

Fundamentals of Convective Heat Transfer
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Module – 12
Boiling and Condensation
Lecture – 42
Laminar film condensation on horizontal tubes

Hello everyone. So, now, today we will consider the Laminar film condensation on horizontal tube. So, in many applications, industrial application, you will find that the condensers are horizontal and over that there will be film flow. So, condensation will take place over the circular pipe, horizontal pipe and those film will fall in downward direction due to gravity. So, in condenser used in power plant, so those are cooled using this technique.

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Laminar film condensation on horizontal tubes

Assumptions:

- The liquid film has laminar flow and constant properties at the mean film temperature.
- The vapor reservoir is at rest and is everywhere at the saturation temperature.
- The vapor exerts no shear stress at the liquid vapor interface.
- Pressure change perpendicular to the wall across film is negligible.
- Momentum and energy transfer by convection in the condensate film are negligible.

$0 = -\frac{\partial^2 T}{\partial y^2} + \mu_L \frac{\partial^2 u}{\partial y^2} + \rho_L g \sin \theta$

$\frac{\partial^2 T}{\partial y^2} = \rho_L g \sin \theta$

$\frac{\partial^2 u}{\partial y^2} = -\frac{g \sin \theta}{\mu_L} (\rho_L - \rho_v)$


Liquid film velocity,

$u(x, y) = \frac{g \sin \theta (\rho_L - \rho_v) \delta^2}{\mu_L} \left[\frac{y}{\delta} - \frac{1}{2} \left(\frac{y}{\delta} \right)^2 \right]$

Mass flow rate per unit width,

$\dot{m} = \int_0^\delta \rho_L u dy = \frac{g \sin \theta \rho_L (\rho_L - \rho_v) \delta^3}{3 \mu_L}$

Temperature profile, $T_{sat} - T(x, y) = \left(1 - \frac{y}{\delta}\right) (T_{sat} - T_w)$



So, first we will consider only single horizontal tube and similar to earlier analysis, whatever we have done for film condensation on vertical plate, we will use here and we will find the Nusselt number. So, you can see if you consider a single horizontal tube, so obviously if the surface temperature is T_w and vapor temperature is T_{sat} and $T_w < T_{sat}$; obviously, the condensation will take place on the surface of the tube and those film due to gravity, it will just travel in the downward direction.

So, the assumptions whatever we made in earlier study, we will use the same assumptions here and the at the vapor liquid interface, we will use no shear stress boundary condition. So, you can see in this particular case, θ will measure from here and x is measured along the surface in this direction. So, at any point, it is tangential direction and perpendicular to this surface, we are measuring y .

So, at any point if you draw the perpendicular directions, so that is your y and tangential direction is your x . So, at an angle θ , we are taking one elemental volume, so that is your $d\theta$. So obviously, so this is your condensate. The surface temperature is T_w and vapor temperature is T_{sat} . So, in this particular case g is acting in this direction, vertically downward direction.

But when we consider the governing equation, you can see this will have two components; one in the tangential direction and another in perpendicular direction. So, when we calculate the pressure gradient from the hydrostatic relation for the vapor. So, these component; x direction component of the gravity, we have to take because we are considering the governing equation in x direction.

So, you can see that we can just left hand side in the momentum equation, it will be 0. You

have $0 = -\frac{\partial p}{\partial x} + \mu_l \frac{\partial^2 u}{\partial y^2} + \rho_l g \sin \theta$. So, in this case if you see if you consider the film here. So,

this is your x direction gravity and this is your y direction gravity and this is your g acting.

So obviously, you can see this will be $g \sin \theta$ because this is your θ and this will be your $g \cos \theta$. So, in the x momentum equation, we have written $\rho_l \times g \sin \theta$; the x component of the gravity. So, here now you can write from the hydrostatic equation in the vapor $\frac{\partial p}{\partial x}$, you can consider.

So, $\frac{\partial p}{\partial x} = \rho_v g \sin \theta$ similar way, in the x direction we are considering. So, if you substitute it

here, you will get $\frac{\partial^2 u}{\partial y^2} = -\frac{g \sin \theta}{\mu_l} (\rho_l - \rho_v)$. So, if you integrate this equation with the

boundary condition that at $y = 0$, your velocity 0 and at $y = \delta$, you have shear stress 0.

So, putting the boundary condition, I am going to write the final expression of the velocity profile. So, velocity profile we will get; it is the same expression as earlier case except g is replaced with $g \sin \theta$ and next, we want to calculate the mass flow rate. So, we have to

integrate from 0 to y . So, mass flow rate per unit width; so, $\dot{m} = \int_0^{\delta} \rho_l u dy$.

So, if you put u expression here finally, we will get $\frac{g \sin \theta \rho_l (\rho_l - \rho_v) \delta^3}{3\mu_l}$. And if you consider

the energy equation, so you will get only $\frac{\partial^2 u}{\partial y^2} = 0$ and the temperature profile will remain same as the case of vertical plate. So, temperature profile will get,

$$T_{sat} - T(x, y) = \left(1 - \frac{y}{\delta}\right) T_{sat} - T_w.$$

So, now, we have calculated the velocity mass flow rate and the temperature profile, now we need to calculate the film thickness or we are interested to calculate the heat transfer coefficient. So, in this particular case, you can see that although your mass flow rate is 0 at $x = 0$; that means, at this particular case, so, your mass flow rate is 0; condensation is taking place ok. But at this place your δ is not 0. The film there is a finite film thickness at $\theta = 0$.

So obviously, we do not know that but we know that their mass flow rate will be 0 because it is a symmetric line. Because you can see in this particular case at the vertical line, it is symmetry because whatever way it will flow here, this way also it will flow. So, we know that due to symmetry at this line, your mass flow rate will be 0.

So, in earlier case, we evaluated the film thickness; but here, it is difficult to find the film thickness because you do not know what is the thickness of δ at $x = 0$. So, hence, we will use a separate route, where we will find what is the mass flow rate.

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neglecting the effect of subcooling, the energy balance can be written as

$$q_w'' dx = d\dot{m}_v h_{fg}$$

$$d\dot{m}_v = \frac{d\dot{m}}{dx} dx$$

$$x = R\theta$$


$$dx = R d\theta$$

$$q_w'' = K_L \frac{\partial T}{\partial y} \bigg|_{y=0} = \frac{K_L (T_{sat} - T_w)}{\delta}$$

$$\Rightarrow \frac{K_L (T_{sat} - T_w)}{\delta} \frac{dx}{dx} = \frac{d\dot{m}}{dx} h_{fg} \frac{dx}{dx}$$

$$\Rightarrow h_{fg} \frac{d\dot{m}}{R d\theta} = \frac{K_L (T_{sat} - T_w)}{\delta}$$

$$\dot{m} = \frac{2 \sin \theta \rho_L (\rho_L - \rho_v) \delta^3}{3 \mu_L}$$

$$\delta = \left[\frac{3 \mu_L}{2 \sin \theta \rho_L (\rho_L - \rho_v)} \right]^{1/3} \dot{m}^{1/3}$$


In this particular case for simplicity, we will neglect the sub cooling effect. So, we will use just latent heat instead of modified latent heat. So, neglecting the effect of sub cooling, the energy balance can be written as you can see in the left hand side, it will be $q_w'' dx$.

So, what is area? So, if you consider this elemental volume, so this is your $d\theta$ and in this direction, this is your x and this is your δ ; this is your δ . So, $q_w'' dx = d\dot{m}_v h_{fg}$. So, the sensible heat part, we are not adding here; we are neglecting it and from the mass balance you can show that, $d\dot{m}_v = \frac{d\dot{m}}{dx} dx$.

Now, you see that in this case, so $x = R \theta$. So, that means, $dx = R d\theta$. So, you will get

$$q_w'' = K_L \frac{\partial T}{\partial y} \bigg|_{y=0}. \text{ If you evaluate it, you will get as, } \frac{K_L (T_{sat} - T_w)}{\delta}.$$

So, you can see this we will put $\frac{K_L (T_{sat} - T_w)}{\delta} dx = \frac{d\dot{m}}{dx} h_{fg} dx$. So, these dx , this dx you cancel.

So, you will get. So, this dx now you write, $dx = R d\theta$ and you take in the left hand side. So,

you will get as $h_{fg} \frac{d\dot{m}}{R d\theta} = \frac{K_L (T_{sat} - T_w)}{\delta}$. So, now we have already know that

$\dot{m} = \frac{g \sin \theta \rho_l (\rho_l - \rho_v) \delta^3}{3\mu_l}$. So, from here, you can find what is δ . So, this we have to replace

in terms of \dot{m} so that we can integrate it. So, $\delta = \left[\frac{3\mu_l}{g \sin \theta \rho_l (\rho_l - \rho_v)} \right]^{\frac{1}{3}} \dot{m}^{\frac{1}{3}}$. So, now, you put

this expression of δ in this equation. So, what you will get? Here $\dot{m}^{\frac{1}{3}}$ is there.

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$$\dot{m}^{\frac{1}{3}} d\dot{m} = \frac{RK_L(T_{sat} - T_w)}{h_{fg}} \left[\frac{g \sin \theta \rho_l (\rho_l - \rho_v)}{3\mu_l} \right]^{\frac{1}{3}}$$

Integrating the above equation between $\theta = 0$ and $\theta = \pi$.

$$\frac{3}{4} \dot{m}_t^{\frac{4}{3}} = \frac{RK_L(T_{sat} - T_w)}{h_{fg}} \left[\frac{g \rho_l (\rho_l - \rho_v)}{3\mu_l} \right]^{\frac{1}{3}} \int_0^\pi \sin^{\frac{1}{3}} \theta d\theta$$

$$\int_0^\pi \sin^{\frac{1}{3}} \theta d\theta = 2 \int_0^{\pi/2} \sin^{\frac{1}{3}} \theta d\theta = 2 \times 1.2936$$

The total condensation rate for one side of the tube,

$$\dot{m}_t = 1.923 \left[\frac{g \rho_l (\rho_l - \rho_v) R^3 K_L^3 (T_{sat} - T_w)^3}{\mu_l h_{fg}^3} \right]^{\frac{1}{4}}$$

So, if you rearrange it, you are going to get $\dot{m}^{\frac{1}{3}} d\dot{m} = \frac{RK_L(T_{sat} - T_w)}{h_{fg}} \left[\frac{g \sin \theta \rho_l (\rho_l - \rho_v)}{3\mu_l} \right]^{\frac{1}{3}}$.

So, you can see that now it will be easier to integrate because you know that at $x = 0$, \dot{m} is 0 because it is a symmetry line and at $x = l$ that means, at the $\theta = \pi$, it will be total mass flow rate.

So, \dot{m} total. Because all will be integrated from $\theta = 0$ to $\theta = \pi$, so you will get the total mass flow rate at $\theta = \pi$. It is for the one side. So, if you consider both sides, then it will be just $2 \dot{m}$. So, here \dot{m} is the mass flow rate per unit width in one-half; but you can see, it is due to symmetry in the other side also you will get \dot{m} . So, total mass flow rate at the bottom you will get $2 \dot{m}$ total. So, integrating the above equation between $\theta = 0$ and $\theta = \pi$. So, what you will get?

So, now at $\theta = 0$ we are telling that at $\dot{m} = 0$ and $\theta = \pi$ it is \dot{m} total. So, this \dot{m} total will just represent as \dot{m}_t . So, you can if you integrate, you will get

$$\frac{3}{4} \dot{m}_t^{4/3} = \frac{RK_l(T_{sat} - T_w)}{h_{fg}} \left[\frac{g \rho_l (\rho_l - \rho_v)}{3 \mu_l} \right]^{1/3} \int_0^\pi \sin^{1/3} \theta d\theta. \text{ So, now you have to integrate this. So,}$$

$$\text{this if you integrate, you will get } \int_0^\pi \sin^{1/3} \theta d\theta = 2 \int_0^{\pi/2} \sin^{1/3} \theta d\theta.$$

If you integrate it, you will get as 1.2936. So, now if you put these value here and rearrange it, so you will get the total condensation rate for one side of the tube, you will get as,

$$\dot{m}_t = 1.923 \left[\frac{g \rho_l (\rho_l - \rho_v) R^3 K_l^3 (T_{sat} - T_w)^3}{\mu_l h_{fg}^3} \right]^{1/4}.$$

So, now, we have calculated the total mass flow rate appearing at $\theta = \pi$ means at the bottom and in one side, we have written \dot{m} . So, if you consider both sides, then it will be $2 \dot{m}$. So, now, you use the energy balance. So, if you use the conservation of energy, then you will be able to calculate the heat transfer coefficient.

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The total condensation rate for one side of the tube is given by

$$\dot{m}_t = 1.923 \left[\frac{g \rho_l (\rho_l - \rho_v) R^3 K_l^3 (T_{sat} - T_w)^3}{\mu_l h_{fg}^3} \right]^{1/4}$$

Now the average heat transfer coefficient for the entire tube (both sides) can be evaluated as

$$2 \dot{m}_t h_{fg} = \bar{h} (2\pi R \times 1) (T_{sat} - T_w)$$

$$\bar{h} = \frac{\dot{m}_t h_{fg}}{\pi R (T_{sat} - T_w)}$$

Substituting the value of \dot{m}_t , we get

$$\bar{h}_0 = 0.729 \left[\frac{h_{fg} g \rho_l (\rho_l - \rho_v) K_l^3}{\mu_l (T_{sat} - T_w) D} \right]^{1/4}$$

The average Nusselt number based on the tube diameter D

$$\bar{Nu}_D = \frac{\bar{h}_0 D}{k_l} = 0.729 \left[\frac{h_{fg} g \rho_l (\rho_l - \rho_v) D^3}{\mu_l k_l (T_{sat} - T_w)} \right]^{1/4}$$

Considering laminar film condensation on a **sphere**, you can show that the average Nusselt number is

$$\bar{Nu}_D = \frac{\bar{h}_0 D}{k_l} = 0.826 \left[\frac{h_{fg} g \rho_l (\rho_l - \rho_v) D^3}{\mu_l k_l (T_{sat} - T_w)} \right]^{1/4}$$

So, you can see that this already we have derived. Now, the average heat transfer coefficient for the entire tube, both sides can be evaluated as so $2 \dot{m}_t h_{fg}$. So, that is the total heat transfer rate is $2 \dot{m}_t h_{fg} = \bar{h} (2\pi R \times 1) (T_{sat} - T_w)$.

So, from here your average heat transfer coefficient, you can calculate as $\bar{h} = \frac{\dot{m}_t h_{fg}}{\pi R (T_{sat} - T_w)}$.

So, now, you see here \dot{m}_t expression is here. So, you substituted it here and rearrange. So, you if you rearrange and based on diameter if you define the average heat transfer coefficient,

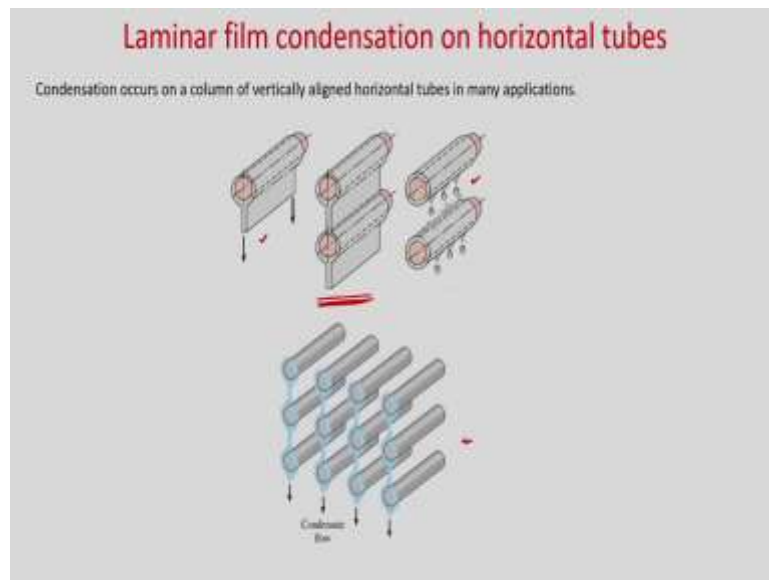
then you will get, $\bar{h}_D = 0.729 \left[\frac{h_{fg} g \rho_l (\rho_l - \rho_v) D^3}{\mu_l K_l (T_{sat} - T_w)} \right]^{1/4}$.

So, now we can calculate the average Nusselt number. So, $\overline{Nu}_D = \frac{\bar{h}_D D}{K_l}$. So, now, in this

particular case, we have defined the average Nusselt number based on diameter. So, if you put it and rearrange it, so, you will get this expression. You if you consider, so this is the average Nusselt number for a single horizontal tube; based on the diameter of the tube.

So, if you compare this equation with the expression for the vertical tube, you can see D is replaced with l ; l³ and this coefficient will be different, that is the just change. Now, if you consider laminar film condensation on a sphere and if you do the similar analysis, you can get the average Nusselt number, similar expression only this constant is 0.826; 0.826 and it is based on the diameter.

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Now, this average Nusselt number, we have calculated for a single tube; but in industry you will get a different columns of tubes. So, in the power plant these are used. So, if you see for this single tube, we have already considered and this is the film falling vertically downward direction.

If you have 2 horizontal tubes, just one above the other; then, these film whatever is coming out from the first tube, it will go to the second tube and it will just fall. Sometimes, it will there will be no continuous film. So, you might get this type of phenomena. But now, if you consider that this columns of horizontal tubes and we will assume that you have a continuous film thickness like these.

So, for this condition, where you have a continuous film and it is falling on over the other; so, you can see here. So, it is falling to the next and next and so on. So, for these; so, for these we need to calculate what will be the heat transfer coefficient and Nusselt number.

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Laminar film condensation on horizontal tubes

We already derived the total condensation rate for one side of the tube as

$$\dot{m}_t = 1.923 \left[\frac{g \rho_l (\rho_l - \rho_v) k_f^3 R^3 (T_{sat} - T_w)^{3/4}}{\mu_l h_{fg}} \right]^{1/4}$$

$$\dot{m}_t^{4/3} = 2.394 \frac{k_f R (T_{sat} - T_w)}{h_{fg}} \left[\frac{g \rho_l (\rho_l - \rho_v)}{\mu_l} \right]^{1/3}$$

Let us apply the above expression to the n^{th} tube in the vertical array. Now the mass flow rate at $\theta = 0$ for the n^{th} tube is the total mass flow rate from the $(n-1)^{th}$ tube denoted as

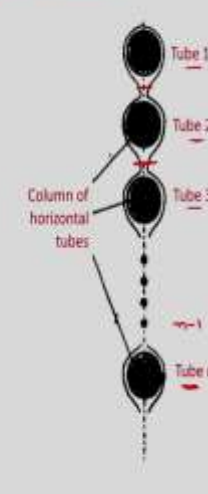
$$\dot{m}_{t,n}^{4/3} - \dot{m}_{t,n-1}^{4/3} = B$$

where $B = 2.394 \frac{k_f R (T_{sat} - T_w)}{h_{fg}} \left[\frac{g \rho_l (\rho_l - \rho_v)}{\mu_l} \right]^{1/3}$

For tube 1, $\dot{m}_{t,1-1}^{4/3} = 0$ So, $\dot{m}_{t,1}^{4/3} = B$

For tube 2, $\dot{m}_{t,2}^{4/3} = \dot{m}_{t,1}^{4/3} + B = 2B$

For tube n , $\dot{m}_{t,n}^{4/3} = \dot{m}_{t,n-1}^{4/3} + B = nB$

$$\dot{m}_{t,n} = (nB)^{3/4}$$


So, you can see if you consider total n tubes in a single column; so, this is the first tube, then second tube, third tube, then this is the n^{th} tube. So, these are column of horizontal tubes. Now, you can see from here when the condensation is taking place; obviously, due to symmetry, mass flow rate will be 0. But when it will come and fall on the second, there is a some mass flow is coming from the tube 1.

So, similarly tube 3, it will get whatever condensation happened in tube 1 and tube 2. Similarly, when you consider tube n , so, some mass flow rate will come whatever is collected from the previous tubes. So, these analysis, now do in this way. We have already calculated this total condensation rate for a single tube and if you write in terms of

$$\dot{m}_t^{4/3} = 2.394 \frac{K_f R (T_{sat} - T_w)}{h_{fg}} \left[\frac{g \rho_l (\rho_l - \rho_v)}{3 \mu_l} \right]^{1/3} \text{ you will get this expression.}$$

Now, let us apply the above expression to the n^{th} tube in the vertical array. Now, the mass flow rate at $\theta = 0$ for the n^{th} tube is the total mass flow rate from the $(n-1)^{th}$ tube denoted as

$$\dot{m}_t^{4/3}; \dot{m}_{t,n} - \dot{m}_{t,n-1} = B; \text{ where } B \text{ is represented with this expression. So, this is your } B.$$

So, what we are telling that whatever total mass flow rate is coming from the previous tube, so that will be just at $\theta = 0$, it will be it will fall to the n^{th} tube. So, if you consider this is the n^{th} tube, then in the n^{th} tube whatever mass flow rate is there, so that will be just minus the previous $n-1$ tube whatever mass flow rate if you would deduct. So, you are going to get a

single tube whatever condensation is taking place. So, that is the total mass flow rate and that is your B .

So, $B = 2.394 \frac{K_l R (T_{sat} - T_w)}{h_{fg}} \left[\frac{g \rho_l (\rho_l - \rho_v)}{3 \mu_l} \right]^{1/3}$. Now, you see when you are considering tube

1, this mass flow rate is 0. So, when you are coming here, so you can see that $\dot{m}_{t,1}$ from the tube 1 whatever you are getting, $\dot{m}_{t,1} = B$, from this expression because n -1 is 0. Now, if you consider tube 2; so, for tube 2, $\dot{m}_{t,2}$.

So, at this place whatever mass flow is coming, it will be just B plus whatever mass flow rate came here. So, $\dot{m}_{t,2} = \dot{m}_{t,1} + B = 2B$. And similar way if you go and if you go to tube n, then you can write $\dot{m}_{t,n} = \dot{m}_{t,n-1} + B = nB$ and this will be nB.

So, total will be nB. So, you can see that $\dot{m}_{t,n} = (nB)^{3/4}$. So, n is the nth tube. So, you can write $\dot{m}_{t,n} = (nB)^{3/4}$ and where, n is the number of tube and B is the this expression. Next, let us do the energy balance.

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Laminar film condensation on horizontal tubes

The average heat transfer coefficient for the tube bank

$$2 \dot{m}_{t,n} h_{fg} = \bar{h} (n \times 2\pi R \times L) (T_{sat} - T_w)$$

$$\bar{h} = \frac{\dot{m}_{t,n} h_{fg}}{n \pi R (T_{sat} - T_w)} = \frac{(nB)^{3/4} h_{fg}}{n \pi R (T_{sat} - T_w)}$$

$$\bar{h}_B = 0.729 \left[\frac{h_{fg} g \rho_l (\rho_l - \rho_v) k_l^3}{\mu_l (T_{sat} - T_w) D} \right]^{1/4}$$

The average Nusselt number for the entire column of tubes is given by

$$\bar{Nu}_D = \frac{\bar{h} (nD)}{k_l} = 0.729 \left[\frac{h_{fg} g \rho_l (\rho_l - \rho_v) (nD)^3}{\mu_l k_l (T_{sat} - T_w)} \right]^{1/4}$$

It will be noted that the above equation is identical to the equation for condensation on a single horizontal cylinder except that D is replaced with nD .

$$\bar{h}_{D,n} = \bar{h}_{D,1} n^{-1/4}$$

$$\bar{Nu}_{D,n} = \bar{Nu}_{D,1} n^{3/4}$$

So, you can see the average heat transfer coefficient for the tube bank, we can calculate 2 into $\dot{m}_{t,n}$. So, at the n^{th} tube whatever this mass flow rate is collected, so that is in both sides. So, $2\dot{m}_{t,n} h_{fg} = \bar{h}(n \times 2\pi R \times 1)(T_{sat} - T_w)$. So, this is the Newton's law of cooling. So, now, what is your total area? So, now, you have n number of tubes in a single column.

So, for a single tube, what is the area? This is $(2\pi R \times 1)$ per unit width. So, $(2\pi R \times 1)$ that is your for area for single tube. Now, in a single column, you have total number of tubes n ; so, $(n \times 2\pi R)$. So, that we have written as the heat transfer area. So, from here your average heat

transfer coefficient, just rearrange, you will get $\bar{h} = \frac{\dot{m}_{t,n} h_{fg}}{n\pi R(T_{sat} - T_w)}$ and $\dot{m}_{t,n} = (nB)^{3/4}$. So, this

is the expression and B expression if you know, so that B expression you just put it and you get the average heat transfer coefficient based on the diameter as,

$$\bar{h}_D = 0.729 \left[\frac{h_{fg} g \rho_l (\rho_l - \rho_v) K_l^3}{n \mu_l (T_{sat} - T_w) D} \right]^{1/4}.$$

So, now let us calculate the average Nusselt number. So, here average Nusselt number based on the diameter we will consider; but how many numbers of tubes are there? So, there are n

number of tubes. So, it will be based on nD . So, you can write $\overline{Nu}_D = \frac{\bar{h}(nD)}{K_l}$. So, now, this is

your $\bar{h}D$. So, if you put this expression, then n if you take inside; so, it will be n^4 and in the denominator $1/n$ is there. So, it will be n^3 . So, you can put it in D .

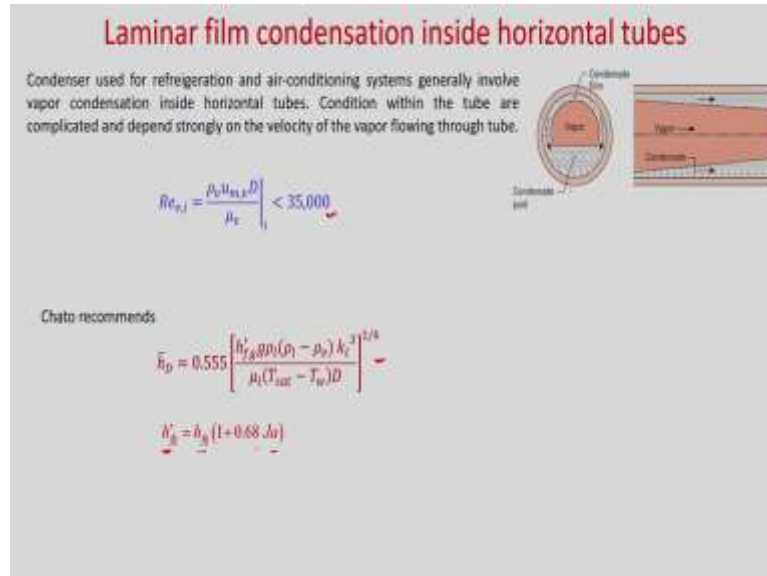
So, it will be $\overline{Nu}_D = 0.729 \left[\frac{h_{fg} g \rho_l (\rho_l - \rho_v) (nD)^3}{\mu_l K_l (T_{sat} - T_w)} \right]^{1/4}$. So, you can see this is the expression

for average Nusselt number for the entire column of tubes and the expression is similar to the single tube except D is replaced with nD and you can show that, $\bar{h}_{D,n} = \bar{h}_{D,1} n^{-1/4}$. Because this is your for the n^{th} tube and if you put for a single tube whatever we have calculated, so that is the first tube. So, you can see this relation.

And similarly, for the Nusselt number also, you can write $\overline{Nu}_{D,n} = \overline{Nu}_{D,1} n^{3/4}$. So, till now you considered the condensation outside the tube. Now, if the steam is condensing inside the tube

when it is flowing through the tube, then your condensation will take place inside the tube and the condensate will fall due to gravity in the downward direction.

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So, if you see condenser used for refrigerator and air conditioning systems generally involve vapor condensation inside horizontal tubes; condition within the tube are complicated and depend strongly on the velocity of the vapor flowing through the tube. So, you can see this is your tube inside this condensation will take place from the surface but due to gravity condensate will come and come down.

So, the film thickness obviously will increase in the downward direction and this is the condensate pool and the Reynolds number of the vapor at the inlet, $Re_{v,i} = \frac{\rho_v u_{m,v} D}{\mu_v} \Big|_i$; i is for

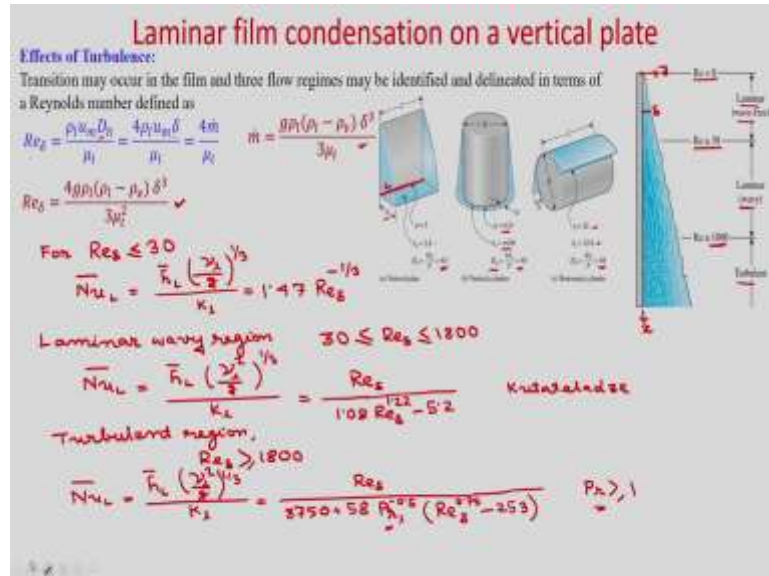
inlet, v for vapor. So, Reynolds number of the vapor at the inlet, you can calculate as density of the vapor into mean velocity of the vapor, the diameter of the tube divided by the viscosity of the vapor.

So, $Re_{v,i} = \frac{\rho_v u_{m,v} D}{\mu_v} \Big|_i < 35,000$, then scientist Chato recommends that,

$\bar{h}_D = 0.555 \left[\frac{h'_{fg} g \rho_l (\rho_l - \rho_v) K_l^3}{\mu_l (T_{sat} - T_w) D} \right]^{1/4}$ and here obviously, $h'_k = h'_{fg} (1 + 0.68 Ja)$. So, all these

study, we assume that it is a laminar flow; but it may become turbulent flow, if your length of the vertical plate is more.

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So, if you consider again the vertical plate and you see how the condensation is taking place at $x = 0$, you have film thickness 0 and gradually in the x direction, it will fall and perpendicular direction is y and δ is the film thickness. So, film thickness gradually, it will increase along the downward direction.

It is seen that the up to Reynolds number 30. So, how the Reynolds number is defined?

Reynolds number is defined based on the film thickness $Re_\delta = \frac{\rho_l u_{m,v} D_h}{\mu_l}$. So, what is

hydraulic diameter here? So, if it is a vertical plate, you can see here. So, the perimeter, perimeter you can see that heat transfer a where it is taking place. So, that is L .

So, $L \times 1$ and what is the flow cross sectional area? So, if this is the L . So, you can see the L is the width of the vertical plate and the perimeter if you see, so it will be L_p ; it will be l at any cross section and the flow area A_c will be $L \times \delta$ because at a particular x , if you see that thickness is δ ; then, cross-sectional flow cross sectional area will be $L \times \delta$.

So, $D_h = \frac{4A_c}{p}$; p is the perimeter of the heat transfer. So, you can see at this particular position, if we consider so this is your L and this is your perimeter. In this particular case, this is your perimeter and at this place your flow cross sectional area is $L \times \delta$.

So, the heat transfer, where it is taking place, so that is your perimeter and these perimeter is in this particular case at any location, you can see that is L. So, $D_h = \frac{4A_c}{p}$. So, if you see, here it will be 4δ . If you consider condensation on a vertical cylinder, so $p = \pi D$. At particular location if you see, it will be πD and flow cross sectional area is $\pi D \times \delta$.

So, D_h will be just 4δ and if it is a horizontal tube, in this particular case, $p = 2L$. So, if it is your L, in both side if you consider, it will be $2L$ and flow cross sectional area will be $L \times \delta$ n; both side, it is $2L \times \delta$. So, hydraulic diameter will be 4δ .

So, Reynolds number, we are defining now this hydraulic diameter we are putting 4δ and if you see that your in place of mass flow rate, if you write; so, it will be $\frac{4\rho_l u_m \delta}{\mu_l}$. So, that will

be your $\frac{4\dot{m}}{\mu_l}$. So, this \dot{m} we know, this expression, so if you put it here, you are going to

get, $Re_\delta = \frac{4g\rho_l(\rho_l - \rho_v)\delta^3}{3\mu_l^2}$. So, Reynolds number is defined in this way.

Now, if you see at $x = 0$, at this point what is the Re_δ is 0? So obviously, Reynolds number will be 0. So, Reynolds number 0. So, it is seen that up to Reynolds number 30, you will get a laminar and it is wave free. So, you can see this interface, it is almost straight line. So, there is no waviness on the interface.

If you see from $30 \leq Re_\delta \leq 1800$, you will get a laminar flow; but on the interface, you will get some waviness. So, this is your laminar wavy region and if $Re_\delta > 1800$, then it will be a turbulent flow. So, inside also you will see that gravity motion is taking place.

So, this is some. So, you will get turbulent zone. So, based on this Reynolds number, if you find the Nusselt number then just we will present what will be the Nusselt number in different

regime. So, Nusselt number already we have calculated based on the length average. So, you

can actually write in this way after rearranging $\overline{Nu}_L = \frac{\bar{h}_L \left(\frac{\nu_l}{g} \right)^{1/3}}{K_l}$.

So, this expression we have not derived, but in some book, you will get this expression and this you can write as four point will get $1.47 Re_\delta^{-1/3}$ and this is valid for laminar zone; wave free zone. So, for $Re_\delta \leq 30$.

So, if your this temperature difference is unknown, $T_{sat} - T_w$ then, actually you can represent the Nusselt number in this way. So, you can see here temperature difference is not coming ok. So, that derivation you can do as a homework and you can show that wavy free laminar zone where $Re_\delta < 30$, you can use this average Nusselt number expression.

Now, for laminar wavy zone; laminar wavy region in the range of $30 \leq Re_\delta \leq 1800$. So, this

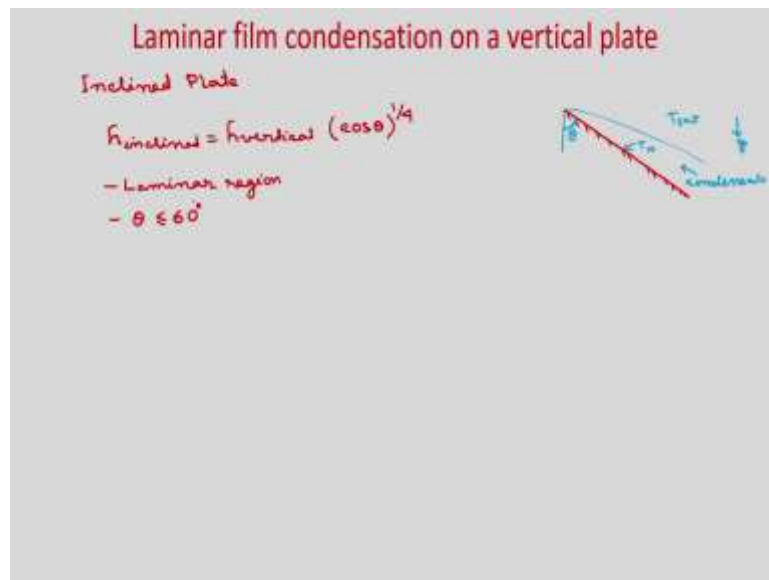
you can use as, $\overline{Nu}_L = \frac{\bar{h}_L \left(\frac{\nu_l^2}{g} \right)^{1/3}}{K_l} = \frac{Re_\delta}{1.08 Re_\delta^{1.22} - 5.2}$. So, this is the correlation actually.

So, it is proposed by the scientist Kutateladze and in turbulent zone, you can get for Re_δ

> 1800 . So, you will get, $\overline{Nu}_L = \frac{\bar{h}_L \left(\frac{\nu_l^2}{g} \right)^{1/3}}{K_l} = \frac{Re_\delta}{8750 + 58 Pr_l^{-0.5} (Re_\delta^{0.75} - 253)}$.

So, in this expression for turbulent region, you can see in the expression Prandtl number is coming and this is valid for $Pr \geq 1$.

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Now, if you consider inclined plates, so you have a plate. So, your condensation will take place. So, you will get this the condensate. And if θ is measured from here, so, this is your T_w , this is your T_{sat} and this is your inclined plate, this is your condensate. So, you can see in this case only the gravity will be different. Because gravity is acting in the vertical downward direction, but plate is inclined with angle θ .

So, you can put $h_{\text{inclined}} = h_{\text{vertical}} (\cos \theta)^{1/4}$ and for it is valid for laminar zone, laminar region and $\theta \leq 60^\circ$. So, this gives satisfactory results specially, for $\theta \leq 60^\circ$.

So, today, we considered the laminar film condensation on a horizontal tube. So, we did the same analysis as we did for the vertical plate, except the gravity in the x direction we considered as $g \sin \theta$ because we consider the x in the tangential direction on the surface of the tube and y in the perpendicular direction.

And from there, we calculate the velocity distribution, mass flow rate and temperature distribution. Then, we used the conservation of mass and conservation of energy. While using the conservation of energy, we evaluated the total mass flow rate at the bottom of the tube. Because in this particular case δ is the film thickness at $x = 0$ is unknown; but due to symmetry at $x = 0$, $\dot{m} = 0$; the mass flow rate \dot{m} will be 0 at $x = 0$.

Hence, we evaluated we the m_t at the bottom of the tube and from there, we calculated the average heat transfer coefficient and average Nusselt number. Then, we considered column of vertical column of horizontal tubes. So, we considered n tubes and for that case also, we evaluated the average heat transfer coefficient and average Nusselt number.

Then, we considered the Reynolds number based on the film thickness δ and depending on different regime whether it is laminar region wave free or laminar region with waviness and turbulent region, we wrote the correlation for Nusselt number.

And finally, if we considered the inclined plate and in this particular case, you know that your gravity in the flow direction will be $g \sin\theta$ and hence, h which is your locally transferred coefficient, you can write $h_{inclined} = h_{vertical} (\cos\theta)^{1/4}$ and generally, it gives satisfactory results for $\theta \leq 60^\circ$ and for laminar region.

Thank you.