Fundamentals of Convective Heat Transfer
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Module – 12
Boiling and Condensation
Lecture – 43
Solution of example problems

Hello everyone. So, in last few classes, we have learnt Boiling and Condensation. Today we will solve few example problems on boiling and condensation.

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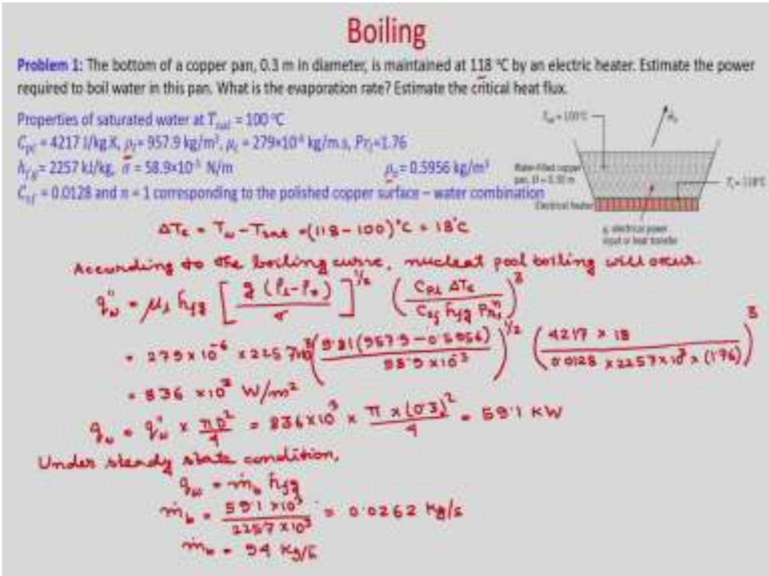

So, let us take the first problem. The bottom of a copper pan, 0.3 meter in diameter, is maintained at 118 ^{0}C by an electric heater. Estimate the power required to boil water in this pan. What is the evaporation rate? Estimate the critical heat flux. So, you can see, this is the boiling phenomena and we need to find the evaporation rate and the critical heat flux, and also the power required to boil water in this pan.

So, you can see this is the copper pan, which is having this bottom diameter as 0.3 m and properties of saturated water at T_{sat} = 100 ^{0}C these are given; C_{pl}, ρ_l, ν_l, Pr_l for liquid. And from steam table, you will get h_{fg} as well as the vapour density ρ_v.

σ is the surface tension; you can see in this particular case, the $\rho_l \sim 1000$ and $\rho_v \sim 1$. So, you can see that $\rho_l >> \rho_v$. In this case, we also need the value of C_{sf} and the n corresponding to the polished copper surface and water combination and that is given.

Now, let us first see what is the excess temperature. ΔT is the excess temperature is just T_w - T_{sat}. So, in this particular case you can see, T_w is 118 ^{0}C and T_{sat} is 100 ^{0}C. So, 118 - 100 = 18^{0}C.

So, if you see the (Refer Time: 02:52) curve of boiling; then you can see for this excess temperature of 18 ^{0}C, it will fall in nucleate boiling. So, we need to use the expression of heat flux from this nucleate boiling region. So, according to the boiling curve, nucleate pool boiling will occur.

So, $q_w^{"} = \mu_l h_{fg} \left[\frac{g(\rho_l - \rho_v)}{\sigma} \right]^{1/2} \left(\frac{C_{pl} \Delta T_e}{C_{sf} h_{fg} \Pr_l^n} \right)^3$. So, for this particular case, you can see C_{sf} and the n value are given.

So, you, if you put all the values here so you see $279 \times 10^{-6} \times 2257 \times 10^3 \left(\frac{9.81(957.9 - 0.5956)}{58.9 \times 10^{-3}} \right)^{1/2} \left(\frac{4217 \times 18}{0.0128 \times 2257 \times 10^3 \times (1.76)} \right)^3$. So, you can write $\Pr_l$, because it is for liquid. So, to calculate this, you will get approximately $836 \times 10^3 W / m^2$. So, this is the heat flux on the wall.

So, now we need to calculate the power required. So, power required will be just your heat flux into the heat transfer area. So, what is the heat transfer area in this particular case? It is the bottom of the pan. So, that is $\frac{\pi D^2}{4}$. So, $q_w = q_w^{"} \times \frac{\pi D^2}{4}$.

So, this is $836 \times 10^3 \times \frac{\pi \times (0.3)^2}{4}$. So, if you calculate it; it will come around 59.1 kW.

Next we need to calculate the evaporation rate and we will use the steady state condition. So, under steady state condition, the energy balance you can see that, q_w whatever is heat transfer rate will be just your mass flow rate, which is your evaporation rate into h_{fg}.

So, this is the latent heat. So, $q_w = \dot{m}_b h_{fg}$, that is your $\dot{m}_b = \frac{59.1\times10^3}{2257\times10^3}$, So, if you calculate this, you will get it as 0.0262 kg/s. If you calculate in kg /hr; then $\dot{m}_b$, you will get as 94 kg/hr. Now, we need to calculate the critical heat flux. So, you know the expression of it.

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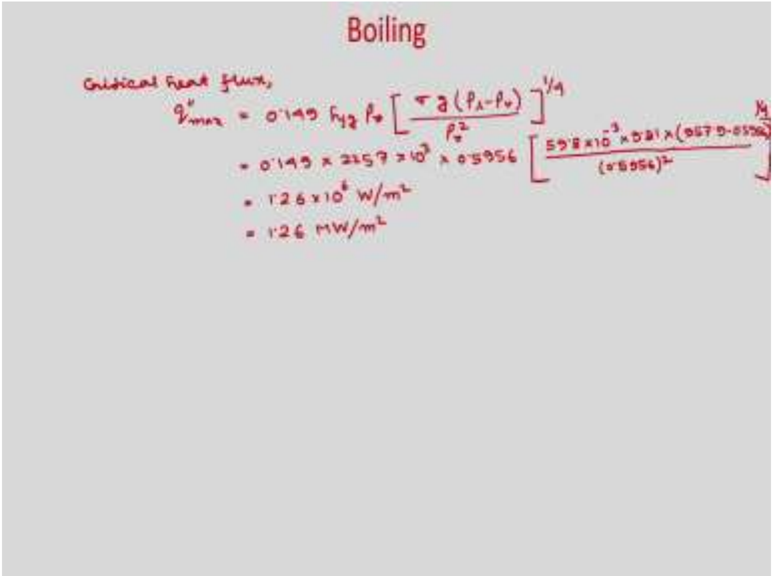

So, critical heat flux $q''_{\max} = 0.149 h_{fg}\rho_v\left[\frac{\sigma g\left(\rho_l-\rho_v\right)}{\rho_v^2}\right]^{1/4}$. So, you put all the values. So, you will get, $0.149\times2257\times10^3\times0.5956\left[\frac{59.8\times10^{-3}\times9.81\times\left(957.9-0.5956\right)}{\left(0.5956\right)^2}\right]^{1/4}$. So, if you calculate, you will get as $1.26\times10^6 W/m^2$. And this is 1.26 MW/m^2.

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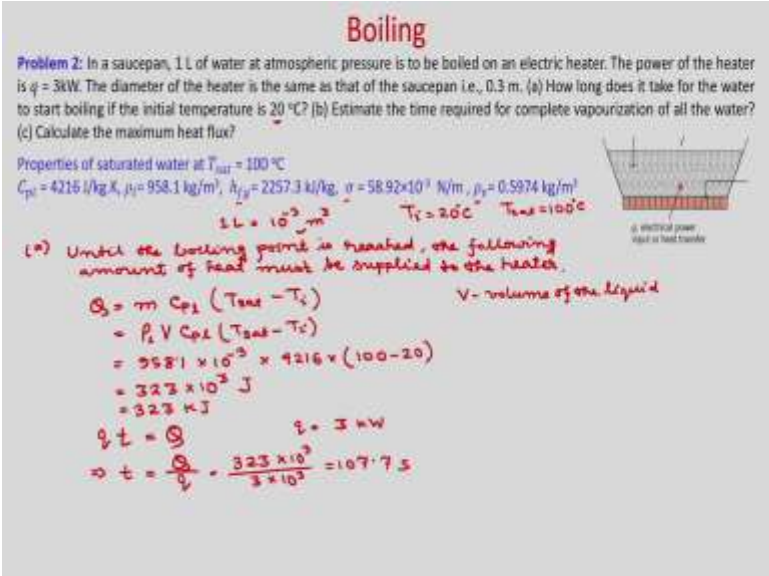

Now, let us take another problem on boiling. In a saucepan, 1 litre of water at atmospheric pressure is to be boiled on an electric heater. The power of the heater is q = 3 kW. The diameter of the heater is the same as that of the saucepan and that is 0.3 m. How long does it take for the water to start boiling if the initial temperature is 20 ^{0}C?

So, you can see in this particular case, the water is at 20 ^{0}C, so that is your initial temperature. So, to boil this water, you have to increase the temperature from 20 ^{0}C to 100 ^{0}C; then only the boiling phenomena will take place.

Next you have to estimate the time required for complete vaporization of all the water? Calculate the maximum heat flux? So, in this particular case, first you have to find what is the power required to increase the temperature of the water from 20 ^{0}C to 100 ^{0}C; 100 ^{0}C is the saturation temperature, then the boiling will start. Once boiling starts, then how much time it will take to evaporate all water, complete water? And we have to also find the critical heat flux.

The properties are given at T_{sat} = 100 ^{0}C; these are the properties and from the steam table, you can find h_{fg} and ρ_v and the surface tension σ is also given. And 1 litre of water; that means it is 10^{-3} m^3. So, this is the volume of the water.

So, first calculate what is the total heat supplied to increase the temperature from 20 ^{0}C to 100 ^{0}C. So, until the boiling point is reached, the following amount of heat must be supplied to the heater. So, that is your Q. So, it will be $Q = mC_{pl}\left(T_{sat} - T_i\right)$.

So, T_i let us say, where T_i in this case it is 20 ^{0}C. And T_{sat} obviously it is 100 ^{0}C. So, what is m? Total mass of the liquid. So it will be just ρ_l into volume of the liquid; so, $\rho_l \times$ V, where Vis the volume of the liquid.

So, in this particular case it is, $958.1 \times 10^{-3} \times 4216 \times (100 - 20)$. So, if you calculate it, you will get around 323×10^{3} J.

So, you can write 323 kJ. So, now, you need to calculate the time; because how long does it take? So, you know that, heat transfer rate is given that q = 3 kW, from the bottom, so from the heater. So, you can write q t = Q. So, you can find, $t = \dfrac{Q}{q}$.

So, $\dfrac{323 \times 10^{3}}{3 \times 10^{3}}$. So, you will get around 107.7 s. So, now, the water has reached to temperature 100 ^{0}C. Now, we have to find the time for complete evaporation of the liquid. So, now as it has reached to the saturation temperature 100 ^{0}C, it will evaporate.

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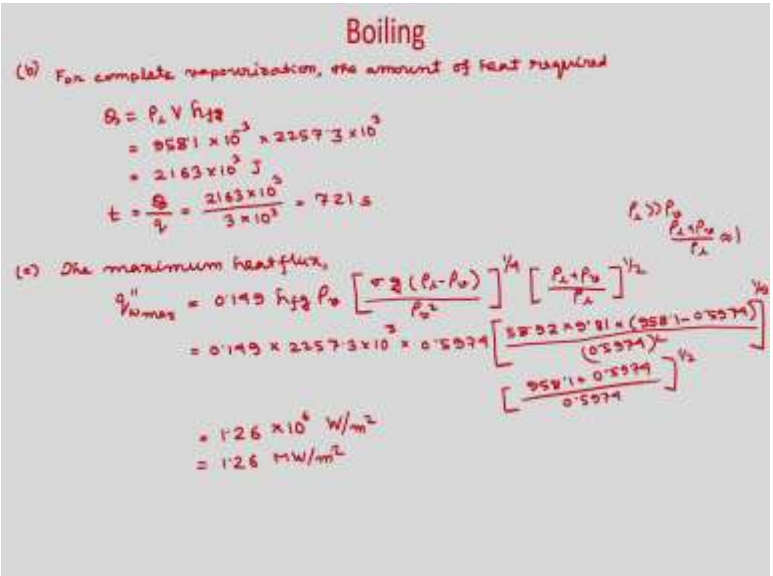

So, now, (b) for complete vaporization the amount of heat required. So, that is $Q = \rho_l V h_{fg}$. That is, $958.1 \times 10^{-3} \times 2257.3 \times 10^3$.

So, this if you calculate, you will get around 2163×10^3 J. So, time you can calculate. Similarly, $t = \frac{Q}{q}$. So, it will be $\frac{2163 \times 10^3}{3 \times 10^3}$. So, you will get around 721 s.

And the maximum heat flux will use the expression what we used for earlier problem. So, the maximum heat flux; $q''_{\max} = 0.149 h_{fg} \rho_v \left[\frac{\sigma g (\rho_l - \rho_v)}{\rho_v^2} \right]^{1/4} \left[\frac{\rho_l + \rho_v}{\rho_l} \right]^{1/2}$.

So, if you see in the earlier case, we have not taken $\left[\frac{\rho_l + \rho_v}{\rho_l} \right]^{1/2}$; because $\rho_l \gg \rho_v$. So, obviously you can see that, $\frac{\rho_l + \rho_v}{\rho_l} \approx 1$. So, for that reason we did not consider.

But if you take also, there will be not much difference in results. So, we put all the values here. So, you will get,

$$0.149 \times 2257.3 \times 10^3 \times 0.5974 \left[\frac{58.92 \times 9.81 \times (958.1 - 0.5974)}{(0.5974)^2} \right]^{1/4} \left[\frac{958.1 + 0.5974}{0.5974} \right]^{1/2}.$$

You can calculate without these terms and you can see what is the difference you are getting in the result. So, if you calculate this, you will get around 1.26 $\times 10^6$ W/m^2 that is you can write 1.26 M W/m^2.

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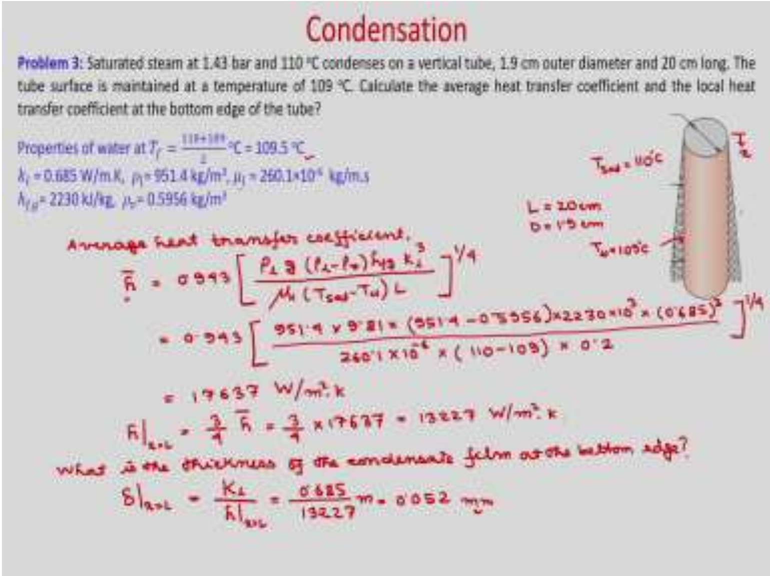

So, now let us take three problems on condensation. First we will discuss this problem; saturated steam at 1.43 bar and 110 ^{0}C condenses on a vertical tube, 1.9 cm outer diameter and 20 cm long. The tube surface is maintained at a temperature of 109 ^{0}C.

Calculate the average heat transfer coefficient and local heat transfer coefficient at the bottom edge of the tube? So, you can see the surface temperature. So, this is the surface temperature T_w is your 109 ^{0}C; whereas the saturation temperature of the steam is 110 ^{0}C.

So, this vapour will condense on the surface of the vertical tube, but you can see that it is a vertical tube; so you can use the same correlation of vertical plate. The expression whatever we derived, it will be same for this particular case; only the total volume of this condensate you can calculate in different way.

So, properties of water at film temperature, which is your mean film temperature as 109.5 ^{0}C; the thermal conductivity of the liquid, density of the liquid, dynamic viscosity of the liquid, are given. And from the steam table, you can find what is h_{fg} and the density of the vapour.

So, you know the average heat transfer coefficient. What is the expression? This is your $\bar{h} = 0.943\left[\dfrac{\rho_l g\left(\rho_l - \rho_v\right) h_{fg} K_l^3}{\mu_l\left(T_{sat} - T_w\right) L}\right]^{1/4}$. So, we put all the values. So, $0.943\left[\dfrac{951.4\times 9.81\times\left(951.4-0.5956\right)\times 2230\times 10^3\times\left(0.685\right)^3}{260.1\times 10^{-6}\times\left(110-109\right)\times 0.2}\right]^{1/4}$. So, if you calculate, you will get this as 17637 W/m^{2}K.

So, you can see; when phase change occurs ok, the heat transfer coefficient is very high. So, you can see it is of the order of 18000 right, whereas when you see the heat transfer coefficient, average heat transfer coefficient for a fully developed case in side circular pipe, it is for constant heat flux 4.36.

So, it is you can see for this case; when you consider the phase change, the heat transfer coefficient is much much higher than the single phase heat transfer. So, as you know the average heat transfer coefficient; now you will be able to calculate what is the heat transfer, local heat transfer coefficient at the bottom edge. So, at the bottom edge means, x we have measured from here.

So, obviously at x = l, you can find what is the local heat transfer coefficient and that you can calculate h, as $h|_{x=L} = \frac{3}{4}\bar{h}$. This relation already we have derived, right.

So, you can right $\frac{3}{4} \times 17637$ = 13227 W/m^{2}K. Now, if you are asked to find the thickness of this condensate at the bottom edge; then how will find it? So, if I ask that what is the thickness of the condensate film at the bottom edge.

So, $\delta|_{x=L} = \frac{K_l}{h|_{x=L}}$. So, that is $\frac{0.685}{13227}$. So, if you calculate, you will get around 0.052 mm.

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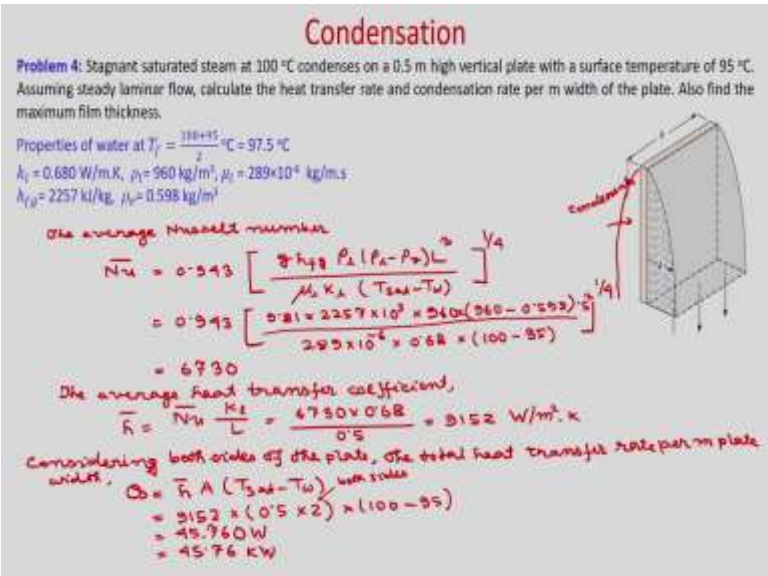

So, it is very small thickness. So, next we will discuss about this problem. Stagnant saturated steam at 100 ^{0}C condenses on a 0.5 m high vertical plate with a surface temperature of 95 ^{0}C.

Assuming steady laminar flow, calculate the heat transfer rate and condensation rate per meter width of the plate? Also find the maximum film thickness?

So, properties of water at mean film temperature 97.5 ^{0}C; k_l, ρ_l, μ_l, h_{fg} and ρ_v. So, if you now calculate the average Nusselt number. So, $\overline{Nu} = 0.943\left[\frac{g h_{fg}\rho_l\left(\rho_l-\rho_v\right)L^3}{\mu_l K_l\left(T_{sat}-T_w\right)}\right]^{1/4}$.

So, if you put all these values. So,

$0.943\left[\frac{9.81\times 2257\times 10^3\times 960\times\left(960-0.598\right)\times 5^3}{289\times 10^{-6}\times 0.68\times\left(100-95\right)}\right]^{1/4}$. So, average Nusselt number if you calculate, you will get 6730. Now, you will be able to calculate the average heat transfer coefficient.

The average heat transfer coefficient, $\bar{h} = \overline{Nu}\frac{K_l}{L}$; because based on length we have calculated the Nusselt number. So, it will be $\frac{6730\times 0.68}{0.5}$. So, you will get 9152 W/m^{2}K.

So, once you know the heat transfer coefficient, you will be able to calculate the heat transfer rate. So, here if you see, we have shown only in one side how the condensate is flowing downward direction; but in the other side also you can consider the similar condensate, other side of the plate.

So, in the both side of the plate if you calculate, the heat transfer rate, then you have to multiply by 2. So, considering both sides of the plate, the total heat transfer rate per meter plate width ok; we can write as, $Q = \bar{h}A\left(T_{sat}-T_w\right)$.

So, $9152\times\left(0.5\times 2\right)\times\left(100-95\right)$. So, you will get it as 45760 W or around 45.76 kW. Now, you will be able to calculate the condensate rate; because you know at steady state, $Q = \dot{m}h_{fg}$.

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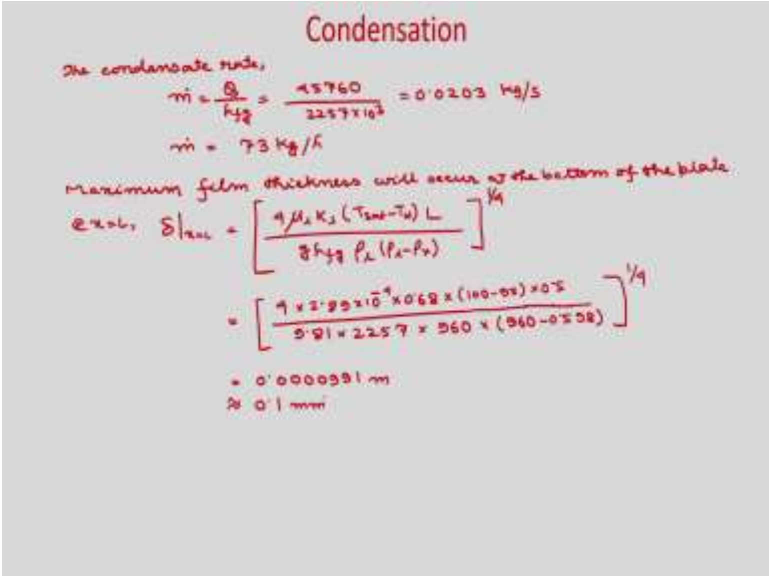

So, the condensate rate you can calculate as. So, $\dot{m} = \frac{Q}{h_{fg}} = \frac{45760}{2257 \times 10^3}$. So, you will get 0.0203 kg/s or you can write as 73 kg/hr. And now, maximum film thickness you can calculate maximum film thickness will occur at the bottom of the plate, ok.

So, at x = L , $\delta|_{x=L} = \left[\frac{4\mu_l K_l \left(T_{sat} - T_w\right) L}{g h_{fg} \rho_l \left(\rho_l - \rho_v\right)} \right]^{1/4}$. So, this is the expression. So, in earlier problem we have seen that, we calculated the local heat transfer coefficient at x = L, at the bottom of the plate.

So, from there easily we could calculate the film thickness; but in this particular case we did not calculate the heat transfer coefficient at x = L. So, for that reason just we are writing the full expression. So, if you put all the values; so you will get, $\left[\frac{4 \times 2.89 \times 10^{-4} \times 0.68 \times (100 - 95) \times 0.5}{9.81 \times 2257 \times 960 \times (960 - 0.598)} \right]^{1/4}$. So, if you calculate, you will get the film thickness at the bottom of the plate as 0.000991 m and you can write it is as ≈ 0.1 mm.

So, you can see that usually the condensate film thickness is very small; in both the cases, we have calculated the film thickness at the end of the plate and it is very very small.

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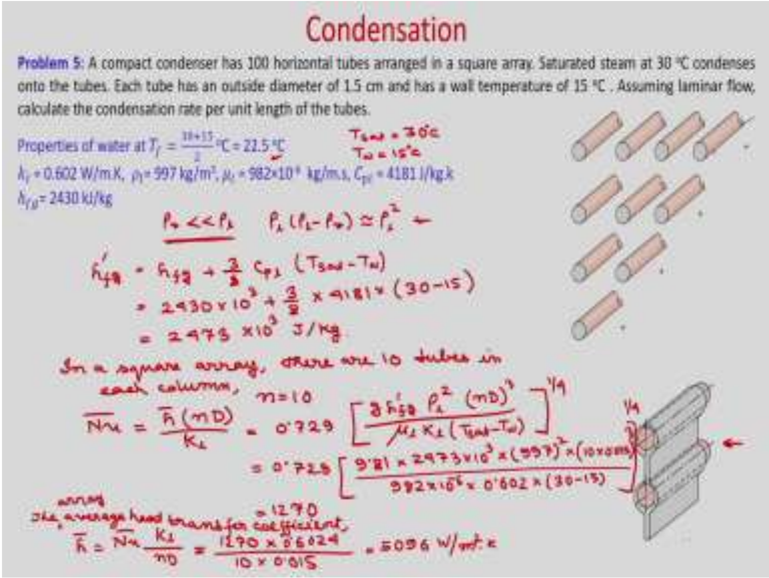

So, now let us take this last problem. A compact condenser has 100 horizontal tubes arranged in a square array. So, you have 100 horizontal tubes and these are arranged in square array, ok. Saturated steam at 30 ^{0}C condenses on to the tubes. Each tube has an outside diameter of 1.5 cm and has a wall temperature of 15 ^{0}C. Assuming laminar flow, calculate the condensation rate per unit length of the tubes?

So, if there are 100 horizontal tubes and these are arranged in a square array; then you can see in 10 by 10 you can arrange. So, in the vertically, you will have 10 tubes and horizontally you can have 10 tubes. So, if you consider only one vertical array; then we will calculate the condensation rate, per unit length of the tubes.

And in this particular case, you can see that T_{sat} is your 30 ^{0}C and T_w temperature is 15 ^{0}C, and at mean film temperature 22.5 ^{0}C, the properties are given. So, you can calculate the here you can see that vapour density obviously, we you know most of the cases we have seen that $\rho_v << \rho_l$.

So, we will just use $\rho_l \left(\rho_l - \rho_v\right) \approx \rho_l^2$. So, in the expression we will use this one, as $\rho_v << \rho_l$; you can consider, but here we are going to neglect. So, in this particular case, the modified h_{fg} you have to calculate; because you have this wall temperature 15 ^{0}C and T_{sat} 30 ^{0}C.

So, here you need to consider the sensible heat. So, you can see $h'_{fg} = h_{fg} + \frac{3}{8}C_{pl}\left(T_{sat} - T_w\right)$.

So, this if you see. So, $2430\times10^3 + \frac{3}{8}\times4181\times\left(30-15\right)$. So, if you calculate it, you will get 2473×10^3 J/kg.

So, you can see that in the square array. So, we will have 10 tubes in each column. So, in a square array, there are 10 tubes in each column. So, n will be 10. So, $\overline{Nu} = \frac{\bar{h}\left(nD\right)}{K_l}$.

So, this you can write as $0.729\left[\frac{gh'_{fg}\rho_l^2\left(nD\right)^3}{\mu_l K_l\left(T_{sat} - T_w\right)}\right]^{1/4}$. So, if you put all the values here. So, you will get $0.729\times\left[\frac{9.81\times2473\times10^3\times\left(997\right)^2\times\left(10\times0.015\right)^3}{982\times10^{-6}\times0.602\times\left(30-15\right)}\right]^{1/4}$. So, this is the average Nusselt number for this particular case considering a single column. So, you will get this around 1270. So, from here $\bar{h}$ you can calculate.

So, the average heat transfer coefficient. So, $\bar{h} = \overline{Nu}\,\frac{K_l}{nD}$. So, it will be $\frac{1270\times0.6024}{10\times0.015}$. So, this you will get as 5096 W/m^{2}K.

So, you can see that, we have calculated the average heat transfer coefficient. So, it is one for, one column. So, it is array average heat transfer coefficient. Now, we need to calculate the total heat transfer rate.

So, when we need to calculate total heat transfer rate is equal to your heat transfer coefficient into area into the temperature difference. So, in this particular case, what is the area? So, we have to consider total area per unit width. So, total area means, it is your $\pi D\times l$; 1 is the unit width and we have 100 cylinders, so we have to multiply with 100.

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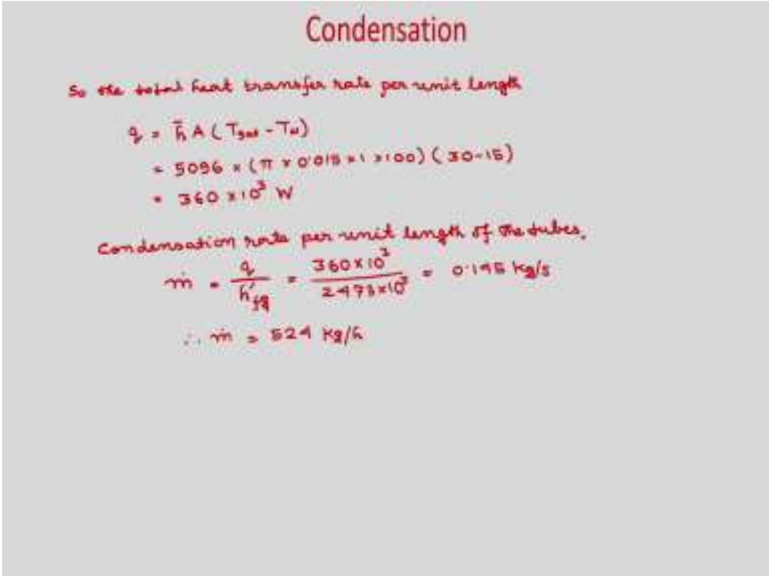

So, the total heat transfer rate per unit length. So, $q = \bar{h}A\left(T_{sat} - T_w\right)$. So, you can see this heat transfer coefficient already we have calculated. So, $5096 \times \left(\pi \times 0.015 \times 1 \times 100\right) \times \left(30 - 15\right)$. So, if you multiply, you will get approximately, 360×10^3 W. So, now, if we need to calculate the mass flow rate; so we have to just use this q divided by your latent heat of condensation.

So, your condensation rate per unit length of the tubes, so that will be $\dot{m} = \frac{q}{h'_{fg}}$; so, this we have calculated as $\frac{360 \times 10^3}{2473 \times 10^3}$.So, if you calculate it, you will get 0.145 kg/s. And if you convert it into kg per hour, then you will get as 524 kg/hr. So, today we solved two problems from boiling and three problems from condensation. You have seen that to solve the boiling problems, you need to remember the expression of heat flux in a particular regime.

So, you need to calculate the excess temperature ΔT and you have to find that in which region it falls and that expression of heat flux you need to use. Also to calculate the critical heat flux, you need to remember the expression which we taught in the first lecture of this module.

When you solve the problems of condensation, then you need to remember the expression for heat transfer coefficient, both local and average and Nusselt number expression. Then from there, you can also calculate the mass flow rate or condensation rate and that expression you should remember, as well as from there you need to calculate the thickness of the film.

So, thickness of the film in terms of your heat transfer coefficient, you can remember the expression or as a whole whatever we have used in today's lecture; so those expression you need to remember to solve these problems. Few problems will be given in assignments. You solve those problems and practice more problems from any undergraduate heat transfer book.

Thank you.

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