structure must be different as compared to the base material.

So, typical characteristic of the heat source can be like that there is a in case of the any welding process it can be arc welding process of course, it can be laser welding process also it can be electron beam welding process. So, it is like that heat source interact with the material creates the molten pool and whether the molten pool that its molten pool is not stationary in practical because depending upon the different driving forces there must be some amount of the material flow happens from normally from centre to some outward periphery it happens during the small weld pool.

So, it means that movement of the material also happens within this weld pool. So, it creates in general we can define this weld pool just looking into the microstructure definitely we will be able to measure the microstructure also ah, but dimension of the weld pool just looking at the microstructure. The there is a solidified structure is different from the heat affected zone and different from the base material structure.

And then once the solidification happens definitely the solidification happens not in the welding process not as a constant cooling rate, in that cases the cooling rate can be different. Cooling rate means that rate of change of temperature it can be different under different zones, may be heat affected zone and the even this there is a variation of the cooling rate even within the weld pool also.

So, its representation is like that heat flux from the heat source. So, that heat flux applied to the heat source it means that the laser is interacting on the surface or arc is interacting with the surface or of the substrate material.

The mathematically what we represents that interaction; that means, what I can represents the neither mathematical form that can be representation of the heat source in a particular welding process. For example, it is like that only even into the arc if you see the arc welding process the intensity of the energy supplied to the substrate material may be intensity of the arc.

At the centre point it is maximum and if away from this thing there is gradually the intensity decreases. But, this we can represent this there is a variation at the centre point at the maximum to the minimum to the boundary. And boundary means at the particular zone in this within that zone it will creating the molten pool. So, therefore, this intensity variation can be represented in the form of a Gaussian distribution kind of equation we can follow this Gaussian distribution. It means that mathematically representation of the Gaussian distribution we will show what way we can represent the mathematical.

Just it means that any kind of the heat source we can show in the form of a distribution on the surface it can be on the surface. But at the same time the distribution may also happen or intensity may also happen within the volume of the weld pool. So, that is called the volumetric heat source so; that means, the distribution the intensity may not vary only on the over the surface the distribution may vary within the volume in that cases we can say volumetric heat flux.

That means we can estimate the heat flux per unit volume and their distribution and what may be the geometric shape of the weld pool; that means, over which the this intensity distributed. So, from these two point of view we can develop the different kind of the heat sources and that is the point of discussion in this particular module.

Now, localized melting and solidification happens in this zone and then flow of the molten material also happen during this process. And of course, even after the welding process; that means, even after the not only its associated only the heat transfer and the material flow within the weld pool, but at the same time there is a differential expansion and thermal expansion and contraction also happens during this process, that actually leads to some amount of the distortion in a welded structure. And then this distortion also associated or this distortion along with the residual stress is actually part of any kind of the welded structure.

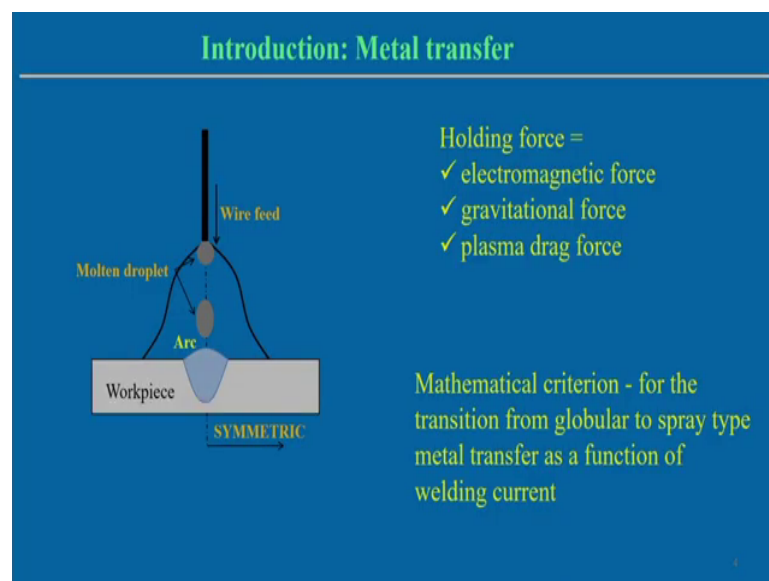
So, it means the structural changes after solidification also associated in case of welding process. So, in that case so why we are looking for this a different model of the heat sources because, correct representation of the heat source is basically helps to estimate even not only the temperature distribution, because at the same time this temperature distribution is also useful to estimate the residual stress and distortion in a welded structure.

So, in that from that point of view it is necessary to represent to understand the interaction of the heat source with the material. And then what way we can mathematically represents this interaction in the simple terminology that is called the heat source model that will be the main point of discussion in this particular module. So, see one cases we have shown that the interaction of the laser or arc with a substrate material, but the at the same time it is also possible that there may be some amount of the metal transfer. For example, if we look into the gas metal arc welding process we use the consumable electrode. So, from the consumable electrode so mass transport will be there.

So, in that cases that apart from that heat transfer and material flow at the same time it is associated with the mass transfer. And of course, these mass transfer or thermal behaviour will also influence by the nature size of the droplet; that means, molten droplet from the consumable electrode and shape size and their impact velocity on the to the substrate material and what is the influence of the shielding gas all actually matters to if we try to look into very precisely the modeling approach of a particular welding process.

But we will try to look into in general in this case the heat source representation its normally excluding the effect of this the metal transfer. Because representation of the heat source is simplified in the sense that only the looking into the in the form of a heat flux intensity to the substrate material without considering the metal transfer all these things phenomena.

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So, here we can see the metal transfer the associated even with the welding process one when we using some kind of the consumable electrode. So, holding force you can see that it is associated some kind of the electromagnetic force, because there is a flow of the current and that will create some kind of the electromagnetic force even in case of the arc welding process. And gravitational force also there the molten material gravitational force and plasma drag force that mean within this medium the that everything the metal transfer happens within the shielded material.

So, plasma drag force electromagnetic force and the gravitational force all are actually influencing parameter to decide the criteria of the molten material detachment their shape and size that will transfer to the actual the to the work piece. So, therefore, this is the one particular direction of the study and with there is one module also we will discuss the, what way we can model the metal transfer mechanism in case of the welding process? So, therefore, mathematical criteria for the transition from the globular to spray type of metal transfer we have already explained that there may be the different pattern of the metal transfer.

So, what may be the mathematical criteria, such that the transition from the globular to the spray type of metal transfer. And as a function of welding current that can be one site of investigation and that is required and that improves the modeling capability during the welding process when it is associated with some amount of the metal transfer. But this metal transfer phenomena is not included in the development of the heat source model.

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Analysis of heat flow

Heat conduction – **Fourier's law**
Non-Fourier heat conduction
Representation of heat source
point, line and distributed

Heat transfer and fluid flow – **Energy transport**
Navier-stokes equation

Keyhole formation – **Laser welding**
Plasma welding
Electron beam welding

Evolution of liquid/vapour interface over time considering the effect of interfacial phenomena like evaporation, homogeneous boiling, and multiple reflections

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Now, in general before starting the heat source model we have to understand that and the what perspective on what way we can simplify the heat source model and what way we can apply this heat source model in an actual problem. So, in general we know that if we want to know what are the temperature distribution in a substrate material during the welding process.

So, we simply use the heat conduction equation that is the Fourier heat conduction equation normally solve. And of course, and then after to get the temperature distribution, but in this case if we considered temperature distribution based only on the heat conduction equation. It means that we are not considering the material flow inside the molten pool, which is maybe significant to alter the actual shape and size of the weld pool.

So; that means, the heat source model is actually more significant specifically I am talking about the volumetric heat source when we are using only the heat conduction analysis. So, in this cases if we neglect the metal transfer that part there is a lots of scope of the development of the heat source model under the frame of the conduction based heat transfer model in case of welding process.

So, we normally use the heat conduction equation the Fourier's law of heat conduction, but in some cases when there is ultrasonic pulse laser is associated then in that cases we use the non Fourier heat conduction model. And of course, in these cases there is a need to represent the heat source. So, that is why there is a representation of the heat source is required to if you want to solve this heat conduction equation. This representation of the heat source can be incorporate in this conduction wire heat transfer analysis either in the form of a surface flux and that is normally associated with the boundary condition and or we can represent either volumetric heat flux.

So, that is associated or volumetric heat source and that is associated with the governing equation there is a one term that is the internal heat generation term. From that term we can incorporate the internal this volumetric heat source term to solve the heat conduction equation to get the temperature distribution. So, these representation of the heat source there is a very simplified when we started the simulation of the welding processes. So, only looking into the analytical solution then started with the point then representation of the line source and then finally, nowadays we represent this heat source model in the form a distributor heat source model.

Which is more realistic approach which depending upon the practically if we observe the welding process and their source of the heat, it is actually that distributed heat sources. But, to simplify the calculation we can start with the point and line heat sources. Now, not only heat conduction analysis it is also necessary to some extent the heat transfer and fluid flow analysis associated with the link process in this cases the energy transport equation need to be solved and; that means, it is associated with conservation of the mass momentum and energy.

And this momentum equation is normally follow Navier-Stoke equation if we solve they will be getting the both temperature distribution as well as the material flow field. So, in this case its not necessary explicitly to define or to develop some kind of the heat source model, if we do the heat transfer analysis heat transfer and fluid flow analysis. Because the fluid flow is the more realistic phenomena it is associated with this the welding process.

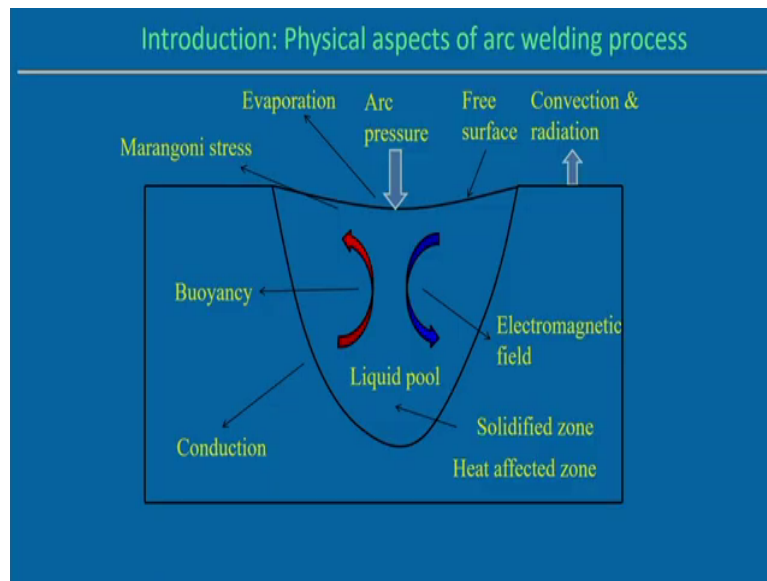
Therefore once we incorporate the heat fluid flow analysis then it is not necessary to consider the explicitly defining that volumetric heat source or volumetric heat source model in this case. Now, if we look into the keyhole formation which is normally associated with the laser welding process not only laser welding process even in plasma welding process and as well as the electron beam welding process this keyhole formation is associated with this thing.

Keyhole formation is nothing, but the there is a formation of the liquid vapour interface over the time considering the effect of the interfacial phenomena. For example, the evaporation homogeneous boiling and the multiple reflection associated with the laser. And that make the dynamic balance of the liquid vapour interface creating the this interface within during the laser welding process

So, that beyond that liquid vapor interface that it is filled with the vapour of the particular material. So, in this keyhole mode laser welding process not only laser welding process even plasma welding processes can create the keyhole formation. But in this case the maximum temperature is much maybe above the boiling point temperature of a particular material. So, actually keyhole formation is necessary when we try to achieve the very high depth of penetration although there are some amount of the material loss, but finally, it is possible to achieve the joint strength when there is a need of the high depth of penetration.

So, we will see also, what way we can represents the laser material interaction in case of the keyhole mode laser welding process as well.

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Now, once we talking about talking about the conduction mode heat transfer analysis in this case there we are not considering the liquid molten material flow. And which is there are so many driving forces the material flow if we look into the all the driving process then we can get the typical shape of a well profile like that. Of course, in this case there is no addition of the electrode or no extra electrode material in this case.

So, this is the typical shape of the weld pool, we can see that there is electromagnetic field will be acting in case of the arc welding process, but in case of laser welding process we neglect the electromagnetic field, which is not associated with the electromagnetic field. So, then electromagnetic field is one of the driving force to drive the molten material from one point to another point within the weld pool. Apart from that buoyancy force although

magnitude of the buoyancy force is very small due to the density differences, but its having some influence on the molten material flow within the small weld pool.

But main the contributing force or maybe main driving force for the molten material flow is the Marangoni stress or maybe I can say the surface tension force acting between the two interface the liquid medium and the shielding gas so, other medium. So, gaseous medium in the liquid medium at the interface it must act some amount of the surface tension force. So, surface tension force is the most significant driving force to drive the molten material within the small weld pool. And that is why if you want to know the what is the flow field within the molten material flow then we need to necessary to do the fluid flow analysis along with the heat transfer analysis.

And of course, from the surface it is the free surface its not exactly straight, even if we use the consumable electrode if we do not use the non consumable electrode then surface profile may not be very flat. So, some sort of curvature will be associated with these things because during this welding and solidification process there may be the expansion and the introduction of the shielding gas flow and arc pressure also there shielding gas pressure also there that influence the free surface profile in a welded structure.

So, apart from that from the outside surface there is a convection and radiation heat loss from the outside surface and then it creates the this is the molten pool liquid pool is associated with the solidification. Because it is solidified then we will get the solidified structure, but remaining part up to certain extent you will get the heat affected zone and remaining will unaffected base material.

So, these are the typical structure and where there is a need of the material flow associated with the welding process ah, we have to look into that aspect.

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Introduction: Physical aspects of solid state welding

- ✓ Frictional heat generation – sticking and sliding
- ✓ Plasticization of material (stirring action)
- ✓ Heat conduction and material flow
- ✓ Solid state phase transformation
- ✓ Distortion and residual stress

Now, if we look into the solid state welding process it is also associated with some amount of the heat generation definitely the normally frictional heat generation is there. And in this case the frictional heat generation theory can be associated with the sticking and the sliding friction condition.

And based on them whether we can apply either surface flux in the heat generation can be applied as a surface flux or heat flux can give the volumetric flux, but in these cases maybe very thin layer of the shear layer is maybe associated with it when there is a finite amount of the shear layer thickness is there, then we can assume some kind of the volumetric heat source and that can be this can be represented by developing a particular volumetric heat source associated with the solid state welding process.

But apart from that the plasticization of the material is there. So, that during the plasticization of metals it may generate some amount of the heat also and heat conduction and material flow, but in these cases the heat conduction is also associated because frictional heat generation is there.

So, heat will be conducted then in these cases it is also necessary to solve the heat conduction equation and then apart from that material flow also there, but in this cases material flow happens due to the plasticization of the material normally in case of ssw solid state welding process one of the solid state welding process we assume the viscoplastic material flow. So, which is not the material flow not like the molten material flow which we normally considered in case of the fusion welding process.

Apart from that the some parties associated with some solid state phase transformation effect and it may also associated some amount of the distortion and the may be magnitude may be small, but certain amount of the distortion and residual stress is associated even in case of the solid state welding process. So, all these phenomena if we analyse there is a link from these things, but we start with the even if we want to know what is the residual stress in a welded structure.

Then it is also necessary to do conduct the thermal analysis then once we want to know we want to do the thermal analysis then it is also necessary to define the interaction of the heat source with the material. And that can be represented in a different form of the heat source model. So, therefore, in any aspect if we want to mathematically represents or if we want to analyse any kind of the welding process we want to know the end result either in the metal flow field or maybe in the form of a residual stress and distortion field it is also necessary to do all the cases the thermal analysis is needed.

And once we want to do some kind of the thermal analysis then correct representation of the heat source is basically useful to increase the prediction capability of a particular mathematical model associated with the welding process.

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Introduction: Significance of heat transfer and fluid flow model

Condition based model - heat source model
Fails if material contains surface active elements

Consider only heat transfer and fluid flow

- o momentum transport due to
 - surface tension force (material specific)
 - buoyancy force
 - electromagnetic force (current)
- o solve conservation of mass, momentum and energy equations

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Now, significance of the heat transfer and the fluid flow model that what way we can differentiate between the conduction based model as well as the fluid flow model. So, conduction base model then in this definitely there is a we will be using all of the heat conduction equation along with the boundary proper boundary conditions or maybe to tension problem. So, initial condition may be required in this case.

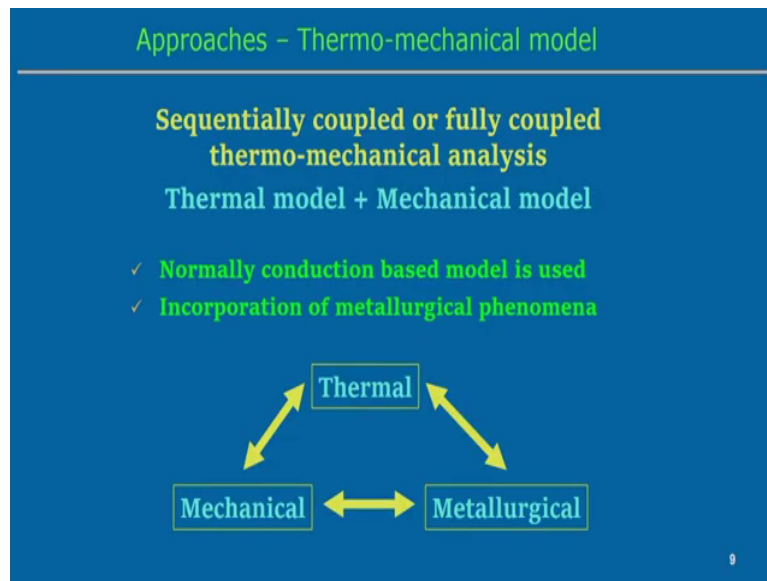
But whatever we do heat conduction analysis it is necessary to define very precisely to define the heat source model. So, heat source model is more important more associated with the conduction based model. But if we do not focus much on the different heat sources model then in that cases we can it is possible to do combine that heat transfer and the fluid flow model together. And in this case it is necessary to solve the momentum transport equation due

to the surface tension force already explained this in buoyancy force and electromagnetic force.

This these are the main responsible driving force for the momentum transport in case of the heat transfer and fluid flow model and necessary to solve conservation of the mass momentum and energy equation. So, basically the energy equation the in this case when we consider the heat transfer and fluid flow equation in this case the energy conservation equation is also associated the material flow term.

So, that convection term is important in these cases to that actually influence the temperature distribution as well also. It means that even we consider the heat transfer and the material flow analysis then in that cases the temperature distribution is also modified by incorporating the effect of the material flow. And, but in this in the only heat conduction based model there is no scope of the consideration of the material flow then heat source in that case is correct representation of the heat source is the most important part to get a good result associated with the welding process.

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But normally we know that even thermo mechanical model we can do the analysis the sequentially coupled or fully coupled thermo mechanical analysis will in case of in welded structure. And that case is the combining the integration of the thermal model as well as the mechanical model is also required, but the mechanical model is most dependent on the thermal model in this case; that means, if the results are very accurate or accuracy of results are there in case of the thermal model then we can expect the mechanical model prediction capability of residual stress and distortion will be good enough.

But apart from this thing thermal model and the mechanical model in this case when we do the develop the model thermo mechanical analysis then we normally use the conduction based model analysis. Because thermo fluid then thermal and fluid flow model is basically computationally expensive. So, in that case even if we thermo mechanical model also computationally expensive because stress analysis model we can get the three degrees of

freedom stress analysis in each and every node point. So, in that sense we normally to reduce the computational time we normally use the only thermal model not the combining thermal and the thermo fluid model.

So, therefore, thermal mechanical model can be combined, but apart from that even there is a scope to incorporate the effect of the metallurgical aspect also micro structural changes or metallurgical aspect can be incorporated and that can enhance the results of the mechanical model. So, that combining thermal mechanical and metallurgical model is also possible in case of the welding process.

We will discuss in detail that what way we can incorporate the different metallurgical effect and in basically when we explicitly explaining the thermo mechanical model in a particular different module. But point of view that we are analysing even if the thermo fluid thermo mechanical model. So, in this case the everywhere there is a need of correct representation of the heat source; that means, heat source model once we are considering only the conduction based heat transfer analysis.

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Significance of heat source model

Solution for the conservation of Thermal Energy

$$\frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + \dot{Q} = \rho C_p \frac{\partial T}{\partial t}$$

Boundary Conditions

$$k_n \frac{\partial T}{\partial n} + q_s + h(T - T_0) + \sigma \epsilon (T^4 - T_0^4) = 0$$

Symmetric surface $\rightarrow \frac{\partial T}{\partial y} = 0$

$$q_s = \frac{P \eta_{\text{gau}} d}{\pi r_{\text{eff}}^2} \exp \left(-\frac{d \cdot x^2}{r_{\text{eff}}^2} - \frac{d \cdot y^2}{r_{\text{eff}}^2} \right)$$

Surface heat flux or volumetric heat flux ?

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But what way we can incorporate this heat height source or how the heat source model is basically in this cases influence the different model calculation. And we can see the solution for the conservation of thermal energy is required to in case of even in case of conduction mode welding.

And if we see there is equation the heat conduction equation we have simply written three dimensional heat conduction equation. In this case there is a term the x y and z the k is the thermal conductivity and Q dot is the internal heat generation term and rho C p specific heat and del T by del small t; small t indicates the time variable in this case and capital T represents the temperature variable.

So, if we wants we solve this equation along with the boundary condition then we will be able to get the what is the temperature distribution. Even in a if you the solution even if we in

particular the welding process and then analytical solution is basically difficult if we consider all the realistic phenomena distributed heat source then getting the analytical solution is little bit difficult. So, in this case we normally use the some kind of numerical modeling approach to solve this particular heat conduction equation. But, what is the significance of the heat source model in this particular when we try to solve this heat conduction equation?

Now, try to look into this representation of this problem welding problem that what way the heat transfer phenomena we can analyse this thing. So, suppose this is the domain, I think I have showing the two dimensional domain to make you understand easily. So, this domain the heat input from the arc. So, heat input from the arc you can see there is a distribution can be like this. So, at the centre it is the intensity is the maximum and gradually it is reducing this is some simply it can follow the Gaussian distribution.

Then apart from the remaining part there will be the heat loss by the convection and radiation and the from the top surface and remaining other surfaces. And bottom surface also if it is open then convection radiation otherwise if can be attached with some kind of the fixture. So, heat will be some kind of the conductor to the fixture so some kind of the contact resistance will be there.

So, the anyway there are some heat loss from the different surfaces all the surfaces in mathematically we can represents. So, then in a compact form in general form this boundary this all the surface heat loss from the boundary and the heat interacting on the through the boundary all can be represented in the form of a boundary condition. Now, just look into this equation of the boundary condition.

So, this first term this $k n \frac{\partial T}{\partial n}$, it means that within this domain we are solving the heat conduction equation, but at the boundary what is the heat conducted exactly at the boundary? So, what is the heat conducted to the boundary? The same amount of the heat in the steady state situation same amount of the heat will be lost by the this is the convection term and this is the radiation, this term was the heat loss by the convection and radiation.

And q_s minus q_s terminology it indicates this is the heat flux it indicates the q_s . So, this is input to the domain so that is why we are taking the minus sign and heat loss we can assume the outward from the domain. So, therefore, we consider this is the plus and then all are making balance, what is the heat conducted exactly at the boundary? So, in the combined form this boundary condition can be written in this way.

So, in this case the heat source through the boundary condition we can incorporate the effect of the heat source because from the boundary condition we can replace the q_s with the heat flux so this heat flux is interacting through the boundary so; that means, definitely it will be acting over the surface. But in some cases there may be need not only the surface flux may not be sufficient to represent the particular welding process or to represent the heat source and the material interaction. In that cases it is necessary to represent then not only the heat flux interacting only on the surface rather, we can say the volumetric flux which is also acting over the particular volume.

In this case when you try to represent the volumetric heat flux then we can incorporate the this volumetric heat flux through this term this governing equation. So, it is mathematically it is not actually at the internal heat generation rather, we can use this term internal heat generation term to incorporate the volumetric heat source through this term; that means, through this governing equation.

So, in general we can say there are two option to incorporate the heat flux or heat input to the substrate material in the mathematical sense. One is through the boundary condition at the surface flux or through the heat internal heat generation term associated with the governing equation and that is the this internal heat generation term is the volumetric heat. So, these are the two ways to incorporate the heat input to the substrate material. So, we will see subsequently that what way we can represent the surface flux as well as the volumetric flux.

Now, there are other conditions for example, the this surface flux can be represented like that; that means, surface heat flux. So, we can say that this is the equation for that P and some efficiency and effective term; that means, effective radius of the arc and then P is the power,

that what it may be arc it may be laser and they exponentially d is the distribution parameter and it is varying with respect to x and y coordinates and r effective is the effective radius of the arc.

So; that means, it is a function of the distribution parameter and then we can estimate this p the effective heat input. Actually, what are the when there is a creation of the arc what is the input voltage and current. The multiplication of the voltage and current represents the what is the power source is from the source this is the power. But not all power all energy may not go into the substrate material there may be some loss also and even there is a creation of the arc there is a huge loss in the due to the radiation term.

So, therefore, effectively some amount of the arc energy will be passes to the substrate material and that will try to melt the work piece material. So, that effective power can be calculate in the different way. So, we will show this is the typical distribution so then we exponentially decaying direction so; that means, its intensity is the maximum at the maximum part.

Then it is gradually decreasing to the out area and that we make a boundary and that boundary define this is the effective arc radius and up to that we can define, what we can calculate the what is the maximum intensity what are the distribution parameter. Actually, the distribution parameter is defined whether this arc can we represent the very stiff or shallow type arc that has to be decided by the distribution parameter.

We will see what way we can estimate the distribution parameter there are the different strategy to different estimation parameter. Now, point is that although we can represent the heat input or maybe heat source in the form of a surface heat flux or in the form of a volumetric heat flux. Now, at which cases we can apply the surface heat flux or which cases we can apply the volumetric heat flux that we will try to look into that aspect.

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Heat source representation of RSW

Governing equations: Heat conduction equation

Boundary condition

$$k_n \frac{\partial T}{\partial n} - q_s + h_{eff}(T - T_0) = 0$$

Initial condition $T(x, y, z, t = 0) = T_0$

Surface heat flux at the EL/WP interface

$$q_s = \pi r_e^2 j^2 R_{ce}$$

Bulk resistivity (volumetric) $\dot{Q}_v = j^2 R_b(T)$

Volumetric heat flux at the faying surface

$$\dot{Q}_f = \frac{\pi r_f^2 j^2 R_{cf}}{t_g}$$

Heat generation due to contact resistance occurs at the contact surfaces as well as bulk electrical resistivity due to current flow.

J - current density, R - resistance
 t - thickness, r - radial distance

For example, we take an example of the resistance spot welding process. So, in principle we know the resistance spot welding process acts like that there is a two sheet metal is there. And there is electrode is attached with the electrode force between these two sheet metal and there is application of the current for a small time such that at the centre point there may be some heat generation at the interface.

And then it will melt this particular zone and then finally, it will create the weld pool at the interface is associated with the particular process parameter maybe the electrode force the current electrode current and the amount of the heat generation depending upon the contact resistance between the surface.

But, at the same time some heat generation can be here also in the between these two surfaces because this is also contact surface. So, contact resistance will be there and then heat

generation will be there. Now, there are different resistance we can represent there are so many resistance the resistance can be at the contact resistance or it can be bulk resistance also.

So, the bulk material volume is there. So, that will even passage of the current it will create the. So, bulk resistive because of the bulk resistivity. So, bulk resistance is there also inside them. So, there are so many resistance components are there, but we will see that our objective here in this case that what way we can represent the heat source which cases the whether it is surface heat flux or volumetric heat flux that we will try to look into that aspect. Similarly, other part also there is some heat loss from the remaining part so heat loss from the convection radiation.

So, in general the solution domain can be represent the similar way only thing is that we have to look that what way we can represent the heat source; that means, how the source of the heat is interacting during this process and then accordingly we can decide the boundary condition. And, in this case is boundary condition also similar kind of boundary condition we can apply because through the boundary there is a loss of the heat by the convection radiation. But, what may be the what way we can input the heat in this case?

So, let us look into thus this equation the we similar heat conduction equation it is necessary to solve in this particular case also and then boundary condition we can represent the similar way this is heat conducted exactly to the boundary the boundary surface. Then heat input and the effective means we can it is also possible to consider the combined effect of the convection and this is the another form of the to take in consider the combined effect of the convection and radiation heat loss.

So, in terms of this we can find out the effective heat transfer coefficient that consider both the effect of the convection and radiation from the surface then we can represent this kind of equation. And of course, it is a transient problem then it is necessary to define some kind of the initial condition. Now, in this case we can see the surface heat flux at the electrode and

workpiece interface. So, this is the electrode and the workpiece interface. So, definitely intimate contact is there.

So, that represent the surface flux in this particular zone; that means, surface heat flux. So, that we can estimate what is the surface heat flux at this particular zone and it is associated with the this current density and of course, the contact resistance between these two parts.

So, we can estimate the typically this πr^2 effective radius the electrode and the current density and the contact resistance that indicates the what is the surface heat flux. And this surface heat flux normally it can be uniform or it may not be uniform also it can vary from centre point to outer periphery depending upon the welding condition or situation.

Now, it means that we can apply the heat flux on the surface, but at the same time bulk resistivity which is bulk resistivity the resistivity of the material to current flow. So, therefore, that bulk resistivity is basically when there is a passage of the current through the electrode even within the workpiece material also. So, the current flow that will create some amount of the bulk resistivity and that will be acting over the volume.

So, we can estimate this volumetric then the bulk resistivity has to be represented in the form of a volumetric heat. So, then current density and then this resistance which is normally the bulk this resistance term is, R is the resistance that is a function of the temperature and if we know this data is available particular material we can estimate the volumetric heat flux in these cases.

So, therefore, volumetric heat flux at the faying surface at the even so that this bulk resistivity may acting over this volume also this work piece material, but at the interface between these two surfaces, if we assume there are some finite layer thickness is there between these two surfaces then we can represent this at the contact surface between the two work pieces in the form of a volumetric heat flux if we consider the finite thickness.

So, in that cases the volumetric heat flux at the finite faying surface; that means, between these two work piece surface there also the application of the volumetric heat we can

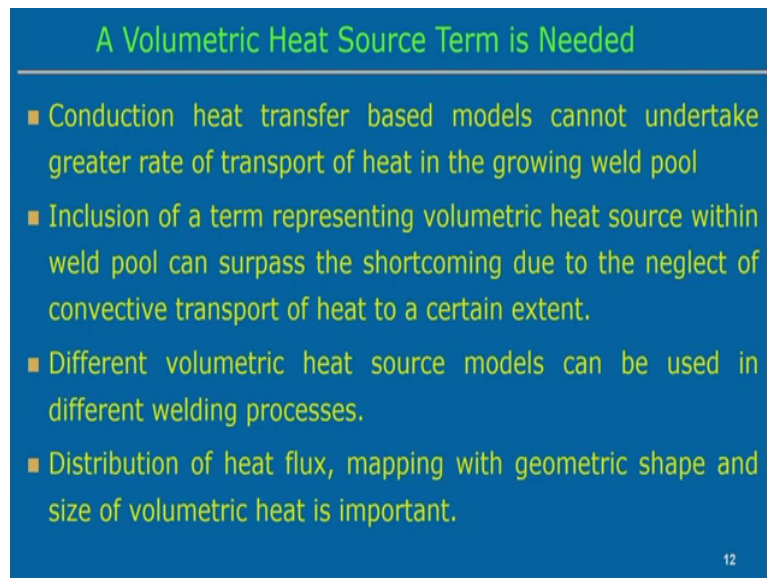
represent these things the t_g is basically thickness of the sheet. So, it means that although we are looking into the particular process then we have to understand that what way we can represent the different.

At which part practically associated with the process the heat flux is acting whether it is surface heat flux or whether it is volumetric heat flux. Accordingly we can incorporate this surface heat flux and volumetric heat flux. But when we solve the basic heat governing equation it means the heat conduction equation in this cases.

If it is volumetric heat flux then we can simply incorporate the volumetric heat flux in the this from the in the governing equation that Q dot term. So, this volumetric heat flux we can use that Q in internal heat generation term we can incorporate through this term or if it is surface flux then we can incorporate in this to the boundary condition.

So; that means, in this particular process resistance spot welding process there is a combining the heat source can be surface heat source can be as in the form of a volumetric heat force. So, therefore, both type of the heat sources there and accordingly we can incorporate the heat source either through the surface or through the governing equation. So, that way we can incorporate the both the volumetric and the surface heat flux associated with the resistance spot welding process.

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A Volumetric Heat Source Term is Needed

- Conduction heat transfer based models cannot undertake greater rate of transport of heat in the growing weld pool
- Inclusion of a term representing volumetric heat source within weld pool can surpass the shortcoming due to the neglect of convective transport of heat to a certain extent.
- Different volumetric heat source models can be used in different welding processes.
- Distribution of heat flux, mapping with geometric shape and size of volumetric heat is important.

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So, therefore, it is necessary the when there is a need of the volumetric heat source term. So, conduction heat transfer based model cannot undertake greater rate of the transport of the heat in the growing weld pool. So, in this case representation of the surface heat flux during the welding process that it is acting only on the surfaces.

But for example, high penetration welding process in case of laser or in the very high thickness in even in case of the plasma arc welding process the heat can be penetrate throughout the depth. So, in this cases only incorporation of the surface heat flux may not sufficient to represents the source of the heat. So, in that case there is a need of the volumetric heat source.

Therefore, inclusion of a term representing the volumetric heat source within the weld pool can suppress the shortcoming due to the neglect of the convective transport of the heat to a

certain extent. It means that, see once we are analysing the conduction based heat transfer model. So, in these cases we are neglecting the material flow, but in actual welding process there must be some kind of the material flow.

But, if we want to incorporate the artificially the effect of the material flow; that means, the influence of the material flow in that cases it is also necessary to incorporate the volumetric heat flux not the surface heat flux. Then volumetric heat flux to some extent compensate the material flow in a conduction based modeling approach. So, that is why there is a need if we neglect the material flow then it is necessary to incorporate the volumetric heat flux to incorporate the effect of the material flow within the weld pool.

But different volumetric heat source models can be used in the different welding processes if welding processes are different then different volumetric heat source can be used can be developed depending upon. For example, it not only different welding processes rather if their parameters are different a very high velocity low velocity in that cases it is also we can develop the different kind of the volumetric heat flux. We will show that what way we can develop all this kind of the heat source models.

So, therefore, actually the alloy this representation of the heat source model actually depends on the what way there is a heat flux distribution is there. That means, is the always necessary to uniformly heat flux is distributed throughout the volume or throughout the surface or there is a variation at the intensity is the very high at the centre point and gradually it will decreasing.

So, depending on the distribution of the heat flux and then mapping with the geometric shape. So; that means, these distribution is exactly mapping of the geometric shape because when there a when we defining some kind of the volumetric heat source in that cases we need to define what is the volumetric shape of this particular geometry, in this particular volumetric heat source the geometric shape has to be defined it can be regular or it can be irregular can be arbitrary shape also, but it is necessary.

But how this distribution is comply with this particular geometric shape that is also has to be look into this thing. And the size of the volumetric heat is important; the size means, what are the different geometric term involve to defining the volumetric heat source in case of the different welding processes? So, therefore, it is all these parameters all these significant these parameters are important when you try to look into the volumetric heat source in a particular welding process.

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Representation of heat source

Nature or type of heat source:
Arc, laser, electron beam, resistance
Representation: **point, line and distributed**

Distributed heat source: **Surface, volumetric**

Surface – **Gaussian distribution**

Volumetric – **Geometric shape and distribution**

Spot welding – **Symmetric**

Linear welding –
Non-symmetric either in geometry or distribution

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Now, in general representation of heat source representation of the heat source; that means, what are the nature or type of the heat source? We can see the arc laser electron beam resistance spot welding process all the different different cases the heat source representation can be different. So, for example, arc normally we can expect in arc welding process the depth of penetration is low or maybe can aspect ratio not much. And in case of the laser

welding process normally depth of penetration is much more as compared to the width of the structure.

Even electron beam welding process which occurs in vacuum it is a long string of the electron passes through the thickness of the particular metal. It means that the electron can be created the keyhole position in the electron beam can be can create the keyhole formation during this structure also. And resistance spot welding process we have already explained the resistance spot welding process what way we can represent the surface and volumetric heat flux and in this cases the distribution can be uniform.

So, there is a variation depending upon the welding process then the variation can comes on the either in the form of a distribution or nature of the heat source. Maybe one cases in certain cases there maybe the formation of the heat source in the it is the one we analysing the heat conduction equation and then in that cases we can develop the heat source model through the internal heat generation term.

But in certain cases it can create the keyhole formation in that case the heat source representation can be different. So, all these different welding processes and the process parameters also influence the different kind of the interaction of the source to the workpiece material. And accordingly we can decide the nature and type of the heat source model.

But the heat source can be represented in the simplest form is the point heat source we can assume the point's heat source we can assume in the line heat source and finally, we can assume the distributed heat source. So, we will see that what are the different representation of the heat source and what way we can it affect the solution and the quality of the solution also; that means, I am talking about the temperature distribution as an output from this governing equation during this process.

So, distributed heat source can be surface distributed over the surface or it can be distributed over the volume. So, both surface distribution and over the volumetric distribution we have to look the different source of the different heat source model depends on that. Then in surface distribution it can be uniform or normally in arc welding processes the surface distribution

can be Gaussian in nature. So, we normally follow the distribution follows the Gaussian distribution

Even if it is volumetric also then in that cases we assume the geometric shape and particular distribution and even we can follow the Gaussian distribution or other kind of the distribution can follow even once we try to develop some kind of the volumetric heat source model. In certain cases the distribution can be symmetric in certain cases, in case of spot welding process; that means, stationary welding process the distribution can be symmetric with respect to the central point.

But, in case of the moving arc so in that cases the distribution cannot be the symmetric in this case it can be non symmetric with particular plane. So, it seems its moves one particular direction then the distribution can be non symmetric that is called the linear welding. So, non symmetric either in the geometry or distribution it means that this non symmetric nature can be incorporate in this heat source model or overall analysis either in the form of a geometry.

The geometry can be a non symmetry the geometry cannot be similar with respect to the particular plane or distribution can be similar both way we can incorporate the effect of the non symmetric energy distribution in case of the linear welding or moving welding problem.

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Analytical solution of Infinite body

Governing equation:
$$\frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + \dot{Q} = \rho C_p \frac{\partial T}{\partial t}$$

Infinite body: the effect of BC can be neglected
Instantaneous point heat source
Initial temperature = 0

$$T(R, t) = \frac{Q}{\rho C_p (4\pi a t)^{3/2}} \exp\left(-\frac{R^2}{4at}\right)$$
$$R = \sqrt{x^2 + y^2 + z^2}$$

Q (J) – source of energy in an elementary volume at time t = 0

Isotherm contours – series of spheres with radius R

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Now, we start with the analytical solution of the infinite body, because if we remember first we start with the that point and line heat source. This is the point and line heat source is not actually representation of the heat source in welding process, but which if we want to get some kind of the very simplified solution analytical solution then some assumptions of the heat source representation is required.

So, very simplified way we can represent the heat source can be a point heat source and sort of if we want to solve this equation we can get some analytical solution. Otherwise if we consider the distributed heat source then it is necessary to solve this governing equation numerically. So, in that case numerical solutions are available some using even in this particular cases we will discuss about the solution using the finite element analysis.

So, this governing equation if we assume that in finite body so definitely if once we are expecting some kind of the analytical solution of this 3D heat conduction equation in this cases lots of assumptions are required to get the analytical solution. But, all these assumptions may not be the actual representation of the actual welding processes.

But we can imitate the actual simulation of the welding processes more close to the consider the practical aspect of the welding process in that case it is necessary to do some kind of the numerical solution. So, in this case analytical solution of the infinite body means the we assume the solution geometry is the infinitely long in x y and or z direction.

So, therefore, since we are assuming the infinite body the assumption is that the effect of the boundary condition can be neglected so not necessary to put the boundary condition. Instantaneous point heat source in a particular the heat source can be represented as a in a particular point. So, instantaneous point heat source with the heat source representation and initial temperature is assume as 0.

Then we can get the solution temperature distribution from this governing equation this in this way that T R are the radial distance can be the on the from the particular reference point R equal to root over x square plus y square plus z square this domain we can get the solution R . And then in terms of R we can say the Q by ρC_p is the density specific heat and $4 \pi \alpha t$ is the thermal diffusivity and t time variable exponential R square minus $4 \alpha t$. So, we can get reach this kind of the solution with this particular assumption.

Now, in this case Q J represent the source of energy in an elementary volume at time t equal to 0. So, basically Q J represent the heat source, but volume elements at the time t equal to 0. So, in this case is the representation of the heat source as a point representation, but we get the isotherm contours is the series of the sphere with particular radius R that is output from the this kind of the solution its analytical solution.

So, in this cases we are getting the analytical solution at a temperature distribution R and we can see the R can be represent this x y and z variables are there; that means, so 3D 3

dimensional solution. We are getting and the representation of the physical representation solution is simply the contour isotherm representation of the weld pool in the form of a series of the sphere with the radius R. So, this kind of solution we can get using this particular assumption.

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Analytical solution of Infinite body

Infinite body: the effect of BC can be neglected
 Instantaneous line heat source (J/m) - 2D temperature distribution
 Initial temperature = 0

$$T(R, t) = \frac{Q_1}{\rho C_p (4\pi a t)} \exp\left(-\frac{R^2}{4at}\right) \quad R = \sqrt{x^2 + y^2}$$

Isotherm contours - series of cylinders

Instantaneous plane heat source (J/m²) - 1D temperature distribution

$$T(x, t) = \frac{Q_2}{\rho C_p (4\pi a t)^{1/2}} \exp\left(-\frac{x^2}{4at}\right)$$

Isotherm contours - series of planes

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But if we change the infinite body differently if we assume the infinite body there is a we are neglecting the boundary condition is neglected. Then in this case instead of the point if we assume the instantaneous heat source line heat source which is more realistic in case of the laser welding process, we assume the heat source is represented over a line.

So; that means, Joule per metre for example, in the SI unit and in this cases 2D we will be able to get the 2D 2 dimensional temperature distribution. And initial temperature is 0 we are assuming this thing and get the solution is like that, that T temperature distribution as a

function of R and time we are getting this is the form of a solution. Analytical solution of temperature distribution, but since we are representing the line heat source; that means, in this case it is possible to get not the 3 temperature distribution we will be getting rather 2 dimension arc is the function of x and y also.

So, isotherm contours is basically series of the cylinders that is representation of the isotherm contours. And if we assume the instantaneous plane heat source. So, instantaneous plane heat source at time t equal to 0 heat source is applied; that means, to this thing instantaneous plane heat source that is plane heat source means the heat source representation is the Joule per unit area plane heat source and in this case it is possible to get analytically the 1 dimensional temperature distribution.

So, it is the temperature distribution T is a function of only one variable x and t and we are getting this kind of the expression. And isotherm contours represent the series of the planes. So, it means that even there is a heat source representation in the form of plane in the form of a line or in the form of a point, then we are getting some we are getting some solution analytical solution, but it is a lots of with the particular assumptions assuming the infinite body.

So, this kind of the assumptions and presentation of the heat source then there is a limitation of getting the temperature distribution whether it is 3D whether it is 2D or whether it is 1D kind of temperature distribution we will be getting.

It means that the representation of the heat source is also matters and this simplified solution even analytically we can estimate the temperature distribution, but this temperature distribution not exactly the very much realistic, if we consider the distributed heat source. Even we can go ahead with this thing the analytical solution of the other cases also.

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Analytical solution of semi-infinite body

Semi-infinite body: Adiabatic in one plane
Instantaneous point heat source
Initial temperature = 0

$$T(R, t) = \frac{2Q}{\rho C_p (4\pi\alpha t)^{3/2}} \exp\left(-\frac{R^2}{4\alpha t}\right)$$

Semi-infinite body of thickness H
Instantaneous line heat source $Q_1 = Q/H$
Initial temperature = 0

$$T(r, t) = \frac{Q}{4\pi k H t} \exp\left(-\frac{r^2}{4\alpha t} - Bt\right)$$

$r = \sqrt{x^2 + y^2}$ $B = \frac{2\alpha}{\rho C_p H}$

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So, if we look into the analytical solution of the semi infinite body. So, semi infinite body means not all the dimension is extends infinitely long, but one particular direction is there is a adiabatic in one particular plan, there is a finite dimension and other two direction can be the infinite direction. Even if we assume the instantaneous point heat source then the solution of the temperature distribution can be different in these cases when as compared to the infinite body.

So, if it is a semi infinite body we can get the similar kind of the distribution the different kind of the distribution as a function of the R and as a function of time t. So, now if we assume the semi infinite body of thickness H. So, now, we use the semi infinite body having the finite thickness H instantaneous line heat source can be represented that Q_1 equal to Q by H. So, this is the representation of line heat source Q is the; Q is the estimate of the energy

supplied, and then we divide this energy supplied not exactly at the point rather the energy supplied over the length particular length.

Then this is the representation of the energy the to this and to get the analytical solution and we will be able to get this analytical sources something like that T as a function of radial distance r and time. Getting the all term k thermal conductivity H is the that thickness of a semi infinite body thermal conductivity and T is the time variable and B can be represented this way also the in terms of the other parameters and r is the root over x square by y square.

It means that that although we represent the particular heat source in the line heat source and then we are getting the different solution of the temperature distribution assuming that in case of the semi infinite body. So, in that sense it is possible to get the different kind of the solution even if we assume the different kind of heat source representation as well as the assumption of whether it is finite body, whether it is semi infinite body or whether it is infinite body.

So, we will see other solution also then moving point heat source now if we consider the moving point heat source on a semi infinite body. So, in these cases what kind of the analytical solution we can get. So, this is the well known the Rosenthal's equation steady state heat flow situation.

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Analytical solution of semi-infinite body

Moving point heat source on a semi-infinite body

Rosenthal's equation (steady-state heat flow):

- Point heat source and no heat losses
- 2D heat flow in welding of thin sheets (h - thickness) of infinite width

$$T = T_0 + \frac{Q}{2\pi kh} \exp\left(\frac{Vx}{2\alpha}\right) K_0\left(\frac{Vr}{2\alpha}\right)$$

Heat source is moving along X axis (opposite) with velocity 'V'

K_0 modified Bessel function of second kind and zero order

$r = \sqrt{x^2 + y^2}$ radial distance from origin

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In this case we are assuming that point heat source and no heat losses with the no heat losses we are assuming and 2 dimensional heat flow in the welding of the thin sheet H is equal to thickness of the sheet in the of infinite. So, 2D of thin sheet H equal to thickness and of infinite width so infinite width is infinite, but there is a finite thickness, but sheet thickness is very small only H.

So, in that case we can get the solution of temperature distribution that is called the Rosenthal's solution equation, T equal to some T 0 is the initial temperature. And in terms of the other parameters we can get this kind of the solution. And where K 0 is the modified Bessel function of the second kind and the 0 order. So, in this case heat source is moving along x axis opposite with the velocity v representation of the heat source with opposite of the x axis v.

So, it means that it achieve the different from more simplified to the more realistic situation associated with the welding process. In this case that there is a heat source moves one particular direction with the velocity v then we will be able to get this kind of the analytical solution along with the other assumption. So, r is the radial distance in this case so; that means, the temperature distribution can be different as compared to the other cases.

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Analytical solution of temperature distribution

Moving point heat source on a semi-infinite body

Rosenthal's 3D equation in semi-infinite workpiece

➤ Point heat source and no heat losses

$$T = T_0 + \frac{Q}{2\pi kR} \exp\left[\frac{-V(R-x)}{2\alpha}\right]$$

Heat source is moving along X axis (opposite) with velocity 'V'

$R = \sqrt{x^2 + y^2 + z^2}$ radial distance from origin

Singularity problem at the origin of the coordinate system caused by the point heat source assumption

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Similarly, moving point heat source on a semi infinite body. If there is a moving point heat source on a semi infinite body Rosenthal's 3 dimensional equation in the semi infinite workpiece that can be represented like that, but assumptions was the point heat source representation of the heat source point heat source and no heat loss. With these cases we can achieve the temperature distribution is like that T equal to some initial temperature T_0 Q by

twice $\pi k R$ at that rate R is the radial distance from the origin. And this X is incorporated here because we are assuming the heat source is moving along the x direction.

So, that way we can achieve this kind of the solution. It means that if we change the assumption condition then further improvement of the solution is there. So, this is the most important solution of the temperature distribution which can be derived analytically, but the basic assumption in this case is the point heat source that mean the arc and laser we are representing the heat source is acting over a point. That is not exactly the realistic phenomena so, but till we can get some solution using the point heat source analytical solution.

But one difficulty of the solution is the singularity problem arises because we not be able to achieve the temperature distribution at the origin basically of the coordinate system, caused by the point heat source assumption because we are assuming the point heat source then singularity problem is there exactly at the origin of the coordinate system. So, that kind of the difficulty limitation is there with this particular solution.

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Analytical solution of temperature distribution

Cooling rate and temperature gradient from Rosenthal's 3D equation

$$T = T_0 + \frac{Q}{2\pi kR} \exp\left[\frac{-V(R-x)}{2\alpha}\right] \quad \left(\frac{\partial x}{\partial t}\right)_T = V$$

Along the x-axis, $y = z = 0$ and $R = x \rightarrow T - T_0 = \frac{Q}{2\pi kx}$

Temperature gradient $\left(\frac{\partial T}{\partial x}\right)_t = \frac{-Q}{2\pi kx^2} = -2\pi k \frac{(T-T_0)^2}{Q}$

$$\left(\frac{\partial T}{\partial t}\right)_x = \left(\frac{\partial T}{\partial x}\right)_t \left(\frac{\partial x}{\partial t}\right)_T = -2\pi kV \frac{(T-T_0)^2}{Q}$$

- ✓ Cooling rate reduced significantly by preheating
- ✓ Cooling rate decreases with increasing Q/V
- ✓ Temperature gradient decreases with increasing Q

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Now, is the solution it is possible to analytically estimate the cooling rate and the temperature gradient from the Rosenthal's 3 dimensional equation. In this; this is the equation we can get this is the solution of the temperature and in this case since the heat source is moving; that means, particularly in one particular direction; that means, along the x axis.

So, dx by dt can be represented as velocity V and along the x axis y equal to z equal to 0 if we assume that in this cases if we put the along the x axis x equal to y equal to 0 then R can be x, because R we are representing root over of x square plus y square plus z square. So, x and y and z are 0 so R can be x in this case. Now, we can simplify this solution R equal to x and they are simply d x by d t represent the velocity V then we can reach the solution, T minus T 0 equal to my simply Q by twice pi k x. Now, temperature gradient keeping this t means the keeping time as a constant here keeping temperature as a constant.

So, temperature gradient can be estimated from this equation expression is that minus Q twice $\pi k x$ square and so from here we can estimate the twice πk the manipulation of this. And the variable we can the temperature gradient can be estimated like this. So, finally, we can estimate the cooling rate along the x axis; so, cooling rate along the x axis we can represents the $\frac{\Delta T}{\Delta t}$ this indicates the rate of the cooling, that is associated the $\frac{\Delta T}{\Delta x}$ by Δt .

So, this term we have already estimated and multiply this thing then we will be getting this kind of the cooling rate twice $\pi k V$ and the $T - T_0$; that means, one particular temperature and T_0 is the initial temperature and Q is the heat input to this thing. So, we can make this simply simple calculation basically we from this analytic solution it is possible to estimate the cooling rate. But, what can be incorporate from this make some conclusion from this cooling rate is that, cooling rate reduce significantly by pre heating.

That means, if pre heating means the initial temperature becomes much more the T_0 is more than the melting more than the room temperature; that means, if some pre heating is applied that it integrates the cooling rate is reduced significantly by the pre heating the this kind of the conclusion we can make from this expression. Cooling rate decreases with increasing Q by V ; Q by V is the heat input. And Q by V is actually represent the heat input per unit length, it is the combining effect of the what is the heat input from the arc or laser and what is the velocity we are applying.

So, that indicates that Q by V is the heat input per unit length. So, that cooling rate we can see the Q by V is reduces decreases with increasing the Q by V ; that means, if heat input increases then cooling rate also decreases during this process. And temperature gradient decreases with increasing Q that mean temperature if the heat input is more then temperature gradient is actually decreases so less temperature gradient.

So, this all this kind of we can get this information this we can link that what is the high heat input or low heat input and accordingly what may be the what way it can affects the cooling rate. Because finally, this cooling rate is decides the what are the solidification behaviour

because it or finally, microstructure of a weld joint. So, in that sense this even if we estimate the temperature during this even from the analytical solution, but that from that further what we will do with this temperature distribution we can further check. The we can estimate the other estimate the what is the cooling rate from the temperature distribution as well.

And we can get the relative prediction of these things what may be the structure microstructure or solidification behaviour to some extent from this information of the temperature. Of course, there is a need of the other kind of information may be solidification kind it is also necessary to understand the solidification behaviour of the microstructure.

But to some extent we can estimate the cooling rate and that cooling rate is basically able to predict the microstructure during the welding process. So, that is why it is important and all this the; but how what is the correctness of this particular welding process the rate estimation of the temperature as well as the cooling rate that actually depends on the correct representation of the heat source. So, in that since the heat source having always some influence for the correct estimation of the temperature distribution.

So, that is all today thank you very much for your kind attention maybe next part we will look into in details about the distributed heat source it associated with the both surface plus as well as the volumetric heat source.

Thank you.