Finite Element modeling of Welding processes
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Module – 02
Fundamentals of finite element (FE) method
Lecture – 12
Material Non-linearity

Hello everybody, today we will discuss the Material Non-linearity in Finite Element Modelling of when you try to solve the equation, the matrix equation that we have already shown. So, in that case, probably in that for example, if we consider the thermal analysis then the output is the temperature distribution. But, if you look into the formulation of this matrix formulation from the governing equation boundary condition, the matrix itself depends on the temperature.

So, once we dependent on the output of a particular this equation, in this particular equation, then the non-linearity comes into the picture, then we will see in this particular module, how we can overcome or what way we can consider the non-linearity in finite element base a modelling approach.

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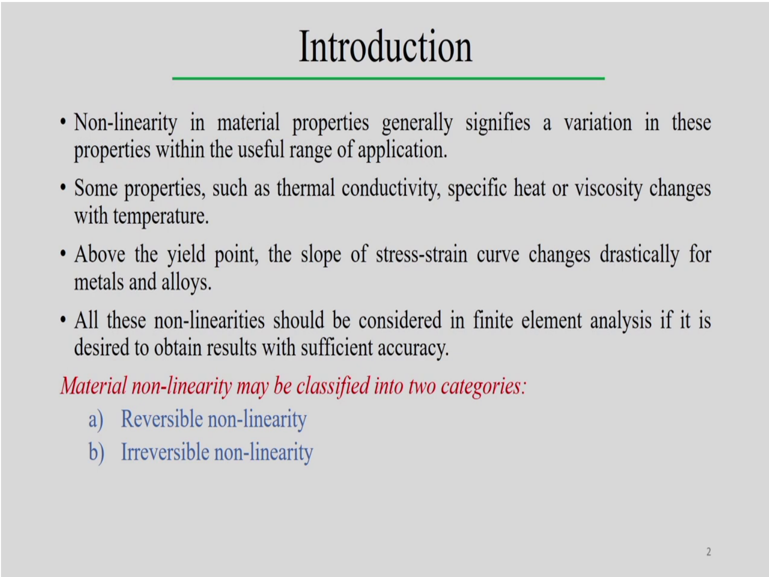

So, non-linearity in material properties generally signifies the variation of the properties within the useful range of the application, within a particular range of the application what are the variations of the properties is as simple as that.

If we consider the we know the properties it is not during this process also properties we cannot consider as a constant because most of the mechanical or thermal or any physical properties most of the time it depends on the temperature. And that temperature within the particular range of this temperature, there is a variation of the properties. So, if you want to incorporate these things, then the non-linearity comes into the picture.

Some properties such as thermal conductivities, specific heat viscosity changes with temperature that we know, there is a drastic change in temperature. So, very precisely, if we consider, the development of the finite elements model then it is necessary to consider. This temperature dependent phenomena, but once we consider this temperature dependent phenomena, then non-linearity comes into the picture.

Even the yield point, the slope of the stress-strain curve also changes even with the change of the temperature in case of particular metal alloy. For example, if you look into this thing the Young's modulus or maybe the other parameters associated with the if you look into the uniaxial tensile testing diagram from that diagram that we normally produce that diagram as the room temperature.

So, but if you do the same material, if you do the universal tensile testing at the very high temperature for example, maybe 500 degree centigrade or 600 degrees centigrade's. Then you will get the time, the stress strain curve is completely different from what we obtain in case of the room temperature. So, that means, it means that the slope, the yield point everything changes with respect to the temperature.

Therefore, all these nonlinearities because of the change of the temperature dependent properties in general are in this particular case also, non-linearity may also arise in the different way also, but if you consider this simple the temperature dependent properties of a particular material, then these non-linearities should be considered in finite element-based analysis if it is desired to obtain the results with sufficient accuracy.

So, accuracy level of the result depends on the how effectively you are considering, the non-linear behaviour which is associated with some sort of temperature dependent properties. Now, material non-linearity can be classified into two different categories, one is the reversible non-linearity, another is the irreversible non-linearity. So, we can see that how, what is this thing, do reversible non-linearity or irreversible nonlinearity.

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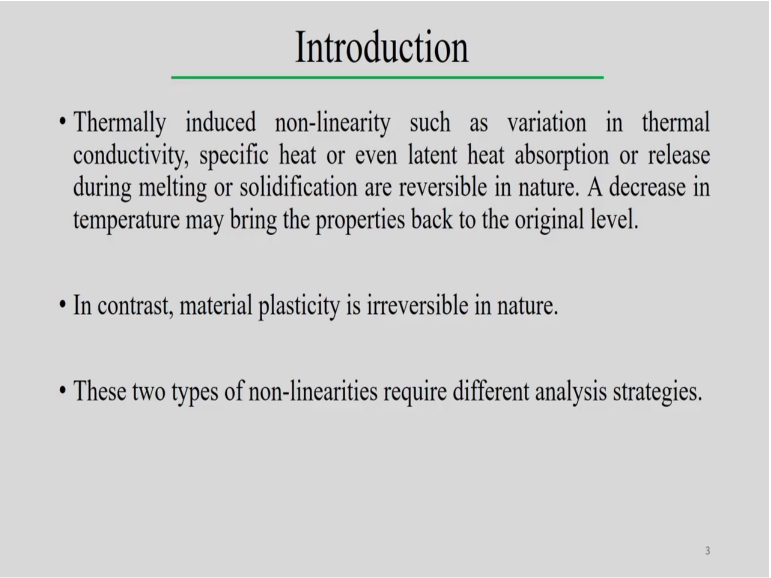

For example, thermally induced non-linearity such as variation in the thermal conductivity, specific heat this is a we can say the thermal properties or even latent heat of absorption, solidification during melting or solidification. These are the actually reversible in nature because in this case, a once we increase the temperature, then once it gives the temperature, we can these properties.

But again we there is a after increasing the temperature, then if there is a decrement of the temperature once he comes to come backs to the room temperature so, we can bring back these material properties so that is why it is called the reversible non-linearity.

Or in contrast, for example, the material plasticity. So, once we plastically deform the material so that means, one deforming the material within the elastic limit also, if we remove the load, then it will come back to the original position. But if once it cross the yield point during the deformation process that means, it enters the plastic deformation zone, then you will not be possible to come back to the initial position so, that is why, the metal plasticity is basically is reversible in nature.

So, therefore, these two types of nonlinearities require the different analysis strategy. So, definitely wants to consider the finite element-based model. So, strategy to consider all these non-linearity will be different way reversible non-linearity or irreversible non-linearity. So, let us look into all this aspect.

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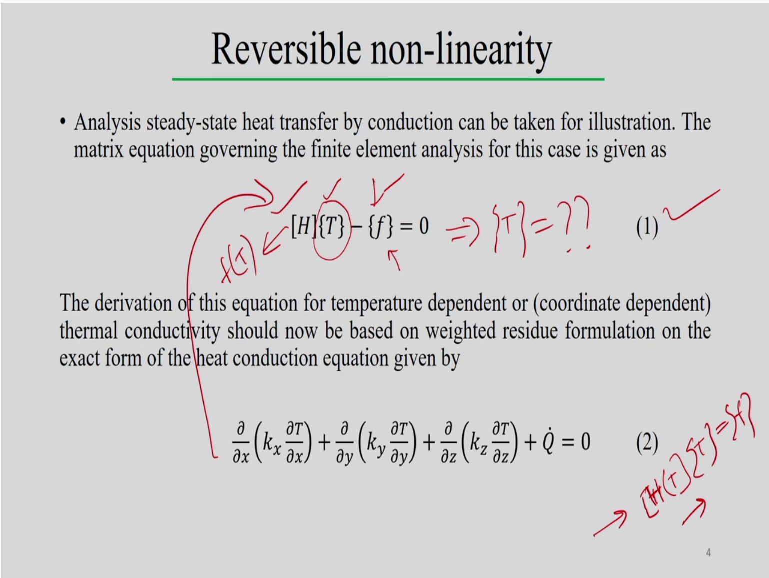

Now, first we look into the reversible non-linearity part. In this case, first we consider the analysis of steady-state heat transfer equation by conduction that means, heat transfer equation when there is only conduct to heat transfer is there neglecting the material flow all these things in this particular case, can be taken as an example such that we will be able to understand that the how we can takes care of the reversible non-linearity in a finite element-base heat conduction analysis.

So, therefore, we have already seen the heat conduct matrix equation in general the after discretizing the finite element domain and assembling from the contribution for the each element. Then we get the final equation in this particular form that H T equal to f equal to minus f equal to 0, if you know this is the basically load vector or it is a column vector we can see. But, it accounts only on the load that means, the thermal loading part in this particular case and this is the temperature variable.

So, in this case, we are desired to solve this equation temperature T and it is a matrix form of the equation and this this will be follow this; this is column, this components of the temperature that means, this temperature is defined at the different node point. So, then T 1, T 2, T 3 up to T n so, there is n number of nodes are there in this finite element base this in the domain.

So, therefore, this equation if we solve it, then we will get the temperature of these things, but point is that, once you consider the temperature dependent properties also, then from here, we always are looking for what is the output. Then what is the temperature distribution that is the by solving the execution that is the our intended objective to get the temperature distribution that H is or maybe defining the temperature at each and every node point.

But once you form this H matrix or maybe even load vector also, then H matrix itself is a function of temperature that means, although temperature you are looking for, temperature as a output by solving this equation. But at the same time, this H, this matrix also dependent on the temperature that is why because of that, this non-linearity actually comes normally this the equation becomes non-linear during this whole new procedure.

Now, this derivation of this equation definitely we come back, we come this final form of this equation 1 by the this equation for temperature dependent or sometimes a coordinate dependent thermal conductivity should now be based on the weighted residue formula on the exact form of the heat conduction equation.

So, we have seen that weighted residue formula, the exact form of the conduction equation in that cases, we consider k, we can consider the k as a constant assuming that and then, we just keep the k outside of the integral. And then, we derive all these things and residual technique and we can reach some kind of this formula.

But this k can be the spatial variation of the thermal conductivity also possible because if material is not isotropic material, an isotropy. And then in that cases, probably we can consider the thermal conductivity and particular direction can be different also so, then, spatial variation of thermal conductivity is there or thermal conductivity can also depend, this is the properties of material that can also depend on the temperature.

So, then in that case, if we look into this formula in simple way, we can reach this from here to this kind of formulation in the matrix form after discretization in the using the finite element base method. So, it is simple that H you are considering H, but this H can be as a function of temperature T and again, we are looking for the output as a temperature equal to say this is the equation we need to solve.

Now, here, if we look into that temperature dependent properties or some sort of spatial dependence the conductivity; thermal conductivity, then we can reach this particular equation, shape of the equation. We assume this that type of the equation now, we look into how we can solve this equation and the different methodology that we will look into this thing.

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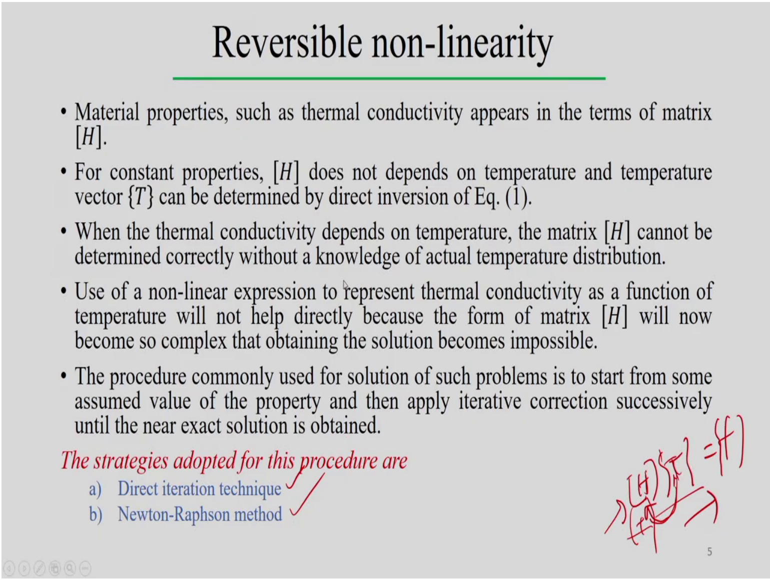

So, therefore, material properties such as thermal conductivity appears in terms of the matrix H. So, definitely, thermal conductivity also when you form for when you do the formulation these things and definitely H includes the one elements of the H is basically, it comes from the apart from the other things, other contribution, then this must account the thermal conductivity also.

So, therefore, for constant material properties, it can be thermal conductivities or also specific it also comes into the picture or other properties also thermal properties that constant properties, then H does not depends on the temperature so, directly, we can solve this equation H equal to H T equal to f, if you directly solve this equation, then we will get the output.

But when thermal conductivity depends on the temperature, then straight-forward way, the matrix H cannot determine correctly without the knowledge of the actual temperature distribution. That means, since we are solving for temperature, but we do not know what is solving for temperature, but we do not know; if you do not know what is the actual temperature distribution, then very correctly we cannot form the H matrix. So, therefore, we need the information of this things.

So, therefore, to do that, some sort of iterative calculation is required otherwise there is no other way to solve this particular equation because this H also depends on the temperature that is the reason. Therefore, use of a non-linear expression to represents the thermal conductivity as a function of the temperature will not also help; because the form of the matrix H will now become so complex even also that obtaining the solution becomes impossible.

Because once you have tried to obtain H matrix even then, we have to incorporate they picked up the temperature also so, it is a temperature so, they need the information what is the temperature, then only very precisely we can define the value of the H. Otherwise, if we do not know the temperature, then it is very difficult to obtain in this particular time step or particular load step or particular any particular situation so, therefore, if that information is require.

And on the other way also, until and unless we are not solving this equation, we will not be able to get the what is the temperature distribution with this particular step. So, that is why non-linearity becomes more complex even if we have the some data that H, thermal conductivity as a function of temperature, then also we cannot very precisely estimate the value of the H.

So, the procedure commonly used for the solution of this kind of problem, we start with the some with some assumption, some assume value, initial start of the some initial value of the property and then, we follow some kind of the iterative calculation. So, once you come back, we assume some value solve for temperature, then next step, we use the same temperature to evaluate the value of H, again we solve for temperature using that properties with a particular temperature. So, this way, we can follow the successive iterative techniques.

So, once that in between two successive iterative techniques, we put some convergence criteria, then we can solve for the then, we can say the solution has converged and that temperature distribution is the actual or very accurate temperature distribution from that by incorporating effect of the temperature dependent properties. So, that is how, we can solve all these things.

But the strategies are adopted for this procedure are maybe there are two different way the if you consider the reversive non-linearity; one is the we have discussed, we will discuss one is the direct iterative technique another is the Newton-Raphson method. We will see how they are different or different way we can consider the iterative technique, we can consider the nonlinearity in finite element base a modelling approach.

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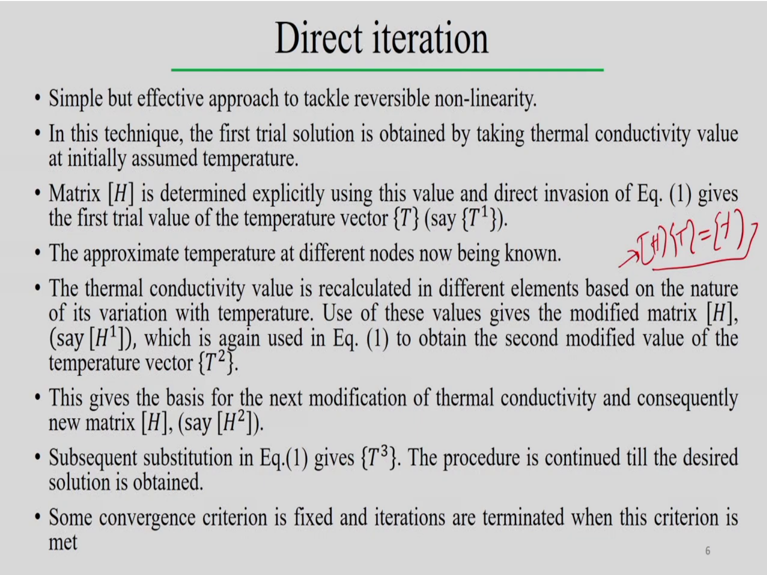

Now, direct iteration. So, direct iteration is very simple, but effective approach to tackle the reversible non-linearity, but in this technique, we start with the first trial solution. Trial solution means we assume some solution is obtained by taking the thermal conductivity as a maybe you can consider the thermal conductivity the room temperature value at initial assume temperature. So that means, it can be initial maybe at room temperature value, we can start with this thing, we can trial solution.

Then, once we assume some room temperature value of the thermal properties, then we obtain what is the value of H matrix. H is determined explicitly using this this room temperature value at the initial phase and then, direct inversion of equation 1 that means, direct inversion of the equation 1 so, from equation 1 is this one. So, direct inversion of equation 1, simply we can estimate. So, H value we can obtain that assuming the all the properties are at the room temperature value, then we can from here, direct inversion of this thing you can find out what is the value of temperature.

Say once you obtain, solve this equation, we get the what is the temperature T 1 for example, particular step, initial step at step 1 that is the T 1. Now, once you get the temperature, actual temperature, then the approximate temperature at the different nodes now being node so, we cannot one single step, we cannot get the exact solution of this what exact temperature distribution for this particular problem because of this temperature dependence, then properties.

So, therefore, now once we get one step, get the temperature distribution again, we recalculate the thermal conductivity, we consider thermal conductivity at that particular temperatures. For example, T 1, the T 1 assuming the approximate temperature so, we recalculate or consider the thermal conductivity at temperature T 1 depending upon this thing.

And the also we again we consider a contribution from each of the different element depending upon their temperature distribution after first step. Then, use these values on these things, we again obtain the matrix H say we can say that the step one it is H 1 which is again the again we put the H 1, then we again we solve this equation. Then, once we solve this equation to obtain the next modified value of the temperature distribution, there may be some changes.

So, there is a modified value of the temperature distribution T 2. Now, this gives ones you get the different temperature distribution again, this temperature distribution take as a input to calculate the temperature different properties to form the H matrix for the different element. And then, we assembly all these things procedure again, we solve this particular equation to get the another temperature next temperature distribution. So, this way the successive way iterative procedure we can estimate the temperature distribution.

So, this gives the basis for the next modification for the thermal conductivity and consequently the new matrix say H 2 like that similarly substitute the H T value and we obtain the T 3 and the this positive continue until the particular solution is not obtained. So, therefore, some convergence criteria is fixed and iterations can be terminated depending upon the convergence criteria. But what kind of convergence criteria we can put in this case.

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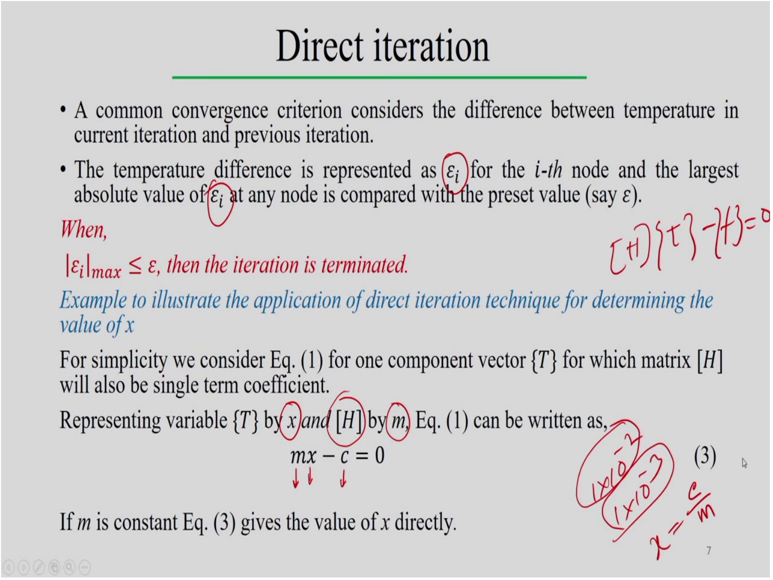

Very common convergence criteria we can put that consider the differences between the temperature distribution of the current iteration and the previous iteration. We can find out the what are the differences of temperature distribution between the current iteration and previous iteration.

And based on that, if this difference between these two is less than that the predefined value so, then in that cases, we can say that it reach the converse solution. And then, we terminate the iteration at this particular point and we assume the accept the latest temperature distribution what is the value as a final temperature distribution for this particular case.

Now, the temperature differences represented for example, the temperature differences between the current iteration and the previous iteration say for example, epsilon i and that i is correspond to the particular node point because every time. We are solving the temperature distribution, that temperature distribution means each and every node point temperature we are getting as a output.

Now, this temperature distribution E epsilon i is indicates that for particular node point, the temperature. Now, we compare the what is a largest absolute value of epsilon i. So, maybe temperature distribution we are getting, but the between the two successive iteration and then, each and every node point out of that with which cases we can find out where is the difference is maximum. So that means, we can find out what is the maximum value of epsilon i, i should value between 1 to n, the n indicates the number of nodes this particular discretized geometry.

Now, once this max epsilon i max is less than equal to epsilon, epsilon then the iteration epsilon is basically predefined value. So, I can give some for example, you can define the predefined value say 1 into 10 to the power minus 2 or 1 into 10 to the power minus 3, if you give the predefined value.

That means, then difference between the temperature distribution between the two successive iteration is some either 10 to the power minus 2 or 10 to the power minus 3 value, then we can stop the solution this and we are accept this approximate solution on this case ah, in this particular problem.

Now, if we increase the accuracy level for example, 10 to the power minus 3 and if you group convergence criteria is the that 10 to the power minus 3, one cases you have given the 10 to the power minus so, definitely if you want to the difference can be very small difference if you as a predefine you want to achieve between the two successive within that cases, the number of iteration can be more. So, it depends upon the what are the convergence criteria, what are the absolute value of the convergence you are putting in this particular case.

Example to illustrate the application of the direct iterative techniques for the determining the value of x, we can see in that way also. Let us see equation 1. So, I am repeating the equation 1; 1, 2nd says therefore, H T minus f equal to 0. So, this is the equation 1. Now, for simplicity to understand how the direct iteration techniques or Newton-Raphson methods is working ah, we can go into the basis of this thing.

Assume that we consider the equation on the for one component rather than the multi-component at the define at the node point, we can simply assume there is a single component, component vector T for example, for which metrics is the H will also be the single term coefficient. So, in that cases, say representing temperature T as a variable quantity by defined by x and H for example, defined by m and single quantity. So, that equation 1 can be written like that mx, m is equivalent to H, x equivalent to T and c equivalent to f equal to 0 for example, this is the equation.

Now, if m equal to constant so, we can if m is constant, it does not depend on this temperature ah, this value so, simply we can estimate what is the value of x, simply x equal to c by m. So, this is the simple way we can estimate the x equal to value c by m. So, there is no need of kind of iterative technique two because of the non-linearity or because of the temperature dependent properties. So, therefore, m is constant, equation 3 directly give you the absolute value of the x directly.

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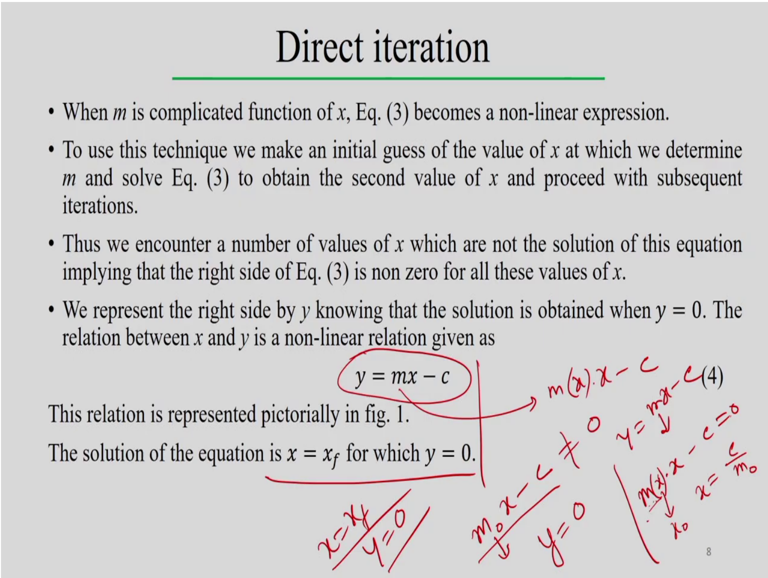

Now, when m is complicated function of x, now if m is itself as a function of x so, equation something that m into x minus c equal to 0. Now, m itself is a function of x, then mx into x minus c equal to 0. So, even m is a itself as a function of x, then definitely this equation becomes a non-linear, it is not a linear equation. So, then, we will see how these non-linear equation we can handle or what we can put some strategy such that iterative calculation is possible to achieve the converged solution of a particular, this particular problem.

Now, to use this technique, we make an initial guess value of the x at which we determine the value of m. So, definitely, we can get some initial value of x, say x; x 0 and what that particular x in this? If you define the x is a variable is because it is a function of x so, first we define what is the value of particular initial value of the x and that x value we can define what is the value of m and then, we solve this particular equation, we obtain the value of x and proceed with subsequent iterations. So that means, if you know some value say x 0, define the value of m, then we solve for x equal to say c minus something like that m 0 because m 0 indicates the m is defined at value x equal to x 0.

Therefore, we encounter a number of values of the x. So, we encounter the different number of value of the x say x 0, x 1 and we can obtain the value of which is not exactly the solution of this equation because once we assume the x equal to x 0, but x equal to x 0 in this case is not the solution of the equation.

So, therefore, we can assume the different values of the x to find out the equation and implying that apply in this particular equation 3 and which is also non-zero for all the values of the x so, that means, it may not be the non-zero, it means that since we are not able to take a exact value of the m.

So, therefore, it means that even x 0, we can find out this equation, but even m so, m 0 x minus c so, it means that here, we are equal to 0 that is the actual equation, but if we consider the different value of the m at the different value of the x, say x equal to x 0 it is m 0, x equal to x 1 it is m 1.

Then if you calculate this thing, then it is may not be the equal to exactly equal to 0; because is the this value of the different and this thing we say that it is not the exact solution of this particular equation. So, then, some sort of residual or some sort of residual will be 0 because it is not becomes equal to 0.

Therefore, we represent the right-hand side by y knowing that the solution is obtained when y equal to 0. It means that we say, we define this equation y equal to mx equal to minus c and assuming that, we will reaching the exact when y equal to 0, it means that we are reaching to the exact solution the of this particular equation.

Then, relation between the x and y is a non-linear relation which is given is like that y equal to mx minus c. So, then, we define these things such that m itself is a function of x and we can see like this also. So, in that case, we can say the y represented by mx minus c and such that this relation is the between y and mx minus c that relation is between y and x, it is a non-linear relation because m depends on the x value.

This relation is represented graphically in figure 1 for example, and the solution of the equation x equal to a x f for which y equal to 0. So, once we calculate the different value of m at the different values of the x, but once exactly this becomes equal to 0, then only we reach the exact solution. It means that other way also if when y equal to 0; because y we have already see m into x minus c so, different values of the m and different values of the x in that case, calculate this thing mx minus c and so, many trials you are doing all these things.

Once we get exactly y equal to that mx minus c or y equal to 0 value, then we can say that we are reaching the exact solution of this equation. For example, here it is written also say once x equal to x f is the actual solution in that cases, y equal to 0 it means that we are reaching the solution y equal to 0 means and that solution is that x equal to x f value that is a solution of this particular equation. So, all this comes into the picture because m itself depends on the value of the x.

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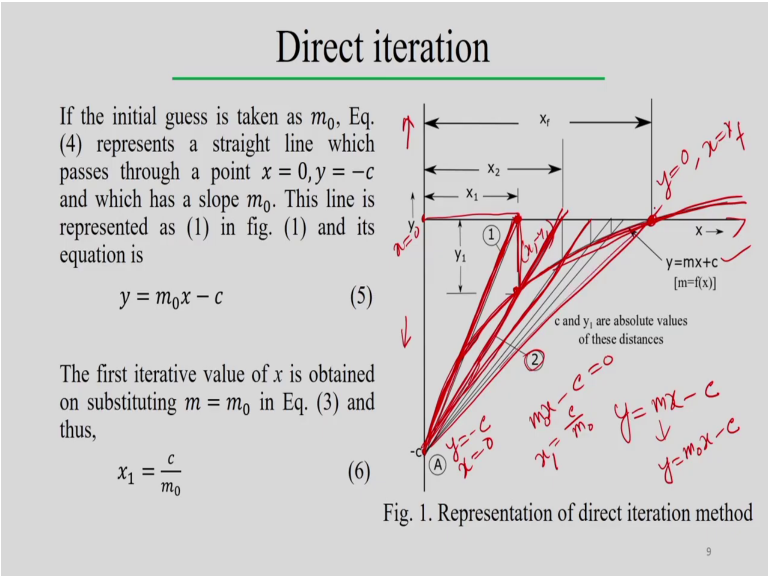

Now, graphically what we can see? We can show all these phenomena. For example, graphically, we assume that, this is the actual the curve, the non-linear curve, it is a non-linear curve, this is the curve, non-linear curve y equal to mx plus c, but y equal to mx plus c is a linear curve if m is constant, but y in this cases, m is not constant because m itself depends on the value of as a function of x.

Even whatever may be the functional form that y becomes non-linear, we assume this is the representation of the that I just highlighted this curve, this is representation of the y equal to mx plus c that is the curve, the actual curve. So, now, this actual curve and this is the y axis sorry this is the y axis, this is negative, and this is the positive and this is the negative axis and this is x, the x axis along the x direction.

Now, once we get, we reach the solution x equal to x f y equal to 0. So, we can reach. So, this point once we reach this point, this is the solution of this equation because at this point, y equal to 0 and at x equal to x f so, x f is the solution of this particular equation because we get the solution of the particular equation means once y equal to 0.

Then that is the, that condition represent the solution of the equation, but it happens y equal to 0 at x equal to x f. So, we can say x equal to x f is the solution of the equation, but how we can reach this particular solution from that particular point by following the direct iterative technique.

So, let us look into it this way. So, let us start with this thing, the initial guess for example, initial guess is taken as m 0 say m equal to m 0. Therefore, equation 4 is represented that equation 4, we can see that equation 4 y equal to mx minus c. So, equation 4 equal to y equal to mx minus c, this is the equation 4.

Now, initial guess value is taken as m 0. So, initially we assume m equal to m 0. Therefore, equation 4 represent a straight line. So, definitely, if once we define the value of m equal to m 0 assuming this a constant, then it represents the as a straight line. So, and line which passes through a point x equal to 0 and y equal to minus 3 which so, this is the point, here if you see y coordinate equal to minus c, y equal to that point indicates y equal to minus c and this is the; this is the x axis so, it is at x equal to 0.

So, at x equal to 0, y equal to minus c this represent the m that this line represent and this is at this point this is the straight line. So, in that straight line represents the y equal to m 0, y equal to m 0 x minus c and it starts with this particular point minus c. So, therefore, which passes through a point x equal to 0, y equal to minus c now, and this which is has a slope, then in this case, the it is a straight line equation so, y equal mx minus we know that m represent the slope so, in this case, the slope equal to m 0.

So, this line is represented in equation in the 1 as 1, if you see the this line represented at 1 so, this is the line and if equation is can be written y equal to m 0 x minus c. Now, the first iterative value of the x is obtained on substituting m equal to m 0 in equation 3. In equation 3 equal to we can see that equation 3 indicates that mx minus c equal to 0.

So, from here, if m equal to m 0 so, then, we can find out when we defining the value of m or making the m as a constant, then x equal to c minus m 0. So, then x is this value equation 3 and thus x 1 equal to c this. So, say assuming that at m equal to m 0, the x 1 is the solution. So, therefore, x 1 is the solution so, x 1 reach the value of c by m 0 and this straight line, it represent this thing it at this point so, it cross this point at x equal to because the x axis.

So, therefore, it is the corresponding the coordinate is x 1, this is the value of the x 1 so, because the slope crosses the line, the x axis so, that represent the value of the x 1. So, initially, we started to with the this is defining that the x equal to 0. Now, it reach to the value of x equal to x 1.

Now, but the actual solution for this case is if we consider this equation, the actual this point, it is a its basically this point where the representation of this it is a x 1 minus y 1, this point.

So, because this is the actual curve going y equal to mx plus c and this at this particular if x equal to x 1, we assume the x equal to x 1 the approximate solution of this equation, then this point is corresponds to y equal to minus y 1 that is very obvious because this in this value indicates the y 1. So, this is the negative direction of the y so, therefore, that is indicates the minus y 1.

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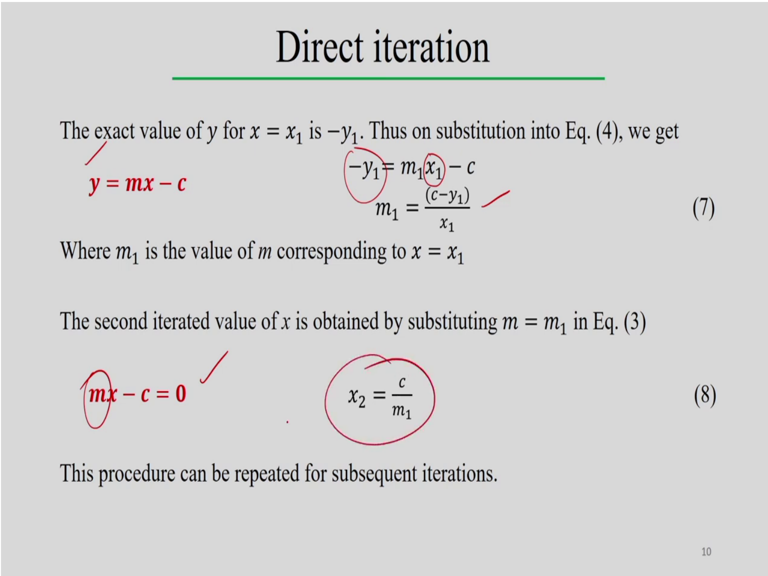

Let us look into this thing. So, therefore, the exact value of y; exact value of y for x equal to x 1 is equal to y equal to minus y 1 so, that is the point I just expressed this thing that y equals to minus y 1 at x equal to x 1. Therefore, the substituting this equation 4, we can get substitution 4 means y equal to mx minus c, there we can substituted all this value what is the value of y at this particular point? y equal to in this particular equation, y equal to minus y 1, x equal to x 1 and there is a slope equal to m 1, the slope can be different in this particular case and the minus c.

So, m 1 can be often that c minus y 1 by x 1 where m 1 is the value of the m corresponding to x equal to x 1 and that is also we can see that at x equal to x 1 so, this is the, at x this point the slope is equal to this one, the slope is changing basically in this case, the slope is changing and this slope is defined that at m 1, the so, m 1 and corresponding to x equal to x 1 that is the slope in this particular case.

Therefore, the second iterative value of x obtained by substituting m equal to m 1 in equation 3, we can find out the f equation 3, there because if we are when we are putting the equation 3, there we are trying to find out what is the value of; what is the value of x? Because this is the solution mx minus c equal 0.

So, now, we put the value of m, modifying the value of m, the recalculate such that, that x 2 equal to in this cases say m equal to m 1 slope, then x 2 equal to c by m 1. So, this is the value of x 2 at this particular point. So, this procedure can be repeated like this. So, therefore, then, the this is the 2, point 2 and then at x equal to x 2 this is; this is the point.

So, slope that there is a change in the slope and again, we recalculate the different slope and the different value, then we estimate what is the value of x 3, x 4 like that changing the slope finally, we can reach that x equal to f, this is the thing. So, we can reach this particular solution at x equal to x f, then y equal to 0, then we reach this solution. So, this is the typical representation of the direct iterative technique to solve this equation.

Now, this is direct iterative technique, we can see that this method is so, every time we are calculating the slope at this particular point and then, we can reach the solution at x equal to x f and a particular point that y equal to 0; that means, once we make y equal to 0. Then we can say we are reach we can reach to this particular solution and then, once we make the reaching the particular solution, then we can get the x equal to x f which is the particular solution.

But when we achieving this solution, it is not directly we are reaching this particular solution, but rather we can following these step, every time we are calculating the, recalculating the slope each and every then there is modifying the slope and we can; this slope is modifying. And gradually, we can reach get directly that y equal to x equal to x f in this particular solution. So, that is the direct iterative technique.

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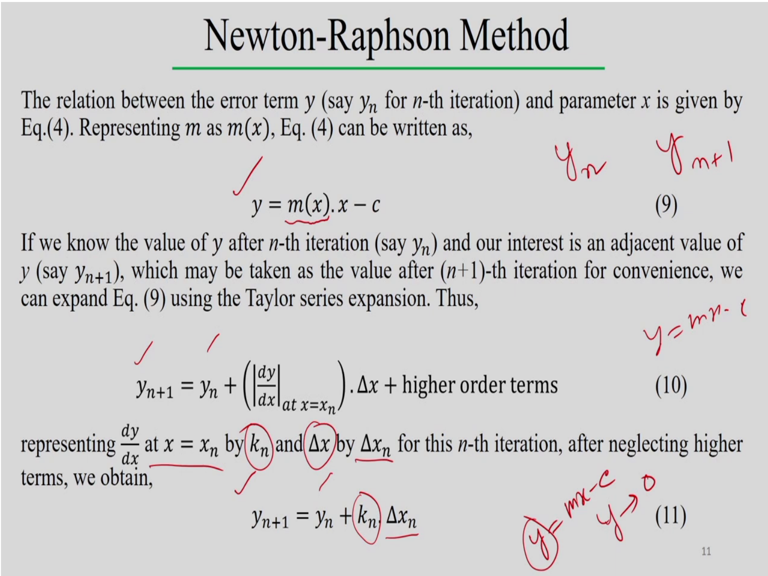

Now, we will try to look into the Newton-Raphson method. So, Newton-Raphson method it comes in the different way or so, it is also iterative technique, but we will try to observe that how it is different from the direct iterative technique. Or whether there is any advantage of using the Newton-Raphson method to accelerate the convergence criteria or to accelerate, to reach to accelerate the convergence of a particular solution that we will see.

So, therefore, we start with this thing. The relation between the error term for example, we define the error term also because once we defining y equal to mx minus c. So, then, we consider y as a error term so, then, error term becomes 0, y equal to 0, then we can reach the exact solution.

So, until y equal to we consider that y equal to error that y equal to consider as a y as a error term and then, for successive iteration, we were trying to reach towards the 0 that means, by one single step, we cannot reach the exactly y equal to 0 which tends to 0, but in successive iteration y actually tends to 0.

Then, once y tends to 0 after n number of iteration, then we can say we have reach the converge solution of this particular equation. So, that therefore, we start with this thing the relation between the error term say for example, y n for the nth; nth iteration and parameter x is given by equation 4 representing m also as a function of mx.

Then equation 4 can be written as, equation 4 we have already seen the y equal to mx minus c, the equation 4 can be written like that y equal to instead of m, we can write y equal to m because m itself as a function of x, x into x minus c. So, this is the non-linear form of the equation and we assume that y is we considered y as a error term in this particular during the iterative calculation.

So, if we know the value of y, after the nth iteration say after the nth iteration, the value of y equal to y n and the our interest is adjacent value of the y so, basically, we always compare what is the value of the y at particular iterate in number of iteration n iteration and what is the adjacent? Maybe next y equal to y n plus 1, what is the value of error such that we can compare between these two error to set some kind of the convergence criteria in a iterative calculation.

Now, so, y equal to n plus 1 which may be taken as the value of the after n plus 1 iteration definitely, for the convenience so, we convenience, the y is in the error term after n number of iteration and y equal to n plus 1, we can do the error term, after n plus 1 iteration.

So, therefore, this suppose, this is the equation 9 ah, this equation and we can consider and then using the Taylor series expansion; Taylor series expansion I am neglecting the higher order term, only the first order we consider, then we can find out that y equal to Taylor a can be written like that y equal to y n plus 1 equal to y n plus dy by dx so, the magnitude at x equal to x n, then into delta x the increment delta x plus a higher order term.

So, first order differences, we have consider in this particular case and neglecting the higher order term, then we can say that y n plus 1 can be written like that. That means, the error term exactly at the y, the value of we can see the other way also that value of this y at n plus 1 iteration is like that y equal to n; n plus this slope at this particular x equal to x n and into delta x that increment plus the higher order terms.

Therefore, dy by dx at x equal to x n maybe you can see the slope at x equal to x n, we can say that it is defined by k n and say delta x defined at x equal to x n, delta x can be represented by say delta x n for the nth iteration techniques. So, after neglecting the higher order obtain. So, therefore, simply we can say that y n plus 1 equal to y n plus this is the slope into that increment the delta x n. So, therefore, we are trying to link the what is; what is happening at the nth of the iteration and the n plus 1 iteration.

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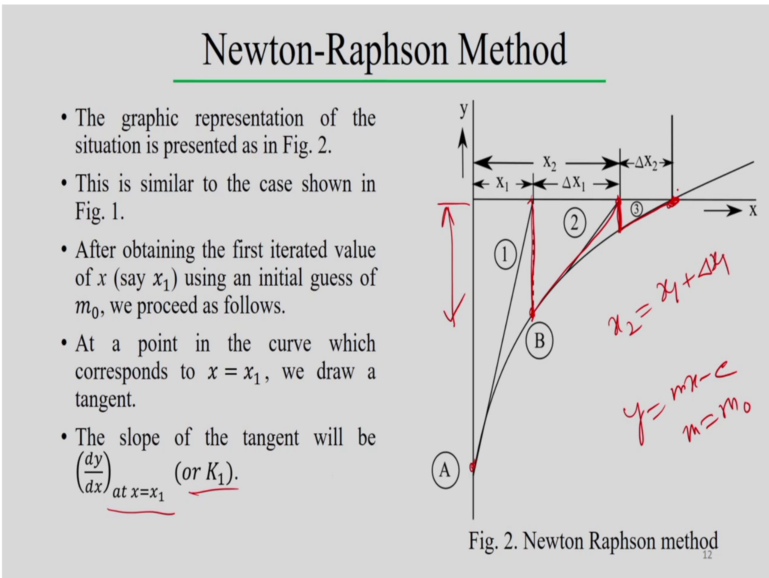

Now, graphic representation of the situation is presented in figure 1. So, this is similar to the case, already we have sown in the direct iterative technique looking into so, assuming that this is the actual curve so, actual curve with this thing and this point is the exact we assume if we the exact functional form, it is possible to define.

Then we can say that this is the exact solution of this particular equation as y is as a function of x in this particular case. So, then when y becomes 0 at certain value of the x, then we can say that we are reaching the solution point of this particular non-linear equation. So, this is the graphically represent this thing, this figure.

Now, after obtaining the first iterative value say of x, say x equal to x 1 and using an initial guess of the m 0, we can proceed the similar way. So, we started with the same thing because y equal to mx minus c, we started with m equal to initial guess m 0 value, then we solve for we estimate what is the value of y and then, at a point in a curve which correspond to x equal to x 1, we can draw a tangent.

So, this basically, this is the initial point we started this thing, this is the initial slope at this point reach these things and then, we get the solution x; x equal to x 1 and there is some error also that y equal to y 1. So, that particular this is the gap between these two. So, till there is a gap between this is the gap, this is the y axis so, this is the gap. So, to reach this.

So, so after first, after the initial guess of the m 0 value, we can reach the solution x 1, y 1 ah, but that y 1 is not exact solution of this particular iteration. So, then, in this particular case, if you look into the formulation how the from Taylor series expansion by considering only the first order term by neglecting the second order and higher term, then we can; we can reach that every time this particular calculation we have to calculate what is the value of the slope at this particular point.

So, now these, these things then, the slope of the tangent will be that dy by dx the slope at x equal to x 1 say for example, K 1, then next time once you calculate the next iteration, the iteration 2, then once we estimate the first point here, then at this point we can estimate the slope x, x equal to x 1 or y equal to minus y 1 there is particular point we can estimate the slope, then at this point we reach the this particular point. So, such that x 2 equal to x 1 plus some increment value, we can see also.

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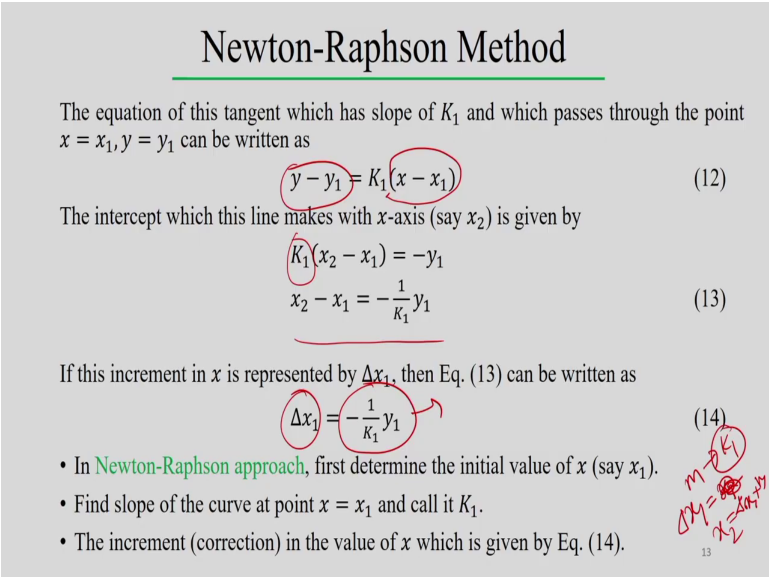

The equation of this tangent which has a slope K 1 and which passes through the point x equal to x 1 and y equal to y 1 can be written like that. So, x equal to x 1 and y equal to y 1, it passes through that, then y minus y 1 and x minus x 1 ratio, it indicates the slope K 1.

So, this intercept which this line makes x axis say for example, x 2 is given by this thing. So, from here, we can see the K into K 1, this is the slope and K 1 x 2 minus x 1 equal to minus y 1, the slope from there, x 2 minus x 1 we can find out in terms of the K 1 so, x 2 minus x 1 equal to 1 by K 1 into y 1.

So, now, this increment between x 2 minus x 1 can be considered as the delta x 1. So, therefore, equation 13 can be rewritten like that. So, delta x 1 is the this is the increment is some 1 minus 1 by k 1 into y 1. So, you can see the k 1 is define the slope at the point 1 during this thing and y 1 is defined this thing from here also, we can estimate this is the value of y 1 sorry this is the value of y 1. So, from there, we can estimate what is the value of delta x 1.

So, now once you reach the delta x 1, then this is the x 1 plus delta x 1 then we can find out this is the so, actually the to reach x 2 so, what is the value of x 1? Started plus delta x 1 that indicates the value of the x 2. So, then, x 2 we can reach this point again, we can what is the value of difference the y, the slope at this particular point this thing, then we can reach this directly solution ah, then again x 2 plus increment delta x 2 and x 2 plus delta x 2, then we can assume that it is the x 3 value.

So, in general, in representative way that basically we are reaching the this three; three steps to the exact through the solution of this particular equation. So, in that sense, I can say the Newton-Raphson method of course, in this approach, first determine the initial value of the x say x 1, then from that find the slope of the curve at particular point.

Once you determine the x equal to x 1, then find the slope of the curve at so, we start with the initial guess, find that the slope at this particular initial guess so, x equal to x 1, then estimate the call it as a K 1 so, basically, K 1 indicates the slope at this particular point, then we estimate the incremental value, the correction. So, incremental value can be delta x 1.

So, at x equal to x 1, we define the m, they are basically slope from m slope or maybe K 1, we can define k 1 once we estimate the slope, then we can estimate what is the increment value delta x 1 so, the initial guess plus I think from here, we can estimate the delta x 1. So, then, once we get the delta x 1, then we reach, we can consider the next point x equal to delta x 1 plus x 1.

So, that is how we can reach the next point. So, therefore, the increment, the correction value of the x which is given by the equation 14. So, equation 14, we can estimate the incremental value and we can find out the value of the x.

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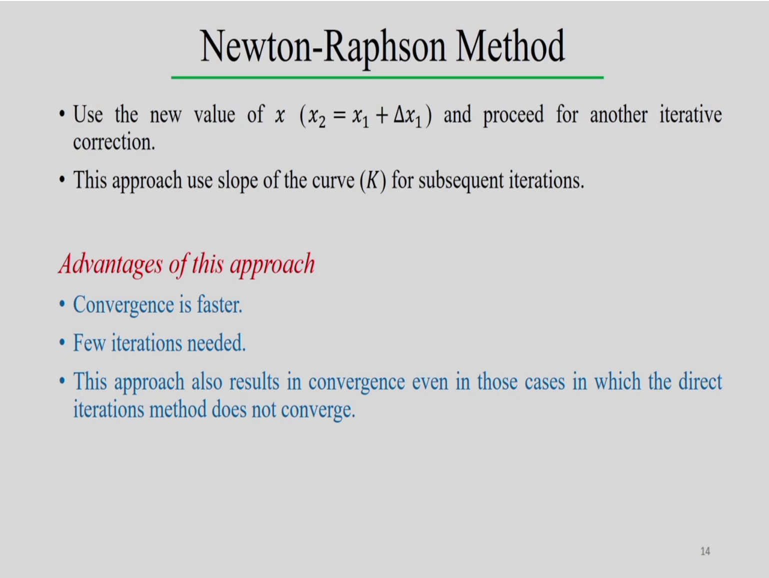

Use the new value of the x so, then, x equal to x 1 plus delta x 1 and proceed with another so this that way successive way, we can improve the value of the x; x 1, x 2, x 3, x n this thing and every time each and every point, we have to estimate the slope at the particular point x in the successive iteration.

So, once we reach this particular two successive iteration, we can compare, then all we can put some convergence criteria, then we can reach the solution of this particular equation, but what is the advantage of this particular process as compared to the direct iterative technique?

So, in this case, the convergence is very fast, so, very quickly we can reach the convergence as compared to the direct iterative technique.

But, direct iterative technique, the implementation of direct iterative technique is very in other way, it is the very simple the simpler you can calculate this thing or, but in this case, Newton-Raphson method every time we have to estimate the slope at this particular point so, that is not there in the direct iterative technique.

And few, but other way also, very few iteration is needed in case of the Newton-Raphson method. So, quickly we can reach the up to the solution. So, this approach also results in convergence even those cases in which the direct iteration method does not converge, in some cases also when the direct iterative technique may not converge this, but we can use the Newton-Raphson method to get the converge solution of a particular problem.

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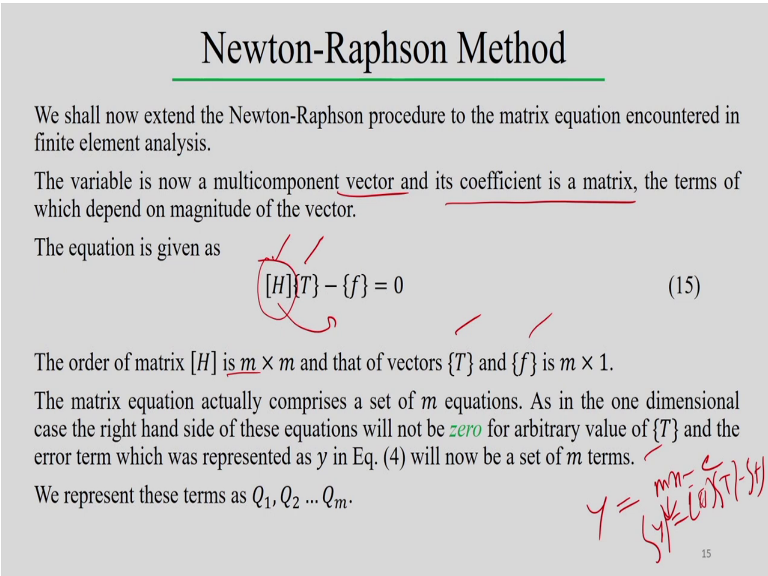

Now, we shall now extend the Newton-Raphson procedure to the matrix equation encounter in the finite element analysis. So, now, finite element analysis, what we can encounter the Newton-Raphson method? In this case, the variable is the multicomponent so, definitely vector and its coefficient is a matrix that we can see the multicomponent vector T and it is a coefficient matrix is the H.

So, T is having so many values. So, therefore, terms which depends on the magnitude of those vectors that means, even H itself it depends on the magnitude of the vector. So, therefore, equation H T minus f, we have already defined this equation.

Now, the actual finite element solution or finite element method, the order of matrix is assuming m H equal to say m into m by 1 m into m so, m by m. So, it means that m indicates the number of node point and that the vectors T and f equal to m by 1. So, once it is it is defined m by m, other vectors T and f is the m by the dimension of m by 1. So, therefore, the matrix equation actually comprises of basically set of equation that we have linear set of the equation we can find out.

As the one-dimensional case the right-hand side of these quantities will be 0 for arbitrary value of the T so, that is the case and the error term which is represented by the y so, y is basically represented that we have already minus c.

In this case, m is replaced by H T minus f so, that we can define the error term and we represent this will be now the set of the m terms also even if we consider the error term, we introduce the y equal to say H T minus f and that is the y so, it is a suppose as a in the vector form. So, that indicates the error term. So, we represent these terms as a Q 1, Q 2, Q m something like that.

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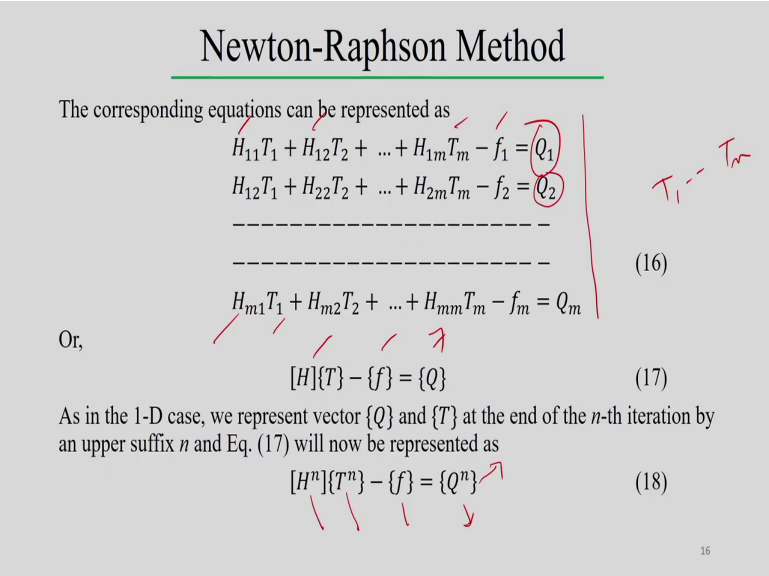

Now, the corresponding equation can be represented like that say H 11 into T 1 so, T 1 is this thing, H 12 into 2 so, different is H term f 1 equal to Q 1, Q 1 the error term, it means that if it is not 0, that is not the exact solution of these things of this particular equation. So, this is equivalent to y, maybe we can assume like that.

So, similarly, this set of equation we can form also H m1 T 1. So, T 1 to T m is the variable that is defined each and every node point. So, therefore, once we get this variable and then finally, we can from this equation H T minus f equal to Q, Q represents the error term, or we can say that equivalent to u.

Now, as in 1-D case, we represents the vector Q and T at the end of the nth iteration by the upper suffix n and equation 17 will now be represented as like that also. So, therefore, at the nth iteration, this H n T n minus f equal to Q n. So, therefore, once we look into the non-iterative solution.

In this case, we can take this in the matrix form and similar strategy what we have just described in case of the Newton-Raphson method or direct iterative technique, we can form the every time this error we can calculate the error a Q n and we just try to minimize this value of Q n and such that this error will always try to tends to value of the 0 or a very small value.

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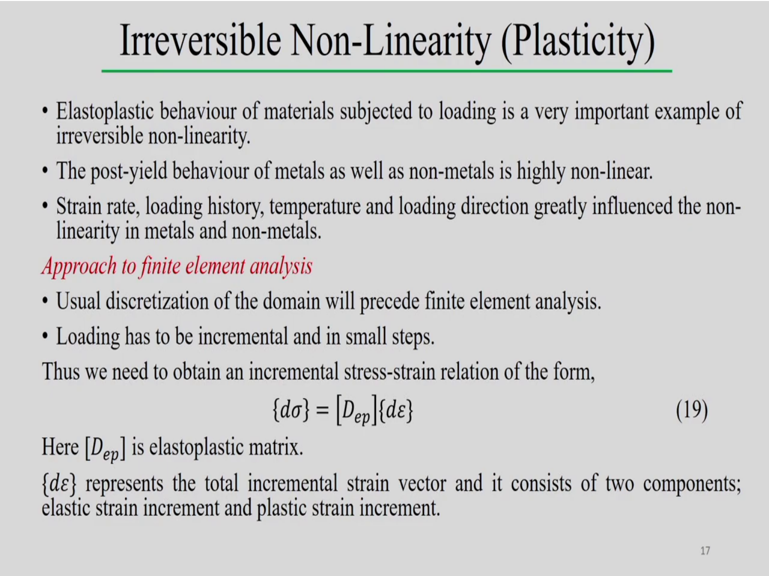

The in that cases, definitely, we will try to reach the converge solution of a particular, but it is not possible to achieve these things in a single iteration. So, in that cases, there is n number of iteration is required and it is also possible both way also, we can follow direct iterative technique or we can follow the Newton-Raphson method to achieve the converge solution of a this particular problem. In this particular context, basically we can consider the heat transfer problem where the each and every node point, there is a only ones degrees of freedom is defined that is the temperature.

So, thank you very much for your kind attention. I think that is all for today. Maybe next class, we will discuss the other part irreversible non-linearity, maybe plastic irreversible non-linearity, we try to look into in more much details and their application in the different problem using the finite element method.

So, thank you.