**Finite Element modeling of Welding processes**

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# Lecture – 10
# Three–Dimensional element

Hello everybody, now we will discuss the stress analysis part; but considering the three dimensional element, what are the stress, strain relationship or displacement strain relationship can be established. And even looking into all these things that, how the finite element this model using the potential energy minimization; then how we can reach the stiffness matrix and you relate between the displacement field, in general the stress analysis for a in case of three dimensional element.

So, let us start with this three dimensional element, almost we will be looking into the similar aspect; that means similar way we can do formulation. Only difference is that, previously we consider the two dimensional element the formulation can be done; now instead of two dimensional, we will consider the three dimensional element and see how this formulation can be done.

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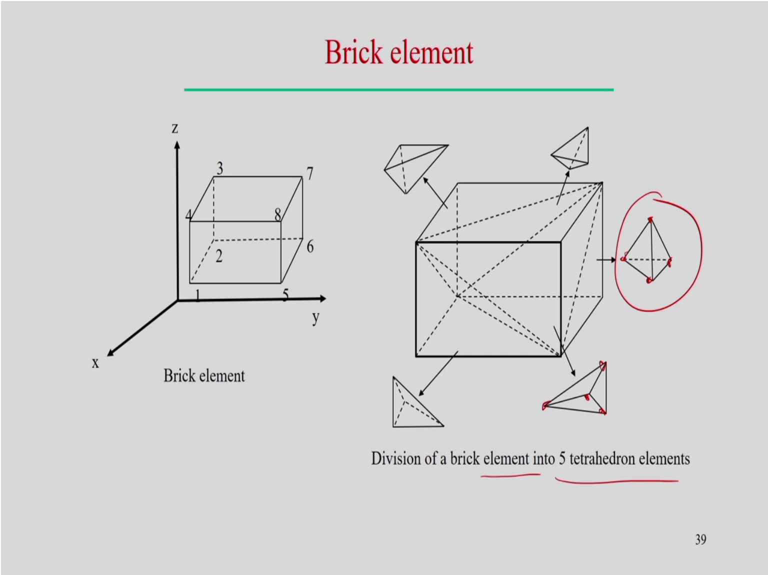


So, let us start with the brick element. If we look into this brick element and we put when you look into one brick element; one particular element for a whole domain we consider and this is normally called the brick element. And you see the numbering is 1, 2, 3, 4; there are 8 node points associated with this brick element and we define in such a way that, we have already explained this thing, numbering can be done in such a way that it should follow some particular sequence.

And the numbering for the each and every element will be following that similar kind of the sequence associated in a particular domain; then it will be easy to keep track on this thing, accounting or the contribution from the different element. So, here this brick element defined in x, y, z coordinate system and if you see the brick element which can be divided into the so many of this thing division of the brick element into five tetrahedron element.

So, this is one tetrahedron element; tetrahedron elements you can see that the node point is 1, 2, 3, 4, these are the node points. So, I think 1, 2, 3, 4; four node points associated with the tetrahedron. And of course, tetrahedron can be, it is also three dimensional element; but at the same time, this brick element also three dimensional element. So, depending upon the problem, we can do the formulation the different type of the elements that is possible.

Just to show you that here the even if we consider the brick element that can also be considered in a particular analysis; at the same time it is also possible to consider the tetrahedron elements. But here I can we can see that, even from the brick element to that is basically consists of the 5 tetrahedron elements.

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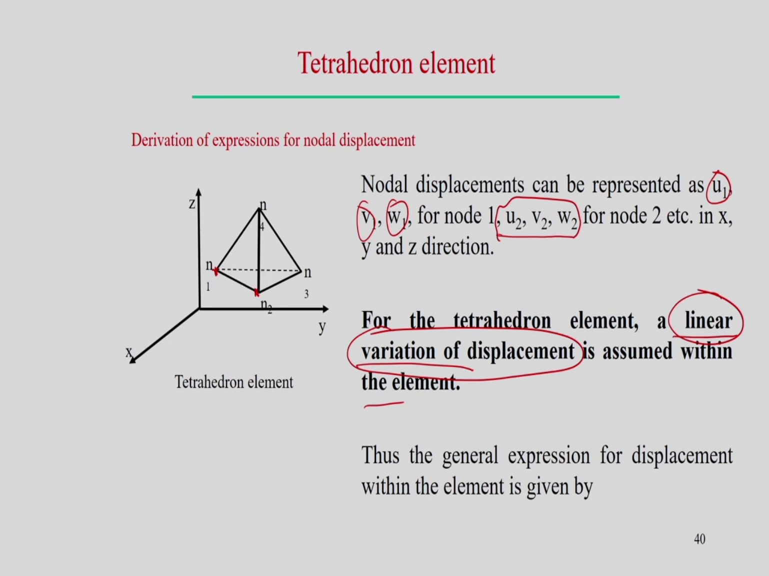


So, we can consider the, even not exactly not necessary always we have to consider the brick element, rather we can consider only on the tetrahedron elements; and that tetrahedron elements is the whole domain; we divide, we mess, put the mess in such a way that it will create only on the tetrahedron element.

Now, let us it will be easy to because it is having the only four node point. So, in this case it will be easy to look into the expression, if we follow, if we look into the tetrahedron element; of course we will be using the similar kind of the strategy or similar kind of the formulation can be done, even if we consider the brick element.

Now, let us look into the expression for the nodal displacement and nodal displacement can be represented u 1; that we already seen that u 1, v 1, w 1. Because for each node point, there are three components one x, y and z direction; that is the that displacement field can be represented u 1, v 1, and w 1 for particular node 1.

Similarly, for node 2, we can see the u 2, v 2, w 2. So, these are the u 2 is the displacement field along x axis, v 2 is the along y axis, and w 2 is the along z axis. But it is the displacement in for particular node 2. Like that we can put the all the numbering in the different node points 2, 3 and 4 similar we can put the nomenclature in particular way.

Now, for the tetrahedron element, a linear variation of the displacement is assumed though. So, linear variation of the displacement is assumed and within the elements. So, within the element, all the variation linear displacement vary linearly. So, that is important, linear variation of the displacement field within the this thing, within one element.

So, it is not necessary to define some kind of the node point in between this point. For example, the between the node 1 and 2; the displacement field can be considered as a vary linearly, linear way. So, therefore, this node 1, consideration node 1, node 2 is maybe sufficient; but if we assume that the quadratic variation of the displacement field, then it is necessary to define another node point in between 1 and 2.

So, that is the difference. So, therefore, say in this case, we are assuming the simple linear variation of the displacement field. So, therefore, defining the two node point is sufficient to represent the displacement field in a finite element based model. So, therefore, general expression for the displacement within a particular element, we can define in this way also.

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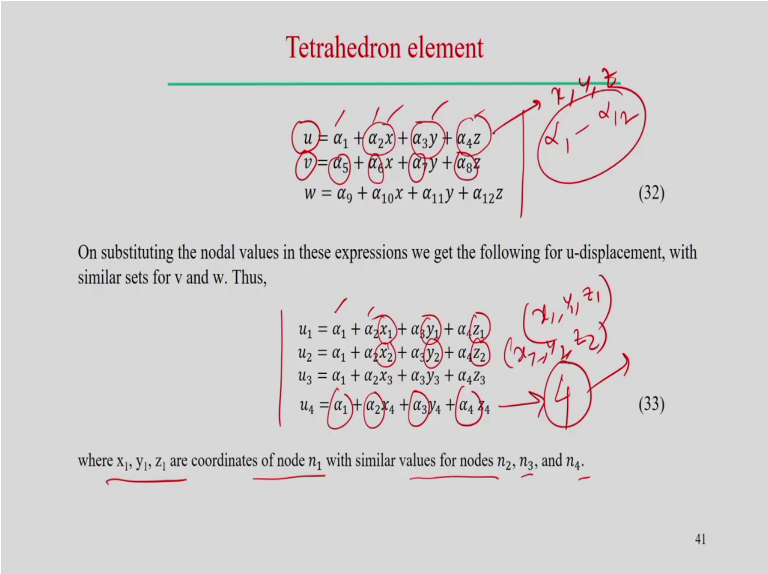


That u, that we have already discussed that u is the general variable, the displacement field and this displacement field indicates the displacement field along x axis. And then it it consist of the alpha 1 is the constant term, alpha 2 is a constant term and since it is involve x, y, z; so therefore, alpha 2 x, alpha 3 y, and alpha 4 z. So, here we can see, it is a variation of the displacement field is the say linearly varying. So, the such that we define this function that in that way that, alpha 1 alpha 2 into x that alpha 3 into y plus alpha 4 into z.

Similarly, v can be defined in such a way you have to introduce, it is the other constant term that alpha 5, alpha 6, alpha 7 and alpha 8; but all is a function of in x, y and z, but it is a linear variation of the displacement field within the element. Similarly, w also can be represented in such a way.

So, here we can see that, we define the; it is a general variable u, v, w the displacement field along x, y and z, but it is consist of, when we define this general variable as a function of x, y and z. But it if you see, it constitutes basically the alpha 1 to alpha 12; there are, so there are 12 constant term.

So, therefore, 12 constant terms can be defined or have to be defined and from the geometric values of the or maybe coordinate of this particular node point; from that point of view we can derive these value of the alpha 1 to alpha 12 in this case. So, therefore, from substituting the nodal values in this expression; so use the general variable u is a variation of alpha 1 this thing as x, y and z, you can see that x, y and z is the variable in this case.

So, therefore, if we stick up the particular node point that, particular define this particular node you this is the one node point. So, for node point 1 it can be represented like this alpha 1, alpha 2 into x 1. Similarly, that constant means y 1 is the variable, because z 1. So, basically x 1, y 1 and z 1 is the, this is defined over the geometry or we can say the coordinate of the node 1.

So, then it is a known term we can say that, that x 1, y 1 and z 1 defined this thing. Similarly, u 2, this u 2 is the variable along y axis, the displacement along x axis sorry, displacement along x axis; it is also as a function of x 2, y 2, and z 2. So, here x 2, y 2, and z 2 these are the three the coordinate of the node point 2; so that we assume this is a known quantity.

Similarly, u 3, u 4 can be done. So, in this case there are four node. So, for all the node point, we look into the u displacement field; that means along x axis the displacement field can be represented there are these things. So, we can see that there are the alpha 1, alpha 2, alpha 3 and alpha 4; this associated with the 4 constant terms alpha 1, alpha 2 and there are 4 equations.

So, if we simply solve this linear system of the equation, we will be able to find out what is the value of alpha 1. So, definitely this alpha 1, alpha 2, alpha 3 and alpha 4, these are the four constant terms can be represented in the form of a x 1, y 1, z 1 or maybe in the coordinate of all these node points; in that term, it is possible to find out the what is the values of the all these different constant term.

So, where x, y, z 1 is the coordinates of the node n 1 and with the similar values of the node n 2, n 3, and n 4; basically n 2 indicates the node 2, n 3 is the node 3, and n 4 is the node 4. So, like that we can solve this equation, we will be able to find out what is the value of the; that means we can represent the all the constant term in terms of the other parameter; so, u 1, u 2, u 3, 4 as well as the coordinate of this particular node point.

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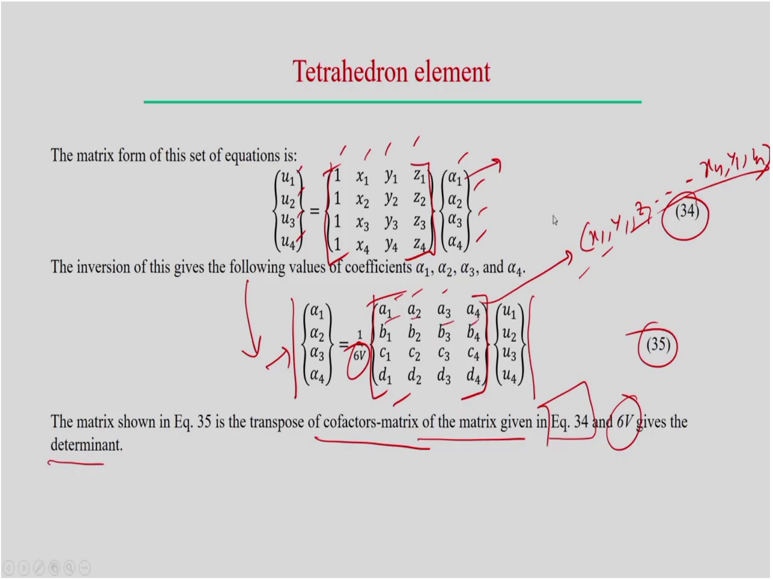

So, therefore, in the matrix form, we can write this equation like that. So, in this case we can see the u 1, u 2, u 3, and u 4; but it should be like that, it is a third bracket. And here the matrix form we write that, put according position alpha 1, alpha 2, alpha 3, alpha 4; this is simply representation of the 4 linear system of the equation in the form of a matrix.

So, now, once we form up the matrix and if we know the matrix operation; so then even this we need to know what is the value of alpha 1, alpha 2, alpha 3. So, this can be represented simply by inversion of this given expression and from that simply inversing this thing, we will be getting the value of the alpha 1, alpha 2, alpha 3 in terms of this, I think it can be like this, in terms of this a 1, a 2, a 3 are actually a 1, a 2, a 3 this is this can be represented that x 1, y 1, z 1 and x 4, y 4, z 4.

But, it is represented all these in terms of x 1, y thing; but to make it shorter, to represent the better representation all these thing, we have we use this terminology a 1, a 2, a 3, a 4. But a 1, a 2, a 3, a 4 which is simply that it can be expressed in the form of a different values of the x 1 to x 4, y 1 to y 4, z 1 to z 4 that is we can represent this thing. So, once we inverse this matrix and then we are getting this expression like that. So, this alpha 1, alpha 2 in terms of u 1, u 2, u 3, u 4 and this is the matrix from a 1, a 2 in the from that.

So, V is the. So, this matrix shown in the above equation, equation 35 we can see that is the transpose of a cofactor matrix and of the matrix given in the equation 34. So, from this 34, we can simply do the matrix operation and 6 V gives the determinant. So, if we operate this further matrix operation, if we perform these things; you say it is simple way, we have to find out this from this the inverse of this matrix and we will get this variable like that.

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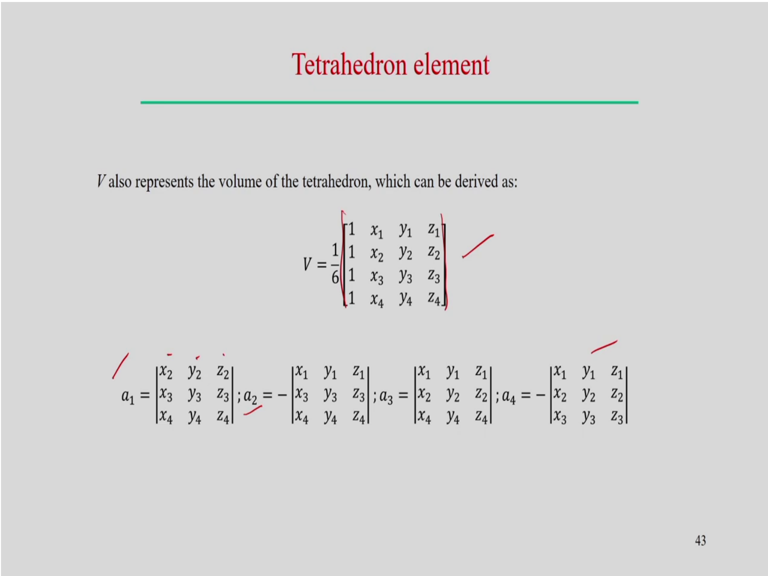


So, now V represents the volume of the tetrahedron; in a physical interpretation of V, we have written the 6 V there. If we look into this, that we have written the 6 V the 6 V; but V indicates the volume of the tetrahedron. The expression of the V is the 1 by 6, I think the determinant of this 1, x 1, y 1, z 1; x 2, y 2, z 2 like that.

So, V represent the volume of the tetrahedron, which can be derived in this particular form. Now, a 1 if we see the a 1, determinant of x 2, y 2, z 2. So, is a 2, 3, 4; a 2 represented in terms of the x 1, y 1. So, basically except the values of the node 2, other involves in this particular expression; similarly a 3 also the in terms of 1, 2 and 4 and a 4 also 1, 2 and 3.

So, this way we represent this a 1 in this form of a this thing, in the form of other coordinate. So, all these if we look into this expression; so a 1, a 2, a 3, a 4 all can be represented in the form of a the only simply the coordinate of this axis. Similarly, V also in the form of that, volume can be represented in the form of a all the values of the known values; we are assuming the simply coordinates of the four node points.

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## Tetrahedron element

Similarly, expression for $b_1$, $c_1$, $d_1$ etc. will be

$$b_1 = -\begin{vmatrix} 1 & y_2 & z_2 \\ 1 & y_3 & z_3 \\ 1 & y_4 & z_4 \end{vmatrix}; c_1 = \begin{vmatrix} 1 & x_2 & z_2 \\ 1 & x_3 & z_3 \\ 1 & x_4 & z_4 \end{vmatrix}; d_1 = -\begin{vmatrix} 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \\ 1 & x_4 & y_4 \end{vmatrix}$$

Similarly, b 1, c 1, d 1 etcetera can be represented like that; see the b 1 can be represented, c 1 can be represented and d 1 can be represented in this particular expression. So, if you perform this matrix manipulation in this case, you will be able to find out all this kind of expression.

So, here we can see there is a 1, a 2, a 3; b 1, b 2, b 3, b 4; c 1, c 2, c 3, c 4 like that. So, therefore, similar way it for therefore, b 1, c 1, d 1; similarly b 2, c 2, d 2, so in that similar way, we can find out all these value of the this thing in the form of a x 1, x 2, x z.

Now, we try to all these expression; why we are explaining all these things? Because how this general expression is coming before that, we should know how this expression that a 1, b 1, c

1 in terms of the other parameter it is coming, such that it will be easy to understand the in general expression, when you try to look into the shape function all this form.

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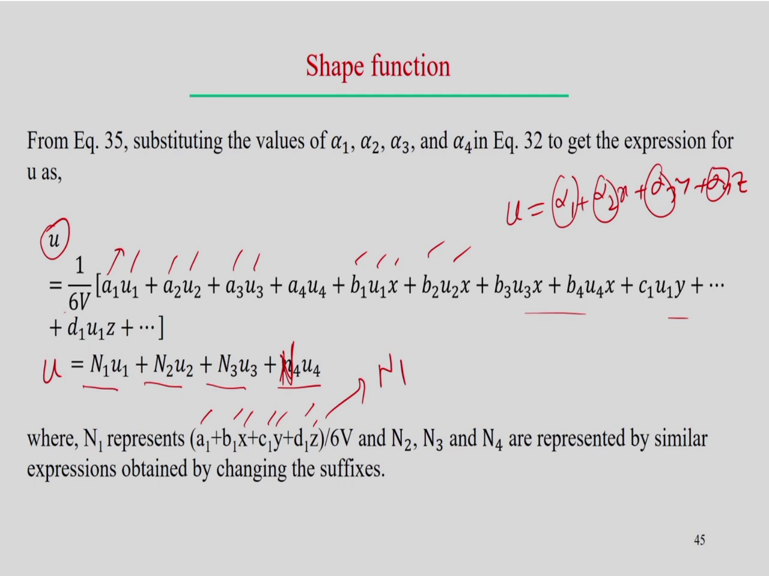


Therefore in equation 35, if we substitutes the value of alpha 1, alpha 2; these are the constant term, we assume these are the constant term alpha 1, alpha 2, alpha 3, alpha 4, you replace this value get the expression. So, u can be represented in the in that way also; in general way u can be, u equal to, this is the u.

So, use the general displacement variable, so that general displacement variable which is within the element this is the; this we are representing in the form of a this thing a 1. So, I can say a 1, a 1 also in the form of a expression in the form a x 1 to the coordinates u 1; similarly a 2 u 2, a 3 u 3 like that we can see that b 1 u 1 x and the b 2 u 2 x.

So, this way if we look into the general expression of the u, if we look into the general expression of the u and if you remember u is basically I think alpha 1 plus alpha 2 x plus alpha 3 y plus alpha 4 z. And then you replace alpha 1, alpha 2, alpha 3, alpha 4 with the known quantity and x, y, z also here there.

So, finally, we represent in the more generalized way that, u can be represented like that N 1 u 1, N 2 u 2, N 3 u 3, n 4 u 4; it should be I think it should be capital N 4 u 4. So, therefore, this N we represent in general way; u is this, this value. So, now N 1 we know that N 1, N 2, N 3, N 4 is the basically this thing are represented by the similar expression by changing the suffix. So, N 1 can be represented now a 1 b 1 x, c 1 y, d 1 z divided by 6 V.

So, this is the expression for N 1 from here, we can find out this expression. So, N 1 is this one, this is the N 1. Similar way we can look into the strategy that, if we follow the similar kind of expression; N 2 can also be represented the similar way, similarly N 3 can be represented, N 4 can be represented by similar this thing.

We know this N 1, N 2, N 3, N 4 is basically associated with the shape function that, we have already discussed in the when you describe the 2 dimensional. The similar way and similar direction we are trying to finding out all this expression of the shape function in case of the tetrahedron element.

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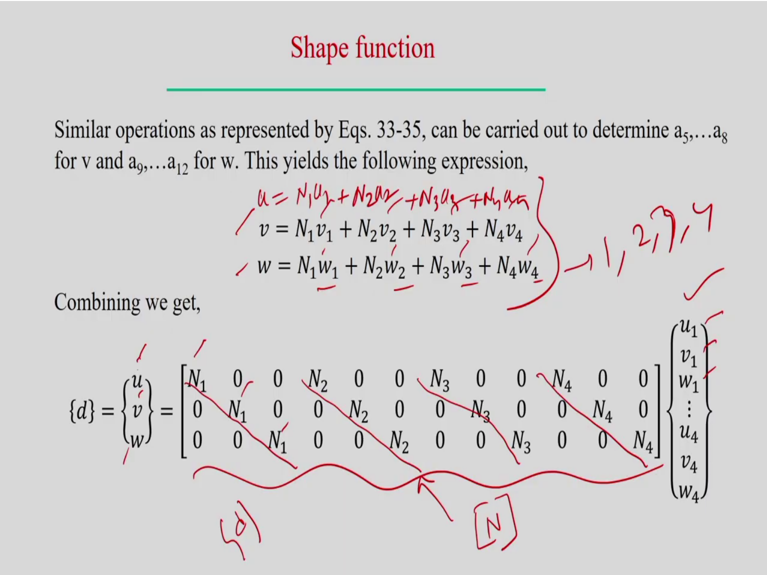


Now, similar operations if we perform that, same kind of to find out the value of other values of maybe a 5 to a 8 for v, similarly a 9 to a 12 for w. So, finally, we can represent this thing, v 1 can be represented also in the shape function N 1, N 2, N 3, N 4; w also N 1, N 2, N 3, N 4, but the variable are different. These are the displacement field at the defined which v 1, v 2, v 3, v 4 is the basically the y displacement, which is defined exactly at the node point.

So, node 1, 2, 3, 4 it is corresponds to that; similarity w 1, w 2, w 3, and w 4 which is defined the displacement field of the node 3. So, that way we can find out, sorry all the nodes point w 1, w 2, w 3 is the node point node 1, 2, 3, 4 and the w indicates the displacement field along the z axis.

So, therefore, once we get all these value of general expression for u, v, w the displacement field; in general d is the displacement field, d consists the displacement vector u, v, and w all three components, because displacement field in general u, v, w is corresponds to the x component, y component, z component. So, therefore, this column vector represent the u, v, w is the general displacement field.

So, then in terms of the shape function, we put the N 1, N 2, N 3; it is simply representing in the matrix form, so that the system of the equation can be represented in the matrix from. So, we know that, u I think here you can write also u equal to N 1 u 1 plus N 2 u 2 plus N 3 u 3 plus N 4 u 4. So, from here this equation, if we put u, v, w equal to in the different position N 1, N 2, N 3; N 1 and N 2, N 3 remaining is the 0, N 4. And finally, this column vector represents all the displacement u 1 v 1 w 1 and u 2 v 2 w 2, u 3 v 3 w 3, and u 4 v 4 w 4.

So, that is why we represent this expression. So, this is the general expression for d displacement field in the form of a shape function and the displacement. Displacement value which is defined exactly on the different node point; there is 1, 2, 3, 4 these are the three different node points we can define.

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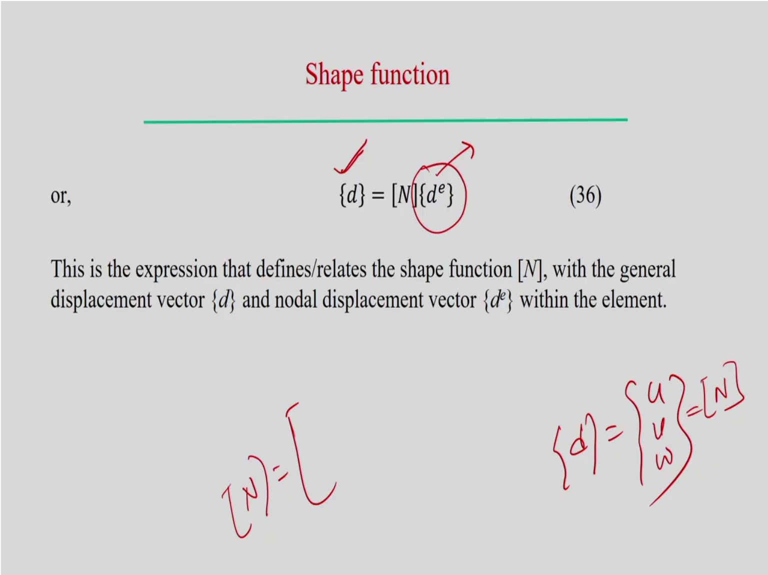


So, once we get this thing, but general expression we can see d; displacement filed the d is basically indicates the displacement field u, v and w. And this can be represented the shape function N and d e; d is the basically the nodal displacement, we can see the displacement is defined each and every of the node point for this particular element. So, this is the expression for the defines, relates the shape function N, with the general displacement vector d and the nodal displacement vector.

So, therefore, d is the general displacement vector and d e we define, that indicates the nodal displacement vector; it means that, it is a general variable expression which is defined over the element. But when you talking about the d e defining, it is these displacement vector is exactly defined exactly on the node point. So, the values of the node point is defined in this by this particular expression, so within this element.

So, therefore, this is the general expression and N the shape function; we have already seen the expression for the N equal to this one that, this is this represents the value is the shape function N. So, N can be replete in terms of the N 1, N 2, N 3 and N 4. So, that are the expression. So, what if we have already look, we look the N 1 can be represented this thing that, N 1 is represented in this way also that is the variable x, y, z are also there.

So, in the associative a 1, b 1; a 1, b 1, c 1 and d 1 we can consider this as a constant value and because it is the expression in the form of a the coordinate of the different node point. So, that is the coordinate for that one, but x, y, z is a general variable in this case.

So, therefore, accordingly N 1, N 2 can be displacement. So, therefore, N can be considered as a shape function, which is the variable quantity we can say, this can be also variable within the particular element. So, this is the general expression for the displacement, general displacement field to the nodal displacement field in the, by through the shape function of a particular element.

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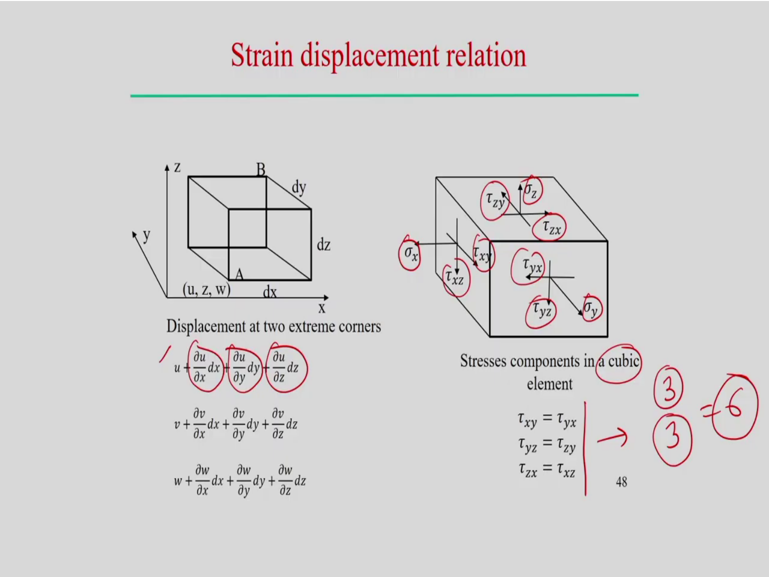


Now, strain displacement relation. We have already seen the strain displacement relation in case of, then two dimensional element, how we can relate between the strain and displacement; because this is also necessary. And we have already explained this thing when you are looking into the strain displacement relation; we are assuming the small deformation theory.

So, basically we are neglecting the rotation and higher degrees of the deformation normally, we are neglecting. And then once we neglect this thing, we can simply looking into this expression the displacement at the two extreme corner maybe like that only. So, u is the displacement field then del u by del x into dx del u by del y into dy dz. So, same this kind of expression displacement of the two extreme corners can be represented like that and then we will try to look into the how the displacement field can be relate with the other components.

So, strain versus displacement field that already you have shown in case of the two dimensional elements. So, I am not repeating the same thing again here also. Similarly in a stress component also we can see that, stress component in a cubic element; the two dimension element we have seen that there are two normal stress and one shear stress component.

But here we can see, there are three normal stress component; sigma x, sigma this is for a particular element sigma y, and sigma z these are the three components, normal stress component and there are other shear stress component z x, z y, x z, x y, y x, y z. So, but if we assuming there is no rotation and that means, this thing particular element or maybe symmetric stress tensor; then in that cases we can assume that, the x y, x y equal to y x.

So, particularly we reduced to 3 normal stress components and the 3 shear stress components. So, total 6 components of the stress in a stress tensor. That in case of three dimensional analysis we have already shown that, how these six components are coming in the stress analysis part.

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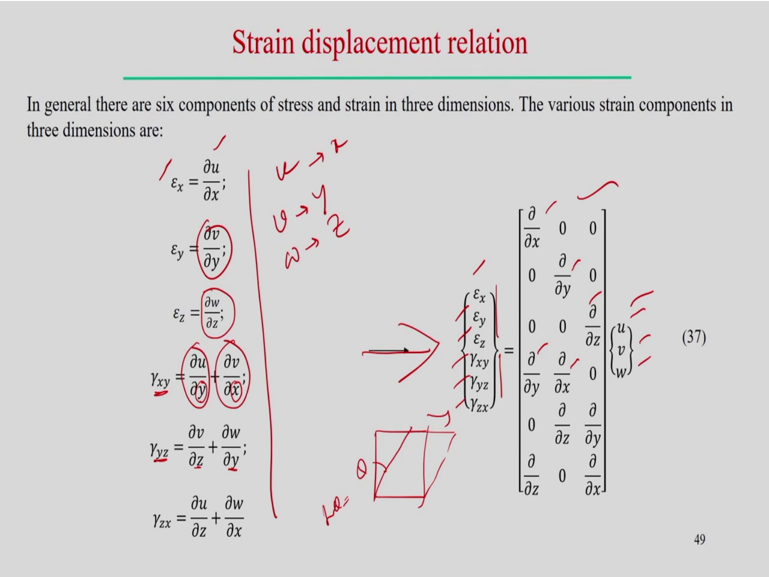


Now, look into the other aspect here you can see the relation with the general expression for the six components of the stress and strain in the three dimensional form. So, therefore, we know that strain the small strain, but in the small deformation is there; then strain epsilon x along the x axis this can be relate the between the displacement field and the strain is something like that del u by del x. But you remember use the variable, is the displacement along x axis.

Similarly, epsilon y can be represented del v by del y, so that v is the general displacement variable along y axis. And epsilon z equal to del w by del z, it is a the w equal to; in this case is the displacement variable along the z axis. So, that means, strain is it can be represented this in the form of a this del u by del x epsilon y del v by del y and epsilon z equal to del w by del z. So, this way we can represent the strain versus the displacement field.

Now, the expression for the shear strain is little different in this case; but we have already shown that how the shear strain component can be represent the this thing. But in general gamma x y the shear strain component, both component del u by del y; not the del x here and del v by del x, because normal stress is acting normal to the normal strain along the direction of the load, there is a normal strain is acting.

But shear strain is acting parallel to the application of the this thing; we define the shear strain or shear stress is basically applying the parallel to the applied forces. So, therefore, the shear strain component and if we remember the shear strain component high difference; if suppose this is the element, if application of the load is deformed or something like this and this is the take the shape and this theta, tan theta we represent these are the shear strain in this particular position. So, that is why the expression are different in this case.

So, del u by del y, del v by del x is the gamma x y. So, this is the gamma x y, this del u del y and v x. Now, when you talking about the y z, the del v by del z, del w by del y. So, like that we can look into the strain displacement versus strain relation we can establish from here, the all this expression; but in the matrix form, we can write this expression like that, epsilon x, epsilon y, epsilon z; gamma x y, gamma y z, and gamma z x.

So, these are the all the strain components. So, we can say the in the column vector, we represent in this all the three normal strain components and the three shear strain component and right hand side represent the displacement field. So, you u v is the v w is the general variable the displacement field, in this cases u v w is the displacement field. And then in terms of the del by del x, del by del y, del z del by del y del z. So, basically these three system six equation we representing in the matrix form in this particular way.

So, then if you do this matrix multiplication between this and this; then we will be established to this similar kind of equation in other way also. We can cross check also that, if we do multiplication that matrix multiplication; then we can reach going back to this kind of expression.

So, it is simply the nothing else the simply representation of this equation in the matrix form, so that it will be easy to handle this matrix manipulation or maybe further different matrix operation will help to do further calculation. So, that is why it is always necessary to represent the, there is number of equations are there to represent in the matrix from; it is easy to track all these things.

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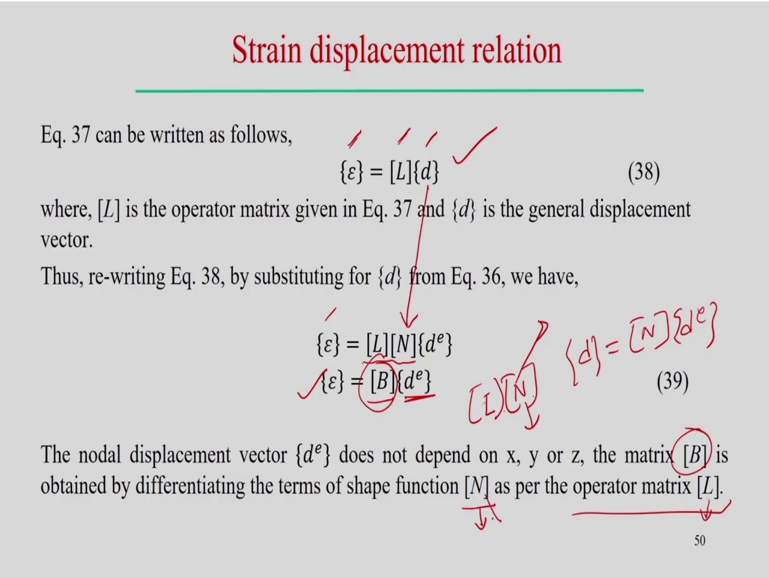


So, these are the expression the strain versus displacement field. Now, more general way we can see that this that can be written in the previous equation can be written e epsilon is the in general strain field variable and L is the operator matrix and d is the. So, general displacement vector, L is the operator matrix and epsilon is the strain field.

Now, operator matrix if we remember, the two dimensional also we can express the operator matrix L; but the expression for the two dimensional and in this case and in case of three dimensional are different, can should be different, I think it may not be the same. Now, L is the operator matrix equation, it is the general displacement vector.

Now, we can rewrite the equation 30; that means this equation we can rewrite in other way also. So, epsilon is the L into d and d is basically that, d represent the general displacement field; we have already shown that in the form of a shape function N into and d is the nodal displacement.

So, therefore, once we replace this is the N into d e. So, therefore, L into N can be represented in the form of B; B is the depend on the x, y, z. So, therefore, B is the strain displacement, B matrix that actually relate the between the strain and displacement relation.

So, the strain variable general, d is the nodal displacement vector and B is the in between stress strain displacement matrix. So, B and B is basically B once we look into that B is the operator L and the shape function N. And shape function we can see is a variable, because shape function represents apart from the constant term; that means coordinate of the different node point, there are also variable x, y, z; x, y, z variable also involve in this case.

So, therefore, B is the; we can obtain we can do the differentiation of this N shape matrix with respect to operator matrix L and differentiating in terms of the shape function at as per the operator matrix L. So, therefore, it is possible, it is necessary to sometimes to find the differentiation of the say the N matrix with respect to this thing, but operator matrix L. So, that is one we can do the further calculation on that.

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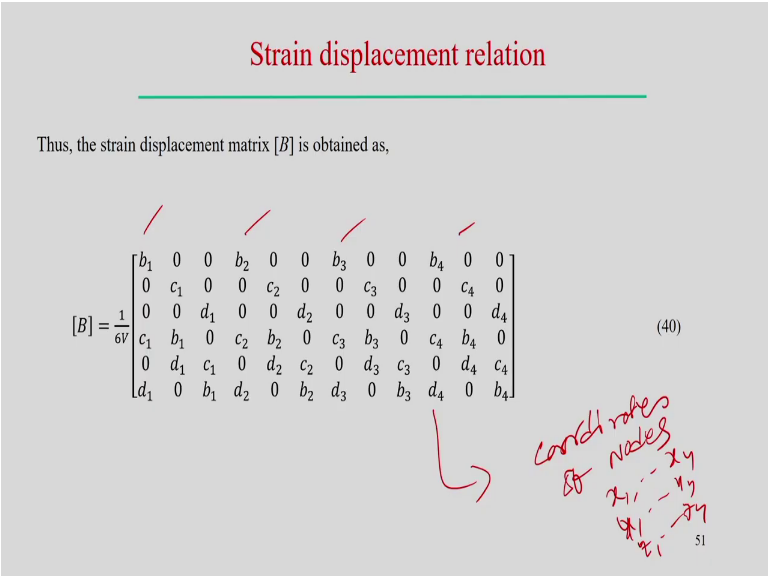


So, therefore, the strain displacement matrix B can be obtained as like that; so now, if we look into that, previous thus for linear element. So, in this case is the linear element in the sense, the displacement vary linearly within this element. So, in that case, the shape function if we remember the; it is a linear function of x, y and z and L is the operator matrix.

So, once if we do the first order differentiation L with that with respect to x, y and z; then it becomes that constant term. So, that means constant term it means that; if you see the B can be represented in the form of a b 1, b 2, b 3, b 4. So, there is no variable in this particular case, strain displacement matrix. So, strain displacement matrix simply representing b 1, b 2, c 1 all is basically the represent in the coordinates of all the node point, so of nodes.

So, that means in terms of x 1, x 4; y 1, y 4; z 1 up to z 4, so basically that b 1 b 2 all these things can be represented in the form for x 1 to x 4; y 1 to y 4 and z 1 to z 4. So, finally, you can see, this is the kind of strain displacement if a linear variation of the displacement within the element; we can represent the B matrix, displacement matrix is a basically constant, that means is in the form of a only the coordinates of the node point.

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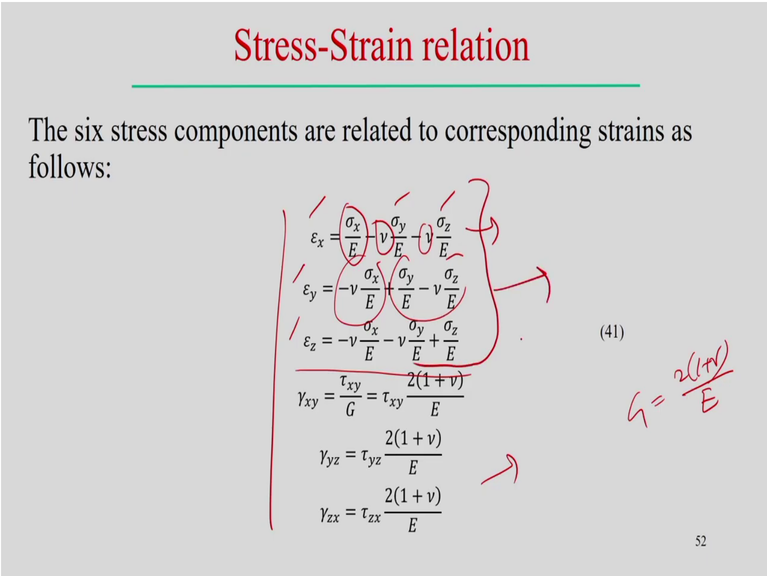


Now, stress strain relation, we have already seen that the; we have already seen that how the displacement and strain field can be related. Now, similar way the stress versus strain can be related and we have already shown that, how the, what is the in case of if we look into the elastic up to the elastic, within the elastic limit, what way the stress and strain can be relate. Those strain can be relate in this five six component of the strain epsilon x, epsilon y and epsilon z.

And we have already seen that sigma x by E and the Poisson's ratio we introduce here and such that total effective strain along x axis is equal to in terms of the; because there are normal stress components are also there and the three shear stress components are there in this particular state of the stress.

So, then strain accounting the this Poisson's ratio and what are the influence of the other along the x direction; what is the strain component, because of the stress along the y axis or along the z axis. So, that can be introduced by simply using the concept of the Poisson's ratio. So, once we look into all this Poisson's ratio and the expression all these things, this is the effective strain along the x axis.

Similar way effective normal strain along y axis also can be represented this way also, and epsilon z also can be represented like that. Now, gamma x y shear strain and versus the, I think shear strain can be just shear stress and the shear modulus G. And even there is a relation between the shear modulus and G and that this can be replaced in other expression also. Or G can be represented, basically G here we can see the G can be represented 2 into 1 plus nu stripping material by E Young's modulus in terms of the Poisson's ratio and the Young's modulus; G can be represented like the shear modulus.

So, that is why the relation between the shear stress and shear strain, if we consider in terms of the G or other parameters, in terms of the Poisson's ratio or the Young's modulus; then basically the six strain components and it is associated with the six stress components. So, then I looking into all this expression which is valid within the elastic limit, we can relate between the different stress versus strain component.

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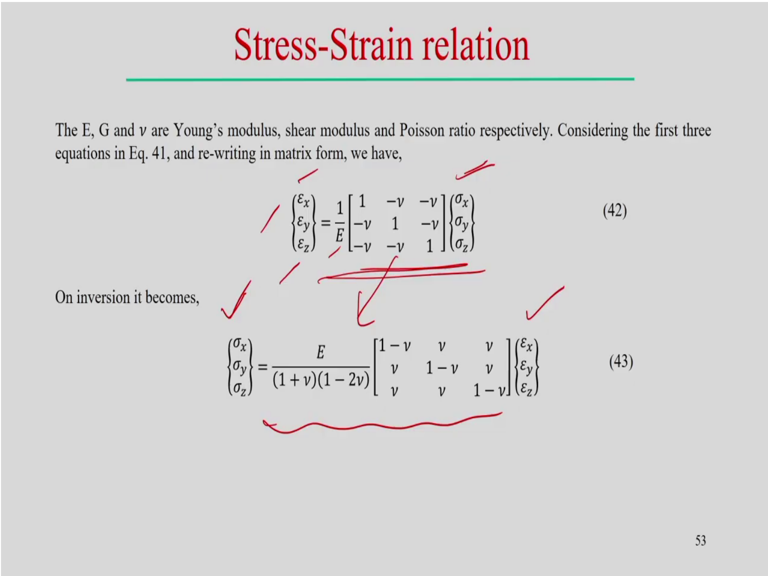


So, we can see that E, G and nu are the Young's modulus, shear modulus, Poisson's ratio respectively. So, considering the first three equation of the equation 41; that means if we look into only the normal strain component, then first three equation if we consider, then we can relate between the strain versus stress is the in terms of the Young's modulus in the matrix form.

So, that, but in this cases we consider the first three equation; maybe we can see that, these three equation we have considered. So, first we consider these three equation and from this equation in we write the matrix form, then we can reach this kind of expression; this is strain all the property the Young's modulus and Poisson's ratio and this is the stress I represented in the form of a column vector.

Now, similarly once we do this, if we want to know stress relate with the strain; that means simply if we do the inversion, it becomes that stress equal to this expression. So, and this is the strain component, this is the stress component. So, then inversion can be the, you have to find out the inverse of this particular matrix and from there we can relate, what is the value of the stress in terms of the strain components; so, that if we do this calculation, we can reach this kind of the expression.

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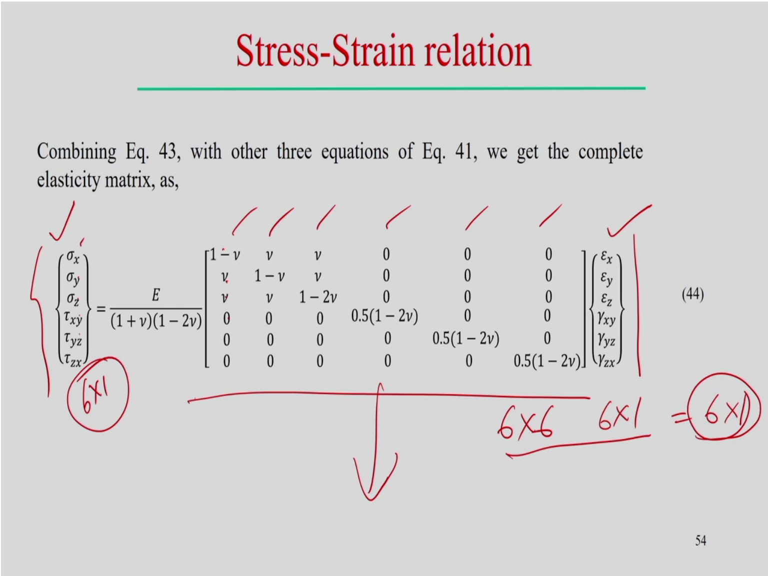


Now, combining this expression in terms of the shear stress and shears all the stress components, this stress components sigma x, sigma y, sigma z, tau x y, y z and z x all the stress components are there. And in this particular case, we write in this in the matrix form and this is the strain component; all the strain components are there.

So, now, this accordingly we can put the simplest; if consider the six equation, stress is the left hand side, and right hand side is the strain components in terms of the other constant term, basically material properties here. And these three material properties here we can see that, only even two material properties we are considered; but three means in the sense the Young's modulus and the Poisson's ratio are there.

But we can, we have the relation between the shear modulus Poisson's ratio with the Young's modulus. So, if you once, you use this relation the shear modulus in terms of the Young's modulus and the Poisson's ratio; then finally, we are reaching all this expression in the two different properties, one is the Young's modulus, other is the Poisson's ratio.

So, in that form, we can represent this matrix form of this particular expression and then this is the sixth system equation and if we see that 1, 2, 3, 4, 5, 6 and 1, 2, 3, 4, 5, 6. So, it is basically 6 by 6 matrix and in this other one, it is a 6 by 1. So, finally, it is reaching the expression is basically the; if you manipulate this, it should be 6 by 1 and this is that it is like 6 by 1, it should be 6 by 1. So, here you can see the, it is the 6 by 1.

So, matrix that is the looking into all this expression and we can see we can cross check also that, the what is the order of this matrix and that we can see from this expression. So, finally, we are reaching the stress in terms of the strain that, is if you see the stress that only the it is a basically material properties; but there is a particular way to or arrangement is of this particular material properties involved to find the relation between the stress and strain.

So, this we can relate all in this case, this stress versus strain in particular to three dimensional. And we have already shown in case of two dimensional also and then in two dimension also the expression will be different, and but it is the nature of the equation or maybe involvement of the number of constant will be the same.

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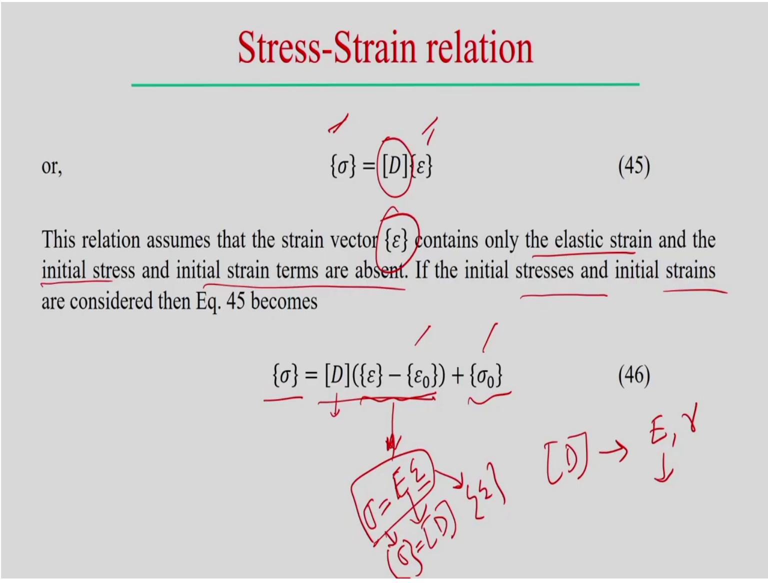


Now, stress strain finally relate the stress equal to, this is the strain and D. D is basically takes care of all this accounting the D is basically associated with the material properties. So, basically Young's modulus and the Poisson's ratio and different expression of this D matrix the form, shape of the D matrix will be different in case of two dimensional or in case of the three dimensional.

So, finally, this is the relation between the stress and strain in between these thing, but remember this relation we consider; but assuming that the relation between the stress and strain, which is valid below the yield points. So, that means, because we consider the elastic part only. So, therefore, elastic strain is related to the elastic strain. So, that is why the strain vector are contains only the elastic strain, and the initial stress and initial strain terms are absent.

So, therefore, initial stress and initial strain are considered; then finally, we can modify the, introduce the if there is a existence of the initial strain or existence of the initial stress value. Then this can be represented like that; D matrix is there properties and this is the strain and this is the stress value.

So, this is the initial stress value. So, we can takes care of this thing in the particular problem; when we if we look into the what is the value of the initial stress or initial strain value, then expression can be little bit modified, such that we can reach this particular expression.

So, that is why this dimensionally also it is balanced, because this is the stress component and then D into strain that is equivalent to stress; because we know that stress equal to even if we know uniaxial stress state also there this thing, Young's modulus into strain, assuming stress and strain in uniaxial and valid within the elastic limit.

So, stress is proportional to the strain. Then sigma equal to E into epsilon we this thing. So, it is a similar nature of the equation, but instead of E; when it is three dimensional case there, we replace E in terms of the D and E replace in terms of the vector and C in terms of the also vector.

So, that is why it is the difference from one dimensional case to the three dimensional case. So, that is why definitely then D matrix will always contains the only the material properties; but entry in the particular element of this matrix, the position or entry will be of this expression will be different.

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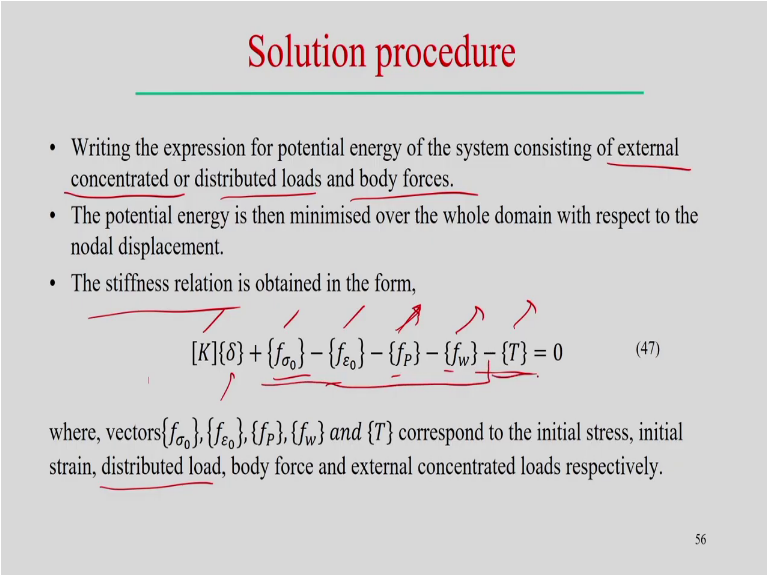


Now, similar solution procedure if we follow, the potential energy minimization if you follow the similar kind of the other potential minimization problem, and consisting of the external load, distributed load, even three different external load, concentrated load, and distributed load and body forces.

Then, if we consider this particular situation and if we follow the potential energy minimization; that we have already shown how the potential minimization to reach some kind of expression between the displacement all these things, the nodal displacement. And finally, the stiffness in the matrix; the stiffness relation in the form of matrix can be represented like that, K is the stiffness matrix in the delta, delta is the general displacement field for the whole domain.

F sigma 0 if we consider, the initial stress exist in the particular system; epsilon initial strain existence, this is the load vector and this is the load vector for the either body forces or distributed load. I think p we consider distributed load. So, it is a kind of due to the distributed load this column; column vector comes and f w is the basically the due to the body forces. And if there is a concentrated load, the t implies in the concentrated load; but that concentrated load should be defined exactly on the node point.

Then we can use this relation that K into the stiffness matrix and for this case and corresponding to all this initial stress, strain and the concentrated load (Refer Time: 40:02); this is the final expression of this matrix. In general any kind of the stress strain problem or maybe displacement if you find the, or you can say the stress strain analysis in a finite element based model, so in using the finite element method.

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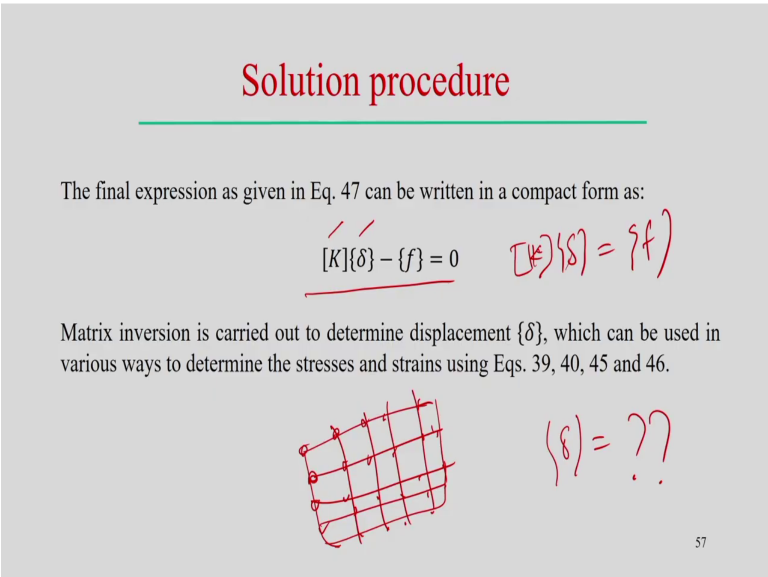

Now, general expression is that, if you look into the dimension of this matrix or column vector, all this can be represented in general way that, K delta equal to the stiffness matrix, delta is the displacement field and f counts all the this thing; f counts all the basically the load acting on the solution domain.

So, maybe the load can be in the form of a only concentrated load, distributed load, even consider the body force also or combining a all these three different types of the loads. So, finally, it is possible to reach this kind of expression, K delta equal to f. So, once is basically K delta equal to f. So, once we solve this equation, is it necessary to solve. So, definitely matrix inversion should be carried out to determine the displacement field.

So, finally, what is the delta that is the, that we can estimate looking into all the in associated with any kind of the particular problem. So, therefore, once we form this expression assuming the potential energy minimization in a particular system, then we reach. Finally, the a x equal to b in that particular form of the equation, where a indicates the or K indicates the stiffness matrix with the associated with this particular problem.

Then we solve for displacement field, so basically delta and displacement field for the whole domain; whole domain means for example, this is the domain of the analysis. So, if initially we create the element number of elements. Now, displacement we will define; so if you getting the displacement each and every node point displacement, delta each and every node point will be getting the displacement field.

So, once the displacement field for the each and every node point. So, once you get the displacement field, then we know the relation between the displacement and the strain. So, then we can calculate what is the strain value, strain of the you can estimate either strain each and every node point or with the strain for a particular element also, both it can be done the strain value.

Once we get the strain value, then from the strain value we can estimate the what is the stress value for each and every node point or it is possible to stress value on particular element based analysis also, that is also possible. So, all this, that means finally, we have to solve for the displacement field in any kind of general stress-strain problem; then once from the displacement field, we have to estimate what is the value of the stress and what is the value of the strain.

Then we will get the stress distribution and strains distribution in a particular problem. So, this is the general procedure for any kind of the stress analysis problem using the finite element method. So, thank you very much for your kind attention. Next class, I will try to look into the what are the particular problem maybe; in this cases we discussed only on the stress analysis problem, maybe next class we will discuss the other problem, maybe the heat transfer problem or the material flow problem, what way we can do the finite element formulation of a particular problem, so.

Thank you very much, for your kind attention.