Fundamentals of Convective Heat Transfer
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Module – 11
Turbulent Flow and Heat Transfer
Lecture – 36
Convection in Turbulent External Flow

Hello everyone, in last class we derived the Reynolds average Navier-Stokes equations as well as the time averaged energy equation. Today, we will consider external flows and we will derive the turbulent boundary layer equations. Then we will discuss about different turbulent layers inside the boundary layer, and we will find the universal velocity profile and universal temperature profile.

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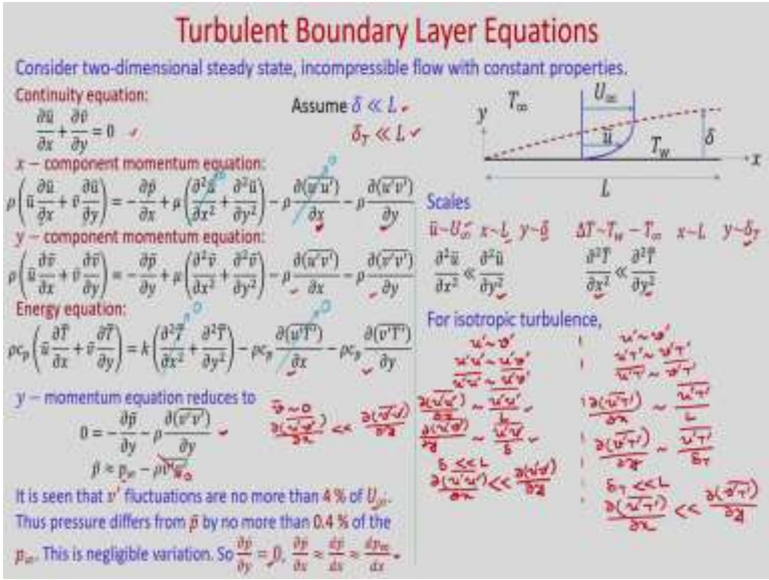

So, you can see that if we consider two-dimensional steady state incompressible flow with constant properties. Then you have this continuity equation. This is the x component momentum equation. You can see we have additional terms. And in y momentum equation, here also we have two additional terms. And in energy equation, we have also two additional terms these are coming due to the fluctuations in velocities and temperature.

So, now in boundary layer approximation, if we assume that $\delta << L$ and δ_T which is your thermal boundary layer is much much smaller than the length of the plate, then using scale analysis we have already carried out for external flows we can do the similar analysis and we can derive the boundary layer equations. Here the other terms will have the similar derivation as we did earlier, but the fluctuating terms we will see here specially.

So, you can see if you use the scales of velocity $\bar{u} \sim U_\infty$, $x \sim L$, and $y \sim \delta_T$, then obviously, this you can show right that $\frac{\partial^2 \bar{u}}{\partial x^2} \ll \frac{\partial^2 \bar{u}}{\partial y^2}$. So, you can see that these already we have derived earlier, and these inner shear terms will be comparable, so we cannot neglect this.

But if you do the similar analysis for the energy equation and the $\Delta T \sim \left(T_w - T_\infty\right)$ and y as you scale of thermal boundary layer thickness δ_T, then obviously, you can show that $\frac{\partial^2 \bar{T}}{\partial x^2} \ll \frac{\partial^2 \bar{u}}{\partial y^2} T$.

Now, what about these terms. can we neglect any term from these two terms? So, let us see that. So, first let us assume that it is a isotropic turbulence. So, for these you know that there is no preferred direction of the fluctuation. So, we can write $u' \sim v'$.

So, now if we multiply u' both side, so you can write $u'u' \sim u'v'$. And if you take the average of this quantity, then you can write $\overline{u'u'} \sim \overline{u'v'}$. So, this will be same order, but now let us see what will be this gradient, the order of these gradients then you can see that this first fluctuating term if you write.

So, $\frac{\partial\left(\overline{u'u'}\right)}{\partial x} \sim \frac{u'u'}{L}$. And this term if you see, then it will be $\frac{\partial\left(\overline{u'v'}\right)}{\partial y} \sim \frac{u'u'}{\delta}$. So, now you can see we have already assumed that $\delta << L$. So, obviously, if you compare these two terms, so you can say that you have $\frac{\partial\left(\overline{u'u'}\right)}{\partial x} \ll \frac{\partial\left(\overline{u'v'}\right)}{\partial y}$.

So, you can see that in the x momentum equation, the last term you can consider because it is having a higher order than the this term. So, you keep this term and this term you can neglect. Now, you can also show in the y momentum equation that $\bar{v} \sim 0$.

And from there you can see for this particular case, your y momentum equation keeping the $\bar{v} \sim 0$, you can also show that $\frac{\partial\left(\overline{u'v'}\right)}{\partial x} \ll \frac{\partial\left(\overline{v'v'}\right)}{\partial y}$. So, from there, you can reduce this y momentum equation as this ok.

In laminar flow, it was $\frac{\partial p}{\partial y} = 0$. But as it is turbulent flows, you will have this term. So, you can see $\bar{p}$ will be if you integrate this equation, so at outside the boundary layer you have the free stream pressure p_∞. So, $\bar{p} \approx p_\infty - \rho\left(\overline{v'v'}\right)$. But it is seen that v' fluctuations are no more than 4 % of the free stream velocity U_∞.

Thus pressure differs from $\bar{p}$ by no more than 0.4 % of the p_∞. So, you can actually neglect this term, and you can write from here that $\frac{\partial \bar{p}}{\partial y} = 0$. So, you can see as laminar flow we have derived that pressure does not vary perpendicular to the wall.

So, $\frac{\partial \bar{p}}{\partial y} = 0$. And say hence you can write that $\frac{\partial \bar{p}}{\partial x} \approx \frac{d\bar{p}}{dx}$. And you can equate it also with the free stream pressure p_∞. So, we have written $\frac{d\bar{p}}{dx} \approx \frac{dp_\infty}{dx}$.

Similarly, now if you do the scale analysis of these two terms. So, you can see from for isotropic turbulence $u' \sim v'$. Then you can write $u'T' \sim v'T'$. So, if you take its time average, so you can write $\overline{u'T'} \sim \overline{v'T'}$.

So, now, let us see the derivative , what is the scale. So, $\frac{\partial\left(\overline{u'T'}\right)}{\partial x} \sim \frac{\overline{u'T'}}{L}$. And here you can write, $\frac{\partial\left(\overline{u'T'}\right)}{\partial y} \sim \frac{\overline{u'T'}}{\delta_T}$.

So, now, we have already assumed that δ_T << L. Hence, you can write $\frac{\partial\left(\overline{u'T'}\right)}{\partial x} \ll \frac{\partial\left(\overline{v'T'}\right)}{\partial y}$.

So, now you can see whatever equations we have written for two-dimensional steady turbulent flow equations, we can neglect few terms for the boundary layer equations. So, you can see, here we can drop this term. We can drop this term from scale analysis we have shown. And these you can write as $\frac{d\overline{p}}{dx}$ and anyway all these term will become 0.

So, $\frac{\partial \overline{p}}{\partial y} = 0$. And in energy equation, similarly you can neglect this term and also this term ok, because its magnitude is very small compared to the other terms. So, if you can neglect, then you can write the continuity equation as this. This is the x momentum equation and this is the energy equation after dropping low magnitude order terms.

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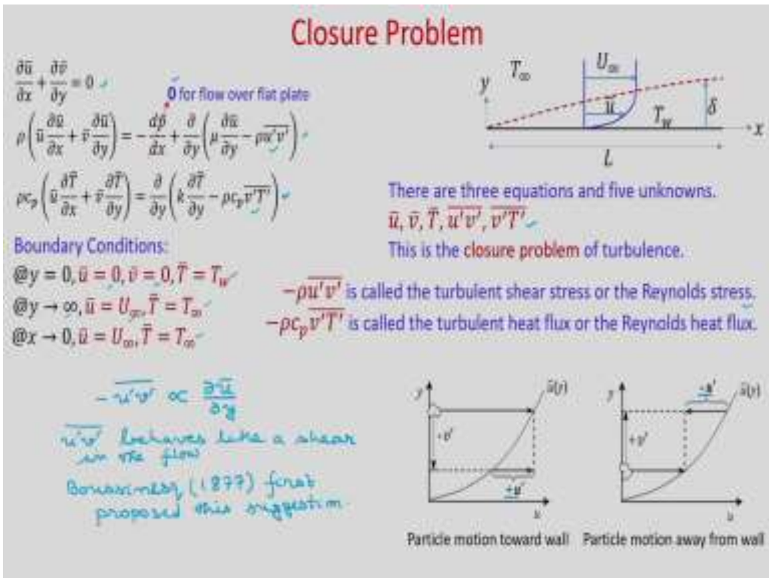

But if you consider flow over flat plate, so if you consider a flow over flat plate of length L, your $\overline{u}$ will be the mean velocity which varies from 0 to U_∞, U_∞ is the free stream velocity, and free stream temperature is T_∞, and wall temperature is T_W . So, if you consider flow over flat plate, obviously, $\frac{d\overline{p}}{dx}$ = 0.

Now, let us discuss about the boundary conditions. So, what are the boundary conditions? So, obviously, at x = 0, you have $U = U_\infty$, and $T = T_\infty$. And y = 0 this is the wall, so again $\bar{u} = 0$ and $\bar{v} = 0$, and $\bar{T} = T_w$.

And $y \to \infty$; you have free stream velocity and free stream temperature. So, you can see at y =0, you have $\bar{u} = 0$, $\bar{v} = 0$, $\bar{T} = T_w$; $y \to \infty$ $\bar{u} = U_\infty$, $\bar{T} = T_\infty$, and $x \to 0$; $\bar{u} = U_\infty$, $\bar{T} = T_\infty$.

Now, if you see these equations, there are three equations. And how many unknowns are there? You can see u', v', then you have T'. You can see there are three equations, and we have how many variables unknown variables $\bar{u}$, $\bar{v}$, $\bar{T}$. And you have two more terms $\overline{u'v'}$, and $\overline{v'T'}$. So, you can see these are the five unknowns and we have three equations.

So, this is known as closure problem of turbulence. So, we need to model these two terms these $\overline{u'v'}$ and v prime $\overline{v'T'}$. So, you can see these term in the momentum equation is called the turbulent shear stress or the Reynolds stress. And this term in the energy equation is called the turbulent heat flux or the Reynolds heat flux.

Now, these we need to model with the known parameters. Now, you see in the turbulent flows in the inside the boundary layer, one particle is here. Now, due to fluctuation it is forced to move at this position. So, you will have the v' will be negative and it will come here. So, the particle if you see here it has higher velocity than here. So, your local velocity is low and this particle will obviously feel low velocity when it will come, but it is having higher velocity than the local velocity.

So, it will have some fluctuation of plus u', so that means, you can see when this particle is coming towards the wall, obviously, it is experiencing one velocity fluctuation as plus u'. So, obviously, the value of this u' will depend on the velocity gradient. So, we can model this $\overline{u'v'}$ with the velocity gradient of the time average velocity.

Similarly, if particle motion away from the wall, if it is going from here to here, so you can see obviously here when it will come, it will experience a higher velocity. So, it will have a minus u prime fluctuation, and it will also depend on the velocity gradient. So, you can see this $u'v'$, obviously, will give a negative value.

Because when v' is positive your u' is negative, and when v' is negative u' is positive. So, $u'v'$ is a itself a negative quantity because if one is positive other will be negative.

So, from these analysis, we can say that $-\overline{u'v'} \propto \frac{\partial \bar{u}}{\partial y}$, so that means, $\overline{u'v'}$ behaves like a shear in the flow. So, this suggestion was first made by Boussinesq, Boussinesq first proposed this suggestion. So, now you can see that we can actually write $\overline{u'v'}$ in terms of the velocity gradient.

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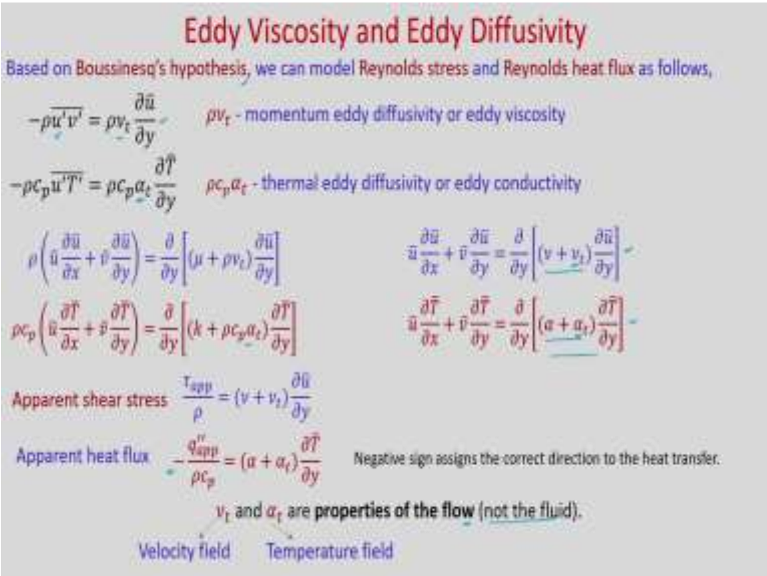

So, based on Boussinesq hypothesis we can model Reynolds stress and Reynolds heat flux as follows. So, this is your Reynolds stress. So, $-\rho\overline{u'v'}$ that we are relating with the velocity gradient $\frac{\partial \bar{u}}{\partial y}$ and $\rho\nu_t$. So, $\rho\nu_t$ is known as momentum eddy diffusivity, and it is known as eddy viscosity also.

And $-\rho c_p\overline{u'T'}$, you can model it as $\rho c_p\alpha_t \frac{\partial \bar{T}}{\partial y}$. And this $\rho c_p\alpha_t$ is the thermal eddy diffusivity or eddy conductivity. So, this is your eddy viscosity commonly known, and this is commonly known as eddy diffusivity.

Now, you can see still here ν_t and α_t are unknown . So, now, our task is to model these eddy viscosity and eddy diffusivity. Now, if you put this in the momentum equation and energy equation, what you will get? So, in the right hand side, now this term you are replacing with this term.

So, you can write $\rho\left(\bar{u}\frac{\partial \bar{u}}{\partial x}+\bar{v}\frac{\partial \bar{u}}{\partial y}\right)=\frac{\partial}{\partial y}\left[\left(\mu+\rho\nu_t\right)\frac{\partial \bar{u}}{\partial y}\right]$. So, you can see we have written in terms of some shear stress. And similarly you can see in the energy equation if we take this term, then we can write $\rho c_p\left(\bar{u}\frac{\partial \bar{T}}{\partial x}+\bar{v}\frac{\partial \bar{T}}{\partial y}\right)=\frac{\partial}{\partial y}\left[\left(k+\rho c_p\alpha_t\right)\frac{\partial \bar{T}}{\partial y}\right]$. So, this also represents some heat flux.

So, if you rearrange it. So, after rearranging you can write, $\bar{u}\frac{\partial \bar{u}}{\partial x}+\bar{v}\frac{\partial \bar{u}}{\partial y}=\frac{\partial}{\partial y}\left[\left(\nu+\nu_t\right)\frac{\partial \bar{u}}{\partial y}\right]$. And in energy equation, $\bar{u}\frac{\partial \bar{T}}{\partial x}+\bar{v}\frac{\partial \bar{T}}{\partial y}=\frac{\partial}{\partial y}\left[\left(\alpha+\alpha_t\right)\frac{\partial \bar{T}}{\partial y}\right]$ where terms ν_t and α_t are unknown.

So, these now together you can say that this is the apparent shear stress, $\frac{\tau_{app}}{\rho}=\left(\nu+\nu_t\right)\frac{\partial \bar{u}}{\partial y}$. And this you can say that it is apparent heat flux it is $-\frac{q_{app}}{\rho c_p}=\left(\alpha+\alpha_t\right)\frac{\partial \bar{T}}{\partial y}$. So, you can see here we have put negative sign, because it assigns the correct direction to the heat transfer.

So, now we can see now ν_t and α_t are properties of the flow. You remember not the fluid. So, this eddy viscosity and eddy diffusivity are properties of flow, because ν_t depends on the velocity field and α_t depends on the temperature field. Now, the question is that how to model this ν_t and α_t? So, first let us discuss how to model ν_t.

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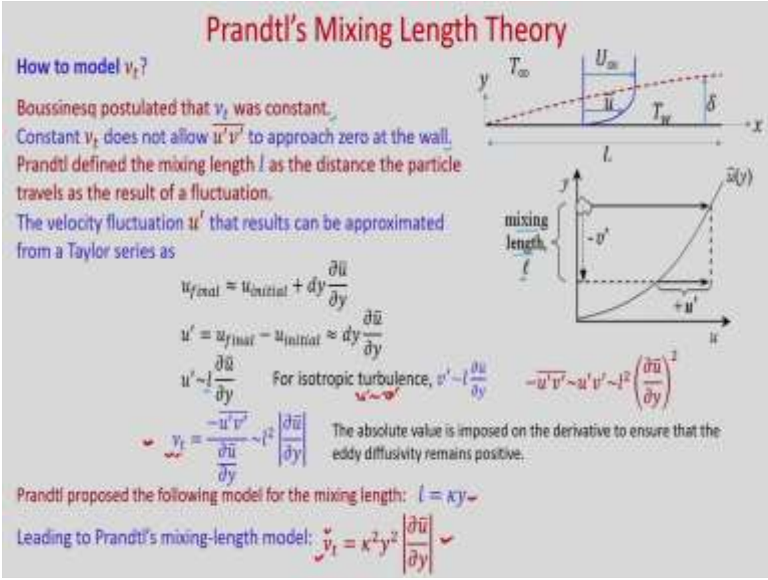

So, first Boussinesq postulated that ν_t was constant . If ν_t is constant, then you can see near to the wall it will have some constant value, but there your it should be 0 right the fluctuation velocity fluctuation should be 0.

So, this model will give problem when you go closer to the wall. So, constant ν_t does not allow $\overline{u'v'}$ to approach zero at the wall. So, Prandtl defined the mixing-length L as the distance the particle travels as the result of a fluctuation.

So, the velocity fluctuation u prime that results can be approximated from a Taylor series as. So, you can see now this particle is forced to move here. So, as we discussed earlier, so obviously, it will have negative v velocity and due to that it will have some fluctuation in u, and that will be u'. And whatever distance it travels, so that is known as mixing length L.

So, now if you see if you tell that velocity here is u_{final} and it is $u_{initial}$ using Taylor series, you can write $u_{final} \approx u_{initial} + dy\frac{\partial \bar{u}}{\partial y}$ and neglect the higher order terms. So, the difference between u_{final} and $u_{initial}$ will be your velocity fluctuation. So, $u' = u_{final} - u_{initial} \approx dy\frac{\partial \bar{u}}{\partial y}$.

So, this distance now whatever it travelled that we are telling mixing length, so $u' \sim l\frac{\partial \bar{u}}{\partial y}$.

So, now, for isotropic turbulence, you know that for isotropic turbulence you know that $u' \sim v'$. So, obviously, $v' \sim l \frac{\partial \bar{u}}{\partial y}$. So, you can see this minus $-\overline{u'v'} \sim u'v' \sim l^2 \left(\frac{\partial \bar{u}}{\partial y} \right)^2$.

So, now if you write ν_t, so you can write $\nu_t = \frac{-\overline{u'v'}}{\frac{\partial \bar{u}}{\partial y}} \sim l^2 \left| \frac{\partial \bar{u}}{\partial y} \right|$. So, these modulus we are giving or absolute value is imposed on the derivative to ensure that the eddy diffusivity remains positive because this is a positive quantity. So, it remains positive.

So, this is the way we can model using Prandtl's mixing-length theory. It is the simplest model we can have. And Prandtl propose the following model for the mixing-length $l = \kappa y$, and κ is constant. And it differs for different types of flows. And leading to Prandtl mixing length model now ν_t, you can write Prandtl proposed the following model for the mixing-length $l = \kappa y$.

So, this κ value depends on different types of flow. And leading to Prandtl's mixing-length model now eddy diffusivity, you can model as $\nu_t = \kappa^2 y^2 \left| \frac{\partial \bar{u}}{\partial y} \right|$. So, you can see this is the simplest model for to determine the eddy diffusivity or eddy viscosity.

So, now, we have found the eddy viscosity ν_t using the Prandtl's mixing-length hypothesis. Inside the boundary layer close to the wall, you can see you can neglect the fluctuating velocities $u'v'$. And if you are away from the wall, then the effect of molecular viscosity can be neglected. So, in based on that, you can differentiate two different layers inside the boundary layer.

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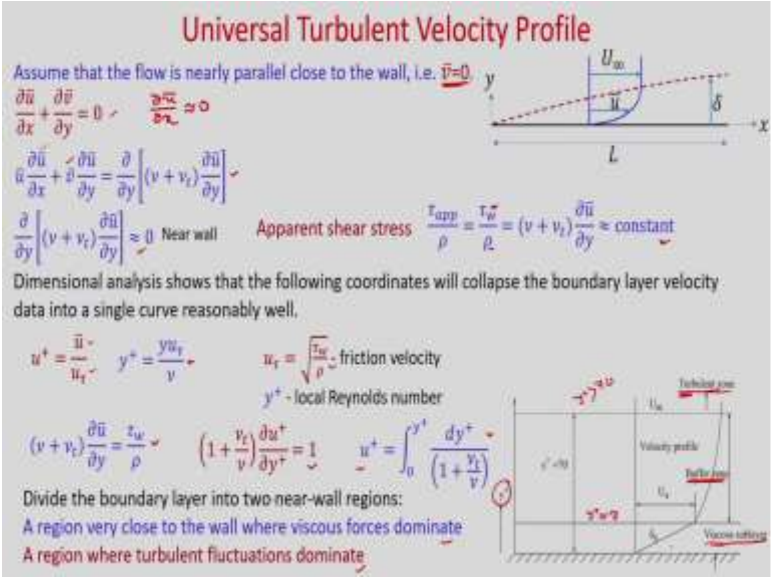

So, you can see if this is the flat plate and you can have one layer very close to the wall where you can neglect the velocity fluctuations, and you can have the viscous sub layer. And away from the wall where you can have fully turbulent layer, where you can neglect the effect of this molecular viscosity. And in between this zone is known as buffer layer or buffer zone.

So, you can see that we have these equations. This is the continuity equation and this is the momentum equation. Now, we are assuming that the flow is nearly parallel close to the wall. So, if close to the wall if we assume nearly parallel, that means, v bar will be 0 ok. And if $\bar{v}$ = 0, then from the continuity equation, you can say that $\frac{\partial \bar{u}}{\partial x}$ ~ 0.

So, now, you can see if this is 0, then in the momentum equation you can see this term is 0 and $\bar{v}$ = 0. So; obviously, inertia terms you can neglect. So, if you neglect the inertia term, so very near to the wall you can say that $\frac{\partial}{\partial y}\left[\left(\nu+\nu_t\right)\frac{\partial \bar{u}}{\partial y}\right]\approx 0$.

So, now we have already defined apparent shear stress as $\frac{\tau_{app}}{\rho}=\frac{\tau_w}{\rho}=\left(\nu+\nu_t\right)\frac{\partial \bar{u}}{\partial y}$. And obviously, τ for steady state flow, τ_w will be constant, density of the fluid is constant. So, this term will be constant.

So, now, if we define this non-dimensional quantities $u^+ = \frac{\bar{u}}{u_\tau}$ where u_τ is known as friction velocity which is defined as $u_\tau = \sqrt{\frac{\tau_w}{\rho}}$ and τ_w is the shear stress at the wall. And $y^+ = \frac{yu_\tau}{\nu}$. So, this term you can see it is related to Reynolds number, because the $\mathrm{Re} = \frac{U_\infty L}{\nu}$. So, similarly it is known as local Reynolds number.

So, if you define it and this equation whatever we got because this if you integrate then you will get $\left(\nu + \nu_t\right)\frac{\partial \bar{u}}{\partial y} = \frac{\tau_w}{\rho}$. And these if you use these non dimensional quantities, you can write as $\left(1 + \frac{\nu_t}{\nu}\right)\frac{\partial u^+}{\partial y^+} = 1$. So, you can see that $u^+ = \int_0^{y^+} \frac{dy^+}{\left(1 + \frac{\nu_t}{\nu}\right)}$, then you will be able to find the velocity.

Now, we are actually dividing the boundary into two near wall regions, a region very close to the wall where viscous force dominant, and a region where turbulent fluctuation dominate. So, you can see away from the surface, it will be fully turbulent zone. And effect of molecular viscosity, you can neglect and turbulent fluctuation will dominate. And generally you can see this y^+ if you take in this way which is your non-dimensional coordinate.

So, here near to the wall , it is around $y^+ = 7$, you can say that it is viscous sub layer. Then away from 70 if $y^+ > 70$, then it is fully turbulent layer. And between 7 and 70, you will get buffer layer. Now, let us consider the near wall region which is your known as viscous sub layer. So, in the viscous sub layer, we can neglect the fluctuating components. So, generally your viscous effect will dominate the flow.

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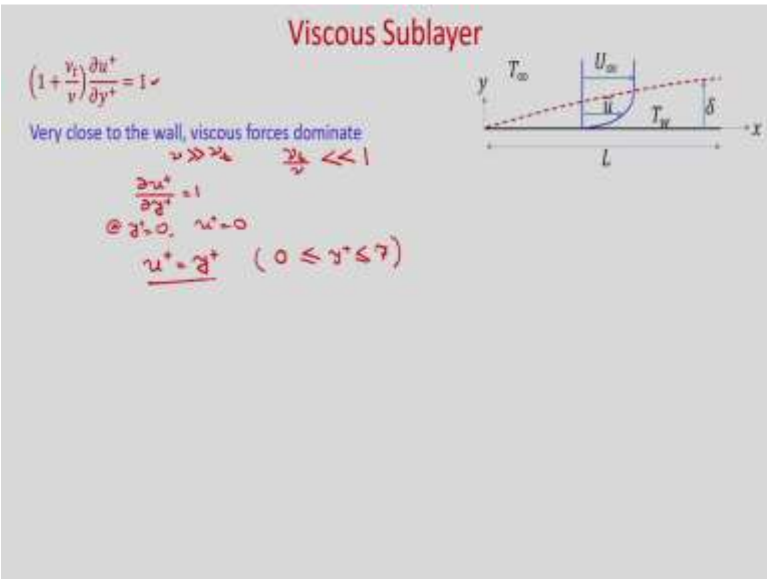

So, in this equation, now you can say that $\nu >> \nu_t$. If $\nu >> \nu_t$, then this equation you can write as $\frac{\partial u^+}{\partial y^+}=1$, because $\frac{\nu_t}{\nu}$ << 1. So, you can neglect this term. So, you will get, $\frac{\partial u^+}{\partial y^+}=1$.

So, now if you integrate it as and put the boundary condition as $y^+ = 0$, obviously, your $u^+ = 0$. So, if you integrate it, you will get $u^+ = y^+$. So, it is valid in the viscous sub layer in the range of $0 \le y^+ \le 7$.

And you can see it is a linear profile in terms of non-dimensional quantities. So, you can see here this varies linearly in the viscous sub layer, because if it is u^+ and this is your y^+, then it varies linearly inside the viscous sub layer. Now, if you go further away from the wall, then your fluctuations will dominate, fluctuating velocities will dominate.

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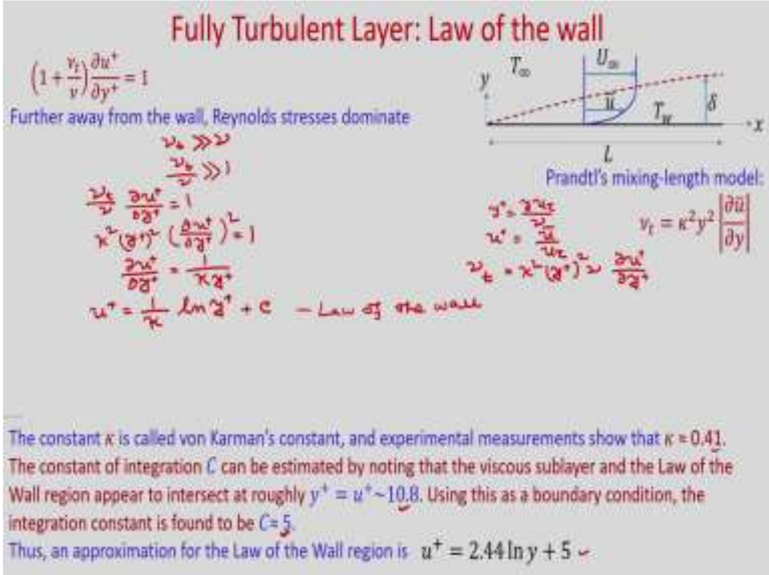

So, in that case, you can say that $\nu_t >> \nu$. So, if it is so, then you can write that $\frac{\nu_t}{\nu} >> 1$.

So, this equation we can write as $\frac{\nu_t}{\nu}\frac{\partial u^+}{\partial y^+}=1$. Now, ν_t we know from the Prandtl's mixing-layer hypothesis. What is that? $\nu_t=\kappa^2 y^2\left|\frac{\partial u^+}{\partial y^+}\right|$.

So, now if you use the non dimensional quantities, $y^+=\frac{y u_\tau}{\nu}$, and $u^+=\frac{\bar{u}}{u_\tau}$. Then this, $\nu_t=\kappa^2\left(y^+\right)^2\nu\frac{\partial u^+}{\partial y^+}$. So, this ν_t value now you put .

So, $\frac{\nu_t}{\nu}$ will be this quantity. So, you can write it as $\kappa^2\left(y^+\right)^2\left(\frac{\partial u^+}{\partial y^+}\right)^2=1$. And you can write, $\frac{\partial u^+}{\partial y^+}=\frac{1}{\kappa y^+}$.

Now, if you integrate it, you will get $u^+=\frac{1}{\kappa}\ln y^+ + C$. So, this is known as law of the wall. Now, how to find this κ and C? So, this you need to find empirically you need to find it from the experimental conditions. So, you can see that in this equation you need to know the value of κ as well as the constant C.

The constant κ is called von Karman's constant and experimental measurements, so that κ = 0.41. And the constant of integration C can be estimated by noting that the viscous sub layer and the Law of the Wall region appear to intersect at roughly $y^+ = u^+ \sim 10.8$.

So, if you put $y^+ = 10.8$, then you will be able to find the constant $C \approx 5$, so that is an approximation for the Law of the Wall region is putting the values of κ and see you can get $u^+ = 2.44 \ln y + 5$. So, now, you can see that these two layers viscous sub layer as well as fully turbulent layer will intersect through the buffer layer.

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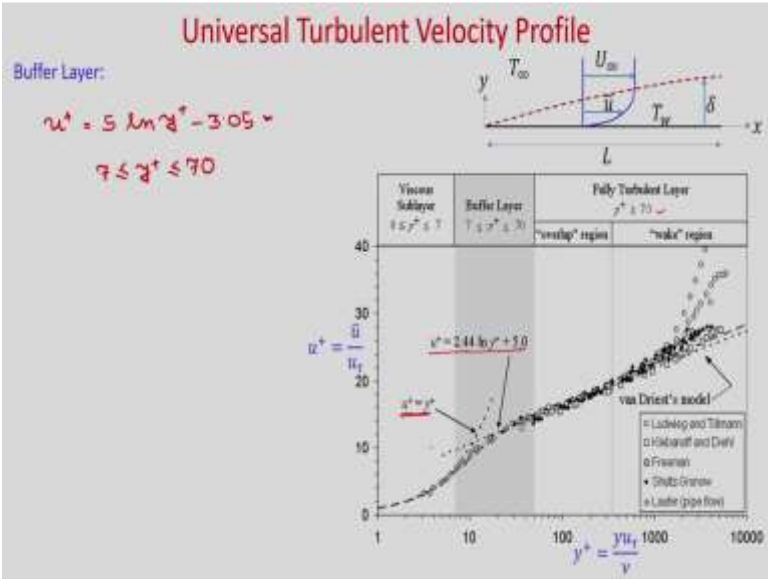

So, you can see in this curve u^+ verses y^+. So, here you can see that this is your viscous sub layer where $y^+ < 7$, and here $u^+ = y^+$. And you have fully turbulent layer, so that is already we have derived as $u^+ = 2.44 \ln y + 5$. So, you can see that these two model meets here in the buffer layer right.

So, in the buffer layer because this is valid in the range of $y^+ > 70$. So, in between 7 and 70, we have buffer layer. And in the buffer layer, you can use $u^+ = 5 \ln y^+ - 3.05$ in the range of $7 \le y^+ \le 70$.

So, this is actually connecting your viscous sub layer as well as fully turbulent layer model. Now, we have discussed about the universal velocity profile. Now, let us discuss about the universal temperature profile. So, now, we need to find the eddy diffusivity which is your α_t.

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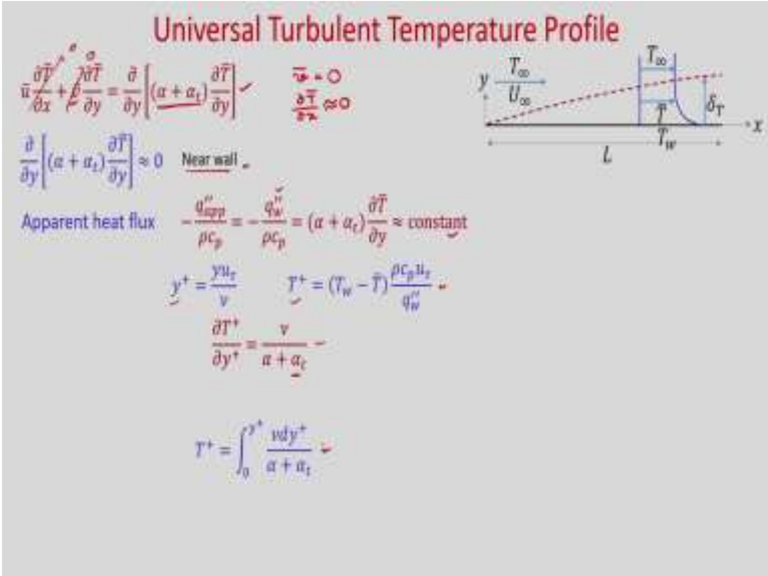

So, you can see this is the equation we have derived. So, this is your $\alpha + \alpha_t$. And similarly in near wall region, we can have this $\bar{v}$ = 0 , because nearly flow is parallel flow is nearly parallel. So, $\bar{v}$ = 0. And your axial heat conduction you can neglect. And you see the variation of $\bar{T}$ along x is very small, so $\frac{\partial \bar{T}}{\partial x}$ ~0.

So, if this is 0 and this is 0, then you can write $\frac{\partial}{\partial y}\left[\left(\alpha + \alpha_t\right)\frac{\partial \bar{T}}{\partial y}\right] \approx 0$ near to the wall. So, obviously, you can write in terms of apparent heat flux $-\frac{q''_{app}}{\rho c_p} = -\frac{q''_w}{\rho c_p}$ which is your wall heat flux, and obviously $\left(\alpha + \alpha_t\right)\frac{\partial \bar{T}}{\partial y} \approx$ constant.

Now, similarly you define y^+, and $T^+ = \left(T_w - \bar{T}\right)\frac{\rho c_p u_\tau}{q''_w}$. So, using these non-dimensional quantities if you put it here, you are going to get $\frac{\partial T^+}{\partial y^+} = \frac{\nu}{\alpha + \alpha_t}$. So, you can see here α_t is unknown . So, $T^+ = \int_0^{y^+} \frac{\nu dy^+}{\alpha + \alpha_t}$.

Now, similarly we can have the conduction layer which is very very near to the wall, where you can neglect the fluctuating components. And away from the wall, you can have fully turbulent region the fluctuating components will dominate.

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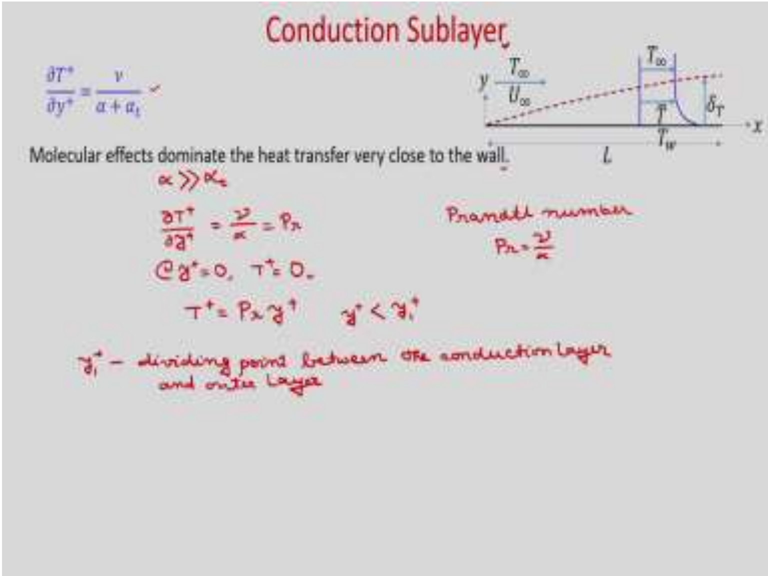

So, with this you can see the conduction sub layer which is very near to the wall. So, this is the equation. So, we are telling that molecular effects dominate the heat transfer very close to the wall. So, fluctuating components you can neglect, that means, here $\alpha \gg \alpha_t$.

So, from here you can see that if $\alpha \gg \alpha_t$, then you can write $\frac{\partial T^+}{\partial y^+} = \frac{\nu}{\alpha}$.

And what is $\frac{\nu}{\alpha}$ you know, Prandtl number right? So, Pr= $\frac{\nu}{\alpha}$. So, you can write this is equal to Prandtl number. Now, if you integrate it and put the boundary condition at $y^+ = 0$ $T^+ = 0$ because T^+ is having one quantity $\overline{T} - T_w$. So, at y = 0, you have T = T_w. So, T^+ will become 0.

So, if you integrate it, you will get $T^+ = Pr\ y^+$. And let us say that it is valid in the range of $y^+ < y_1^+$, where y_1^+ is the dividing point between the conduction layer and outer layer. So, now let us consider the outer layer. So, in the outer layer, obviously, your it is a fully turbulent flow and fluctuating components dominate.

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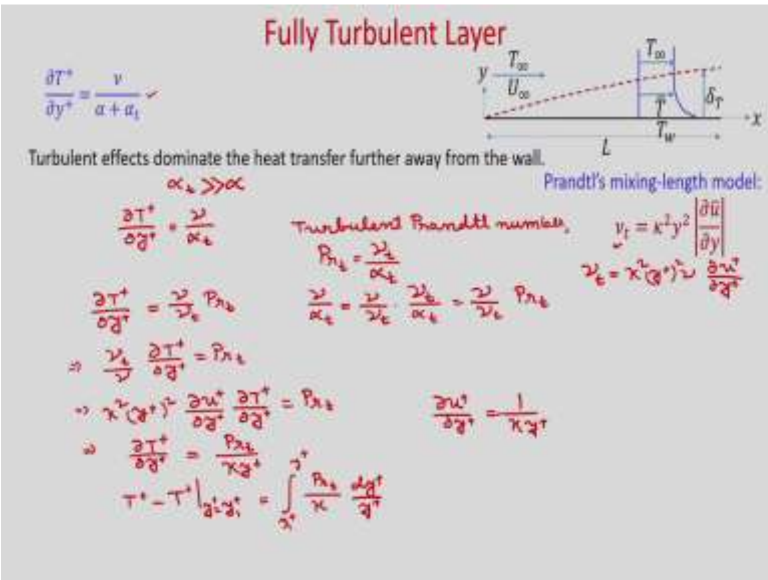

So, in this case you can see this is the equation. So, turbulent effects dominate the heat transfer further away from the wall. So, you can see that $\alpha_t \gg \alpha$. So, you can see you can write $\frac{\partial T^+}{\partial y^+} = \frac{\nu}{\alpha + \alpha_t}$. So, α_t is unknown. So, now, we will write α_t in terms of turbulent Prandtl number. And we will use the Prandtl mixing-length model and we will substitute this ν_t.

So, we can see here turbulent Prandtl number we are defining as general $\Pr = \frac{\nu}{\alpha}$. So, turbulent $\Pr_t = \frac{\nu_t}{\alpha_t}$. So, you can see here $\frac{\nu}{\alpha_t}$. You can write as $\frac{\nu}{\alpha_t} = \frac{\nu}{\nu_t}\frac{\nu_t}{\alpha_t}$. So, you can write $\frac{\nu}{\nu_t}\Pr_t$. So, and this ν_t in non-dimensional form if you write it will be, $\nu_t = \kappa^2 \left(y^+\right)^2 \nu \frac{\partial u^+}{\partial y^+}$.

So, now if you write this equation, $\frac{\partial T^+}{\partial y^+} = \frac{\nu}{\nu_t}\Pr_t$. If you take in the left hand side you get, $\frac{\nu_t}{\nu}\frac{\partial T^+}{\partial y^+} = \Pr_t$. So, now you see, $\kappa^2 \left(y^+\right)^2 \frac{\partial u^+}{\partial y^+}\frac{\partial T^+}{\partial y^+} = \Pr_t$.

Now, from universal velocity profile for the fully turbulent layer, $\frac{\partial u^+}{\partial y^+} = \frac{1}{\kappa y^+}$. So, we will use $\frac{\partial u^+}{\partial y^+} = \frac{1}{\kappa y^+}$. So, if you put it here, so you are going to get as, $\frac{\partial T^+}{\partial y^+} = \frac{\Pr_t}{\kappa y^+}$.

So, now we have $\frac{\partial T^+}{\partial y^+} = \frac{\Pr_t}{\kappa y^+}$. Now, if you integrate it, so you will get $T^+ - T^+\big|_{y^+=y_1^+} = \int_{y_1^+}^{y^+} \frac{\Pr_t}{\kappa} \frac{dy^+}{y^+}$. So, you see if we assume $\Pr_t$ and κ as constant, then you will be able to integrate it.

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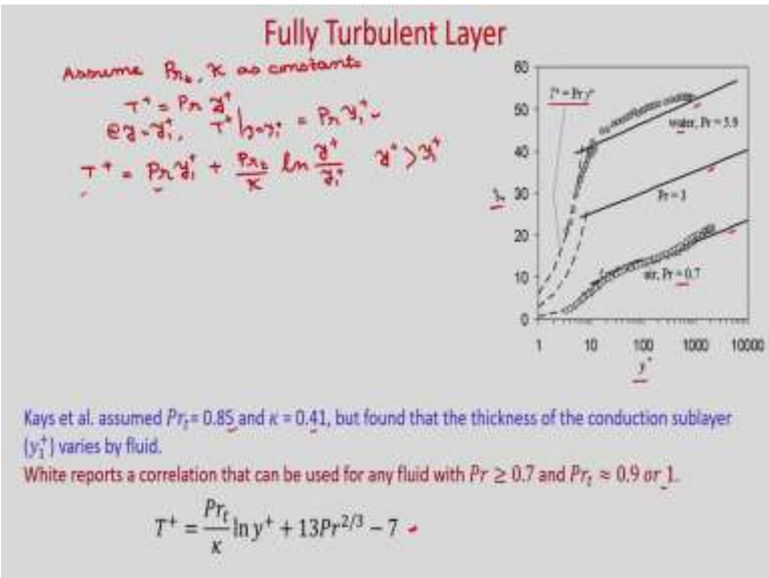

So, let us assume $\Pr_t$ and κ as constants. And you can see we have in the conduction layer $T^+ = \Pr y^+$. So, at $y = y_1^+$, you can write $T^+\big|_{y^+=y_1^+} = \Pr y_1^+$.

So, if you put all these values and if you integrate keeping $\Pr_t$ and κ constant, then you will get $T^+ = \Pr y_1^+ + \frac{\Pr_t}{\kappa} \ln \frac{y^+}{y_1^+}$ and it is valid for $y^+ > y_1^+$.

So, you can see that your temperature profile depends on this fluid that means for Prandtl number, and also it depends on $\Pr_t$ and κ. So, Kays et al. assumed this $\Pr_t$ = 0.85 and κ =

0.41, but found that the thickness of the conduction sub layer y_1^+ varies by fluid. So, if you have different fluid, this y_1^+ varies. So, depending on the value of y_1^+, you can use these conduction layer model as well as fully turbulent layer model.

Why it reports a correlation that can be used for any fluid with $\Pr \geq 0.7$, and $\Pr_t \approx 0.9$ or 1. So, if you put all these values, you will get $T^+ = \frac{\Pr_t}{\kappa} \ln y^+ + 13 \Pr^{2/3} - 7$.

So, you can see here in this curve T^+ versus y^+. So, in viscous sub layer region or in conduction sub layer, you have $T^+ = \Pr y^+$; and in fully turbulent region, you can have this model.

So, for different Prandtl number, you can see for here 0.7. So, these are the solid line you can see for different Prandtl number Prandtl number 0.7, 0.3, and what are Prandtl number 5.9. So, this is the model for fully turbulent layer. And the results of kays et al. also here it is shown for air as well as water, and this is the Kays et al. model. So; obviously, you can see with increase of Prandtl number, your value of T^+ increases.

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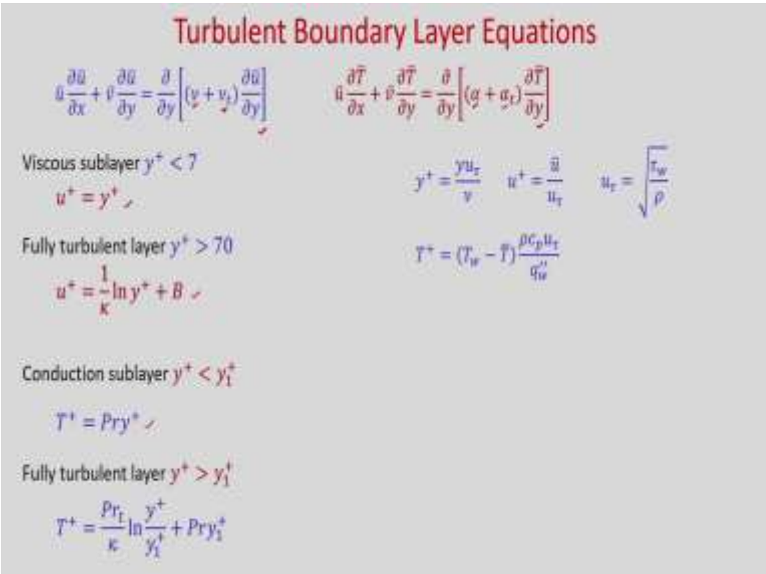

So, you can see in today's class we have derived these boundary layer equations for turbulent flows, where ν is your kinematic viscosity, ν_t is your eddy viscosity, and α is your thermal diffusivity, and α_t is your eddy diffusivity, and ν_t is your eddy viscosity.

So, you can see in viscous sub layer we have derived $u^+ = y^+$ and in fully turbulent layer; $u^+ = \frac{1}{\kappa}\ln y^+ + B$ where $y^+ = \frac{yu_\tau}{\nu}$, $u^+ = \frac{\bar{u}}{u_\tau}$ and $u_\tau = \sqrt{\frac{\tau_w}{\rho}}$, and $T^+ = \left(T_w - \bar{T}\right)\frac{\rho c_p u_\tau}{q_w^{"}}$. And in conduction sub layer, we have derived this $T^+ = \Pr y^+$; and in fully turbulent layer, you have, $T^+ = \frac{\Pr_t}{\kappa}\ln\frac{y^+}{y_1^+} + \Pr y_1^+$.

In today's class, we considered a steady state two-dimensional in incompressible fluid flow equations and for turbulent flows, and we used the scale analysis and we have written the boundary layer equations for turbulent flow. When we write these boundary layer equations, we have seen that you have the fluctuating components $\overline{u'v'}$; and in energy equation we have $\overline{u'T'}$. So, these are the unknowns and that we need to model some way.

So, from the Prandtl mixing-length hypothesis, we have seen that your eddy viscosity ν_t you can write in terms of the mixing length. And this mixing length also you can write as κy, where κ is constant for any fluid flow and you need to determine experimentally.

Now, we have seen that to solve these equations, you need to know the eddy viscosity as well as the eddy diffusivity which are unknown. And these unknowns you need to find with some assumptions as well as from the experimental conditions. When we considered the velocity profile, we have taken two different layers; one region is very near to the wall where you can neglect the effect of the fluctuating components.

And that is known as a viscous sub layer. And one is away from the wall where your fluctuating components dominate and you can neglect the effect of viscosity or effect of wall in those region and in between you have you will have the buffer zone.

So, in these two different zones in viscous sub layer and fully turbulent layer, we have derived the non-dimensional velocity profile u^+. And similarly for the energy equation, we considered two layers one is very near to the wall that is your conduction layer and away from the wall that is your fully turbulent layer.

So, in these layers also we have derived the non-dimensional temperature profile T^+. And these velocity profile and temperature profiles are known as universal velocity profile and universal temperature profile.

Thank you.