Finite Element modeling of Welding processes
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Lecture – 13
Material nonlinearity – II

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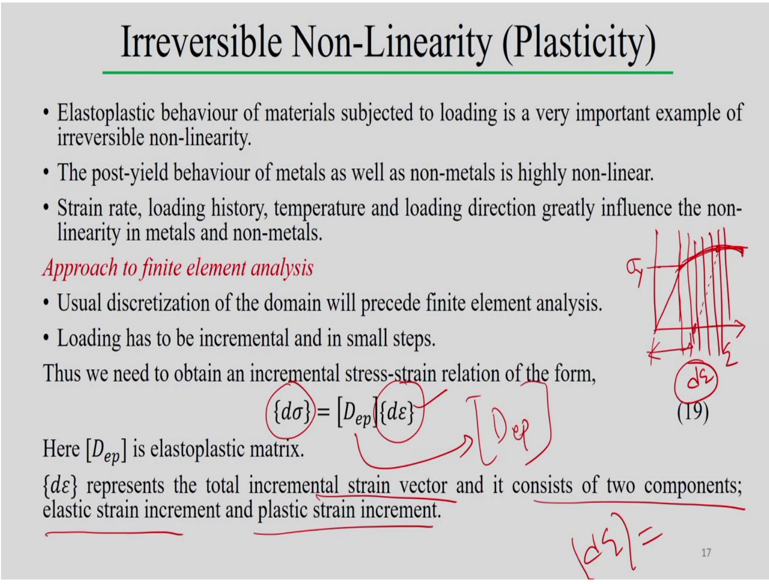

Hello, everybody. We have already discussed this Material Non-linearity and we have shown that how Newton Raphson method, or direct iterative techniques is normally works. In any kind of the finite element based model or finite element based analysis. So, therefore, that was the irreversible non-linearity. Now, we will try to look into that irreversible non-linearity, in the light of the plasticity model.

Basically we try to explain the irreversible non-linearity, in the platform of plastic deformation in a or maybe we can see we can tell that elastoplastic deformation. In any kind of the thermomechanical model associated with the welding fusion welding processes.

So, let us see how it works. So, elastoplastic behavior of materials is basically subjected to loading it is a very important example of the irreversible nonlinearity. So, we have taken this as an example to understand that how irreversible non-linearity normally works. So, if you look into the uniaxial tensile testing diagram also, if you observe that up to the yield point the elastic this is common engineering materials.

For example, you assume for example, stainless steel or any other common engineering materials if we consider, see up to the yield point then there is a relation between stress and strain is linear. But, once we cross the yield point then, if you see there is a curvature. That means, its not it becomes non-linear relation so; that means, once you cross the yield point. Then, there is some sort of permanent deformation happens and because of the plastic deformation of this particular material.

So, then that means, post weld behavior of the metals as well as the non-metals is basically highly non-linear and that is very much obvious. If you see this thing this up to this we can reverse non-linear; that means, if this is the, this is the yield point corresponding particular material. So, up to this point the analysis can be done, assuming the simple because there is a linear relation between the stress and strain.

So, we can use some kind of the reversible non-linearity maybe in that particular problem, or maybe we can say that the non-linearity comes from different aspect in this up to that yield point. But, once you cross the yield point if you see there is some deformation. So, even if we remove the load at this particular point, then the state will come back up to this point. So, some sort of permanent strain will always be there even if once we cross the yield point.

So, that is the typical plastic deformation or deformation or associated to the any kind of the material. So, that is why this is very difficult to come back to the initial position and we can say this kind of non-linearity arises, due to the plasticity we can consider the irreversible non-linearity associated with this problem.

Now, what way what are the different way we can solve. We can take care of this particular irreversibility in a any kind of the material model, or where you assume the material we were follow some sort of elastoplastic behavior. So, therefore, strain rate loading history temperature and loading direction greatly all this actually influence the non-linearity in the metal, associated with other metals even for non-metals as well also. So, we have to look into all this aspect.

Now, what are may be the approaches to the finite element analysis. So, we have seen we have already shown that how the discretization in a domain is normally done, when you do some kind of the mechanical analysis or we can say the thermomechanical analysis associated with the fusion welding problem.

So, usually discretization of the domain will proceed finite element analysis definitely. So, first we in in particular we decide the domain, we follow the discretization of this domain. It creates the elements and node their connectivity up to all alls are as usual, if we look even for the thermal analysis also.

So, same kind of discretization we can use, now in thermal analysis or may be in other kind of analysis we can follow a particular, if we do only only the thermal analysis. May be depending upon the thermal cycle or thermal load, we can do the analysis and can get the output as a temperature distribution in the discretized domain. But, in stress analysis normally what happens, because of the non-linear nature of the beyond the yield point. So, in that particular zone, it is better to follow some kind of the incremental mode of analysis.

It means that, we consider on this non-linear zone the small increment of the strain. So, small increment of the strain and then what may be the associated increment of the stress, we can calculate looking into the constitutive relation between the stress and strain. As well as the we know the relation between the displacement as well as the strain field also we use all this relationship, but analysis can be done in the incremental mode. Because, of this non-linear nature of this thing.

But now, we will try to understand how this incremental mode analysis stress analysis can be done, in a using the finite element method. So, therefore, loading has to be incremental and in the small steps so; that means, when increment happens. So, its not possible to analyze in a single step to reach into particular point.

Rather we in the plastic domain we can reach small step, small increments. So, all small increment then we will look into that what is happening within small increment, what way we can utilize the plasticity theory different plasticity law also. And, then how for this small increment of the strain, what is the update of the stress and update of the displacement all can be updating in the each and every steps.

So, that is of all incremental mode analysis we can see. Therefore, we need to obtain an incremental stress strain relationship also from this form. So, that we can see we know that we have already shown, that relation between the stress and strain in the, that form, that stress components is there and this is the strain component. Now, that is related to if uniaxial we can see put simply the Young's modulus, or some other modulus even it is plastic plastic modulus if we use it. Then, basically we can relate between this incremental stress and strain.

But, once a three dimensional state of the stress may be in that cases, we know that it is already we can convert into in the form of a matrix form even there is need column and columnar vector form, these variables we can represent these things. And, such that in this case also, then instead of single value for against the uniaxial tensile testing problem, rather in the matrix form we have already seen the D ep means it is the elastoplastic matrix. So, everything related to the, it is basically relate the material properties.

But, once we look into the elastoplastic analysis, then D ep not exactly relates only the material properties in terms of the Young's modulus and the Poisson's ratio not in that way. Maybe we have once we consider the plastic model also, then some deviatoric stress component comes into the picture to develop this D ep matrix, we will see how this D ep matrix can be made.

So, in general we relate between the stress and strain looking into that perspective. If the analysis limited to below the yield point, then we simply use the D ep can be replaced by the elasticity matrix only. Even it is cross the, if you if you do the analysis for the plastic domain, then we can use the D ep. So, we can say the D the elastoplastic matrix. So, we can use so, we have to look how this elastoplastic matrix can be formed in any kind of the stress analysis problem.

D epsilon total strain incremental this d epsilon is the total incremental strain vector that we have already shown, and it consists of the two components one is the elastic strain increment, another is the plastic strain increments. So that means, total strain increment consists of the elastic part as well as the plastic.

Because, once we are talking about the, we are doing some material, we were assuming the elastoplastic in nature. So, therefore, we have to incorporate it to assume that total strain increment consists of these two components, elastic component as well as the plastic components will be there.

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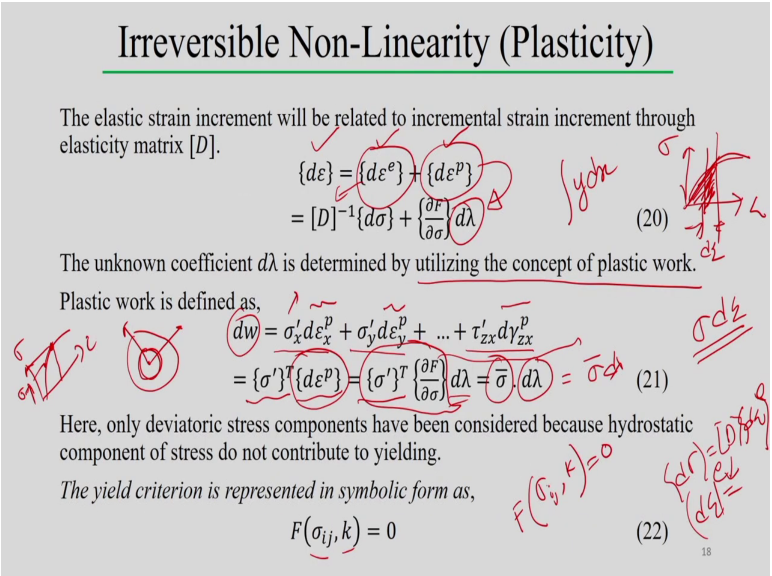

Now, if we assume the only the elastic and plastic component is there which is corresponds to the total strain increment. Then, elastic strain increment will be related to the incremental strain through elasticity matrix. So, therefore, we can see that this total strain increment consists of these two parts, one is the elastic component, other is the plastic component plastic strain increment.

Now, the straightway elastic strain increment we can use the, this is related to the D matrix because, in this case d sigma equal to D matrix into D epsilon so, in e. So, this is the elastic because this thing. So, in that cases we replace instead of D ep we use the D because, within the elastic that the analysis of the domain within the below the elastic point.

So, therefore then if we look into the epsilon elastic component and then it is correspond to the d inverse and thus sigma, stress component incremental of the stress. So, this is corresponds to the strain increment associated with the elasticity part. Now, the d epsilon e p so, plastic strain increment d epsilon p. We can say the incremental plastic strain also it is associated with this thing del F by del sigma into d lambda.

So, del F by del sigma into d lambda is it is like that only. So, we introduce the, this functional form of F that actually able to predict the potentiality of the how the plastic. That means, yield surface in case of 3 dimensional problem also how the yield surface evolves. So, F is exactly representing the potentiality of the or maybe I can say the functional form of the yield surface.

And then d lambda with the plastic multiplier so, d lambda is called the plastic multiplier. Therefore, d lambda is the theoretically I can introduce the d lambda what is the; that means, what may be the incremental of the. It is a broadly indicates the what amount of the evolving say of the plastic that means, yield surface in case of the 3 dimensional state of the stress. So, accordingly the plastic multiplier this comes into the picture.

But, there are the several ways to identify what is the value of what may be the value of the d lambda; we will see how it can be express this thing. So, it is a expression for this del F by del sigma into the d lambda. So, del F by del sigma is basically the it is a F is the it indicates the functional form of the yield surface, during a three dimensional analysis. So, once it enters the plastic domain.

So, now we have to look into the other theoretical aspects also. So, how what way we can estimate this value the second part. Basically, I am talking about the plastic incremental of plastic strain that part. So, therefore, the unknown coefficient d lambda is the determined by utilizing the concept of the plastic work. So, utilizing the concept of the plastic work, maybe we can estimate what is the value of the d lambda. So, that expression we will try to look into that.

So, plastic work is defined by if you remember plastic work is defined by dw for example it is also assuming within that. For this incremental plastic strain what may be the incremental plastic work done in that way, we can think about. Now, see we can this is the components of all strain component, we have we have considered. And, if you remember that from a simple stress strain diagram what way we can estimate the, this is suppose this is stress strain diagram. Suppose this is the curve.

So, what is the work done? Work done simply represents the area of this curve, in this case area of this curve that actually indicates or we can see it can be represented by that y. So, y into d s is the elemental form and if you integrate over that indicates the work done, that calculation we can find out if we have the data of the simple stress strain diagram. From there we can estimate what is the work done over a particular limit. So, from epsilon 1 to epsilon 2 like that.

So, why? So therefore, elemental in this if we consider every elemental part. So, what is this elemental area? If it is assumed this is the in strain component and corresponding stress is there. So, sigma d epsilon is incremented for this the work done for this elements, but this work done represents the per unit volume. So, simply the stress into strain we multiply stress into incremental strain that actually indicates, the value of the work done per unit volume for this element.

So, therefore, there are so many in a different 3 dimensional state there are six components of the strains are there. Even that we have already shown that how what way we can estimate the plastic work done. So, this is the stress component, and this is the incremental strain. This is one component epsilon x, epsilon y, similarly there are all other components and finally, the even shear strain component also. And shear stress corresponding, shear strain component all indicates the elemental work done and that per unit volume.

So, combining contribution from all these things simply adding all this thing that indicates the total plastic work done. So, that can be represented finally, in this way also. But, in this case we have considered the sigma x dot; that means, only you have the why only we have considered the deviatoric stress component. Because, the in the plasticity law also even if you look into the von Mises yield condition once we try to develop remember once we discuss this relative to the stress analysis part also. So, you can see that von Mises yield condition says that, that once you try to establish the von Mises yield criteria.

So, therefore, hydrostatic stress component do not influence the plasticity of a particular. Therefore, there is no need to consider the hydrostatic component. So, in this cases the deviatoric stress component is basically we are considering to estimate the plastic work done. So, if you remember or maybe I hope you have maybe one of the module, I have explained in this particular part, that any state of the stress we can divide in the deviatoric stress component and the hydrostatic stress component.

So, therefore, hydrostatic stress component is the plasticity law also even the independent of the hydrostatic stress component. So, that is why we have considered the work done only the considering the deviatoric stress component. So, that is why sigma dot is considered? So, sigma dot here indicates the deviatoric stress part.

So, now in the matrix form or in the form of a column vector also if we consider that, this is the stress and this is the strain. And that is a single matrix form or single columnar vector in that form, if we consider all these components. Then, in general we can write the stress into incremental strain plastic strain that indicates the plastic work done.

Now, sigma dot T transpose and d epsilon p, that we have seen already the d epsilon p we can see that its a delta F by delta sigma, this is the and into the plastic multiplier that this term. So, that is equal to the, we can see that is equal to the sigma bar into d lambda. So, sigma bar into d lambda so, therefore, in this case we can see the only the deviatoric components has been considered because, hydrostatics component of the stress do not contribute to the yielding.

So, that is why we have considered only the deviatoric stress component. Now, d the in this case sigma dot T the all this form we can see that this part we are representing this some value of sigma bar, which is we can see the say you can see what we can see we can say in that way also the sigma bar may be the this yield stress component, but it is a single component and into the D multiplier.

So, we will see further how it can be different way we can derive the, we can relate the different parameters. So, therefore, in general the yield criterion is represented in the symbolic form in that particular sigma i j k. So, that functional form if you represent that this functional form introduce it is a sigma i j. So, state of the stress and k we can see k, we can say kind of relate to the hardening effect. So, strain hardening effect maybe if you see that that in terms of the k and sigma state of the stress that actually the functional form of a yield surface.

So, it means that, when if you try to look into different hardening thing isotropic hardening or kinematic different hardening rule if we apply all these things. So, there is a it means that there is a evolve of the yield surface, due to the when it is try to follow this thing. So, maybe this is the initial yield point yield surface defined. So, which is if a single component, a single uniaxial tensile test maybe you can consider this as the yield point first yield point.

But, once we reach this particular point stress strain so, uniaxial so the suppose this is correspond to sigma uniaxial yield. Now, if we further applying the load the it will deform, then it will try to reach this particular point. So, once we reach this point this act as a because of the strain hardening effect. The maybe we can consider that this is the next yield point corresponding to this thing because of the strain hardening effect. So, therefore, that way the yield surface actually evolves or yield point evolves in this particular either uniaxial or 3 dimensional state of the stress.

So, therefore, that yield parameter which is considered this thing or maybe we can say because of the hardening effect. So, k actually takes care of this particular hardening effect and sigma, which is the particular state of the stress. And combining these things we can represent the functional form of the yield surface.

So, that is why symbolic form we can represent this functional form of the yield surface. And, within this functional form the yield surface the incremental strain depends on this. What are the functional form of the yield surface? Based on that we can define the plasticity law or yield criteria we can it is possible to define.

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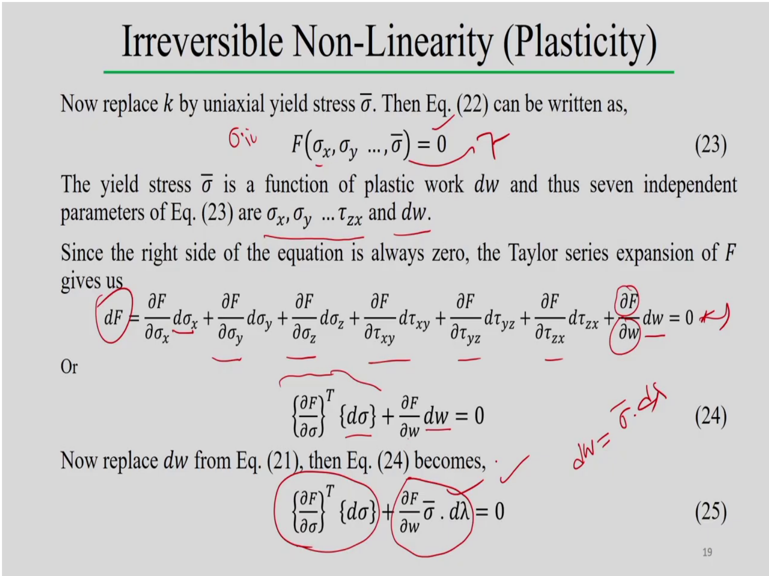

Now, replace k by uniaxial yield stress sigma bar. So, now, assuming the k uniaxial yield stress value k, we can replace this thing then equation 12, 22 can be written like that. So, F is exclusively we have seen the sigma i j. So, state of the stress sigma x sigma y and may be shear stress component.

And k we are assuming the k can be replaced by the yield stress value may be you know particular process uniaxial yield stress value sigma bar. Then, this functional form can be in general written as this sigma x sigma y tau sigma z tau x y, tau y z tau z x and sigma bar.

So, therefore, we can see there are 7 components are there, 6 is the stress components and other k can be replaced by the uniaxial stress value sigma bar. Now, this yield stress sigma bar is a function of the plastic work dw at the 7 independent parameters of r. Now, if you look into the sigma bar is the function of the plastic work dw. So, therefore, seven independent parameters of this equation are there sigma x up to sigma y and dw, we can assuming the dw is the plastic work done.

So, therefore, all these 7 independent parameters we can consider. Now, right side of the equation is always zero. So, you can see the right side of this equation is always zero the Taylor series expansion of F. So, F this functional from assuming that expression of the functional from F, can be written in that way also d F that means, that partial differentiation if we consider delta F by delta x into d sigma x, d sigma x and delta F by delta x with that form we can we can write this expression.

So, therefore, finally del F by del w into dw. So, because dw and other stress components we can assume the independent parameters. So, that we look into delta F; that means, since they are independent parameters, then we consider this as a partial derivative. And, then total derivative d F can be represented in this particular functional form that equal to 0.

So, finally, or in other way also del F by del sigma transpose into d sigma so; that means, in the matrix form we can write. So, if we see the previous equation also from here in the matrix form, we can write in this particular way d sigma plus del F by del w into dw equal to 0. So, this way we can further express this way.

Now, replace dw from equation 21 and then equation 24 becomes like this. So, therefore, d w value if you remember d w equal to previous slide also, we can see d w equal to sigma bar into d lambda. So, that is equal to that in general c sigma bar into d lambda. Actually, this d w although, it is represented in the different stress component into stress into strain that component. That is also equal to sigma bar into d lambda in this case the sigma bar can be represented like that.

So, plastic work done overall which corresponds to the all the stress component. Now, if we represent this stress component as a single component of the stress value, then this part replaced by the single component of the stress value into d lambda. So, therefore, dw equal to sigma bar into d lambda. So, from here we can see that dw equal to replace by the sigma bar into d lambda, sigma bar into d lambda and we can reach this kind of expression.

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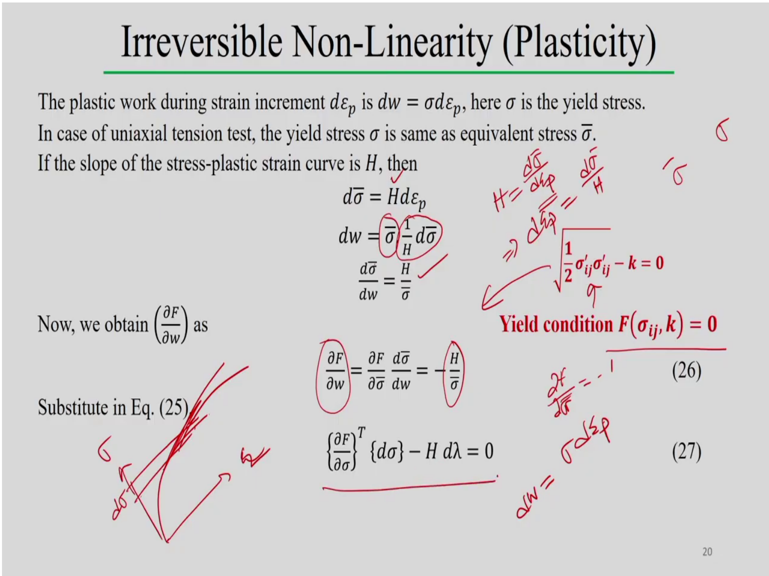

Now, plastic work during the strain increment d epsilon p is also can also be represented like that dw equal to sigma into d epsilon p. But, in this case sigma is the yield stress. So, therefore, in case of uniaxial tensile testing the yield stress sigma is same as the equivalent stress sigma bar.

So, what is the difference between sigma bar or in this case is sigma see, we can see that in general dw the plastic work done sigma into d epsilon p. So, in this cases point, but if you do the uniaxial tensile testing so, uniaxial tensile testing we have the only the yield stress value yield stress component. So, plastic work done simply we can estimate the replacing the single value of the plastic stress into the strain component, and then we can estimate what is the plastic work done.

But, when there is a three dimensional state of the stress so, therefore, the yield stress itself can be represented and the in the equivalent form also. Even if you follow the von Mises yield criteria. The sigma bar can be represented in this sigma bar, we can consider this as a single component, but it consists of the expression in terms of the different stress components say sigma 1 2 3; 3 in the all the principle stress components. Or even principle stress components can be represented in the form of the original state of the stress.

That means, there are sigma x sigma y sigma z tau x y tau y z and tau z x. So, that way that in that sense that sigma bar can be considered as an equivalent stress. So, therefore, once there is a need to replace the single component of the stress value, then we can replace in the form of the equivalence stress. Even that equivalence stress itself is a function of the all the involve all the other stress components. So, therefore, uniaxial tensile strain sigma is same as the equivalence stress sigma value.

So, if the slope of the stress plastic strain curve equal to H, then we can find out that maybe if we look into this stress strain curve also. The slope so, we can consider also that any this is the increment of the, this this sigma. And H is the this slope plastic domain the slope is the there H is the slope.

Therefore, H is basically the slope equal to d y by d x d sigma bar by d epsilon p, even if we consider this is the strain component. So, H and the slope plastic strain come H in the slope of the stress strain, stress plastic strain curve, then H can be the slope. So, then in that cases H equal to d y by d s d sigma by d epsilon p this is the plastic strain component.

Now, for example, in a particular material the stress strain curve this value of the H can be well defined, because the uniaxial tensile testing curve we can easily evaluate, we can easily find out even may be the available of the data is there. So, from there we can get all this information also that what is the value of H. So, we assume that H is known may be for a particular material.

Now, dw is what is the work done equal to sigma bar, then this is the value into d sigma bar this 1 by H. So, we have used this relation equal to because dw was the sigma bar sigma into d epsilon p. So, from here we can see the d epsilon p equal to H equal to this so, d epsilon p equal to d sigma bar by H. So, from here if we put d sigma bar by H so, then sigma bar into D ep.

So, that we have already seen the, this dw equal to sigma bar into d epsilon p. So, we can see d epsilon p, but d epsilon p replace in the form of a, the plastic strain stress curve. Because, this H actually we assume that H is a known quantity, or we can see that H can be the most well defined quantity. So, that is why we can replace this d epsilon p in the form of H. So, that is mean finally, we are getting d sigma bar by dw equal to H by sigma bar in that way.

Now, we obtain del F by del w so then del F by del w, this partial derivative or del F by del sigma bar into del sigma bar by del w. So, such that its equal to del F by del w. So, from here we can find out del F by del sigma bar d sigma bar by d w. So, d sigma bar by dw equal to H by this thing and del F by del sigma bar is actually minus 1 that can be proved in this case.

So, that will this expression maybe the expression may be beyond the scope of two analysis, but del F by del sigma we can evaluate the do the calculation finally, it will become equal to minus 1. Now, we can see that yield condition we have seen, that yield condition can be represent this equal to k. And finally, in that form it can be equal to H so, from this from here if you derive del F by del sigma bar. So, there we can find out equal to finally, it will be coming to the minus 1.

Now, what may be the equation 25, if we remember the what is the equation of the 25 equal to del F by this thing. So, we can estimate this value and we put this value in the equation 25 to put this value del F by del sigma valve T transpose into this sigma equal to this value equal to minus H into d lambda, you see minus H into d lambda. So, this expression we can reach that equation 27.

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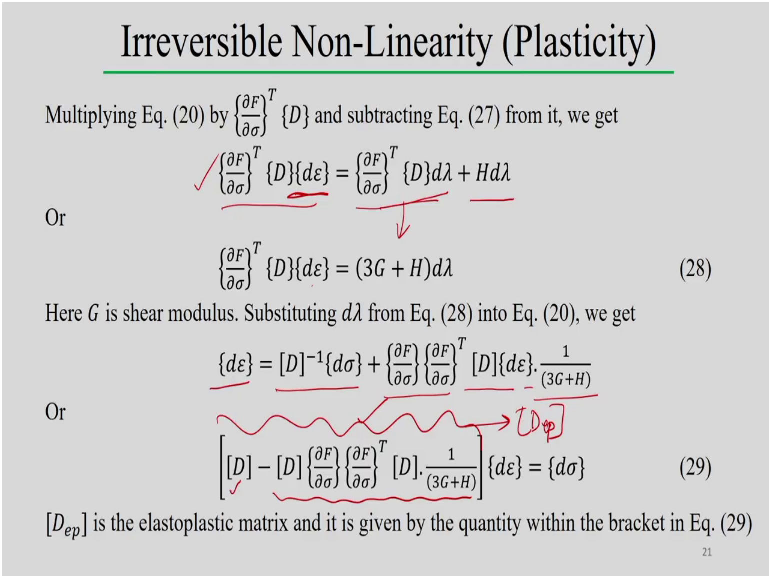

Now, multiplying equation 20 by both the side del F by del sigma transpose into D and subtracting equation 27 from it. So, if we look into the equation 20 also, we can see that equation 20 this equation what now we are talking about coming back to, what is the total strain increment that expression equation 20 also. So, here we can multiply this particular term and del F by del sigma D and subtracting equation 27 from it.

Now, we can get this expression can be represented like this, if you see the total strain equal to some D inverse and some expression was there if you multiply this thing. Finally, we are getting this expression. Now, del F by del sigma into D into d epsilon equal to 3 G plus H into d lambda. So, del F by del so this part equal to this thing. So, it means that H d lambda and 3 G into d lambda. So, this expression is coming that 3 G into d lambda.

So, it can be is proof or you can follow any kind of the standard book also, that expression is available that how it is coming actually if you will follow all this expression. It is possible to evaluate this del F by del sigma, transpose into D can be reached to the 3 G d lambda for this this particular analysis. So, therefore, yes G is actually the shear modulus. And subtracting d lambda from equation 28 also, you can show the manipulation 20 accrues into the 20 equation, 20 means the expression for the total strain component.

Then, total strain component can be represented like this D this value in terms of the del F by del into del F by del sigma transpose d epsilon 1 by 3 G plus H. So, that is why basically we are replacing the plastic multiplier d lambda value actually. And, then now you can see this expression in terms of the D and the del F by del sigma F is the we can see already told that F F corresponds to the may be expression for the plastic, maybe in terms of the different stress component that its a functional form of the yield surface mean.

So, therefore, D into d epsilon into 1 by 3 G plus H from here, we can manipulate this thing and finally, we are reaching this expression into d epsilon equal to d sigma. So, that is the expression. So, in this case D ep is the elastoplastic matrix and it is given by the quantity within the bracket of equation 29. So, therefore, if you see this expression you can see we can write it is equal to D ep.

So, D ep means in this case if you see D is there, D matrix is there and I assume this is the plasticity matrix D ep. So, D minus D ep we can say that this is equal to the D ep so, elastoplastic analysis. So, therefore, elastoplastic matrix so once we do some kind of the if we assume the material model follow some sort of elastoplastic analysis or elastoplastic deformation is following particular material in the thermomechanical analysis, then we can construct this D ep matrix.

And, accordingly the depending upon the yield point or yield criteria, we can decide the some part may be associated with goes through only the elastic deformation. So, therefore, that is in that case we use the only elastic matrix D. And which part is associated with the plastic deformation in that cases we can use the D ep.

So, since elastoplastic analysis, if we assume that this analysis corresponds to the only the plasticity there is no elastic part in this during the deformation model of a particular material. In that cases maybe we can (Refer Time: 32:14) to the elasticity part and then only we can consider plasticity. So, in that cases when you do the analysis thermomechanical analysis, then in that only the plastic part is important. So, in that cases no elastoplastic component will be there. So, only the plastic component will be there.

So, in that cases only D ep may be useful. But, since most of the material follow engineering materials follow initial state the some small part may be associated with the elastic deformation, then it goes through the plastic deformation state. Also, even if you look into the recovery part is also associated with the even we assume the metal material deformation or material weaver follow elastoplastic. In that case even if it is plastic domain also, if you release the load some recovery will be there. So, that recovery associated with the we can say the elastic recovery will be there.

So, since it is associated with some sort of elasto recovery elastic recovery. So, therefore, the analysis should be considered the elastoplastic analysis and accordingly the D ep. Then, elastoplastic matrix is basically has to be considered, even if you do the analysis in the plastic domain.

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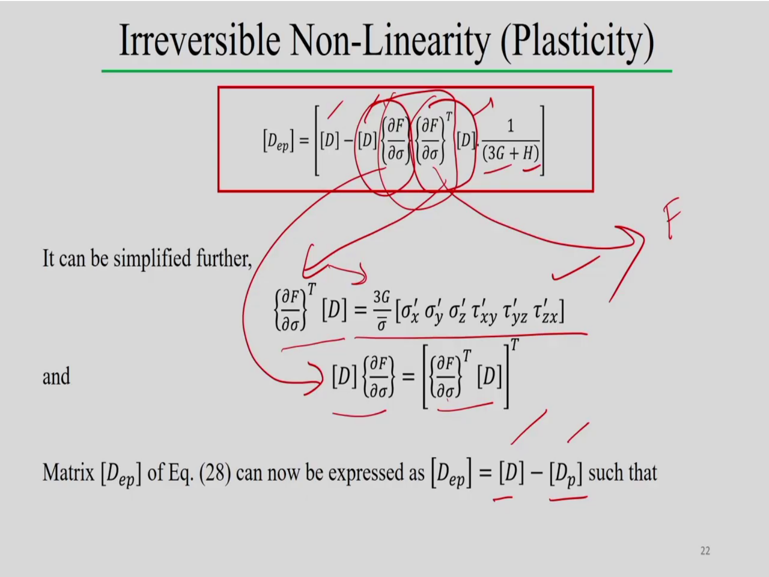

So, finally, D ep elastoplastic analysis in the form of the D associated with that G and H is the modulus shear modulus and H is the plastic modulus also, already told the shear modulus plastic modulus, these are well defined quantity for a particular material.

Now, thing is that how what are this expression can vary also, because D is also defined the particular material D is associated the all the components the last Young's modulus as well as the Poisson's ratio also. So, that values is also known for a particular material from the material properties but, how to look into this part.

So, del F by del sigma; that means, what way we can represents the elastics; that means, yield surface and maybe even if we by considering the strain hardening effect of a particular material, then accordingly the expression can be done.

So, del F by del sigma can be simplified further like that, because del F by del sigma into D, we can see that it can be expressed like this. This way also it is associated with 3 G by sigma bar sigma bar is the yield, yield point value this thing and it is associated to all the deviatoric stress components. So, therefore, D del F by del sigma equal to del F by del sigma transpose D, the transpose of over of or this value equal to this of transpose of that.

Now, matrix D ep of equation 28 we can represent this thing can now be expressed. So, because in this case we are looking at what is this value and other cases what is the this value. So, transpose this value is can be represented like this expression, all the deviatoric stress components are involved in this case and, but what is the value of D del F by del sigma D del F by del sigma can be the terms represent the del F by del sigma transpose into D. The transpose of all this quantity that represents the D del F by del sigma. So, this is, the this del F by del sigma is the this quantity.

So, therefore, accordingly the transpose of basically transpose of this quantity. Now, this matrix D ep we can express this equation 28 count. And, now we express the D ep is basically we are representing D minus D p.

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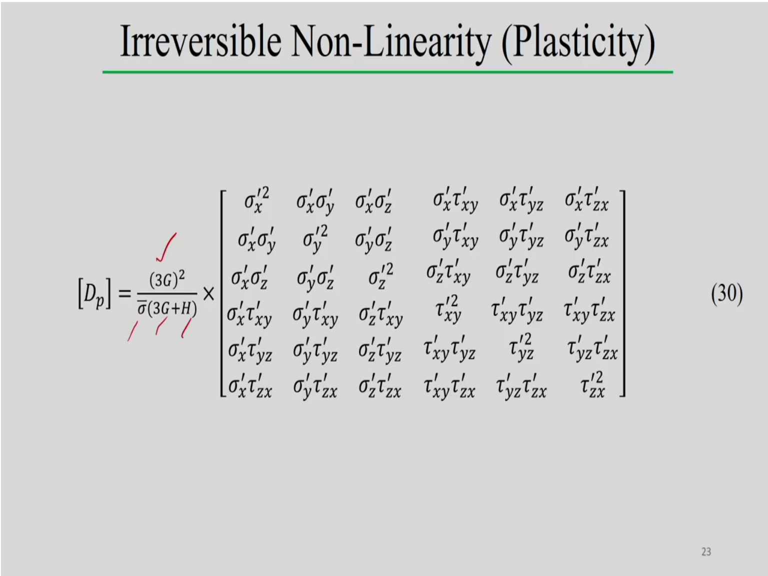

So, such that D minus D p is in such a way that plasticity matrix. And these we can say elasticity matrix also. So, D p you can see this material property G sigma bar G and H modulus is there, but the remaining components in the form of if you see that is the deviatoric stress component.

So, if you want to evaluate the plasticity matrix say any particular state also. So, therefore, we need to look into what are the deviatoric stress component of this particular. So, that is why elastoplastic analysis always we can do in the form of a incremental mode. So, each and every increment, what is the state of the stress that information is required to estimate the this plasticity matrix D p matrix.

So, that information can be obtained, but it is function of the deviatoric stress component only, not the total stress component. So, we can remove the hydrostatic part and the deviatoric component we consider. Because, deviatoric components comes into the picture assuming that the hydrostatic stress component is having no role in the plasticity, that is why only you consider the deviatoric stress component.

So, this is the plasticity matrix this is the typical expression of the plasticity matrix so, in terms of this value. So, once we know that D p matrix then, we can easily estimate what is the value of the D ep matrix. So, here you can see the D ep matrix the D value minus of D p. So, D value D matrix we have already shown that what is the expression for the d matrix in a 3 dimensional state. Even different cases also. So, same expression we can use here to estimate the D ep matrix. So, that is way it works.

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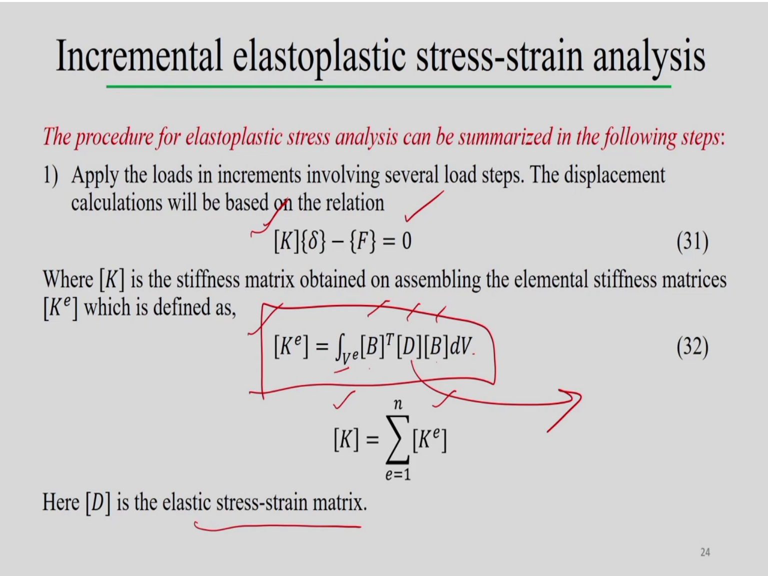

Now, incremental elastoplastic stress analysis, how it can be done also, before that the procedure for elastoplastic stress analysis can be summarized in the following particular steps also. So, first step is the apply the loads in the increment involving the several load step, the displacement calculation will be based on this relation.

So, if you remember once we develop the stress analysis model. In that case we first we look into the stiffness matrix, we try to form the stiffness matrix and from the stiffness matrix we solve for the displacement field. And that displacement field, we converted to the strain state and then finally, that strain from the strains to the, we convert in the stress field.

So, that is the usual procedure for the analysis what finally, we actually make the final equation in the form of a displacement so, from this relation. So, K we can see the K delta minus F equal to 0. So, that is the K is the stiffness matrix obtained from by assembling the of elemental stiffness matrices. And we have seen the K expression of the K for a particular element, the over the volume integral over the volume B transpose D B matrix into d V.

Now, once we form the elemental matrix one particular element, we make the assembly for the all the node all the elements, then we can find out what is the K matrix. So, here D is the elastic stress strain curve. So, that is the, that is the normal procedure we do, in this case the K D is the elastic stress strain curve and we estimate the K. And, then once we do estimation, then if we solve for this thing solve this equation 31. Then, we will be able to find out what is the value of the displacement field that means, delta.

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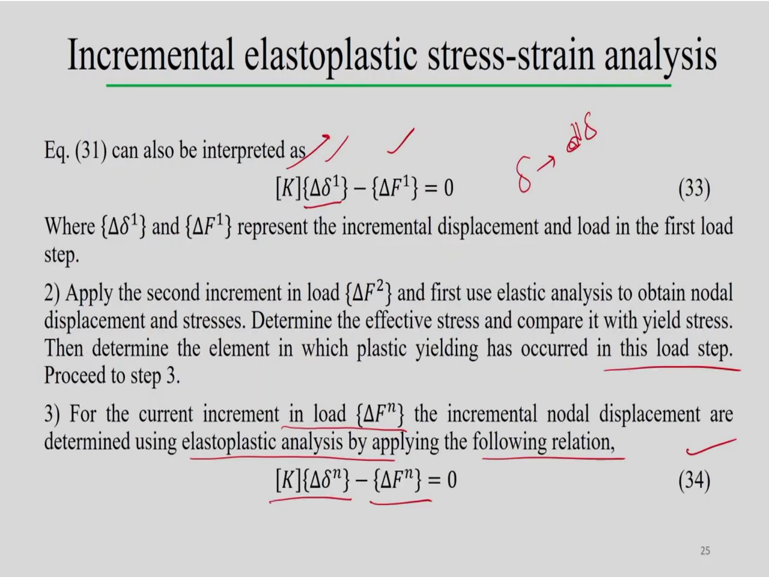

Now, 31 can be interpreted like that also ok. So, K the stiffness matrix, but not exactly the delta in a single step; that means, displacement for a single step. Rather we can see the delta of that means, we can take a small stress incremental in the incremental way, or instead of only direct field delta we can use the d delta maybe you can see like that also. So, that means small increment and corresponding to the, what is the load vector, not a total load vector rather total load vector, we can divide into the small incremental way small incremental point.

So, assuming that this is the total load step is divided into n number of steps. So, for one each and every step what is the incremental load, accordingly we can estimate what is that increment of the displacement. And, then once you solve this equation, then we can reach to the one incremental displacement field. Then, we convert to the strain and then after that we can convert to the stress in the incremental way. And we can add all this thing with respect to the previous what we got in the case of the previous iterations.

So, therefore, delta 1 and the delta F 1 represent the incremental displacement and load in the first load step. Now, once we extract for the first load step, then apply the second increment of the load. So, second increment of the load, we can divide and the first use the elastic analysis to obtain the nodal displacement and stresses.

So, therefore, it is also necessary to consider first the elastic stress analysis elastic component. So, therefore, then we find out what is the effective stress value. Determine the effective stress and we compare it with the yield stress value yield stress means, we have the data of the yield stress for example, we do the elastoplastic analysis for the say low carbon steel.

So, for that particular metal we have the yield stress value and the plastic deformation zone everything is well defined. So, that is the, that we can get the data what we know that material properties what is the yield point value. Now, we compare the yield stress value. And, once we after doing performing the elastic stress analysis, then then we have to check whether any of the element is crossing this particular yield point or not, if it is particularly crossing the yield point value particular element. So, in that cases we will be using the D ep matrix.

So that means, D ep matrix means elastoplastic matrix, if it is cross the yield point for the for the, this thing or if it is do not cross the yield point. Then, we simply use the only the elasticity matrix only the d matrix not necessary to consider the D ep matrix. So, that is why it is necessary to compare each and every steps also, whether effective stress for a particular element is crossing the yield point or not.

Now, once we determine the effective stress and compare it with the yield stress, then determine the element in which the plastic yielding has occurred in this particular load step. Whether it is not on happens or not then we proceed to the next step. So, for the current increment in the load delta F n so, for example, any particular step and for incremental nodal displacement are determined using the elastoplastic analysis by applying the following relation.

So, K into this delta F n so, from this relation we can use the elastoplastic analysis, we can perform the analysis, but to construct this thing K matrix or this stiffness matrix also. We have to look whether the state of the particular element is with the below the yield point or above the yield point. Accordingly we can construct the elastoplastic matrix or only elastic matrix.

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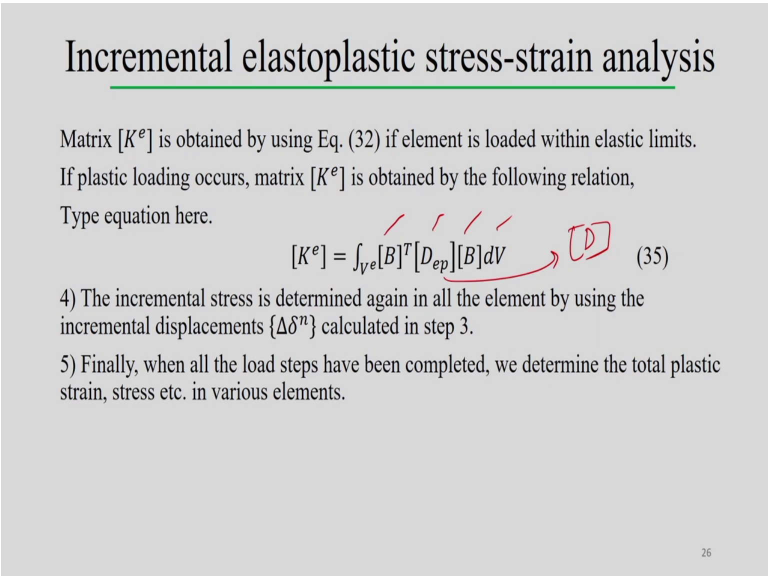

Now, matrix K is obtained by using 32 also element, if the element loaded within the elastic limit. Let us look into the thirty two also equation so, this equation. So, we can see this equation also K is the B transpose D B d V. So, D so D if the particular element within the elastic domain, we use the D expression of the D. If it is not elastic domain it is in the plastic domain, then we use the D ep instead of only the elasticity analysis. So, that is the difference.

So, therefore, if the element is loaded within elastic limit then can be use equation 32. But, if the plastic loading occurs, then matrix K stiffness matrix can be obtained by using the following relation. If you see that in this particular element, this B transpose D ep B. So, in this cases D matrix should be replaced by the D ep and over the elemental volume. So, that is the stiffness matrix.

So, therefore, the incremental stress is determined again all the elements by using the incremental displacement delta n calculated in the step 3 also. So, same way we can estimate once we form this K matrix stiffness matrix, whether it is that particular element within below the yield point or above the yield point. Accordingly, we can decide construct this matrix assemble for all the elements and we solve for the displacement field.

Then, that is the step every time we repeat this thing until and unless there is the complete application of the all the load step. So, finally, all the load steps have been completed, we determine the total plastic stress strain in the various particular elements. So, that is the way to analyze the elastoplastic overall looking into all these things.

So, in general we can say that when you try to do develop some elasto plasticity model, or thermomechanical analysis associated with the welding process also. First in stress analysis we have to construct the stiffness matrix. So, once we construct the stiffness matrix, then we solve for the displacement field.

And then once we solve for the displacement field, then look into the, if it is incremental mode. Then keep on updating the displacement field also and from the displacement field also, you can link with the, we can estimate what is the strain field and from the strain field we can estimate the stress field.

So, that is the usual way to do the analysis. Apart from that once we look into that when you try to develop the K matrix also, then stiffness matrix then we have to look whether particular element crossing the yield point or not. Accordingly the K matrix of K formation expression will be different, that is why in these cases we have expressed equation 32 is usable when the element is within the elastic limit.

If equation 35 is usable when the domain, when the particular element is in the above the yield point value. So, plastic domain so therefore, based on that we have to construct the K matrix. So, this is the overall procedure to do any kind of the thermo mechanical analysis, associated when you try to use the finite element method.

So, thank you very much for your kind attention for this particular module.