Fundamentals of Convective Heat Transfer
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Module – 11
Turbulent Flow and Heat Transfer
Lecture – 39
Convection in turbulent pipe flow

Hello, everyone. So, today we will study Convection in turbulent pipe flow. In last classes we have already derived the universal velocity profile for flow over flat plate and also we have derived the heat transfer analogy relations or correlations. We will use those universal velocity profile for pipe flow with slight modifications. First let us discuss about the entry length for turbulent pipe flow.

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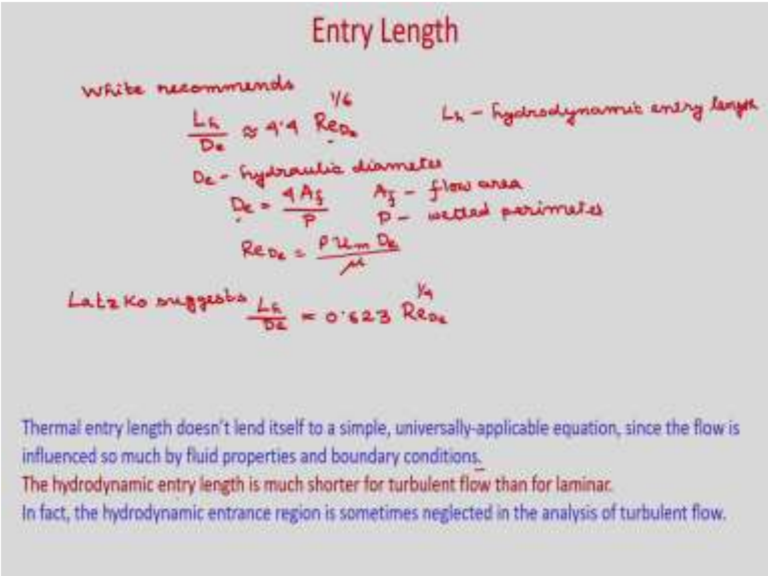

White recommends $\frac{Lh}{D_e} \approx 4.4\,\mathrm{Re}_{D_e}^{1/6}$. So, you know that D_e is your hydraulic diameter and it is obviously, you know how it is defined. It is defined as $D_e = \frac{4A_f}{P}$, where A_f is your flow area and P is your wetted perimeter wetted perimeter and this Reynolds number is defined based on this hydraulic diameter. So, Re_{D_e} is defined as $\mathrm{Re}_{D_e} = \frac{\rho u_m D_e}{\mu}$ and in this case we are considering internal flow.

Another scientist Latzko suggests $\frac{Lh}{D_e} = 0.623 \mathrm{Re}_{D_e}^{1/4}$. So, L h obviously, it is hydrodynamic entry length hydrodynamic entry length. In general, in turbulent flows it is very small compared to the laminar flow and open this hydrodynamic entrance length is neglected. However, it is very difficult to calculate the thermal entrance length for turbulent flows.

Thermal entry length does not lend itself to a simple, universally-applicable equation, since the flow is influenced so much by fluid properties and boundary conditions. The hydrodynamic entry length is much shorter for turbulent flow than for laminar. In fact, the hydrodynamic entrance region is sometime neglected in the analysis of turbulent flow.

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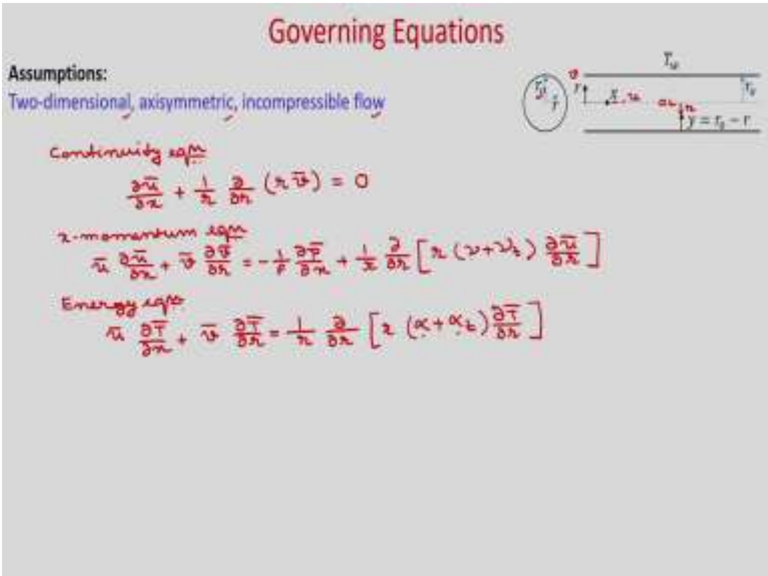

So, let us write the governing equation for this internal flow with these assumptions: two-dimensional, axisymmetric and incompressible flow. And, as we are considering boundary layer flows so, you can see this is the pipe of radius r_0 and x is measured in axial direction; r is measured from the centerline.

So, this is your C L centerline and y if we tell it is measured from the boundary then it is r_0 - r. So, obviously, $y = r_0 - r$. So, in this case you can see we will define the velocity u in axial direction and v velocity in r direction ; for convenience we are just defining these velocities u and v.

So, now, you can write the governing equations after Reynolds averaging. So, you will get continuity equation. So, we can write as $\frac{\partial \bar{u}}{\partial x}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\bar{v}\right)=0$. And, Reynolds averaged x momentum equation you can write as $\bar{u}\frac{\partial \bar{u}}{\partial x}+\bar{v}\frac{\partial \bar{v}}{\partial r}=-\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x}+\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\nu+\nu_t\right)\frac{\partial \bar{u}}{\partial r}\right]$. So, this is boundary layer flow so obviously, $\frac{\partial^2 u}{\partial x^2}$ we can neglect. And, Reynolds average energy equation will be $\bar{u}\frac{\partial \bar{T}}{\partial x}+\bar{v}\frac{\partial \bar{T}}{\partial r}=-\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x}+\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\alpha+\alpha_t\right)\frac{\partial \bar{T}}{\partial r}\right]$. So, these equations we have already derived for the flow over flat plate.

These we have written for the circular pipe case and you can see ν is your kinematic viscosity and ν_t is your eddy viscosity, and α is your thermal diffusivity and α_t is your eddy diffusivity and these are coming due to the turbulent fluctuations.

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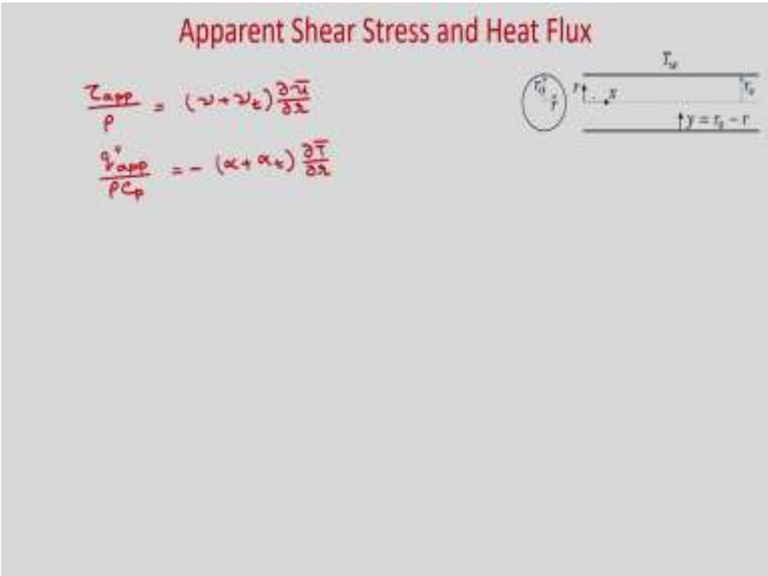

Now, whatever we have derived the apparent stress and apparent heat flux flow over flat plate those will be applicable for pipe flow. So, you can write $\frac{\tau_{app}}{\rho}=\left(\nu+\nu_t\right)\frac{\partial \bar{u}}{\partial r}$ and

$\frac{q''_{app}}{\rho C_p} = -\left(\alpha + \alpha_t\right)\frac{\partial \bar{T}}{\partial r}$. So, you can see for flow over flat plate we have derived it has $\frac{\partial \bar{u}}{\partial y}$. So, in this case we are writing $\frac{\partial \bar{u}}{\partial r}$.

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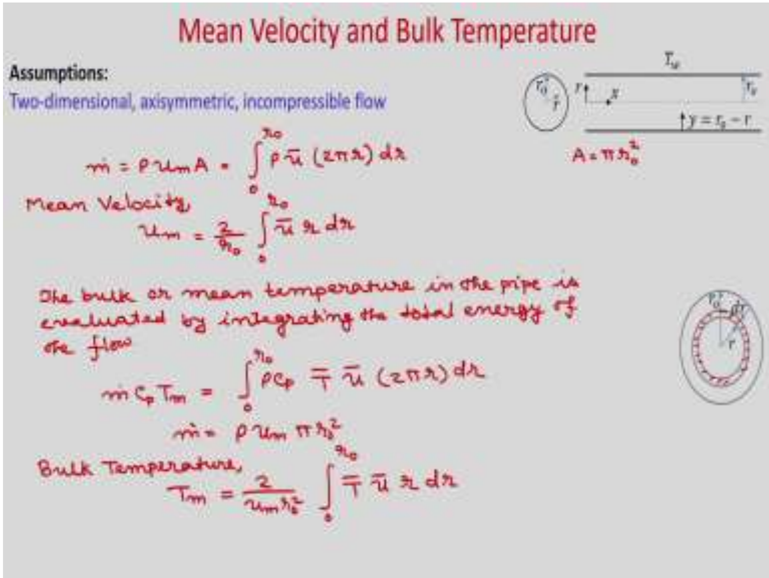

So, in pipe flow generally we deal with the mean velocity and bulk temperature. When we did find the Nusselt number we write it based on the mean bulk mean temperature, as well as when we define the Reynolds number we define based on the mean velocity. So, let us write the expression for mean velocity as well as the bulk temperature.

So, we can write mass flow rate $\dot{m} = \rho u_m A$. So, in this case you can see; obviously, $A = \pi r_0^2$. So, it is your flow area. So, this will be $\int_0^{r_0} \rho \bar{u}\left(2\pi r\right) dr$.

So, if you put here $A = \pi r_0^2$. So, you can write the mean velocity, $u_m = \frac{2}{r_0}\int_0^{r_0} \bar{u} r dr$. So, the bulk or mean temperature in the pipe is evaluated by integrating the total energy of the flow. So, you can write $\dot{m} C_p T_m = \int_0^{r_0} \rho C_p \bar{T}\bar{u}\left(2\pi r\right) dr$.

So, from here you know that $\dot{m}$. So, you can write, $\dot{m} = \rho u_m \pi r_0^2$. So, if you put it here and ρC_p are constant. So, you can cancel. So, you will get bulk temperature $T_m = \frac{2}{u_m r_0^2}\int_0^{r_0} \bar{T}\bar{u} r dr$. So, it is same expression as laminar only difference is that this velocity and temperature are evaluated as mean value.

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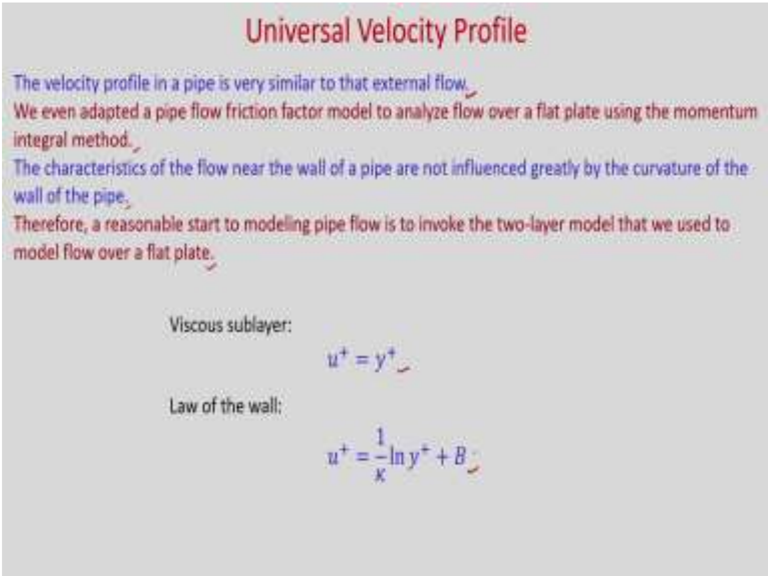

For flow over flat plate case already we have derived the universal velocity profile, we considered very small region near to the wall and we assumed that their shear stress remain constant. So, that is your viscous sub-layer and away from the wall you have law of the wall. So, the velocity profile in a pipe is very similar to the external flow.

We even adapted a pipe flow friction factor model to analyze flow over a flat plate using the momentum integral method. The characteristic of the flow near the wall of a pipe are not influenced greatly by the curvature of the wall of the pipe. Therefore, a reasonable start to modeling pipe flow is to invoke the two-layer model that we used to model flow over a flat plate.

So, you can see for viscous sub layer we have derived $u^+ = y^+$ and law of the wall $u^+ = \frac{1}{\kappa}\ln y^+ + B$. Here now, the definition of y^+ will be somewhat different in case of pipe flow.

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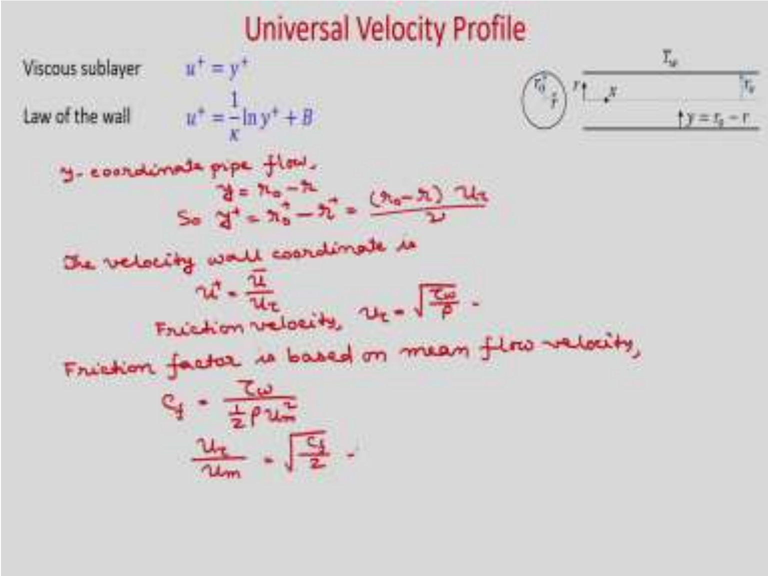

So, you can see in case of flat plate this y is measured from the wall. So, in this particular case now if you measure the distance from the wall, this is your y then you have to replace this $y = r_0 - r$. So, if you see y coordinate pipe flow so, $y = r_0 - r$.

So, $y^+ = r_0^+ - r^+ = \frac{(r_0 - r)u_\tau}{\nu}$. u_τ is your friction velocity and the velocity wall coordinate is, $u^+ = \frac{\bar{u}}{u_\tau}$. This expression is same. So, your fiction velocity is $u_\tau = \sqrt{\frac{\tau_w}{\rho}}$.

And, friction factor based on the mean flow velocity you can write friction factor is based on mean flow velocity. So, $C_f = \frac{\tau_w}{\frac{1}{2}\rho u_m^2}$ and now you can write $\frac{u_\tau}{u_m} = \sqrt{\frac{C_f}{2}}$. So, if you put these expression in this friction velocity then you will get, $\frac{u_\tau}{u_m} = \sqrt{\frac{C_f}{2}}$. Now, let us see that in pipe flow how the shear stress varies inside the domain.

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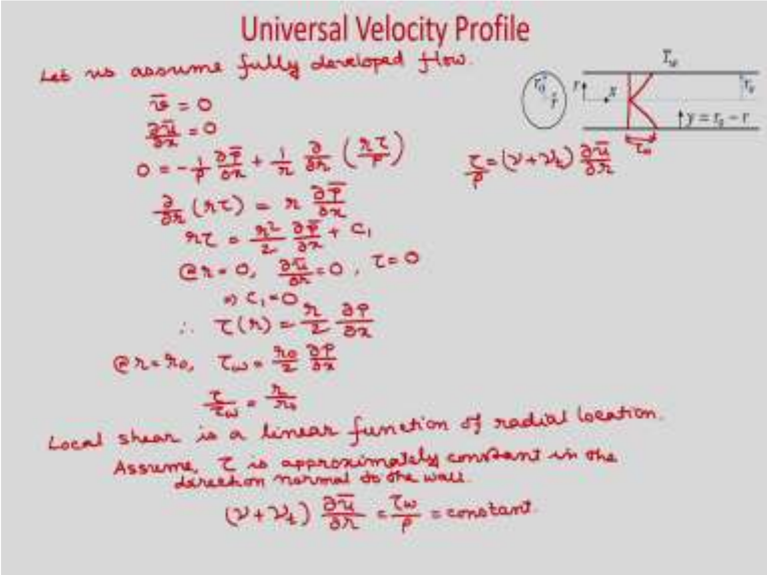

So, for that let us assume fully developed flow. Let us assume fully developed flow. So, if it is a fully developed flow obviously, the velocity $\bar{v}=0$ and from continuity equation you can write, $\frac{\partial \bar{u}}{\partial x}=0$. So, if you put these in the boundary layer equation whatever we have written so, you can write, $0=-\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x}+\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r\tau}{\rho}\right)$.

So, $\frac{\tau}{\rho}=\left(\nu+\nu_t\right)\frac{\partial \bar{u}}{\partial r}$. So, τ is your shear stress. So, if you rearrange it you will get and ρ is constant. So, you can write $\frac{\partial}{\partial r}\left(r\tau\right)=r\frac{\partial \bar{p}}{\partial x}$. So, if you integrate it you will get, $r\tau=\frac{r^2}{2}\frac{\partial \bar{p}}{\partial x}+C_1$.

Now, you know at r = 0, $\frac{\partial \bar{u}}{\partial r}$ = 0, right? Because this is your at the center it is changing its gradient. So, obviously, $\frac{\partial \bar{u}}{\partial r}$ = 0 and hence shear stress will be 0. So, that means, C_1=0. So, if you see that $\tau\left(r\right)=\frac{r}{2}\frac{\partial p}{\partial x}$.

So, you can see that shear stress varies linearly inside the flow domain maximum will be at the wall and this will be your τ_w at r = r_0, τ will be τ_w and 0 will be at r = 0. So, at r = r_0. So, $\tau_w = \frac{r_0}{2}\frac{\partial p}{\partial x}$. So, the ratio $\frac{\tau}{\tau_w} = \frac{r}{r_0}$. So, local shear is a linear function of radial location.

So, here you can see that shear stress linearly varies with radius. So, it contradicts with the assumptions whatever we have taken for the flow over flat plate case. So, here also we will assume that wherein close to the wall shear stress remain constant and that is equal to τ_w. So, the assume that τ is approximately constant in the direction normal to the wall.

So, universal velocity profile that resulted from this assumption works well for flat plate flow as well as pipe flow. So, in this case we can write $(\nu + \nu_t)\frac{\partial \bar{u}}{\partial r} = \frac{\tau_w}{\rho}$ obviously, it is constant. So, this is the assumptions we are taking.

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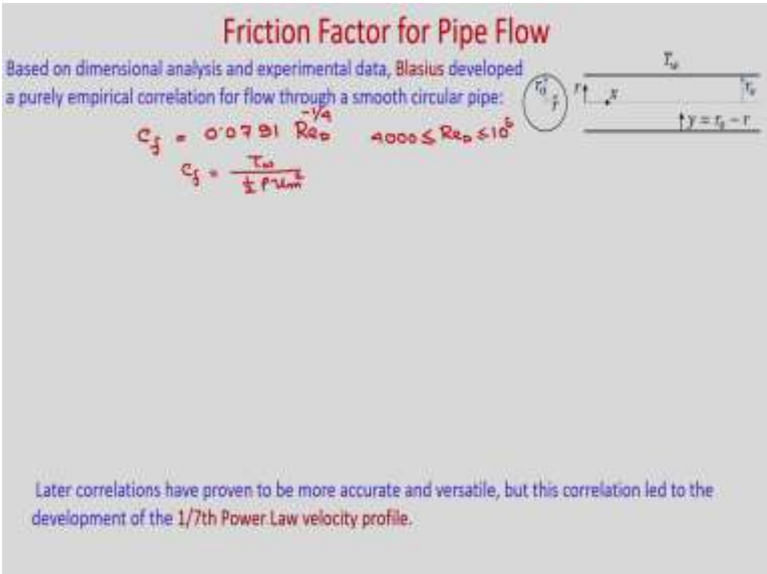

So, now, let us discuss about the friction factor for the pipe flow. So, we have already seen the Blasius correlation for friction factor right. So, based on dimensional analysis and experimental data, Blasius developed a purely empirical coordination for flow through a smooth circular pipe.

And you know that it is $C_f = 0.0791\text{Re}_D^{-1/4}$ and it is valid in the range $4000 \le \text{Re}_D \le 10^5$ and C_f is defined based on the mean velocity. So, it will be $C_f = \dfrac{\tau_w}{\frac{1}{2}\rho u_m^2}$.

So, now if you use the 1/7th velocity profile then you can write the expression for the shear stress. So, later correlations have proven to be more accurate and versatile, but this correlation lead to a development of the 1/7th power law velocity profile.

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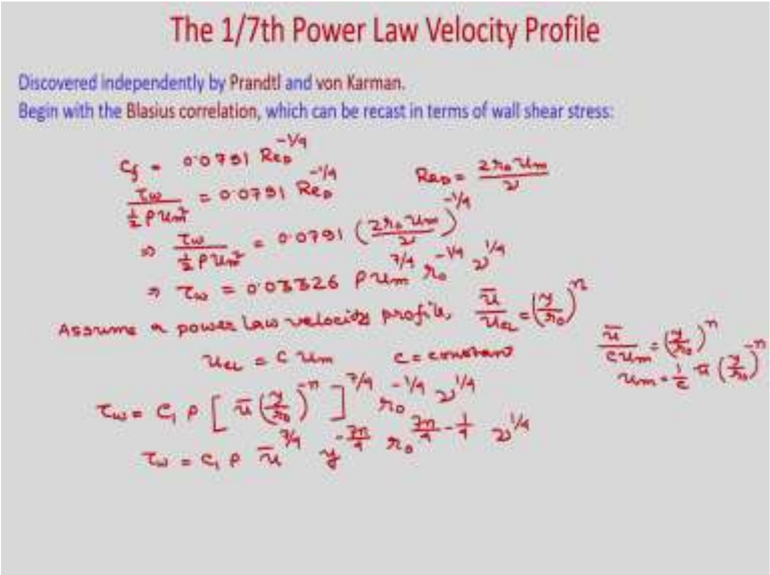

So, let us assume that velocity profile mean velocity profile $\dfrac{\bar{u}}{u_{CL}} = \left(\dfrac{y}{r_0}\right)^n$ and let us find what is the value of this exponent n. So, we have already seen that $C_f = 0.0791\text{Re}_D^{-1/4}$ and if you put the expression of C_f, then you will write $\dfrac{\tau_w}{\frac{1}{2}\rho u_m^2} = 0.0791\text{Re}_D^{-1/4}$. So, this,

$$\text{Re}_D = \frac{2r_0 u_m}{\nu}.$$

So, you can see that you can write as, $\dfrac{\tau_w}{\frac{1}{2}\rho u_m^2} = 0.0791\left(\dfrac{2r_0 u_m}{\nu}\right)^{-1/4}$. So, you can rearrange and you can write $\tau_w = 0.03326\rho u_m^{7/4} r_0^{-1/4}\nu^{1/4}$. Now, assume a power law velocity profile ok. So, we will assume that $\dfrac{\bar{u}}{u_{CL}} = \left(\dfrac{y}{r_0}\right)^n$. So, let us find the value of this exponent n.

You put this velocity profile in the expression of shear stress and find the value of n. So, your centerline velocity will be $u_{CL} = C\ u_m$. So, now, you can see your C is your constant. So, what is u m? So, $u_m = \dfrac{u_{CL}}{C}$. So, you can see it will be. So, here if you put then, $\dfrac{\bar{u}}{Cu_m} = \left(\dfrac{y}{r_0}\right)^n$ and $u_m = \dfrac{1}{C}\bar{u}\left(\dfrac{y}{r_0}\right)^{-n}$. So, some constant. So, these constant will be involved here.

So, $\tau_w = C_1\rho\left[\bar{u}\left(\dfrac{y}{r_0}\right)^{-n}\right]^{7/4} r_0^{-1/4}\nu^{1/4}$. So, now, if you simplify, $\tau_w = C_1\rho\bar{u}^{7/4} y^{-\frac{7n}{4}} r_0^{\frac{7n}{4}-\frac{1}{4}}\nu^{1/4}$.

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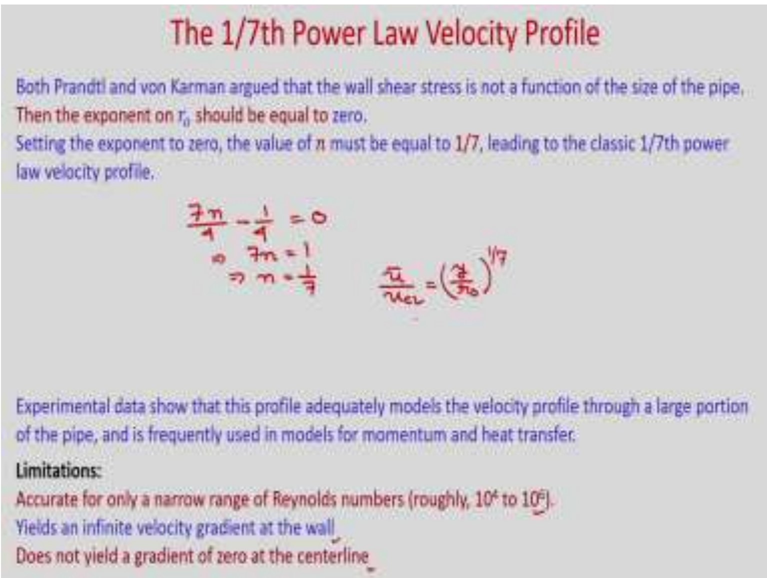

Now, we need to find the value of exponent n both Prandtl and von Karman argued that the wall shear stress is not a function of the size of the pipe then the exponent on $r_0 = 0$.

So, you can see in this relation whatever we have written it should not depend on the shear stress should not depend on the size of the pipe. So, here you can see only r_0 is there which is your radius of the pipe. So, we will put its exponent as 0. So, $\frac{7n}{4}-\frac{1}{4}=0$.

So, setting the exponent to 0 the value of $n=\frac{1}{7}$ leading to the classic 1/ 7th power law velocity profile. $\frac{7n}{4}-\frac{1}{4}=0$. So, you can see 7n = 1. So, $n=\frac{1}{7}$. So, you can see the velocity profile $\frac{\bar{u}}{u_{CL}}=\left(\frac{y}{r_0}\right)^{1/7}$ and this is known as 1/7th power law velocity profile.

Experimental data show that this profile adequately models the velocity profile through a large portion of the pipe and is frequently used in models for momentum and heat transfer. But, it has some limitations. You can see that if you use this velocity profile the velocity gradient at r = 0 will not be 0. So, you cannot find the shear stress directly from these velocity profile.

So, the limitations are accurate for only a narrow range of Reynolds number roughly 10^4 to 10^6 yields an infinite velocity gradient at the wall and does not yield a gradient of 0 at the centerline . So, these are the limitations.

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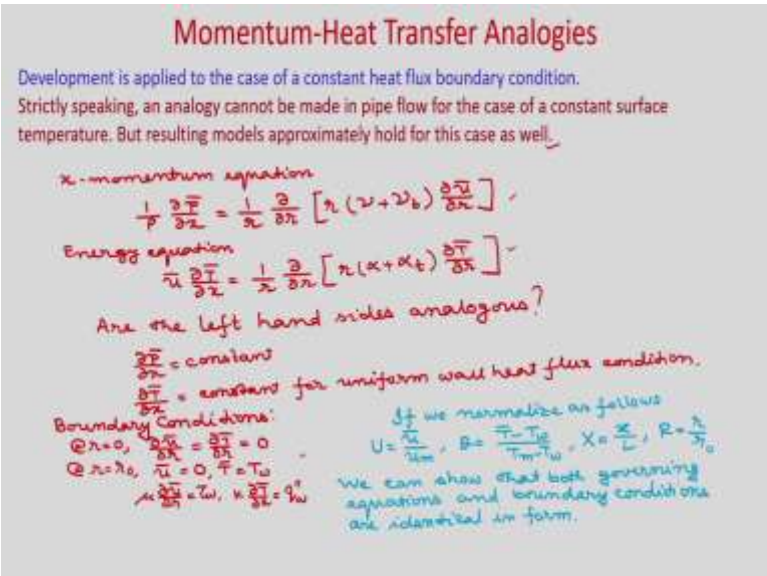

Now, let us discuss about the momentum and heat transfer analogies. So, we have already written the expression for apparent shear stress and apparent heat flux and, let us see that both are analogous to each other or not. Development is applied to the case of constant heat flux boundary conditions.

So, whatever we will be discussing, so, it is directly applicable for the thermal condition with uniform heat flux boundary condition. Strictly speaking, an analogy cannot be made in pipe flow for the case of constant surface temperature. But resulting models approximately hold for this case as well.

So, you can see that your x momentum equation whatever we have written we can write the inertia terms as 0. So, $\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\nu+\nu_t\right)\frac{\partial \bar{u}}{\partial r}\right]$. And, if you write the energy equation, so, it is, $\bar{u}\frac{\partial \bar{T}}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\alpha+\alpha_t\right)\frac{\partial \bar{T}}{\partial r}\right]$.

So, you can see as we have assumed that it is fully developed flow so, obviously, $\bar{v}=0$. So, for that reason the second term in the energy equation is 0. So, now, the question is that are these left hand sides analogous the question is that are the left hand sides analogous? So, now, let us see if you consider pipe flow so, obviously, you see that pressure varies linearly in the axial direction. So, that means, your $\frac{\partial \bar{p}}{\partial x}$ = 0.

So, in the momentum equation left hand side is constant because $\frac{\partial \bar{p}}{\partial x}$ is constant. So, in x momentum equation left hand side is constant. Now, if you come to the energy equation we have derived while discussing about laminar internal flows that for a constant wall heat flux boundary condition $\frac{\partial \bar{T}}{\partial x}$ is constant.

So, you can see $\frac{\partial \bar{p}}{\partial x}$ is constant and $\frac{\partial \bar{T}}{\partial x}$ is constant for uniform wall heat flux condition and now, let us check about the boundary conditions. So, boundary conditions if you check. So, boundary condition at r = 0, $\frac{\partial \bar{u}}{\partial r}$ = $\frac{\partial \bar{T}}{\partial r}$ = 0 at r = r_0 , $\bar{u}$ = 0 .

And, $\overline{T}$ = T_w as well as you have shear stress $\mu \frac{\partial \overline{u}}{\partial r} = \tau_w$ and we have, $k \frac{\partial \overline{T}}{\partial x} = q_w^{"}$. Then we can show that the both the governing equation and boundary conditions are identical in form.

So, if we normalize as follows, $U = \frac{\overline{u}}{u_m}; \theta = \frac{\overline{T} - T_w}{T_m - T_w}$; $X = \frac{x}{L}$ and $R = \frac{r}{r_0}$ we can show that both governing equations and boundary conditions are identical in form.

So, we can use the analogy whatever we are writing for momentum equation that also you can use for energy equation. Using these normalized variables, we can show that both governing equations and boundary conditions are identical in form. So, momentum heat transfer analogy is possible and we can apply analogy method for pipe flow.

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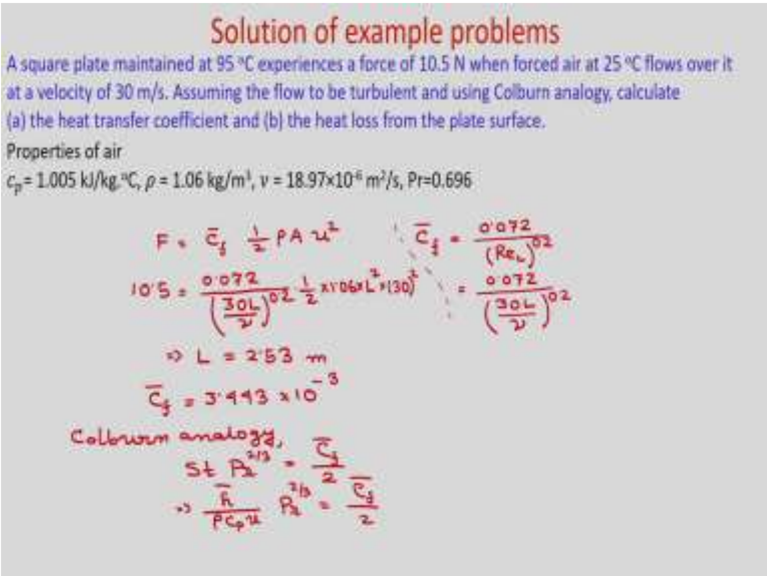

Now, let us solve two problems. A square plate maintained at 95 ^{0}C experiences a force of 10.5 N when forced air at 25 ^{0}C flows over it at a velocity of 30 m/s. Assuming the flow to be turbulent and using Colburn analogy, calculate (a) the heat transfer coefficient and (b) the heat loss from the plate surface.

Properties of air are given – you can see c_p, ρ, ν and Pr. So, what we can do you can see the force is given. So, from here you will be able to calculate what is the friction coefficient. So, you can see that the force is given 10.5 N. So, you can write $F = \overline{C}_f \frac{1}{2} \rho A u^2$ and this C_f you know from the analogy that, $\overline{C}_f = \frac{0.072}{(\mathrm{Re}_L)^{0.2}}$.

So, this is average friction coefficient $\overline{C}_f = \frac{0.072}{\left(\frac{30L}{\nu}\right)^{0.2}}$. So, if you substitute it here from here you will be able to calculate the L. So, you can see $10.5 = \frac{0.072}{\left(\frac{30L}{\nu}\right)^{0.2}} \times \frac{1}{2} \times 1.06 \times L^2 \times (30)^2$. So, if you evaluate it you will get length as 2.53 m. So, once you know L then you will be able to calculate Reynolds number and $\overline{C}_f$. So, from here you can see your $\overline{C}_f$ you can calculate from here $\overline{C}_f$ average friction coefficient if you put the value of L=2.53m, you will get, $\overline{C}_f = 3.443 \times 10^{-3}$.

So, now you use the Colburn analogy. So, Colburn analogy if you use then you will be able to calculate the average heat transfer coefficient. So, this is your $St\,\mathrm{Pr}^{2/3} = \frac{\overline{C}_f}{2}$.

So, this Stanton number you can write as $\frac{\overline{h}}{\rho c_p u} \mathrm{Pr}^{2/3} = \frac{\overline{C}_f}{2}$. Now, you put the values ρ c_p u Pr and $\overline{C}_f$ are known so, you will be able to calculate the heat transfer coefficient.

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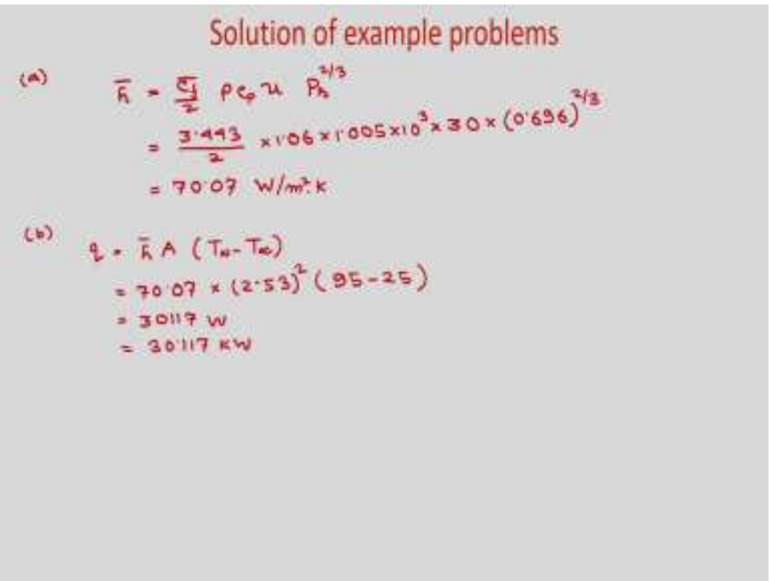

So, $\bar{h}=\frac{\overline{C}_f}{2}\rho c_p u \Pr^{2/3}$. So, you can calculate $\bar{h}=\frac{3.443}{2}\times 1.06\times 1.005\times 10^3\times 30\times (0.696)^{2/3}$.

So, if you calculate you will get $\bar{h}=70.07$ W/m^{2}K. So, this first part we have already calculated. So, this is you're (a) heat transfer coefficient. Now, you have to calculate the heat loss from the plate surface. So, heat loss $q=\bar{h}A(T_w-T_\infty)$.

So, what is your temperature difference? $q=70.07\times(2.53)^2(95-25)$. You will get as 30117 W or 30.117 kW.

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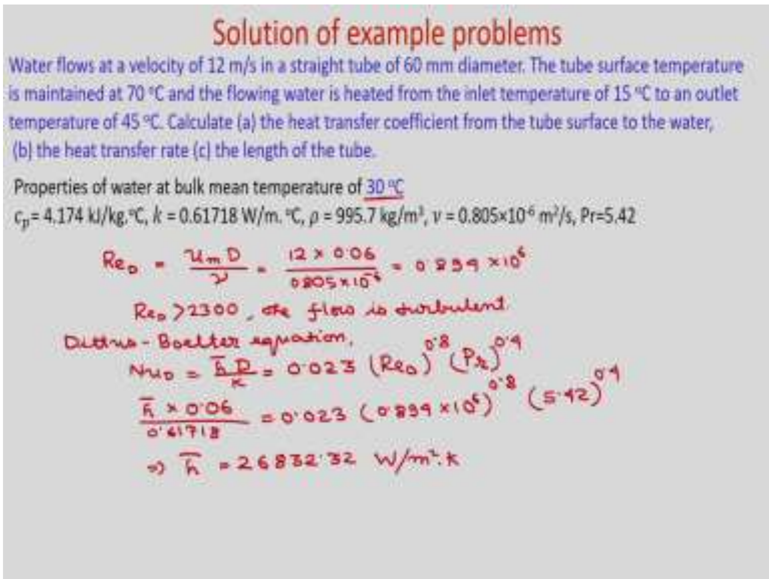

Now, let us discuss about the next problem. Water flows at a velocity of 12 m/ in a straight tube of 60 mm diameter. The tube surface temperature is maintained at 70 ^{0}C and the flowing water is heated from the inlet temperature of 15 ^{0}C to an outlet temperature of 45 ^{0}C.

Calculate (a) the heat transfer coefficient from the tube surface to the water. Calculate the heat transfer coefficient from the tube surface to the water, the heat transfer rate and the length of the tube. Properties of water at bulk mean temperature of 30 ^{0}C are given. So, you can see bulk mean temperature is 30 ^{0}C. So, c_p, k, ρ, ν, Pr are given.

So, from here now first you calculate the Reynolds number. So, $\mathrm{Re}_D = \frac{u_m D}{\nu}$. So, based on mean velocity so, it will be 12 m/s, D= 60 mm. So, $\frac{12 \times 0.06}{0.805 \times 10^{-6}}$; so, it will be around 0.894×10^6. So, you can see your $\mathrm{Re}_D > 2300$.

So, obviously, the flow is turbulent. So, we discuss about Dittus-Boelter equation so, that we can use and find the heat transfer coefficient. So, Dittus-Boelter equation so, here you can see it is a heating case because T_w is higher. So, you can use $Nu_D = \frac{\bar{h} D}{K} = 0.023 \left(\mathrm{Re}_D\right)^{0.8} \left(\mathrm{Pr}\right)^{0.4}$.

So, $\frac{\bar{h}\times 0.06}{0.61718}=0.023\left(0.894\times 10^{6}\right)^{0.8}\left(5.42\right)^{0.4}$. So, if you calculate $\bar{h}=26832.32W/m^{2}K$.

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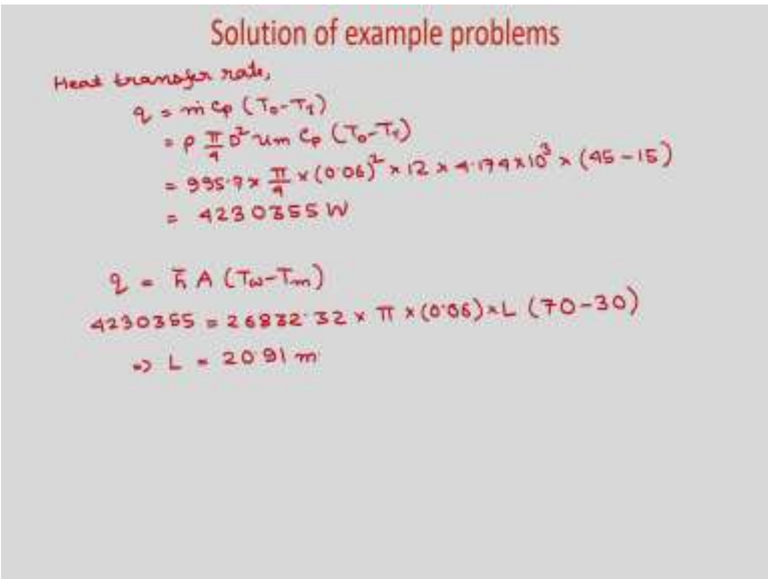

So, next you need to calculate the heat transfer rate. So, heat transfer rate you can calculate $q=\dot{m}c_{p}\left(T_{o}-T_{i}\right)$. So, $q=\rho\frac{\pi}{4}D^{2}u_{m}c_{p}\left(T_{o}-T_{i}\right)$. So, you put all these values density as, $995.7\times\frac{\pi}{4}\times\left(0.06\right)^{2}\times 12\times 4.174\times 10^{3}\times\left(45-15\right)$. So, if you calculate then you will get as 4230355 W.

Now, you need to calculate the length of the tube. So, we will use now the Newton's law of cooling. So, $q=\bar{h}A\left(T_{w}-T_{m}\right)$ because T_m is your bulk mean temperature it is given. So, $4230355=26832.32\times\pi\times\left(0.06\right)\times L\times\left(70-30\right)$. So, if you calculate from here you will get length as 20.91 m.

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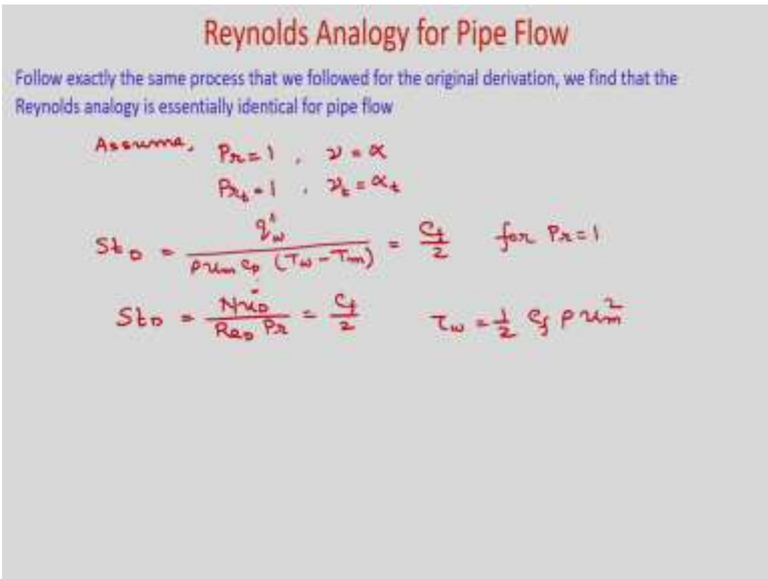

Now, first let us discuss about the Reynolds analogy because we have already derived for laminar flows and for a special case when Pr =1 and turbulent Pr_t =1; that means, your kinetic viscosity is equal to turbulent viscosity and also your thermal diffusivity is equal to your eddy diffusivity.

So, in that case you can use the Reynolds analogy. So, follow exactly the same process that we followed for the original derivation we find that the Reynolds analogy is essentially identical for pipe flow and you assume Pr =1; that means, your $\nu = \alpha$ and Pr_t =1; that means, your $\nu_t = \alpha_t$.

So, the, $St_D = \dfrac{q_w^{"}}{\rho u_m c_p \left(T_w - T_m\right)} = \dfrac{C_f}{2}$ for Pr =1. So, $St_D = \dfrac{Nu_D}{\mathrm{Re}_D \,\mathrm{Pr}}$. So, $\dfrac{C_f}{2}$ And, you know that tau w we have already found. So, $\tau_m = \dfrac{1}{2} C_f \rho u_m^2$. So, from here you will be able to find what is the Nusselt number in case of pipe flow. For Pr =1, you can use Colburn analogy that also we have discussed in detail when we considered laminar internal flow.

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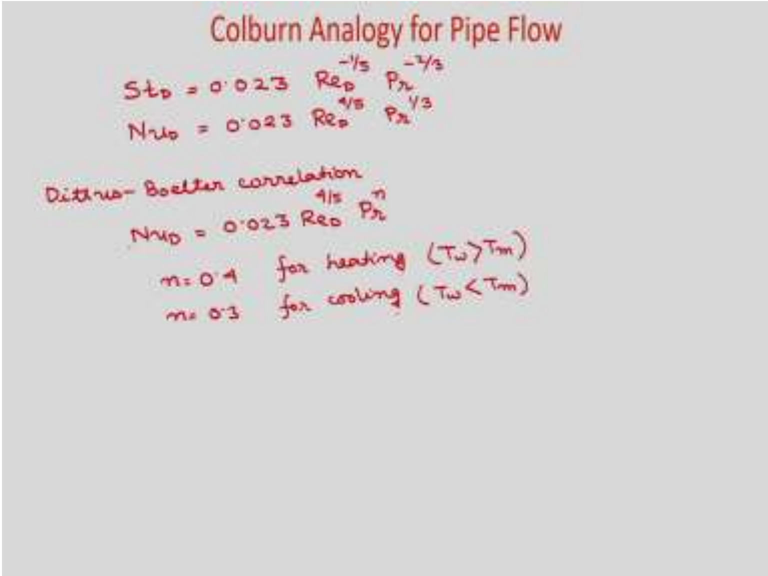

So, in this case you can write the, $St_D = 0.023\,\mathrm{Re}_D^{-1/5}\,\mathrm{Pr}^{-2/3}$ and $Nu_D = 0.023\,\mathrm{Re}_D^{4/5}\,\mathrm{Pr}^{1/3}$.

So, this is your Colburn analogy and you can use this relations when $\mathrm{Pr} \neq 1$. Another analogy you can write it is a popular correlation $Nu_D = 0.023\,\mathrm{Re}_D^{4/5}\,\mathrm{Pr}^{n}$ where n = 0.4 for heating.

So, when $T_w > T_m$ and n = 0.3 for cooling. So, this you can write as $T_w < T_m$. So, means depending on the whether wall temperature is greater than T_m that means, it is a heating case and if it is a cooling case $T_w < T_m$. So, you can use different value of n and it gives a reasonably good results using this correlation.

So, today we discussed about the convection in a turbulent pipe flow. We started with the universal velocity profile for the flow over flat plate case, and those are also applicable for the pipe flow. Then, we use the Blasius correlation for the friction factor and from there we have derive the exponent for the power law velocity profile. So, n = 1/ 7.

Then, we also we have seen the shear stress varies linearly inside the flow domain, but when we use the universal velocity profile we near to the wall we need to assume tau w as constant. After that we have discuss about the momentum and heat transfer analogy. So, we have seen that the equations governing equations and the boundary conditions in non-dimensional form both are identical.

So, we have used the Reynolds analogy for Pr =1 and we have found the Nusselt number expression; as well as for Pr $\neq$ 1, we use Colburn analogy and also we have written the expression for Nusselt number.

Thank you.