Fundamentals of Convective Heat Transfer
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Module – 11
Turbulent Flow and Heat Transfer
Lecture – 36
Derivation of Reynolds Averaged Navier – Stokes Equations

Hello everyone. So, today we will study convection in turbulent flow most flows in nature and in industrial applications are turbulent. You will find applications in mixing of the flows, then in combustion processes as well as in heat exchangers.

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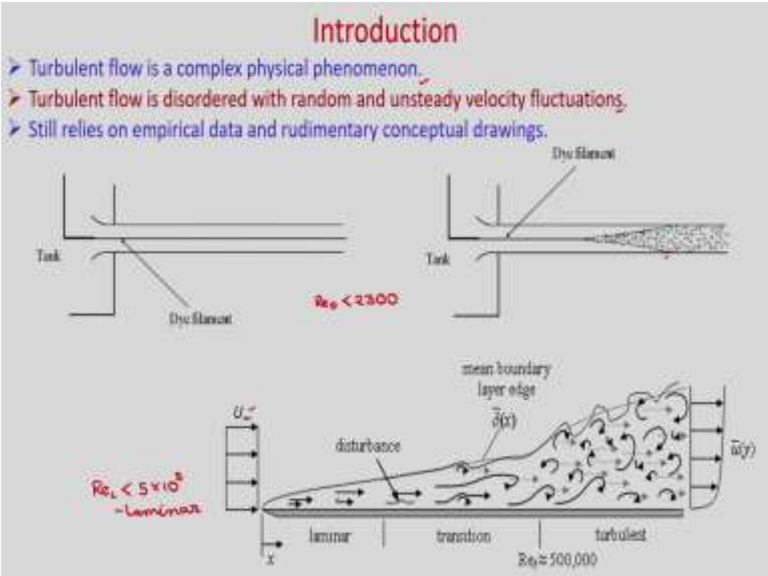

Generally turbulent flow occurs relatively at high Reynolds number and in turbulent flows velocity temperature fluctuate with time. Turbulent flow is a complex physical phenomena, turbulent flow is disordered with random and unsteady velocity fluctuations. In laminar flows we already had the exact solution of many flows with certain assumptions, but in turbulent flows it is very difficult to have the exact solutions.

So, mostly whatever correlations will write that depends on the experimental values and that is why we will write the empirical correlations. You know about the famous experiment carried by Reynolds. So, you can see here Reynolds did this experiment say the ink is injected here in a tank and this is the pipe, when it passes through this pipe you can see this dye filament is almost straight line.

You will get this type of flow when you have low velocity, but if you increase the velocity here then you can see these dye filament will diffuse in other directions. So, it is due to the fluctuation of the velocities at high Reynolds number and it becomes turbulent flows.

So, you will get this kind of structure you know that in pipe flow when Reynolds number based on the diameter is < 2300 then it will be laminar, then it will transform to transition and turbulent flow.

If you consider flow over flat plate which already we have done the exact solutions in earlier classes in external flows, you have seen that near to the leading edge of the flat plate we get laminar flows and we have done the study of this laminar flow.

But if you increase the length then you will find that it will become a transitionism then turbulent. So, you can see here. So, you have a free stream velocity U_∞; when it will come and flow over this flat plate so; obviously, you know due to the viscous effect there will be formation of boundary layer and near to the leading edge you will get laminar flows.

You will see that very streamline flow but after that there will be some disturbances near to the wall and it will propagate away from the wall slowly and after if you go ahead and you will get a fully turbulent flows and in this case you see if Reynolds number based on the length of the flat plate if it is > 5×10^5, then you will get laminar flows.

So, turbulent flow is very complex and chaotic. So, it is very difficult to define turbulent flows. So, most of the researchers they are given the characteristic of the turbulent flows.

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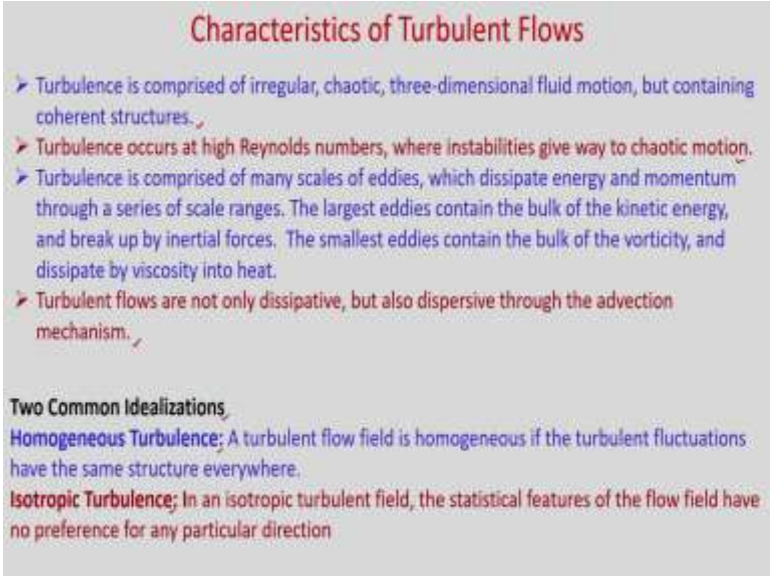

So, you see turbulence is comprised of irregular chaotic three dimensional fluid motion, but containing coherent structures. So, you can see that turbulent flow inherently three dimensional and unsteady.

But with certain assumptions again we may consider as two dimensional flow as well as steady flow. When we consider the velocity components or temperature as comprised of mean value, time average mean value and the fluctuating components.

Problems occurs at high Reynolds number where instabilities give way to chaotic motion we have already seen in external and internal flows; obviously, it is a high Reynolds number flow then the turbulence occur.

Turbulence is comprised of many scales of eddies which dissipate energy and momentum through a series of scale ranges. The largest eddies contain the bulk of the kinetic energy and break up by inertial forces, the smallest eddies contain the bulk of the vorticity and dissipate by viscosity into heat.

So, you can see that in general you will have a larger eddy which will contain bulk of the kinetic energy and it will be transported by the velocities and during this transport it will divide into smaller eddies and it will keep on decreasing the size of the eddies and those eddies will contain bulk of the vorticity and then it will actually dissipate into heat by the viscous effect. Turbulent flows are not only dissipative, but also dispersive through the advection mechanism.

So, when we study the turbulence we make two common idealizations one is homogeneous turbulence and another is isotropic turbulence. What is homogeneous turbulence?

If the turbulence has the same structure quantitatively in all parts of the flow field then the turbulence is said to be homogeneous turbulence and in isotropic turbulence, the statistical features have no directional preference, then it is called isotropic turbulence. So, when we do the numerical simulation of this turbulent flows, we need to have some prediction methods.

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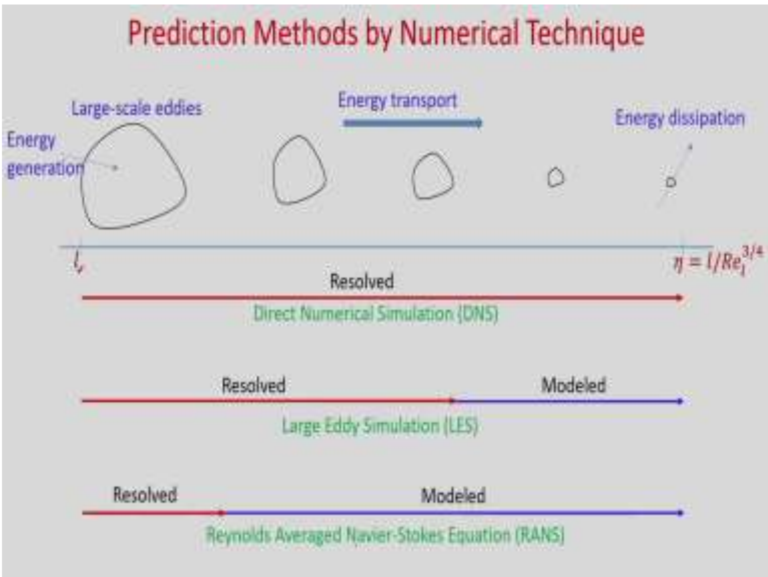

So, here you can see as we discussed you have a large scale eddies let us say its scale is of the order of l. So, it contains bulk kinetic energy due to the inertial effect it will be transported and due to the during the transport it will divide into smaller eddies.

Then again it will become smaller and after that it cannot be smaller than this eddy and that time this eddy will contain bulk of the vorticity and these vorticity or this eddy will dissipate heat due to the viscous effect and the scale of this eddy is known as Kolmogorov scale and η is known as Kolmogorov scale and $\eta = l / \mathrm{Re}_l^{3/4}$.

So, you see Re is based on the large scale eddy length and if Reynolds number is very high then η which is your Kolmogorov scale will be very low and to actually compute or to capture these smaller eddies it is very difficult because you need to have very very small grid size to capture the small eddy.

So, you can see if you solve the Navier stokes equations using direct numerical simulation without modelling any of the eddies, then you need to have very fine mesh so, that you can capture the smaller eddies in the Kolmogorov scale length scale.

Now, direct numerical simulation for three dimensional flows and unsteady flows obviously, it is very difficult computational time will be huge and generally for industrial fluid flow direct numerical simulation is very difficult.

We can have some smaller scales we can model and rest of the eddies we can compute using large eddy simulation. So, we can see in the large eddy simulation up to certain eddy size we resolve and rest we model and another simplified way to solve this turbulent flow is just using the Reynolds average Navier stokes equation.

So, whatever velocity and temperature you have you can decompose into two components one is time average min component and another is fluctuating component and we can solve these governing equations in a average sense. So, here you will model only the large scale eddies, but other you need to model it.

So, in this course we will derive the Reynolds average Navier stokes equations and later we learn how to solve for flow over flat plate and the pipe flow.

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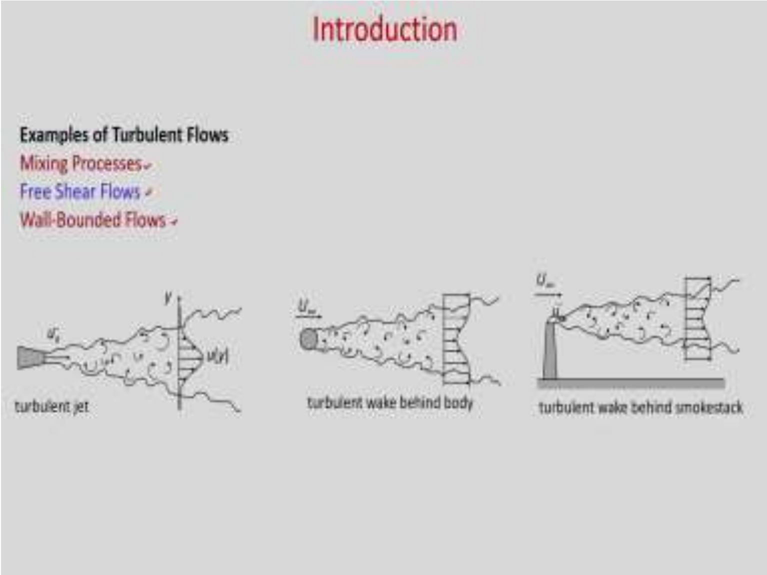

You can see examples of turbulent flows in mixing processes, free shear flows, wall bounded flows. So, these are some examples you can see in turbulent jet turbulent wake behind a body.

So, if this circular cylinder is heated then obviously, there will be convection in turbulent flow and here obviously, you can see turbulent wake behind smokestack. So, in the smokestack you will have high temperature and ambient will be at lower temperature and there will be convection and that is your turbulent convection you will get.

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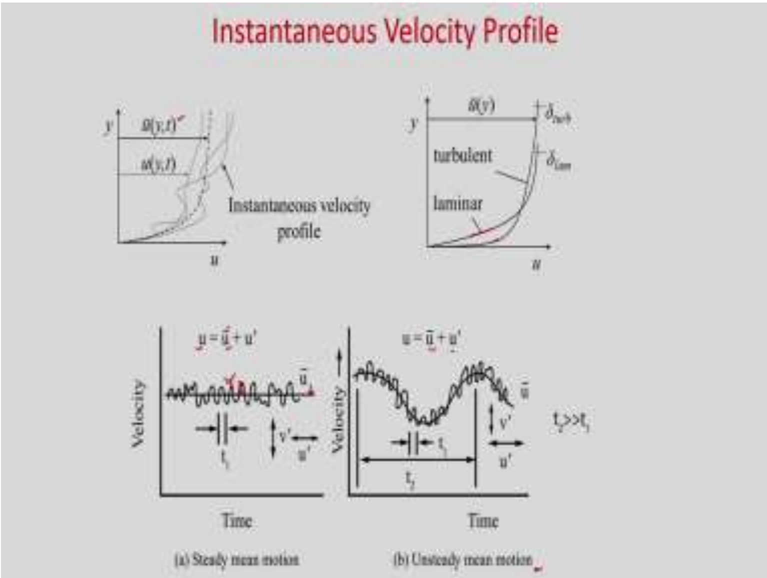

Here you can see the instantaneous velocity profile. So, velocity will vary with time . So, these are the velocity profile over a flat plate and if you time average the quantity then you will get this dotted line.

So, you can see this is the time average velocity profile and obviously, you will get a smooth curvature of this velocity and here we have compared the velocity profile for laminar and turbulent flow. So, this is your laminar flow velocity profile and this is your turbulent flow velocity profile.

So, you can see in turbulent flow near to the wall you have more gradient. So, obviously, you can see from this velocity profile now we have discussed that these velocity and temperature fluctuate with time and we can decompose into two components one is time average mean component and another is your fluctuating component deviating from the mean value.

So, you can see this is some velocity varying with time and we can decompose this velocity into two components one is time average mean component. So, we will denote with u bar and plus your fluctuating component that is your u'. So, you can see if we take the mean. So, this is your u bar. So, this straight line is u bar and whatever fluctuating component you have whatever it is deviating from the mean value.

So, this is known as steady mean motion because your mean velocity is not varying with time, but you can have unsteady mean motion as well. So, here you can see these are fluctuating components and if the time period is t_1 of the fluctuation. So, here if you time average in this period and you will get that your mean value will also vary with time.

So, in this case you can see that t_2 is the time period of mean velocity variation and t_1 is the time period for the fluctuating velocity variation and you can see here velocity is also varying with mean velocity is varying with time. So, this is your unsteady mean motion. So, Reynolds first proposed that we can have this decomposition of this velocity into two; one is mean velocity and the fluctuating velocity.

So, this is known as Reynolds decomposition. So, first we will see that you if you take any scalar f and if we decompose into 2, then what are the identities we can have?

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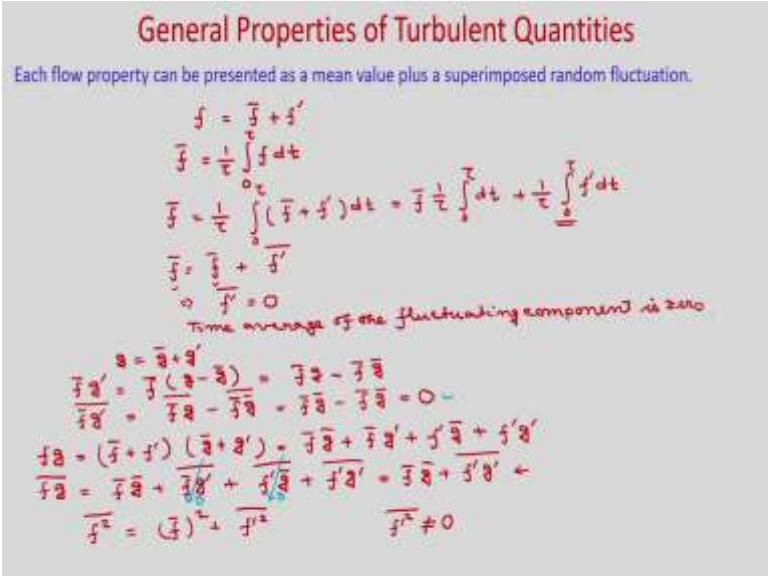

So, if we consider a scalar property f, then we can have $f = \overline{f} + f'$ where, $\overline{f}$ which is your time averaged value and f' which is your fluctuating component.

So, you can see this is superposition of this time average and fluctuating components. Now if you calculate the mean value; that means, it is if you have a time period τ, then it is defined as, $\overline{f} = \frac{1}{\tau}\int_0^\tau f dt$. So, this way actually you calculate the time average component. So, you can see in this case that if you put this $f = \overline{f} + f'$ then what you will get?

So, now $\overline{f} = \frac{1}{\tau}\int_0^\tau \left(\overline{f} + f'\right) dt$ then what we can have? So, you can see. So, $\overline{f}$ which is your average quantity. So, you can bring it outside the integral. So, $\overline{f}\frac{1}{\tau}\int_0^\tau dt$.So, we assume that in that time period $\overline{f}$ is constant.

So, you can take it out of the integral and we can have $\frac{1}{\tau}\int_0^\tau f' dt$. So, you can see you can write $\overline{f}$. So, this $\overline{f}\frac{1}{\tau}\int_0^\tau dt$ will give only $\overline{f}$ and what does it mean? This actually give the definition of this average quantity; that means, it is $\overline{f'}$. So, you can see this quantity and this quantity are same.

So, you can see $\overline{f'}=0$; that means, time average of the fluctuating component is 0. So, time average of the fluctuating components is 0. Now, let us consider another scalar property $g=\overline{g}+g'$. So, if you calculate $\overline{f}g'$ what does it mean?

So, $\overline{f}g'=\overline{f}\left(g-\overline{g}\right)=\overline{f}g-\overline{f}\,\overline{g}$. So, if you take the time average of this quantity left hand side then what you will get? So, you will get $\overline{\overline{f}g'}=\overline{\overline{f}g}-\overline{\overline{f}\,\overline{g}}$. So, you can show that this quantity will be $\overline{f}\,\overline{g}-\overline{f}\,\overline{g}$.

So, this is equal to 0. So, you can see $\overline{\overline{f}g'}=0$ and now if you calculate f g; that means, $fg=\left(\overline{f}+f'\right)\left(\overline{g}+g'\right)$. So, what you will get? $\overline{f}\,\overline{g}+\overline{f}g'+f'\overline{g}+f'g'$. Now, you take the time average of this quantity.

So, if you take the time average of this quantity. So, this will give $\overline{fg}=\overline{f}\,\overline{g}+\overline{\overline{f}g'}+\overline{f'\overline{g}}+\overline{f'g'}$. So, you can see this quantity already we have shown that it is 0.

So, this is 0 and this is 0. So, you will get, $\overline{fg}=\overline{f}\,\overline{g}+\overline{f'g'}$ and if you write f^2. So, from here you can write $\overline{f^2}=\left(\overline{f}\right)^2+\overline{f'^2}$ and $\overline{f'^2}\neq 0$, the time average of the fluctuating component is 0 that we have already shown.

That means, $\overline{f'}=0$, but $\overline{f'^2}\neq 0$ why? Because this is a fluctuating component. So, it is the deviation from the mean value. So, it is having the positive value as well as negative value so, but f'^2 where when you are making. So, it is always positive. So, $\overline{f'^2}\neq 0$.

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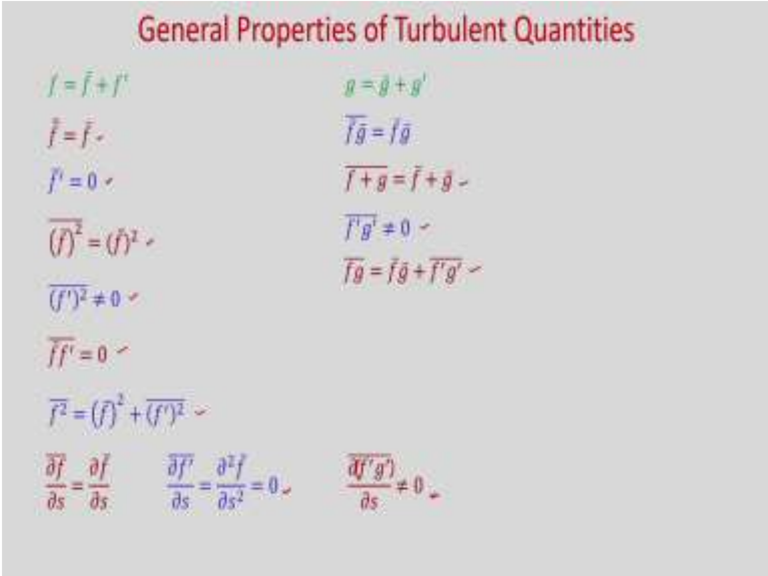

So, now you can see the general properties of turbulent quantities. So, if we have two scalars f and g and we have $f = \overline{f} + f', g = \overline{g} + g'$. So, you can see $\overline{\overline{f}} = \overline{f}$, $\overline{f'} = 0$ that we have already shown then $\overline{\left(\overline{f}\right)^2} = \left(\overline{f}\right)^2$.

But $\overline{f'^2} \neq 0$ and $\overline{\overline{f} f'} = 0$ that we have already shown $\overline{f^2} = \left(\overline{f}\right)^2 + \overline{\left(f'\right)^2}$. So, that also we have shown now if you take $\overline{\overline{f}\,\overline{g}} = \overline{f}\,\overline{g}$, $\overline{f + g} = \overline{f} + \overline{g}$, $\overline{f'g'} \neq 0$.

And $\overline{fg} = \overline{f}\,\overline{g} + \overline{f'g'}$ and if you take the derivative s is any special direction, then $\dfrac{\overline{\partial f}}{\partial s} = \dfrac{\partial \overline{f}}{\partial s}$, $\dfrac{\overline{\partial f'}}{\partial s} = \dfrac{\partial^2 \overline{f}}{\partial s^2} = 0$ and $\dfrac{\overline{\partial f'g'}}{\partial s} \neq 0$.

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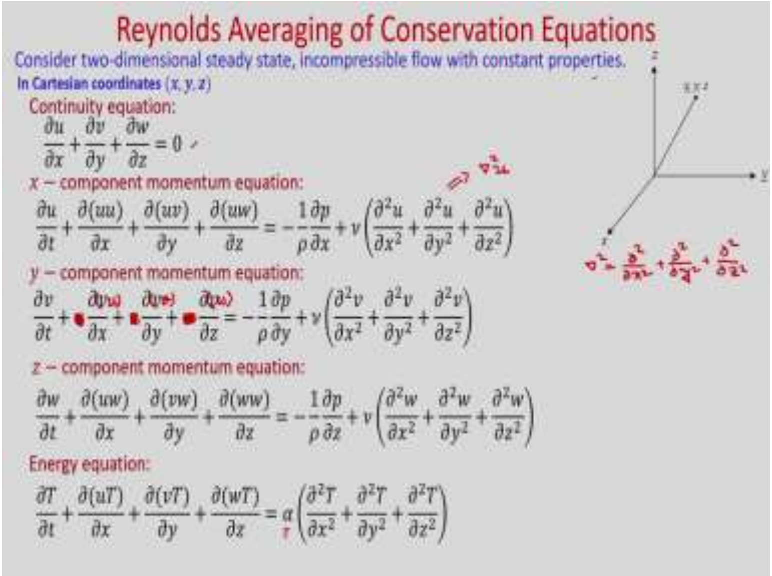

So, now these properties we can use for the when we will decompose the velocities and temperature into mean quantity and the fluctuating component. So, first we will start with the Navier stokes equations, we will write the Navier stokes equation in weak conservative form. So, you can see if you consider two dimensional steady state incompressible flow with constant properties.

So, in Cartesian coordinate you can write continuity equation as $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$, x component momentum equation $\frac{\partial u}{\partial t}+\frac{\partial (uu)}{\partial x}+\frac{\partial (uv)}{\partial y}+\frac{\partial (uw)}{\partial z}$. So, this is your convection terms and you can see we have written in weak conservative form.

So, if you invoke continuity equation you can put from non-conservative form to this weak conservative form and in right hand side you will get $-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$. So, this is the diffusion term. So, this last term actually you can write also $\nabla^2 u$.

So, in the bracket whatever quantities there. So, we can write $\nabla^2 u$ because you know, $\nabla^2 = \left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$.

Similarly y momentum equation you can write in conservative form. So, this you can write in conservative form u v w.

So, this is $\frac{\partial v}{\partial t}+u\frac{\partial(vu)}{\partial x}+v\frac{\partial(vv)}{\partial y}+w\frac{\partial(vw)}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)$ and this is the energy equation in weak conservative form.

So, right hand side is your diffusion term we have neglected the viscous dissipation and α is the thermal diffusivity and obviously, ν is your kinematic viscosity and ρ is the density.

Now, we will use the Reynolds decomposition. So, we will write the velocities u v w and temperature t as superposition of mean quantity as well as the fluctuating quantity.

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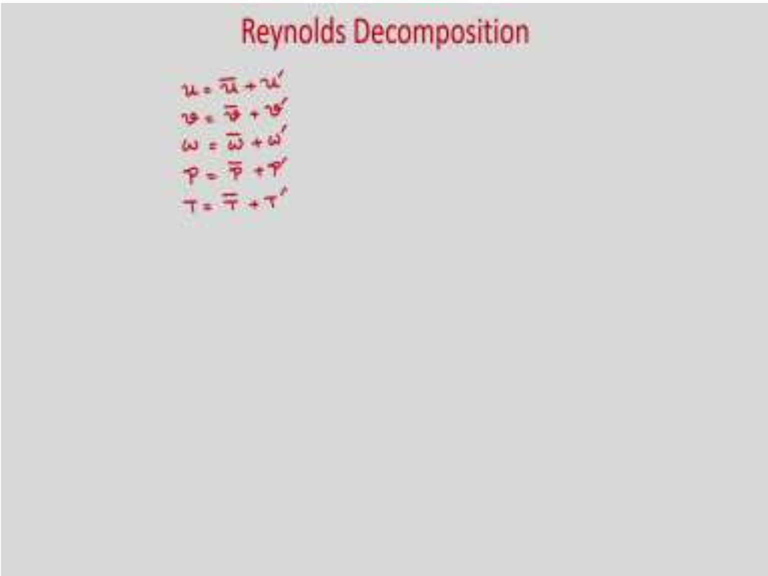

So, we can use $u=\overline{u}+u'$, $v=\overline{v}+v'$, $w=\overline{w}+w'$ you have pressure. So, $P=\overline{P}+P'$ and temperature $T=\overline{T}+T'$.

So, first let us consider only x momentum equation and we will do the derivation and similar way you can do for the y and z momentum equations.

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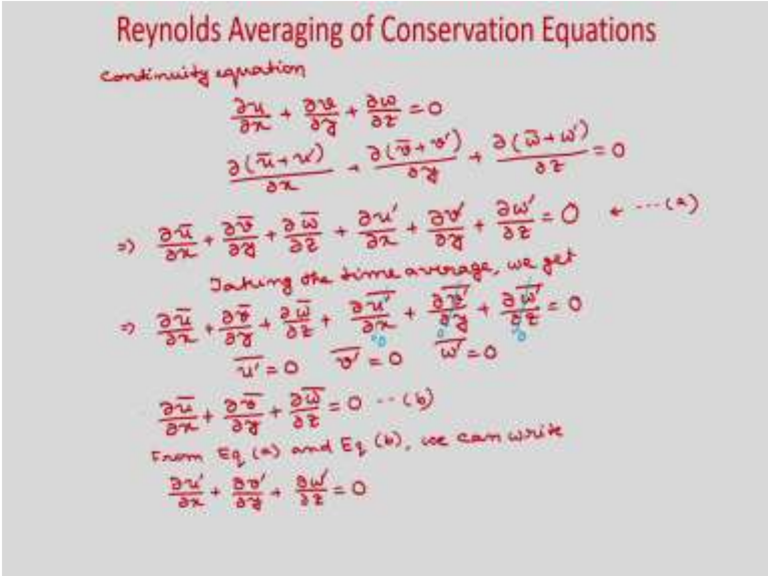

So, we have the continuity equation. So, continuity equation is $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$.

So, if you substitute you get, $\frac{\partial\left(\bar{u}+u'\right)}{\partial x}+\frac{\partial\left(\bar{v}+v'\right)}{\partial y}+\frac{\partial\left(\bar{w}+w'\right)}{\partial z}=0$. So, you can write it as $\frac{\partial \bar{u}}{\partial x}+\frac{\partial \bar{v}}{\partial y}+\frac{\partial \bar{w}}{\partial z}+\frac{\partial u'}{\partial x}+\frac{\partial v'}{\partial y}+\frac{\partial w'}{\partial z}=0$ now you take the time average.

So, taking the time average we get. So, you can see if you take the time average $\frac{\partial \bar{u}}{\partial x}+\frac{\partial \bar{v}}{\partial y}+\frac{\partial \bar{w}}{\partial z}+\frac{\partial \overline{u'}}{\partial x}+\frac{\partial \overline{v'}}{\partial y}+\frac{\partial \overline{w'}}{\partial z}=0$ and if you take the time average, then you can write, $\frac{\partial \overline{u'}}{\partial x}+\frac{\partial \overline{v'}}{\partial y}+\frac{\partial \overline{w'}}{\partial z}=0$.

We have already shown that the time average of the fluctuating components are 0. So; that means, $\overline{u'}=0$, $\overline{v'}=0$, $\overline{w'}=0$ so; that means, these quantities will be 0. So, this will be 0, this will be 0 and this will be also 0. So, you can have $\frac{\partial \bar{u}}{\partial x}+\frac{\partial \bar{v}}{\partial y}+\frac{\partial \bar{w}}{\partial z}=0$.

So, you can see the time average velocities satisfy the continuity equation. So, this is the continuity equation you need to solve when you are using Reynolds average Navier stokes equations.

So, here you can see if you invoke these in this equation what you will get. So, let us say this is your (a) and this is your (b). So, from equation (a) and equation (b) we can write. So, if you put this equal to 0.

So, first three terms will become 0. So, you will get $\frac{\partial u'}{\partial x}+\frac{\partial v'}{\partial y}+\frac{\partial w'}{\partial z}=0$ what does it mean? It means that the fluctuating component satisfy the continuity equation now let us consider x momentum equation.

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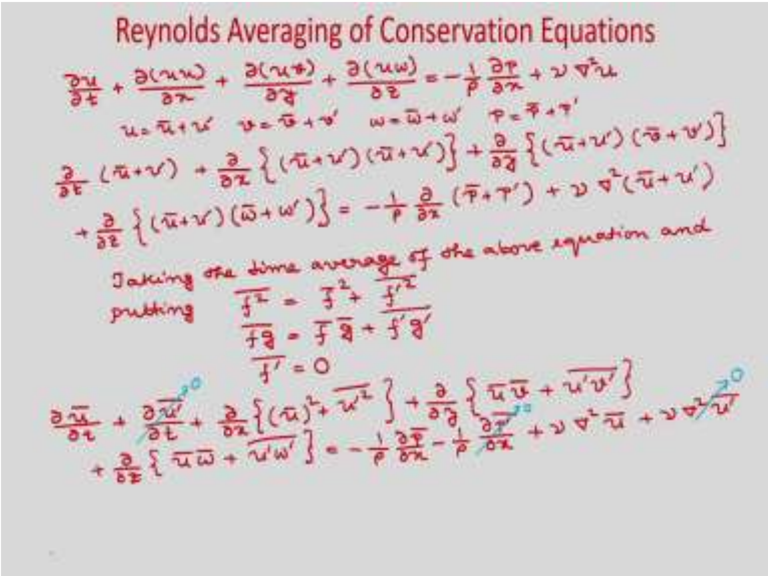

So, let us write the x component momentum equation, $\frac{\partial u}{\partial t}+\frac{\partial(uu)}{\partial x}+\frac{\partial(uv)}{\partial y}+\frac{\partial(uw)}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\nabla^2 u$.

So, now you invoke $u=\bar{u}+u'$, $v=\bar{v}+v'$, $w=\bar{w}+w'$, $P=\bar{P}+P'$. So, if you put it here. So, you will get,

$$\frac{\partial}{\partial t}\left(\bar{u}+u'\right)+\frac{\partial}{\partial x}\left\{\left(\bar{u}+u'\right)\left(\bar{u}+u'\right)\right\}+\frac{\partial}{\partial y}\left\{\left(\bar{u}+u'\right)\left(\bar{v}+v'\right)\right\}+\frac{\partial}{\partial z}\left\{\left(\bar{u}+u'\right)\left(\bar{w}+w'\right)\right\}=-\frac{1}{\rho}\frac{\partial}{\partial x}\left(\bar{P}+P'\right)+\nu\nabla^2\left(\bar{u}+u'\right)$$

So, now, you take the time average of this equation and invoke some properties which we have already derived. So, taking the time average of the above equation and putting, $\overline{f^2}=\bar{f}^2+\overline{f'^2}$.

And also you write $\overline{fg} = \bar{f}\,\bar{g} + \overline{f'g'}$ and $\overline{f'} = 0$. So, we will use these properties and we will simplify the above equation. So, if you put it. So, you will get this as,

$$\frac{\partial \bar{u}}{\partial t} + \frac{\partial \overline{u'}}{\partial t} + \frac{\partial}{\partial x}\left\{(\bar{u})^2 + \overline{u'^2}\right\} + \frac{\partial}{\partial y}\left\{\bar{u}\bar{v} + \overline{u'v'}\right\} + \frac{\partial}{\partial z}\left\{\bar{u}\bar{w} + \overline{u'w'}\right\} = -\frac{1}{\rho}\frac{\partial \bar{P}}{\partial x} - \frac{1}{\rho}\frac{\partial \overline{P'}}{\partial x} + \nu\nabla^2\bar{u} + \nu\nabla^2\overline{u'}$$

So, here you can see this will be 0 because time average of the fluctuating components will be 0, this will be 0 and this will be 0.

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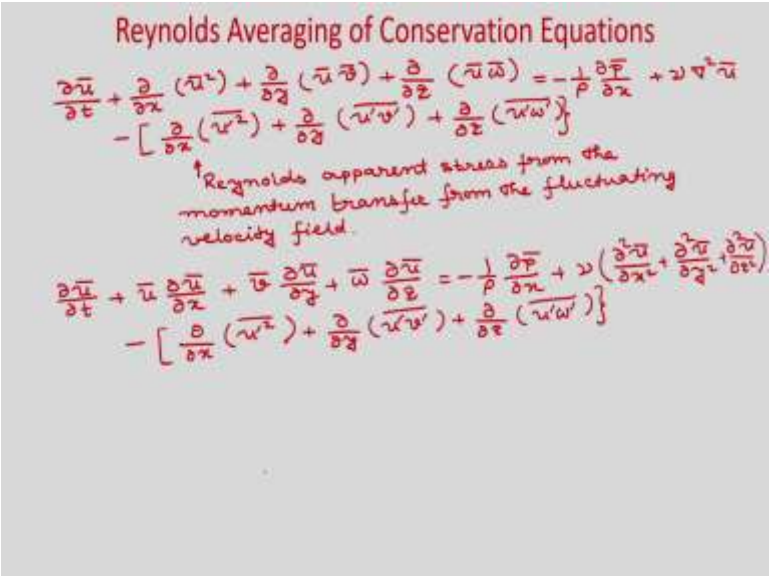

So, now you can write this equation as,

$$\frac{\partial \bar{u}}{\partial t} + \frac{\partial}{\partial x}(\bar{u})^2 + \frac{\partial}{\partial y}(\bar{u}\bar{v}) + \frac{\partial}{\partial z}(\bar{u}\bar{w}) = -\frac{1}{\rho}\frac{\partial \bar{P}}{\partial x} + \nu\nabla^2\bar{u} - \left[\frac{\partial}{\partial x}\left(\overline{u'^2}\right) + \frac{\partial}{\partial y}\left(\overline{u'v'}\right) + \frac{\partial}{\partial z}\left(\overline{u'w'}\right)\right].$$

So, you can see these are the new unknowns. So, these three terms are not zero because we have already shown that your $\overline{f'g'} \neq 0$.

So, as these are not zero. So, these three unknowns are appearing during the Reynolds decomposition due to the fluctuating components of the velocities. So, you can see these are known as Reynolds apparent stress later we will discuss in detail Reynolds apparent stress from the momentum transfer from the fluctuating components.

So, now these we can write in nonconservative form. So, you can write,

$$\frac{\partial \overline{u}}{\partial t}+\overline{u}\frac{\partial \overline{u}}{\partial x}+\overline{v}\frac{\partial \overline{u}}{\partial y}+\overline{w}\frac{\partial \overline{u}}{\partial z}=-\frac{1}{\rho}\frac{\partial \overline{P}}{\partial x}+\nu\left(\frac{\partial^2 \overline{u}}{\partial x^2}+\frac{\partial^2 \overline{u}}{\partial y^2}+\frac{\partial^2 \overline{u}}{\partial z^2}\right)-\left[\frac{\partial}{\partial x}\left(\overline{u'^2}\right)+\frac{\partial}{\partial y}\left(\overline{u'v'}\right)+\frac{\partial}{\partial z}\left(\overline{u'w'}\right)\right]$$

Now, let us consider the energy equation and we will use the Reynolds decomposition in similar way whatever we have done for the x momentum equation.

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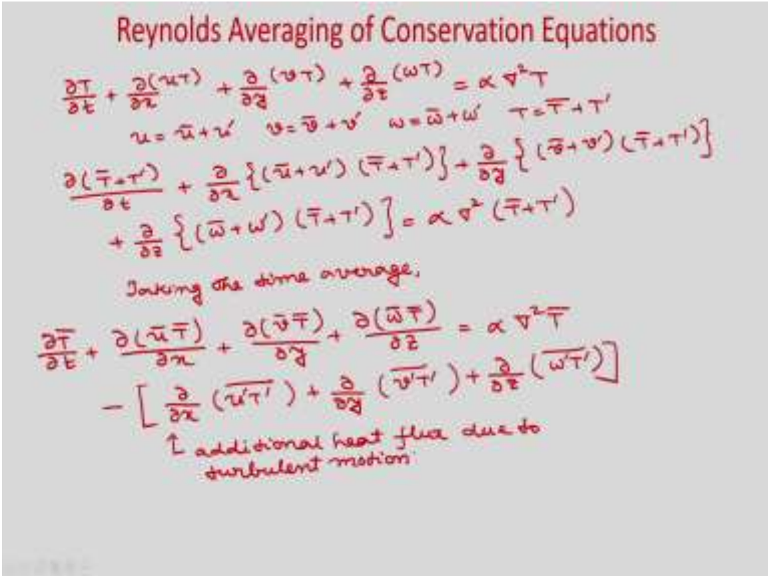

So, your energy equation is $\frac{\partial T}{\partial t}+\frac{\partial}{\partial x}(uT)+\frac{\partial}{\partial y}(vT)+\frac{\partial}{\partial z}(wT)=\alpha\nabla^2 T$. So, now, you use $u=\overline{u}+u'$, $v=\overline{v}+v'$, $w=\overline{w}+w'$ and $T=\overline{T}+T'$.

So, if you invoke and do the similar way if you take the time average of these quantities. So, you can write,

$$\frac{\partial\left(\overline{T}+T'\right)}{\partial t}+\frac{\partial}{\partial x}\left\{\left(\overline{u}+u'\right)\left(\overline{T}+T'\right)\right\}+\frac{\partial}{\partial y}\left\{\left(\overline{v}+v'\right)\left(\overline{T}+T'\right)\right\}+\frac{\partial}{\partial z}\left\{\left(\overline{w}+w'\right)\left(\overline{T}+T'\right)\right\}=\alpha\nabla^2\left(\overline{T}+T'\right)$$

So, taking the time average and using the properties you can write, $\frac{\partial \overline{T}}{\partial t}+\frac{\partial\left(\overline{u}\overline{T}\right)}{\partial x}+\frac{\partial\left(\overline{u}\overline{T}\right)}{\partial y}+\frac{\partial\left(\overline{u}\overline{T}\right)}{\partial z}=-\frac{1}{\rho}\frac{\partial \overline{P}}{\partial x}+\nu\nabla^2\overline{T}-\left[\frac{\partial}{\partial x}\left(\overline{u'T'}\right)+\frac{\partial}{\partial y}\left(\overline{v'T'}\right)+\frac{\partial}{\partial z}\left(\overline{w'T'}\right)\right]$.

So, these are again unknown terms. So, you have additional heat flux due to turbulent motion.

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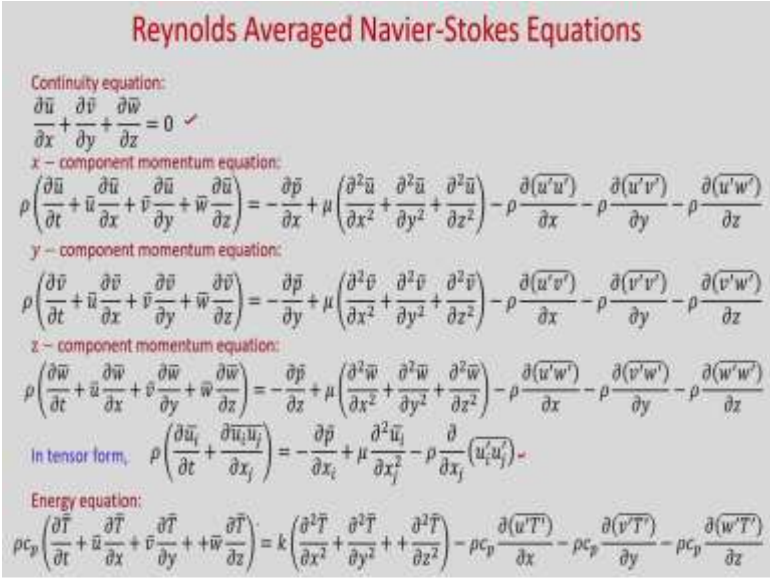

So, now if you carry out the time averaging of y momentum and z momentum equation similar way then we can write the Reynolds average Navier stokes equation as. So, this is your continuity equation this is the x component momentum equation and these are the three additional terms in y component momentum equation these are the three additional terms, in z component momentum equation these are the three additional terms.

So, now, these three equations you can write in tensor form as $\rho\left(\frac{\partial \overline{u_i}}{\partial t}+\frac{\partial \overline{u_i u_j}}{\partial x_j}\right)=-\frac{\partial \overline{p}}{\partial x_i}+\mu\frac{\partial^2 \overline{u_i}}{\partial x_j^2}-\rho\frac{\partial}{\partial x_j}\left(\overline{u_i^{'}u_j^{'}}\right)$. So, you can see this is the additional term and energy equation also you can see these are the additional terms.

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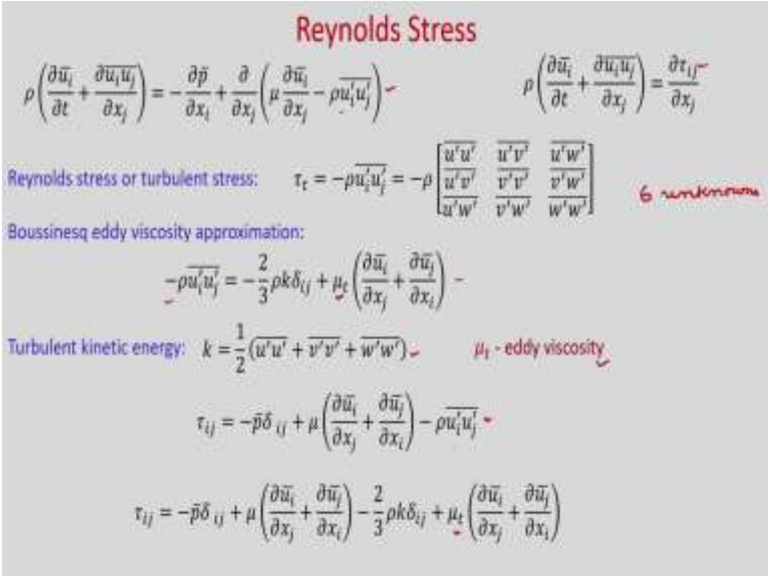

So, if you see you can rewrite the momentum equation in tensor form as these where $\frac{\partial}{\partial x_j}$ we have taken. So, $\mu \frac{\partial \overline{u_i}}{\partial x_j} - \rho\left(\overline{u_i^{'} u_j^{'}}\right)$.

So, these right hand side quantity if we put as the total stress $\frac{\partial \tau_{ij}}{\partial x_j}$, then this stress term this is known as Reynolds stress or turbulent stress. So, we can write $\tau_t = -\rho\left(\overline{u_i^{'} u_j^{'}}\right) = -\rho \begin{pmatrix} \overline{u'u'} & \overline{u'v'} & \overline{u'w'} \\ \overline{u'v'} & \overline{v'u'} & \overline{v'w'} \\ \overline{u'w'} & \overline{v'w'} & \overline{w'w'} \end{pmatrix}$. So, this is the Reynolds stress tensor and here you can see how many unknowns are there because it is a symmetric tensor.

So, there are six unknowns there are six unknowns. So, 1, 2, 3, 4, 5 then 6. So, these six unknowns now we have to model to find these unknowns. So, we can use Boussinesq eddy viscosity approximation where this Reynolds stress is written in this way $-\rho \overline{u_i^{'} u_j^{'}} = -\frac{2}{3}\rho k \delta_{ij} + \mu_t \left(\frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i} \right)$ where k is the turbulent kinetic energy and δ_{ij} is the Kronecker delta.

Kronecker delta you know that if i = j then its value is 1 otherwise 0 +μ_t which is your turbulent viscosity or eddy viscosity it is known as eddy viscosity it is known as eddy viscosity and in terms of your the gradient of the mean velocity. So, you can write these Reynolds stress in terms of the gradient of the mean velocity with a unknown parameter eddy viscosity.

So, you can see now this μ_t is unknown. So, that you need to model. So, here turbulent kinetic energy is given by this expression and if you see the total stress, then total stress will be your laminar stress plus the turbulent stress we already know from the constitutive relation and this is the additional stress. So, that is your turbulent stress.

So, this expression if you put it here and rearrange, then you will get in terms of the time average velocity gradient and mu t is unknown.

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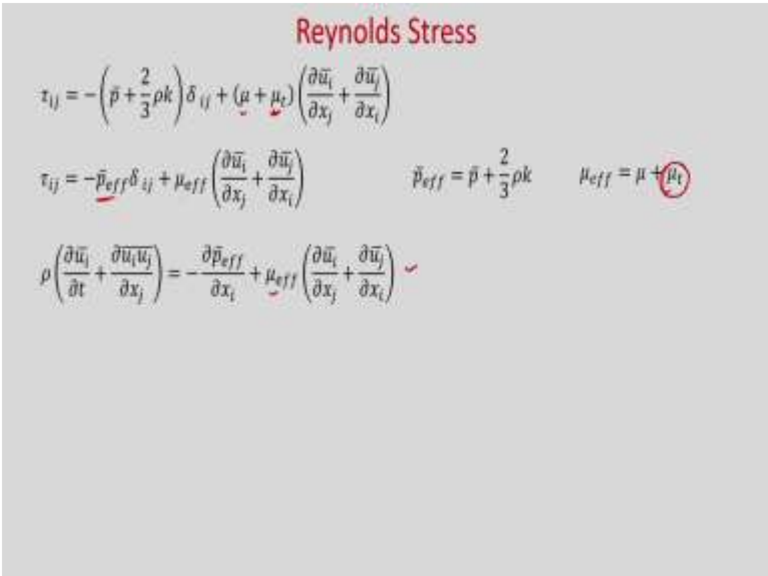

So, if you rearrange it. So, you can see $\tau_{ij} = -\left(\overline{p} + \frac{2}{3}\rho k\right)\delta_{ij} + \left(\mu + \mu_t\right)\left(\frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i}\right)$.

So, you can see here only unknown is μ_t. So, we have expressed the Reynolds stress in terms of the gradient of a time average velocity. So, this $\overline{p}_{eff} = \overline{p} + \frac{2}{3}\rho k$ you can write as and μ_{eff} which is your dynamic viscosity molecular viscosity and this is your turbulent viscosity.

So, these two together you can write as μ_{eff}. So, if you invoke in the tensor equation. So, you will get in this form where $\mu_{eff} = \mu + \mu_t$ and this μ_t is unknown. So, this μ_t is to be modelled in this regard we will discuss about the turbulence intensity.

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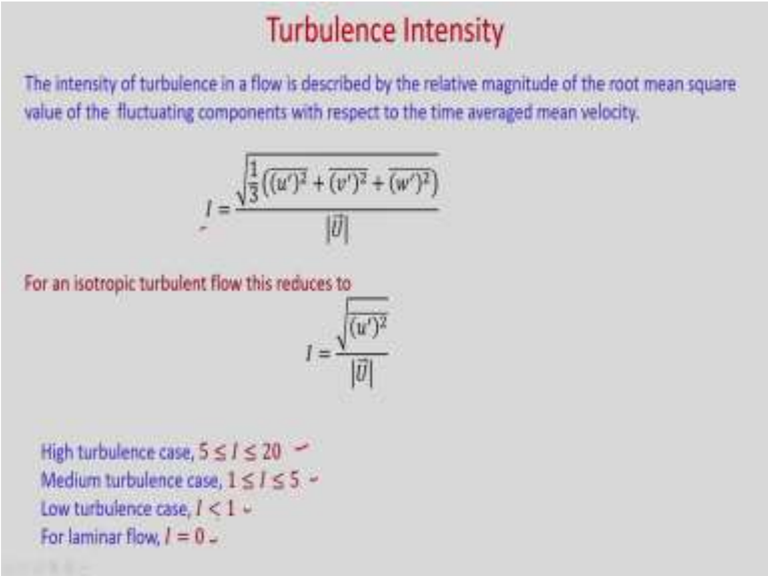

So, it is a some measure about the turbulence and you can see it is defined as the intensity of turbulence in a flow is described by the relative magnitude of the root mean square value of the fluctuating components with respect to the time average mean velocity. So, if I is the turbulent intensity, then it is root mean square value of this fluctuating component.

So, $I = \frac{\sqrt{\frac{1}{3}\left(\overline{(u')^2} + \overline{(v')^2} + \overline{(w')^2}\right)}}{|\vec{U}|}$. So, if you have a flow over flat plate then it will become u_∞ and if you consider isotropic turbulent flow. So, it is not having any directional preference. So, $\overline{u'} = \overline{v'} = \overline{w'}$.

So, you will get this intensity $I = \frac{\sqrt{\overline{(u')^2}}}{|\vec{U}|}$. So, you can see if this intensity is between 5 and 20, then you can say that it is a high turbulent case if it is in between 1 and 5, then it is medium turbulence case and if it is > 1 then it is low turbulence case and; obviously, you can see for laminar flow these fluctuating components will be 0 then your turbulent intensity will be 0.

So, when you numerically solve this Reynolds average Navier stokes equations then you need to give the turbulence intensity at the inlet so, that you can impose some turbulence at the entry.

So, in today's class we started with the characteristic of the turbulent flows then we have discussed about two main characteristic of the turbulent flows one is a homogeneous turbulence, another is isotropic turbulence, then we discuss about the Reynolds decomposition and we started with the continuity equation and using the Reynolds decomposition we have shown that mean velocity satisfy the continuity equation as well as the fluctuating components also satisfy the continuity equation.

When we used the Reynolds decomposition for x momentum equation and taking the time average of this equation there are three additional unknowns appear.

So, these three unknowns are coming from the fluctuating components of the velocity and if you have these three momentum equations u, v and w momentum equations then obviously, you will get nine additional terms out of which six are unknowns and you can write in as a Reynolds stress or the apparent stress.

We have also cut out the time averaging of the energy equation and using Reynolds decomposition we have written the time average energy equation there also we have seen there are additional three unknowns.

Later we use the constitutive relation and the Boussinesq approximation for the Reynolds stress and we have written the Reynolds average Navier stokes equation in tensor form in terms of the eddy viscosity. So, you can see that here it resembles with the laminar flow Navier stokes equations except one additional term in the viscous term.

So, that is your Reynold stress and from the Reynold stress using this Boussinesq approximation and the constitutive relation we have shown that you will get the effective viscosity as the as a summation of molecular viscosity and the turbulent or eddy viscosity and here you can see only one unknown term is there that is your eddy viscosity μ_t. So, in later classes we will try to find these unknown eddy viscosity.

Thank you.