Finite Element modeling of Welding processes

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Module – 02
Fundamentals of finite element (FE) method
Lecture – 14
Fluid flow

Hello, everybody. Today, we will look into the Finite Element Modelling of the Fluid Flow problems actually. We have already done the heat conduction problem the how what you can develop some kind of the finite element base model also. Now, try to look into the fluid flow problem also it is mostly associated with the fusion welding process, maybe it is confined into very small zone.

If you look into if you understand the fusion welding process there is a small weld pool. So, within that weld pool there is a material flow. So, significance of the material flow I have already explained these things because definitely when there is a look into effect of the surface active elements, then there is a need to understand the fluid flow as particular associated with the fusion welding process also. So, in from that perspective we will try to look into the fluid flow.

First we will cover the basic formulation the this governing equation of the fluid flow problem and then we will try to look into that what we can discretize in the particular governing equation and then what we can develop particular element, how we can form in this elemental matrix also. This is the basic objective of this today's this particular module.

But, details analysis of the fluid flow and the in overall the broad idea of the finite element formulation of the fluid flow if there is a one separate module. So, in that particular module we will discuss about the in details of the fluid flow problem also, but in this particular module we will try to look into the basics of fundamentals towards the what I can convert to the look into the finite element formulation for a particular element that we will try to look.

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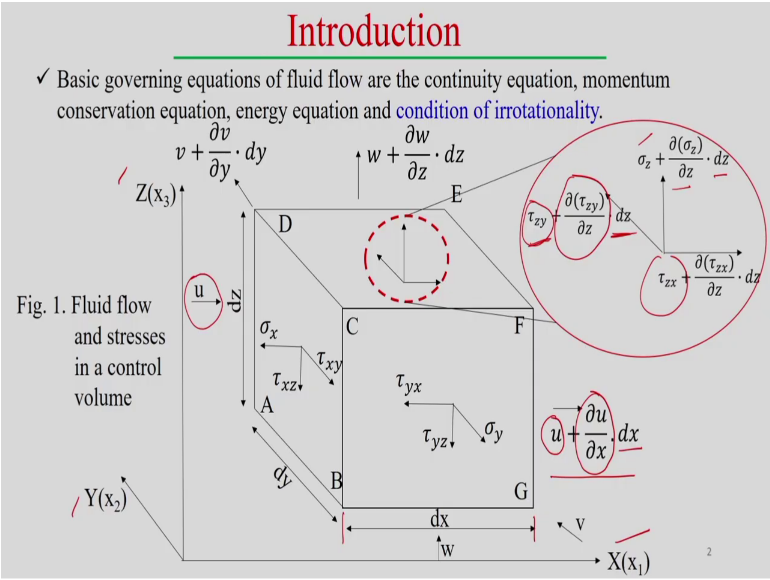

So, to start with this first we will look into how basic governing equation of the fluid flow are in the normally develop. So, there is a if you see look into this basic aspects associated with the fluid flow also is the first is the continuity equation, momentum equation even also energy conservation equation as well as the condition per irrotationality.

This is this basically associated with this basic fluid flow governing equation, but we will not discuss about the condition of the irrotationality. So, that maybe we try to look into only the conservation of the mass, momentum and energy equation; how it forms in associated with the fluid flow phenomena.

Now, to understand this basic governing equation we have to consider for one small element. We can see that it is a control volume. Within this control volume what is the different phenomena phenomenological things we normally express in the mathematical form also.

So, in terms of the stress also even when you do for stress analysis we consider small element and with that element what are the different stress components which is acting. So, with the similar concept we will be using here also to develop the continuing education as well as the momentum equation and even for energy equation also.

So, now say this is the small element and in this particular element we define the Cartesian coordinate system X, Y and Z direction, so, this X Y and Z direction; this element on the element if you define the different stress components because if you look into the sigma y, sigma x and sigma z all actually acting on the faces.

And, then normal to the faces the normal stress is acting that is represented by either sigma x, sigma y or sigma z and remaining other two are the which is parallel to the surface that is defined as a shear stress component. So, all this normal stress, shear stress components, we can see even also.

Now, we consider let us look into one particular direction x direction. So, this is the face one faces there is a velocity fluid velocity for example, this is u it is entering to the control volume as the velocity u and uniform throughout this faces also and then exit this thing u plus. So, maybe del u by del x into dx because dx is the length along the x-axis. So, this elemental length that we have already defined this is the dx.

So, similar way so, here see we assuming the there is a change of the velocity in between one entering the ah fluid velocity on the face of the within the control volume that is the velocity u, but inside this control volume there is a there may be change of the velocity field and that changes we can mathematical form we can write.

So, del u by del x or whatever changing with respect to the along the x axis. So, del u by del x and this is happening, this changing happening over the or you can say this gradient also del u by del x in this things, but this is happening over the length elemental length dx. So, that is why when is exits from this other faces also within his control volume then we can see that u plus del u by del x into dx.

So, this is the this corresponds to the along the X axis. So, similar kind of expression we can write in the along the Y axis, even along the Z axis also. So, similarly way if you see the stresses also acting this one particular and one particular faces if you see; the stress also acting for example, we are showing only the Z direction.

So, state of the stress can be like that tau y this is the stress components no shear stress components on the on the Z face are normal to the Z axis and then this is the changes actually del that change of the stress state with respect to del z into dz because this is happening along the Z direction over the elemental length dz.

So, same thing this is all the corresponding changes with respect to Z axis also sigma z also changes to this del sigma z by del z into dz similarly other shear stress components. So, same kind of expression we can write on the other faces also. So, maybe normal to the X axis even the faces and another faces normal to the Y axis.

So, similar way we can express all these changes. So, it means that within this control room there is some entry velocity within this counter entering and there is some exit also, but exit there is a changes all this mathematical changes we are considering all these things. So, this is the situation looking into one particular control volume. Now, what we can look into the continuity equation and momentum equation and energy equation can be formed.

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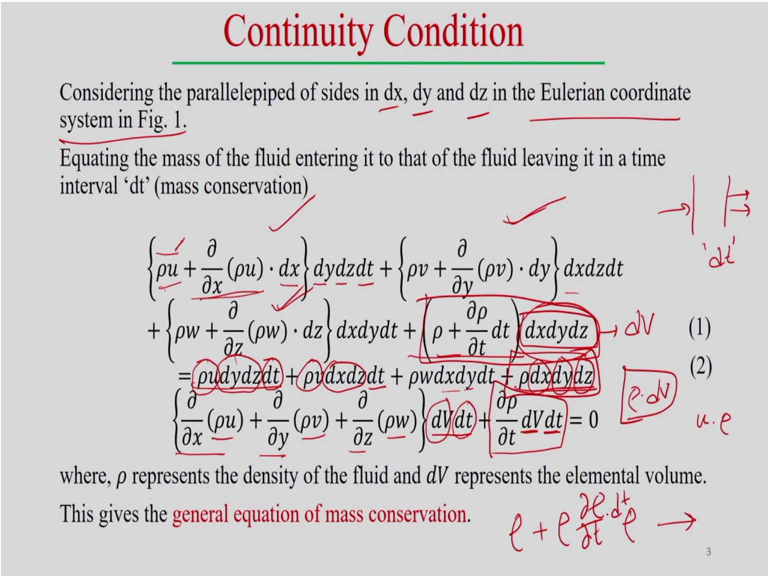

So, let us considering the parallelepiped of the sides of the dx, dy and dz that way. The Eulerian coordinate system I already explained in the figure one. Now, equating the mass of the fluid entering to that of the fluid and which is entering to it and that of the fluid living it at time interval dt.

So, which is entering one particular phase and what is a living the other phase over the time span equal to dt then what is happening. So, that is called the equating this mass entering and equating the mass living over a time dt that may consider as a that conserves mass conservation for this particular system.

So, let us look into this. So, first what is the exiting the this is the velocity u. So, now, u is the velocity we can we can multiply by rho and we know the rho is the represent the density of the fluid and density of the fluid density that means, mass per unit volume this is representation of this thing. So, basically you mean to when you multiply by the rho, then it is converting to this per unit volume.

Now, this is the rho u and this rho into u and then del by del x rho into u. So, this is the changes that if remember that what is the exit face there is a change of the velocity field. So, that is the this change of the velocity field and that is happening over the dx. Now, it is per unit volume it representation is per unit volume and then it is a dx dy dz this over the face; that means, it is this exit from the face.

So, that area of this particular face equal to dy by dz because it is the normal to the X axis. So, normal to the x axis is the area of the face equal to dy into dz and this is happening over the time dt. So, that multiply by dt. So, same thing this is for the x component, this is for the y component and this is for the z component and this thing also rho plus del rho by del t.

So, this is changing of the so that means, initially the density was rho the velocity, but once the exit then changing the modifying the density rho into rho into del rho by del t rho into over the time dt. So, this is a changing of the density also you can consider over the volume dx, dy and dz this is the equivalent to the elementary volume dV.

So, this we are looking the entering of the velocity and all these things over the surface; that means, over the faces and this we part we can considering that over the volume this is the changes within this counter volume. Now, that is the after the time dt. Now, over that sorry over the time dt now that is the exit part. So, what is the entering inlet part? So, that was rho u and dy into dz into dt.

So, that is the once it is entering velocity within to the control volume. So, the same thing also rho v dx dz into dt and x component, y component, z component also and even for the density also density if you see dx dy and dz. So, it means that once the entering fluid also this thing in that case the this is the over the volume so, rho into basically d into V.

So, this is the mass all these things changing thing, but this thing, but over the time dt there is a change of the density in this particular expression. That is why we consider this as a entry a exit and this as the entering rho into this volume from mass per unit volume is basically equivalent to the mass.

Now, once we make this look mass conservation what is exit and entering this part to make it equal these things. So, in that cases we will be able to get this thing del x del y del by del y into rho e rho u del by del z into rho w dV into dt. So, over the elemental volume and over the time dt. And, this also changing of the density over the time this volume elemental volume over the elemental volume over the time dt.

Now, the density of the fluid dV represent the elemental volume and this keeps the general equation of the mass conservation, but if you take the one particular element or maybe the control volume also what is entering what is outside what remains the inside or changing all these things. Looking into all these aspects we can follow the mass conservation equation.

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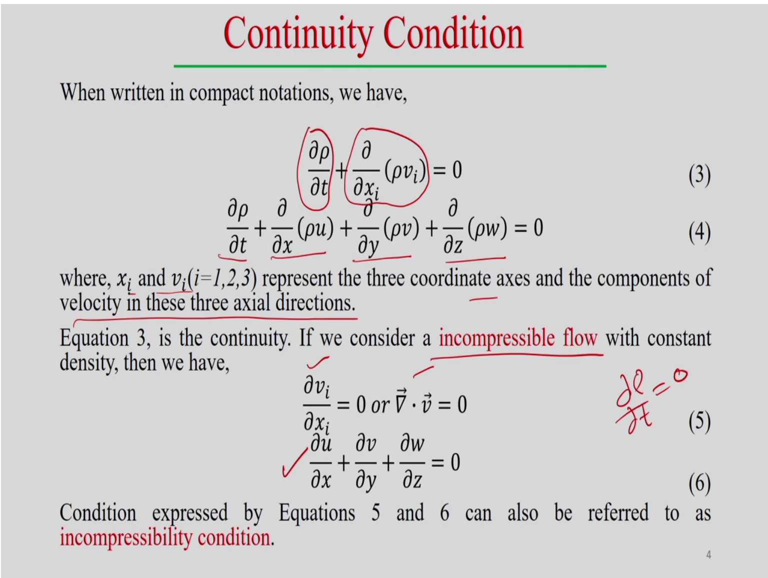

And, then we are getting in the compact from the notation we can find out the del by del rho by del t and this del by del x i equal to v i, the i equal to 1, 2, 3. So, that means, x 1, x 2, x 3 or x, y, z. Now, expand this one also other way also in terms of x, y, z. So, the x component del by rho dz and del rho by del t equal to 0; so, that comes from the elemental volume.

And, of course, we are considering the time phenomena also. So, tangent part also we are considering in this case then we are getting the following the mass conservation equation and from there we can getting this particular equation.

Now, where x i is the x and v i is the one represent the three coordinate x is that is already and the components of this velocity in this axial direction; so, v i basically v 1, 2, 3 the component of the velocity along the x, y and z direction.

Now, this equation 3, is the continuity if we consider equation 3 is the call the continuity equation and then if we consider the incompressible flow. So, incompressible flow means there may not be any change in the density of the fluid; that means, within the over the time dt or maybe in that case if it is not changing the density over the time dt the density remains constant then that is called the incompressible flow.

So, if density remains constant over time with a constant density, then del rho by del t can be equal to 0 also and even rho becomes constant rho just came out of this that is within this integration and then finally, it becomes del u by del x plus del v by del y delta del w by del z equal to 0.

So, this equation we can say the continuity equation and different form this continuity equation I confirm either in the index form also del v i by del x i equal to 0, in the index from in the vector terminology. So, dot product of this gradient into velocity vector equal to 0.

So, that represents the continuity equation. Now, condition x is by equation 5 and 6 can also be referred to as the incompressibility condition as well. So, because in this case incompressibility conditions simply we understand the density remains the constant.

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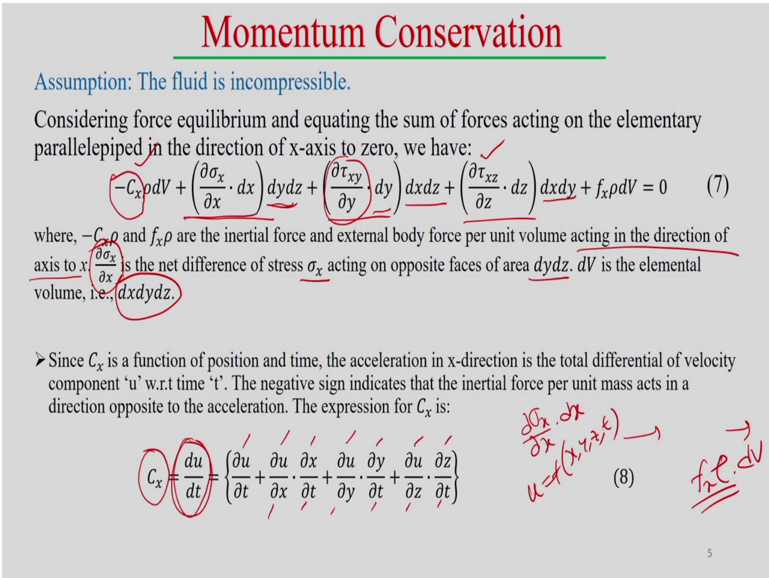

So, that is that once you get the continuity conservation equation then we look into the momentum conservation equation. So, in momentum conservation equation when you derive particular cases we are assuming the fluid is incompressible fluid. Now, considering the force equilibrium and equating the sum of the forces acting on particular element elementary part what we have considered in this particular element.

And, we can look into the individual direction what is the force balance along the x, y and z direction and if it becomes 0, then we can say the we can reach the force equilibrium condition. Then from that we can find out by following the momentum conservation because we can find out the momentum equation associated with this fluid flow problem.

Now, what are the different components are there we can do the similar kind of the exercise maybe what is the entering and what is the exit from this thing. You can do make the difference between those two and equating it equal to 0, then it is possible to conserve the momentum conservation is possible to consider.

Now, what are the different components? So, in this case you can see the del sigma x by del x. So, the in terms of the stress also del sigma x by del x into dx over the length dx. And, this is acting over the faces also, then we can multiply the area of this face normal to the x axis the dy dz. So, similar kinds of things can also be considered because there are all this is only for the normal stress component.

So, similar kind of phenomena may happen due to the shear stress component. So, the shear stress component also do if we do the similar way because remember the normal stress and shear stress component the difference is that normal stress acting normal to the face and shear stress is acting parallel to the face. So, accordingly we have to consider the area.

So, in this case changing over the dy and then, but this is the happening over the particular parallel to the face is the dx dz. Here also, other cases for this particular stress component dx dy. So, these are the normal stress these are the two shear stress components and plus two other component internal or component also maybe we can say that the inertial forces we can consider changes of the inertial forces C x into rho.

C x into rho C x inertial forces and rho into dV basically we represent the rho is the mass per unit volume and then when you multiply by the volume so, it becomes mass. So, then this is the inertial forces, C x into rho and f into rho in the basically any kind of the external body forces per unit volume.

So, then this f x into rho, that represents the external body forces per unit volume. So, when you multiply by the volume so, it represents the forces, so, external body forces. So, that means, there may be different form of the energy or forces maybe there driving forces maybe they are within this weld pool, so that we can consider in the form of volume the distributed over the volume. So, from that point of view we can consider this as a body forces also acting on this thing.

So, apart from that, there may be some sort of the inertial forces also because there is a flow of the motion fluid motion is there. So, looking into all this aspect the changes in terms of the stress and strain also, then we can find out the momentum conservation equation and such that unit volume acting in the direction and normal to the axis del sigma del is the net difference in the stress sigma x acting opposite faces of the area dy dz.

So, that means, normal to the x axis and dV is the elemental volume. So, dx dy and dz that is a elemental volume represents the dV. Now, this is the expression for the momentum conservation equation from that point of view. Now, since C x is a function of position and time so, inertial forces we are talking about because there is a flow of the motion there is a fluid flow thing is there.

So, and inertial forces can be considered like that the along the x-direction, so, it is the acceleration in the x-direction is the total differential velocity component with respect to time t. So, that means, we can represent this inertial forces C x along x-axis C x is simply du by dt, simply change of the velocity component with respect to time dy dt. So, that is the total differential of the velocity respect to time.

Now, negative side indicates that the inertial force per unit mass acts in direction opposite to the acceleration. So, that is why we have considered the minus sign. Here also if you see this negative sign is there negative sign we assuming that the negative that inertial forces per unit mass acts in direction opposite to that of the acceleration. So, that is why negative sign is there.

But C x, how we can represent the C x also? Simply du dt. Now, this is the total derivative du dt. So, now, look into this total derivative into in turn form of a partial derivative also. So, that means, u can be a function of the u and face because u can be a function of maybe we can say that x, y, z and t. So, from that point we can see the du by dt so, del u by del t plus del u by del x into del x by delta t.

So, finally, it is coming a del u by del t. So, that even del u by del t also. So, dy del u by del t inside this del z also there. So, this is the from total derivative to the in the form of a partial derivatives. So, then velocity u can be represents like that.

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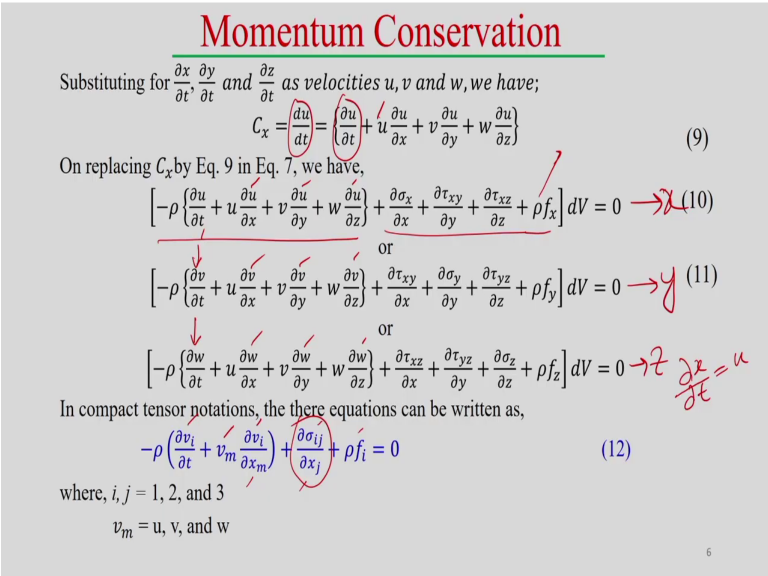

So, now substitute d del x by del t. So, similarly in that case is del x by del t we can see this also velocity along the x-axis this is can be represented like the u velocity component. So, similarly del y by del t, del z by del t all the velocity components; so, u v and w along the x, y and z direction. So, from there we can find out what is available C x.

C x du by dt. So, del u by del t is there one partial derivative simply del x by del t equal to u. So, u into del u by del x v into del u by del y and w into del u by del z. So, that is the way we can represents all these u, v and this all velocity components also.

So, now we replace the C x in terms of the u, v, w and t also then replace this part also is there and even there is a stress components also there and this is the body forces over the elemental volume dV equal to 0. Or we can see this thing also del sigma x by del x and this is the I think corresponds to the along the x-axis.

This equation and this equation correspond to the y-direction, this is the x-direction and this is correspond to the z-direction. So, basically you are getting the three different equation making the momentum conservation with along the x, y and z-direction, then we will reach this kind of equation.

Now, we can find out the difference of this expression also, del u by del t the here del v by del t here del w by del t and other courses also u here all the cases there is v and the z-direction z. So, that is the w. So, that are the different expression we can find out the similar way what if we perform this thing what is happening for along the x-axis, the similar strategy you can find we can add up to find out the components of make this kind of equation along the y as well as the z-direction.

Now, thus compact tensor notation using all these three equations can be written in this particular way also rho del v i by del t v m velocity component x m and del sigma ij x j and rho f i, but till we are representing where i is equal to 1, 2, 3 and v m represent the velocity components say u, v and w.

So, then actually till we are representing this equation in the in term till we are using this stress also; so, in this case we are using the stress notation, in terms of the stress we are representing this momentum conservation equation. Now, we will try to look into the different form of the momentum conservation equation which actually we will be using to solve the to develop the finite element model in case of the fluid flow problem. Before that we try to look into that the energy equation.

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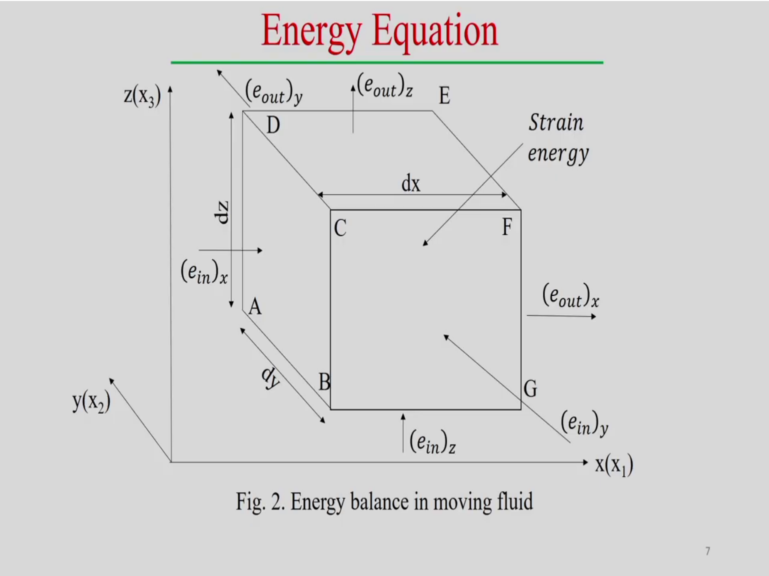

Now, energy equation we can see the similar kind of one small element we consider and we are considering what is the energy input from one side, what is the energy output from the other side also and whether is there any internal strain energy is there inside the element also that we have to consider.

From that similar point of view similar kind of the exercise what we have done in case of the momentum equation also only this. There the momentum we look into the force equilibrium for this particular element, but in this cases we try to look into the energy balance within this what way we can within the particular element and from that point of view we can estimate the energy conservation equation follow these things.

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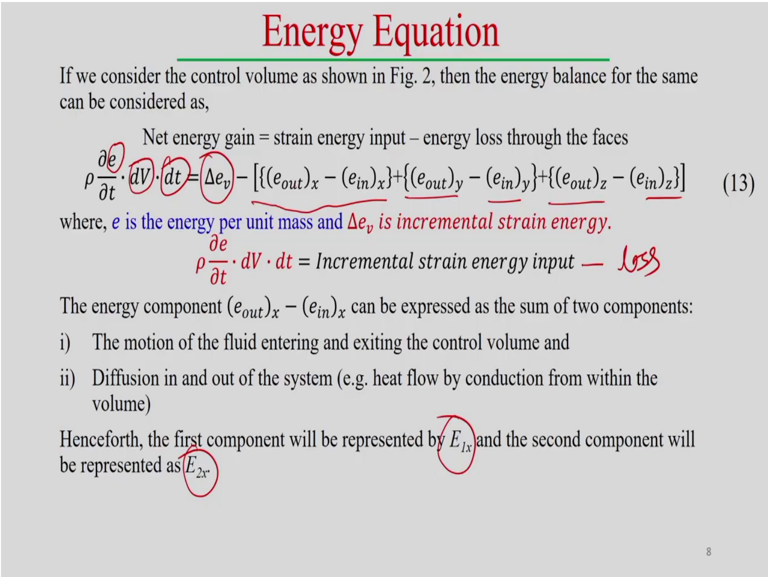

Now, if we consider the control volume for the particular figure 2 also, then energy balance for the same can be considered like that what is the net energy gain that can be strain energy input minus energy loss through the faces; so, net strain energy gain during this maybe passage of the fluid flow passage of the material from this one point to the another continuously flowing the material is there and we pick up one particular focus on the small element and we are try to look into what is happening on this particular element.

Then in that particular element for the energy conservation equation we can see that what is the net energy gain through the passage of the fluid through this control volume that is there any strain energy input or maybe an increment of the strain energy or the strain energy incremental strain energy within this control volume minus the what is the loss of the energy through the faces because material is continuously flowing from one face to another faces.

So, because of that what is the loss of the energy through this thing. So, it is a simply like that only net energy gain rho into del e by e is the energy per unit u can be considered small e can be considered energy per unit mass. So, del e by del t over the time the this thing del e by del t the change of the this thing over energy and with respect to time dV over the elemental volume and dt over the time dt. So, that is a total net energy gain that is equal to the strain energy is there within this element and this is this indicates what is the energy loss.

So, energy loss means input minus out minus in, out minus in, out minus in it means the considering the x respectively along the x, y and z direction. So, e is the energy per unit mass and delta e v is the incremental strain energy. So, now del e by del t minus into dV into dt is the equal to the incremental strain energy input and definitely it is in these cases minus the loss, loss of the energy.

Now, once if we look into the energy component the output e out minus e in along the particular direction can be expressed by the sum of the two components the motion of the fluid entering and exiting the control volume because of that energy component we can consider other is the diffusion in and diffusion out of the system.

For example, diffusion in and diffusion out of the system can be considered like is there any heat flux heat flow by conduction with from within the volume. So, we consider the what is the heat conduction in terms of the flux within this control volume also that we can consider as a diffusion in and diffusion out within this control volume.

Therefore, the first component will be represented E 1x and the second component will be E 2x. So, this mean this is the 1x means is the simply because of the fluid entering and exiting of the control volume what is the basically energy gain or loss within this control volume that we have to look.

And, second point is the because of the diffusion in and out of the system by considering the heat conduction because in fluid flow problem also the it is may not when entering and exit of the fluid it is not at the same temperature there is a variation of the temperature. So, that can be considered if there is a variation of temperature.

So, heat conduction may happen within this elemental volume also that part we have to consider. So, that can consider as a energy either loss or gain within this control volume that we will see. So, that means, two components of the energies are there in this particular energy this thing; so, looking into the first one component E 1x because of the fluid motion the energy balance.

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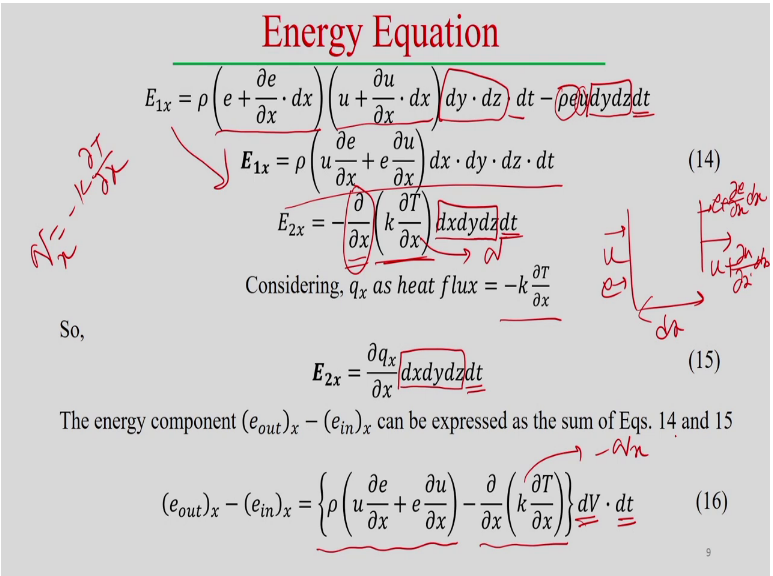

So, rho e into change of the energy with respect to because over the spatial domain dx even this thing and at the same time there is a change of the velocity also, the spatial domain one is the exit. So, here is the velocity was u and there is a one entry level and exit from this particular phase the velocity will change and del u by even it is suppose this is our elemental length dx del u by del x into dx. So, this is the velocity for the exit.

So, this is the velocity this is the energy from this exit and velocity components into dy dz particular area over the time dt. And, that rho e this is the initial energy the input rho here the energy e and they are also variation of the energy e also in this cases e plus del e by del x into dx that is the spatial distribution of the energy and changes of the energy.

Similarly, u is the initial velocity and dy, dz was the area and dt was the time. So, same kind of exercise we can do what is the exit happening and what is the input. So, because of the fluid motion this energy were considering because of the fluid motion. So, now if we do manipulate all these thing we can reach this kind of expression also dx, dy, dz into dt and rho u del e by del x are the from this in the manipulation we can get this kind of expression.

Now, again looking into the diffusion, the heat diffusion within this elemental volume also that can be considered like that. The heat flux so, del by del x; so, basically this thing and these represent the q the heat flux will be the k in to del T by del x we know the Fourier heat conduction equation we from there we can show the q in the heat flux minus k into delta T by del x what is the del T by del x equal to temperature gradient along the x-axis and k is the thermal conductivity a negative sign because the temperature gradient is a negative that is why you consider negative.

So, it basically indicates the heat flux within this thing domain. So, this heat flux and if we look into the heat flux here q k into del t by del x per unit area we can see these things and then once you multiply dx, dy, dz over the volume also we can earn over the time dt and then total energy can be calculated del by del x into change, del by del x into this.

So, basically q x as the heat flux; so, it is sorry, if you write the q x because heat flux along the x axis also. So, minus k into del T by del x. So, E 2x can also be represented that del q by del x into dx, dy, dz elemental volume into dt. So, that way also we can see the effective diffusion energy may be considered like that E 2x this way.

So, therefore, x component e output energy output minus energy in can be expressed in the sum of the E 1x and E 2x and that can be represented like that out minus in. So, E 1x in this one over the elemental volume dV and over the into the time dt over the time dt. So, that is the energy input energy component we can estimate.

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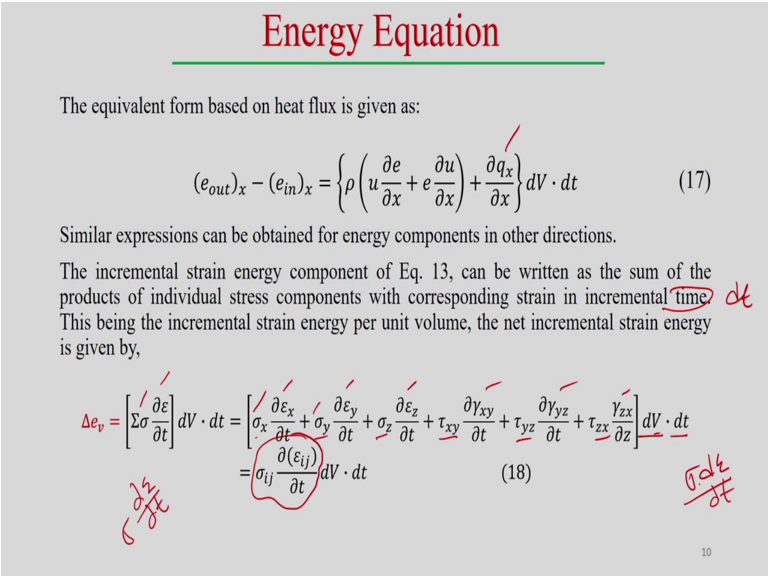

Now, equivalent form of the based on the heat flux also we can see the simply we are not writing k into del T by del x the looking for heat conduction in place directly in terms of the heat flux instead of writing in the k del T by del x so, that equation. So, this equation can be replaced minus q x. So, that we can write in this way also input output this thing. So, similar expression can be obtained in case of other direction or other components may be in y and z-direction.

So, therefore, this is the one part and that is another component of the energy [FL] incremental strain energy component of equation 13 also we can we have seen the incremental strain energy can also be there. So, del e v that it is the incremental strain energy and which is the del incremental strain energy from this particular governing equation that equation 13 also.

And, we can see the incremental strain energy component of 13 can be written as the sum of the heat products of individual stress and components with the corresponding strain in the incremental time. So, incremental over the incremental time dt what is the incremental stress and strain energy we can basically we can consider the incremental strain energy.

This being the incremental strain energy per unit volume, the net incremental strain energy can be given like this. So, summation of sigma, stress state and the del epsilon into del t. So, del the strain del t. So, it is basically represent the strain rate actually and the stress because we know the stress into d epsilon we have already shown the when I make this one if we know the stress field also and the incremental strain.

So, sigma into incremental strain that indicates the strain energy per unit volume and that is incremental form because we are considering the d epsilon. So, incremental strain; so, from there strain energy per unit volume we can see that what once you look into the state energy per unit volume and then when you divided by del t also.

So, strain energy rate physically and then we can multiply into dV elemental volume into dt that indicates the total strain energy. But, the stress state may have not the it is not a single dimensional case was stress state can be the different component different shear stress or normal stress components also.

So, that can be expressed like that also sigma x del epsilon x in del t and sigma y this thing; similarly, the normal stress components and the strain according the shear stress component and the shear strain component also. And, if you see here the all the cases we represent all these things are partial derivative form.

So, that means, this partial derivative form independently maybe individually you can we are considering all the stress and strain components such that this is happening over the dV and into over the time dt. So, this all stress components can be represented in this way also that in general form or index from we can say the sigma ij into del epsilon ij into del t into dV into dt. So, that is the expression of the incremental strain energy.

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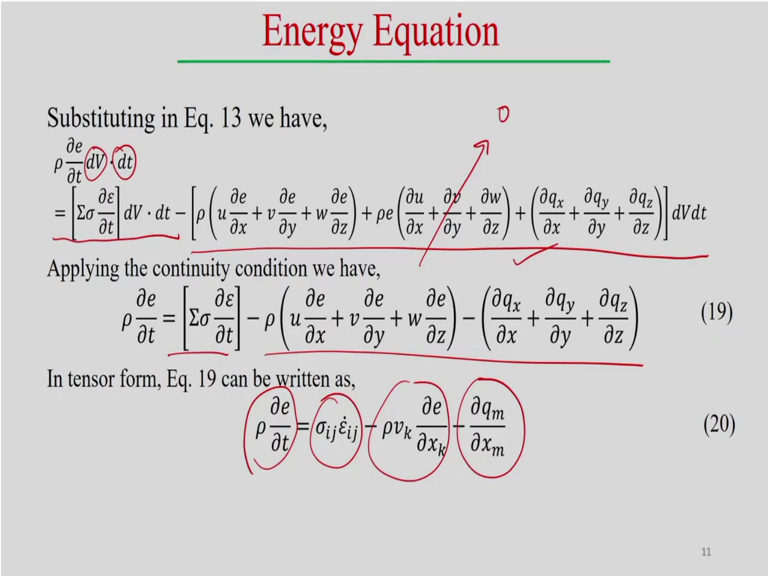

So, now if we substitute all this expression in incremental we have the this thing rho del e by del t into dV into dt. So, that we can see also and that is the; that is the expression 13 we can we are considering from there. This is the incremental strain energy and this is the energy in and out balance and that means, from there.

And, so, this is a expression equation 13 energy equation can be written in this way considering, but if you look into del u by del x this equal to 0 because if we follow the continuity equation also. Now, applying the continuity condition we have we can modify this thing equation rho del e by del t.

So, it is a net gain or strain energy gain of the energy then give it a strain energy in other part and finally, we can rate this is the energy equation. So, you can see that this energy equation in general form can be written like that this one sigma ij, the velocity components also there and the heat flux also there, the diffusion components of the shear stress all this all components are also there.

Now, it is a different way we can represent this energy equation also. Now, this is the general form of the energy equation associated with this any kind of the problem also. Now, looking into the different kind of the problem that from this basic equation we can convert we can simplify further also looking into this particular problem also. Or putting into other condition we can further simplify.

Now, once we develop this we already looked into that what way the this momentum conservation equation it is form. Now, this momentum conservation equation form can be represented in case of the viscous flow problem.

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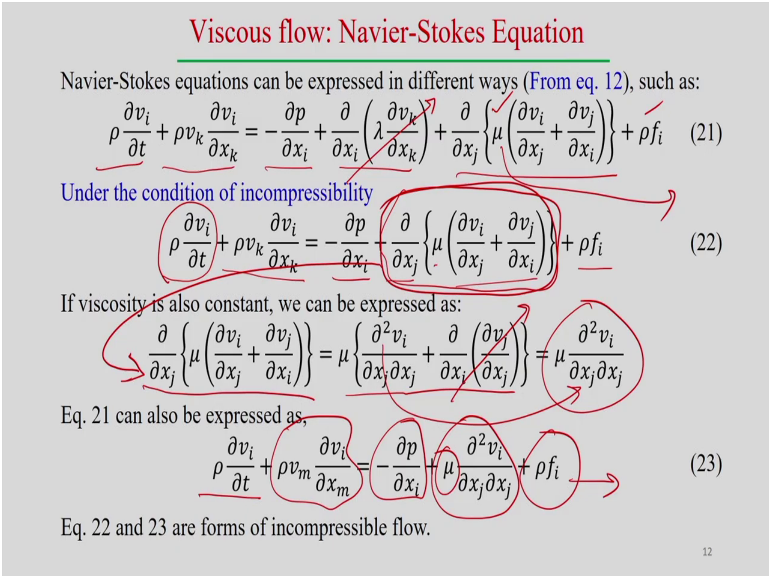

And, in that case what is the Navier-Stokes equation we can find out. We can this Navier-Stokes equation actually can be expressed in a different way from the equation number 12, such that we can get the velocity component and the this velocity components also there and the pressure is also comes into the picture.

The velocity components in terms of the velocity and the this gives the viscosity and then this is the this represents the force per the body force basically per unit volume. So, body force. So, this if you look into this thing the way the we replacing the stress components in the form of the all the velocity components, we can represent the Navier-Stokes equation.

And, this Navier-Stoke equation may be applicable in case of the viscous flow problem. So, that is why you have used these viscosity these properties here also the viscosity is there in this case. Now, this Navier-Stroke equation mostly used in to solve kind of the fluid flow problem and maybe I can say that it is more associated with the welding process also. If you solve this Navier-Stoke equation it is to associated with the fusion welding process.

Now, under the condition of the incompressibility, so, now, if you follow the incompressibility condition when we keeping incompressibility condition is keeping density as a constant. Then we can further modify this equation see this equation is also there the velocity components rho into this, rho v all these things this part is there the pressure component is also there.

And, then del x j mu this thing and rho this is the components and we can see this under when the compressibility condition actually this is not required at all this becomes 0. So, that or maybe we can say that we can neglect this particular part of this equation then we can further simplify this equation Navier-Stokes equation.

So, viscosity is constant we are trying to represent what is this term this term can be further simplify if viscosity is constant. So, viscosity is constant keep, the viscosity outside this integrands and then do the exercise in these cases we will be able to find out this thing del v i by del x, but if you see there is a we basically we this is the component.

But, what about this component? This component becomes 0 because if we put the continuity equation also continuity condition or follow the continuity equation from there this term becomes 0. So, finally, we can simplify this part to this one single component. Now, this equation 21 can be expressed like this you can see that left hand side this component is there that velocity component with varying with respect to time t and this is the also velocity component del v i by del x m; pressure component also there and the viscosity and the this is the body force.

So, all these 22 and 23 are forms on the incompressible flow in this case and then even viscosity remains constant then in that cases we can further simplify this equation. This is the particular this is the Navier this kind of equation is applicable because fluid flow problem we normally consider as a viscous flow problem.

So, if we consider the viscous flow problem and the at the same time if we assume this a laminar flow, in that case we can solve that this Navier form of the Navier-Stokes equation and we will be able to find out what is the velocity field. So, this is the very general expression which may be useful in the fluid flow problem associated with the fusion welding process.

So, here this if you see in the fluid flow problem there is a need for the viscosity property of this particular liquid metal viscosity viscous properties of the liquid metal is basically needed. So, from if we solve this equation, equation 23 will be able to get what is the velocity field also.

But, along with the this momentum equation if we follow if we solve the energy equation also, from there, energy equation we will be able to find out the temperature field also in case of this welding problem. So, that temperature field once we look into this thing, but in if you follow it up the energy equation also there are some velocity components are also there.

So, both we can get the temperature in from the energy equation we will be able to know what is the temperature field also, but in that cases there is a need to know what is the velocity field. So, that is why, combining strategies also required the momentum conservation equation as well as the energy equation such that velocity field full of the velocity field is the input to solve the energy equation with a particular domain also.

So, that is why it is very important or if we ok. So, before doing the next part if we look into the very carefully the energy equation as well the this momentum conservation equation and both the cases there is a need of the velocity field.

So, some sort of strategy is required between the energy equation and the momentum equation to get the temperature field from the energy equation, but at the same time to get the velocity field from the momentum equation and the utilization of the velocity field to the energy equation to get the temperature distribution.

So, that is why if you follow for example, if you do not consider about only the heat conduction equation then if you neglect the velocity part in a fusion welding problem also in that case is when you solving the energy equation so, you will be get output only the temperature distribution. So, and that may not be very accurate.

But, if you consider the velocity field also then there is a need to solve the fluid flow problem, the Navier-Stokes equation has to be solved or momentum conservation has to be solved to get the flow field and that flow field can be fed or input to the energy equation such that once we solve the energy equation will be get the more refined or maybe I can the more accurate the temperature field if we consider the velocity field in the energy equation. So, that is the relation between the velocity field as well as the energy equation associated with the fluid flow problem.

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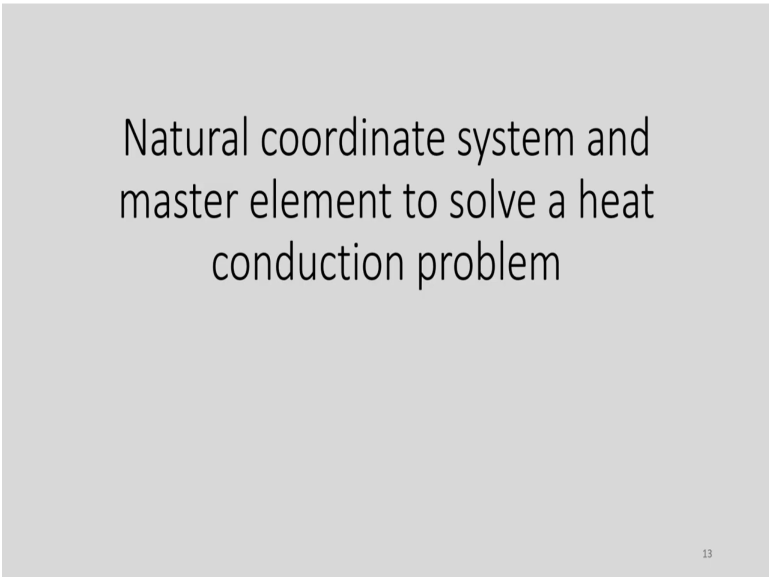

Now, try to look into this natural coordinate system and the master element to solve a heat conduction problem. So, now, we have seen the overall idea that how the elemental things can be form, also, now if you consider a particular element and looking into this particular element how this matrix can be formed and look by introducing the natural coordinate system.

It is simply mapping from the global coordinate system means the domain of the analysis which is you have already said the Cartesian coordinate system that in different node points having the x, y, z value and based on that we have shown that shape function can be formed also. But, each and every element can be represented map with the one local coordinate system.

So, transformation from the global coordinate system to the local coordinate system is necessary to understand to estimate the thing what is happening for one particular element. So, that, we will discuss in this particular problem.

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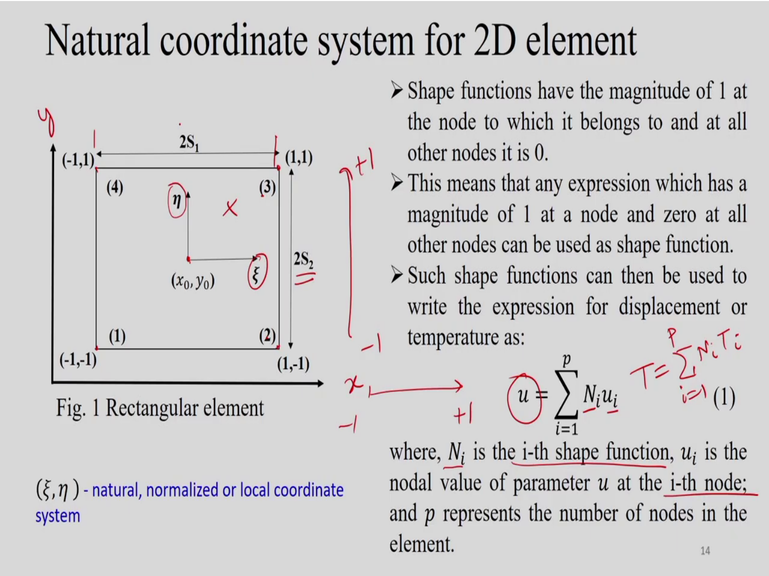

Now, natural coordinate system for 2-dimensional element: let us look into the natural coordinate system. We consider the one rectangular element say for example, along the x and y axis that is represented in the global coordinate system x and y. Now, shape function have the magnitude of the 1 at the node point to which it belongs to and at all other node point it becomes 0. So, shape function becomes 1 and the either 1 or becomes 0. This means that expression which has a magnitude of 1 at a node and 0 at all other node can be used as a shape function.

So, something like that looking into the property of the shape function you have to choosing such a that is a function value one particular node point it is becomes 1 and remaining all other node points it becomes 0 to looking into these aspects. So, we have to can be that can be defined or maybe it is basically easier to mapping to look into the different functional form.

Now, such shape function can be used to write the expression for the displacement or the temperature field. So, u either you can consider as a displacement field or as a temperature field can be like that that i equal to 1 to p N i u i. So, u i either displacement field in the node point or it can be temperature field also, where n i is the ith shape function.

So, particular if you know there is u i is the nodal displacement and shape function for a particular node and u i is the nodal value of the parameter u at a i-th node basically at the i node and p represent the number of nodes in this particular element. So, we are trying to focusing on the what is happening one particular element.

So, one particular element having the p number of nodes for example, and if it is a big element there may be 8 number of nodes also and in that cases p can vary also depending upon the type of the element. Now, so, this is the expression also even T can also be represent like temperature variation can represent I similar way i equal to 1 to p and that N i T i same way we can represent whether T i whether u i.

So, now, here if you see that define this is the this point having some global value also, this is the particular element one particular element and each node point having some coordinate in term global coordination in terms of x, y value with respect to the whole domain of the analysis.

But, with this particular domain we can do in such a way that we can introduce some kind of the x system that x 0, y 0 that is the CG of this particular node point and that CG x 0 y 0 represent at the centre point and we can use the two axis system, the xi and eta system, xi axis and the eta axis such that the variation of this is can vary from this eta from 1 to 1 directs it can vary from minus 1 to plus 1 with respect to this point and other axis also there is a minus 1 to plus 1. So, it says simply clear mapping of this global coordinate system to the local coordinate system.

So, in this cases this axis system xi and eta axis that can be that can be considered as a local coordinate system such that it can vary from minus 1 to plus 1 in that particular range. And, we define the system it is not at the centre point CG, this is the origin of this particular coordinate system and accordingly we can define the node number 1, 2, 3, 4 for this particular element and such that with respect to that coordinate system we can see this is the 1, minus 1, 1, 1.

So, this is the positive 1 1 plus plus minus plus plus minus minus minus that way we can do the coordinate of this different part of the coordinate. So, accordingly we can decide this mapping. Now, but how what we can map this thing? So, assuming this the edge length equal to twice S 2 and this edge length is basically for example, twice S 1.

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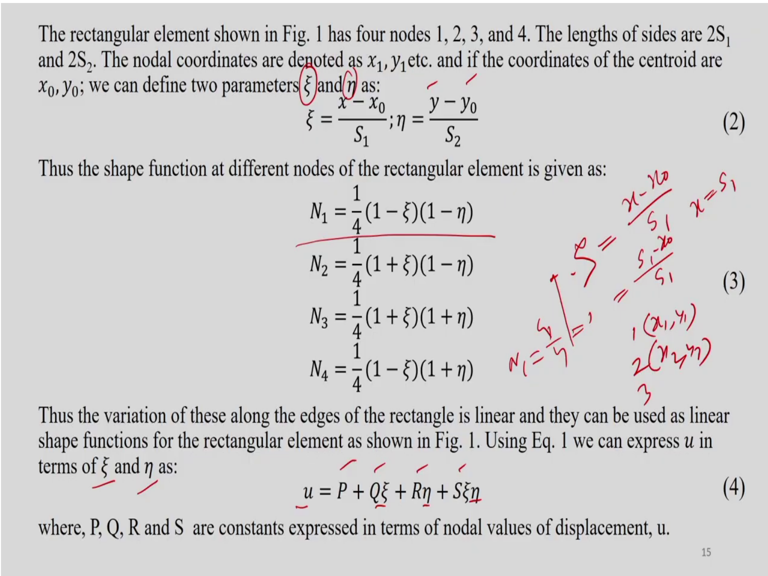

Now, we will see what we can map from global to the local coordinate system. Now, the rectangular elements shown in this figure has the four nodes 1, 2, 3, 4 the length of the sides are 2S 1 and 2S 2 that we have already seen. The nodal coordinates are denoted by the x 1, y 1 etcetera.

So, different if it is node 1 then we can say the coordinate is the x 1, y 1 that is the global value, x 2, y 2 some like that 3 also x 3, y 3 and 4 x 4 y 4 like that. But, the if the coordinates of the centroid are x 0, y 0. So, for example, coordinate of the centre x 0, y 0 then we can define the two parameters that is the xi and eta in such a way the xi can be mapping in this way the x minus x 0 by S 1 and eta equal to y minus y 0 by S 2. So, it is like that only xi and eta can be map in this way.

So, once we equal to x minus x 0 by S 1. Now, you can see that the xi if it is x equal to x 0 and then centre then xi equal to 0. And, but if x equal to S 1 then what will happen it become S 1 minus x 0 by S 1.

Now, what is the value of x 0 in terms of S 1, S 2 and we can see that it is can vary from 0 to 1; xi becomes 1 at the edge at the corner point or even node point similarly either 1, plus 1 or minus 1 depending upon the position whether node point 1, 2, 3 or 4. Similar eta can be mapped like that y minus y 0 my by S 2.

Now, thus the shape function of these different nodes of the rectangular element can be given like this only. So, that means xi and eta in terms of the xi and eta so, we can say this is the local coordinate system and then using this local coordinate system we can represent the shape function N 1 equal to 1 by 4 1 minus this particular element rectangular element and then 1 minus xi 1 minus eta, such that that if xi equal to 1 plus 1 eta 1 then N 1 becomes 0 like that.

So, that means, variation is there different values of the xi eta and we can see the xi equal to minus 1 eta equal to minus 1, then N 1 can become and N 1 can become it is both are minus 1 then the plus 2 into 2 by 4, then N 1 become 1 by 4 by 2 into 2 it becomes 1, 4 by 4 it becomes 1.

Similarly, the this thing if xi becomes 0, eta becomes 0, then what is happening? N becomes 1 by 4. So, like that there is a shape function at the different nodes of a rectangular element can be written in the in terms of the local coordinate system like this so, different forms.

So, plus minus depends on the in general form it can be written, but plus minus depends on the which part whether it is what we can define the different coordinate also we can see. Whether this part is the plus plus this part is the minus plus you can divide in this four. This is the minus, minus, this is the plus minus.

So, accordingly these coordinates can be there plus plus, minus plus and minus minus plus minus. So, that is way we can map also with the local coordinate system we can represent the differential function in terms of this parameter. Now, thus the variation of this along the edge of the rectangle is linear.

So, they can be used as a linear shape functions for the rectangular element, even it is shown in figure 1. So, therefore, using equation 1, we can express u in terms of xi and eta. So, u can be represent the displacement in terms of xi and eta.

So, P, Q, R is other constant term and in terms of the xi, eta we can evaluate the value of the P, Q, R, S constant, express in terms of the nodal values of the displacement u. So, this way we can treat a simply we can introduce the some local coordinate system such that each and every element we use the local coordinate system then it will be easier to make the integrand over this particular problem.

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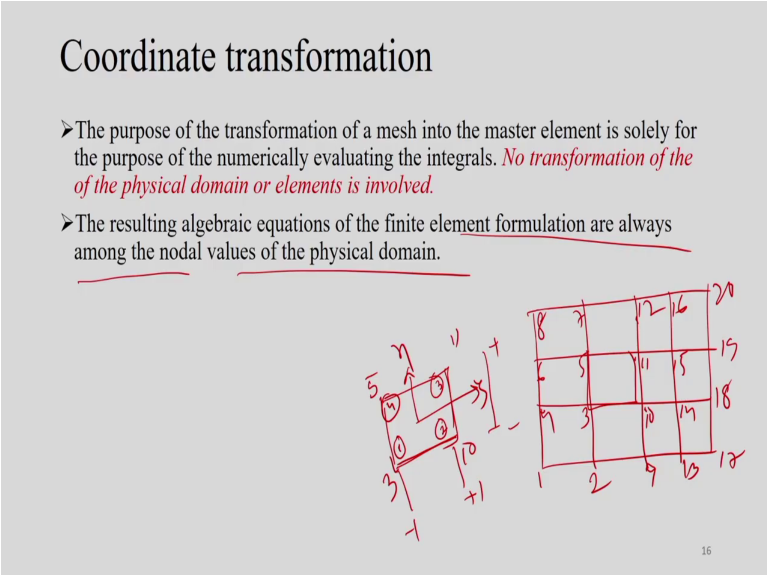

Now, coordinate transformation is something like that. The purpose of the transformation of the mesh into a master element is the solely for the purpose of the numerically evaluating the integral. So, that is that will be easier. So, all the coordinate transformation is required in this case.

So, no transforms of the physical domain of elements is involved. So, it is like them only. Suppose, this is the domain of the analysis we have divided the different elements and then we have the different node points 3, 4, say 5, 6, 7, 8. So, we can consider one particular image.

Similarly, some names are also there 8, say 9, 10, 11, 12, say 13, 14, 15, 16 like that 17, 18, 19, 20. So, these are the neighbour. So, we pick up one particular node shape the one particular element. So, that 5 this is the global number 11, 3 and 10. So, then in this particular element we can make the strategy for say this is given the local numbering one, this is the second one, this is the third one, this is the fourth one particular sequence we can follow. Then you can follow the local coordinate system for this case such that map mapping in such way that it can vary from minus 1 to plus 1 and here also plus 1 to minus 1 such that xi and eta local coordinate systems.

So, we can develop with the particular sequencing local numbering and local system we can follow on the one particular element and then we will be following for the all element the similar kind of the strategy, such that it will be easy to conduct do some evaluating the integrals over this particular case. So, that is the purpose of transforming all this thing global system into the local coordinate system or introducing the local coordinate system.

So, resulting algebraic equation of the finite element formulation are always among the nodal values of the physical domain. That is always there also once you get all these things we can once for one element we consider then contribution for the all the node point can be divided during the assembly in the global position.

So, the global position in this particular element equal to say 3, 10, 11 and 5. So, according to the global position we will put all the nodal values according to the global position when you try to assemble for the contribution for all the elements.

So, that is the procedure in these cases we are not actually transforming for the all the I think not among values of physical equation of the finite domain formation always among the nodal values of the physical domain. So, there is no transformation of the physical domain or elements is basically not involved.

Only thing is that one master element we identify and we do all this local coordinate system this particular system developed and we will be following or particular sequence for local numbering for all the elements. And, then once we (Refer Time: 54:37) for the all the elements then keep on assembling for a particular for all the elements, then we get the equation.

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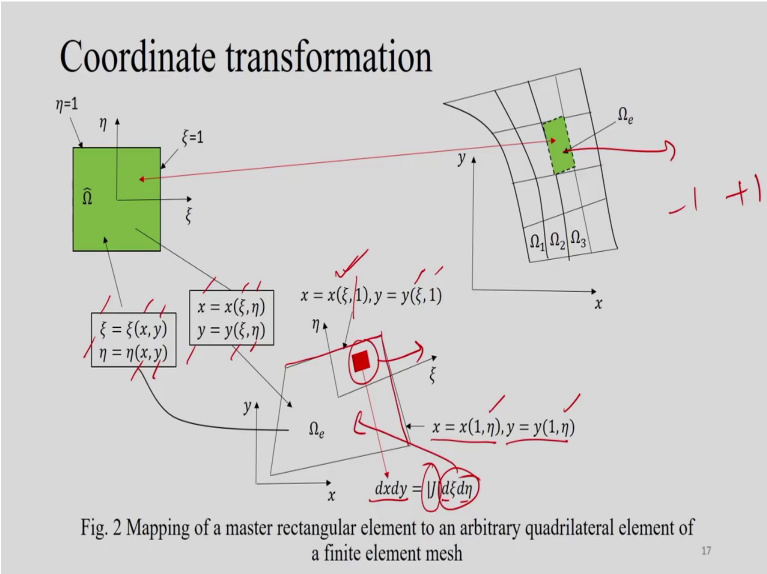

Fig. 2 Mapping of a master rectangular element to an arbitrary quadrilateral element of a finite element mesh

You can see the also coordinate transformation the similar way the mathematically we can express like this suppose this is the; this is the actual domain one particular domain pick up this domain. We can transport this thing local coordinate system coordinate transformation here we can see the this thing.

So, domain the xi and eta can be a function of x and y also because x and y is the actual global coordinate system and we have the information the data of the x and y. So, x can be represented also the as a function of the xi eta and y can be represent at the function of the xi eta in this particular and along the edge also we can see that x equal to in terms of the eta equal to 1, but there is a variation of the xi.

Even along the y edge along the y-axis, the variation of the same thing variation of the eta equal to 1 xi variation of these things. So, along this particular x the variation x is the this only variation of this eta here the y variation of the eta this thing and this particular element dx dy, but in actual these things with Jacobian j which can be represented in the form of a in the coordinate terms of this thing and d xi d eta is the elemental.

So, this is the so, here we will be doing the area represent the d eta and d xi and d eta. So, that represent that element in terms of the local coordinate system, we can take our, but this j determined of this thing there is a need to consider the global information; that means, the actual coordinate system information is required.

So, this way we can transfer from the global things with the local coordinate system and then after that we will perform the integration. Because this once you particular the volume integral or surface integral in this particular domain so, this is well defined Gauss quadrature rule.

Then this particular value the domain is defined because domain is minus 1 to plus 1 with this particular domain we can put the because each and every domain we represent this thing in the local coordinate system between the minus 1 to plus 1, such that we put the weightage value of the when you try to do numerical integration of this particular over the area or over the volume of a particular element.

So, that is the advantage of transforming from the global to the local or picking up looking into one master element to convert into the local coordinate system and then utilize the same concept or same thing for all the elements. So, that is the purpose of doing the coordinate transformation for or defining one master element during these particular analysts.

Next we will see that how one particular element we can calculate we will pick up one particular element we will calculate the what contribution of the different contribution say conductivity matrix or other kinds of matrix in case of the heat conduction process. So, thank you very much for your kind attention. I think that is all for today.

Thank you.