Finite Element modeling of Welding processes
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Module – 02
Fundamentals of finite element (FE) method
Lecture – 11
Weighted residue technique

Good afternoon everybody. Today we will discuss the different aspect of the Finite Element Method, last class what we discussed that we develop the stress analysis model the relation of the stiffness matrix the formation of the stiffness matrix for a particular element. And we can see all that stress analysis part we normally use the constitutive relation between the stress versus strain and the stress strain versus displacement.

So, we use some relation and by following the potential energy minimization we finally, develop the formulation of that we bring the stiffness matrix and we solve for displacement field. So, that was the general approach in case of the stress analysis model, but in any kind of the physical process or maybe that can govern some kind of the physical law also.

For example it is a heat transfer problem, in any kind of the manufacturing process that it may associate with some kind of the heat transfer. So, once we looking into the heat transfer or we are interested to know what is the temperature distribution, then we should know what are the governing laws or what are the governing equation basically drives this kind of the physical mechanism.

Or maybe that is why when any particular we try to solve in general the heat transfer problem. So, associated with the boundary interaction or boundary condition, then governing equation along with the boundary condition there is a need to look into the some techniques so; that means, we normally use the weighted residue technique to find the finite element based formulation.

So, today we will discuss that part, how we can using the weighted residue technique we can do finite element based formulation in the cases where there is a governing equation is there, as well as the mathematical interpretation of the boundary condition is also there. So, in that case we look into that how the finite element formulation can be done.

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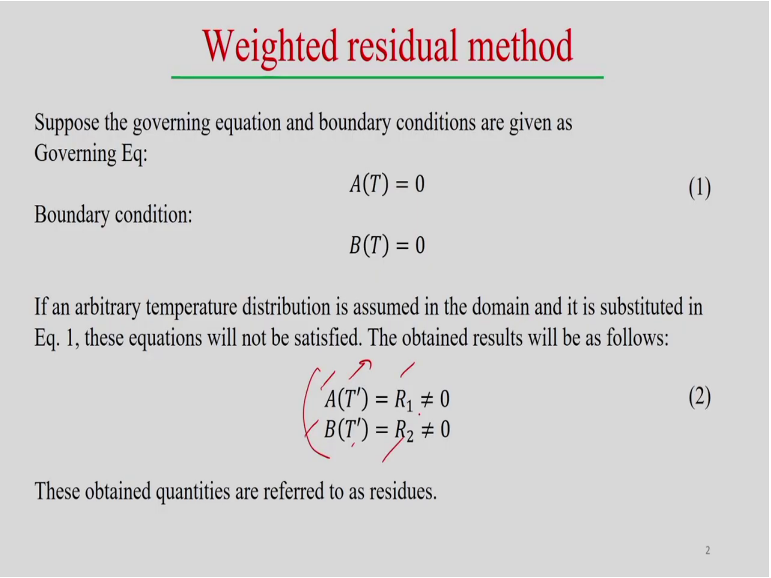

But before that what is the weighted residual method or weighted residue technique we normally use in any kind of the finite element formulation?

Suppose the governing equation and boundary condition represented by these two different form of the equation. So, equation 1 we can see that A the equation the functional form of a governing equation in general and B that is the functional form of a boundary condition. And both as a function of the T is the basically the solution variable we can say for example, heat transfer problem it is a it is equivalent to the temperature.

So, once we get these two equation what is the governing equation, what is the boundary condition for this particular problem? Then we assume that some arbitrary temperature distribution in for example, in this case we assume some arbitrary temperature distribution in the domain. And that arbitrary temperature distribution we consider as a solution of this particular equation.

By assuming that if it is substituted in this equation this particular equation in this case is it is equation 1, then this equation will not be satisfied may not be satisfied also. Because, in this case we do not know the exact solution, but assuming some arbitrary solution and then that arbitrary solution we are fitting to the equation we are just try to fitting this arbitrary solution equation it is not necessary it will satisfy this particular equation.

So, therefore, in the mathematical form we can write the obtained results once we put we assume this particular solution of this equation. Then put this mathematical form of the equation we can see that A T dot; T dot we assuming this is the solution of this particular equation. So, it should satisfy, then or B is the this is a boundary condition and it is a the solution is the T dot.

So, then both should satisfy if it is the exact solution, but we do not know the beforehand we assuming some solution that is why it may not satisfy. So, once we put this thing this there exist an some sort of the residuals. So, that is the quantities called R 1 and R 2 residuals and in the mathematical form it is not equal to 0; that means, residuals some residuals are there; that means, definitely it is not equal to 0.

If it is equal to 0, then our assumption was correct and we are getting the solution of this equation by assuming this particular solution, but it is not possible to get this solution in general. So; that means, we arbitrarily by assuming some arbitrary solution if we want to satisfy this particular equation there must be some amount of the residuals must be there and that residuals can be represented this way R 1, R 2. So, then this equation two is represented like that.

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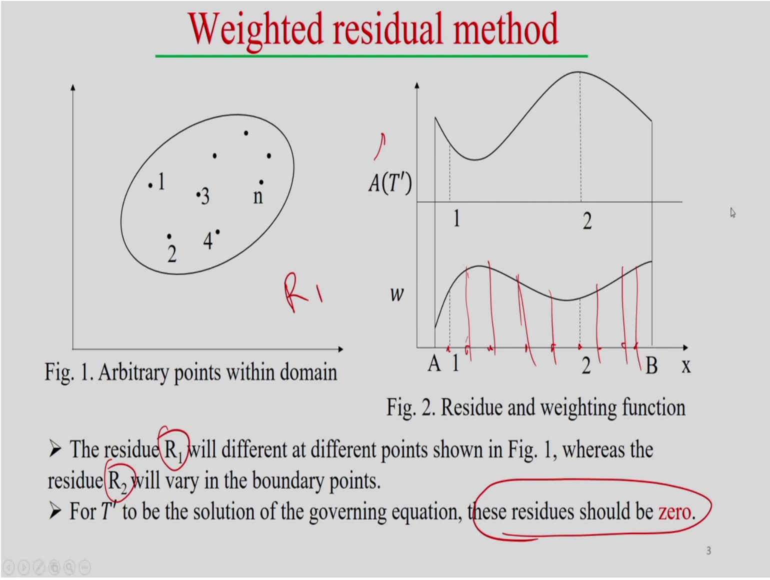

Other way also if you look into this figure 1 that arbitrary points within the domain. So, that so many arbitrary points are there 1 2 3 4 up to dot dot n.

So, this residue R 1 will different so; that means, that this different at the different points R 1 value can be different at the different points shown in the figure. Whereas, the residue R 2 will vary on the boundary points actually, the R 1 residue R 1 the residual residue is defined within the domain, but R 2 is thus the residue existence of because on the boundary. So, therefore, R 2 will vary on the boundary points.

So, but it is not necessary R 1 can be fixed value, so R 1 can vary the residue can vary at the different points. So, in even we are looking in the from the point of the finite element base method, because in finite element what we do we just discretize the domain; discretize the domain means we create the elements small small elements within this particular solution domain. And we are trying to look into what is happening the this looking into the residual in the particular node points.

So, that is why different different point 1 2 3 is the domain that we assume that we can it indicates the kind of the node point in this domain. And we are trying to satisfy this equation with the particular defining we are defining the equation in the node points. And based on that we can say that when you try to satisfying some arbitrary solution of this particular equation, then we feed the value looking into what are the values what looking into what are the residuals is exactly on the different node points.

Now, this T dot, for T dot for we are assuming T dot is a arbitrary solution for this particular equation governing equation and boundary condition as well. So, T dot to be the solution so; that means, T dot to be the solution of this governing equation. So, then if it is not T dot it is exactly the solution of this governing equation then definitely there may not be any kind of the residuals, so residue should be 0; so then in that cases residue should be 0.

If T dot is exactly satisfying this equation and if each and each and every node point then that is the exact that, when solution it is possible to that we consider this T dot is the solution of this particular equation, otherwise there exist an some sort of the residuals. So, from that concept we can finally, analyse these things.

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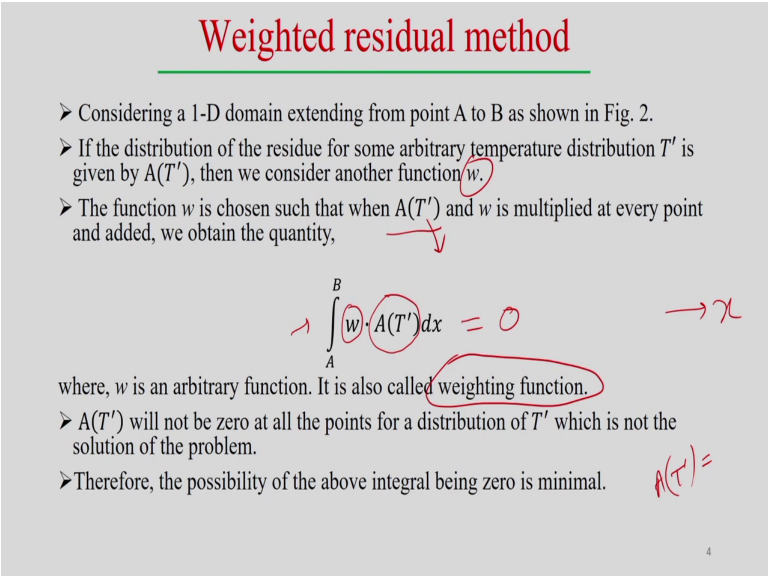

So, now to understand these things how this residual weighted residual technique works on that we assume some one dimensional domain. Extending from point A to and point B we can see that one dimensional domain and this is a x direction one dimensional domain and A T dot is the this functional form T dot is the solution this particular case.

And w is the weighted residue again weighting function and this we discretize this domain in the different thing or maybe that small elements. So, it is a one dimensional element such that we consider, what is the this discrete point as basically node point we are looking into the solution or we are looking into the residuals for this particular equation.

Now, if the distribution of the residue then we just distribution of the residue means in the discrete point, what are the value of the residue? And this distribution we consider for the for some arbitrary temperature distribution T dot is given by A T dot; that means, this functional form A T dot the particular distribution of the T dot or a functional form of this A T dot we are assuming this is the solution of this equation.

Then we consider the another function. So, in that case we introduce the another function w which is called the weighting function. Now, this function w is chosen in such a way that when A T dot; that means, this is the T dot is the solution for the particular equation and w is that the another function if we consider and w is multiplied at every point and added we obtain the quantity.

So, integration of this thing A to B, but in this case this is the weighting function and this is the actual assuming the T dot is the solution arbitrary solution of this governing equation and this is the functional form of the A T dot. So, this weighting function and multiply this A T dot this is one dimensional problem, so one only the variation along x axis.

So, therefore, d x integrand of this we can edit this quantity we can represent this thing. So, this where w is the also arbitrary function that weighting arbitrary function it is called the weighting function. So, w is called the weighting function. So, to satisfy this the solution or to bring some kind of the solution of a particular equation we just multiply with another arbitrary function that is called the weighting function. And in the form of a integral this thing, but definitely this where integration, but this integral part is defined or maybe we can look into the numerical integration of this thing.

So, in that integral form we can represent this multiplying weighting function and the actual governing equation for that governing equation we assuming some solution arbitrary solution. Then A T dot will not be 0 at all the points, definitely that this functional form A T dot the A T dot is basically it is in if you look into the analytical solution it follows some kind of the distribution or some functional form it follow.

So, therefore, A T dot will not be 0 at all the points for a distribution of T dot which is not solution of the problem. So, if A T dot is 0 each and every point; that means, each and every we looking into the for one dimensional problem each and every node point if it is equal to 0 then this is the solution, but if it is not equal to 0 then this is not the solution of this particular equation.

So, therefore, the possibility of the integral being 0 is minimal. So, the chance's is very that being the 0 of being the 0 is a minimal. So, that is why we multiply this thing the weighting function in the form of a integral.

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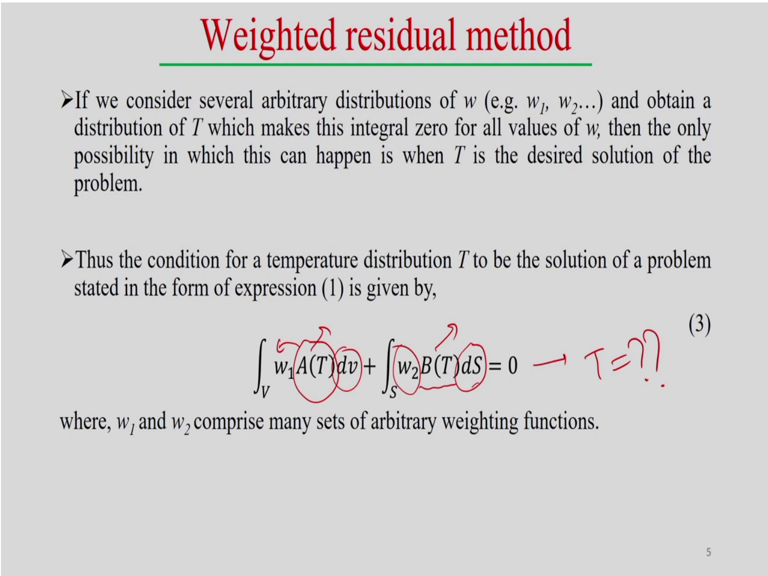

Now, if we consider several arbitrary distribution of the weighting function w. So, different form of the w several arbitrary different types of the different functional form of the weighting function and w 1 w 2 or maybe different weighting function at the different discrete points.

If you consider and obtain a distribution of the T which makes this integral 0 for all values of w then only the possibility we see in which this can happen when the T is the desired solution of the problem. It means that in other way we can see that define and if it is equal to 0, then it indicates that it means that the by putting the weighting function we choose the weighting function in such way that it becomes 0 then we can say that T dot is the solution of this particular problem.

So, we looking into this thing desired solution T dot, T is the desired solution of this particular problem and we assume that T is the desired solution of the particular problem the other way also. So, whatever functional form we can see that solution of this particular equation and this we can obtain with the multiplying of the different weighting function. So, therefore, thus the condition for temperature distribution.

So, basically we are looking for the temperature distribution T the that distribution to be the solution stated in the form of the expression of one expression of the equation one we can look back to the equation one also that this is the expression of the.

So, this A T equal to 0 means, T will be the solution of this particular temperature distribution if get the T solution if this is the this multiply by the weighting function w 1, but this equation one means both are ah when you are talking this kind of problem both it is important to consider the governing equation in the domain of the solution domain as well as the what are the boundary condition is important and combined together we can define the in a particular problem.

So, that means we have giving some weightage weighting function introduce some weighting function which is multiplied by the governing equation and this since the governing equation is valid over the volume over the domain then whole domain; that means, over the volume. So, that is why we put the d v plus integration we put the another weighted function w 2 it is not necessary weighting function can be same also can be different also, but that different weighting function if we consider w 2 and B T represents the boundary condition.

So; that means, it is a boundary interaction. So, which is defined over the surface because boundary is represented with three dimensional problem, boundary is represented over the surface. So, then this weighting function into this boundary condition mathematical form of the boundary condition assuming that T is the solution and what satisfying this should equal to 0.

It means once this satisfying a condition this particular condition equal to 0 then we can say that T from this expression we can get the T is the solution; that means, temperature distribution in this case is the solution for this particular problem. So, that other way we just in this case. So, w 1 and w 2 comprise of the many sets of arbitrary weighting functions. So, there are so many possibilities of defining the what is the w 1 and w 2 or both can be the same also.

But, finally in the mathematical form when you are multiplying the governing equation boundary condition with some weighting function do doing the integration which is integrated over the volume for the governing equation, and integrated over the surface area for the boundary interaction. Then the whole system equal to 0 then we can assume the by solution of this equation is the from here what is the temperature distribution that is the solution of this particular problem.

So, in the other way by introducing the weighting function we can form the mathematical form of a any particular problem. And which is basically looking into that particular problem and then we can formulate the by using the finite element method any kind of the problem, which is governed by the one particular governing equation and along with the boundary interaction or boundary condition.

Now, we can see w and w 2 may be have different sets or different weighting function there are so many possibility are there for w 1 and w 2. How looking into that the so many possibilities of w 1 w 2. And I think that is not discussing what are the mathematical aspect the what are the different forms of the weighting function and then what are the limitation and all these things.

And how we can form this weighting function that is not beyond the scope of this particular course rather we can focus on the very simplified way to understand the concept of the weighted residue technique in this particular case and then how this what way we can put the weighting and more in the we can define the heat transfer problem associated with the welding process that is the focus of this particular course.

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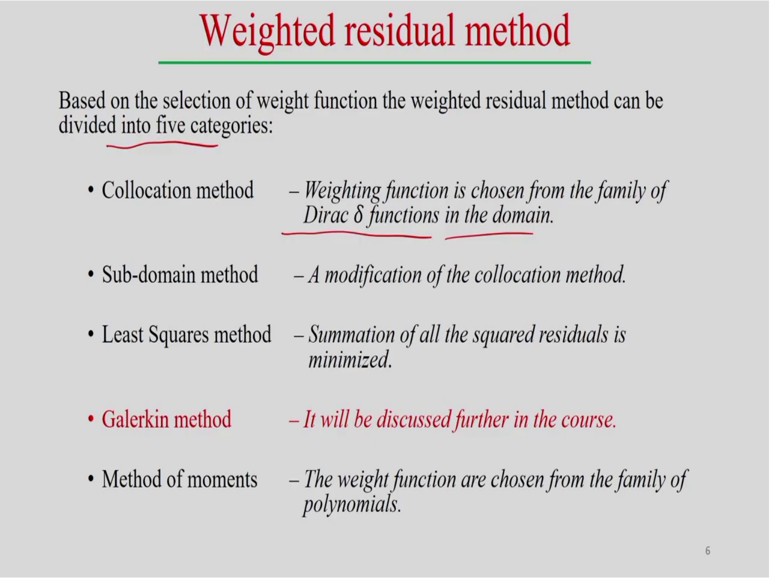

So, therefore, based on the selection of the weighting function and there may be different techniques weighted residual method can be divided into 4 5 categories. You can see there are in general there are 5 categories problem can be divine defined based on the different nature of the weighting function.

So, one is the collocation method, collocation method the weighting function is chosen from the family of the direct delta function in the domain. So, it is a basically simply in general sense we can say that this is the this method is basically the particular weighting function we can choose and accordingly this method has been developed or this method is normally used to get the desired finite element base equation or may be the to defining the problem using some governing equation boundary condition.

Similarly, sub domain method it is a simply some modification of the collocation method is required in this particular cases. Even least square methods also in this case the summation of the all the square residuals.

So, we estimate the residuals so summation of the all the residuals; that means, we putting the weighting function for a particular node point and this governing equation though we can assume some solution. Then multiply this thing summation of this thing all the residuals and square root may be summation of all the residuals and square of them can be minimized to get the finite element formulation for this using this least square method.

So, that is the methodology is beyond our scope of analysis in this particular course. Then Galerkin method, so Galerkin method basically we will be discussing and we will be for this particular courses and based on this Galerkin method what we can develop the different finite element formulation of the associated the welding process. And that is the main focus of this course we will discuss later on this thing.

Then method of movements in this cases the weight function are chosen from the family of the polynomial. So, from the family of the polynomials the weighting function can be chosen and based on that the method movements ah techniques has been developed.

So, we can see that all this methodology and with weighted residue technique in general sense we can say that how we can choose the this thing, how we can choose the weighting function and what way we can minimize the residuals and based on that the different methodology different techniques of the finite element formulation has been developed.

Now, let us look into what is the Galerkin method which we are supposed to use in this particular course.

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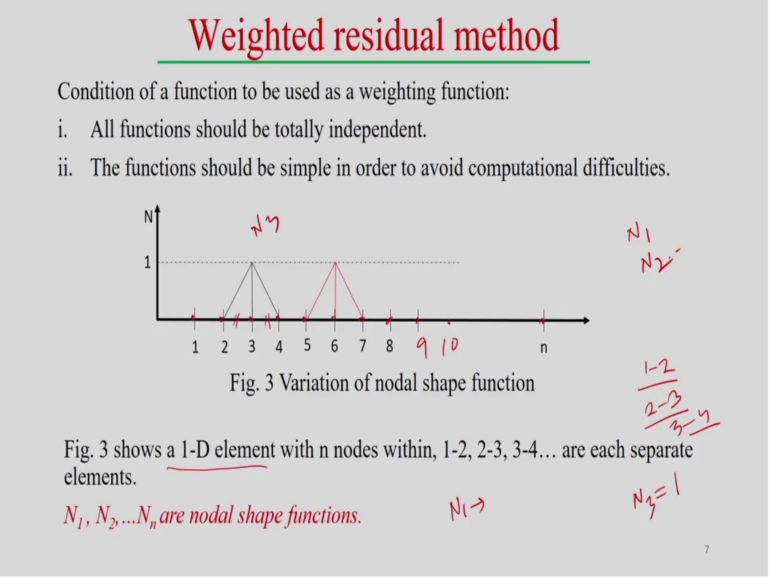

Now, condition weighted residual method condition of a function to be used as a weighting function, two things are there two conditions are there that what are the as a, which functional form can be used as a weighting function? One is the, first point is the all function should be totally independent; we have to choosen the weighting function in such a way that all the functions should be totally independent.

And second thing is that the function should be very simple. So, in order to avoid any kind of the computational difficulty; so, it is a definitely there are lots of possibilities to choose the different form of the functional form, but it is as simple as possible other such that we can avoid some kind of the computational difficulties.

So, this case you can see the figure before going into the details of this analysis we understand we should understand these things in a particular 1 D element and suppose there are n nodes and there are so many elements n. I think if there are n nodes I think there are N minus 1 elements will be there are each separated it is separate elements. So; that means, this is the node number 1 2 1 and 3 node number 4 5 6 7 8 9 like that 10 like that these are the different node numbers and this is the node number n particular n number.

Now, 1 2 this indicates the one element in a one dimensional problem. Similarly 2 3 is the another element 3 4 is another element like that. So, this way we can define the different elements nodes also in a one dimensional problem. But, we can show the variation on shape function we have seen we have already discussed that shape function value even if we look into the expression of the shape function in the that we can see that where from shape function from one particular associated with node 1. So, we can use the several shape function N 1, N 2 something like that for a particular element we use the different shape function.

So, if one element they are having the tetrahedron element in that cases there are 4 nodes so we can find out that there are 4 shape function N 1, N 2, N 3 and N 4 that we have already discussed in the last class also. Now, in this case we can assume there is a 2 1 element and there is a node 1 element have associated with the 2 nodes N 1 and N 2. But on the particular case this if it is a N 3 that node N 3 value of the shape function will be 1 for this particular node, but its 0 corresponds to the other surrounding zone 0.

So, therefore, N 1, N 2, N n are the nodal shape functions. So, that is why this shape function we can say the nodal shape function because we are N 1 which basically one is the node number so accordingly we can define the shape function also. So, N 1, N 2, N 3 this all represents the these are the nodal shape function. Now, we will see how it works in this case.

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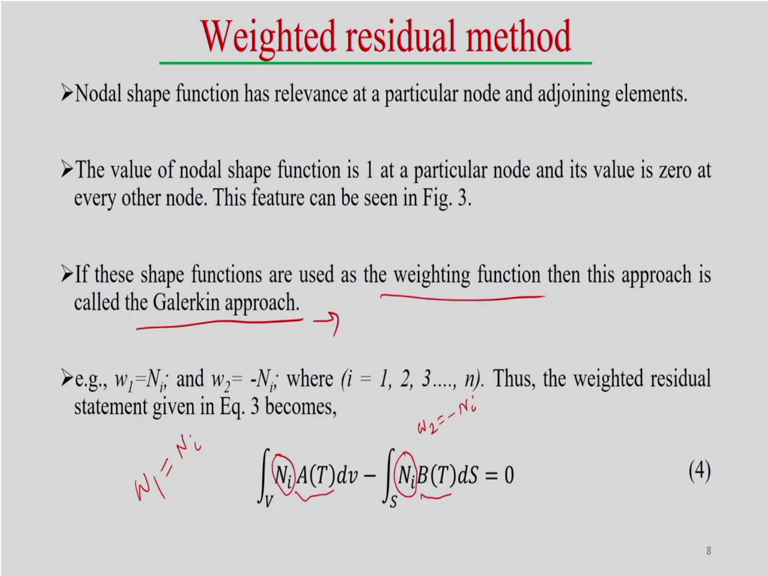

Now, nodal shape function has the relevance at the particular node definitely nodal shape function is a particular node and the adjoining elements. So, one particular node and the element.

For example in this case this is the particular node N 3 equal to 1, but adjacent elements for these cases are the adjacent node 2 3 and adjacent element 3 4. The at the node, at the node value at 4 basically node point 4 equal to 0 for this particular case, shape function will be 0. So, the value of the nodal shape function is 1 at a particular node and its value is 0 at every other node.

So, we can see that for the particular N 3 value N 3 equal to 1 for this particular node, but it is 0 the shape function the nodal shape function value equal to 0 for other nodes that is very obvious also ah. Now, if this shape function can be used as a weighting function then this approach is called the Galerkin method that is to if this shape function are used as a weighting function.

For example we have seen that that when you try to establish the weight weighted residue technique we have to choose some weighting function. So, particularly the Galerkin weighted residue technique; that means, in Galerkin method we normally use the weighting function as a shape function. Same shape function if you use then this approach is normally called the Galerkin approach.

So, is the same the weighting function is the same as the shape function. For example, in this case if we consider the one weighting function associated with the governing equation as a N i. So, N i is the weighting function is the N i, i indicates the 1 to n its basically indicates the node ah; basically indicates the node point. Now, w 2 that means, another weighting function it can be used the also minus N i.

So, if we consider the governing equation in such a way that it is a plus N i; that means, shape function similarly weighting function w 2 can be considered as a minus N i. So, N i corresponds to the basically the nodal shape function for each and every node point particular node point.

Thus the weighted residual statement is given equation becomes that, if you see that this is the weighting function and this is also weighting function. So, this weighting function w 2 we considered the minus N i. So, we simply put the N i here and this is the governing equation this is the boundary condition and because it is defined over the surface and this is over defined over the volume.

So, multiply by the shape function and this equal to 0 in this case once we establish this equation and the in the integral form. It means that once we get the solution if we find out what is the temperature distribution from here then that indicates the solution of this particular problem or we can say this is the solution of this particular problem by just putting in the weighting function. So, that is the importance of the putting some kind of the weighting function in a finite element analysis.

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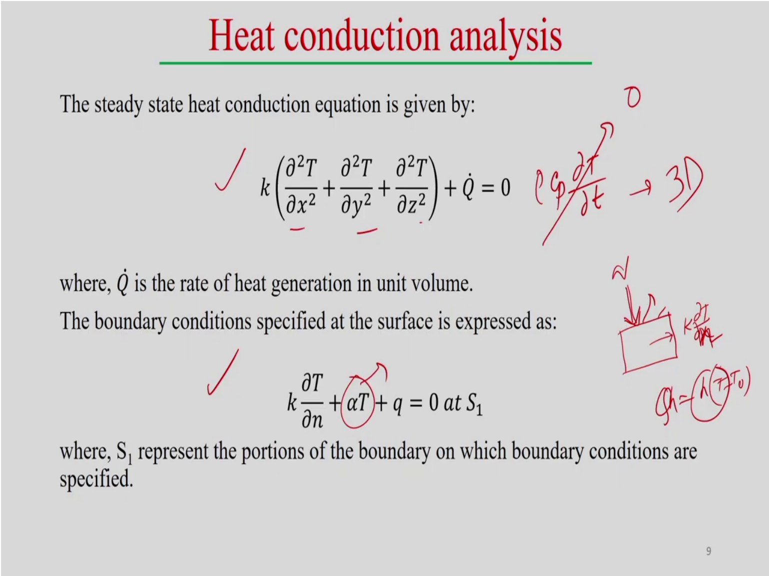

Now, look into this particular problem the heat conduction analysis. So, we assume the steady state heat conduction equation we know the steady state heat conduction equation and k thing k into del T by del x this thing Q dot and the, I think I equal to rho C p del T by t.

So, basically this term equal to 0. So, then there is no time component here in this particular governing equation. So, we can say this is a steady state equation just simply neglecting or maybe make it equal to 0 because temperature variation is not independent of the time component. So, that is why it becomes the steady state heat conduction equation.

So, this is the steady state equation and it is also the x y z; that means, we can say the it is a three dimensional steady state heat conduction equation where Q dot is the rate of internal heat generation a per unit volume.

So, this equation what way we can see that how this is we assume this is the governing equation any kind of the heat transfer problem, it can be associated with the welding any manufacturing process or it can be associated with the some other problem also. But in general this is there is a heat transfer problem we consider the Fourier heat conduction equation or three dimensional heat conduction equation in the steady state condition.

But the boundary condition specified at the surface so; that means, in a welding problem we can see the boundary condition. So, suppose this is the domain and we have seen the boundary condition in a welding problem also it is as in general the heat is conducted a on the surface that indicates the k into del T by del n that normal to, n equal to n vector represents the normal to the surface. So, it is defined on the exactly the heat conduction, but that is defined exactly on the surface.

Alpha T is basically maybe heat loss by convection and radiation from here, and maybe other surfaces and the q is the basically heat flux, the impose there may be some in particular problem there may be the heat is applied heat flux is applied to the surface and that is the q in general. So, q is the heat input to the surface and in general I am talking about and alpha T is the heat loss though h equal to h sorry the heat loss by convection per unit area T minus T ambient temperature. So, basically h T in general alpha T we can say in the generalized form that alpha T is the variable in this case.

So, therefore, it is a general form of the boundary interaction normally associated with the heat conduction problem if you assume that and that is defined only on the surface S 1. So, let us see let us look into this all this defined over the surface S 1 and this boundary interaction. Now, S 1 represent the portion of the boundary on which boundary conditions are specified. So, it is not necessary the whole boundary is subjected to some sort of the boundary interaction.

So, it is a particular boundary interaction may be possible even some other boundary condition can be imposed in the other part of the boundary. So, we assume that maybe we assuming that it is a small part of the boundary the this boundary condition in general form of the boundary condition is there.

So, therefore, we can see this is the governing equation this is the boundary condition. So, from here what we can develop the using the application of the Galerkin residue technique and we can formulate the heat transfer problem.

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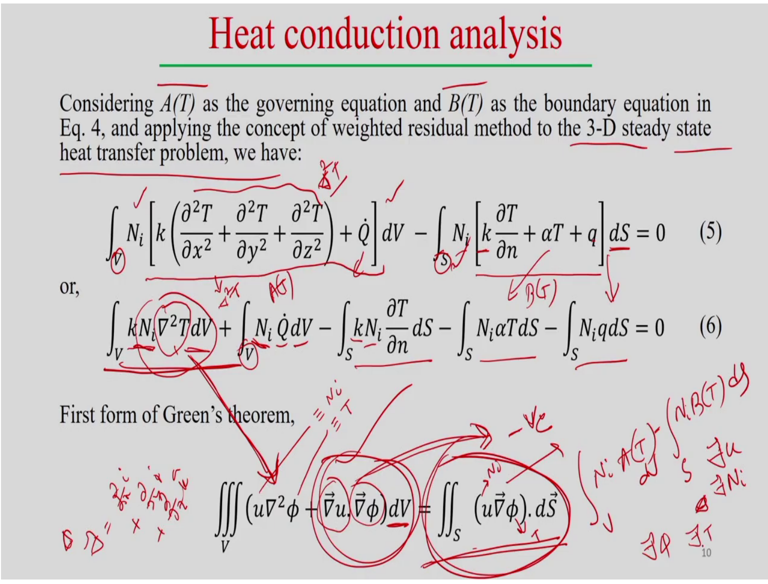

Now, considering A T if you look remember equation 1 A T is the governing equation and B T we consider the boundary condition the T is the is the functional form T is the as a function of T we can say like that. As a function of T means as a function of temperature this is the functional form. As the boundary equation in equation 4 and applying the concept of the weighted residual method to the three dimensional steady state heat transfer problem, so three dimensional heat transfer problem.

So, this is the if you see that A T this weighting function N i, other is the B T and weighting function N i and this is the plus and this is our volume over surface and this is d v it is d s. The same way we have considered, so this is the governing equation up to Q dot this third bracket within the third bracket N i is the weighting function as the same as shape function over the volume integrand over the volume and this is the integration over this volume the volume integral.

So, now similarly for boundary interaction this is the B T in this case and this was A T this case and this is the B T and this is the shape function surface. Because boundary interaction over the surface and minus shape function we have defined. If it is a plus for the governing equation the minus for the boundary interaction. So, this becomes 0.

Now, this is the formulation this thing in that cases the what are the temperature distribution we will be getting, then this is the solution of this particular equation. And of course, in this case we are using the we following the weighted residue technique. Now, this look into the individual elements for this particular equation.

So, this can be represented also this equation can be represent also 2 into T the differences are del to T gradient in if you look into the gradient divergence color in the vector form we can represents this second del 2 T by del x square plus delta 2 T by del y square they are simply represented by this equation can be written the in general sorry 2 T.

So, that is T is the governing equation here. Now, k we just keep it outside k. So, we assuming this in k the thing N i is the variable and d V is the volume over the volume. Second term this Q dot term Q N i Q dot d V over the volume, this without expanding this different elements of this particular integral.

Then N k, k is the this is the from boundary interaction N i del T by del n into d S. Second term is coming like this N i alpha T d S over the surface, third term it comes like that it is also N i q d S this is also over the surface.

So, from here now we apply first form of the Green’s theorem. So, Green’s theorem is simply we can use the relation from the volume integral to the surface integral in that formula. From here you can see that we apply the Green’s theorem here because in this case from this term we can use the Green’s theorem.

So, this term is equivalent to this term and then from here we can see that this Green’s theorem over the volume thus volume integral to conversation from volume integral to the surface integral. So, if we apply the greens theorem here, then this part will come to this side, but it is will be negative sign. And the another surface components will be there another this if you see over the volume. So, this is volumetric component will be there on the surface components will be there if we apply the Green’s theorem for this particular part.

Now, see from here we can expand by Green's theorem that two terms will be there. So, this will come other side; that means, this term will be there as well as this surface term and this volume term will be there. Now, looking into this equation how this can form. Then this term terminology the first term from here we can converted to this term.

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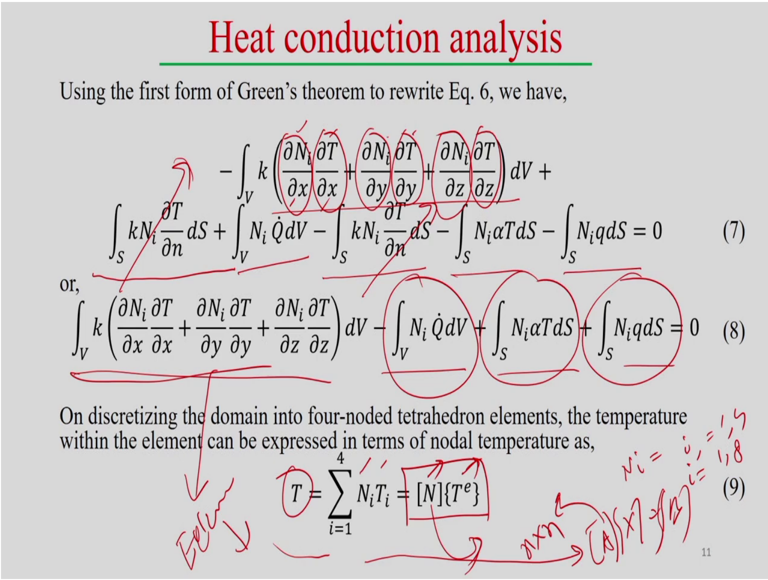

So, this term; so this term represent that here you can see minus k del N i by del x del T by del x, del N i by del y del t by del y, this terms will be coming into the picture d V. And another application of the Green's theorem also we can find out this is other surface term will be also be there.

So, u is equivalent to here we can see the shape function u is equivalent to here shape function and phi is basically u is basically equivalent to the shape function N i and phi is equivalent to basically temperature T here. So, N i N T in terms of that, we can find out k del N i by del x. So, this is in the gradient basically del this indicates the sorry; this is the u it indicates the gradient of u or we can in this case the gradient of u means here N i.

So, gradient of N i and gradient we can see the gradient is basically del by del x i this is a vector quantity del by del y j plus del by del j k. So, from here the gradient of that it represents the this indicates that 2 gradient is there and gradient of phi.

So, in this case gradient phi means it is the it will be simply the phi is represented by T basically the yeah T and then dot product of that indicates that del N i by del x the first term del T by del x del N i by del y del T by del y del i N i by del z del T by del z. So, this is the term first term is coming into this picture over the d V.

And second another surface terminology will be coming that plus this is the k N i del T by del n k we assume it is a constant this thing. So, del N i or other parameters are variable del T by del n del S. So, then it is coming from the from here this term the this term will be coming the surface term surface integral term will be come into the picture u is equivalent to the T here, u is equivalent to sorry shape function N i and phi is equivalent to T.

So, del the gradient of phi means gradient of T in this case phi equivalent to T. So, therefore, we will be getting this expression k N i del T by del n into d S. So, this why del T by del n because we define the surface means, but we defining del T by del n; that means, this temperature gradient which is defined normal to the surface. So, del T by del n into d x this term is coming after the composition from the volume integral to the surface integral.

Now, look into the other term this is already was there and this is from the surface term and already from the boundary condition there. Now, next line this term from; the this comes from the governing equation and this term from the governing equation by applying the Green's theorem, but this comes from the boundary condition though. So, that will balance if we look into this thing. So, finally, we the expression is like that d v N i this one N i equal to 0.

So, therefore, on discretizing the domain the domain into the 4 noded, so these are the different terminology we can use and this kind of expression we reach after the application of the Galerkin weighted residue technique. So, on discretizing the now it is necessary to discretize the domain 4 noded. Now, this is the general form of the equation now depending upon the N i, what should be the value of i? It depends on the what kind of the element how many nodes are there in the particular element that has to be.

For example if it is the 4 noded tetrahedron elements; 4 noded tetrahedron elements i can vary from 1 to 4, but if it is a brick element then i can vary from 1 to 8. So, depending upon the what kind of the element we consider or you have discretized the domain accordingly i can vary. So, therefore, if we assuming that on discretizing the domain into 4 noded tetrahedron elements, then the temperature within particular element can be written like that. So, temperature variable here T.

Expression in the temperature T itself within the in the form of a nodal temperature as well as the shape function form. So, that N i T i summation of i equal to 1 to 4 which is also equal to the shape function into T that we have already seen. That even we have shown in case of the displacement field also if you remember correctly in the stress analysis part there we have developed that we have seen that displacement field can be represented in the form of a nodal value. So, T in this cases also temperature can also be represent in the form of this is the shape function and this is the in the form of a nodal temperature value so T e; so that form we can represent.

So, but in this case assuming that N is the variable quantity so whatever variable quantity means we can do the derivative first derivative it can be if it is a linear shape function if you do the first derivative it becomes constant also. The T is the nodal temperature we can assume this is a nodal value.

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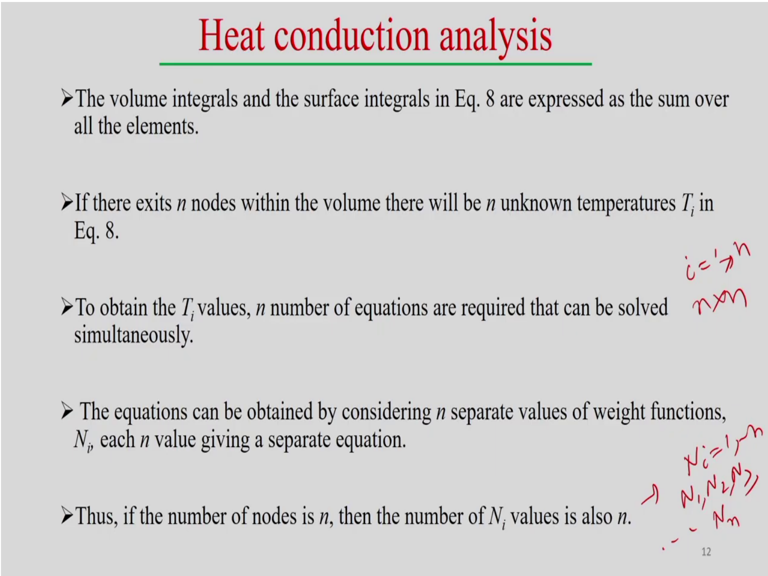

So, therefore, the volume integrals and the surface integrals of the last equation 8 are expressed as the sum of the over all the elements. So, once we do all this term this in this particular expression here we can discretize we can do in integrand, but integration we can perform over the numerical integration, then we can perform for a particular element, then once we get the particular value of the sorry particular element where you can distribute the value in each in node point.

But once we assemble for the contribution for the all elements then accordingly we can put the their nodal value which point the node point is common 1, 2 or 4 different element.

So, which keep on adding the contribution basically we calculate all this thing one particular element. Then we call one by one element and accordingly we can put the position the position of the particular node point. Because a one particular node point may be the common node point more than one or more than one elements in that particular cases.

So, once we call the particular element what are the contribution for the node point and the when you consider other element and both the element having some common node point, then we just keep on adding for this particular common node point and. In that way we can form the contribution for the each and every node point and we form the linear system of equation or we can say the form of the matrix that size is normally that number of depends on the number of nodes.

If there is n number of nodes the size of the matrix A X equal to size of the matrix we can see that, A X equal to B that is the final form of the equation. So, A matrix if there is n nodes the a can be n into n node, but this calculation we can do for particular element. And then we call all the element and in a form that we met we make the assemble one big matrix A which is dimension of this matrix is the n by n it depends on the number of nodes in this particular domain, then we solve for the equation.

Therefore if there exist n nodes within the volume. So, there will be n unknown temperature definitely each and every nodes are there. So, each and every node point the temperature can be considered as a unknown quantity and that is temperature can be considered as the T i. So, where i can vary from one to n in that particular equation 8.

Then to obtain the T i values; that means, T i equal to 1 to n; n is the number of nodes. So, in that cases n by n matrix has to be solved equation so we can get the n number of equations and n number of variables. So, in that case we can solve this equation and then solve simultaneously solve for the values. So, in that cases we will be able to find out what is the value of the temperature for each and every node point.

So, therefore, the equations can be obtained by considering the n separate values of the weighting function N i each n value giving a separate equation. So, that is also possible the equation can be obtained considering the n separate values of the weighting function that is also possible. In this case the weighting function is equal to N i the each n value giving the one separate equation. So, if the number of nodes is n then the number of n y N i values will be the n.

So, N i also value it can be also n 1 to n, so N 1, N 2, N 3 that up to N n. So, that are the different different shape function will be able to feed in this particular case and then we will be able to solve this particular problem.

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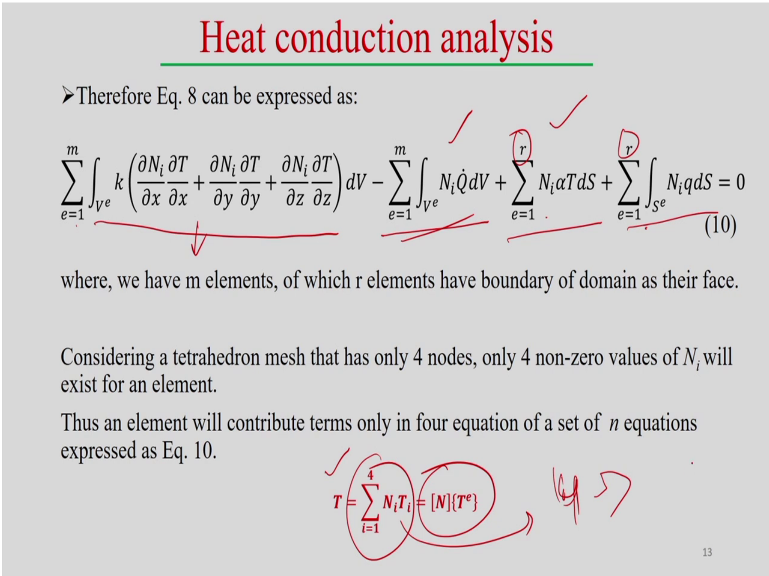

Therefore equation 8 can be expressed like that contributing from all the elements then we can find out the v e element for this we expressed is first we estimate for a one element and we assuming the one element having the 4 nodes.

So, contribution from the 4 nodes or in general one element we can form this matrix first and then we assemble for the all the elements so the summation of the all the elements. So, same way it can be done for other cases also volume in this the surface over the surface and all this thing the different contribution. So, therefore, we have the m elements of which r elements have boundary domain as their face.

So, if you look into have to be look into that that whether this that one particular component of this particular expression of finite element formulation this part; that whether it is defined over the volume or it is defined over the surface. So, contribution from the surface boundary condition that is over the surface.

So, in that cases this particular heat interaction with the boundary the, which part of the boundary is basically or how many number of elements are is associated with this particular that has to be taken care. For example, in this case r number of elements exactly on the boundary which is may be interaction through the boundary.

So, therefore, r can be different as compared to the m; m is defined for the whole domain total number of elements. So, then N i q over the domain so that is how we consider the m in this cases the whole number of elements the whole domain having total m number of elements. So, accordingly the contribution can be considered summation can be considered from vehicle to 1 to m.

Here contribution from 1 to r contribution from r, because r is the number of boundary elements which is boundary interaction is basically happening in this particular case. Now, considering a tetrahedron mesh that has only 4 nodes so therefore, only 4 nonzero values of N i will exist for an a particular element definitely, in a tetrahedron elements mesh that are only 4 nodes. So, therefore, four non zero values of N i will exist for an element.

So, basically one particular element one is from this the interaction from the each and every node to other node within the element will be 4 because there are 4 number of nodes one particular element and particular in case of tetrahedron mesh. Therefore, element will contribute terms only in 4 equations of a set and of a n equation expressed in equation 10.

So, therefore, element will compute terms only the 4 equations. So, therefore, we count one of the m of although we are doing the summation for a e equal to 1 to m. So, total m number of elements, but nodes number are different is node number this is not equal to the element number. So, node numbers can be different as compared to the node number.

So, once we assemble this thing, so accordingly the node tracking of the node numbering is also there in their particular domain such that finally, we will be able to find out what is the temperature of each and every node point, so that is the node point.

So, that is a dimension of the equation in such a way that it depends on the what is the total number of nodes not total number of elements in this particular case. Now, this expression you have already seen the temperature variable can be expressed as a function into T the nodal temperature or summation of i equal to 1 is to for N i T i depend i equal to; i equal to 4 up to 4 because it is a tetrahedron element, if it is weak element then 4 can be replaced by the 8, number 8.

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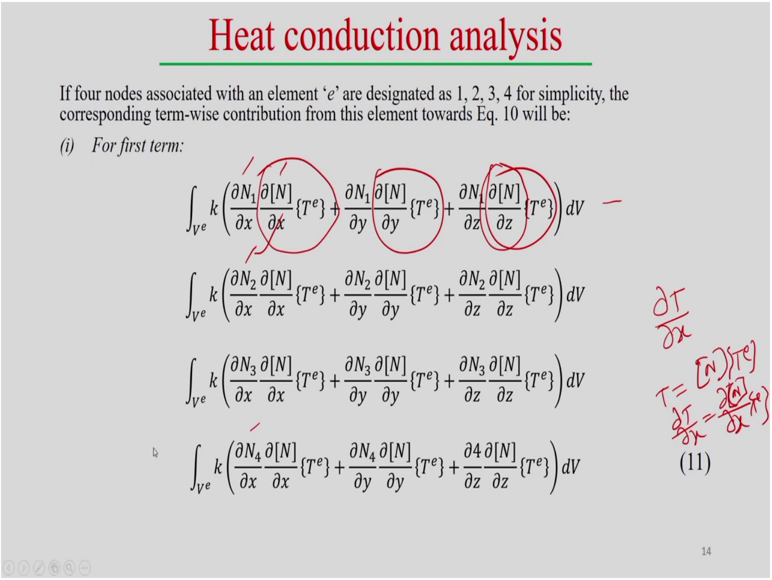

So, now look into this thing if four nodes associated with the particular element e and they start designated as 1, 2, 3, 4 for the simplicity, the corresponding term wise contribution for this element contribution from this element for the in more explore this particular part how thus formulation can be done.

So, if you look into there are 4 node and a particular element. So, term can be represented like that delta N 1 this N is the shape function because this if you see d to the del T by del x that was the term actually. So, T we represent T equal to the shape function into nodal temperature.

So, then we can say the del T by del x can be write that del N, because N is the variable in terms of N we remember that when we express the shape function N can be represented in the form of a x y z if it is three dimensional two depending upon three dimensional two dimensional problem, N can be expressed in the in terms of the coordinates and the as a variable x y and z.

So, therefore, it can be derivation can be possible here and in this case equal to T e. So, in from that sense it is coming del T by del x is replaced by this value. Similarly, for this thing N 1 and this is the x, this is for the del N 1 by del z. All these cases we can see, but in same way that is the node 1, so if it that is change in the node 2, then N 2 will come node 3 N 3 will be coming node 4 N 4 will be coming. If it is a 8 node then N 5, N 6, N 7 up to N 8 will be there. So, that needs to be accounting look into this thing.

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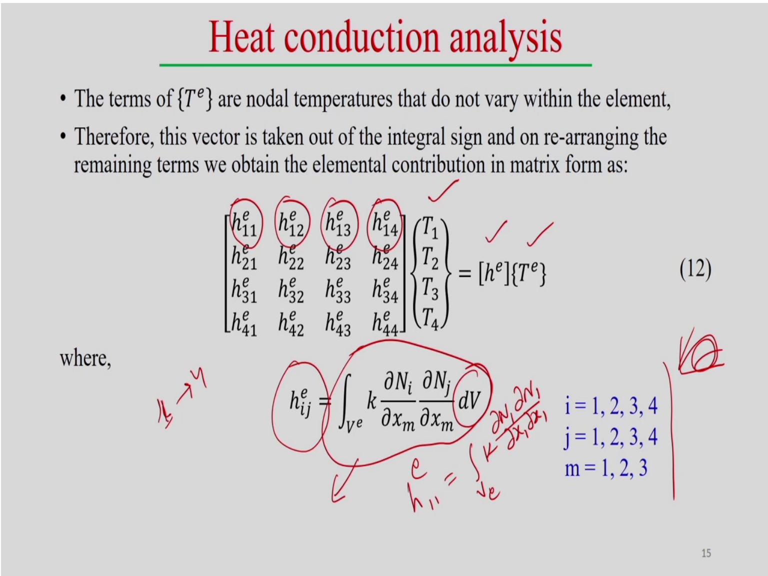

So, all terminology first term we can explore further looking into all these components. Therefore, the terms T e are nodal temperature that we have already explained this thing that do not vary within the element. So, nodal temperature does not vary within the element.

So, that is why we consider this as a constant, but n shape function we are assuming it is a variable. So, therefore, this vector is taken out of a integral sign that is true sign and rearranging the remaining terms we obtain the elemental contribution for elemental contribution in the matrix form like that.

So, therefore, rearranging all these things we can see that looking into this thing h 1 1, h 1 2 term will be there, h 1 3 and contribution these things 1 4. 2 1, 2 2, 2 3, 2 4. So, basically the arrangement so it is interaction between the node 1 to node 4. So, different way you can see the interaction between node within the one element the interaction between 1 1, 1 2, 1 3 like that 1 4.

Similarly, 2 1, 2 2, 2 3, 2 4 and the 4 1, 4 2, 4 3, 4; like that there is interaction and each term can be like that, h 1 1, h 1 2 this way. And all this contribution from the e indicates the elemental form and finally, this T 1 is the basically that tetrahedron element that is the having the 4 node.

So, therefore, the nodal temperature and 4 node is T 1, T 2, T 3 and T 4. And in general we can say that h e and the T is the nodal temperature in that form, but the general expression for h i j equal to the integration over the v e k del N i del N j del x m x m, but in this cases m equal to 1 2 3; that means, and i equal to 1 2 3 4 in this particular case.

So; that means, h 1 1, I can say that h 1 1 e can be like that v e and then k del N 1; del N 1 by del x m 1 x 1 or x 1 means x 1 means x x 2 means y direction x 3 means z direction. So, that is why this formulation can be form also it is a j equal to 1 in this case 1 1. Similarly, 1 2 1 3 like that, it can be represent in this way also in general. So, I think we can see this part also here.

So, 1 1, 1 2, 1 3, 1 4 like that 1 1. So, that is way we can represent in general way, but remember the bottom side is either a combination of the 1 1, 1 2, 1 3. So, this is the general expression, but this one find out this thing there is a need to do the integration. So, over the volume integration, but the numerical integration is actually required in this particular problem.

We will may be next time we will show how we can do the numerical integration or one particular form of the element how we can form this how we can form one particular element of this matrix in any kind of the problem. Specifically I try to explain in the heat transfer problem how this one element is formed.

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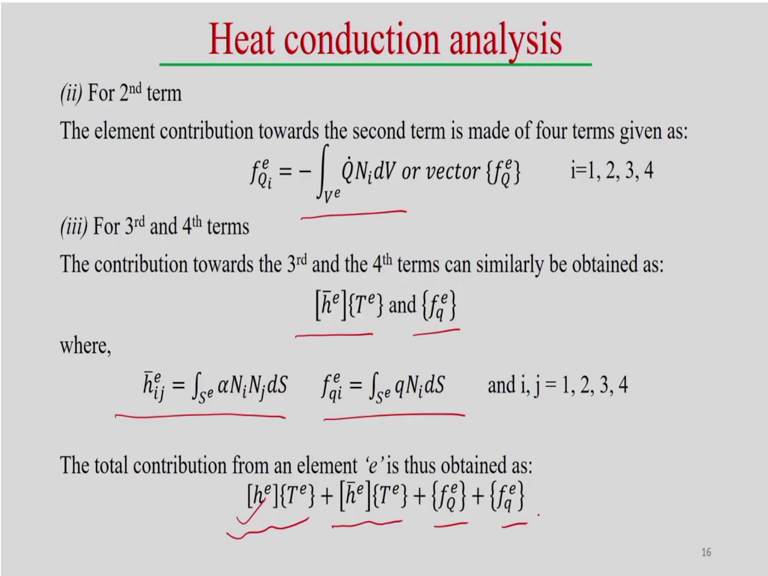

Now, for the 2nd term also we can see that the f Q i similarly this expression can be represented like that the 3rd and 4th term also can be represented in generally h e t e or you can see that f Q; that means, it is a expression see this is the this is all these cases is basically required to do some kind of the either integration over the surface or integration over the volume, but all these cases we can do the numerical integration this thing, but how we can do the numerical integration we can show in the other lectures also.

So, therefore, total contribution in the matrix form can be write that h e this part, this is also part and this is the load part. So, that in the by with the application of the Galerkin weighted residue technique and in this cases we using this weighting function same as the shape function and from that shape function we can form the matrix form of a particular equation.

So, definitely with the weighting function all this application all this thing is required once you try to do the analysis the any kind of the problem which is defined in some governing equation which is driven by some governing equation or at the same time some boundary interaction is there.

So, that kind of problem we can put the weighted residue technique and the different matrix form we can form. Now, in more details one particular element what are the different elements; that means, this matrix can be formed that will show in the next class.

So, thank you very much for your kind attention