Fundamentals of Convective Heat Transfer

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Module - 02
Preliminary Concepts
Lecture - 04
Derivation of boundary layer equations

Hello everyone. So, in last class, we derived the energy equation. Now, in today's class, first we will just summarize what we have done in last class, and we will write the governing equations in cylindrical and spherical coordinates, then we will simplify these equations for Cartesian coordinate for the boundary layer flows.

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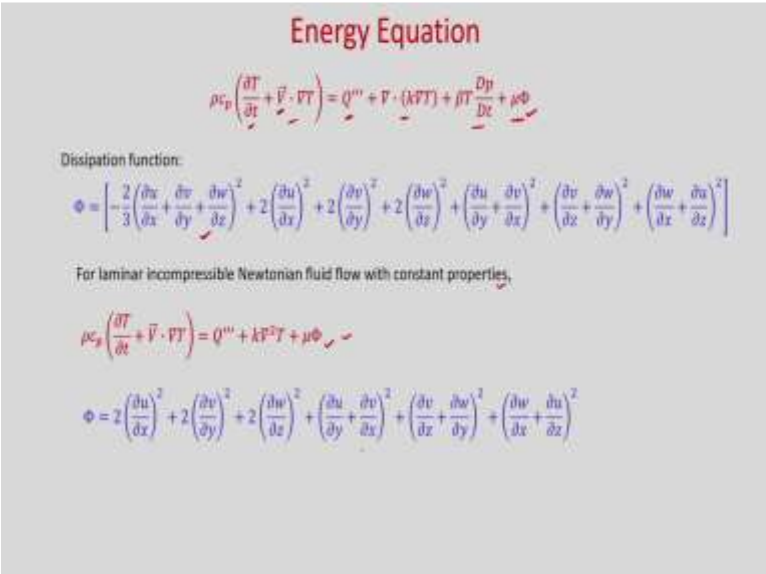

So, we can see we have derived this equation in general, and Φ is the dissipation function, and $\mu\Phi$ is the dissipation term, here this is the temporal term, this is the inertia term where temperature is convected due to the velocity v, this is the heat generation per unit volume, and this is the diffusion term.

And you can see for laminar incompressible Newtonian fluid flow with constant properties, you can write these equations, because constant properties so k you can take it outside.

And you can see for incompressible flow velocity will be low, so you can see these term you can neglect, and in general you can write this equation with this assumptions, where dissipation function you can see this is the term you can neglect because for incompressible flow divergence of v will be 0; that means, $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$. So, dropping this term, you can write the dissipation function as this.

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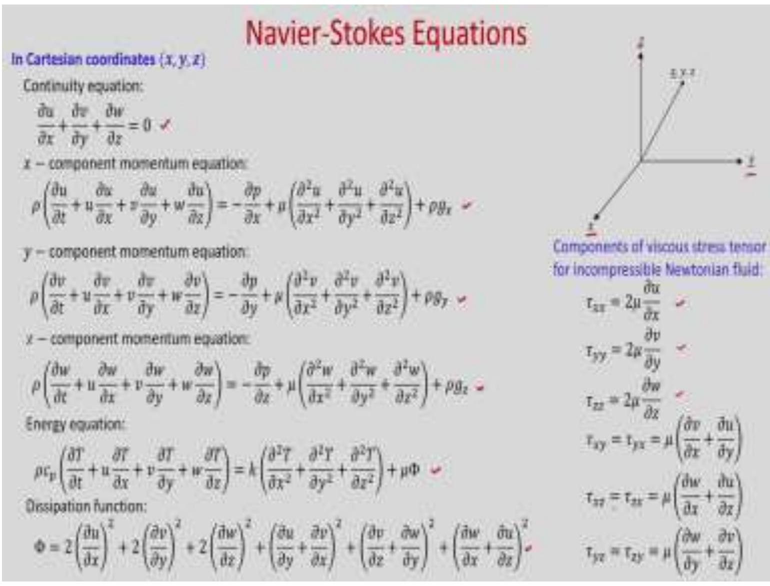

So, in Cartesian coordinate now let us write the governing equations for laminar incompressible Newtonian fluid flow with constant properties. So, this is the coordinate system Cartesian coordinate x, y, z. So, this is the continuity equation. This is the x component of momentum equation, this is the y component of momentum equation, and this is the z component of momentum equation, where g x, g y, g z are the gravitational acceleration in x, y, z direction respectively. And this is the energy equation, and this is the dissipation function.

And for these equations you can see you can find the components of viscous stress tensor like this. So, there will be six components, because this stress tensor is symmetric. So, $\tau_{xy}=\tau_{yx}$, so obviously, we will have total six components in the stress tensor.

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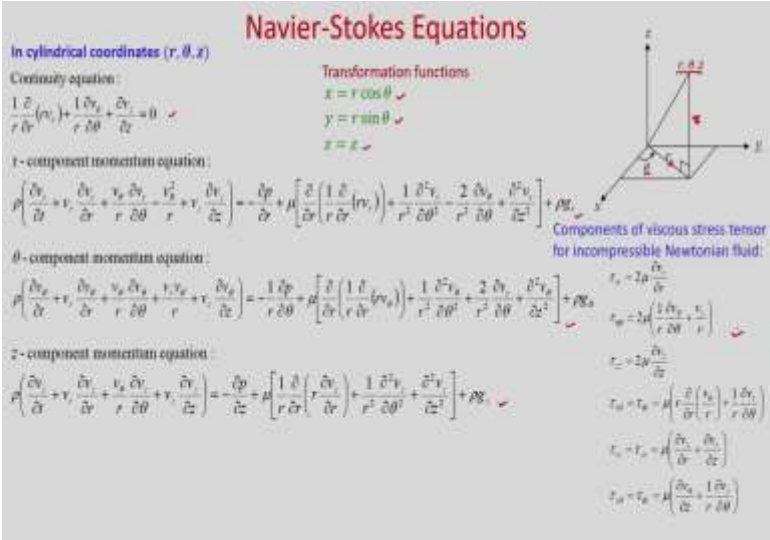

Now, if you use some suitable transformation function, then you can convert these equations in Cartesian coordinate to the equations in cylindrical coordinate. So, if you use this as a cylindrical coordinate, so you can see this is the r, and this is the θ, and this is the z.

Then for r, θ, z coordinate in cylindrical coordinate, if you use the transformation function as x = r cosθ, y = r sinθ, and z =z, then you can write the continuity equation as these; r component of equation as this; θ component of equation as this; and z component of momentum equation as this. And corresponding viscous stress tensor will be this.

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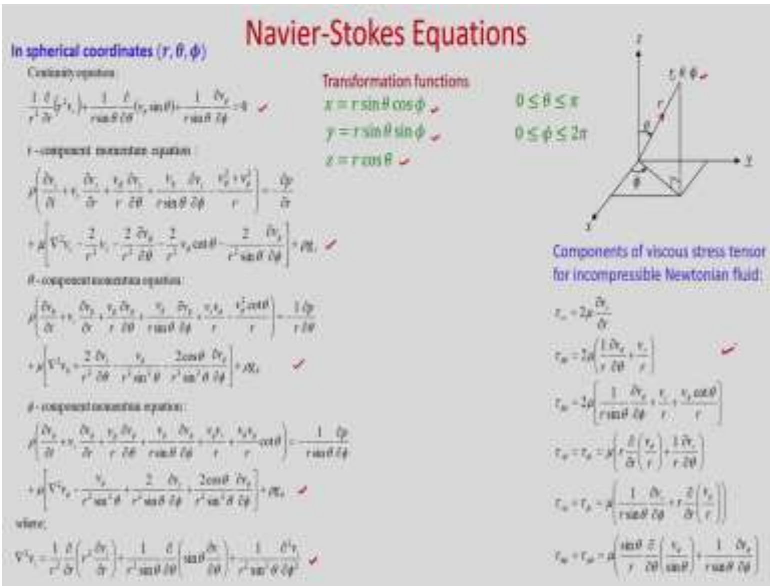

And in spherical coordinate, so we are considering r, θ, Φ, so this is the Φ. So, Φ obviously, you can see it will vary 0 to 2 π; this is the θ, it will be 0 to π; and this is the r. So, if you use this transformation functions x =r sinθ cosΦ; y = r sinθ sinΦ; and z = r cos θ. Then you can write the continuity equation in spherical coordinate like these.

This is the r component of momentum equation; this is the θ component of momentum equation; and this is the Φ component of momentum equation, where $\nabla^2 . v_i$ is given by this expression. And corresponding components of viscous stress tensor can be written like this.

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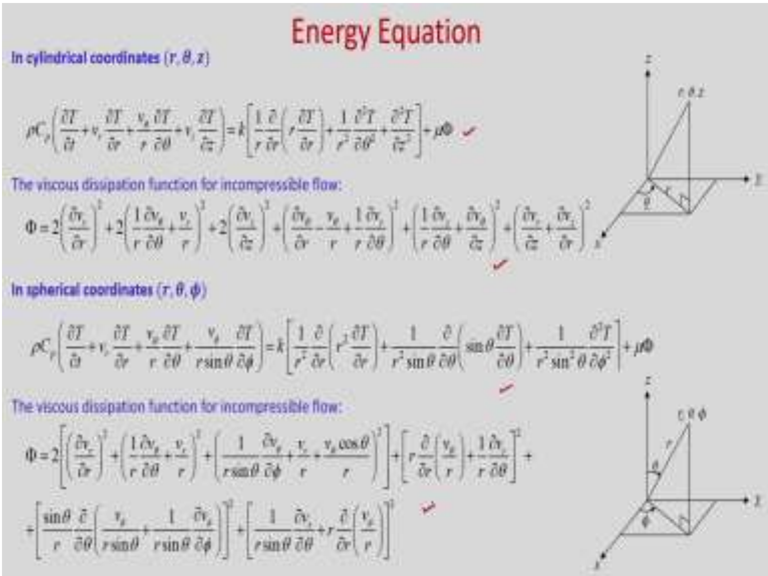

And energy equation now in cylindrical and spherical coordinate you can write like this. And this is the viscous dissipation function. And similarly in spherical coordinate this is the energy equation, and this is the viscous dissipation function.

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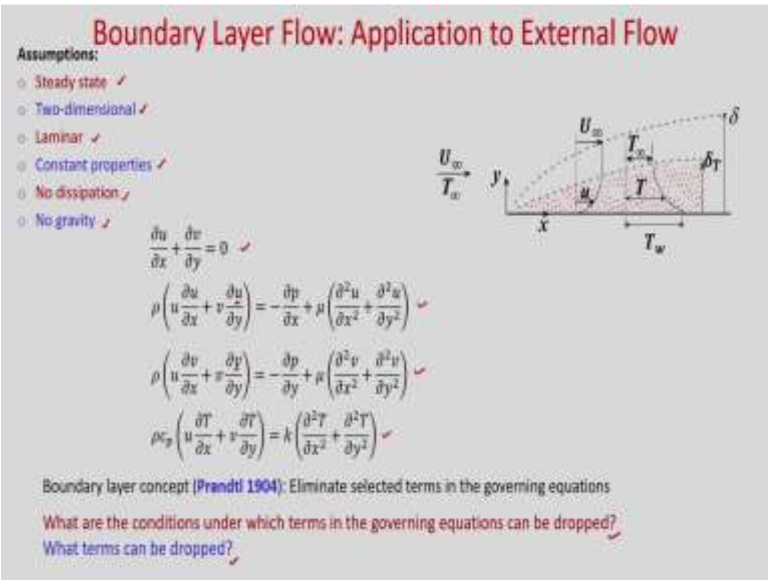

Now, we will discuss about the boundary layer flow. We will consider Cartesian coordinate and we will consider external flows. First we will assume that it is a steady, two-dimensional and laminar flow with constant properties and neglecting dissipation and gravity.

So, invoking these assumptions, you can write the governing equations as this. This is the continuity equation it is a two-dimensional flow. So, this is the continuity equation. This is the u_x component of momentum equation, and this is the y component of momentum equation, and this is the energy equation. And obviously, you can see u is the velocity in x direction; v is the velocity in y direction.

So, in today's class, we will use boundary layer concept, and we will see that if you can drop some term from these equations for boundary layer flows. So, scientist Prandtl actually used scaling analysis, and showed that few terms in the governing equation can be dropped because those terms are very small compared to the others.

So, we will ask these questions now. What are the conditions under which terms in the governing equations can be dropped, and what terms can be dropped? So, these questions now we will answer one by one.

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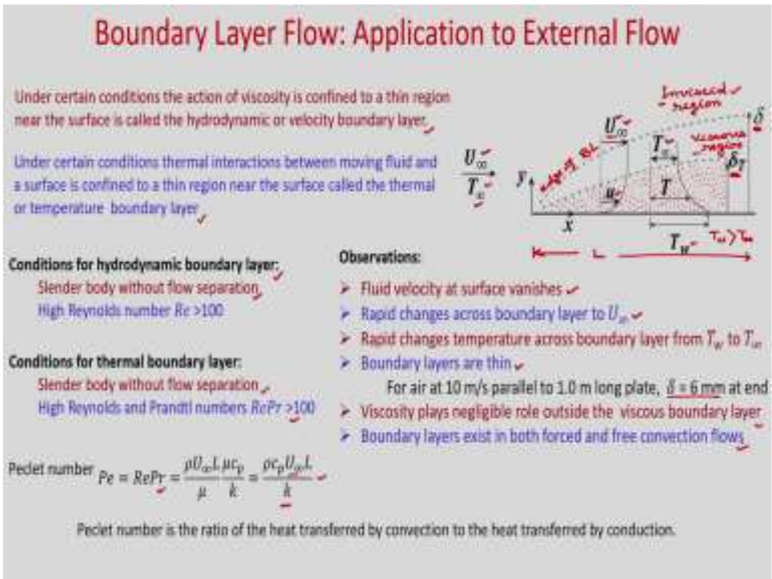

First let us understand what is boundary layer. Let us consider that you have a external flow over a flat plate. If you see that there will be some region close to the wall, where viscous effect is significant, and some other region away from the surface you will find that there is no effect of viscosity.

So, you can see here you have a flat plate let us say of length L, and you have a uniform flow U_∞, and temperature T_∞. This velocity is known as free stream velocity and this temperature is known as free stream temperature; x is measured in the axial direction, and y is measured perpendicular to the plate.

Now, if you see that the velocity boundary layer, so there is some region where you have effect of viscosity, and this is known as viscous region. And some region away from the surface, there will be no effect of viscosity and that is known as inviscid region.

And if you see the velocity profile obviously to invoke the no slip condition at the wall velocity will be 0. So, the fluid flow which is residing on the top of this flat plate, the velocity will be 0. And gradually this velocity will increase from 0 to U_∞, where U_∞ is the free stream velocity.

Now, we can see that there is a some region which is known as boundary layer; inside that you have viscous region, and outside that you will have inviscid region. So, if you consider that this is the edge of the boundary layer, then you can see this distance from the normal distance from the flat plate is known as hydrodynamic boundary layer thickness that is denoted by δ.

δ is the normal distance from the plate at which these velocity U becomes almost 99 % of U_∞, so that is known as hydro dynamic boundary layer thickness and this fictitious line where at every distance, we have the hydro dynamic boundary layer thickness, so that is known as edge of boundary layer.

So, the inside of this edge of boundary layer you can see there is a viscous region, and outside it is inviscid region. So, you can see under certain conditions the action of viscosity is confined to a thin region near the surface is called the hydrodynamic or velocity boundary layer.

Now, if you consider the thermal part, so you can see you have free stream temperature T_∞ and let us say wall temperature is T_w, and $T_w > T_\infty$. Then you can see that there will be some region where thermal effect will be there, and temperature gradient will exist. And this temperature will vary inside this layer from T_w to T_∞.

And outside this region you can see it will maintain the free stream temperature T_∞. So, the normal distance from the plate up to which you have the effect of this temperature gradient, so that is known as thermal boundary layer thickness and denoted as δ_T.

So, you can see that these fictitious line inside, there is a temperature gradient, but outside you have free stream temperature T_∞. So, this is the edge of thermal boundary layer. So, you can see under certain conditions, thermal interactions between moving fluid and a surface is confined to a thin region near the surface called the thermal or temperature boundary layer.

So, here we have to consider two important assumptions, one is that there is no flow separation. Then under that condition we can derive the boundary layer equations and you have a slender body that means thickness of the body is much, much smaller than the length of the body.

So, you can see conditions for hydro dynamic boundary layer, we have slender body without flow separation, and we have to consider high Reynolds number flows. And Reynolds number should be >100. Conditions for thermal boundary layer, slender body without flow separation and high Reynolds and Prandtl numbers flow that means, the Peclet number which is the product of Reynolds number and Prandtl number should be >100.

And you can see Peclet number you can write in this expression. So, you can see that Peclet number is the ratio of the heat transferred by the convection to the heat transferred by conduction. So, let us list down the observations of these boundary layer flow. Fluid velocity at surface vanishes rapid changes across boundary layer to U_∞.

So, from 0 to U_∞, these changes occurring inside the boundary layer. Rapid change temperature across boundary layer from T_w to T_∞. So we can see it is changing from T_w to T_∞ inside the thermal boundary layer.

Another observation is that boundary layers are thin. From the experiment it is seen that for air at 10 m/s parallel to 1 m long plate this boundary layer thickness will be of the order of 6 mm at the end; at x equal to 1 m, you will get 6 mm.

So, you can see that you have the length of the plate as 1 m and the boundary layer thickness is 6 mm, so it is very very small compared to the length of the plate. So, you can see boundary layers are very thin. Viscosity plays negligible role outside the viscous boundary layer which is your inviscid region; and boundary layer exist in both forced and free convection flows.

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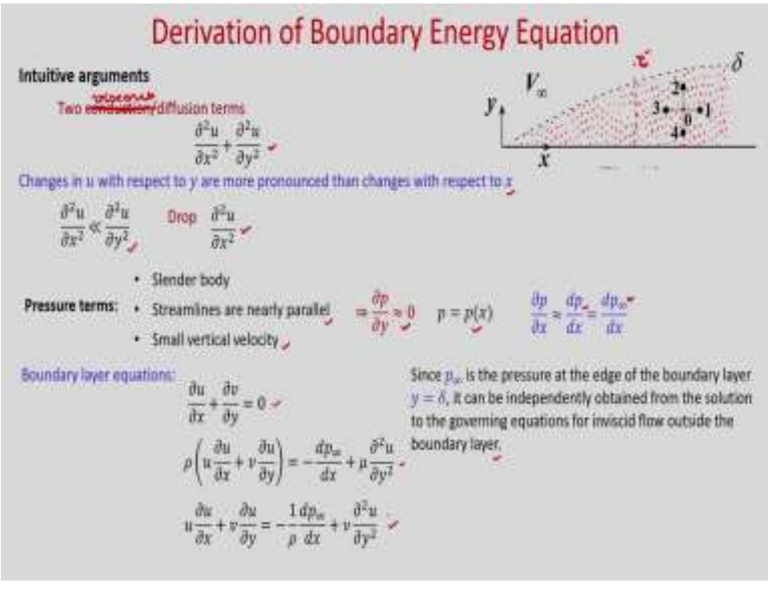

Now, we will use two different approaches to see that which term we can drop from the governing equations. So, first let us use the intuitive arguments. So, you can see that in the right hand side of the momentum equations, we have viscous terms. Let us see that if you can drop one term. So, there are two viscous terms, $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}$. So, this is not conduction. This is diffusion or viscous. So, these are the two terms.

Now, let us consider one small insect which is sitting inside the boundary layer, and it is not disturbing the flow. And let us say that it is flying at this position 0. Now, it is experiencing a high velocity and it wants to shift to a lower velocity region. So, by intuitive arguments what can you say that where it will go 1, 2, 3 or 4 positions.

So, you can see obviously that it will travel to position 4, because it will have less velocity right, because if it is travels from 0 to 4, then it will have less velocity. And if it is ultimately goes to the surface, then it will not feel any velocity.

So, you can see that changes in u with respect to y are more pronounced than changes with respect to x. So, it does not answer that the change of these gradient $\frac{\partial u}{\partial x}$ is which one is smaller than the other. Now, you see that the insect is just one step away from the surface. So, you can see that if it goes to the surface, then obviously, the velocity will become 0, and the changes in the velocity gradient will be more.

If you consider that insect is at the edge of the boundary layer and if it goes away from the surface, then you will find that there will be not much variation in the velocity or velocity gradient. And if you consider in the axial direction, if it moves then obviously there will be not much change in the velocity gradient. So, that means, that your velocity gradient with respect to y is much higher than the velocity gradient with respect to x.

So, considering that you can see that $\frac{\partial^2 u}{\partial x^2}$ will be much less than $\frac{\partial^2 u}{\partial y^2}$, and you can drop the term from the momentum equation. So, now, what about the pressure terms. So, if you consider slender body, so obviously, streamlines are nearly parallel and you will have very small particle velocity.

So, if you have very small vertical velocity and if you consider the y momentum equations, then all the inertia terms and viscous terms will become 0. So, you can consider that the pressure gradient $\frac{\partial p}{\partial y}$ will be almost 0 very small.

So, obviously, $\frac{\partial p}{\partial y}$ will be close to 0. So, you can write p is only function of x, and $\frac{\partial p}{\partial x}$ you can write as $\frac{dp}{dx}$. And as pressure gradient along the y direction almost 0, whatever pressure is there outside the boundary layer p_∞ so that will also be impressed inside the boundary layer because there is no change in the pressure in y direction.

So, whatever pressure is there p_∞ here so inside also at this x location will be same pressure everywhere. So, you can see that you can write $\frac{dp}{dx} = \frac{dp_\infty}{dx}$. And since p_∞ is the pressure at the edge of the boundary layer that means at $y=\delta$, it can be independently obtained from the solution to the governing equation for inviscid flow outside the boundary layer.

So, you can see that in the momentum equation, then we can write $\frac{dp}{dx} = \frac{dp_\infty}{dx}$. So, we have these boundary layer equations, this is the continuity equation. And $\frac{dp}{dx}$ we have written $\frac{dp_\infty}{dx}$, and dropping the term $\frac{\partial^2 u}{\partial x^2}$ this is the term we have written. So, now you can see that if you divide by ρ, then you can write in right hand side $-\frac{1}{\rho}\frac{dp_\infty}{dx}$ and $\frac{\mu}{\rho}$ is your kinematic viscosity. So, $\nu\frac{\partial^2 u}{\partial y^2}$.

Now, let use another approach which is your mathematical approach that is your scale analysis. So, here we will use the order of magnitude analysis. And we will see the order of magnitude of each term in the governing equations and which term is having less order of magnitude compared to the other, we will drop from the governing equations.

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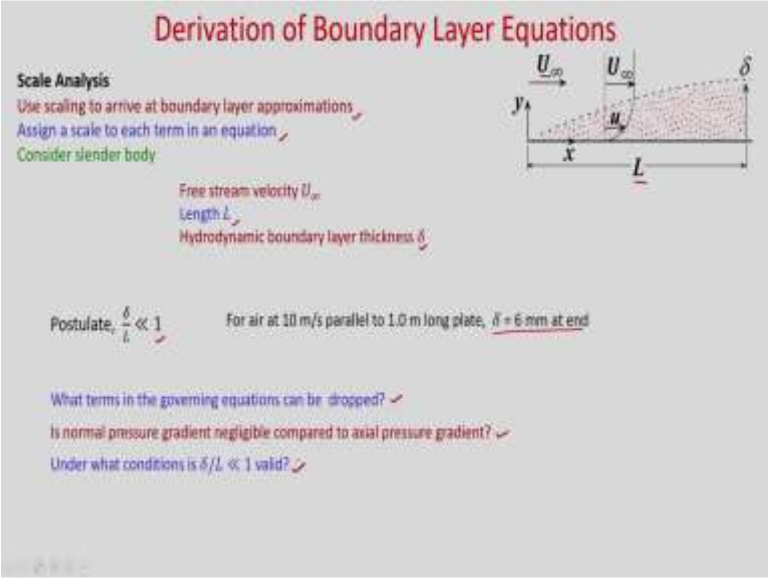

So, use scaling to arrive at boundary layer approximations, assign a scale to each term in the equation. So, obviously, you have free stream velocity U_∞, length of the plate as L, and hydrodynamic boundary layer thickness δ. Now, we will postulate that $\frac{\delta}{L}$ is much, much smaller than 1, because we have already seen that for air at 10 m/s parallel to 1 m long plate δ is of the order of 6 mm at the end. So, obviously, we can assume that $\frac{\delta}{L} << 1$.

Now, let us answer these questions. What terms in the governing equation can be dropped? Is normal pressure gradient negligible compared to the axial pressure gradient? And under what conditions is $\frac{\delta}{L} << 1$ valid? So, we will answer these questions one by one.

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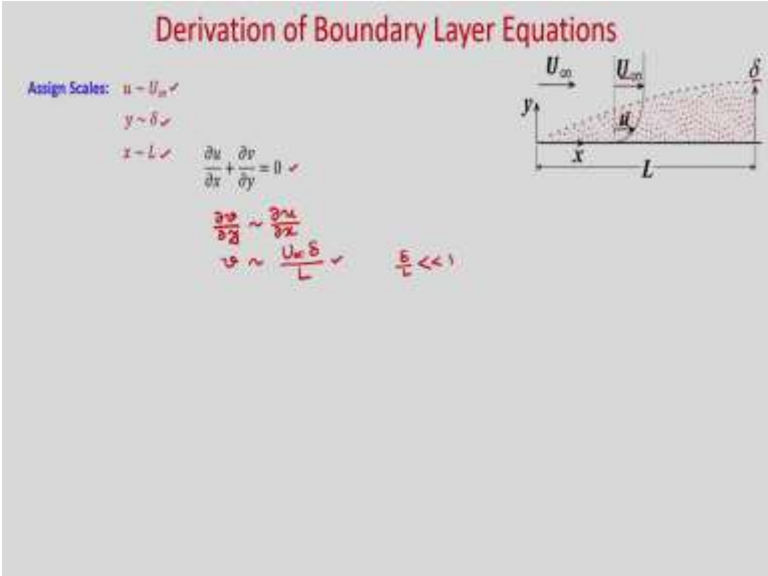

So, first assign the scales. So, velocity scale you can see the free stream velocity is U_∞. So, we can assign the scale for these velocity u as U_∞. We have hydro dynamic boundary layer thickness δ. So, in y direction, so obviously, y we can write as order of δ. And length of the plate is L, so the order of x, we can write as L. Now, let us consider the continuity equation, and find what is the order of velocity v. So, this is the continuity equation. So, you can see $\frac{\partial v}{\partial y}$, you can write as order of $\frac{\partial u}{\partial x}$.

So, you can write the order of v as this is the $\frac{\partial u}{\partial x}$, what is the order of u? It is U_∞. What is the order of δ_y, this is δ and divided by x. So, you can see the order of velocity v is $\frac{U_\infty \delta}{L}$. So, here you can see we have already assume that $\frac{\delta}{L}$ is much smaller than 1. So, we can see v will be much smaller than U_∞. So, the velocity in y direction v is very small compared to U_∞. And we got the scale for v velocity as this.

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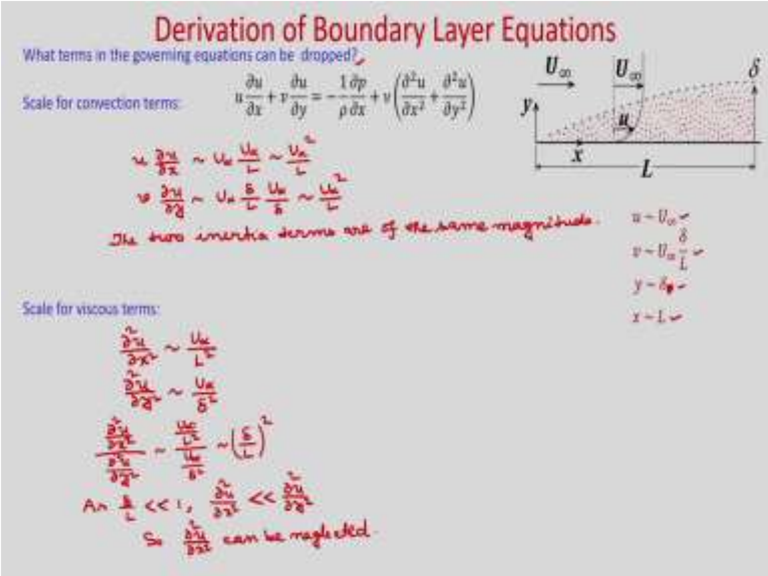

Now, let us answer this question what terms in the governing equations can be dropped. So, we have already assigned the scales, and we have found the scale for velocity v. So, you can see this is the u scale, this is the v scale, this is the y scale which is δ, and this is the x scale L.

Now, let us consider the convection terms. So, $u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}$. So, first term is $u\frac{\partial u}{\partial x}$. So, what is the order? u is U_∞, this is your U_∞ and this is L, so that means, $\frac{U_\infty^2}{L}$. Now, consider the other convection term $v\frac{\partial u}{\partial y}$.

So, what is the order of v you see this is your $U_\infty\frac{\delta}{L}$, and u is U_∞ and y is δ. So, you see this is $\frac{U_\infty^2}{L}$. So, you can see both are of same order. So, you cannot drop any terms. So, the two inertia terms are of the same magnitude. So, we cannot drop any term in the convection terms.

Now, let us consider the viscous terms. So, we have $\frac{\partial^2 u}{\partial x^2}$. So, you can see this is the order of $\frac{U_\infty}{L^2}$, and you have $\frac{\partial^2 u}{\partial y^2}$ is order of $\frac{U_\infty}{\delta^2}$. So, now you can see that we can write $\frac{\frac{\partial^2 u}{\partial x^2}}{\frac{\partial^2 u}{\partial y^2}}$.

So, this ratio will be order of $\frac{\frac{U_\infty}{L^2}}{\frac{U_\infty}{\delta^2}}$. So, you can see you can write these as order of $(\frac{\delta}{L})^2$.

So, we have already assume that $\frac{\delta}{L}$ is much smaller than 1.

So, from here you can see that $\frac{\partial^2 u}{\partial x^2}$ is much smaller than $\frac{\partial^2 u}{\partial y^2}$. So, as $\frac{\delta}{L} << 1$. So, $\frac{\partial^2 u}{\partial x^2} << \frac{\partial^2 u}{\partial y^2}$. So, $\frac{\partial^2 u}{\partial x^2}$ can be neglected from the viscous terms.

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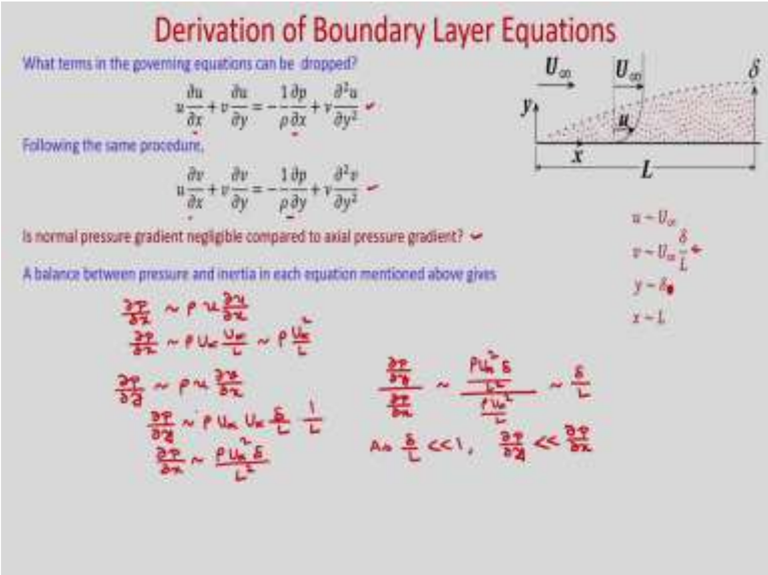

So, you can see that if $\frac{\delta}{L} << 1$, then you can drop the term $\frac{\partial^2 u}{\partial x^2}$. So, dropping the $\frac{\partial^2 u}{\partial x^2}$ term, you can write the x component momentum equation as this. And following the same procedure you can drop $\frac{\partial^2 v}{\partial y^2}$ from the y momentum equation and you can write like this.

Now, the next question is that is normal pressure gradient negligible compared to the pressure gradient? So, next question is that is normal pressure gradient negligible compared to axial pressure gradient? So, now, you can see that in the u momentum equation we have the term $\frac{\partial p}{\partial x}$, and in the v momentum equation we have $\frac{\partial p}{\partial y}$. Now, let us consider these each pressure gradient term with corresponding inertia term.

So, in the u momentum equation compare these term with the inertia term and v momentum equation compare this pressure gradient term with the inertia term, because both terms are of the same order. So, we have already seen. So, you can take any one term. So, we can see that $\frac{\partial p}{\partial x}$ will be order of $\rho u \frac{\partial u}{\partial x}$. So, you can see $\frac{\partial p}{\partial x}$ will be ρU_∞, this is U_∞ divided by x length is L. So, this is $\rho \frac{U_\infty^2}{L}$.

Now, if you see the pressure gradient term in the y component of momentum equation, so you can write $\frac{\partial p}{\partial y}$ will be order of $\rho u \frac{\partial v}{\partial x}$. So, $\frac{\partial p}{\partial y}$ will be ρU_∞, and v is the order of $U_\infty \frac{\delta}{L}$, so $U_\infty \frac{\delta}{L}$ and $\frac{1}{L}$. So, $\frac{\partial p}{\partial y}$ you can write order of $\frac{\rho U_\infty^2 \delta}{L^2}$.

So, now see the ratio $\frac{\frac{\partial p}{\partial y}}{\frac{\partial p}{\partial x}}$. So, this is the order of so $\frac{\partial p}{\partial y}$ is this one $\frac{\rho U_\infty^2 \delta}{L^2}$ and divided by $\frac{\partial p}{\partial x}$. So, it is $\frac{\rho U_\infty^2}{L}$. So, you can see this will be order of $\frac{\delta}{L}$.

So, from here you can see that $\frac{\delta}{L}$, we have assumed as much smaller than 1, so obviously, your $\frac{\partial p}{\partial y}$ will be much smaller than $\frac{\partial p}{\partial x}$. So, you can see as $\frac{\delta}{L}$ is much smaller than 1.

So, obviously, $\frac{\partial p}{\partial y} << \frac{\partial p}{\partial x}$. So, you can see that $\frac{\partial p}{\partial y}$ will be order of 0, because it is very small. You can see that we have done the balance between the pressure gradient and the inertia term you can also balance the pressure gradient with the viscous term and you will get the same result.

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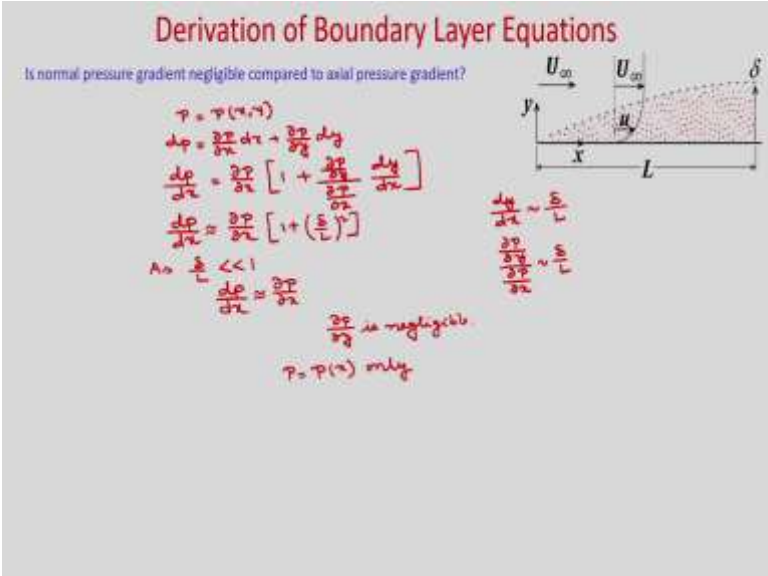

So, now, you can see for the two-dimensional flow p is function of x and y. So, for 2D flow we know that p is function of x and y. So, you can write. Now, you can write $\frac{\partial p}{\partial x}$.

So, we have divided by d x. So, you can write $\frac{\partial p}{\partial x}$.

And if you take common then you will get $1+\frac{\frac{\partial p}{\partial y}}{\frac{\partial p}{\partial x}}\frac{dy}{dx}$. So, now, you can see what is the order of $\frac{dy}{dx}$; $\frac{dy}{dx}$ is order of y is δ, and x is L. And $\frac{\partial p}{\partial y}$, and the $\frac{\frac{\partial p}{\partial y}}{\frac{\partial p}{\partial x}}$ that is also we have derived in the last slide as $\frac{\delta}{L}$.

So, you can see $\frac{dp}{dx} \approx \frac{\partial p}{\partial x}[1+(\frac{\delta}{L})^2]$, because $\frac{dy}{dx}$ is order of $\frac{\delta}{L}$ and $\frac{\partial p}{\partial y}$. So, it will be $\frac{\delta}{L}$. So, you can write this as order of this. So, as $\frac{\delta}{L}$ is much smaller than 1, so you can write $\frac{dp}{dx} \approx \frac{\partial p}{\partial x}$. So, now you can see that in this as $\frac{\partial p}{\partial y}$ is negligible. So, $p = p(x)$.

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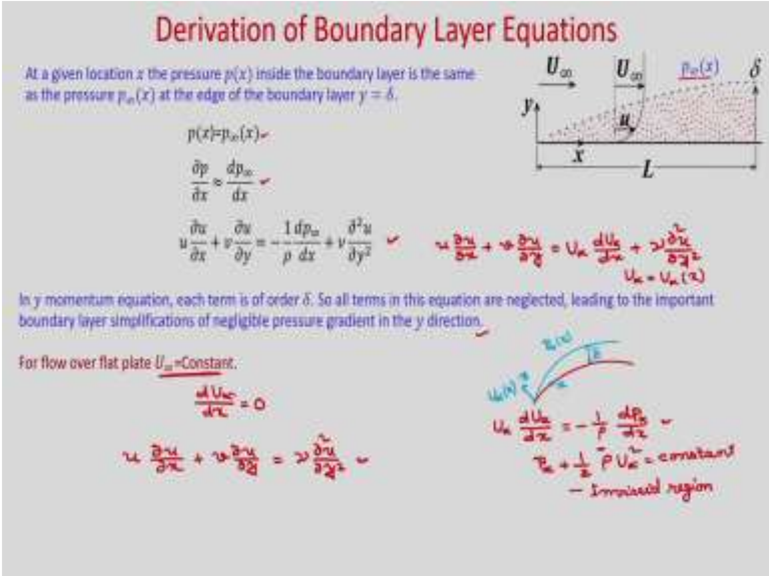

So, now, at a given location x the pressure $p(x)$ inside the boundary layer is the same as the pressure p_∞ at the edge of the boundary layer y = δ. So, if you can see that outside the velocity boundary layer, if you have a pressure gradient p_∞ which is function of x, then $p(x) = p_\infty(x)$, and $\frac{dp}{dx} \approx \frac{dp_\infty}{dx}$.

And in the u momentum equation, you can see that it will become $u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dp_\infty}{dx} + v\frac{\partial^2 u}{\partial y^2}$. So, this is the boundary layer equation.

What about the v momentum equations? So, we have seen that v is much smaller than U_∞ right, and it is having very small value. And from there we have derived the $\frac{\partial p}{\partial y}$ also is very small value. So, in the y momentum equation, if you see all the terms will be order of δ.

So, you can neglect the y momentum equation. So, in y momentum equation, each term is of order δ. So, all terms in this equation are neglected leading to the important boundary layer simplifications of negligible pressure gradient in the y direction.

So, now we can see that this equation we have derived in general, where p may be function of x, and as $\frac{\partial p}{\partial y} = 0$. So, obviously the pressure at the outside the boundary layer will be also function of x. So, you can see that p_∞ (x), so that can be impressed inside the boundary layer and we will have at a certain x location it will have the same pressure.

Now, this pressure gradient if you consider for a let us say you have a in general curved surface, and you can have the boundary layer like these where this is the boundary layer thickness delta, and x is measured along the surface, and y is measured perpendicular to the surface. In this particular case, actually you will get that outside pressure p_∞ (x). And for that reason your velocity free stream velocity ∞ will be function of x.

So, you can see that for this particular case from this equation if you apply outside the boundary layer, then what will happen? So, obviously, you can see that from here you can write U_∞ and $\frac{\partial u}{\partial x}$ so it will be u(x), so you can write $\frac{dU_\infty}{dx}$.

And v is very small negligible, so you can drop this term as $-\frac{1}{\rho}\frac{dp_\infty}{dx}$. And obviously, outside the boundary layer u is function of x only, so $\frac{\partial^2 u}{\partial y^2}$ will be 0. So, you will get this equation. So, $U_\infty \frac{dU_\infty}{dx} = -\frac{1}{\rho}\frac{dp_\infty}{dx}$.

So, if you write it here then you can get, so you can write it here $u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U_\infty \frac{dU_\infty}{dx} + \nu \frac{\partial^2 u}{\partial y^2}$, where U_∞(x). And if you integrate this equation what you will get? If you integrate this situation you can see you will get $p_\infty + \frac{1}{2}\rho U_\infty^2 =$ constant. And it is valid in the inviscid region outside the boundary layer.

Now, if you consider a special case that flow over flat plate. So, in case of flow over flat plate, this U_∞ will be constant; it is not function of x. So, for flow over flat plate, U_∞ is constant. So, here you can see that if U_∞ is constant, then $\frac{dU_\infty}{dx}$ = 0, because U_∞ is constant.

So, $\frac{dU_\infty}{dx}$ will be 0. So, if you have a flow over flat plate where U_∞ is constant, then you can write your boundary layer equation as $u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}$, so that means, your $\frac{dp_\infty}{dx}$=0.

So, you can write this equation as a special case for flow over flat plate. But if it is a curved surface like wedge or flow over a cylinder, then you have a curved surface, then in that case your U_∞ is function of x, and you have to consider the pressure gradient term $\frac{dp_\infty}{dx}$ in the momentum equation.

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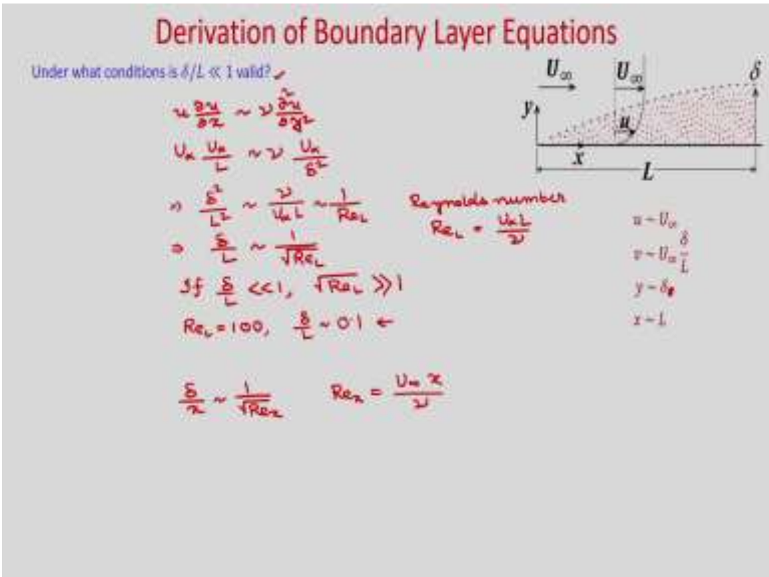

Now, the last question is that under what conditions is $\frac{\delta}{L}$ much small than 1 valid? So, let us answer this question. So, what will do now? Now, you can see in the left hand side the inertia terms are of the same order. So, we will take one inertia term, and we will compare it with the viscous term because we have only one viscous term right $\nu\frac{\partial^2 u}{\partial y^2}$. So, this we will compare.

So, you can see $u\frac{\partial u}{\partial x}$ one inertia term we will compare with the viscous term $\frac{\partial^2 u}{\partial y^2}$. So, what is the scale? So, $U_\infty \frac{U_\infty}{L} \sim \nu \frac{U_\infty}{\delta^2}$.

So, if you take delta square in the left hand side, if you divide both side by L, then you will get L^2, then you can see here you will get $\frac{\nu}{U_\infty L}$. And we can define Reynolds number based on free stream velocity ∞ and plate length L, then we can write $R_{e_L} = \frac{U_\infty L}{\nu}$. So, you can see from here it will be $\frac{1}{R_{e_L}}$, and $\frac{\delta}{L} \sim \frac{1}{\sqrt{R_{e_L}}}$.

Now, we will answer this. So, if $\frac{\delta}{L}$ delta by L is much smaller than 1, so what will be this term. So, if $\frac{\delta}{L}$ is much smaller than 1 then, $\frac{1}{\sqrt{R_{e_L}}}$ will be much greater than 1. So, you can see that it is a high Reynolds number flow. So, if you have high Reynolds number flow, but not in the range of turbulent, then you will get the hydro dynamic boundary layer thickness is very small compared to the length of the plate.

So, if you consider Reynolds number 100, then let us see that what is the $\frac{\delta}{L}$ order. So, if consider R_{e_L} as 100, then you can see from here $\frac{\delta}{L}$ will be order of, so it will be 100, so $\sqrt{100} = 10$. So, it will be 0.1.

So, you can see that it is very small. So, generally we say that these equations are valid when Reynolds number is greater than 100. Now, for any length x, this expression you can write as $\frac{\delta}{x} \sim \frac{1}{\sqrt{R_{e_x}}}$, where the Reynolds number $R_{e_x} = \frac{U_\infty x}{\nu}$.

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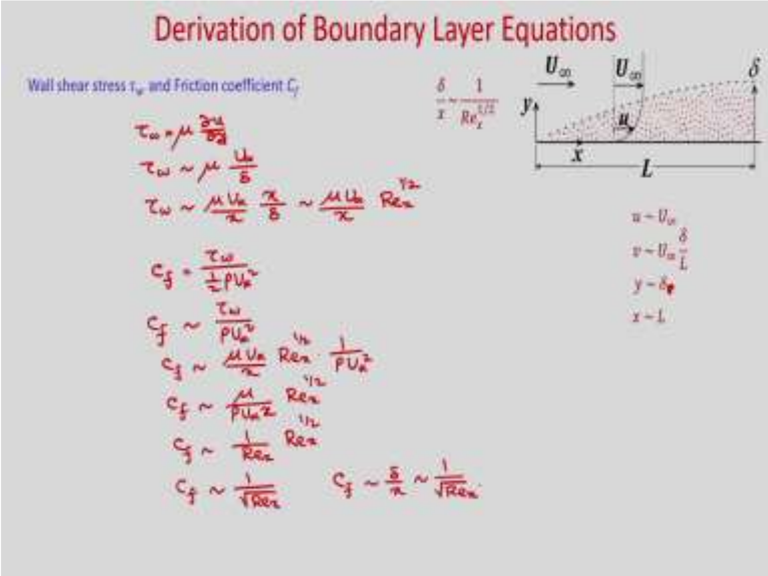

Now let us find what is the wall shear stress and the friction coefficient. So, what is the order of these terms? So, now, we know that for two-dimensional flow, wall shear stress $\tau_w = \mu \frac{\partial u}{\partial y}$.

So, for 2D flow we know $\tau_w = \mu \frac{\partial u}{\partial y}$. So, you can see that $\tau_w \sim \mu \frac{U_\infty}{\delta}$. And we know that $\frac{\delta}{x} \sim \frac{1}{\sqrt{R_{e_x}}}$. So, $\tau_w \sim \mu \frac{U_\infty}{x}\frac{x}{\delta} \sim \mu \frac{U_\infty}{x} R_{e_x}^{1/2}$.

And we know that friction coefficient $C_f = \frac{\tau_w}{\frac{1}{2}\rho U_\infty^2}$. So, you can see that friction coefficient C_f will be generally $\frac{\tau_w}{\frac{1}{2}\rho U_\infty^2}$. So, if you see $C_f \sim \frac{\tau_w}{\rho U_\infty^2}$, we will not consider half because we are doing the order of magnitude analysis.

So, you can see that $C_f \sim \frac{\mu U_\infty}{x} R_{e_x}^{1/2} \frac{1}{\rho U_\infty^2}$. So, if you see from here that if you $C_f \sim \frac{\mu}{\rho U_\infty x} R_{e_x}^{1/2}$. And C f will be $C_f \sim \frac{1}{R_{e_x}} R_{e_x}^{1/2}$.

So, if you see that $C_f \sim \frac{1}{\sqrt{R_{e_x}}}$. So, if $\frac{1}{\sqrt{R_{e_x}}}$ and $\frac{\delta}{x}$ is also $\frac{1}{\sqrt{R_{e_x}}}$. So, obviously, you can see $C_f \sim \frac{\delta}{x} \sim \frac{1}{\sqrt{R_{e_x}}}$.

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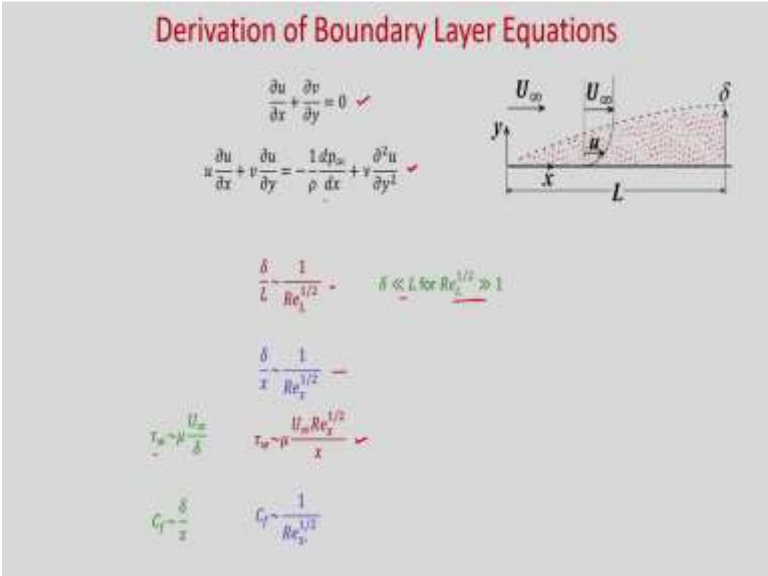

So, let us summarize what we have studied in today's class. So, first we have just written the governing equations in Cartesian, cylindrical and spherical coordinates. Then we considered external flow, and we have discussed about boundary layer.

So, we have seen that near to the wall, we have a small region where you have an effect of viscous region. So, that is known as viscous region then outside this and away from the surface there is a region that is known as inviscid region where there is no effect of viscosity. Then we used intuitive arguments and scale analysis, and we have derived the boundary layer equations.

So, you can see this is the continuity equation and this is the momentum equation. We have derived expressing the term p as p_∞ because from y momentum equation we have seen that $\frac{\partial p}{\partial y} \sim 0$.

So, this is the equation we have seen. And as a special case also we have discussed that if you have a flow over flat plate, then U_∞ is constant and there will be no axial pressure gradient. So, there will be $\frac{\partial p_\infty}{\partial x} = 0$. So, this term you can drop for flow over flat plate.

Then we have derived this relation $\frac{\delta}{L} \sim \frac{1}{R_{e_L}^{1/2}}$. And if $\frac{\delta}{L}$ is much smaller than 1, obviously, it has to be a high Reynolds number flow. And for any x we have written $\frac{\delta}{x}$ as this expression and from here we have written the shear stress $\tau_w \sim \mu \frac{U_\infty}{\delta}$, and we got this expression. And then we have seen that your friction coefficient $C_f \sim \frac{\delta}{x}$ and $C_f \sim \frac{1}{R_{e_x}^{1/2}}$.

Thank you.