Fundamentals of Convective Heat Transfer

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Module – 12
Boiling and Condensation
Lecture – 41
Laminar film condensation on a vertical plate

Hello, everyone. So, today we will study the condensation and particularly, in today's lecture we will study the Laminar film condensation on a vertical plate.

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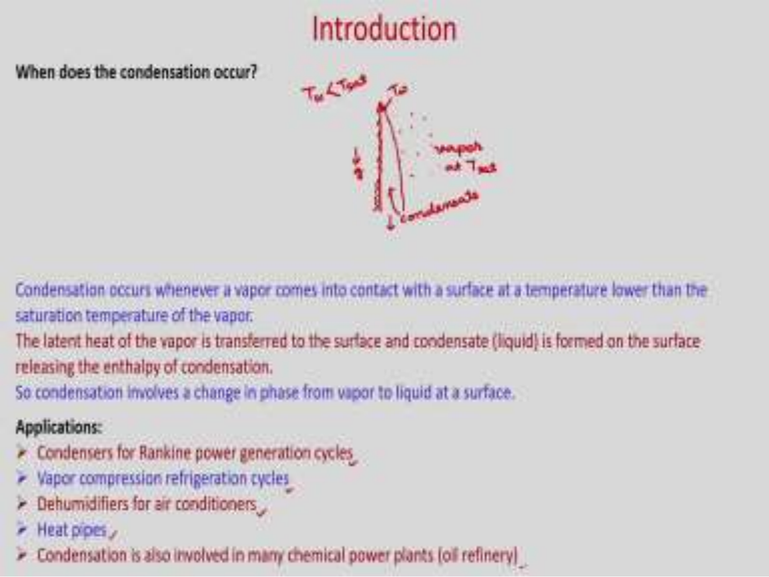

Introduction

When does the condensation occur?

Condensation occurs whenever a vapor comes into contact with a surface at a temperature lower than the saturation temperature of the vapor.
The latent heat of the vapor is transferred to the surface and condensate (liquid) is formed on the surface releasing the enthalpy of condensation.
So condensation involves a change in phase from vapor to liquid at a surface.

Applications:

- Condensers for Rankine power generation cycles
- Vapor compression refrigeration cycles
- Dehumidifiers for air conditioners
- Heat pipes
- Condensation is also involved in many chemical power plants (oil refinery)

We can ask this question, when does the condensation occur? So, when the vapor comes into contact with a liquid or a surface which has less temperature than the saturated saturation temperature of that vapor, then condensation will occur. When the vapor comes into contact with a cold liquid then condensation may occur and it may form fog.

But, in our lecture we will consider only condensation on a surface. So, if you consider that one vapour regime is there and it is at saturation temperature T_{sat}. Now, if you bring one surface whose temperature is lower than the saturation temperature of the vapor, then condensation will take place. So, vapor will release the heat to the surface which is colder than the saturation temperature and condensation will take place and liquid will be formed on the surface of the plate.

So, if you consider one vapor this is your vapor at temperature T_{sat} . Now, if you bring one plate to this vapor whose temperature is T_w and $T_w < T_{sat}$. So, what will happen? This vapor will release the heat to this surface, and liquid will be formed on the surface which is known as condensate and this liquid will form a film on the surface depending on the nature of the surface.

So, this is this liquid is known as condensate and due to gravity it will fall in the downward direction. So, you can see condensation occurs whenever a vapor comes into contact with a surface at a temperature lower than the saturation temperature of the vapor. The latent heat of the vapor is transferred to the surface and condensate is formed on the surface releasing the enthalpy of the condensation.

So, condensation involves a change in phase from vapor to liquid at a surface. So, one good example is that when you see a cold drinks bottle, you bring out from the refrigerator and keep in a ambient then you will see that condensation takes place on the outer surface of the cold drinks bottle. And, generally you have seen that droplets are formed and those droplets just come in downward direction. So, this is one example of condensation.

So, there are many applications of condensation in industry as well as in our daily life. So, you can see here we have noted few applications; condensers for Rankine power generation cycles, vapor compression refrigeration cycles, dehumidifier for air conditioners, heat pipes. So, in heat pipes there are evaporators and condensers. So, in condensers condensation takes place; condensation is also involved in many chemical power plants like oil refinery.

So, as in our lecture we are considering only the surface condensation. So, there are two ways condensation can takes place – one is film condensation and another is drop wise condensation and it depends on the nature of the surface.

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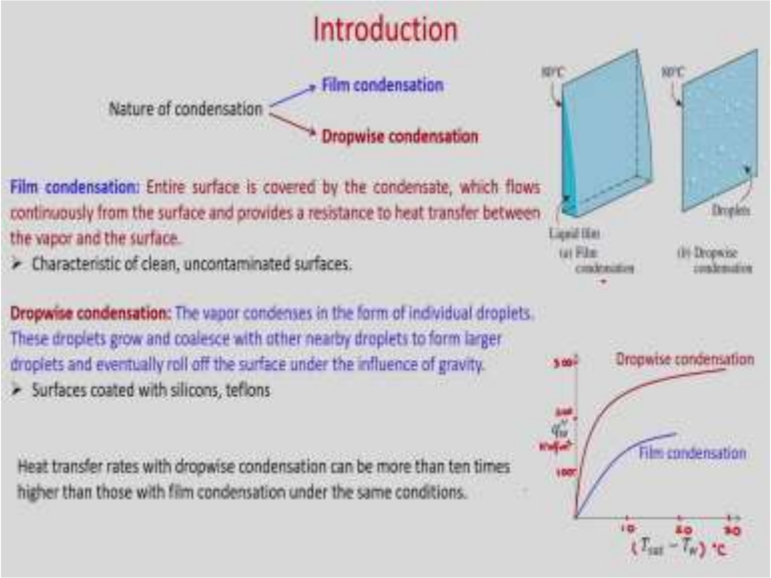

Introduction

Nature of condensation → Film condensation, Dropwise condensation

Film condensation: Entire surface is covered by the condensate, which flows continuously from the surface and provides a resistance to heat transfer between the vapor and the surface.

- Characteristic of clean, uncontaminated surfaces.

Dropwise condensation: The vapor condenses in the form of individual droplets. These droplets grow and coalesce with other nearby droplets to form larger droplets and eventually roll off the surface under the influence of gravity.

- Surfaces coated with silicons, teflons

Heat transfer rates with dropwise condensation can be more than ten times higher than those with film condensation under the same conditions.

So, you can see film condensation generally one film is formed on the surface and this film just come in downward direction due to gravity. So, this is known as film condensation. So, entire surface is covered by the condensate which flows continuously from the surface and provides a resistant to heat transfer between the vapor and the surface.

So, you can see when this condensation takes place on the surface this liquid film is formed. Once the liquid film is formed then this vapor is not in direct contact with the surface. So, this film acts as a thermal barrier. So, generally this vapor gives this heat to this liquid and this heat is transferred from this liquid to the surface.

So, generally, this film condensation is takes place for a clean and uncontaminated surface. Whereas, in dropwise condensation you can see here so, the small drops are formed after realising the latent heat of condensation; then these smaller droplets coalesces and form bigger droplets and these bigger droplets due to gravity just slides in downward direction.

So, you can see here you have more beer surface where more heat transfer can take place because the vapor is in direct contact with the surface and more heat transfer can takes place. But in case of film condensation, there is a resistance because there is a film thickness. So, you can see the vapor condenses in the form of individual droplets these droplets grow and coalesces with other nearby droplets to form larger droplets and eventually roll up the surface under the influence of gravity.

So, surfaces coated with silicones and Teflons. If you have those surfaces then this dropwise condensation can takes place. So, as I told that in case of dropwise condensation vapor is in direct contact with the surface and it can release the latent heat of condensation directly to the surface, but in case of film condensation there is a resistance in heat transfer. So, heat transfer is less compared to the dropwise condensation.

So, if you see the heat transferred on the wall due to condensation versus this the T_{sat} - T_w then you can see that if it is let us say 10 ^{0}C. So, this is your 20 ^{0}C, this is your 30 ^{0}C. So, this T_{sat} - T_w in ^{0}C and y-axis is your heat flux wall heat flux and let us say this is your 100 kW/m^2 this is your 200 and this is your 300 kW/m^2.

So, in case of film condensation so, if you increase this T_{sat} - T_w then gradually it increases. Whereas, you have dropwise condensation then it is much higher heat transfer takes place in case of dropwise condensation. So, you can see here how the heat flux on the wall varies with T_{sat} - T_w. So, heat transfer rates with dropwise condensation can be more than 10 times higher than those with film condensation under the same conditions.

So, now we will study the laminar film condensation on a vertical plate. So, we will consider a vertical plate and the vapor is in quiescent medium. So, when this vapor comes into contact with the surface which is at lower temperature than the T^{sat}, then condensation takes place. And, we are considering only the film condensation, we will use some assumptions.

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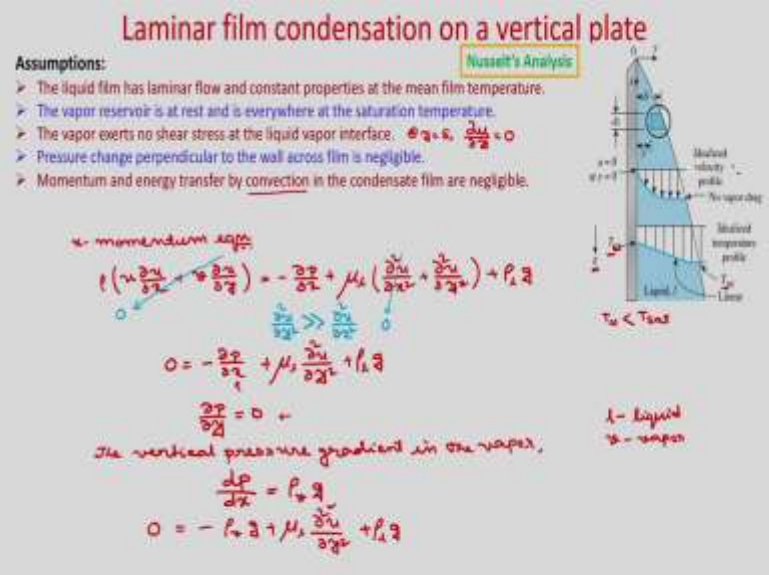

Laminar film condensation on a vertical plate

Nusselt's Analysis

Assumptions:

- The liquid film has laminar flow and constant properties at the mean film temperature.
- The vapor reservoir is at rest and is everywhere at the saturation temperature.
- The vapor exerts no shear stress at the liquid vapor interface. @ $y=\delta$, $\frac{\partial u}{\partial y}=0$
- Pressure change perpendicular to the wall across film is negligible.
- Momentum and energy transfer by convection in the condensate film are negligible.

x- momentum eqn:

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu_l\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\rho_l g$$

0

$$\frac{\partial^2 u}{\partial y^2}\gg\frac{\partial^2 u}{\partial x^2}$$

0

$$0=-\frac{\partial p}{\partial x}+\mu_l\frac{\partial^2 u}{\partial y^2}+\rho_l g$$

$$\frac{\partial p}{\partial y}=0$$

The vertical pressure gradient in the vapor,

$$\frac{dp}{dx}=\rho_v g$$

$$0=-\rho_v g+\mu_l\frac{\partial^2 u}{\partial y^2}+\rho_l g$$

$T_w < T_{sat}$

l - liquid
v - vapor

So, these are the assumptions are made. The liquid film has laminar flow. It is steady laminar flow and constant properties at the mean film temperature. The vapor reservoir is at rest and is everywhere at the saturation temperature. The vapor exerts no shear stress at the liquid vapor interface; that means, if you have, this is the vertical plate.

So, y is measured perpendicular to this plate and x is measured along the vertical plate in the downward direction. Gravity acts in downward direction; the temperature of the plate is constant that is your T_w and the vapor is at rest and its temperature is T_{sat} everywhere. And, in this particular case $T_w < T_{sat}$. So, obviously, condensation will take place and a liquid film will be formed over this surface.

If you see the velocity distribution we will find in this class. So, here we are assuming that the vapor exerts no shear stress at the liquid vapor interface. So, if this is your liquid vapor interface. So, at this $y = \delta$, where δ is the film thickness. So, if δ is the film thickness then at $y=\delta$ you have no vapor drag; that means, at $y = \delta$ you have $\frac{\partial u}{\partial y} = 0$.

Then pressure change perpendicular to the wall across film is negligible. So, the film thickness is very small and you can neglect the pressure change perpendicular to this wall across film. One another important assumptions we will take in this particular case that momentum and energy transfer by convection in the condensate film are negligible.

So, we will see later that the film thickness is very very small and the velocity of the film in downward direction is very small. And due to that we can neglect the inertia terms in the momentum equation as well as in energy equation. And, obviously, if we assume it so, it will simplify the analysis and this analysis actually was first carried out by scientist Nusselt in 1916 and that is why this analysis is known as Nusselt analysis.

And, it is found that with these assumptions if you find the heat transfer coefficient and Nusselt number or the total heat transfer rate, generally it predicts well with the experiments. With this our objective is to find what is the heat transfer coefficient and the Nusselt number.

Both local heat transfer coefficient and average heat transfer coefficient we will calculate and from there we will calculate the local Nusselt number and average Nusselt number. We need to calculate again the condensation rate or the mass flow rate for this particular case. And, from there we need to calculate what is the film thickness δ because δ varies is in the x-direction. So, we need to calculate the thickness film thickness δ.

Let us write the x-momentum equation. So, $\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu_l\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\rho_l g$.

So, in this particular case as we have made these assumptions that your force inertia force is very very small. So, in the left hand side this you can neglect you can neglect and also you can show that the variation $\frac{\partial^2 u}{\partial y^2} >> \frac{\partial^2 u}{\partial x^2}$. Hence you can also neglect this term in the diffusion term.

So, if you simplify it now you can write the governing equation as $=-\frac{\partial p}{\partial x}+\mu_l\frac{\partial^2 u}{\partial y^2}+\rho_l g$, and also you can see the liquid film on the surface is assumed to remain to remain thin. So, pressure change across the liquid film are neglected. So, from you can say the $\frac{\partial p}{\partial y}=0$.

So, you can see so, now, if you consider the vapor region. So, in the vapor region it is at rest. So, you can use the hydrostatic pressure. So, how do you calculate the hydrostatic pressure in the vapor region? So, in the vapor region if you calculate the hydrostatic pressure, so, the vertical pressure gradient in the vapor so, we will use this hydrostatic pressure equation.

So, this is nothing, but, $\frac{dp}{dx}=\rho_v g$. So, v is for vapor; so, density of the vapor. So, l is l suffix l is for liquid and suffix v is for vapor. So, as this vapor is at rest. So, you can see $\frac{dp}{dx}=\rho_v g$ because g is acting in the positive x-direction. So, you can write $\frac{dp}{dx}=\rho_v g$.

So, now this pressure gradient is imposed on the outer edge of the liquid film. So, if $\frac{\partial p}{\partial y}=0$, then $\frac{dp}{dx}$ this you can actually use in this equation. So, you can see that you can write $0=-\rho_l g+\mu_l\frac{\partial^2 u}{\partial y^2}+\rho_l g$. So, this we could do because $\frac{\partial p}{\partial y}=0$. So, you have imposed this pressure gradient inside the liquid film.

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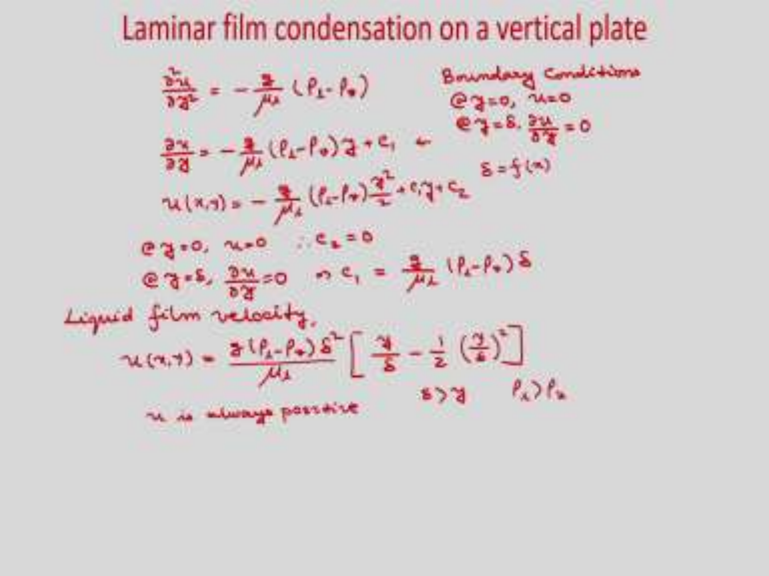

Laminar film condensation on a vertical plate

$\frac{\partial^2 u}{\partial y^2} = -\frac{g}{\mu_l}(\rho_l - \rho_v)$

Boundary Conditions

@ $y = 0$, $u = 0$

@ $y = \delta$, $\frac{\partial u}{\partial y} = 0$

$\frac{\partial u}{\partial y} = -\frac{g}{\mu_l}(\rho_l - \rho_v) y + C_1 \leftarrow$

$\delta = f(x)$

$u(x,y) = -\frac{g}{\mu_l}(\rho_l - \rho_v)\frac{y^2}{2} + C_1 y + C_2$

@ $y = 0$, $u = 0$ $\therefore C_2 = 0$

@ $y = \delta$, $\frac{\partial u}{\partial y} = 0$ $\Rightarrow C_1 = \frac{g}{\mu_l}(\rho_l - \rho_v)\delta$

Liquid film velocity,

$u(x,y) = \frac{g(\rho_l - \rho_v)\delta^2}{\mu_l}\left[\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^2\right]$

$\delta > y$ $\rho_l > \rho_v$

u is always positive

So, now from here you can see that you can write the simplified equation as $\frac{\partial^2 u}{\partial y^2} = -\frac{g}{\mu_l}(\rho_l - \rho_v)$. So, this is the equation now you can integrate it and find the velocity distribution, but here you can see what are the boundary conditions at the wall, where y = 0.

Obviously, velocity is 0 and at the edge of the film which is your y = δ you have shear stress is 0; that means, $\frac{\partial u}{\partial y}$ = 0 that assumptions already we have made that vapor is at rest and it exerts no shear stress on the film.

So, boundary conditions at y = 0, u = 0 and at y = δ you have $\frac{\partial u}{\partial y}$ = 0. So, if you integrate this equation so, you will get $\frac{\partial u}{\partial y} = -\frac{g}{\mu_l}(\rho_l - \rho_v) y + C_1$ and u which is function of x and y because as a function of δ where δ is function of x right.

So, we will get $u(x, y) = -\frac{g}{\mu_l}(\rho_l - \rho_v)\frac{y^2}{2} + C_1 y + C_2$. So, now you apply the boundary conditions. So, if you put at y = 0, u = 0 you will get C_2 as 0. So, at y = 0 u = 0; that means, C_2 = 0 and at y = δ, $\frac{\partial u}{\partial y}$ = 0.

So, if you see from here you can calculate $C_1 = \frac{g}{\mu_l}(\rho_l - \rho_v)\delta$ and δ is function of x. So, if you put all these C_1 and C_2 value there. So, you will get as $u(\mathrm{x}, y) = \frac{g(\rho_l - \rho_v)\delta^2}{\mu_l}\left[\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^2\right]$.

So, in this equation you can see in the right hand side $(\rho_l - \rho_v)$ is positive because $\rho_l >> \rho_v$ and $\frac{y}{\delta}$ is also positive. So, because $\delta > y$ and $\rho_l > \rho_v$.

So, you can see the right hand side term will be positive. So, u is always positive. So, u is always positive. Now, you want to calculate the mass flow rate. So, if you want to calculate the mass flow rate you can integrate this velocity. So, over the film thickness δ.

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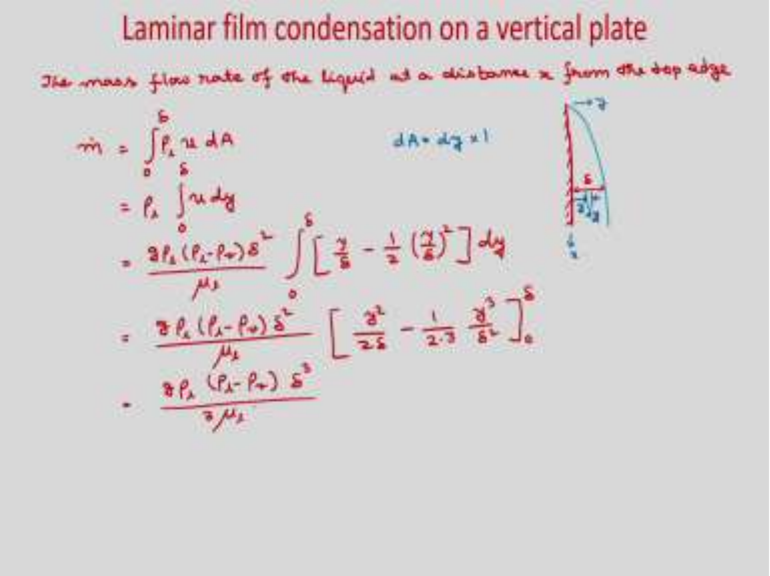

Laminar film condensation on a vertical plate

The mass flow rate of the liquid at a distance x from the top edge

$$\dot{m} = \int_0^\delta \rho_l u \, dA$$

$dA = dy \times 1$

$$= \rho_l \int_0^\delta u \, dy$$

$$= \frac{g\rho_l(\rho_l - \rho_v)\delta^2}{\mu_l} \int_0^\delta \left[\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^2\right] dy$$

$$= \frac{g\rho_l(\rho_l - \rho_v)\delta^2}{\mu_l} \left[\frac{y^2}{2\delta} - \frac{1}{2\cdot 3}\frac{y^3}{\delta^2}\right]_0^\delta$$

$$= \frac{g\rho_l(\rho_l - \rho_v)\delta^3}{3\mu_l}$$

So, the mass flow rate of the liquid at a distance x from the top edge. So, you consider the vertical plate and this is your liquid film over this surface. Now, if you take a distance y from the surface and an elemental distance dy. So, in the perpendicular direction of the surface if you take unit width then your this elemental area will be dy × 1 and this is your y and this is your x.

So, what will be your area? dA = dy × 1; 1 is your unit width . So, now if you want to calculate the mass flow rate, so, $\dot{m}$ you know that you integrate from 0 to δ. So, this is your film thickness δ; so, $\dot{m} = \int_0^\delta \rho_l dA$. So, you can see this will be ρ_l is constant, so $\dot{m} = \rho_l \int_0^\delta u dy$.

So, you can see if you put the expression of u and the constant if you take outside the integral, then you can write this, $\dot{m} = \frac{g\rho_l(\rho_l - \rho_v)\delta^2}{\mu_l} \int_0^\delta \left[\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^2 \right] dy$. So, now you perform the integration and put the limits then you can find the mass flow rate as $\dot{m} = \frac{g\rho_l(\rho_l - \rho_v)\delta^2}{\mu_l} \left[\frac{y^2}{2\delta} - \frac{1}{2\cdot 3}\frac{y^3}{\delta^2} \right]_0^\delta$ and finally, you will get $\dot{m} = \frac{g\rho_l(\rho_l - \rho_v)\delta^3}{3\mu_l}$. So, you can see the in the expression of velocity as well as mass flow rate, δ is there and this δ is function of x and it is still unknown. So, we need to find this film thickness δ.

So, next we will find the temperature distribution. So, if you see in the energy equation we can neglect the inertia term. So, left hand side terms will be 0 and in this particular case also we can neglect the axial heat conduction because it is very very small compared to the $\frac{\partial^2 T}{\partial y^2}$ hence you will get the energy simplified energy equation as.

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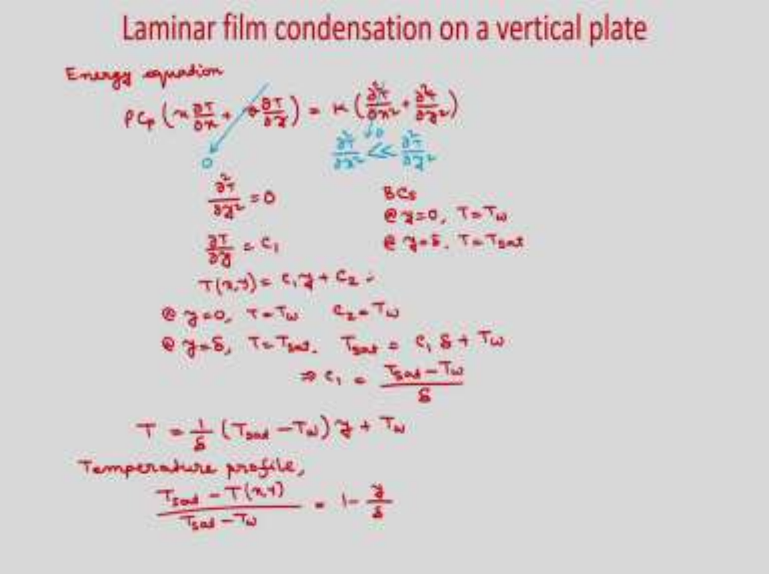

Laminar film condensation on a vertical plate

Energy equation

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$$

$$0 \qquad \frac{\partial^2 T}{\partial x^2} \ll \frac{\partial^2 T}{\partial y^2}$$

$$\frac{\partial^2 T}{\partial y^2} = 0$$

$$\frac{\partial T}{\partial y} = C_1$$

BCs

@ $y = 0$, $T = T_w$

@ $y = \delta$, $T = T_{sat}$

$$T(x,y) = C_1 y + C_2$$

@ $y = 0$, $T = T_w$ $\quad C_2 = T_w$

@ $y = \delta$, $T = T_{sat}$. $\quad T_{sat} = C_1\delta + T_w$

$$\Rightarrow C_1 = \frac{T_{sat} - T_w}{\delta}$$

$$T = \frac{1}{\delta}(T_{sat} - T_w)y + T_w$$

Temperature profile,

$$\frac{T_{sat} - T(x,y)}{T_{sat} - T_w} = 1 - \frac{y}{\delta}$$

So, if you write energy equation $\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=K\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)$. So, all these terms in the left hand side these are negligible because inertia terms and $\frac{\partial^2 T}{\partial x^2}\ll\frac{\partial^2 T}{\partial y^2}$. So, this term also you can neglect.

So, simplified energy equation you can write $\frac{\partial^2 T}{\partial y^2}$=0 so; that means, only heat convection is taking place. So, what are the boundary conditions? At y = 0 you have wall temperature T_w and at y = δ you have T = T_{sat}.

So, if you integrate this equation, what you will get? You will get $\frac{\partial T}{\partial y}=C_1$ or you can write C_3. So, after integrating this equation you will get $\frac{\partial T}{\partial y}=C_1$ and another time if you integrate then you will get, $T(x,y)=C_1 y+C_2$.

So, if you put the boundary condition at y = 0, T = T_w, then you can see that you will get, C_2=T_w and at y = δ, T =T_{sat} . So, you can see you will get $T_{sat}=C_1\delta+T_w$; that means, $C_1=\frac{T_{sat}-T_w}{\delta}$.

So, if you put C_2 and C_1 value here you will get $T=\frac{1}{\delta}\left(T_{sat}-T_w\right)y+T_w$ and you can write temperature profile. So, you can see here only we considered the heat conduction inside the film. So, we will get, $\frac{T_{sat}-T_{(x,y)}}{T_{sat}-T_w}=1-\frac{y}{\delta}$.

So, if only heat conduction is taking place so, what do we expect the temperature profile? It will be linear and you can see here that it is a linear profile ok. So, it is a linear. So, it will vary linearly from T_w to T_{sat} because T_{sat} > T_w. Now, δ is unknown to calculate the δ, now we will use conservation of mass and conservation of energy inside an elemental volume in liquid film.

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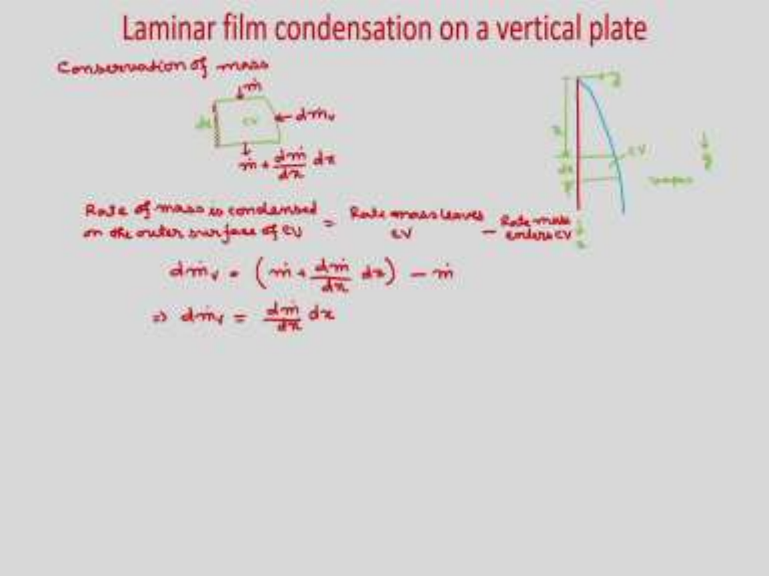

Laminar film condensation on a vertical plate

Conservation of mass

Rate of mass is condensed on the outer surface of CV = Rate mass leaves CV − Rate mass enters CV

$$d\dot{m}_v = \left(\dot{m} + \frac{d\dot{m}}{dx} dx\right) - \dot{m}$$

$$\Rightarrow d\dot{m}_v = \frac{d\dot{m}}{dx} dx$$

So, if you consider the vertical plate and this is your film thickness, this is your y, this is your x, g is acting in downward direction and if you consider these elemental volume where you can see if it is vapor. So, from the vapor it will release the latent heat through this interface and this will conduct to the surface and in the surface it will release the heat.

And, obviously, there will be mass flow rate is coming here and it will mass flow rate will go out and with that your energy also will come in and energy will go out. So, this is the elemental control volume we are considering and these control volume if you see so, these we are considering at a distance x of distance dx here it is dx and this is your control volume. So, we are using conservation of mass now.

So, you can see from the vapor. So, $d\dot{m}_v$ will go because this will convert from vapor to the liquid releasing the latent heat. Your m dot is coming mass flow rate is coming and as $d\dot{m}_v$ is added. So, you will get m dot and if you use the Taylor series expansion then whatever $\dot{m}$ is coming so, that you can write at the exit of this control volume as $d\dot{m}_v = \left(\dot{m} + \frac{d\dot{m}}{dx} dx\right) - \dot{m}$.

So, this is we are using Taylor series expansion only and this is wall. So, there is no mass is going through the surface because this is wall. So, if you consider this and if you do the mass balance so, you will get rate of mass is condensed on the outer surface of CV is equal to rate of rate mass leaves the control volume minus rate mass enters in control volume.

So, what is that? In left hand side it is $d\dot{m}_v$, right hand side what is leaving? So, $\left(\dot{m}+\frac{d\dot{m}}{dx}dx\right)$ and whatever enters that is your $\dot{m}$. So, you can see your whatever vapor is converting to the liquid, $d\dot{m}_v = \frac{d\dot{m}}{dx}dx$.

So, now we will use the conservation of energy. In this particular case as liquid flame act as the thermal resistance, the vapor converts to the liquid releasing the latent heat to the film, and then this heat is conducted through the liquid to the surface. So, if you consider the latent heat whatever it is given to the liquid at the interface, after that there will be a temperature difference that we are assuming. So, that is known as sub cooling.

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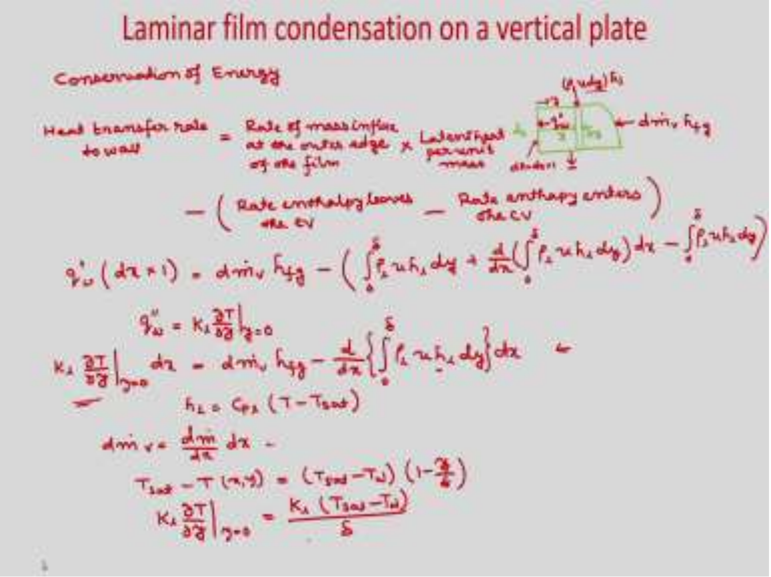

Laminar film condensation on a vertical plate

Conservation of Energy

Heat transfer rate to wall = Rate of mass inflow at the outer edge of the film × Latent heat per unit mass

− (Rate enthalpy leaves the CV − Rate enthalpy enters the CV)

$$q''_w (dx \times 1) = d\dot{m}_v h_{fg} - \left(\int_0^\delta \rho_l u h_l \, dy + \frac{d}{dx}\left(\int_0^\delta \rho_l u h_l \, dy\right)dx - \int_0^\delta \rho_l u h_l \, dy\right)$$

$$q''_w = k_l \frac{\partial T}{\partial y}\Big|_{y=0}$$

$$k_l \frac{\partial T}{\partial y}\Big|_{y=0} dx = d\dot{m}_v h_{fg} - \frac{d}{dx}\left\{\int_0^\delta \rho_l u h_l \, dy\right\}dx$$

$$h_l = c_{pl}(T - T_{sat})$$

$$d\dot{m}_v = \frac{d\dot{m}}{dx}dx$$

$$T_{sat} - T(x,y) = (T_{sat} - T_w)\left(1 - \frac{y}{\delta}\right)$$

$$k_l \frac{\partial T}{\partial y}\Big|_{y=0} = \frac{k_l (T_{sat} - T_w)}{\delta}$$

So, if you consider the effect of sub cooling and write the conservation of energy equation then we can write it as. So, whatever control volume we considered. So, this is the control volume we considered, this is your dx and at a distance y let us take a strip of distance dy.

So, now we will use the conservation of energy in this control volume. So, you can see the, whatever vapor is converted to the liquid at the liquid film interface so, that is your $d\dot{m}_v h_{fg}$.

And, there will be a sensible heat because whatever $d\dot{m}_v h_{fg}$ this is going and here on the surface your q_w'' is released.

And, you have $\dot{m}$ is coming in and this $\dot{m}$ in this particular small elemental volume it is $\rho_l u h_l$ because we are considering the sensible heat. So, if you are considering a sensible heat then $\rho_l u h_l$ is coming here and obviously, again if you integrate this from 0 to δ because δ is your thickness. So, if you so 0 to δ if you integrate, then you will get the total energy coming in and that if you use the Taylor series expansion then you can write at the outlet.

So, when sub cooling of the liquid film is allowed then total enthalpy flowing into and out of the CV must be included in the energy balance. So, if you do not consider the sensible heat then you can see it is simply whatever heat is released; that means, $d\dot{m}_v h_{fg} = q_w''$. But, if you consider the effect of sub cooling and if you consider this sensible heat then this you need to consider whatever energy is coming in.

So, these and whatever is coming out so, that sensible heat you need to consider. So, conservation of energy now if you write so, it will be heat transfer rate to wall is equal to rate of mass in flux at the outer edge of the film in to latent heat per unit mass.

Now, minus you have to write rate enthalpy leaves the CV minus rate enthalpy enters the C V. So, now, left hand side so, heat transfer rate to the wall what is that? So, your heat flux is

$$q_w''(dx \times 1) = d\dot{m}_v h_{fg} - \left(\int_0^{\delta} \rho_l u h_l dy + \frac{d}{dx}\left(\int_0^{\delta} \rho_l u h_l dy \right) dx - \int_0^{\delta} \rho_l u h_l dy \right).$$

So, this we are considering if you are taking into account the sensible heat . So, this is known as effect of sub cooling. So, if you are considering that then only you need to add it. So, if you see that if you do not consider the sensible heat then it is just $q_w''(dx \times 1) = d\dot{m}_v h_{fg}$. This is the simple thing, but as we are considering the sensible heat you need to add this term and you have to integrate because u is function of y. So, x and y, so that you need to integrate.

So, $q''_w = K_l \left.\frac{\partial T}{\partial y}\right|_{y=0}$ you see we are writing plus because q''_w is in the negative x y direction because this is your y direction. So, obviously, this is negative into y direction. So, generally we write $q''_w = -K \left.\frac{\partial T}{\partial y}\right|_{y=0}$, but here q''_w is negative to y. So, this is $+K_l \left.\frac{\partial T}{\partial y}\right|_{y=0}$.

So, this if you write here so, you will get, $K_l \left.\frac{\partial T}{\partial y}\right|_{y=0} = d\,\dot{m}_v\, h_{fg} - \frac{d}{dx}\int_0^\delta \rho_l u h_l\, dy dx$. Now, here the enthalpy you see the enthalpy of this sub cooled liquid is measured relative to that existing at the saturation temperature. I assuming that the liquid specific heat is constant and the enthalpy of the sub liquid film, $h_l = C_{pl}\left(T - T_{sat}\right)$.

So, this expression you can write. So, as we are calculating the enthalpy with respect to the enthalpy at the interface T at T$_{sat}$. So, that is why you are writing $h_l = C_{pl}\left(T - T_{sat}\right)$. So, obviously, your now, $d\,\dot{m}_v = \frac{d\,\dot{m}}{dx}dx$. So, this you can see your temperature distribution is $T_{sat} - T_{(x,y)} = \left(T_{sat} - T_w\right)\left(1 - \frac{y}{\delta}\right)$.

So, from here if you calculate $K_l \left.\frac{\partial T}{\partial y}\right|_{y=0} = \frac{K_l\left(T_{sat} - T_w\right)}{\delta}$. So, in this term you put this value, then d m dot v you put this value.

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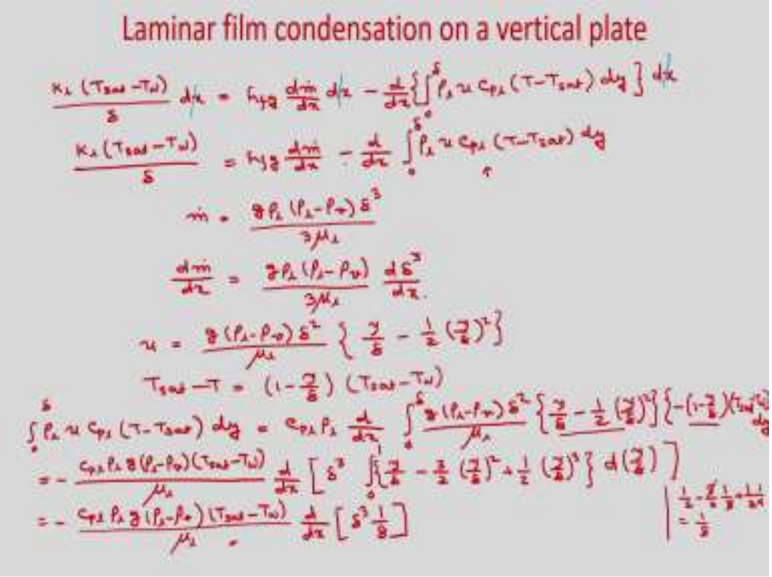

Laminar film condensation on a vertical plate

$$\frac{K_l(T_{sat}-T_w)}{\delta}\cancel{dx} = h_{fg}\frac{d\dot{m}}{dx}\cancel{dx} - \frac{d}{dx}\left\{\int_0^\delta \rho_l u c_{pl}(T-T_{sat})\,dy\right\}\cancel{dx}$$

$$\frac{K_l(T_{sat}-T_w)}{\delta} = h_{fg}\frac{d\dot{m}}{dx} - \frac{d}{dx}\int_0^\delta \rho_l u c_{pl}(T-T_{sat})\,dy$$

$$\dot{m} = \frac{g\rho_l(\rho_l-\rho_v)\delta^3}{3\mu_l}$$

$$\frac{d\dot{m}}{dx} = \frac{g\rho_l(\rho_l-\rho_v)}{3\mu_l}\frac{d\delta^3}{dx}$$

$$u = \frac{g(\rho_l-\rho_v)\delta^2}{\mu_l}\left\{\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^2\right\}$$

$$T_{sat} - T = \left(1-\frac{y}{\delta}\right)(T_{sat}-T_w)$$

$$\int_0^\delta \rho_l u c_{pl}(T-T_{sat})\,dy = c_{pl}\rho_l\frac{d}{dx}\int_0^\delta \frac{g(\rho_l-\rho_v)\delta^2}{\mu_l}\left\{\frac{y}{\delta}-\frac{1}{2}\left(\frac{y}{\delta}\right)^2\right\}\left\{-\left(1-\frac{y}{\delta}\right)(T_{sat}-T_w)\right\}dy$$

$$= -\frac{c_{pl}\rho_l g(\rho_l-\rho_v)(T_{sat}-T_w)}{\mu_l}\frac{d}{dx}\left[\delta^3\int_0^1\left\{\frac{y}{\delta}-\frac{3}{2}\left(\frac{y}{\delta}\right)^2+\frac{1}{2}\left(\frac{y}{\delta}\right)^3\right\}d\left(\frac{y}{\delta}\right)\right]$$

$$= -\frac{c_{pl}\rho_l g(\rho_l-\rho_v)(T_{sat}-T_w)}{\mu_l}\frac{d}{dx}\left[\delta^3\frac{1}{8}\right]$$

$$\frac{1}{2}-\frac{3}{2}\frac{1}{3}+\frac{1}{2}\frac{1}{4}$$
$$=\frac{1}{8}$$

And, write the energy equation as $\frac{K_l\left(T_{sat}-T_w\right)}{\delta}dx = h_{fg}\frac{d\dot{m}}{dx}dx-\frac{d}{dx}\left\{\int_0^\delta \rho_l u C_{pl}\left(T-T_{sat}\right)dy\right\}dx$.

So, just putting all the terms here we wrote this expression.

Now, you can see in every term dx is there. So, these dx term you can cancel. So, you can write it as $\frac{K_l\left(T_{sat}-T_w\right)}{\delta} = h_{fg}\frac{d\dot{m}}{dx}-\frac{d}{dx}\int_0^\delta \rho_l u C_{pl}\left(T-T_{sat}\right)dy$.

So, now let us calculate $\frac{d\dot{m}}{dx}$ you know the mass flow rate $\dot{m}$. So, $\dot{m}=\frac{g\rho_l\left(\rho_l-\rho_v\right)\delta^3}{3\mu_l}$. So, $\frac{d\dot{m}}{dx}=\frac{g\rho_l\left(\rho_l-\rho_v\right)}{3\mu_l}\frac{d\delta^3}{dx}$.

And, we know u, $u=\frac{g\left(\rho_l-\rho_v\right)\delta^2}{\mu_l}\left[\frac{y}{\delta}-\frac{1}{2}\left(\frac{y}{\delta}\right)^2\right]$and here temperature is there. So, you know $T_{sat}-T=\left(1-\frac{y}{\delta}\right)\left(T_{sat}-T_w\right)$. So, if you put let us evaluate this term first. So, if you put u and T -T_{sat}, then you integrate it.

So, if you write this,

$\int_0^\delta \rho_l u C_{pl}\left(T-T_{sat}\right)dy = C_{pl}\rho_l\frac{d}{dx}\int_0^\delta \frac{g\left(\rho_l-\rho_v\right)\delta^2}{\mu_l}\left\{\frac{y}{\delta}-\frac{1}{2}\left(\frac{y}{\delta}\right)^2\right\}\left\{-\left(1-\frac{y}{\delta}\right)\left(T_{sat}-T_w\right)\right\}dy$ .So, now, you integrate this one. So, if you see this you can write as, $=-\frac{C_{pl}\rho_l g\left(\rho_l-\rho_v\right)\left(T_{sat}-T_w\right)}{\mu_l}\frac{d}{dx}\left[\delta^3\int_0^1\left\{\frac{y}{\delta}-\frac{3}{2}\left(\frac{y}{\delta}\right)^2+\frac{1}{2}\left(\frac{y}{\delta}\right)^3\right\}d\left(\frac{y}{\delta}\right)\right]$. So, you can see here δ^2 is there, we have divided by δ. So, it will be δ^3. So, now, easily you can integrate it.

You will get $=-\frac{C_{pl}\rho_l g\left(\rho_l-\rho_v\right)\left(T_{sat}-T_w\right)}{\mu_l}\frac{d}{dx}\left[\delta^3\frac{1}{8}\right]$. Because it will be, $\frac{1}{2}-\frac{3}{2}\frac{1}{3}+\frac{1}{2}\frac{1}{4}=\frac{1}{8}$.

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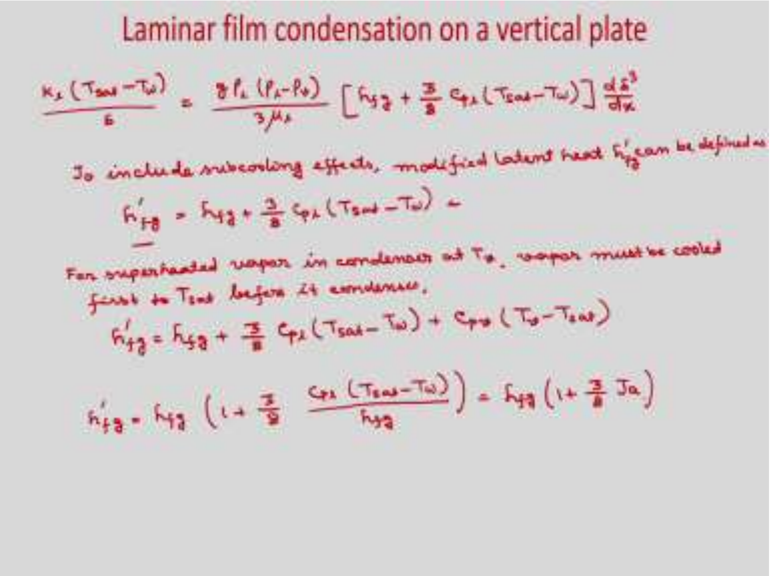

Laminar film condensation on a vertical plate

$$\frac{k_l(T_{sat}-T_w)}{\delta} = \frac{g\rho_l(\rho_l-\rho_v)}{3\mu_l}\left[h_{fg}+\frac{3}{8}c_{pl}(T_{sat}-T_w)\right]\frac{d\delta^3}{dx}$$

To include subcooling effects, modified latent heat h'_{fg} can be defined as

$$h'_{fg} = h_{fg}+\frac{3}{8}c_{pl}(T_{sat}-T_w)$$

For superheated vapor in condenser at T_v, vapor must be cooled first to T_{sat} before it condenses.

$$h'_{fg} = h_{fg}+\frac{3}{8}c_{pl}(T_{sat}-T_w)+c_{pv}(T_v-T_{sat})$$

$$h'_{fg} = h_{fg}\left(1+\frac{3}{8}\frac{c_{pl}(T_{sat}-T_w)}{h_{fg}}\right) = h_{fg}\left(1+\frac{3}{8}Ja\right)$$

And, $\frac{d\dot{m}}{dx}$ you put it here so, finally, you will get $\frac{K_l\left(T_{sat}-T_w\right)}{\delta}=\frac{g\rho_l\left(\rho_l-\rho_v\right)}{3\mu_l}\left[h_{fg}+\frac{3}{8}C_{pl}\left(T_{sat}-T_w\right)\right]\frac{d\delta^3}{dx}$. So, with that you can write as $h_{fg}+\frac{3}{8}C_{pl}\left(T_{sat}-T_w\right)$. So, you can see when we consider the sensible heat one additional term is coming. So, we can say that it is a modified latent heat of condensation h'_{fg} which is your h_{fg} plus the term which is coming considering the sub cooling effect.

You can say that to include sub cooling effects modified latent heat h' can be defined as $h'_{fg}=h_{fg}+\frac{3}{8}C_{pl}\left(T_{sat}-T_w\right)$. So, if you do not consider the sensible heat part, then the last term will be 0. So, simply h_{fg} you can use and that will come from the energy balance where you can write that $d\dot{m}_v\, h_{fg}=q''_w\left(dA\right)$.

So, that is the energy balance, but when we considered the sub cooling effect. So, this additional term is coming. Now, if you consider that your vapor is at higher temperature than T_{sat}; that means superheated vapor then you have to bring this temperature to T_{sat} then only the condensation will take place.

So, that time if you consider the modified latent heat, then you need to consider that effect also to cool down the vapor from the superheated temperature to the saturation temperature. So, in that particular case you can write for superheated vapor in condenser at T_v, vapor must be cooled first to T_{sat} before it condenses. So, in that case, $h'_{fg} = h_{fg} + \frac{3}{8}C_{pl}\left(T_{sat} - T_w\right) + C_{pv}\left(T_v - T_{sat}\right)$.

So, now here we will define one new non-dimensional number. So, when you considering the sub cooling effect $h'_{fg} = h_{fg}\left(1 + \frac{3}{8}\frac{C_{pl}\left(T_{sat} - T_w\right)}{h_{fg}}\right) = h_{fg}\left(1 + \frac{3}{8}Ja\right)$, one new non-dimensional number which is known as Jakob number.

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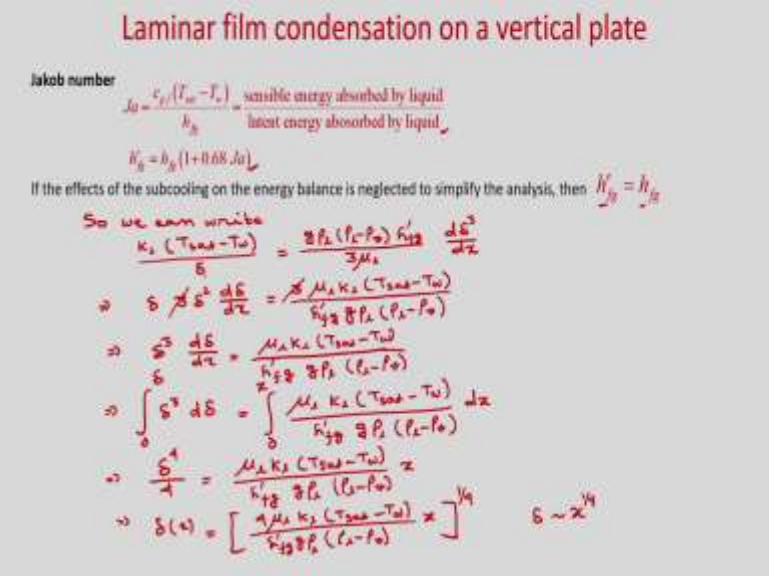

Laminar film condensation on a vertical plate

Jakob number

$$Ja = \frac{c_{pl}(T_{sat} - T_w)}{h_{fg}} = \frac{\text{sensible energy absorbed by liquid}}{\text{latent energy abosorbed by liquid}}$$

$$h'_{fg} = h_{fg}(1 + 0.68\,Ja)$$

If the effects of the subcooling on the energy balance is neglected to simplify the analysis, then $h'_{fg} = h_{fg}$

So we can write

$$\frac{k_l(T_{sat} - T_w)}{\delta} = \frac{g\rho_l(\rho_l - \rho_v)h'_{fg}}{3\mu_l}\frac{d\delta^3}{dx}$$

$$\Rightarrow \delta\,\cancel{3}\delta^2\frac{d\delta}{dx} = \frac{\cancel{3}\mu_l k_l(T_{sat} - T_w)}{h'_{fg}\,g\rho_l(\rho_l - \rho_v)}$$

$$\Rightarrow \delta^3\frac{d\delta}{dx} = \frac{\mu_l k_l(T_{sat} - T_w)}{h'_{fg}\,g\rho_l(\rho_l - \rho_v)}$$

$$\Rightarrow \int_0^\delta \delta^3\,d\delta = \int_0^x \frac{\mu_l k_l(T_{sat} - T_w)}{h'_{fg}\,g\rho_l(\rho_l - \rho_v)}\,dx$$

$$\Rightarrow \frac{\delta^4}{4} = \frac{\mu_l k_l(T_{sat} - T_w)}{h'_{fg}\,g\rho_l(\rho_l - \rho_v)}\,x$$

$$\Rightarrow \delta(x) = \left[\frac{4\mu_l k_l(T_{sat} - T_w)}{h'_{fg}\,g\rho_l(\rho_l - \rho_v)}\,x\right]^{1/4} \qquad \delta \sim x^{1/4}$$

So, now, you can see that we can define $Ja = \frac{C_{pl}\left(T_{sat} - T_w\right)}{h_{fg}}$ where it is the ratio of sensible energy absorbed by liquid to the latent energy absorbed by the liquid. And, already so, if you define this Jakob number like this, then the modified latent heat $h'_{fg} = h_{fg}\left(1 + 0.68Ja\right)$.

And, if the effects of the sub-cooling on the energy balance is neglected to simplify the analysis then whatever now we will derive just in place of h'_{fg} you put h_{fg} then the sub-cooling effect will be neglected. So, now we have done the integration. So, let us put in the original equation and find what is the film thickness δ.

So, we can right, $\frac{K_l\left(T_{sat}-T_w\right)}{\delta}=\frac{g\rho_l\left(\rho_l-\rho_v\right)h'_{fg}}{3\mu_l}\frac{d\delta^3}{dx}$.

So, you can see if you take $\frac{d\delta^3}{dx}$, then you can write it as, $\delta 3\delta^2\frac{d\delta}{dx}=\frac{3\mu_l K_l\left(T_{sat}-T_w\right)}{h'_{fg}g\rho_l\left(\rho_l-\rho_v\right)}$.

So, you can see these 3, 3 will get cancel. So, $\delta^3\frac{d\delta}{dx}=\frac{\mu_l K_l\left(T_{sat}-T_w\right)}{h'_{fg}g\rho_l\left(\rho_l-\rho_v\right)}$. So, we can integrate this now. So, you can write $\int_0^\delta \delta^3 d\delta=\int_0^x \frac{\mu_l K_l\left(T_{sat}-T_w\right)}{h'_{fg}g\rho_l\left(\rho_l-\rho_v\right)}dx$.

So, you can see if you integrate from x = 0 to l; l is the length of the plate, then at x = 0 what is the thickness of the film? It is 0 and at x = l it is δ . So, let us integrate left hand side 0 to δ and this is 0 to x; x at any length x you can put. So, you can write $\frac{\delta^4}{4}=\frac{\mu_l K_l\left(T_{sat}-T_w\right)}{h'_{fg}g\rho_l\left(\rho_l-\rho_v\right)}x$. So, in general we have written the film thickness. So, you can see δ which is function of x you can write, $\delta\left(x\right)=\left[\frac{4\mu_l K_l\left(T_{sat}-T_w\right)}{h'_{fg}g\rho_l\left(\rho_l-\rho_v\right)}x\right]^{1/4}$.

So, you see this film thickness film thickness varies with x. So, what is the order? $x^{1/4}$. So, $\delta \sim x^{1/4}$ and when we will solve some example problems you will see that at the end of vertical plate this δ will be order of less than 1 mm. So, it is very very thin.

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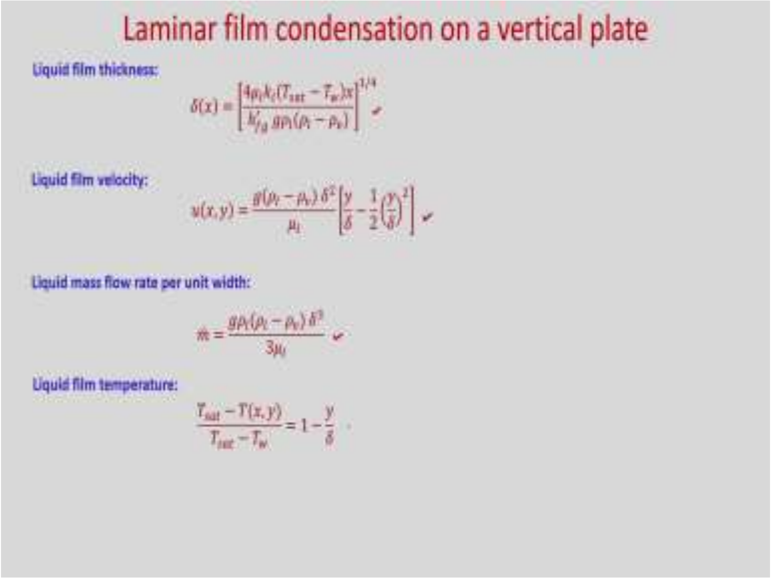

Laminar film condensation on a vertical plate

Liquid film thickness:

$$\delta(x)=\left[\frac{4\mu_l k_l(T_{sat}-T_w)x}{h'_{fg}\,g\rho_l(\rho_l-\rho_v)}\right]^{1/4}$$

Liquid film velocity:

$$u(x,y)=\frac{g(\rho_l-\rho_v)\delta^2}{\mu_l}\left[\frac{y}{\delta}-\frac{1}{2}\left(\frac{y}{\delta}\right)^2\right]$$

Liquid mass flow rate per unit width:

$$\dot{m}=\frac{g\rho_l(\rho_l-\rho_v)\delta^3}{3\mu_l}$$

Liquid film temperature:

$$\frac{T_{sat}-T(x,y)}{T_{sat}-T_w}=1-\frac{y}{\delta}$$

So, now we have found the film thickness δ. So, you will be able to calculate the velocity as well as temperature profile, then the mass flow rate because all these we have represented in terms of δ. So, you can see the δ(x) we have found now. So, the liquid film velocity is this one.

So, δ if you put you will be you will be getting the film velocity and now, m dot which is your mass flow rate per unit width. So, this is the expression where δ you can put this value and liquid film temperature is this one. So, now we are interested to find what is the local and average heat transfer coefficient and then local and average Nusselt number. So, you can see liquid film thickness we have found like this.

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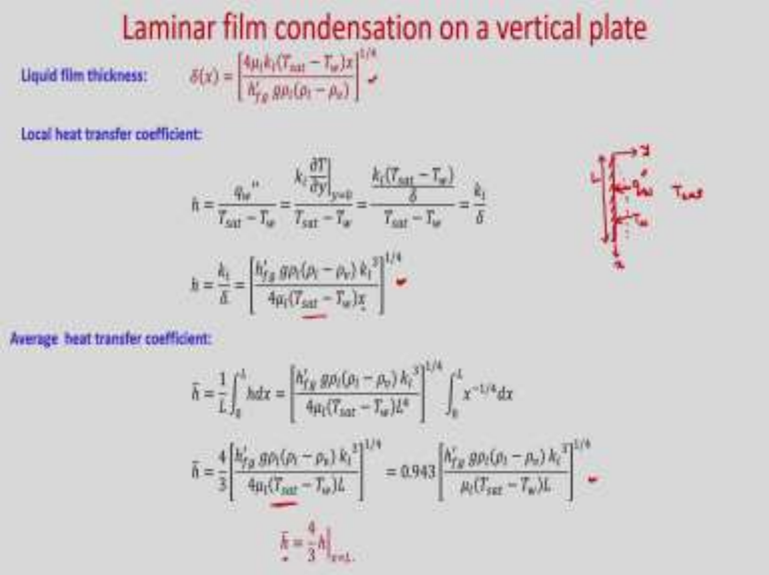

Laminar film condensation on a vertical plate

Liquid film thickness: $\delta(x) = \left[\frac{4\mu_l k_l (T_{sat} - T_w)x}{h'_{fg}\, g\rho_l(\rho_l - \rho_v)}\right]^{1/4}$

Local heat transfer coefficient:

$$h = \frac{q_w''}{T_{sat} - T_w} = \frac{k_l \left.\frac{\partial T}{\partial y}\right|_{y=0}}{T_{sat} - T_w} = \frac{\frac{k_l (T_{sat} - T_w)}{\delta}}{T_{sat} - T_w} = \frac{k_l}{\delta}$$

$$h = \frac{k_l}{\delta} = \left[\frac{h'_{fg}\, g\rho_l(\rho_l - \rho_v)\, k_l^3}{4\mu_l (T_{sat} - T_w)x}\right]^{1/4}$$

Average heat transfer coefficient:

$$\bar{h} = \frac{1}{L}\int_0^L h\,dx = \left[\frac{h'_{fg}\, g\rho_l(\rho_l - \rho_v)\, k_l^3}{4\mu_l (T_{sat} - T_w)L^4}\right]^{1/4} \int_0^L x^{-1/4}\,dx$$

$$\bar{h} = \frac{4}{3}\left[\frac{h'_{fg}\, g\rho_l(\rho_l - \rho_v)\, k_l^3}{4\mu_l (T_{sat} - T_w)L}\right]^{1/4} = 0.943\left[\frac{h'_{fg}\, g\rho_l(\rho_l - \rho_v)\, k_l^3}{\mu_l (T_{sat} - T_w)L}\right]^{1/4}$$

$$\bar{h} = \frac{4}{3}h\Big|_{x=L}$$

So, now, heat transfer coefficient which is your local heat transfer coefficient you can define as $h = \frac{q_w''}{T_{sat} - T_w}$. So, you can see if this is your plate. So, q_w'' is this one and this is your y direction. So, obviously, you can see the heat transfer coefficient you can write as the temperature of the plate is T_w, this is your film thickness and vapor temperature is T_{sat}.

So, obviously, the heat transfer coefficient you can write as $h = \frac{k_l \left.\frac{\partial T}{\partial y}\right|_{y=0}}{T_{sat} - T_w}$ and this is a positive sign; because y direction the q_w'' whatever we have considered in negative y direction.

So, obviously, $\dfrac{\dfrac{k_l\left(T_{sat}-T_w\right)}{\delta}}{T_{sat}-T_w}$.So, here you can see this temperature profile you know. So, you just find $\left.\dfrac{\partial T}{\partial y}\right|_{y=0}$. So, you can see heat transfer coefficient is $\dfrac{k_l}{\delta}$. So, δ if you put in this expression, then you will get the local heat transfer coefficient as this. So, you have to remember this expression.

And, average heat transfer coefficient now you have to just integrate it over the plate length L. So, if plate length L, x is measured from here in the downward direction. So, $\bar{h}=\dfrac{1}{L}\int_0^L h dx$.

So, $\bar{h}=\dfrac{1}{L}\int_0^L h dx=\left[\dfrac{h_{fg}^{'}g\rho_l\left(\rho_l-\rho_v\right)k_l^3}{4\mu_l\left(T_{sat}-T_w\right)L^4}\right]^{1/4}\int_0^L x^{-1/4}dx$. So, if you put it then you will get the average heat transfer coefficient like this. So, $\bar{h}=\dfrac{4}{3}\left[\dfrac{h_{fg}^{'}g\rho_l\left(\rho_l-\rho_v\right)k_l^3}{4\mu_l\left(T_{sat}-T_w\right)L}\right]^{1/4}$. So, if you evaluate it you will get around $0.943\left[\dfrac{h_{fg}^{'}g\rho_l\left(\rho_l-\rho_v\right)k_l^3}{4\mu_l\left(T_{sat}-T_w\right)L}\right]^{1/4}$. Now, you see this is your local heat transfer coefficient. If you put the local heat transfer coefficient x = L, so, you will get just x = L if you put you are going to get similar expression only outside there will be 4 by 3 because you see this is the expression is same as this inside the bracket except x equal to L. So, the average heat transfer coefficient we can write $\bar{h}=\dfrac{4}{3}h\big|_{x=L}$.

Now, we can easily calculate the Nusselt number because as you have found the local and average heat transfer coefficient using that you will be able to calculate the Nusselt number.

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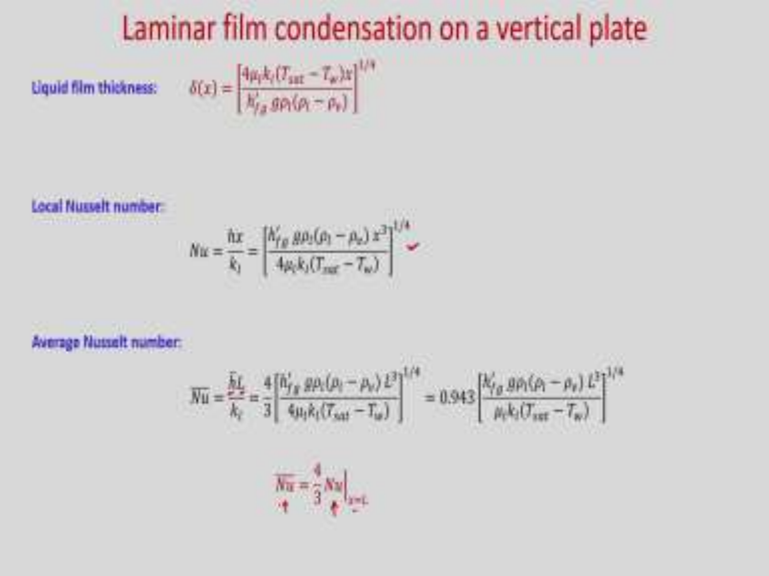

Laminar film condensation on a vertical plate

Liquid film thickness: $\delta(x)=\left[\frac{4\mu_l k_l (T_{sat}-T_w)x}{h'_{fg}\, g\rho_l(\rho_l-\rho_v)}\right]^{1/4}$

Local Nusselt number:

$$Nu=\frac{hx}{k_l}=\left[\frac{h'_{fg}\, g\rho_l(\rho_l-\rho_v)\,x^3}{4\mu_l k_l (T_{sat}-T_w)}\right]^{1/4}$$

Average Nusselt number:

$$\overline{Nu}=\frac{\bar{h}L}{k_l}=\frac{4}{3}\left[\frac{h'_{fg}\, g\rho_l(\rho_l-\rho_v)\,L^3}{4\mu_l k_l (T_{sat}-T_w)}\right]^{1/4}=0.943\left[\frac{h'_{fg}\, g\rho_l(\rho_l-\rho_v)\,L^3}{\mu_l k_l (T_{sat}-T_w)}\right]^{1/4}$$

$$\overline{Nu}=\frac{4}{3}Nu\Big|_{x=L}$$

So, you can see $Nu=\frac{hx}{k_l}$. So, h already we have found. So, this h if you put here and rearrange you will get this expression. So, now if you put this h bar whatever we have put already evaluated. So, average Nusselt number, $\overline{Nu}=\frac{\bar{h}x}{k_l}$.

So, these if you put here and rearrange you will get, $\overline{Nu}=0.943\left[\frac{h'_{fg}\, g\rho_l\left(\rho_l-\rho_v\right)L^3}{\mu_l k_l\left(T_{sat}-T_w\right)}\right]^{1/4}$ and remember here we have defined the Nusselt number based on the length of the plate. Similarly the average Nusselt number you can write $\overline{Nu}=\frac{4}{3}Nu\big|_{x=L}$. So, this is your local Nusselt number this is your local Nusselt number and this is your average Nusselt number.

So, today we considered laminar film condensation on a vertical plate and we neglected the inertia term in the left hand side. So, because the film thickness is very small and it travels very in very slow motion hence we neglected the convection term in the momentum equation and energy equation and from there we calculated the velocity distribution as well as temperature distribution.

Then we calculated the mass flow rate per unit width, then from there we have calculated equating the using the conservation of mass and conservation of energy we have found the film thickness. Once you know the film thickness you know the velocity distribution, temperature profile as well as mass flow rate.

Then we have calculated the local and average heat transfer coefficient as you know the film thickness δ. So, you will be able to calculate easily once you calculate the average heat transfer coefficient and local heat transfer coefficient you will be able to calculate the average Nusselt number and local Nusselt number and that we have found. Now, from there you can conclude that, $\overline{Nu} = \frac{4}{3} Nu\big|_{x=L}$.

Thank you.