Fundamentals of Convective Heat Transfer
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Module – 03
Convective Heat Transfer in External Flows – I
Lecture - 09
Falkner-Skan equation: Boundary layer flow over a wedge

Hello everyone. So, till now we have considered flow over flat plate, with constant or variable temperature boundary condition. In those cases, pressure gradient was 0; right, $\frac{dp}{dx}$ is 0 for flow over flat plate because your free stream velocity U_∞ was constant. Today, we will consider Boundary layer flow over a wedge, where U_∞ is function of x. So, for potential flow you know outside the boundary layer velocity will vary as Cx^m, where m is wedge parameter.

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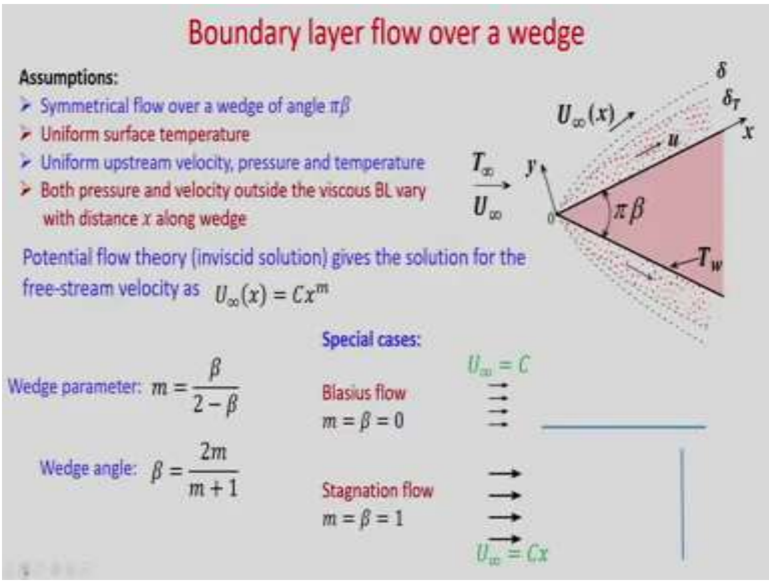

So, let us consider laminar boundary layer flow over a wedge. This is the case. So, you can see this is the wedge, with the wedge angle π β and your free stream temperature is T_∞ and free stream velocity is U_∞, x is measured along the surface and y is normal to the surface. So, obviously, you can see when fluid will flow over this wedge, it will be accelerating; the fluid velocity will accelerate.

So, obviously U_∞ will be function of x. So, these are the assumptions we will consider symmetrical flow over a wedge of angle πβ. Uniform surface temperature; so, we will consider T_w as wall temperature as constant. Uniform upstream velocity, pressure and temperature. And, both pressure and velocity outside the viscous boundary layer vary with distance x along wedge. So, potential flow theory gives the solution for the free stream velocity as $U_\infty(x) = Cx^m$, where m is the wedge parameter and C is constant.

The relation between these wedge parameter m and wedge angle β is given here. So, you can see wedge parameter m, you can write as $m = \frac{\beta}{2-\beta}$ and wedge angle you can write in terms of the wedge parameter $\beta = \frac{2m}{m+1}$. Now, you can see that as a special case, if you put β = 0 m will become 0. And, what will be the flow situation? If β becomes 0, so it will be flow over flat plate.

So, you see if this wedge angle β, if you put 0, then obviously, it will become flow over flat plate. And, if β is 0; that means, m will be 0 and if m is 0, so U_∞ will be constant. So, it will be a Blasius flow over a flat plate. And, if β = 1, so that will means it will become π, so it will be a vertical plate. So, if flow occurs then it is known as stagnation flow and for β = 1, m will be 1 and if you put m = 1 here, you can see $U_\infty = Cx$, so it is a stagnation flow.

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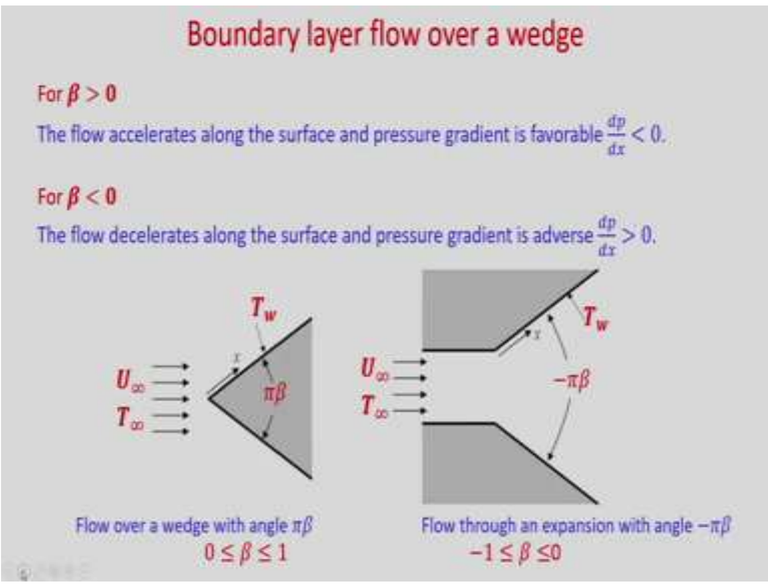

So, here you can see two different situations for $\beta > 0$. So, in this case you can consider that, flow over a wedge with angle $\pi\beta$ and β varies between 0 and 1, then the flow accelerates along the surface and pressure gradient is favourable. Because $\frac{dp}{dx} < 0$, so pressure gradient will be favourable and flow will accelerate.

For $\beta < 0$, so if you consider flow through an expansion with angle $-\pi\beta$. So, β will vary between -1 and 0, so you can consider this expansion. So, it is kind of diffuser. So, fluid is entering with velocity U_∞, once it is comes here, so you can see it is kind of diverging. So, the wall temperature is T_w and x is measured along this wedge surface. And, in this particular case, you can see the flow decelerates along the surface and pressure gradient is adverse that means, $\frac{dp}{dx} > 0$.

And, in this situation, it may happen that there will be a flow separation, and if flow separates boundary layer flow theory will not be valid. So, we will see that, at which wedge angle β flow separates? And, how do you know that flow separation has happened? When you will see the shear stress τ_w or the velocity gradient will become 0, so that time you will know that the, at that point, flow separates. So, after that boundary layer flow theory will not be valid.

So, let us consider laminar boundary layer flow over a wedge and we will use the similarity transformation technique similar to what we have done for solving the boundary layer equation for a flow over flat plate. We will use that, similarity variable as well as the velocity distribution.

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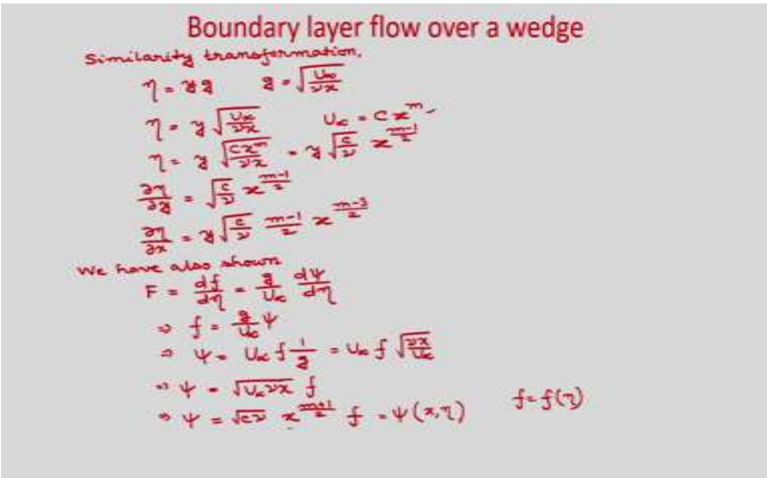

So, you can see what we derived in similarity transformation approach, similarity transformation. So, we have used the similarity variable $\eta = yg$ and g is function of x, and $g = \sqrt{\frac{U_\infty}{\nu x}}$.

So, you can see from our earlier lecture that, we have derived g which is function of x as, $g = \sqrt{\frac{U_\infty}{\nu x}}$. So, you can write $\eta = y\sqrt{\frac{U_\infty}{\nu x}}$. And, what is the U_∞? U_∞ is your free stream velocity and we have considered it as $U_\infty = Cx^m$, where m is the wedge parameter.

So, here if you put Cx^m then what you will get η? η you will get as $\eta = y\sqrt{\frac{Cx^m}{\nu x}}$ and you can write as $y\sqrt{\frac{C}{\nu}}x^{\frac{m-1}{2}}$. Now, let us take the derivative of η with respect to y as well as x. So, what you will get? $\frac{\partial \eta}{\partial x} = \sqrt{\frac{C}{\nu}}x^{\frac{m-1}{2}}$ and $\frac{\partial \eta}{\partial x} = y\sqrt{\frac{C}{\nu}}\frac{m-1}{2}x^{\frac{m-3}{2}}$.

So, now, let us write down the expression for f', whatever we have derived in earlier lecture. So, we have also shown $F = \frac{df}{d\eta} = \frac{g}{U_\infty}\frac{d\psi}{d\eta}$. So, where Ψ is the stream function and we have already told that f is having the physical significance of stream function.

So, from here you can write $f = \frac{g}{U_\infty}\psi$ equivalent to that. And, you can write $\psi = U_\infty f \frac{1}{g}$. And, what is g? $g = \sqrt{\frac{U_\infty}{\nu x}}$. So, you can write $\psi = U_\infty f\sqrt{\frac{\nu x}{U_\infty}}$. So, from here you can write $\psi = \sqrt{U_\infty \nu x}\, f$ and $U_\infty = Cx^m$.

So, if you put it here, so you will get $\psi = \sqrt{C\nu}\, x^{\frac{m+1}{2}} f$. So, you can see Ψ(x,η) because f is function of η we know, right. And, also you can see Ψ is function of x. So, it will be Ψ(x,η).

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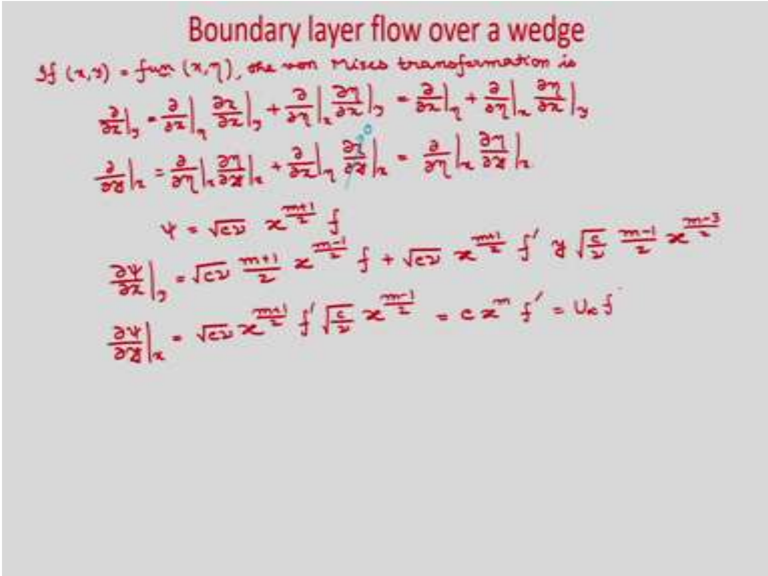

So, now let us write the von Mises transformation. So, what is von Mises transformation? So, if from any book you can see this von Mises transformation, if (x,y) = func (x,η), the von Mises transformation you can write as, $\frac{\partial}{\partial x}\Big|_y = \frac{\partial}{\partial x}\Big|_\eta \frac{\partial x}{\partial x}\Big|_y + \frac{\partial}{\partial \eta}\Big|_x \frac{\partial \eta}{\partial x}\Big|_y = \frac{\partial}{\partial x}\Big|_\eta + \frac{\partial}{\partial \eta}\Big|_x \frac{\partial \eta}{\partial x}\Big|_y$.

Similarly, you can write $\frac{\partial}{\partial y}\Big|_x = \frac{\partial}{\partial \eta}\Big|_x \frac{\partial \eta}{\partial y}\Big|_x + \frac{\partial}{\partial x}\Big|_\eta \frac{\partial x}{\partial y}\Big|_x = \frac{\partial}{\partial \eta}\Big|_x \frac{\partial \eta}{\partial y}\Big|_x$.

So, from here now, $\psi = \sqrt{C\nu}\, x^{\frac{m+1}{2}} f$.

So, from here you can write $\frac{\partial \psi}{\partial x}\Big|_y = \sqrt{C\nu}\,\frac{m+1}{2} x^{\frac{m-1}{2}} f + \sqrt{C\nu}\, x^{\frac{m+1}{2}} f\,' y\sqrt{\frac{C}{\nu}}\,\frac{m-1}{2} x^{\frac{m-3}{2}}$

Similarly, $\frac{\partial \psi}{\partial y}\Big|_x = \sqrt{C\nu}\, x^{\frac{m+1}{2}} f\,'\sqrt{\frac{C}{\nu}} x^{\frac{m-1}{2}}$. So, what you can write from here? So, you can see it will be $Cx^m f\,'$. And, it is nothing but, $U_\infty f\,'$. So, now, let us write the governing equations for flow over wedge.

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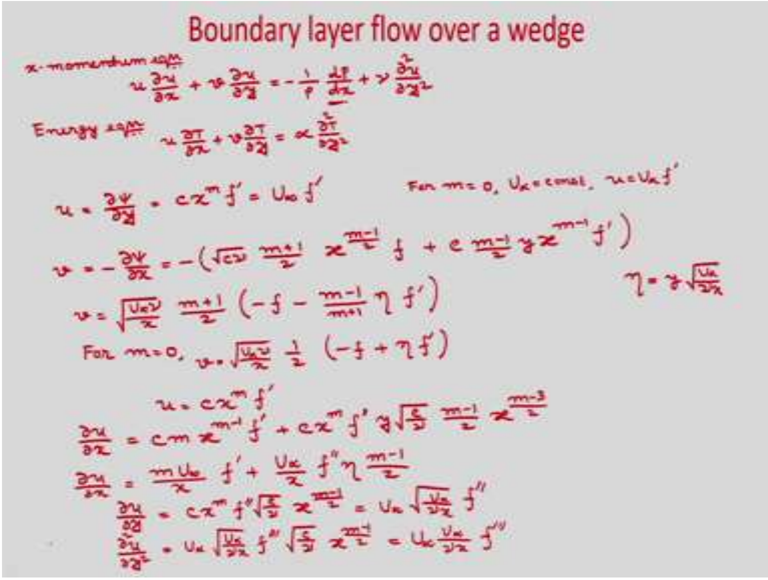

So, we will have the assumptions steady laminar flow, so obviously, you can write the momentum equation as $u\dfrac{\partial u}{\partial x}+v\dfrac{\partial u}{\partial y}$. Now, in this case you will have pressure gradient. So, it will be $-\dfrac{1}{\rho}\dfrac{dp}{dx}+\nu\dfrac{\partial^2 u}{\partial y^2}$. So, this is your x momentum equation.

And, what is the energy equation? Energy equation will remain same, neglecting the viscous distribution you can write $u\dfrac{\partial T}{\partial x}+v\dfrac{\partial T}{\partial y}=\alpha\dfrac{\partial^2 T}{\partial y^2}$. So, these are boundary layer equations, right. So, in this case as pressure gradient is not 0, so we have written $-\dfrac{1}{\rho}\dfrac{dp}{dx}$.

Now, we have found the stream function and its gradient with respect to x and y. So, you will be able to find the value of u, v and its gradient. So, first let us write u. So, $u=\dfrac{\partial \psi}{\partial y}$. So, already we have found it. So, it will be just $Cx^m f'$ and it is same as whatever we have written for the Blasius solution. So, for flow over flat plate we have already defined $f'=\dfrac{u}{U_\infty}$. So, from here you can write the x direction velocity $u=U_\infty f'$. And, for m = 0 obviously, flow over flat plate, U_∞ is constant and $u=U_\infty f'$ we have already shown.

Now, the velocity v you can write as $v = -\frac{\partial \psi}{\partial x}$, and that we have found already. So, if you see it will be $-(\sqrt{C\nu}\,\frac{m+1}{2}x^{\frac{m-1}{2}} f + C\frac{m-1}{2} y x^{m-1} f')$. So, you can see from the previous expression from here. So, that we have rearranged and we have written in this form.

So, if you rearrange it, you can see you can write, $v = \sqrt{\frac{U_\infty \nu}{x}}\frac{m+1}{2}(-f - \frac{m-1}{m+1}\eta f')$

And, if you see this is the similar expression this in the inside the bracket. So, if you put m = 0; m = 0, then what will be v? So, v will be m = 0. So, it will be, $v = \sqrt{\frac{U_\infty \nu}{x}}\frac{1}{2}(-f + \eta f')$. So, this is the expression already we have seen for flow over flat plate.

Now, u, v we have found. Now, we let us find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$. So, you can see from here u you have written $Cx^m f'$, so $\frac{\partial u}{\partial x}$ you can write $Cmx^{m-1} f'$. Again, we are using the von Mises transformation. You refer in last slide and use that expression $y\sqrt{\frac{C}{\nu}}\frac{m-1}{2}x^{\frac{m-3}{2}}$. So, $\frac{\partial u}{\partial x}$ expression you can write as, now again Cx^m will write as U∞. Hence,

$$\frac{\partial u}{\partial x} = Cmx^{m-1} f' + Cx^m f'' y\sqrt{\frac{C}{\nu}}\frac{m-1}{2}x^{\frac{m-3}{2}}$$

So, it will be $\frac{\partial u}{\partial x} = \frac{mU_\infty}{x} f' + \frac{U_\infty}{x} f''\eta\frac{m-1}{2}$.

And $\frac{\partial u}{\partial y} = Cx^m f''\sqrt{\frac{C}{\nu}}x^{\frac{m-1}{2}} = U_\infty\sqrt{\frac{U_\infty}{\nu x}} f''$.

And, $\frac{\partial^2 u}{\partial y^2} = U_\infty\sqrt{\frac{U_\infty \nu}{x}} f'''\sqrt{\frac{C}{\nu}}x^{\frac{m-1}{2}} = U_\infty\frac{U_\infty}{\nu x} f'''$. Now, you see we have written the expression for u, $\frac{\partial u}{\partial x}$, v, $\frac{\partial u}{\partial y}$, $\frac{\partial^2 u}{\partial y^2}$.

Now, we need to find the pressure gradient. So, for pressure gradient, so you can see for boundary layer flow, $\frac{\partial p}{\partial y}$ =0, right. So, whatever pressure will be there outside the boundary layer that will be impressed inside the boundary layer. So, outside you can use Bernoulli's equation and find, what is the temperature gradient.

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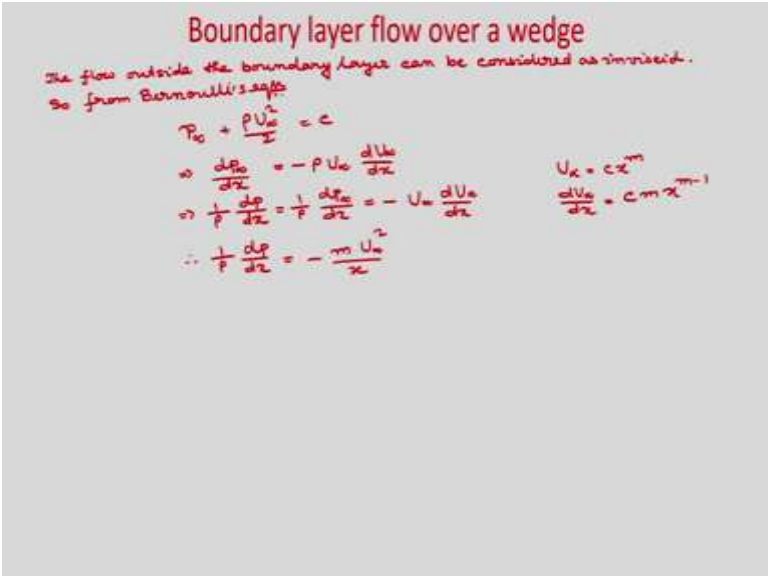

So, you can see the flow outside the boundary layer can be considered as inviscid. So, from Bernoulli's equation you can write $p_\infty + \frac{\rho U_\infty^2}{2} = C$ or you can write $\frac{dp_\infty}{dx} = -\rho U_\infty \frac{dU_\infty}{dx}$. So, you can write $\frac{1}{\rho}\frac{dp}{dx}$.

So, inside the boundary layer whatever pressure gradient will be there, that will be equal to the outside pressure gradient. So, that will be $\frac{1}{\rho}\frac{dp_\infty}{dx}$ which will be $-U_\infty \frac{dU_\infty}{dx}$.

And, we know $U_\infty = Cx^m$. So, $\frac{dU_\infty}{dx} = Cmx^{m-1}$. So, from here you can see $\frac{1}{\rho}\frac{dp}{dx}$ you can write as $-\frac{mU_\infty^2}{x}$. So, now, the expressions for u, v, $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, $\frac{\partial^2 u}{\partial y^2}$ and $\frac{dp}{dx}$ you put in the x momentum equation.

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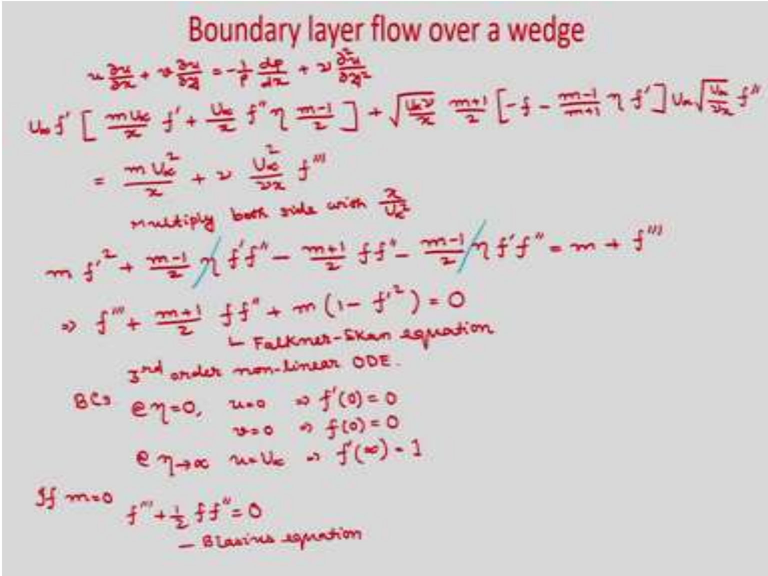

We have x momentum equation $u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{dp}{dx}+\nu\frac{\partial^2 u}{\partial y^2}$. So, all these expressions you put in this x momentum equation. So, we will get, $U_\infty f'[\frac{mU_\infty}{x}f'+\frac{U_\infty}{x}f''\eta\frac{m-1}{2}]+\sqrt{\frac{U_\infty \nu}{x}}\frac{m+1}{2}[-f-\frac{m-1}{m+1}\eta f']U_\infty\sqrt{\frac{U_\infty}{\nu x}}f''$.

Then, we have $-\frac{1}{\rho}\frac{dp}{dx}$, you can write as $\frac{mU_\infty^2}{x}$. And, $\frac{\partial^2 u}{\partial y^2}$. So, you can write, $\nu\frac{U_\infty^2}{\nu x}f'''$. So, this is the expression you simplify it, multiply both side by $\frac{x}{U_\infty^2}$. So, multiply both side with $\frac{x}{U_\infty^2}$ and simplify it.

So, if you do it, you will get, $mf'^2+\frac{m-1}{2}\eta f'f''-\frac{m+1}{2}ff''-\frac{m-1}{2}\eta f'f''$ and that will be just $m+f'''$.

So, you can see here, this terms will get cancelled, this term and this term and you will get, $f'''+\frac{m+1}{2}ff''+m(1-f'^2)=0$. So, you can see we started with partial differential equation, then after using the similarity transformation we could transfer the PD to ordinary differential equation. So, this equation is known as Falkner-Skan equation.

So, you can see this is a third order ODE and non-linear, third order non-linear ODE. And, what are the boundary conditions? Boundary conditions you have, at η = 0, ok, from u = 0; you can write $f'(0)= 0$ and from v = 0 you have the expression of v. So, from there you can write $f'(0)= 0$. It is kind of a stream function where you are assuming the value of stream function on the wall as 0. And, at η →∞, so u = U_∞. So, from here you can write $f'(\infty) = 1$.

If you put m =0 then, you will get flow over flat plate and you will get the Blasius equation back. So, you let us see whether we get it or not. So, if m = 0, then you can see from this equation m = 0, so this is the last term will become 0 and here it will be 1/2. So, it will be, $f''' + \frac{1}{2} ff'' = 0$; which is your Blasius equation.

Now, we have solved for the velocity profile because, these third order non-linear ordinary differential equation if you solve using numerical technique, then you will get the velocity profile for flow over a wedge. Now, let us consider energy equation as a special case we will consider surface or the wall as a uniform temperature, so temperature will remain constant; with that assumption let us find what will be the equation to find the temperature distribution.

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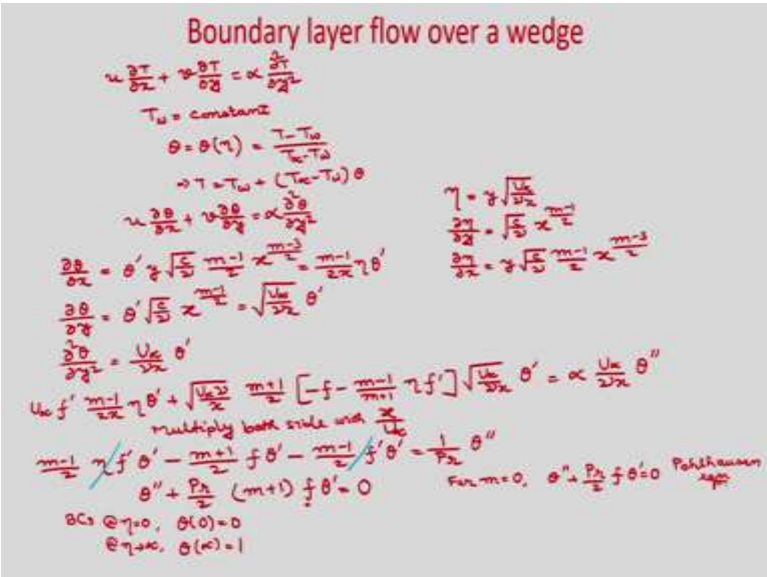

So, if you see we have written the energy equation as $u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 u}{\partial y^2}$. So, now, already we have the transformation details. Here, we are assuming T_w = constant. And, we are assuming θ is function of η only, $\theta=\theta(\eta)=\frac{T-T_W}{T_\infty-T_w}$. So, from here you can see, $T=T_w+(T_\infty-T_w)\theta$.

So, the equation you can see you can write if you put here, so you will get $u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}=\alpha\frac{\partial^2\theta}{\partial y^2}$. And, θ is function of η only and we have the transformation variable η, right. It is $\eta=y\sqrt{\frac{U_\infty}{\nu x}}$. So, let us find the value of $\frac{\partial T}{\partial x}$, $\frac{\partial T}{\partial y}$ and $\frac{\partial^2 T}{\partial y^2}$ and already we have the value for u and v.

So, $\frac{\partial\theta}{\partial x}$ now you find. So, $\eta=y\sqrt{\frac{U_\infty}{\nu x}}$. And, $\frac{\partial\eta}{\partial y}=\sqrt{\frac{C}{\nu}}x^{\frac{m-1}{2}}$ and $\frac{\partial\eta}{\partial x}=y\sqrt{\frac{C}{\nu}}\frac{m-1}{2}x^{\frac{m-3}{2}}$. And again, we will use the von Mises transformation and we will find $\frac{\partial\theta}{\partial x}$.

In this particular case, θ is function of η only, so easily you can find the derivative. So, you can write $\frac{\partial\theta}{\partial x}=\theta' y\sqrt{\frac{C}{\nu}}\frac{m-1}{2}x^{\frac{m-3}{2}}$. So, this you can write as $\frac{m-1}{2x}\eta\theta'$.

And, $\frac{\partial\theta}{\partial y}$ if you write it will be $\frac{\partial\theta}{\partial y}=\theta'\sqrt{\frac{C}{\nu}}x^{\frac{m-1}{2}}$. So, you can write $\sqrt{\frac{U_\infty}{\nu x}}\theta'$. And, $\frac{\partial^2\theta}{\partial y^2}=\frac{U_\infty}{\nu x}\theta'$.

So, if you now substitute this with the value of u v here. So, what you will get? $U_\infty f'\frac{m-1}{2x}\eta\theta'+\sqrt{\frac{U_\infty\nu}{x}}\frac{m+1}{2}[-f-\frac{m-1}{m+1}\eta f']\sqrt{\frac{U_\infty}{\nu x}}\theta'=\alpha\frac{U_\infty}{\nu x}\theta''$, multiply both side with $\frac{x}{U_\infty}$ and rearrange it.

So, you will get as $\frac{m-1}{2}\eta f'\theta' - \frac{m+1}{2} f\theta' - \frac{m-1}{2}\eta f'\theta' = \frac{\alpha}{\nu}\theta''$ that means, it will be, $\frac{1}{\Pr}\theta''$. So, this term this term will get cancelled, so you will get finally, $\theta'' + \frac{\Pr}{2}(m+1) f\theta' = 0$. So, you can see this is the ODE and this is linear equation, because already you know the value of f, right from the velocity distribution. So, this is known. So, from the Falkner-Skan solution you will be knowing the velocity profile. So, from there you can calculate the f.

And, if you put for m = 0 as a special case flow over flat plate, then you can see m = 0; that means, it will be $\theta'' + \frac{\Pr}{2} f\theta' = 0$. This is your Pohlhausen equation we have already derived for flow over flat plate. And, what are the boundary conditions? So, boundary conditions at η = 0 T = T_w. So, if it is T =T_w, then θ = 0. And, at η →∞, so T will be T_∞, so it will be 1.

And, you can see this is the second order ODE. Using some numerical technique you can solve this equation, you can find the temperature distribution and if you know that, already we have derived the expression for temperature non-dimensional temperature for flow over flat plate, similarly, you can do the analysis here and you can find the temperature distribution as non-dimensional temperature.

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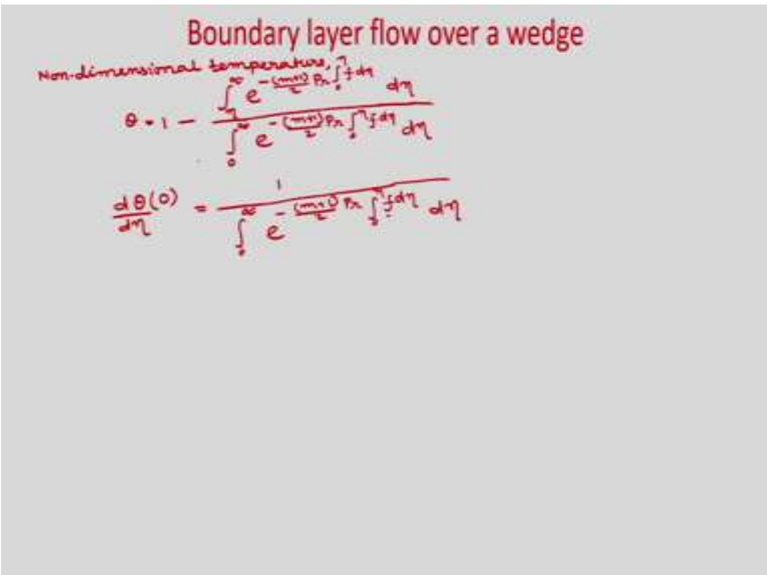

$$\theta = 1 - \frac{\int_{\eta}^{\infty} e^{-\frac{(m+1)}{2}\Pr\int_0^{\eta} f d\eta} d\eta}{\int_{0}^{\infty} e^{-\frac{(m+1)}{2}\Pr\int_0^{\eta} f d\eta} d\eta}.$$

And, if you remember, we have written the expression for $\frac{d\theta}{d\eta}$ it is required to find the Nusselt number or heat transfer coefficient. So, that at η = 0, right; $\frac{d\theta(0)}{d\eta} = \frac{1}{\int_{0}^{\infty} e^{-\frac{(m+1)}{2}\Pr\int_0^{\eta} f d\eta} d\eta}$. So, once you know the f from the velocity distribution you will be able to find, what is $\frac{d\theta}{d\eta}$? And, using some numerical technique you can find the value.

So, if you solve this Falkner-Skan equation whatever we have derived. So, you will get the velocity profile. And, once you know the velocity profile from the energy equation whatever we have derived ordinary differential equation you will be able to find the temperature distribution. And, once you know the temperature distribution you will be able to find $\frac{d\theta}{d\eta}$ = 0 that means, the temperature gradient at the wall and you will be able to find, the local heat transfer coefficient, local Nusselt number, average heat transfer coefficient and average Nusselt number.

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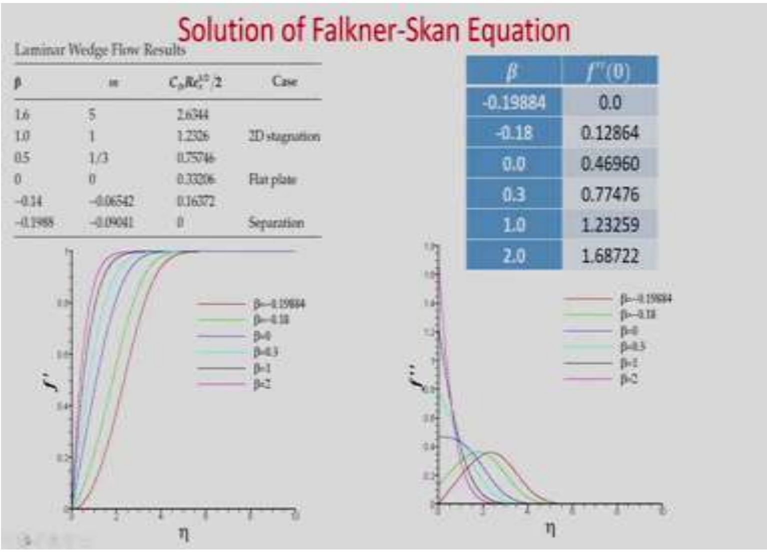

So, if you see laminar wedge flow, so for different β and corresponding m value is shown here and if you see $C_{fx} \mathrm{Re}_x^{1/2}/2$, this value if you see here then, you can see for β=0, m = 0. So, what is your $f''(0)$? That means, your velocity gradient at the wall. So, that is 0.33206. So, that you have already found, right. So, this is the flat plate case.

And, if you see the case, where this is your $f''(0)$; that means, your velocity gradient at the wall. So, velocity gradient at the wall become 0. What does it mean? Shear stress is 0. And, if it become shear stress as 0; then obviously, the fluid particle will just float, and that is the point of flow separation. And, flow separation occurs at that point. So, you corresponding β value you can see an m value you can see where flow separates.

So, after that your boundary layer theory is not valid, because the important assumptions, one of the important assumptions we have taken while deriving the boundary layer equation is that flow does not separate. So, here flow is separating at this β value. So, this is the separation point.

Now, for different β value, so this is the similar case. And, at this point you can see that $f''(0)$ becomes 0 for this particular β, and if you see the f' versus η, what is f'? $f' = \frac{u}{U_\infty}$. So, $\frac{u}{U_\infty}$ and η, so you can see what is the velocity profile at different value of β.

So, this is the case, where flow separates. And, if you plot the f'' versus η then you will be able to see, so for this particular case where flow separates you can see, the value of f'' is 0. And, raised other β values, you can see you have a positive value that means, shear stress is present at the wall, but at β value of -0.19884 your velocity gradient becomes 0 and flow separates.

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Boundary layer flow over a wedge

	$Nu_x / Re_x^{1/2} = \frac{d\theta(0)}{d\eta}$				
m	$Pr = 0.7$	0.8	1.0	5.0	10.0
−0.0753	0.242	2.53	0.272	0.457	0.570
0	0.292	0.307	0.332	0.585	0.730
0.111	0.331	0.348	0.378	0.669	0.851
0.333	0.384	0.403	0.440	0.792	1.013
1.0	0.496	0.523	0.570	1.043	1.344
4.0	0.813	0.858	0.938	1.736	2.236

$$h_x = \frac{k}{x}\sqrt{Re_x}\frac{d\theta(0)}{d\eta} \qquad \bar{h} = \frac{2k}{L}\sqrt{Re_L}\frac{d\theta(0)}{d\eta}$$

$$Nu_x = \sqrt{Re_x}\frac{d\theta(0)}{d\eta} \qquad \overline{Nu} = 2\sqrt{Re_L}\frac{d\theta(0)}{d\eta}$$

Now, if you see the $\frac{d\theta(0)}{d\eta}$; that means, $\frac{Nu_x}{\mathrm{Re}_x^{1/2}}$. So, for different values of m and these for different Prandtl number 0.7, 0.8, 1, 5 and 10, these are the values of $\frac{d\theta(0)}{d\eta}$ at different m.

So, now once you know the value of $\frac{d\theta}{d\eta}$ you will be able to calculate the local heat transfer coefficient because, $h_x = \frac{k}{x}\sqrt{\mathrm{Re}_x}\frac{d\theta(0)}{d\eta}$. So, for at any Prandtl number and any m value you will be able to find local heat transfer coefficient, local Nusselt number, average heat transfer coefficient and average Nusselt number. From this table you can see. So, from the solution of the Falkner-Skan equation and the energy equation the value of $\frac{d\theta(0)}{d\eta}$, at $\eta = 0$, it is diluted.

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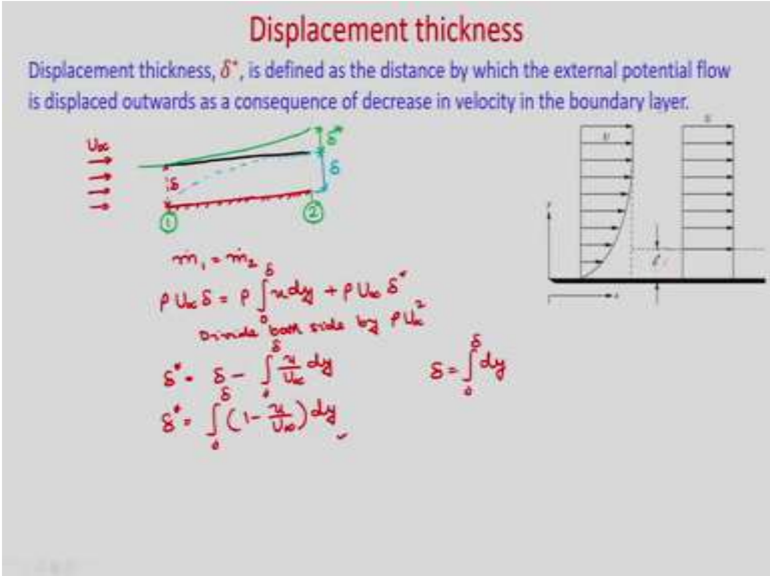

Now, let us define displacement thickness, momentum thickness and shape factor for boundary layer flow. So, first let us discuss displacement thickness. Consider just free stream velocity U_∞. So, if you consider this case then obviously, you can see your stream line will be all parallel to each other, right.

So, if you consider a stream line, so it will be parallel to each other all stream line. Now, if you bring one flat plate here. So, what will happen? So, due to the presence of flat plate there will be formation of boundary layer, over this flat plate, so it will. So, this is the boundary layer. So, this, at this location, this is your boundary layer thickness δ.

Now, you can see that in the presence of flat plate, this stream line will be no longer flat, ok, because it will deflect. Why? Because velocity gradient will be there and due to the velocity gradient to have the same mass flux at each location these stream line will deflect. So, your new stream line will be like this. So, it was earlier flat sorry a straight line, but there will be deflection due to the presence of this flat plate.

Because, you can see at this location 1 and at this location 2, you should have same mass flow rate and as it is a stream line there will be no flow across the streamline. So, mass flow rate at 1 should be equal to mass flow rate 2. And hence, as velocity gradient will be there these stream line will deflect and these deflection is known as displacement thickness δ^*.

So, now, let us find, what is the δ*? So, from here you can see that at section 1 what is mass flow rate, will be equal to mass flow rate at section 2. So, at section 1 it will be just $\rho U_\infty \delta$ because here also, it is same thickness δ. And, at section 2 there will be velocity variation here.

So, it will be $\rho\int_0^\delta u dy + \rho U_\infty \delta^*$. Now, divide both side by ρU_∞ and rearrange. So, you can find $\delta^* = \delta - \int_0^\delta \frac{u}{U_\infty} dy$. Now, this δ we can write $\delta = \int_0^\delta dy$, so δ* if you put it here and you can write $\delta^* = \int_0^\delta (1 - \frac{u}{U_\infty}) dy$.

So, this is the mathematical expression of displacement thickness. And, also you can define in this way say you have a boundary layer and this is the velocity distribution, and if you have a free stream velocity then, how much distance you have to shift this wall to maintain the same mass flux. So, that distance is known as also δ*. So, you can see here displacement thickness δ* is defined as the distance by which the external potential flow is displaced outwards as a consequence of decrease in velocity in the boundary layer.

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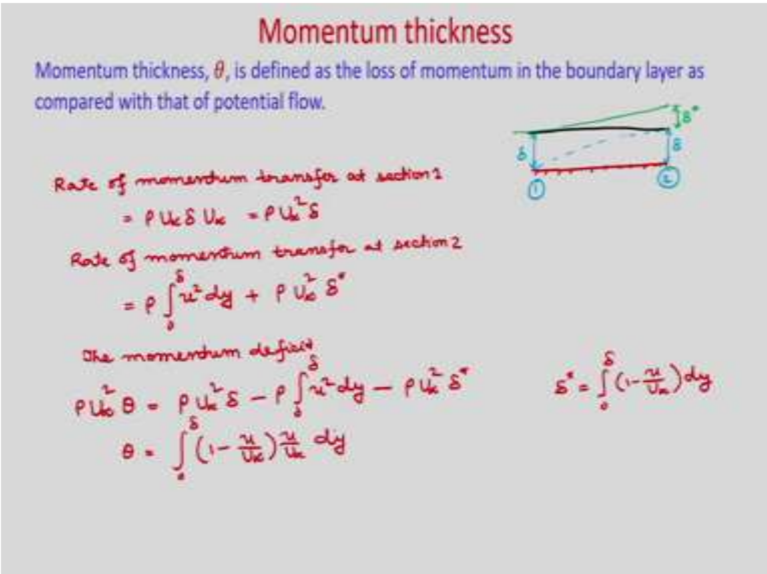

Now, we will talk about the momentum thickness. So, momentum thickness θ is defined as the loss of momentum in the boundary layer as compared with that of potential flow.

So, let us consider the same figure here. So, this is your flat plate and this is your stream line, and there will be formation of boundary layer, and this is your δ, this is also δ, section 1 and this is your section 2, but there is a deflection in a boundary layer and so this will be your δ*, right. This is your displacement thickness.

So, you can see that, if you see the rate of momentum at section 1 and section 2, it will not be same although, mass flow rates are same. So, there will be deficit in momentum at section 2. So, this deficit is known as momentum thickness. So, mathematical if you see here. So, what is the rate of momentum transfer at section 1, at section 1? So, what is that? That will be your $\rho U_\infty \delta U_\infty$. So, that is your momentum.

So, $\rho U_\infty \delta U_\infty$. So, that will be $\rho U_\infty^2 \delta$. And, rate of momentum transfer at section 2. So, at section 2 you can see; so, it will be $\rho \int_0^\delta u^2 dy$. So, this is your momentum transfer in this section and in outside it will be $+\rho U_\infty^2 \delta *$.

So, now, you see there will be a deficit. And, this deficit momentum deficit you can find the momentum deficit. So, this the difference you can write as, $\rho U_\infty^2 \theta = \rho U_\infty^2 \delta - \rho \int_0^\delta u^2 dy - \rho U_\infty^2 \delta *$.

And, you know the value of δ* we have found in last slide. So, it is $\delta * = \int_0^\delta (1 - \frac{u}{U_\infty}) dy$.

So, you put these values in the δ*, you divide by ρU_∞^2, rearrange this, you will get $\theta = \int_0^\delta (1 - \frac{u}{U_\infty}) \frac{u}{U_\infty} dy$.

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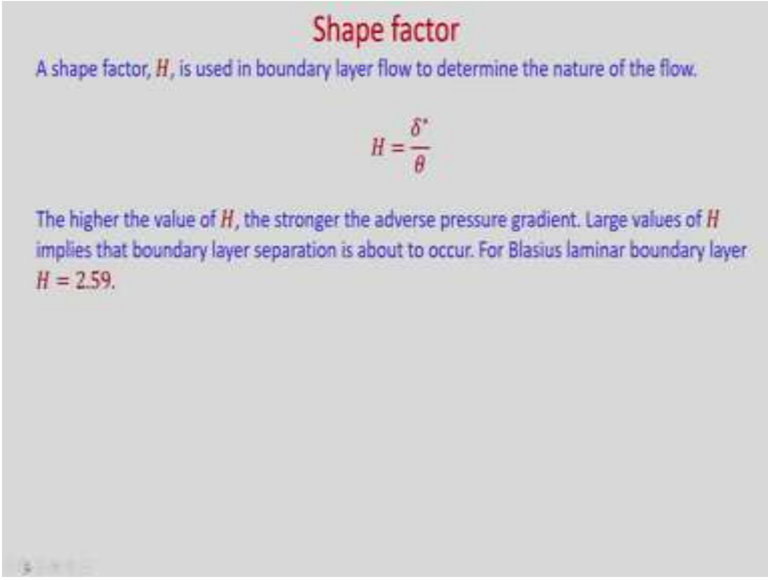

And shape factor, so a shape factor H is used in boundary layer flow to determine the nature of the flow. So, H is defined as ratio of displacement thickness to the momentum thickness. So, $H = \frac{\delta *}{\theta}$. And, you can see it is known as shape factor, because it solely depends on the shape of the velocity. So, the higher the value of H; the stronger the adverse pressure gradient and large values of H implies that boundary layer separation is about to occur. And, for Blasius laminar boundary layer flow you can find H = 2.59.

So, in today's class we considered laminar boundary layer flow over a wedge and we defined the wedge angle and the wedge parameter and velocity varies as Cx^m as a potential flow. And then, we define the stream function and the similarity variable η and from there we found the velocity u and v from the stream function gradient.

And later, we from the momentum equation we substituted this value of u, v and velocity gradients and we derived the Falkner-Skan equation. And, you can see the Falkner-Skan equation is the third order non-linear ordinary differential equation, so using any numerical technique you can solve this ordinary differential equation.

Then, we considered energy equation keeping the T_w as constant. And, again we define the non-dimensional temperature θ which is function of η only, and we converted this partial differential equation to ordinary differential equation using similarity transformation and that is second ordered ODE and Linear.

And, from there we express the non-dimensional temperature and also $\frac{d\theta(0)}{d\eta}$. Then, we have shown the numerical solution for this Falkner-Skan equation, and we have shown that flow separates at a particular value of β and after that your laminar boundary layer theory will not be valid. At last we defined the displacement thickness as well as the momentum thickness and shape factor.

Thank you.