Finite Element modeling of Welding processes
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Lecture - 15
Natural coordinate system in 3D and XFEM

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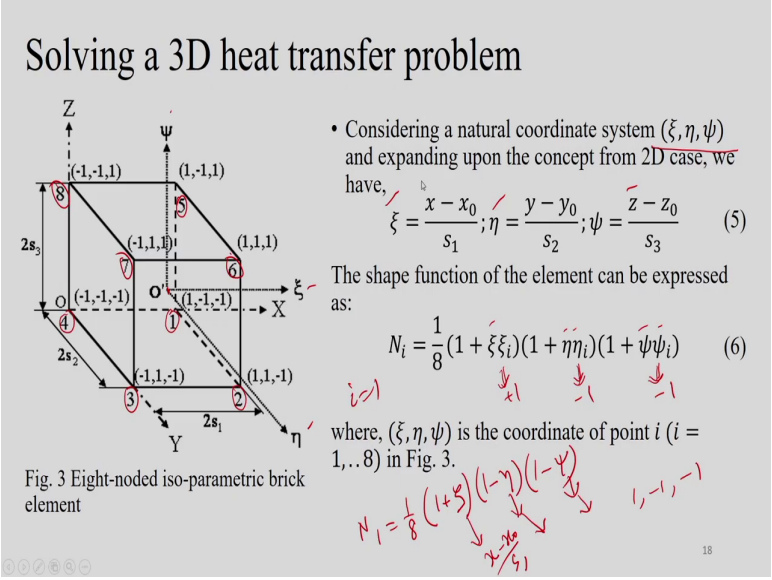

Hello everybody today we will discuss this what way we can look into the natural coordinate system in case of 3 dimensional problem. We can consider a particular brick element. So, it will be easy to understand that how what way we can mapping with the local coordinate system to the global coordinate system in this particular element.

So, therefore, considering natural coordinate system xi, eta, psi. So, that it will be centroid of this thing can be og is the center and this it is mentioned already that 1, 2, 3, 4. So, I think this is 1, 2, 3, 4 and 5, 6, 7, 8. So, its particular sequence this noded eight noded one brick element

we have considered and then we define the natural coordinate system as the xi, eta, psi. So, that is x is xi,eta, psi

So, now the of all the information of all the nodes in terms of the global coordinate system it means that in terms of the XYZ that data are available and that we can get once we look into the discretized geometry. And we have the all information, what is the particular node and what is the corresponding coordinate system.

Now, once you look into one particular element, we can convert this global to in terms of the local system. So, that we can follow the similar kind of the exercise for all the elements and it will easy to handle easy to accounting for this particular element. And I think that way we can link between this local and global coordinate system like that.

So, zeta equal to x minus x 0 by S 1 and similarly xi eta psi also in terms of x y and z these are the variable and x 0 y 0 z 0 is the coordinate of the centroid. So, that is way we can make this variant and S 1 and S 2 is defined in such a way such that twice S 1 is representing the length of the H length. So, here you can see the twice S 1 this is along the X axis the H length a distance between the node 2 and 3. So, that is the twice S 1.

So, in that respect we can consider this is as S 1 S 2 and S 3 similar way. So, in general the shape function of the element can be expressed in general way N i equal to 1 by 8 1 by xi xi i eta eta i psi psi i like that. And such that this actually if we put xi eta and psi value particular with respect to the that what is the value, if you see that we can map in such a way that with respect to the xi eta psi coordinate it values it locally it varies between plus 1 to minus 1 in that within that range.

So, accordingly we can put the coordinate of all this all these 8 points in terms of that 1; 1 in terms of this particular scaling that you need is basically 1, 1, 1 or 1 minus 1, 1 depending upon the sign convention or depending with respect to the origin. Now, if we put for example, N i say n i equal to 1 in this particular case. So, then for i equal to 1, zeta, 1 eta 1 and this

thing. So, accordingly the sign can be is something like that if i equal to 1. So, if you look into the coordinate of 1; 1 is having 1 minus 1 and minus 1.

So, this is the coordinate of the node point 1. So, we put then xi eta equal to plus 1 and eta equal to minus 1 this equal to minus 1 such that it becomes. So, N 1 can be represented like that 1 by 8 you can see that 1 minus 1 plus xi into 1 minus eta into 1 minus psi like that.

So, this way we can generalize this thing and such that xi eta psi is the coordinate of the particular point i and that is i varies equal to 1 to 8. So, this way we can estimate that that N 1 the shape function in the in terms of the local coordinate system, but of course, this local coordinate system can be represented in terms of the global coordinate system. It can be in terms of x minus x 0 by S 1 and similarly for in terms of y and in terms of z. So, that way this term can be defined.

Now, once we convert to the global to the local coordinate system then we can do the formulation. We can see that how the formulation can be done.

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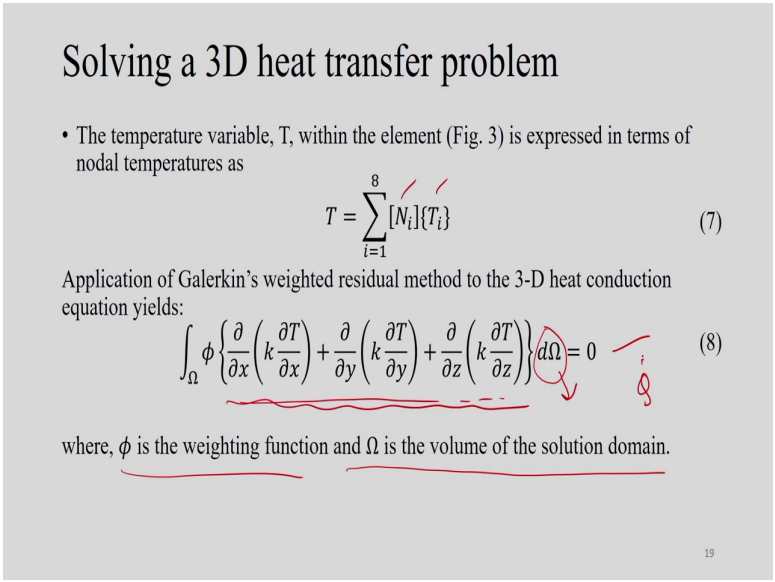

Now, if you look into the 3 dimensional problem heat transfer problem also. So, then in that case the temperature is the variable. So, each and every node point having some temperature value. Therefore, within particular element the this variable temperature nodal in terms of the nodal temperature it can be represented like that T equal to i equal to 1 to 8. So, i equal to 1 to 8 in this cases we have considered the brick 8 noded brick element

So, then Ni corresponds to the shape function and T i is the nodal temperature value T i in that way. Now this is the, this way we can represent the we basically interpolate the variable temperature within the element in terms of the nodal values in the shape function as well as the, what is the value of the temperature at the node point.

Now, we know this we have already shown that what way we can discretize the governing equation along in the boundary condition also, but in this cases if we follow the Galerkin

weighted residue technique. So, in 3 dimensional heat conduction equation. So, this part we can see that if you discretize this part of this equation also. So, here the heat generation term is not considered.

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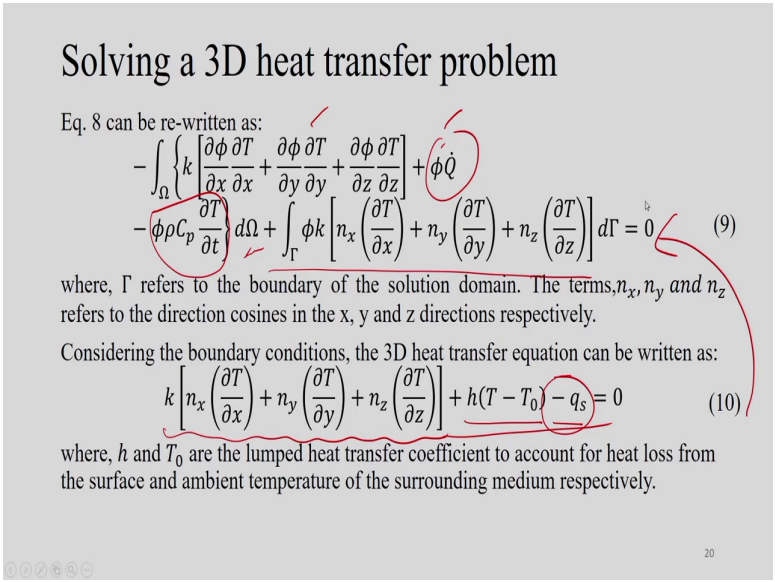

Now, see phi is the weighting function and the volume of the solution domain, this d the this indicates the volume of the solution domain. And then if we do this put this usual procedure of the discretization of these things and then rewritten in such a way that equation 8.

So, equation 8, I think there are this is the only heat conductivity in terms of this thing, but there is a Q dot term and was there also, but then if we look into an even if we tangent heat conduction equation we have the other term also. So, apart from this thing there is a this term also there and there is a this tangent part also there.

So, looking into all the 3 components this thing. Now if we put this in the weighting function and then we discretize putting in such way that if we gains theorem if we apply. So, volume integral to convert to the surface integral in that form. So, then this is volume integral is there this part is there this part plus this plus this volume integral and this correspond to the surface integral.

Now, we replace this surface integral this term also if we put the appropriate boundary condition; that means, in general where the mathematical form of the boundary condition can be written like that. So, this indicates the what is the heat is actually conducted to the at the surface.

Now, once it is heat conduction at the surface and then its basically n x n y n z is the normal vector with on the surface and is the from the surface this is the heat loss from the surface by convection. And then q S is the heat flux applied to the domain. So, that is why it is becomes minus q s. So, this is the mathematically with all the boundary interaction we can represent in terms of the; what is the heat conduction on the surface.

What is the convection on the surface as well as the if there is any heat interaction by the external flux also then this can be counted in this way in opposite sign because q S is the input to the surface in that way the term can be written. And if we look into this boundary condition implement here also, then we can form the this equation that we have already shown or we will be showing in the when it exactly discussing the heat conduction equation. There you can show all this in details that the formulation and then how it is coming the different elemental part also or contribution from the different components in from the governing equation combining with the boundary condition.

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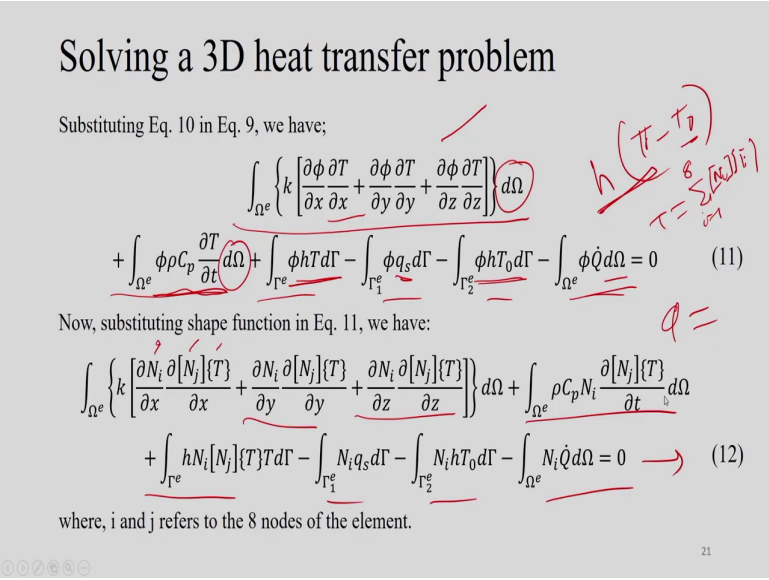

So, then if we do all these things then we can find out discretize this thing and then we will be getting this particular term in the this thing that this second term this all this term. If you see this is because of conduction the thermal conductivity is involved in this critical and this we see over the volumetric domain they are also rolled C p del T; that means, this transient part also there.

And this is the H is there. So, it is a convective part component is also there and qs if there is any internal external flux applied to the domain then it comes into this picture also. And H T 0 is coming this is the H T minus T minus T 0 when you calculate this heat loss by the convection. So, this H T term is there and this H T 0; that means, in reference term is come here also, but that is that accounts on the surface.

And then Q dot if there is any internal heat generation term also and definitely internal heat generation term has to be accounted over the volume. So, then this is the volumetric term. So, then finally, combining this thing discretize the Galerkin weighted residue technique and if we look into. If we weighting function can be replaced in the shape function in the form of weighting function is if it is equal to the shape function.

Then we can reach this particular expression. So, that total derivation can be explained in somewhere else, but here point is that what way we can utilize the shape function to form 1 elemental matrix. Now this once we replace phi as a in the form of a shape function and then we can modify this equation also that way in terms of the shape function del phi equal to del N i by del x. So, del this is the shape function weighting function is equal to the shape function.

Similarly, del T, T can be represented T was the variable. So, T can be interpolated something i equal to 1 to 8 in the 8 noded brick element and that is correspond to the N i and then T i in terms of the nodal shape function as well as the nodal temperature value.

Then we put is the nodal shape function and temperature value del of this thing derivative of this is equal with respect to x. Similar way other term can be written in that way and see in other way also. Now this term we can write in the in the form of a shape function ah. Now we replace in this thing the weighting function in the shape function.

Then for i equal to i or j refers to the 8 nodes of this particular element then we can reach all these components will be coming. Now we can pick up any of the component how what way we can calculate. This one contribution from the each component, we can estimate this thing that we will try to look into this thing.

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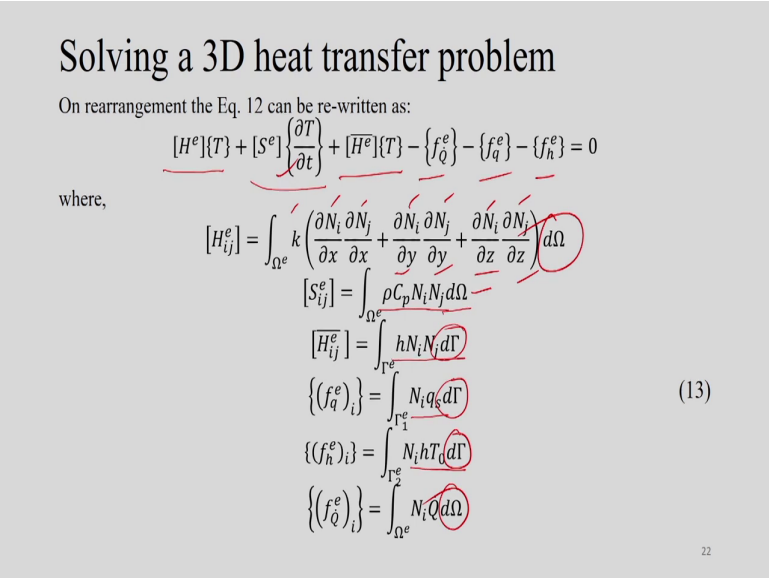

So, if we look into all the components rearrange all this thing in the matrix form. We can write in this way for a particular element H T S del T by del T because in this case the with the Galerkin weighted residue technique, but that discretization we are assuming over the space also not over the time. So, then in that cases we have to time domain you have to consider some shape function then we can follow some discretization scheme also to get this value.

But before discretization ah in the time domain only we look into the what is happening in case of the spatial domain. So, H T f Q all terms are coming in this thing in general. We can see that H i j is basically in terms of the k del N i del N j del x del N i N j del y del y N i N j del z del z. So, this way we are getting all this term and this is over the volume.

S term comes into this thing it is also over the volume rho C p N i N j in this particular way also then H other term is the heat transfer coefficient, but this is over the surface. This is over the surface f Q in any heat flux surface flux is there, if this is the heat loss when account to the reference temperature comes over the surface and this this if there is internal heat generation over the volume.

So, these are the typical expression some integrand is basically expands over the volume in some cases it is over the surface. So, this combining all these terms we can form a linear system of the equation.

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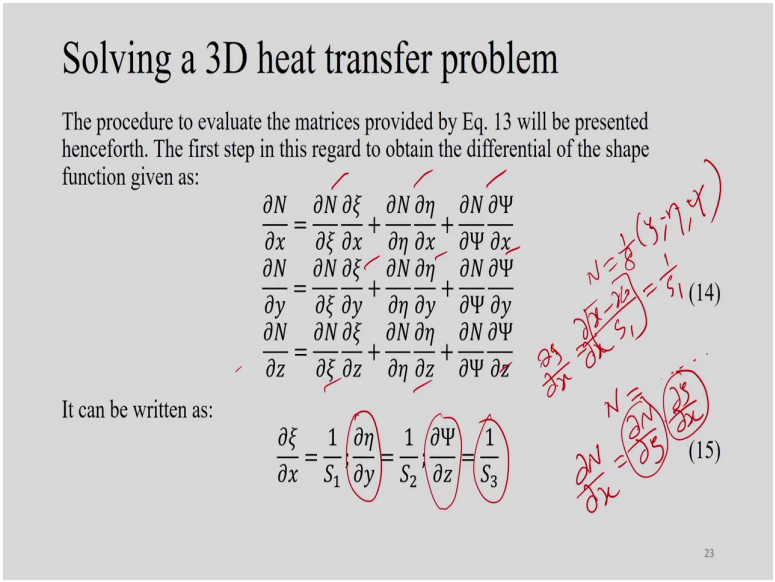

Now we will see what way we can discretize this thing. Now, before doing that discretization thing, maybe we can look into this thing because we have seen that N. See represented this thing when representing the shape function N was represented in terms of x.

So, therefore, del N by del n x del N by del x can be written like that del N by del this thing eta because xi by del x ah. Such that del N by del eta xi and del xi by del x. So, N also in terms of the N can also be represent. We have seen n can be 1 by 8 or that in the particular form xi eta psi also in that form we can represents.

Similarly, plus this del N by del this thing because N is represented xi in terms of and xi eta psi. So, that is why del N by del eta and del N by del psi also and accordingly del eta by del x. So, these are the 3 components we can write in this way also. So, that is represent del N by del N x similarly del n by del y also. You can see there are 3 components are there del n by del z also there are 3 components are there.

Now, if we look into the expression del xi by del x del xi by del x with respect to that we can find out this expression that it has become. If you remember x minus x 0 by S 1; when i represent the xi. So, then del by del x is something like that 0, such that it become 1 by S 1. So, it becomes constant this is del xi by del x. Similarly, xi eta del eta by del y also 1 by S 2 and del xi eta psi by del z equal to 1 by S 3. So, S 1, S 2 and S 3 half of the H length of this particular element.

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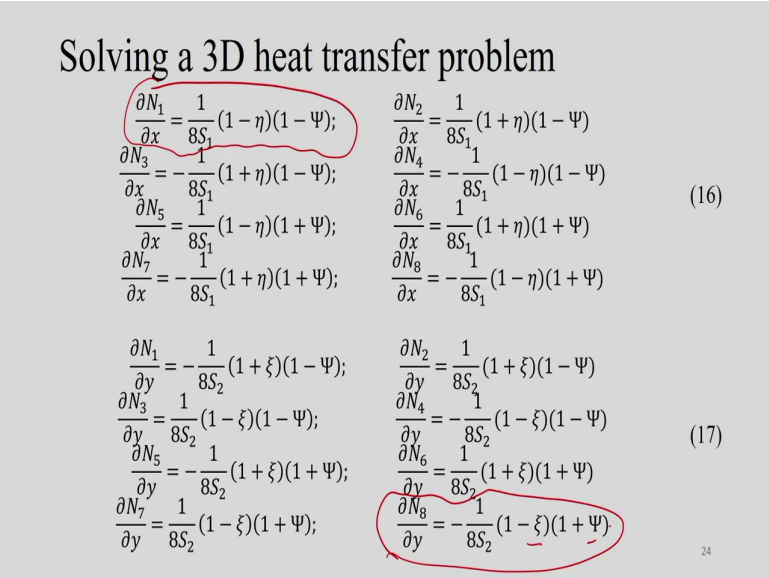

So, then that if you replace these values then we are getting the expression of the all the expressions. So, N equal to 1 to 8 N 1, 2, 3, 4, 5, 6, 7, 8 then del N by particular x or with respect to y with respect to z all this getting in terms of the xi eta psi you can find out all this expression.

So, let us took one example also simply del N 1 by del x. If you see this particular expression see here, it represents 1 by 8 S 1 1 by 1 minus eta 1 minus psi, so like that. So, similarly if one put any other also say for example, the last one also if you look into that del N 8 by del y it is minus 1 by 8 S 2 1 by this thing. So, xi and 1 plus psi. So, it means that all this expression we can see that in terms of the S 1 S 2 and the H length and this local coordinate system we can represent the shape function as well.

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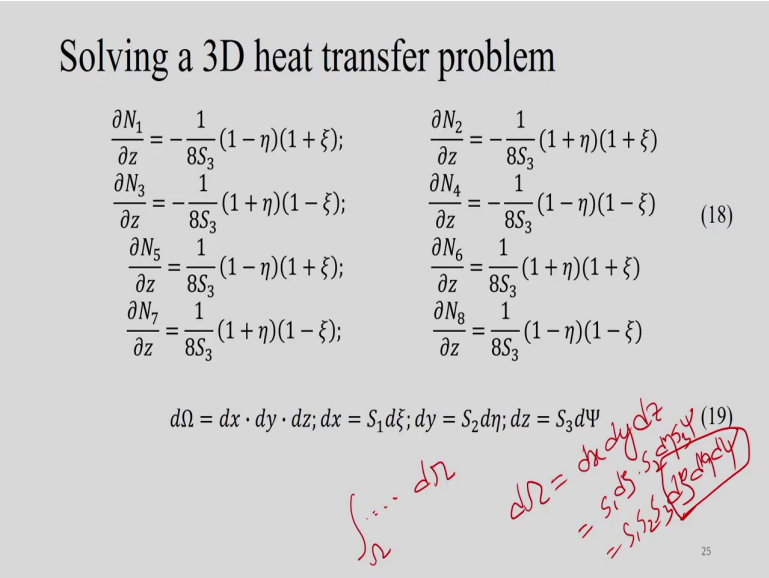

Now, look into this thing even also the so many all the expression are given also if you look into in details all these things. We can get the derivative of all this shape function with respect to the local with respect to x, y, z. That means, the global coordinate system variable, then we can convert in terms of the local coordinate system.

Now, so, this domain once we represent this domain with volume integral over the volume integral this domain and this domain can be represented the elemental domain in the global coordinate system d x, d y and d z. So, then it is the particular domain it is the volume integral over this particular domain.

So, then it is equal to dx, dy and dz such that now we know dx equal to S 1 in terms of local coordinate system xi S 2 eta and S 3 psi. So, it means that S 1, S 2 S 3 d xi eta psi d xi d psi like that. So, it means that this is the in terms of local coordinate system apart from that we

can S 1, S 2 and S 3. So, that means, this elemental volume is converted into the in terms of the local coordinate system, local variable ah, that way we can convert this thing.

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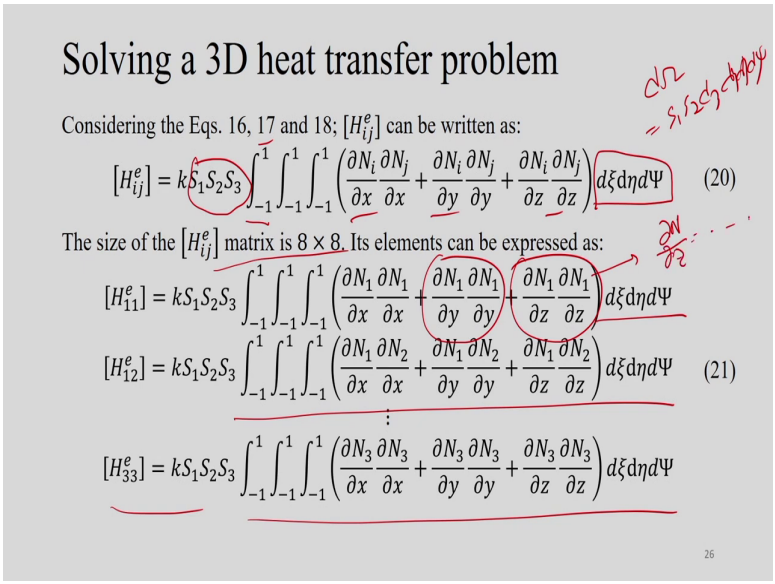

Now, once we convert this thing. Now if you look into any pick up any of the particular item for example, H for this particular H i j e. So, that particular element how particular element how it forms. So, in that cases we replace simply the it is the volume it simply replaces that S 1, S 2, S 3 and d xi d eta and d psi replace this thing this is the things and S 1, S 2 and S 3 there and into k because the k term was there.

And then we put this thing in this way that del N by del N x that we have already shown that it is actually coming in terms of del n by del x del N j by del x plus in terms of del N by del y del N j by del y something like that this way the all these 3 components are there. And this is the integrand is the in terms of the because if you see the local coordinate system it actually

varies minus 1 to plus 1. So, then we can put the limit minus 1 to plus 1. So, this way we can conduct this way.

Now, we particular element say for example, H i j matrix we can 8 by element 8 by 8 matrix it elements can be expressed for example, H 1 1, how we can be represent like this? H 1 1 equal to k S 1, S 2, S 3 will be there in this case i equal to 1 and j also 1. So, del N 1 by del x del N 1 by del x immediately del N 1 by del y del N 1 by del y similarly, z xi eta psi.

Now here similarly H 1 2 can be represented like this H 1 3 3 can be represented like this. So, once you we can convert in terms of the basically the local coordinate system only thing is that it easy to S 1, S 2, S 3 is the basically H length that we can look into this.

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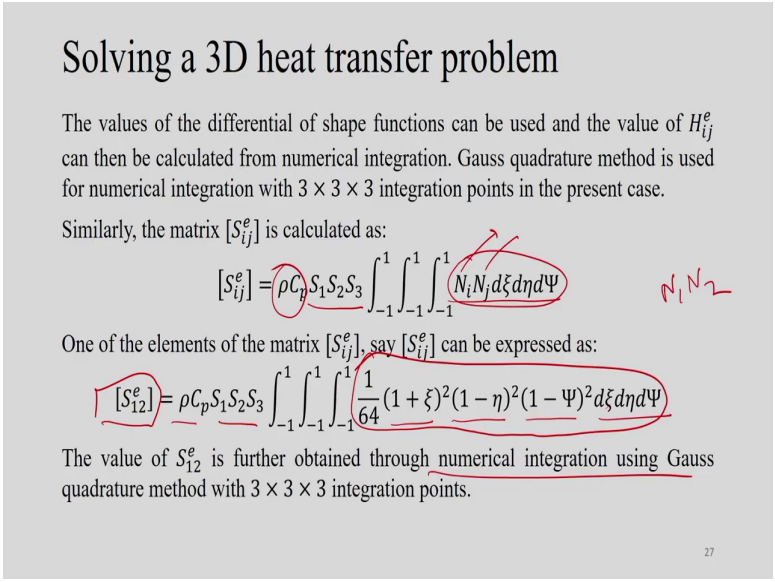

Now, these values of the different shape function can be used and the value of the H i j can then be calculated from the numerical integration. So, once we look into this numerically this once minus 1 to plus 1 and shape function can be represented at N 1 N 2. Basically if you see that N 1 this thing say for example, if you do this if further exercise derivative del N by del z.

So, it is come in terms of the local system; that means, local coordinate system. So, once it comes to the local coordinate system and I can do the numerical integration. And such that assuming the 3 by 3 by 3 integration points and we can follow the gauss quadrature rule to find out to calculate the integrand, so this thing.

So, similar way S can also be calculated like that rho C p the properties S 1, S 2, S 3 and then N i N j it is there also. So, that can be represented if you put replace N i N j in terms of the xi eta psi.

Then we can perform the numerical integration technique and there also we can find out that 3 by 3 by the 27 integration points and though we can we normally do the integration. We can identify what is the this 27 points and what is the weightage of this particular point and there is that way it is having some fixed value if you follow the gauss quadrature rule.

So, from that point we can estimate the numerically integrated then we can estimate what is the S i j term can be calculated. So, similar way we can calculate all the elemental term in any kind of this element it can be S also it can be H also. So, there this particular element for example, S i j. So, for example, S 1 2

Here you can see the row C p S 1, S 2, S 3 and this thing S 1 2 means in this cases it will be the N 1 N 2 put this value and then we can get this particular expression. That 1 by 64 into 1 plus xi square of that and 1 minus eta square of that and 1 minus psi square of that and this this thing.

So, now is this term can be done using obtain through the numerical integration using the gauss quadrature rule and whether we have we can use the 3 by 3 by 3 integration point. So, it

means that we are not discussing then how what way we can do the numerical integration that is the in standard numerical techniques book this is available and this thing.

Standard values are also available, if you follow gauss quadrature rule how many points you are considering and based on that what are the weightage of this particular point all these data are available in the standard format. We can use this one and finally, we can perform. So, finally, during the numerical integration we estimate what is the value of S 1 2. Similarly all the term in the matrix we can estimate the similar way.

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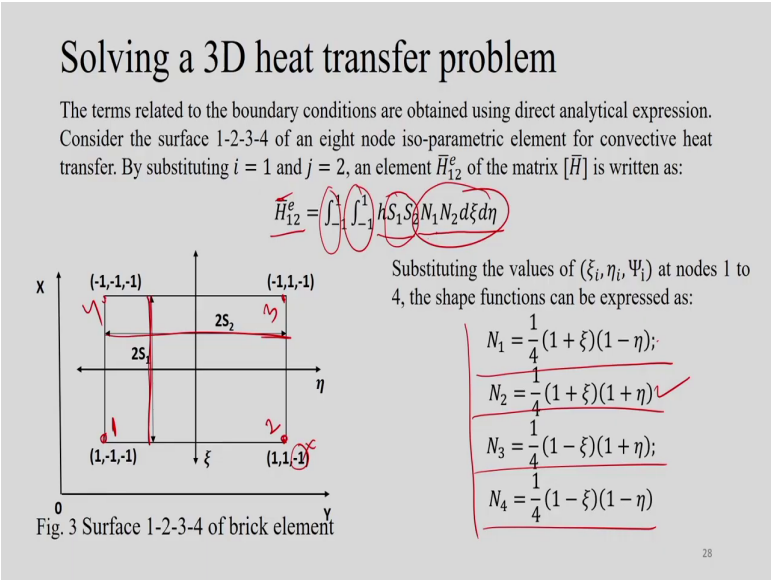

Here you can see the terms related to the boundary condition also. So, we can see the how what way we can that look into the boundary condition, often using the direct analytical expression that can also be done. We can see that for example, the surface say surface is defined by 1, 2, 3, 4 of an 8 noded isoparametric element.

And then on the surface there is a because when we taking the 3 dimensional analysis and it is a. If we consider the brick element also then one phase will be on the surface. So, that phase we can consider say it in case of brick element if we consider in that case surface will be having some 4 noded element out of this brick element.

So, then in that cases H 1 2 on the surface can be estimated like that also H 1 2 over the surface, so H 1 2 bar here. So, do not confuse with the previously that H we calculated over the volume now it is over the surface. So, H 1 2 over the surface. So, same way we can estimate this thing h S 1, S 2, N 1, N 2 this way we can estimate this value.

Now, substituting this thing you can just find out the shape function also on the surface the N 1. Suppose there are 1, 2, 3, 4 that is the particular node on the surface. So, then we can estimate the in terms of the local coordinate system xi eta psi N 1 N 2 N 3 N 4 it is similar way also for example, this is the point 1 1. Say I am not considering the z component z component I am considering only the 1 1. So, then this indicates the this one N 2. So, I think this is point 2 and then plus 1 minus 1. So, plus 1 minus 1 I think this is point 1. So, it is correspond to the necessary function is that 1 2 I think this is point 3 and this is point 4.

So, that is way accordingly we can decide the shape function and then on the surface we can then estimate this similar way also. So, put the value N 1 N 2 correspondingly N 1 N 2 value. And then we know S 1 S 2 is the that is already defined this is total 2 S 1 and this is the total is the twice S 2. So, half of that S 1 and S 2 put that is well defined.

because that we can get from the global coordinate system from this what is we can find out what is the H length for a particular element because we have the all the coordinate of the particular coordinates of the nodes of all the elements. And that information is always there and that information is needed actually to calculate for the elemental matrix.

So, from there you can find out S 1, S 2 H is the material property we define these things and here you can estimate the N 1, N 2 and then do the integration a numerical integration we can perform. Then we can estimate what is the value of H 1 2 and for a particular surface.

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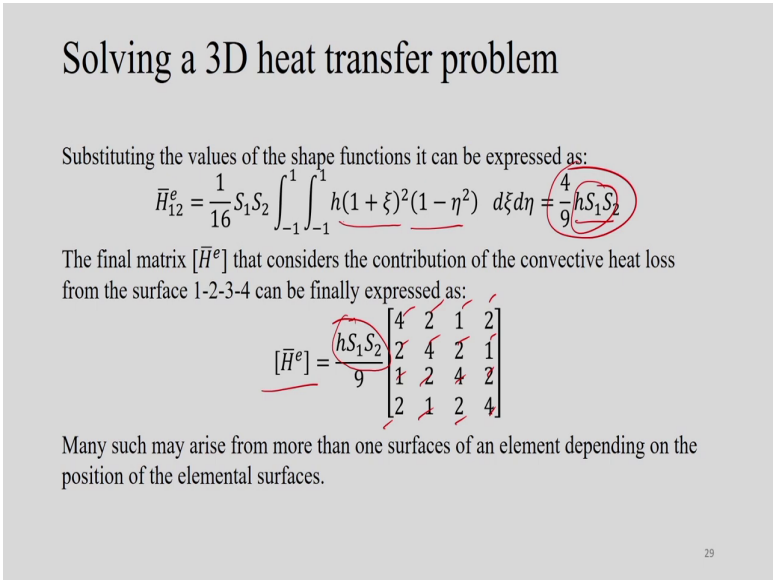

Now, also here we can see the H 1 2 can be estimated analytically other way also we can put this value S 1 S 2 minus 1 h 1 by if in terms of the local coordinate system xi eta psi put all these things, then we can estimate also that its finally, coming. So, the 4 by 9 h S 1 S 2 then finally, express like that only the surface 1 2 3 4 if there is surfaces associated particular 4 node then H e can be estimated like this or say in terms of the C. h S 1 S 2 by 9 of h S 1 S 2 will be the this constant H.

h S 1 S 2 and then 9 and this we can estimate this way also and we can find out and I will directly integrate put the do the integration. And then we will be able to find out this particular coefficients of this matrix and that way we can estimate on the surface also. So, this way we can estimate the one particular elemental all the components of a particular element. Now once we look at count on this thing both the what is the here we are assuming the 1 2 3 4

that is the local numbering of this particular element on this particular surface, but each and every element having some global numbering system also.

So, once we do all these things in the local numbering system it is a follow. Say in case of brick element we follow particular sequence and we construct the elemental matrix for node 1 2 3 4 as counting all these nodes also, but that and similar numbering local number you can follow 1 to 8 also, but each N node also having some global numbering.

So, once you do the elemental formation for all the elements for one elements of a matrix focusing on one particular element. Then according to the global numbering we can put the desired position. When we do the assembly for contribution from by calling from one by one of all the elements.

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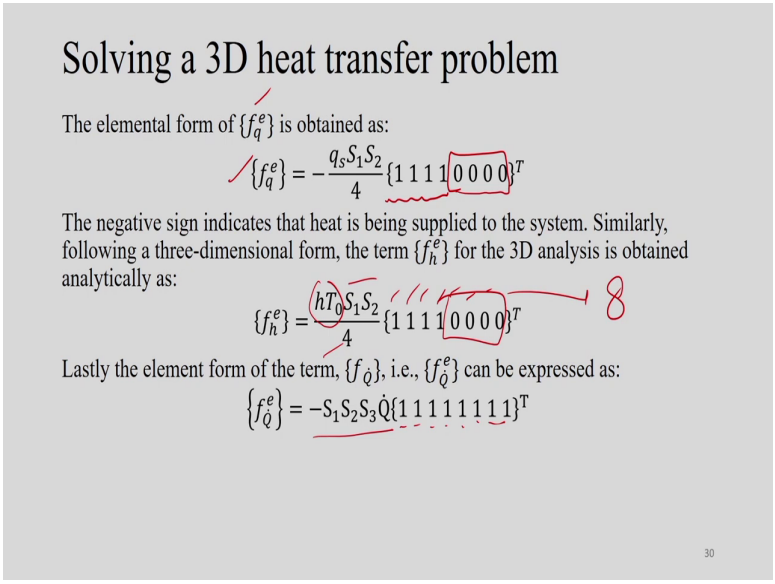

Similarly, the elemental form of f q also we can see f q means the surface flux. We can see in the if we look into there are 8 nodes one particular element, but if there is a contribution only for the surface then only the surface components are there this contribution from the remaining's you can put simply 0.

Because that is not associated on the surface that is the that particular node is the not belongs to this particular surface. So, that is way we can estimate this also the surface contribution. And similarly for the f h is associated with the h T 0 S 1 S 2 4 in terms of this thing here also this is not belongs to the surface.

So, in this case we can see there are 8 nodes contribution 1 2 3 4 maybe it is belongs to the surface, but this is the other not belong to the surface. So, then once we construct this thing though if it is not contributing because it is the inside the volume then we can simply keep that remaining part as 0, but once with the volumetric heat source term. So, then that volumetric heat source term it is the distributed over the volume.

So, then contribution from all the 8 node has to be there. So, that is why you can see even we calculate the that way the similar strategy also, but here we can see all the there is a contribution not we are not putting 0 because this is the volumetric part. So, we have the contribution from all the nodes we have to consider.

So, that way a for in case of the 8 noded brick element we can simply calculate the required elemental matrix and once we do for one particular this thing. Then we go for assembly look into the general form.

Now, that is ok, so once we discuss about this the, how elemental matrix can be form? One particular a particular problem. So, if it is a thermal problem also we have shown that if it is a thermal it is related to the thermal problem.

So, we normally solve the heat conduction equation and from there we look into the what is the governing equation what is the boundary conditions and from that point of view we

discretize we follow some kind of weighted residue technique also Galerkin weighted residue technique.

We form the one what we form all the components that comes from the governing equation along with the boundary condition also. Then we construct the matrix for one particular element once you construct all the different different matrix for a particular element depending we have to look into that because not all the elements belongs to the surface also. Which elements belongs to the surface, we have to treat that is the from the because that contribution from the surface has to be there also, but certain elements which inside the volume also. So, that element do not contribute for the surface interaction

So, therefore, in that cases only the contribution from the volumetric term will be there. So, no surface components will be there for this particular element. So, looking into all this aspect we calculate at one place all one particular element all the components. The convection term also the conductivity term conductivity matrix convective terms also and all other if there is a tangent term or we can consider all the matrix.

So, once it is done for one particular element one single element then we call element one make a loop also. Then put the position that contribution from the each node point accordingly the their global position we can put that contribution. Then we make a big matrix depending upon the, what is the node number for the whole solution domain.

Then for example, the whole solution domain having 100 node point so, but one element is may be having only 8 node point. Then we can construct for the 8 node points every all the matrix. Then then we call this matrix and then we place it the we create the 100 by 100 matrix.

Because that is the global numbering of the node point. Say then from the there we can put the this contribution from the particular global number of this particular node and then we assembly for this is then finally, it becomes 100 by 100 matrix. So, then it is in the form of a x equal to b. So, then we solve for this linear system of the equation to get the variable.

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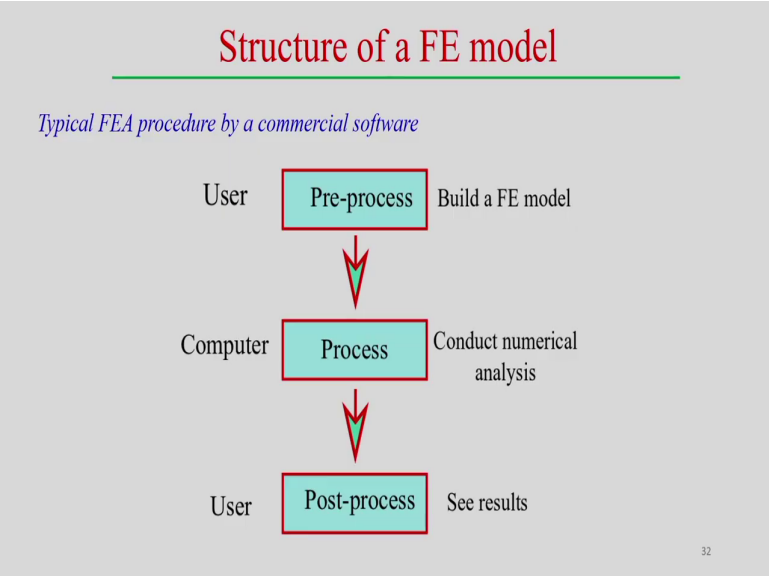

So, this is the usual procedure of finite element. Now we see that, what is the structure of a finite element model? If we look into the structure of a finite element it is like that, typical finite element analysis procedure by a commercial software normally first is the user defined parameter is required.

And that is called the pre processing that helps to build a FE model. So, one pre processing means we have to decide the coordinate system and we have to decide the unit also and then we have to give for a particular problem, what are the required material properties if it is thermal problem it is the mechanical problem.

Also accordingly you have to define all the mechanic all the properties also then we have to create the geometry. That means, what is the define the solution domain and then we have to

follow some kind of that discretization scheme also we can follow we can follow some meshing scheme also.

We can define the meshing for example, we can choose the brick element also for whole the domain or we can choose some automatic meshing also. Some position we can do some kind of the fine meshing is also required and some cases may be coarse meshing is also required.

So, all kind of these things the pre processing to develop built FE model. So, before run the actual code the actual program. So, we do all this pre processing part first then; then we do the use the help of a computer do the processing.

They conduct the numerical analysis processing means it is a its a simply it calculate the contribution from the each element, then assembly for the contribution for all the elements then make a global matrix and then solve for the global matrix assuming the linear system of the equation and then we solve this linear system of the equation.

So, once we solved the equation, then we getting the output what is the variable each and every node point. So, if it is a temperature it is a thermal problem we can get the temperature distribution each and every node point. If it is a fluid flow problem we can get the each and every node point what is the velocity component, so like that.

So, once it is done then use the post processing is also required. So, what way we want to get the result. So, that means, do we look into the result that is distributed each and every node point also or do you want to look into that is in the elemental form also.

That means, on the one particular element what is the particular value that is average value of the particular variable over the element or suppose we want to look into the result one within an element also one particular point we will we want to what is the result.

So, once we look into the particular point then the you simply interpolate or extrapolate this thing, the surrounding data point node point and then give the result in this particular point

which is not belong to a node point. So, that is how we can see the results also. And for example, in case of thermal analysis we want to see the, what is the temperature distribution? Temperature isotherm also different way we can present the result also.

If it is a stress analysis model we can see the distribution of the distortion distribution displacement field also we can see that we can see the strain distribution we can see the stress distribution. And Von Mises stress also the its average value we can see the equivalent stress value we can see also an even individual stress component also we can see the different node form.

So, this way the different way we can look into the result that is called the post processing just to see the result it is also user defined. So, user can decide what kind of the results we can see. So, this is the overall structure of a finite element model.

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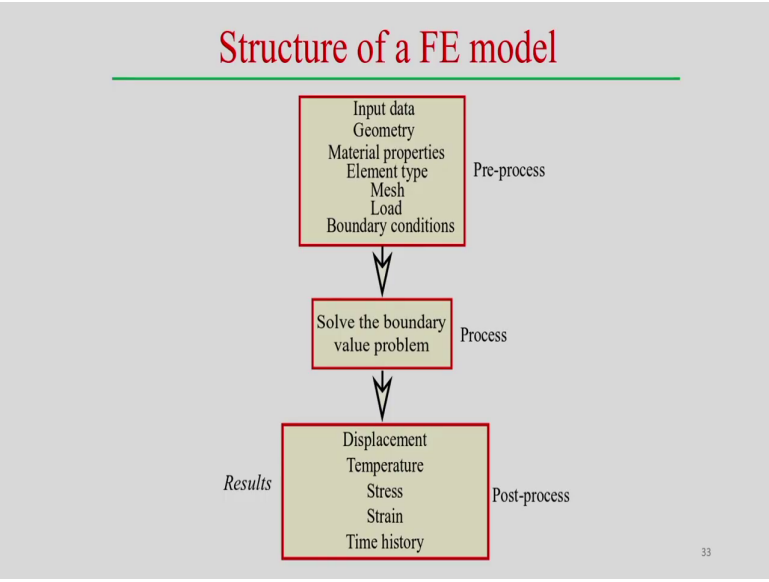

Now, in details the pre processing here you can I think we have may be pointed here that in pre processing we can get the input data required the then we give the geometry then basically domain of the solution domain then, what are the material properties? Then after that type of the element we can use it, what are the mesh? Meshing we can do.

Then how we can apply the load may be thermal load or may be it is mechanical load or then finally, what way we can look into the boundary interaction. So, boundary application of the boundary conditions all this pre processing is required. Now, the processing the solve the boundary value problem. So, looking into the or we can see not only boundary value problem. I can say that finite element problem the whole in the linear it finally, forms in the form of a linear system equation.

And it solve the linear system of the equation to get the output in each and every node point. Now, results getting in the form of a displacement field we can get the in the form of a temperature field also stress analysis strain also and even time temperature history also. We can see all sorts of results we can find out at the post processing part.

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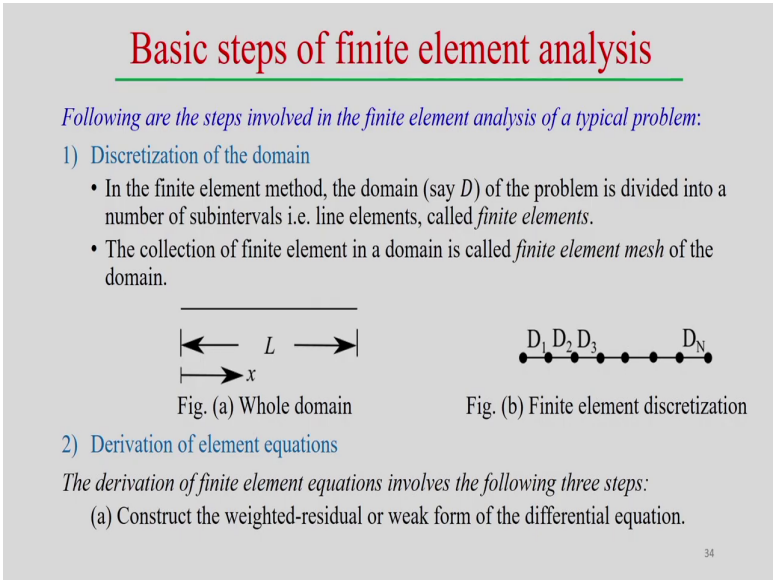

But basic steps of a finite element analysis just look into this thing I just try to look into the overall finite element analysis what way we can do this thing. So, following are the steps involved in the any kind of the finite element analysis of a particular problem. So, first is the discretization of the domain.

So, discretization of domain means in the finite element method it is a ah solution domain is there and say it is divided into small small elements number of sub intervals ah, it means the line elements called the finite elements. So, it can be the if it is single dimensional problem then we can create some kind of the line elements small small segment and then it is called the finite element.

So, basically discretization of the domain is required. Now collection of the finite elements in a domain is called the finite element in a domain is called the finite element mesh of the

domain. So, that is the basically that is in term that is sometimes it is called as a meshing means simply discretization of the geometry total geometry into small small elements.

And then accounting this when you create small small elements mean we have to keep accounting what is the number of elements we have to each and every element has to be give a unique number. So, that we can identify one particular element. Apart from this thing each and every node point has to be an unique number.

So, that globally you can identify this particular node number and then there is a need of each and every node number. We need the information of the, what is the coordinate of this particular node number? So, that is the that coordinate is maybe called the global coordinate system in that form; that means, data each and every node point what is the coordinate data.

Apart from this thing it is also necessary to keep on accounting all this thing for a each and every element. So, one particular element is associated with the so many node numbers. So, then so one what is an element number, and what are the corresponding node number of this particular element? That information is also required. So, all this data is actually stored once we create discretize the geometry in the small small elements and all this data structure is actually required to further processing.

Now, derivation of the elemental equations once we do create the element then we have to derivation of the finite element equation solving the 3 steps. So, it means that we have to then we focus one particular element in general. in general one particular element we have to look into the construct the weighted residue technique form of the differential equation along with the boundary interaction if it is over the boundary.

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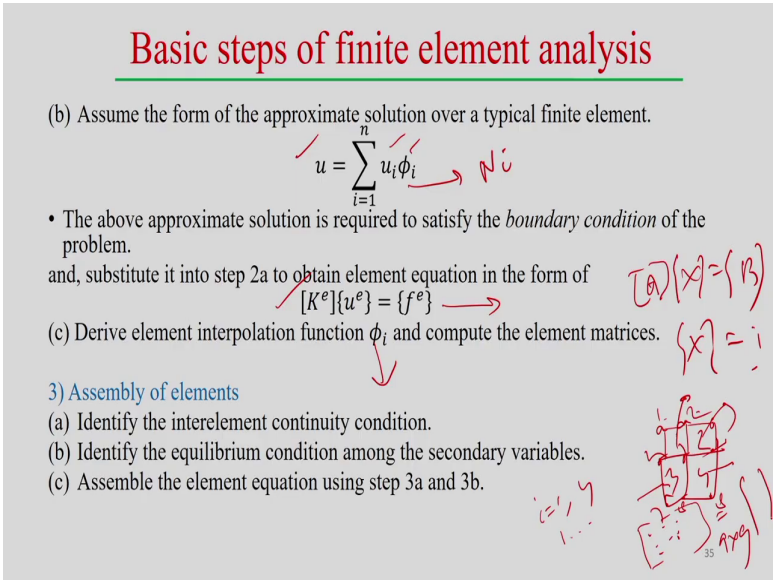

Then that is required to do this performing this particular task. So, be assuming the form of approximate solution of a typical finite element. So, in this this is a field variable use the variable in this case is the that particular variable it can be displacement, it can be temperature also. That can be represented like that also say i equal to 1 to n ui phi i in that particular form ui phi i is that that this some sort of shape function and then representing some sort of the nodal value of this particular variable.

So, this above approximate solution is required to satisfy the boundary conditions of the problem that is also required also it should satisfy the boundary condition of the problem. And substitute it into the step 2a to obtain the elemental equation in that form.

So, once we do substitute all these things then value then we can finally, from this elemental equation. So, maybe in stress analysis model we can say the case the stiffness matrix. We can

form for a particular element and then use the this displacement field and f is the load vector of this particular element. So, then a load in that form equal to a x equal to the b. So, we can solve for x also in this cases to get the this variable means which is node point.

So, so once we form this elemental equation in that particular form and then derive elemental interpolation function phi i and compute the elemental matrix. So, in terms of the if it is phi. So, if we put before and also the interpolation function or shape function phi i replace the shape function for example.

Then this thing we construct the elemental matrixes. Elemental matrix is basically construct for one particular element and that particular element it is also necessary at the next step also that basically we have to look into that local coordinate system on this particular element and we have to make the formulation based on the local coordinate system such that. Same kind of conversation from local coordinate system to can be applied as a master element and it can be applied for all the elements.

And then next step is assembly for the element. So, identify the inter element continuity condition identify the equilibrium condition among the secondary variables and assemble the element equation using the step 3a or step 3b. So, it means that identify the inter element continuity equation.

So, that we have to look into this is also necessary that this particular. In general we can see that one particular element. So, say this is the element 1, 2, 3, 4 it is associated with some node also and some node common to elements. So, we focus on the what is one particular element formulation. So, same kind of can be 2, can be 3, can be 4 also. So, contribution all these things on, then we keep on keep on calling we make a loop say i equal to say 1, 2, 4 in this case.

So, then contribution from element 1 we can put in the global matrix. So, maybe there 1, 2, 3, 4, 5, 6, 7, 8, 9. So, it is the a 9 node. So, it can be 9 by 9. So, we can put contribution 1, 2 maybe 1, 2 3, 4.So, then we can put 1, 2, 3, 4 according to the contribution in the global matrix. So, if it is done for same thing for next element 2nd element 2nd. Same thing we can

do for 3rd element 4th element accordingly the contribution is put in the global matrix form. Then once it forms according to the right hand side also form. Then we solve this equation solve the system of the equation to get the value.

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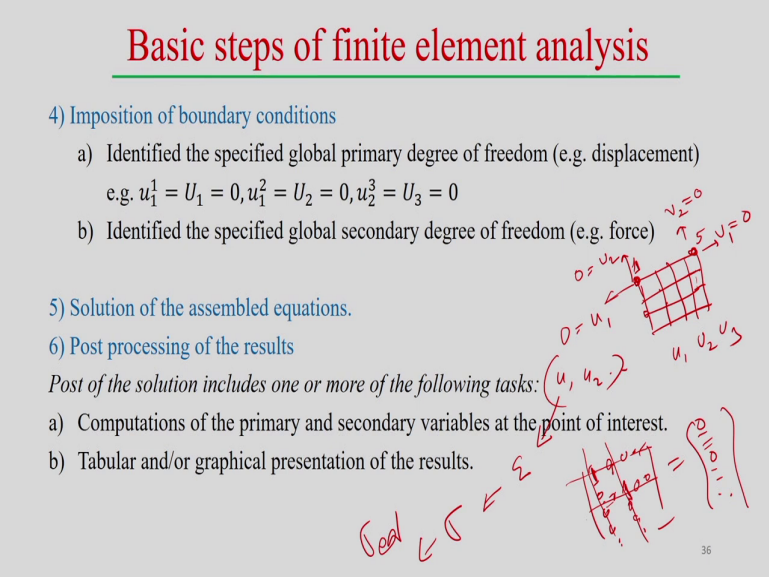

So, this is the general steps basic step of a finite element analysis. Now, before solution so once we look into the governing equation all these things or maybe in displacement problems also even we can look into the boundary interaction, but other way the this it is also necessary to impose the boundary condition. For example, in strain displacement problem also.

In that cases identify the specified global primary degrees of freedom displacement and for example, u u 1 capital U 1 capital U 2 and capital U 3 can be 0. So, that means, suppose one is domain this is the domain analysis discretized, but we need to put some kind of the displacement constant for example, these two point.

And suppose displacement constant say U 1 and U 2 equal to 0 we have to put and here also suppose U 1 U 2 equal to 0 U 1 equal to 0. It means that once is for the elemental matrix then we can impose the boundary condition in such a way then one is from the matrix the elemental matrix. Such that we can impose we can ensure this particular node point it may say suppose this is the global number 1 and suppose 1, 2, 3, 4 suppose this is the global number 5 this particular node.

So, particular node once we look into the contribution particular node global number is 1 in that cases we have to ensure the displacement equal to 0. Accordingly you can modify the matrix such that diagonal of this particular matrix in this position diagonal element should be 1 and remaining can be particular corresponds to the row and column. So, it is like suppose this is the global matrix also and the 1 position 1 and suppose this is the position 5.

So, here also it is position 5. So, here we can ensure 0 and this particular row and column, here this particular row and column we can ensure the remaining's are 0. And the right hand side if it is displacement 0 mean this 0 and fifth position it is 0 and remaining can be some other value. Such that we have to modify this matrix in such a to impose the boundary conditions on this particular node point.

And such that it when you solve this one it will ensure that exactly we will be getting the displacement equal to will be 0 for this particular node point and both the components U 1 U 2 would be 0 accordingly we can ah. So, that we can modify this matrix. So, this is the way to impose the boundary condition normally displacement boundary condition associated with any kind of the stress analysis problem.

Now, once we do this imposition of boundary condition ensure then solution of the assembled equation then we go for the solution of the assembled equation and that is the there are. So, many solution strategy are also there and then looking into we will discuss also the different types of the solution strategy. But finally, we can solve this particular equation then once it is done then processing of the result.

So, once solution is done then we are getting the displacement field each and every node point it if there is a 2; 2 dimensional problem or 3 dimensional problem accordingly. We can get the displacement field along say if it is 3 dimensional problem, we can get the what is the value of U 1 U 2 and U 3 on each and every node point. if it is 2 dimensional problem we can get only the two components U 1 and U 2, so like that.

So, then computation of the primary secondary variables at the node point of interest and graphical representation of the result. It is also possible graphical representation we can extract the data point and from the data point we can graphically we can represents the result, but although we are getting the results in terms of the U 1 U 2 in that displacement field.

But we know that displacement can be converted to the strain field and from the strain also we can estimate the stress field. So, this way we can convert displacement to the stress strain to the displacement field. So, strain to the stress field. So, that it means that we are getting even if you know the stress field also from here we can estimate the equivalent stress field also. That means, single value of the stress components.

Or the individual component of the stress similarly individual component of the strain all kind of information we can will be able to get in the during the post processing of the results. So, in that different way we will be able to get the results from the analysis. So, that is the post processing of this.

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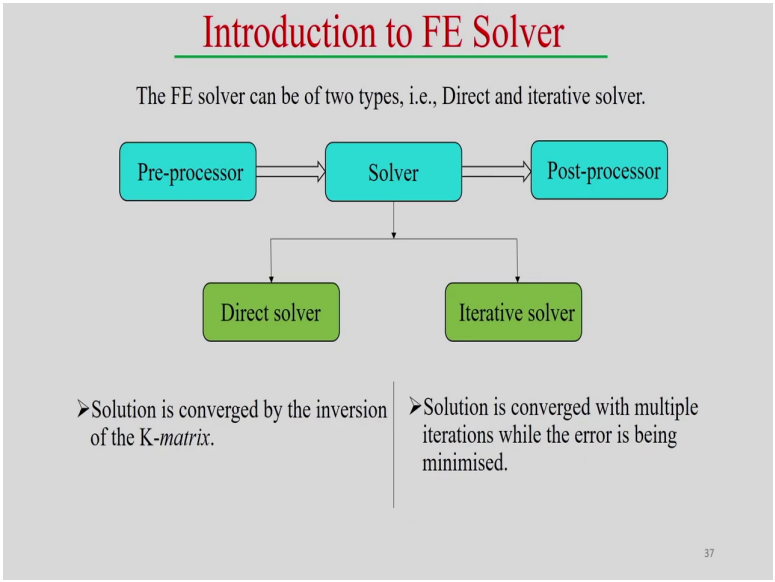

So, that is the basic steps for any kind of the finite element analysis. Now the solver is one of the important part associated with any kind of the finite element based problem and in in this particular problem most of the time consuming part is the, to get the solution of the linear system of the equation.

So, definitely if you look into details about this any kind of the finite element based problem. So, to solve this particular system of the equation, it takes the maximum time this thing or other way you can say the computational time entirely depends on the choice of the solver in any kind of finite element based problem.

So, basically a finite element solver in general can be 2 types one is the direct solver another is iterative solver. So, direct solver means it is a kind of ah, how it works? So, first pre

processor this assemble all this thing is done. So, once the after assembling the global stiffness matrix is ready then we go for solver.

So, then we try to look into the how we can solve this matrix. So, then then there is a need of the solver then one solver will get the output then we look into the post processing of the result the different way we can represent the result. So, this is the user procedure this thing, but solver is the main part in the finite element processing. I think in terms of time in terms of the computational cost in our other computational cost in terms of time.

So, then solver can be direct solver or it can be iterative solver. So, direct solver means solution is converge by the inversion of the K matrix that is simply inversion of the K matrix we can get this direct solver is possible, but iterative solver means in this cases the solution is converge with multiple iteration is required with the error is being minimized.

So; that means, we can assume some initial value of the solution. Then we can follow some strategy or some algorithm we get the next value and then compare with the previous value. So, that is why some iterative way we can calculate the result also.

So, definitely if it seems that if we look into in that way the direct solvers what you can get the exact solution inversion of the K matrix. And maybe so many iteration is involved in the iterative solver, but looking into these two aspect it seems that direct solver can be more suitable in case of finite element based problem.

But in actual practice if the element size or maybe number of nodes is used in a particular domain of analysis for example, say in usual problem it is 50000 or 1 lakh of node points are available. In that cases if you look into any kind of the direct solver also it takes the huge time consuming effect.

But maybe nowadays, some iterative solver has been developed. Such that very quick, although we can getting the approximate solution, but approximate results also using the

iterative solution ah. Then the time may be it is it can take the less time than that of the direct solver.

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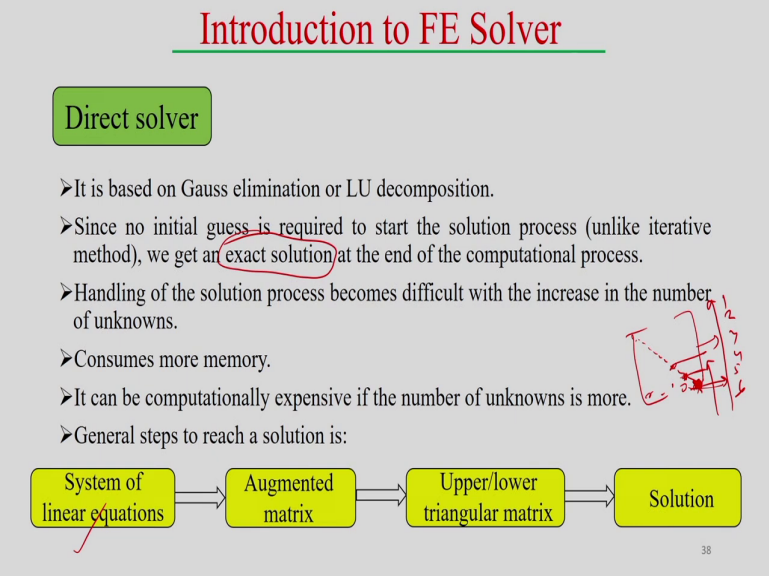

So, let us look into this thing, but what is the difference between direct solver and the iterative solver. So, direct solver actually it is based on the Gauss elimination or lower triangular or upper triangle decomposition. So, we know the Gauss elimination method if we put this thing, but in this case initial no initial guess is required to start the solution process.

So, maybe as compared to the iterative solution method then we can get an exact solution at the end of the computational time that is very important. Because, if you look into the direct solver we can expect the exact solution at the end of the computational process. It says simply following the gauss elimination method also if we follow this basically simply inversion of a particular matrix.

Then you can expect the exact solution of this particular, but maybe if handling of the solution process becomes very difficult with the increasing number of the node point unknowns variables. So, basically or in terms of the node points we understand in the finite element method. So, more number of node points means more computational time. So, that if it is increasing is too much of node point then it is very difficult to get a solution or maybe I other way we can say the computational time is used.

It consumes more memory the solution procedure and it can be computationally expensive if the number of unknowns are more that is already explained this thing, but general steps to reach a solution is that actually the general step system of the linear equation. So, system of the linear equation in the form of a matrix we can for and then we basically have to inverse this matrix to get the solution.

So, it can augmented matrix can be formed and it can be from the forward elimination if we look at the Gauss elimination forward elimination. Gauss elimination processes there are two steps one is the forward elimination as and second one is the back substitution.

Forward elimination means it basically form in the form of a in the particular matrix is converted to in terms of the upper or lower triangular matrix in this particular form. So, when it is converted to the upper or lower triangular matrix then from the back side maybe I can say that if symmetric matrix you can form in this way also, so that triangle.

So, once it is done forward elimination is done. So, it this basically convert this particular way this matrix then back substitution you start the solution from this thing because here is the remaining such 0. And here we can get the solution direct solution this is the first variable. Then once you get the solution use this solution value for the next next next step.

So, then then 2 and next 3 step 3 like that from the back we can get the solution of the 1 variables. So, variables 1, 2, 3, 4, 5, 6 for example,. So, we get the solution for the from back substitution means. We start with the solution from the get the solution of x 6 the variable 6.

One get the variable 6 solution then in the matrix we can use the value 6 value. Then we can get the solution of 5 one for the 4 also we need the solution of 5 and 6 then we can get the solution of the 4, like that back substitution. So, then finally, we are getting the solution we can solve this particular equation.

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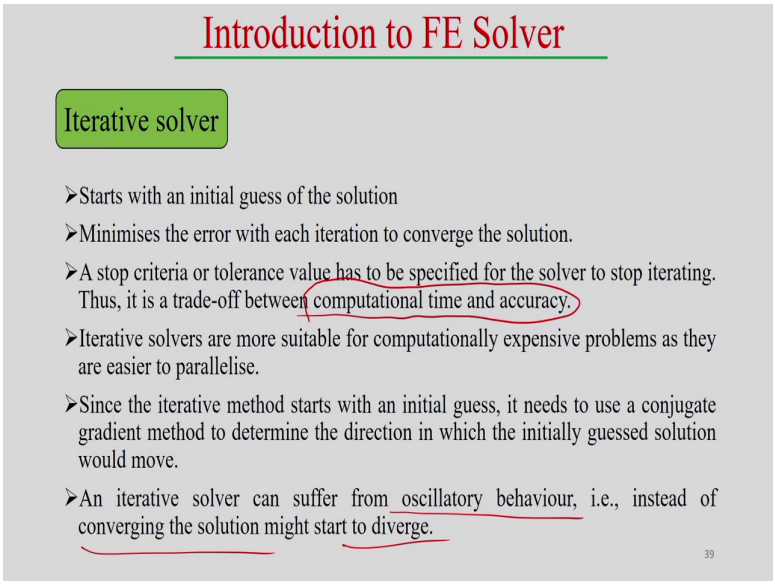

So, I am not going into details how the Gauss elimination works also you can it is available in the any standard book and standard module the this particular procedure is available. Now, iterative solver of course, they start with the initial guess of the solution, but minimizes the error with each iteration to converge the particular solution. So, basically it works the minimization of the error in the with the each iteration 2 converse in the particular solution.

So, that is there, but stopping criteria or tolerance has to be specified to the solver to stop the iterating thus its trade off between the computational time and accuracy. It is very important

because it is a trade off computational time and accuracy depending upon that, what kind of the it stopping criteria you can decide? Or maybe tolerance we can give.

So, accordingly we can assume particular solution which is more close to this satisfying the tolerance criteria or stock criteria then we can accept this is the solution. So, definitely you cannot expect the exact solution of the equation using the iterative technique

So, since we are putting some stopping criteria tolerance value. So, definitely in this cases we can expect the approximate solution of the particular equation. So, that is in that aspect it is different from the Gauss elimination method because Gauss elimination method will be getting the exact solution of the particular equation.

Now, iterative solvers are more suitable for the computationally expensive problem as they are easier to parallelize. So, basically parallel computational method is easy to form if you follow some kind of the iterative calculation. So, that is in that sense it is very important. So, in in that way it is advantageous. So, you can easily do the parallel programming when you handling the iterative solver or iterative solution method.

So, in that cases. So, there is a huge number of elements are there huge number of nodes are there. So, large number of system of equation then in that cases the iterative solver may be it will be advantageous. If it is the if it is possible to solve is the this thing parallel server has to be developed.

So, therefore, since the iterative method starts with the initial guess value. So, it needs the use of conjugate gradient method to determine the direction. Definitely once you start the initial value then what may be the increment of the particular value one particular direction what can be the that is. So, that is why conjugate gradient method can be used to determine the direction in the initial guess solution you will remove.

And apart from that the iterative solvers the convergence of the iterative solver also depends the very good initial guess. So, initial guess is very good also then we can expect the quick convert solution of a particular iterative solver so that is also very important. So, there are

several algorithm also there ah, such that we can quickly have a very good initial guess for the iterative solver. So, indeed iterative solver can be suffer from the auxiliary behaviour some and some that also possible.

So, instead of the converging the solution there may be start to the diverse in that cases there may be iterative solver may not be the effective in this particular situation. So, that is why that we can say the limitation or maybe some kind of the disadvantage of this iterative solver also there ok.

So, now once it done this thing the last part of this module 2 also. Then above apart from this elemental form all this finite element process also. Then that maybe what way we can do this what are the basic steps of the finite element method and how we can do the elemental form elemental matrix. And apart from that what are the basic steps and what kind of the solver we can use in the solution.

So, that is the basic thing that was associated with the basic things of a finite element method, but apart from that sometimes there is a need to look into. In specifically I am talking about the welding problem also. So, in that cases there is a need to track the interface.

So, it means that that it is easy to understand that if your domain is very fixed also and fixed domain and we can do the analysis over the fixed domain also we can get the solution on this particular domain also assuming this thing, but if there is a evolves the interface for example, in the welding with respect to time the weld pool grow. So, weld pool grow and that is the that weld pool domain and at the same time the free surface also may not be exactly the flat what was the initial state.

So, during this after the welding process the initial the you can get some curvature also or. Sometimes if there is a material deposition on the particular domain also, then the this domain which the free surface free surface profile of this particular domain may not be exactly what was the initial case.

So, in that cases it is necessary to look into the what is happening the interface or other way also. Mathematically how we can track this interface. So, to track the interface also there are the several strategy several algorithm also there overall of; that means, the basic frame of the finite element method or we can say that the other has been developed. That is the extended finite element method.

So, this is the extended finite element in method is in particular if there is a discontinuity or some kind of the free surface there is a need to track in a in any kind of the problem then there is a need to further extension of the finite element method can be used. Which is called the, extended finite element we look into that. But apart from that there are several other interface tracking method which can be in which can be integrate over the basic finite element based analysis to simply track the interface in a finite element based problem.

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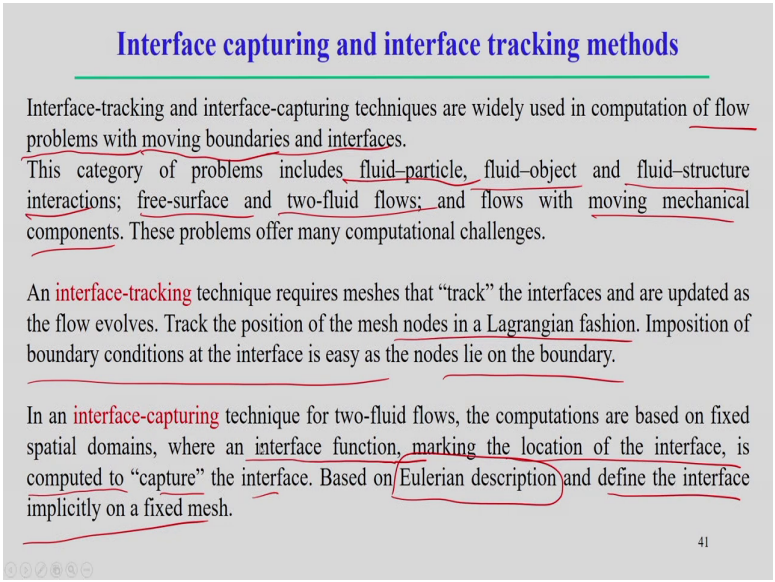

So, let us look into this thing in general the interface interface, but when you looking into the what do you understand by interface there are two different way to look track the interface one is the interface capturing and another is the interface tracking method. So, interface tracking and interface capturing techniques basically widely used in the computation of the flow problems if you see the flow problems. So, with the moving boundaries is there and presence of the interface.

So, in that cases we can use the interface capturing and interface tracking method also and this particular problem say fluid particle interaction fluid object and the fluid structure interaction interaction free surfaces and the 2 fluid flow and 2 phase flow also and flows with the moving mechanical components and flow with the moving. All these particular cases which is very challenging actually computationally challenging in that particular cases. We can find out there is a need to look into the situation of the interface.

Now, in from the point of view mathematical point of view or we can say that analysis of the finite element method. In that case the interface tracking technique requires the meshes the that track the interfaces. So, to track the interfaces either conform to the mesh and updated as the flow evolves.

Track the position of the mesh nodes in a Lagrangian fashion and then imposition of the boundary conditions will be easy in that cases because node lie on the boundary. So, in that cases we have to in the Lagrangian fashion then maybe in that cases the interface tracking means the interface is moving along with the mesh also such that every time its mapping with the particular need.

And there is a need to look into the remeshing of this particular element, if it is a large deformation problem also. So, in that cases the interface lie with the node point. So, that is the called in general that is called the interface tracking method. But interface capturing method is the 2 fluid flow or maybe computational based on the fixed domain. In this case the capturing method is equal to the domain is fixed, but externally where it is possible to capture the interface.

So, without not confirming the every time the mesh along with the interface the movement of the interface. So, in that cases domain is fixed over the domain interface can evolve. So, but interface can be track externally. So, that is called the interface capturing method. So, it means that the where the interface function making the location of the interface is compared to the capture interface.

So, based on in this case Euler and description and that define the interface implicitly on a fixed mesh. So, basically it is it is necessary to define the implicitly the interface over the fixed domain. And then we can track the interface looking into the different strategy.

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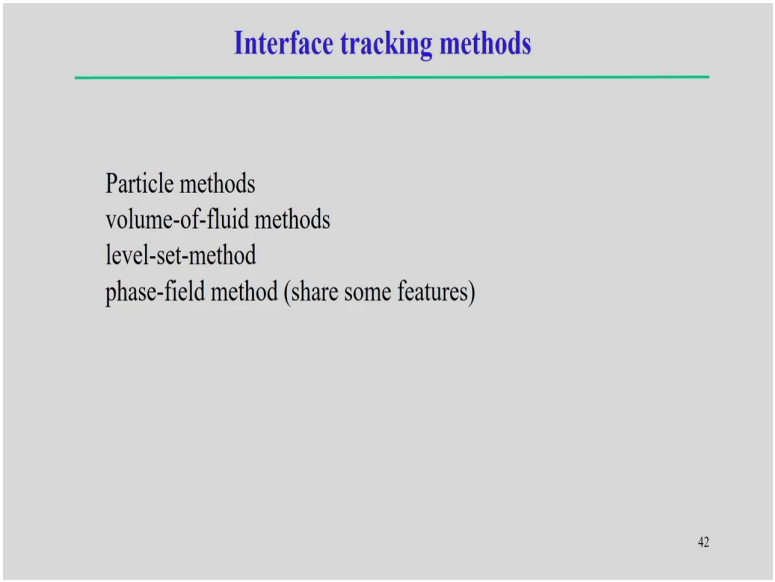

So, there are several interpret tracking methods the practical tracking, volume of fluid methods, level set method, phase field method these are the very common method. So, interface tracking method we normally use, I have discussed this particular different type of

methods one particular module, most probably it is the fluid flow module. So, maybe you can look into this that particular module the, what are the different methods you will be getting in details.

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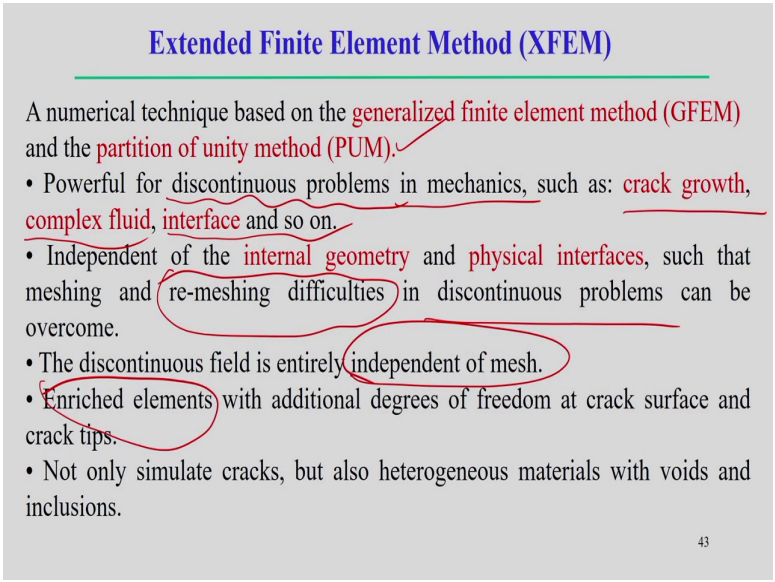

But here we just try to look into the what is the extended finite element how it is different from the classical finite element method. So, extended finite element method is basically numerical technique based on the generalized finite element method. So, that is called the GEFM or it is and the partition of unity method PUM sometimes it is called the partition of unity method.

So, in this case the powerful for discontinuous problem. So, it is more associated with discontinuous problem in mechanics. Such as the crack growth complex fluid and the interface and so on. All this kind of method we can use the extended finite element method.

So, it is basically independent of the internal geometry physical interfaces also because we remember it is a fixed domain we are implicitly tracking the interface. So, in that cases we can use the extended finite element method such that meshing and re meshing difficulties in discontinuous problem are not there basically re meshing is not required in discontinuous problem.

So, discontinuous for example, there is a crack. So, what way we can how crack propagate and what way we can, we can track the mathematically in in a particular problem. So, in that cases the this crack here creates some sort of discontinuous in the domain also. So, then the whatever the usual procedure that is to capture the discontinuous thing discontinuity.

So, simply remeshing in this particular domain. So, this is the one way to capture look into the crack what the evolves of crack in the particular mechanics program also, but other way also if you want to avoid the remeshing again creating the meshing in this particular domain we can follow. No, there is not necessary to rematch this particular domain rather we keeping the same mesh, but with different way we can track the discontinuity. So, that is why there is a need of extended finite element to understand this thing.

So, discontinuous field is entirely independent of the mesh, that is very true. That it is independent of the mesh and enrich elements with the additional degrees of freedom at the crack surface or crack tips may be required. So, some sort of enriched elements has to develop we have to understand the, what is the enriched elements. Not only simulate the cracks, but also heterogeneous materials with the voids and if there is inclusion all kind of this interface with tracking or in discontinuity can be captured using the extended finite element based method.

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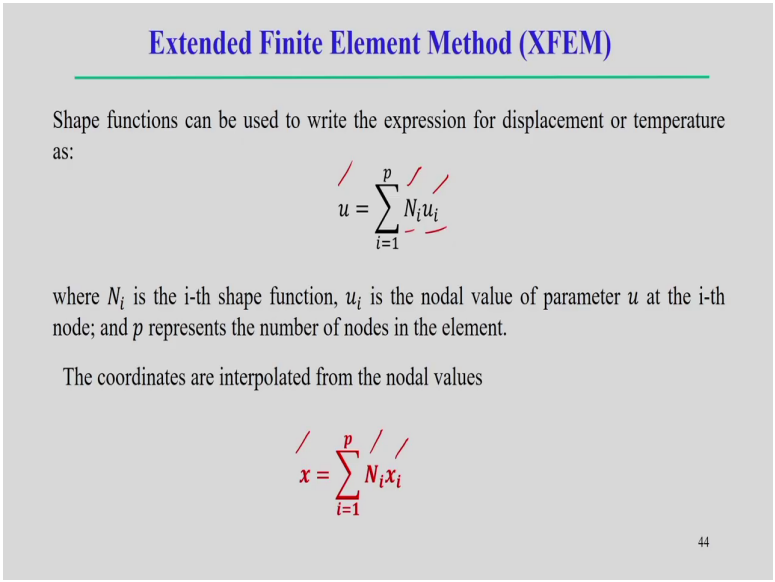

Now, we know the shape function can be used to write the expression for the displacement field in terms of this thing shape function and the displacement u u is the N i and u i. So, where n is the ith shape function and u has the nodal value.

So, this is the u variable and parameter u and the ith node and p represent the number of nodes in this particular element. That we note and we have already seen even it is temperature or displacement whatever we can represent this within particular domain within element is. In terms of the mapping with the shape function as well as the nodal shape function as well as the nodal values.

Now, the even same way coordinate can also be interpolated with respect to the nodal value. So, suppose x is the variable this can be coordinated in terms of the shape function and coordinate of the particular node point. So, N i is the shape function and x i represents the

coordinate of this particular node and p is the number of nodes of this particular element. So, this way is can be implement interpolated. Similarly any kind of the variable it can be temperature it can be displacement all can be in way it can be interpolated in this thing variable.

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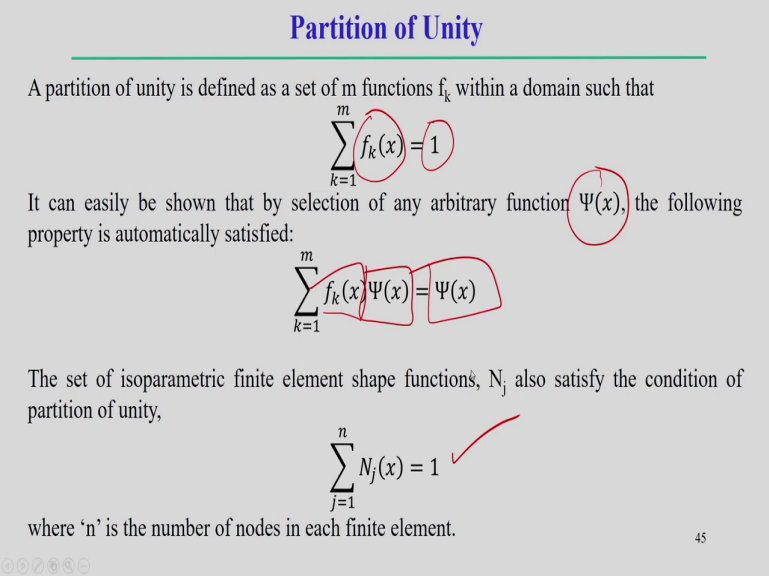

Now, partition of unity we started with these things basically the definite a set of the m functions f k within a domain. Such that k equal to 1 to m within a particular domain we define the functional f k f as a function of x is equal to one. Now it can be easily shown that by selection of the any arbitrary function say xi x xi eta psi x. The following property is automatically satisfied k equal to 1 to m this is the functional form and this is the thing, such that it equal to psi x xi x.

So, based on this thing we can find out the set of isoparametric finite elements shape function N j also satisfied the condition of partition of unity. So, that thing such that j equal to 1 to n N j x is basically 1. That we have already seen the summation of the shape function of this this thing also satisfy the partition of unity. Where n is the number of nodes in each finite element. So, n equal to number of nodes in each finite element. So, that also satisfy this partition of unity method.

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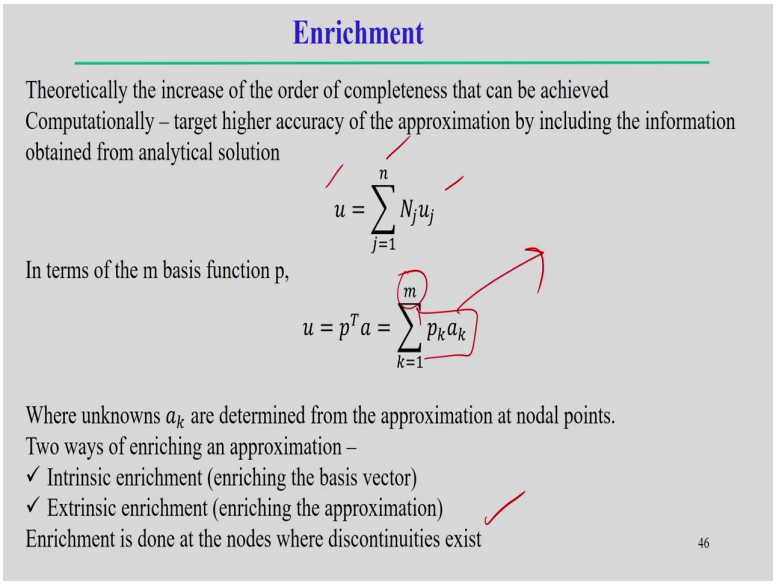

Now, theoretically the increase of the order of completeness that has been achieved. Basically in this particular case and computationally targeting the higher accuracy of the approximation by including the information obtained from the analytical solution. We can say that u can be represented like this. So, in this cases the information from analytical solution also helpful.

Such that theoretically we can increase the order of completeness, then we can achieve this thing particular relation n u equal to j equal to 1 to n N j u j that is way.

So, in terms of the m basis function p then we can write this way also as the m basis function then p k ak in this way also we can write. So, where unknown ak determine the form of the approximation at the nodal point. So, basically ak determine the form of approximation. What are the different form of the approximation in the nodal point? Can be represented.

Two ways of enriching this approximation one is the instinct enrichment that enriching the basic vector otherwise the extinct enrichment that is enriching the approximation. Two different way the approximation can be done. So, enrichment is done at the node point where the discontinuous continuities exist. So, enrichment done. One particular node point there is existence of the discontinuity.

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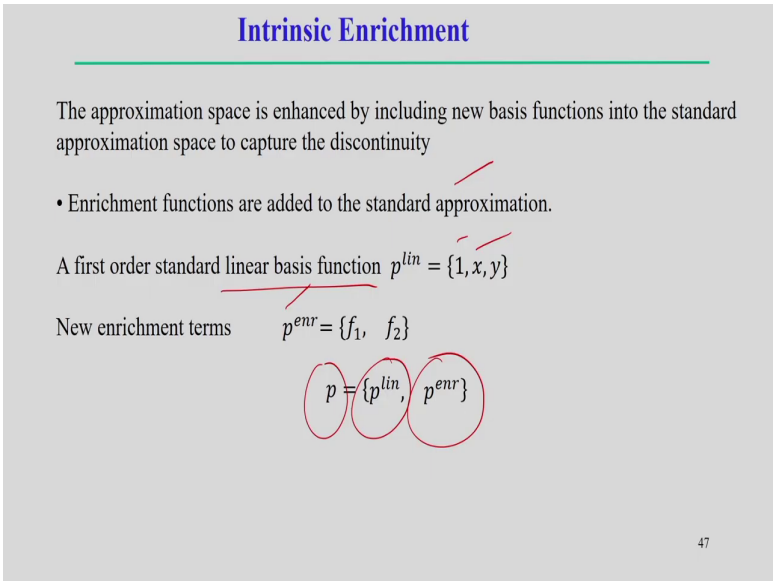

So, in that way the approximation space enhanced by including the new basis function into the standard approximation space to capture the discontinuity. So, in that way a enrichment function are added to the standard approximation such that the basis function linear basis function for example, standard first order linear basis function is this one 1 1 x and y.

Now, new enrichment terms can be like that in terms of the f 1 and f 2; two different functional form such that p consists of the linear basis function and as well as the enrichment. So, it consists of basically the we define the this thing to capture the discontinuity we can use some sort of enrichment function also.

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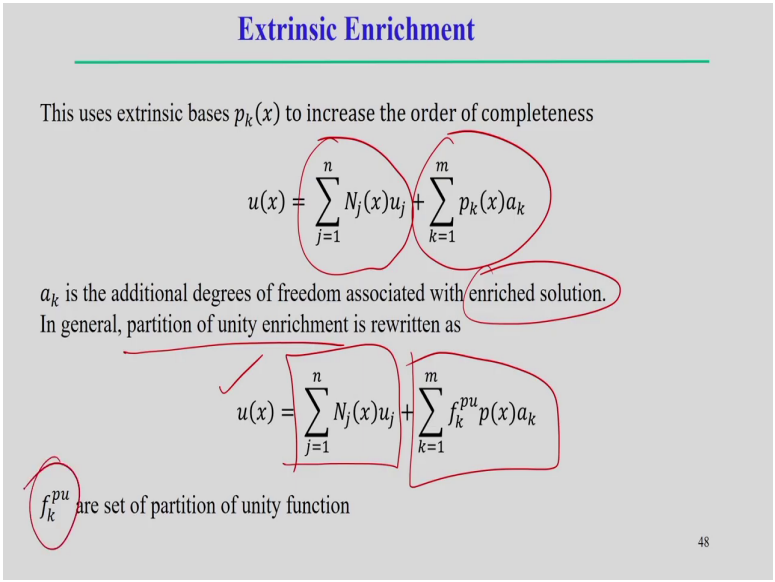

Now, this uses the extrinsic basics to increase the order of completeness it means that to order of completeness that from the theoretical part we can see that. Then apart from this conventional things the this thing we can use the other ah. We can use the pk; that means,

functional form simply to order of completeness we can enhance. Such that ak is the additional degrees of freedom associated with the enriched solution.

So, we can enrich the solution, but that enrichment can be done locally on the selective elements also and then additional degrees of freedom is associated with this particular in this solution. So, in general partition of unity enrichment. Partition of unity enrichment is written something like that consists of this the u x is the u is the variable. This part which is we have seen in the classical finite element method apart from this thing you can use this other functional form. So, here fpu are the set of partition of unity function.

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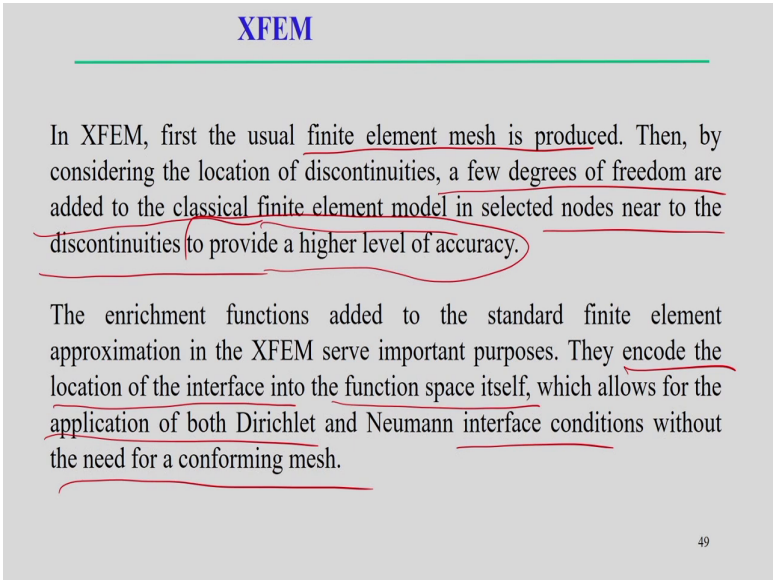

So, based on this theory that extended finite element is has been developed. So, you can see that in XFEM first the usual finite element mesh is produced. So, usual finite element mesh is produced, then by considering the location of the discontinuity a few degrees of freedom are

added to the classical finite element model in selected nodes only near to the crack or discontinuity, to provide a higher level of accuracy.

So, then enrichment normally happens near I think few degrees of freedom is basically added in the selected nodes near about the discontinuity or near about the crack. So, that way we can develop the x extended finite element method.

So, the enrichment function added to the standard finite element approximation in the XFEM extended finite element method, it actually serve the important purposes. So, they encode the location of the interface into the function space itself.

Which allows for the application of the both the Dirichlet and Neumann interface conditions without the need for a conforming mesh. It means that there is not necessary to conforming mesh on the exactly on that, once we look into the enrichment functions are added that may not resist to follow the conforming mesh.

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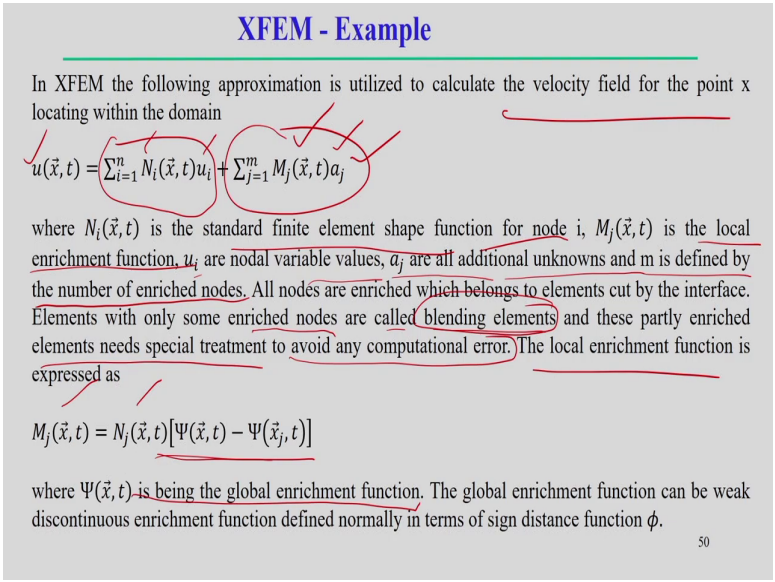

Such that XFEM can be approximation to calculate for example, the velocity field at the point of x location within the domain. So, in this cases use the velocity field you can use that 1 to n is the u u e y is the nodal values particular node point n is the shape function and plus this extra term.

So, in this ni is the standard finite element shape function or the particular node i and, but M j is the local enrichment function u i are the nodal variable and a j are the all additional unknowns and all additional unknowns aj because an M is defined by the number of enriched nodes.

So; that means, these terms is comes because of the enriched nodes and extra degrees of freedom, particularly associated with the enriched node. Now point is that what I can decide

the enriched node. So, that will see later on, but elements with only some enriched nodes are called blending elements.

So, elements which having enriched nodes that is called the blending elements and this partly enriched elements needed the special treatment to avoid any computational error, that is also required ah. The local enrichment function is can be expressed like that also in terms of the shape function and this xi is the global enrichment function in terms of this thing. Global enrichment function we can write the it is a local enrichment function.

So, that is why global enrichment function can be the weak discontinuities enrichment function defined normally in terms of the sign distance phi. So, that is the. Then in that cases also, global independence functions can be on the weak discontinuities. That enrichment function defined normally in terms of the sign distance function phi

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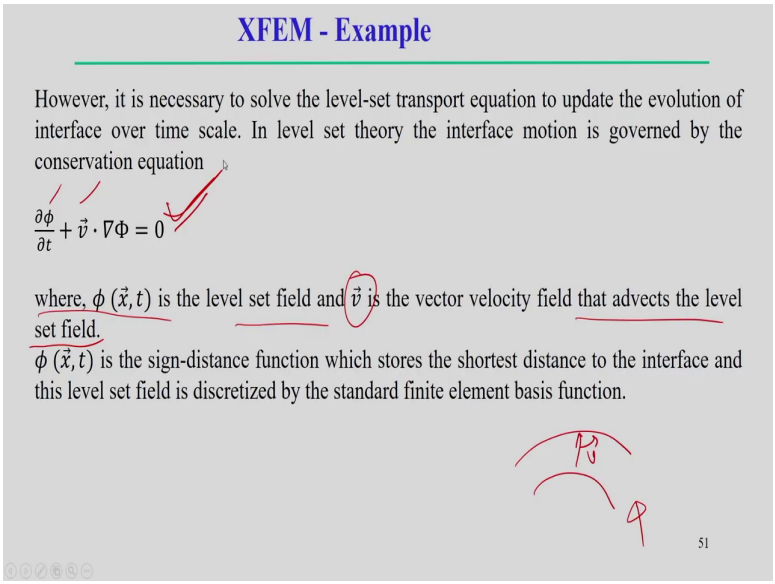

Now, however, it is necessary to solve the level set equation transport equation to update the evaluation of the interface. So, it is also required because to update the evaluation of the interface it is necessary to some sort of the level set transport equation to solve. So, that transport equation can be represented like that. So, phi equal to here phi is the that sign distance function.

So, using the signs distance function we can identify the interface and this governed by the conservation equation though this is the level set theory. In terms of the sign distance function we can see the phi x the level set field and in terms it phi x the level set field may be in terms of the sign distance function. And v is the velocity vector that advects the level of the set field.

So, v is the advects the level of the set field that means how is the one particular interface we capturing using these things ah, that is represented by sign phi functional form. Then what way it moves that with the velocity v also velocity gradient that we can put this.

So, therefore, this equation is necessary to solve to define the interface also. Say therefore, phi x t is the sign distance function in this particular case which stores the shortest distance to the interface. And this level set phase is discretized by using the standard finite element method basis function, standard finite element basis function, we can discretize the domain this particular equation. Then we can solve for this thing this particular field and then that field define the actual interface in terms of the shortest distance to the interface it stores the data in such a way.

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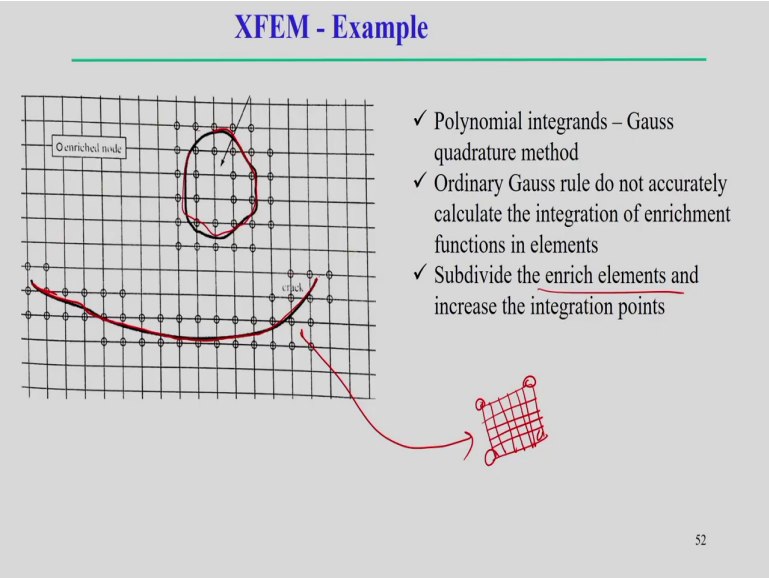

Now, we can able to understand these things example x fem also say suppose this is the crack length. So, what way we can crack length? But, what way we can do the enrichment of this particular? So, then we can identify that from through which a element it's a passes then particular element from this we can identify that element and then associate a node also and such that this associated node and element can be enriched this thing particular.

So, it means it is a locally enriched with a some locally identified nodes element through which the discontinuity exists. Even some inclusion also if suppose this is the inclusion on hole in the particular shape. So, we can identify what is the enriched element and then we can identify what is the enriched node also. And then on that particular enriched node or element we can apply the extra degrees of freedom what we have seen also that there is a one particular terms comes because of this because of the enrichment.

So, once you do the enrichment of this particular node point then we can formula we can develop this thing and one the total matrix stiffness matrix you can solve for it we can get the interface. Even to get the interface also we also it is also necessary we have to track the interface also it is also necessary to solve this particular equation also in a finite element domain in the same way. Such that we can able to track, what is the interface?

and there are the few issues in these things that polynomial integrands we can use the normal Gauss quadrature method also in classical finite element method and you can get the results also, but ordinary Gauss will do not accurately calculate the integration of enriched function.

So, once you use the integration of the enriched function then Gauss ordinary Gauss rule may not be give the accurate result, in this cases. So, in that cases what happens the it is necessary to subdivide the enriched element. So, suppose this is the enriched element in this case suppose this is the enriched element. So, pick up this particular enriched elements.

So, then in this particular enrichment subdivide the enrich element, such that it will increase the integration point. So, you can it is a small small integration point. So, small domain such that we can increase the integration the subdivision is required. So, that integration points can be used then the using the Gauss quadrature method or maybe numerical integration techniques if we follow then we can expect the very good result.

So, that is the two things one is the identification of the enriched elements or enriched node also in this particular. And then applying the enrichment function all this locally all this particular domain, apart from the what we are following in general way the classical finite element methods. Apart from this is the extra thing to track the interface also.

Solve this equation plus we need to solve this particular equation that govern that interface motion equation we need to solve interface motion equation to track the interface also that is also required. And that interface motion equation we can use the ah, we can define some level set function. Maybe you can use the sign distance function to get the to track the interface also, at the same time how it is moves we able to capture using this.

So, this is the in general way the overall the extended finite element works, but if you want to go more details about the extended finite elements you can follow any kind of the standard book to work into this particular direction. So, this is all about the module 2 the fundamentals of the finite maybe of finite element methods also for. In this particular module I have tried to discuss the basic this thing, but only the finite element method itself is a one subject one course also.

So, here I just overview the particular part or emphasize on this particular element. Which may be some sort of working knowledge to develop some kind of the finite element this model is specifically applicable in case of the welding process.

So, thank you very much for your kind attention.