Fundamentals of Convective Heat Transfer

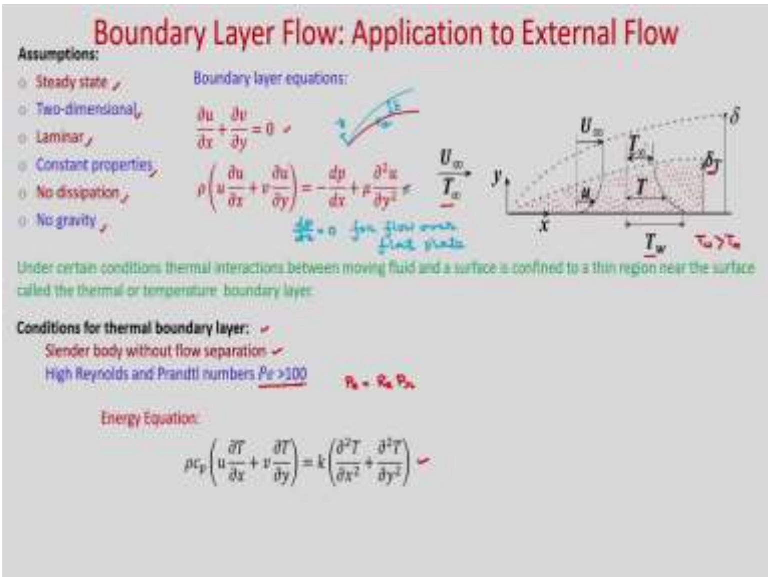
Prof. Amaresh Dalal
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Module - 02
Preliminary Concepts
Lecture - 05
Derivation of boundary layer energy equation

Hello everyone. So, in last class, we discussed about the velocity boundary layer and we have seen that near to the wall, there is a effect of viscosity and this is a viscous region and away from the wall, there is a region where there is no effect of viscosity and that is known as inviscid region and we have discussed about the edge of the boundary layer and boundary layer thickness.

Today, we will discuss about thermal boundary layer. We can see there is a region near to the wall, there will be an effect of this temperature gradient, but away from the surface, it will be at the free stream temperature T ∞.

(Refer Slide Time: 01:18)



So, we will start with the assumptions. So, we will consider steady state, two-dimensional and laminar flow with constant properties and neglecting dissipation and gravity.

So, in last class, you can see we have derived these boundary layer equations so, this is your continuity equation and this is the momentum equation in general. So, if you have any curved surface and over it if there is a velocity boundary layer then obviously, you can see normal to this surface if you measure the distance from the surface to the edge of this boundary layer so, that is known as boundary layer thickness and there will be a change in the pressure along the x direction. So, along the x direction and perpendicular to this surface this is your y.

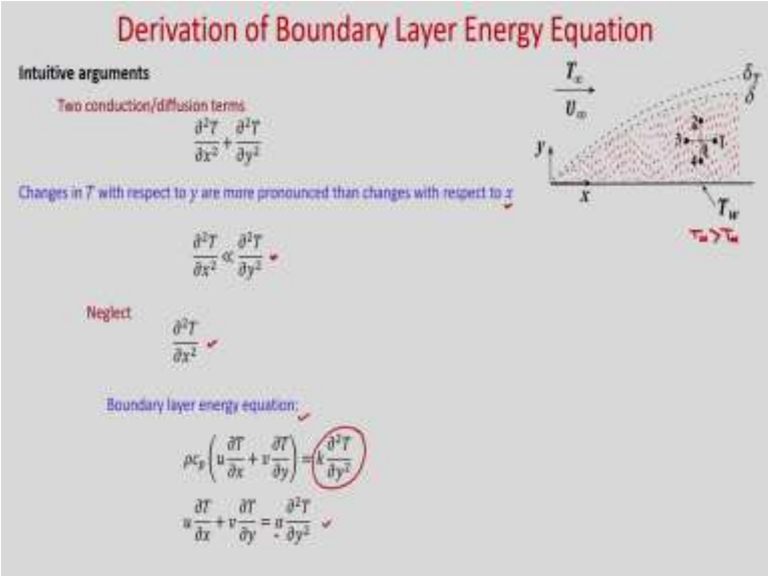
So, in general, we have derived this equation, but as a special case, if you consider flow over flat plate, then this $\frac{dp}{dx}$ becomes 0 for flow over flat plate. Now, let us discuss about the thermal boundary layer. Let us consider that the wall is maintained at a higher temperature than the free stream temperature. In this condition, you can see that you have a wall temperature T_w and free stream temperature is T_∞. Obviously, we have considered $T_w > T_\infty$. So, wall is hot and obviously, your $T_\infty < T_w$.

There will be some region where you can see that there will be change of temperature and at a certain distance from the plate; you will find that this temperature will become equal to free stream temperature T_∞. So, that distance is known as thermal boundary layer thickness δ_T. So, we measure normal distance from the surface.

So, obviously, you can see that under certain conditions thermal interactions between moving fluid and a surface is confined to a thin region near the surface called the thermal or temperature boundary layer and the distance from the normal from the wall the distance at which this temperature becomes almost 99 % of the free stream velocity normal to the surface is known as thermal boundary layer thickness.

So, now we have energy equation, this is your energy equation. Now, under certain conditions can we drop some term from this equation when we consider external flows which we consider as the boundary layer flows. So, in this case also we consider that these are the conditions for thermal boundary layer slender body without flow separation and it is a high Reynolds and Prandtl numbers flows and we know that Peclet number is product of Reynolds number and Prandtl number and generally if it is > 100, then thermal boundary layer is significant.

(Refer Slide Time: 05:30)



So, like earlier class, now we will consider both approaches intuitive approach as well as scaling approach. First, let us discuss about intuitive approach. Let us consider that one insect is there at position 0 and it finds very hot at this position so, it wants to move to some position 1, 2, 3, 4 and we considered that $T_w > T_\infty$.

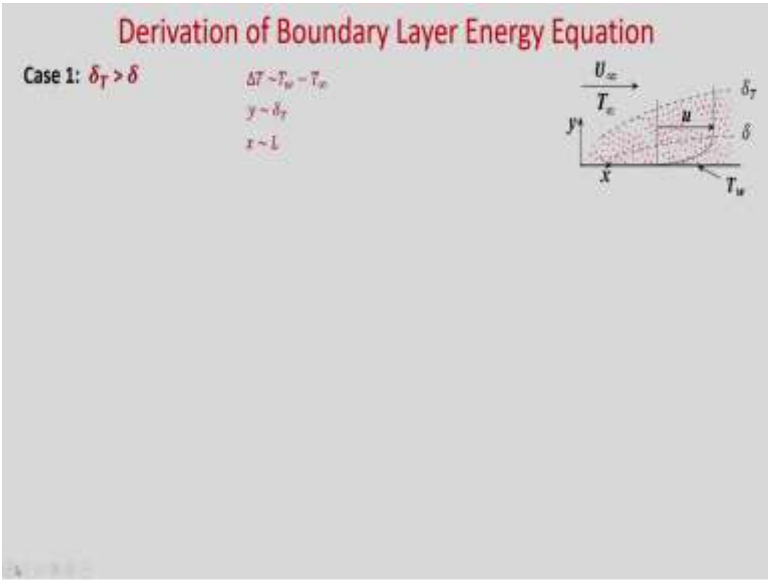
So, under this condition, this insect if it is feeling very hot at this position where it should move? So, you can see that if it moves to 1 position or 3 position obviously, it will not find that much change in the temperature, but if it moves to position 2, then obviously, it will get some relief.

So, from intuitive approach, you can see that obviously, if it is go away from the surface to position 2, then it will feel somewhat cooler than earlier position. Now, let us consider that the insect is near to the wall surface. Let us consider that the insect is near to the wall and it wants to go to a some position where it will feel some significance really, then obviously, you can see that it will go away from the surface because the temperature change is more in that direction, but if it travels in the axial direction, then obviously, there will be not much change in the temperature.

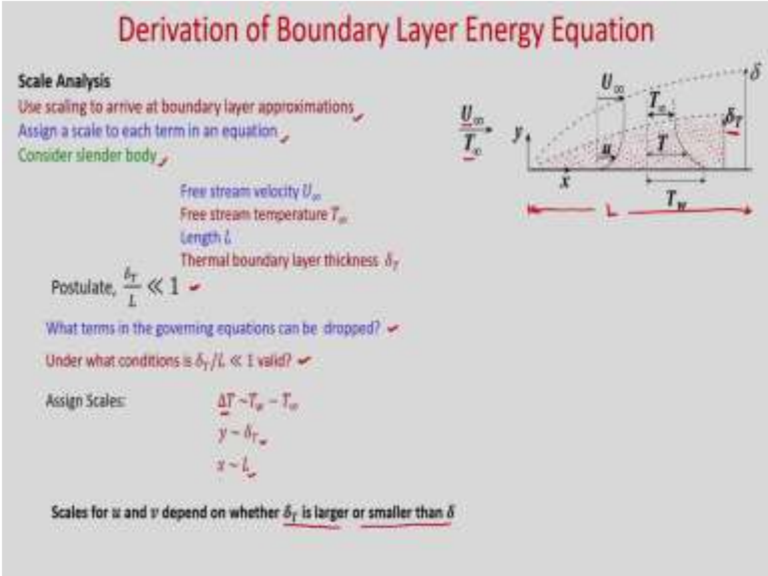
So, now we are considering about the change of temperature right so that means, gradient. So obviously, from the intuitive approach, you can see that your normal direction temperature gradient is higher than the axial temperature gradient. So, from there you can see that $\frac{\partial^2 T}{\partial x^2} \ll \frac{\partial^2 T}{\partial y^2}$. So, changes in T with respect to y are more pronounced than changes with respect to x. So, under this condition, you can neglect $\frac{\partial^2 T}{\partial x^2}$. So, you neglect this term.

So, if you see now the energy equation, we can write as considering only one term in the right-hand side which is your diffusion term. So, this is known as boundary layer energy equation and if you divide by ρ C_p, then you can write $u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial x} = \frac{k}{\rho C_p}$ that is your thermal diffusivity $\alpha \frac{\partial^2 T}{\partial y^2}$. So, we can see that we have dropped one term $\frac{\partial^2 T}{\partial x^2}$ because it is much smaller than the $\frac{\partial^2 T}{\partial y^2}$.

(Refer Slide Time: 08:34)



(Refer Slide Time: 08:38)

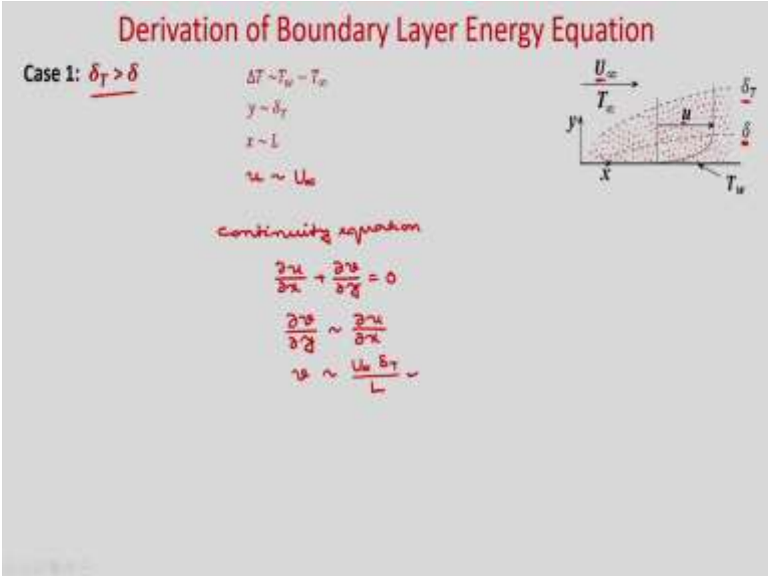


Now, let us consider the scaling approach. In scaling approach, we will examine the order of magnitude of each term and we will see that if some term can be dropped. So, use scaling to arrive at boundary layer approximation. Assign a scale to each term in an equation and obviously, we are considering slender body. So, you know that we have a free stream velocity U_∞. Free stream temperature T_∞ and the plate length is L and we have thermal boundary layer thickness δ_T.

Now, let us postulate that $\frac{\delta_T}{L} \ll 1$. So, with that assumption, let us go ahead and we will see that under what condition this is valid. So, now, let us see what terms in the governing equations can be dropped and under what conditions is $\frac{\delta_T}{L} \ll 1$ valid.

So, now let us assign the scales. We will assign the scales temperature difference $\Delta T \sim T_w - T_\infty$, $y \sim \delta_T$ thermal boundary layer thickness and x in axial direction it is order of the plate length L. In this particular case, now the scale of u will depend on the value of Prandtl number and obviously, your scale of v will also depend on the value of Prandtl number. So, we can see scales for u and v depend on whether δ_T is larger or smaller than δ.

(Refer Slide Time: 10:27)



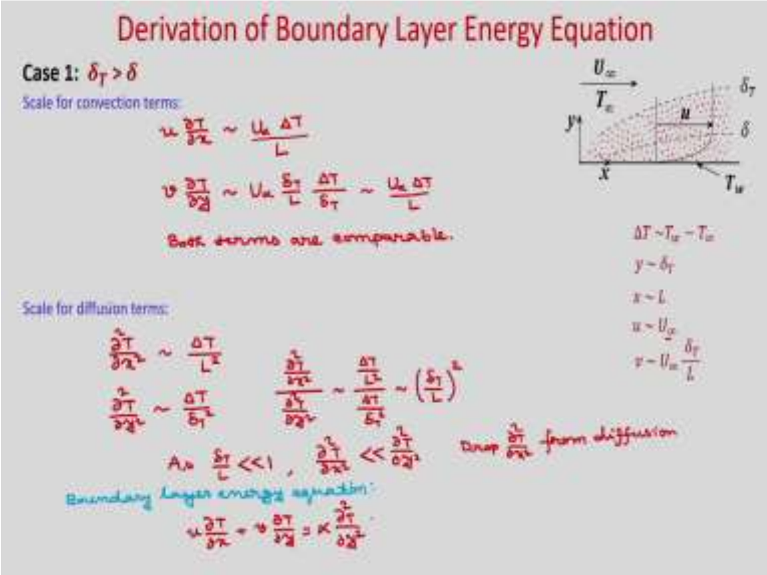
So, first let us consider that thermal boundary layer thickness is higher than the velocity boundary layer thickness. So, you see here. So, this is your thermal boundary layer thickness and it is higher than the velocity boundary layer thickness δ. If you see the velocity distribution obviously, the velocity will vary up to the velocity boundary layer thickness δ which is known as also hydro dynamic boundary layer thickness and after that it will have the same velocity U_∞ right.

So, the scale of u we can consider in this particular case as the order of U_∞ which is your free stream velocity. When $\delta_T > \delta$, we will consider the scale of velocity u as order of free stream velocity U_∞ because you can see that at δ_T , we have the free stream velocity U_∞.

Now, we will use continuity equation to find the order of velocity v. So, we know the continuity equation $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ so obviously, you can write $\frac{\partial v}{\partial y} \sim \frac{\partial u}{\partial x}$. So, the v will be order of so, what is the order of u? It will be U_∞, order of $y \sim \delta_T$ and $x \sim L$. So, you can see $v \sim \frac{U_\infty \delta_T}{L}$.

Now, let us examine the convection terms. We have two convection terms and let us see that what is the order of magnitude of each term.

(Refer Slide Time: 12:31)



So, if you see the scale of convection term so, we have $u\dfrac{\partial T}{\partial x}$ this is the first term and what is the order of this? So, $u \sim U_\infty$, $\delta_T \sim \Delta T$ so, that we will write $\dfrac{\delta_T}{\delta_x}$ so, $x \sim L$.

So, $\dfrac{U_\infty \Delta T}{L}$.

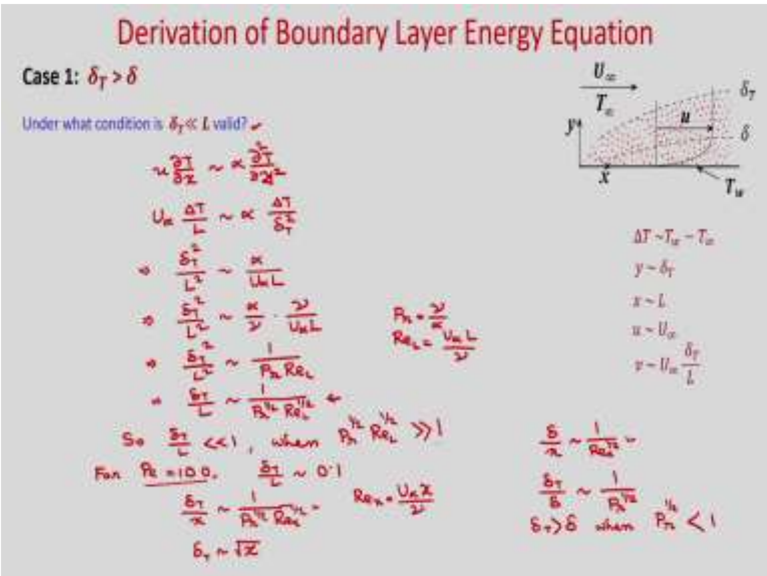
Similarly, the other convection term $v\dfrac{\partial T}{\partial y}$. So, this is the order of v so, v is $\dfrac{U_\infty \delta_T}{L}$ and $\delta_T \sim \Delta T$ which is $T_w - T_\infty$ and your $y \sim \delta_T$. So, you can see these δ_T δ_T will cancel out so, it will have $\dfrac{U_\infty \Delta T}{L}$. So, you can see both the terms are having the same order. So, we cannot drop any term in the convection. So, both terms are comparable. So, you cannot drop any term from this convection.

Now, let us consider the diffusion terms. So, first term is $\frac{\partial^2 T}{\partial x^2}$. So, what is the order of this? So, you can see it is $\frac{\Delta T}{L^2}$ and the next term is $\frac{\partial^2 T}{\partial y^2}$. So, what is the order? It is $\frac{\Delta T}{\delta_T^2}$.

So, now, let us see. So, we know that $\frac{\frac{\partial^2 T}{\partial x^2}}{\frac{\partial^2 T}{\partial y^2}} \sim \frac{\frac{\Delta T}{L^2}}{\frac{\Delta T}{\delta_T^2}} \sim (\frac{\delta_T}{L})^2$.

Now, we have already postulated that $\frac{\delta_T}{L} \ll 1$. So, what does it mean? From here, you can see that $\frac{\partial^2 T}{\partial x^2} \ll \frac{\partial^2 T}{\partial y^2}$. So, as $\frac{\delta_T}{L} \ll 1$ so obviously, you can see $\frac{\partial^2 T}{\partial x^2} \ll \frac{\partial^2 T}{\partial y^2}$. So, drop $\frac{\partial^2 T}{\partial x^2}$ from diffusion. So, if you drop these term, then obviously, essentially you will get the boundary layer energy equation. So, boundary layer energy equation you can write $u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}$ both are comparable so, we did not drop any term is equal to now you have thermal diffusivity $\alpha\frac{\partial^2 T}{\partial y^2}$ dropping the term $\frac{\partial^2 T}{\partial x^2}$.

(Refer Slide Time: 16:16)



So, now, let us see that under what condition is δ_T << L valid. So, for that we will take one convection term, one diffusion term and obviously, as both terms are there in the boundary layer energy equation so, both will be comparable. So, you can see that the convection term $u\frac{\partial T}{\partial x}$, we can compare with the diffusion term $\alpha\frac{\partial^2 T}{\partial y^2}$. So, now put the scale. So, what is the scale of u? It is $U_\infty\frac{\Delta T}{L} \sim \alpha\frac{\Delta T}{\delta_T^2}$.

So, if you see, now we can write δ_T^2 this side and we will write L^2 so, we have divided by L in the left-hand side so, in the right-hand side, we will write $\frac{\alpha}{U_\infty L}$,we have divided by L in the left-hand side so, we will also divide in the right-hand side. So, we can write $\frac{\delta_T^2}{L^2} \sim \frac{\alpha}{\nu}\frac{\nu}{U_\infty L}$, you see.

What is $\frac{\nu}{\alpha}$? $\frac{\nu}{\alpha}$ is nothing but Prandtl number and Reynolds number is $\frac{U_\infty L}{\nu}$. So, Prandtl number is $\frac{\nu}{\alpha}$ and Reynolds number based on the characteristic length L in this particular case, it is plate length is $\frac{U_\infty L}{\nu}$ where ν is your kinematic viscosity. So, you can see, we can write $\frac{\delta_T^2}{L^2} \sim \frac{1}{P_r R_{e_L}}$ based on L.

So, you can see that $\frac{\delta_T}{L} \sim \frac{1}{P_r^{1/2} R_{e_L}^{1/2}}$. So, from here, you can see when this $\frac{\delta_T}{L}$<<1. So, you can see here. So, $\frac{\delta_T}{L}$<< 1 when you can see your $P_r^{1/2} R_{e_L}^{1/2}$ >> 1. So, from this you can see .

So, $\frac{\delta_T}{L}$, we have already postulated that it is << 1 when your $P_r^{1/2} R_{e_L}^{1/2}$ >> 1. So, that means, this is your Peclet number. So, $P_e^{1/2}$ >> 1; that means, Peclet number also will be >>1.

So, in this case, if you consider let us say Peclet number is 100. So, if you consider Peclet number 100, then what will be your $\frac{\delta_T}{L}$? So, if it is Peclet number 100 so, you can see for Peclet number is 100 so, $\frac{\delta_T}{L}$ from here what will be that value? So, it will be $\sqrt{100}$. So, it will be 10, $\frac{1}{10}$ so, it will be 0.1. So, you can see that Peclet number is 100 $\frac{\delta_T}{L}$ will be 0.1. So, it is very small. So, generally we say that for thermal boundary layer to exist Peclet number should be greater than 100.

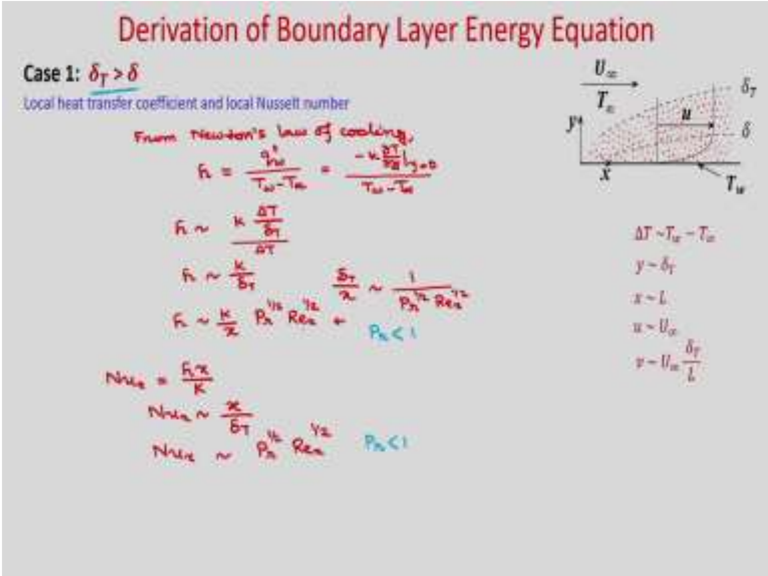
Now, you can see that we have written $\frac{\delta_T}{L}$ as this expression. So, for any x we can write $\frac{\delta_T}{x} \sim \frac{1}{P_r^{1/2} R_{e_x}^{1/2}}$ so that means, R_{e_x} we are defining as $\frac{U_\infty x}{\nu}$. So, for at any x or $\frac{\delta_T}{x}$, we can write $\frac{1}{P_r^{1/2} R_{e_x}^{1/2}}$.

So, from this expression, you can see how your thermal boundary layer thickness varies with x. So, you can see from this expression, it varies with root x and velocity boundary layer also varies with $\sqrt{x}$. So, you can see from this expression that δ_T varies with $\sqrt{x}$. You can see that we have already found the hydrodynamic boundary layer thickness $\frac{\delta}{x} \sim \frac{1}{R_{e_x}^{1/2}}$. So, $\frac{\delta}{x}$ is this expression and $\frac{\delta_T}{x}$ is having this expression. So, you can see $\frac{\delta_T}{\delta}$ what you can write? So, $\frac{\delta_T}{\delta}$ so, this you can write as $\frac{1}{P_r^{1/2}}$.

So, now, you can see $\frac{\delta_T}{\delta} \sim \frac{1}{P_r^{1/2}}$. So, what does it mean that $\delta_T > \delta$ when your $P_r^{1/2} < 1$.

So, you can see that when you have low Prandtl number fluids, then your thermal boundary layer thickness will be higher than the hydro dynamic boundary layer thickness. So, generally you see that liquid metals are having low Prandtl number values. So, for those cases, you can have the thermal boundary layer thickness, you can have higher than the hydro dynamic boundary layer thickness.

(Refer Slide Time: 22:51)



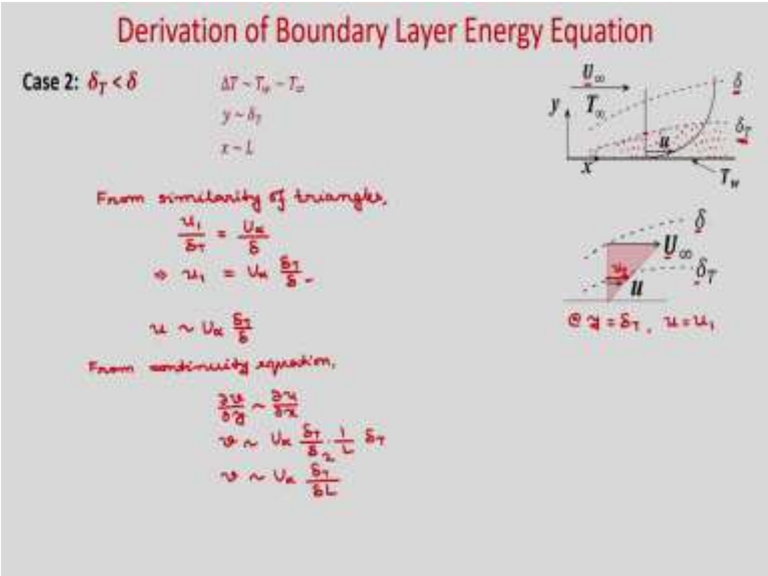
Now, let us discuss about the local heat transfer coefficient and local Nusselt number and what is the order we will find. So, you know local heat transfer coefficient definition from Newton’s law of cooling, you can write; from Newton’s law of cooling you can write $h = \frac{q_w^{"}}{T_w - T_\infty}$ and what is your heat flux? Heat flux is nothing but $-\frac{k\frac{\partial T}{\partial y}\big|_{y=0}}{T_w - T_\infty}$.

Now, let us see what is the order of h. So, now, $h \sim \frac{k\frac{\Delta T}{\delta_T}}{\Delta T}$. So, that means, you can see $h \sim \frac{k}{\delta_T}$ and we know what is the value of $\frac{\delta_T}{x}$ so, from there, you can write $\frac{\delta_T}{x} \sim \frac{1}{P_r^{1/2} R_{e_x}^{1/2}}$ so, from here, you can see that $h \sim \frac{k}{x} P_r^{1/2} R_{e_x}^{1/2}$. So, this is the local heat transfer coefficient order and if you consider local Nusselt number, then the local Nusselt number $N_{u_x} = \frac{hx}{k}$.

So, now you can see $h \sim \frac{k}{\delta_T}$. So, Nusselt number you can see will be order of h if it is order of $\frac{k}{\delta_T}$ so, it will be $\frac{x}{\delta_T}$ and $\frac{x}{\delta_T}$ from here, you can see $N_{u_x} \sim P_r^{1/2} R_{e_x}^{1/2}$ and from here also you can see $N_{u_x} = \frac{hx}{k}$ so, it is $P_r^{1/2} R_{e_x}^{1/2}$ and you can see that these expressions are valid when you have $\delta_T > \delta$ and when it is valid? When you have low Prandtl number fluids; that means, Prandtl number < 1.

So, these expression is valid when you have low Prandtl number fluids; that means, Prandtl number < 1 because we have taken the case when $\delta_T > \delta$ so obviously, it is valid when Prandtl number < 1 and under this condition your h and Nusselt number x will be order of these expressions.

(Refer Slide Time: 26:22)



Now, let us consider when thermal boundary layer thickness is less than the hydrodynamic boundary layer thickness. So, you consider here your thermal boundary layer thickness is δ_T and this is less than your hydrodynamic boundary layer thickness δ.

So, you have free stream velocity U_∞. So, these U_∞ obviously, you can see that when your velocity will develop from 0 to ∞ and at the edge of this velocity boundary layer, it will become free stream velocity U_∞ right. So, you can see that inside the thermal boundary layer thickness at the edge of thermal boundary layer, the velocity will be lower than the free stream velocity because it is not same as U_∞ right. So, at this position, you can see your velocity will be lower than the free stream velocity U_∞ and that will be the scale for u in case of δ_T < δ.

So, in this case, we will have one assumptions. We will assume that your velocity is varying inside the thermal boundary layer linearly. So, we can see. So, in this case, we are considering that this is your free stream velocity $U_\infty \big|_{y=\delta}$; at y = δ. So, at y = δ_T; let us say you have velocity u = u_1 some scale at the edge of thermal boundary layer. So, let us say this is your some scale u_1.

Now, we are assuming that your velocity is varying linearly as you have very small velocity boundary layer thickness, if you assume that it is varying linearly, then whatever expression we will derive you will find that there is not much error. So, in that condition, now you find with the similar triangle what is the velocity at the edge of thermal boundary layer assuming the linear velocity profile.

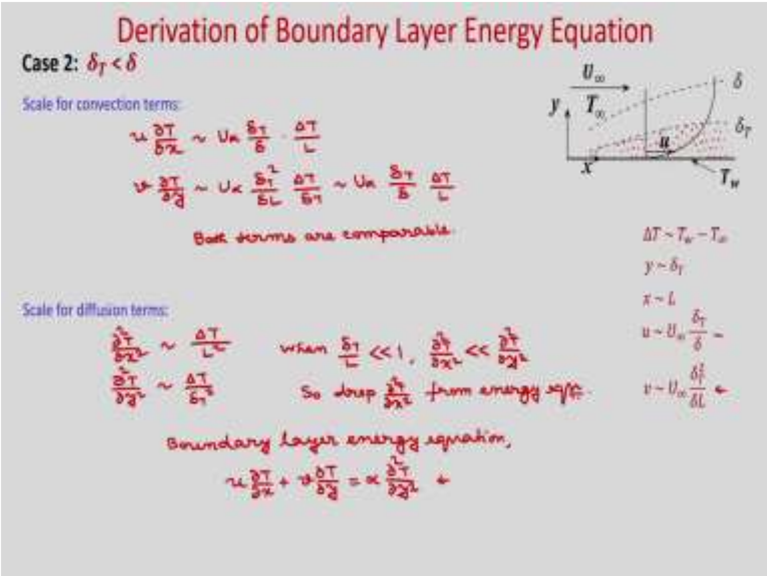
So, you can see from similarity of triangles so, there are two triangles. So, this is one triangle, this is one triangle. So, you can see that $\frac{u_1}{\delta_T} = \frac{U_\infty}{\delta}$ because that y = δ you have U_∞. So, $u_1 = U_\infty \frac{\delta_T}{\delta}$.

So, now when $\delta_T < \delta$, we will consider the scale for velocity u inside the thermal boundary layer as u_1. So, we will consider $u_1 \sim U_\infty \frac{\delta_T}{\delta}$. Here, we cannot consider u as U_∞ because U_∞ is becoming at y = δ, but we are considering thermal boundary layer thickness so, y = δ_T. So, inside this, we are assuming that u will be less than the free stream velocity U_∞ and the order of velocity will be $U_\infty \frac{\delta_T}{\delta}$.

So, we can see when we consider $\delta_T > \delta$ obviously, outside the velocity boundary layer thickness you have U_∞ everywhere. So, you can we took the scale for u as U_∞, but in this particular case, we cannot take because at the edge of velocity boundary layer, you have U_∞, but now $\delta_T < \delta$. So, at the edge of thermal boundary layer, we do not have the velocity scale as U_∞. So, for that reason we considered the velocity scale in this particular case u as u_1 and this u_1 we have found from similarity of triangles as $U_\infty \frac{\delta_T}{\delta}$.

So, now from continuity equation, you can find the scale for v from continuity equation. So, from continuity equation, you can see that $\frac{\partial v}{\partial y} \sim \frac{\partial u}{\partial x}$. So, we can see $v \sim U_\infty \frac{\delta_T}{\delta} \frac{1}{L} \delta_T$. So, you can see $v \sim U_\infty \frac{\delta_T^2}{\delta L}$.

(Refer Slide Time: 31:26)



Now, with these let us see that if we can drop some term in the energy equation. So, first we will consider the convection terms. So, it is $u\frac{\partial T}{\partial x}$. So, it is order of u is scale of $U_\infty \frac{\delta_T}{\delta}$. So, $U_\infty \frac{\delta_T}{\delta}$, δ_T is δ, x is L. So, and $v\frac{\partial T}{\partial y}$ as order of so,

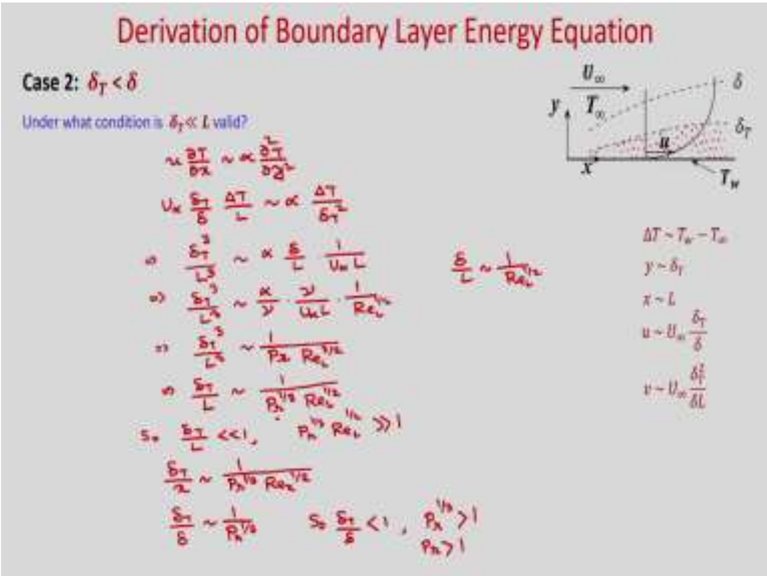
$$v\frac{\partial T}{\partial y} \sim U_\infty \frac{\delta_T^2}{\delta L}\frac{\Delta T}{\delta_T} \sim U_\infty \frac{\delta_T}{\delta}\frac{\Delta T}{L}.$$

So, you can see this is also $U_\infty \frac{\delta_T}{\delta}$ because here δ_T^2 and here δ_T is there so, it will become δ_T and $\frac{\delta_T}{L}$. So, we can see both are having the same order. So, you cannot drop any term from the convection. Both terms is comparable. So, you cannot drop any term.

Now, let us consider the conduction terms or diffusion terms, So, $\frac{\partial^2 T}{\partial x^2}$ so obviously, you can see it will be $\frac{\Delta T}{L^2}$ and $\frac{\partial^2 T}{\partial y^2} \sim \frac{\Delta T}{\delta_T^2}$ and with similar argument, you can see when is $\frac{\delta_T}{L} << 1$, $\frac{\partial^2 T}{\partial x^2} << \frac{\partial^2 T}{\partial y^2}$. So, drop $\frac{\partial^2 T}{\partial y^2}$ from energy equation.

So, if you drop this term from the energy equation, you can write boundary layer energy equation as $u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$. So, this is the same expression when we derived for $\delta_T > \delta$. Now, let us see under what condition is $\frac{\delta_T}{L} << 1$, valid.

(Refer Slide Time: 34:05)



So, again we will compare the diffusion term with the convection term. So, you can see that $u\frac{\partial T}{\partial x} \sim \alpha\frac{\partial^2 T}{\partial y^2}$,now put the scale. So, $U_\infty \frac{\delta_T}{\delta}\frac{\Delta T}{L} \sim \alpha\frac{\Delta T}{\delta_T^2}$. So, here you can see this δ_T^2 if you take in the left-hand side, then it will be δ_T^3 and divide by L^2 both side. So, here L is there so, it will become L^3 will be order of α.

Then, we have $\frac{\delta}{L}$, L^2 we have divided right in right-hand side so, $\frac{\delta}{L}$ will be there and will be $\frac{1}{U_\infty L}$ and δ_T δ_T will get cancelled. So, you can see it will be $\frac{\delta_T^3}{L^3} \sim \frac{\alpha}{\nu}\frac{\nu}{U_\infty L}\frac{1}{R_{e_L}^{1/2}}$. So, we can write $\frac{\delta}{L} \sim \frac{1}{R_{e_L}^{1/2}}$. So, you can see $\frac{\nu}{U_\infty L}$ it is $\frac{1}{R_{e_L}}$ and $\frac{\nu}{\alpha}$ as Prandtl number.

So, you can see $\frac{\delta_T^3}{L^3} \sim \frac{1}{P_r}\frac{1}{R_{e_L}^{3/2}}$. So, you can see $\frac{\delta_T}{L} \sim \frac{1}{P_r^{1/3} R_{e_L}^{1/2}}$.

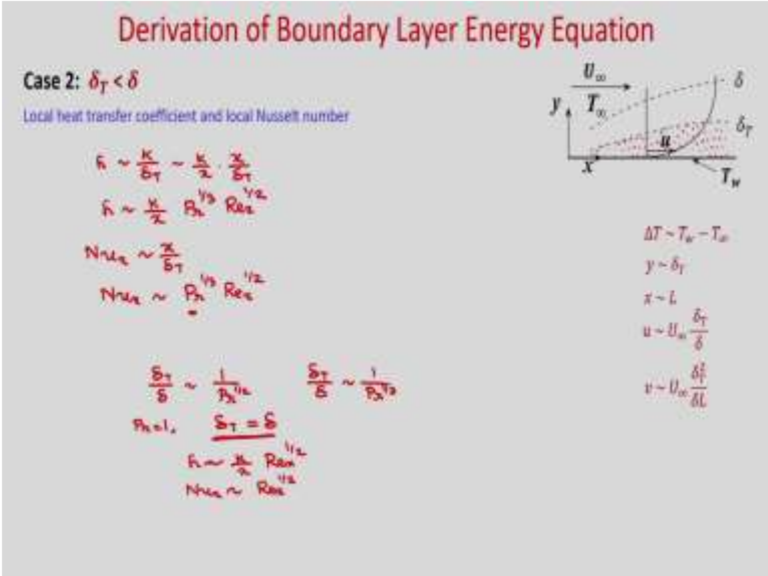
So, if you observe the expression of this and the term which we derive for $\delta_T > \delta$, you can see Prandtl number, $P_r^{1/3}$ and in case of $\delta_T > \delta$, it is Prandtl number $P_r^{1/2}$.

So, you can see so, $\frac{\delta_T}{L}$ << 1 when you have $P_r^{1/3} R_{e_L}^{1/2}$ >> 1. So, generally, you can see that if Peclet number >>1, then δ_T << L.

So, now for any x also you can write $\frac{\delta_T}{x} \sim \frac{1}{P_r^{1/3} R_{e_x}^{1/2}}$ and $\frac{\delta_T}{\delta} \sim \frac{1}{P_r^{1/3}}$. So, now, from this expression you see when you consider $\delta_T < \delta$ that means, your thermal boundary layer thickness is less than the hydrodynamic boundary layer thickness when it will happen? When your $P_r^{1/3}$ > 1 that means, it is high Prandtl number fluids.

So, you can see that. So, $\frac{\delta_T}{\delta}$ will be so, we have consider $\frac{\delta_T}{\delta}$ < 1 so obviously, your $P_r^{1/3}$ > 1. So, if you consider fluid like oils, it will have high Prandtl number. So, for that the hydrodynamic boundary layer thickness will be greater than the thermal boundary layer thickness that means, $\delta > \delta_T$ for oils and you can see that $\frac{\delta_T}{\delta} \sim \frac{1}{P_r^{1/3}}$.

(Refer Slide Time: 38:52)



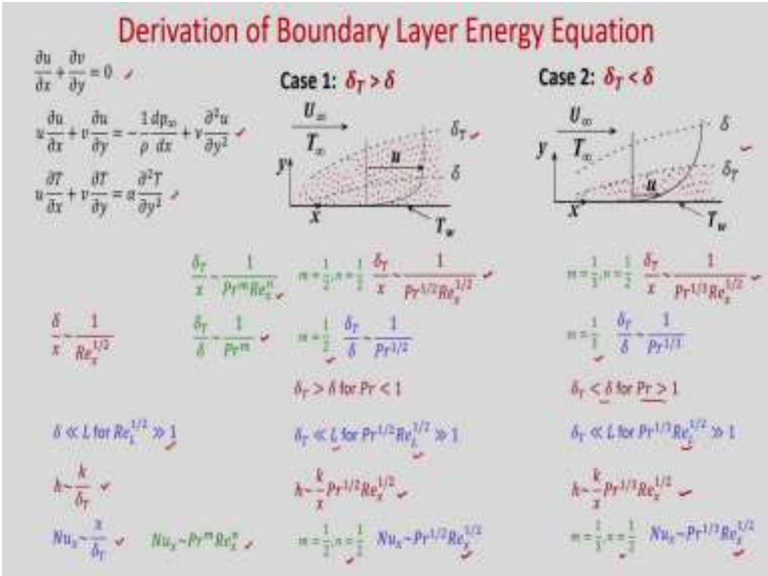
Now, under this condition, let us consider what is the local heat transfer coefficient and local Nusselt number. So, again $h \sim \frac{k}{\delta_T}$. So, we have already derived it. So that means, you can see it will be $\frac{k}{x}\frac{x}{\delta_T}$ and $\frac{\delta_T}{x}$ already we have found. So, it will be $\frac{k}{x}\frac{x}{\delta_T}$ is just $P_r^{1/3} R_{e_x}^{1/2}$ and local Nusselt number $N_{u_x} \sim \frac{x}{\delta_T}$ and so, $N_{u_x} \sim P_r^{1/3} R_{e_x}^{1/2}$.

So, you can see the difference when $\delta_T > \delta$ is in the power of Prandtl number. So, if Prandtl number is 1 so, most of the gases will have the Prandtl number as 1 say let say air, air Prandtl number is almost 0.71 right.

So, it is of the order of 1 and so, for most of the gases, you have Prandtl number is order of 1. So, like air you have Prandtl number as 0.71. So, if Prandtl number is 1 so, we can see for both the cases, we have derived $\frac{\delta_T}{\delta} \sim \frac{1}{P_r^{1/2}}$ and another case we have consider $\frac{\delta_T}{\delta} \sim \frac{1}{P_r^{1/3}}$.

So, if you consider Prandtl number = 1, then obviously, $\frac{\delta_T}{\delta}$ so, because your thermal boundary layer thickness will be equal to hydrodynamic boundary layer thickness and in that particular case, you can see your $h \sim \frac{k}{x} R_{e_x}^{1/2}$ and $N_{u_x} \sim R_{e_x}^{1/2}$.

(Refer Slide Time: 41:06)



So, let us summarize what we have done today. So, we started with the energy equation and using order of magnitude analysis, we have derived the boundary layer energy equation and then, we considered two different cases: first case is when thermal boundary layer is larger than the hydrodynamic boundary layer and next we considered that hydrodynamic boundary layer is greater than thermal boundary layer.

So, we can see. So, finally, we have derived these boundary layer equations, this is the continuity equation, this is the u momentum equation where we have dropped the term $\frac{\partial^2 u}{\partial x^2}$ and this is the energy equation dropping the term $\frac{\partial^2 T}{\partial x^2}$.

You can see here that we have considered two different cases. In this particular case, we have considered $\delta_T > \delta$ and in this case, we have considered $\delta_T < \delta$. So, from here we have derived $\frac{\delta_T}{x} \sim \frac{1}{P_r^m R_{e_x}^n}$ where m = 1/2 and n = 1/2 when $\delta_T > \delta$ so, from here, you will get this expression and when you consider $\delta_T < \delta$, then you can put m = 1/3 and n =1/2, then you will get this expression.

From the velocity boundary layer equation also we have derived $\frac{\delta_T}{x} \sim \frac{1}{R_{e_x}^{1/2}}$. So, if you now divide the thermal boundary layer to the hydrodynamic boundary layer, then you can find $\frac{\delta_T}{\delta} \sim \frac{1}{P_r^m}$. So obviously, you can see m =1/2 when $\delta_T > \delta$ and m =1/3 when you have $\delta_T < \delta$ and from here, you can see δ_T will be δ for Prandtl number <1 that means, low Prandtl number fluids and $\delta_T < \delta$ for high Prandtl number fluids. Also, we have seen that δ << l when Reynolds number is very high and δ_T << L when Peclet number will be very high.

Then, we considered the local heat transfer coefficient and local Nusselt number and you can see that we have derived each of the order of $\frac{k}{\delta_T}$ and this is the expression for two different cases and $N_{u_x} \sim \frac{x}{\delta_T}$ you can see from here you can write $N_{u_x} \sim P_r^m R_{e_x}^n$. So obviously, m =1/2, n =1/2 when $\delta_T > \delta$ and you will get this expression and for $\delta_T < \delta$, you will get m =1/3 and n =1/2 and you will get this expression.

So, today we will just stop here and in the next class, we will start with the solution of these energy equation analytically and also using your approximate solution that means, integral approach.

Thank you.