Fundamentals of Convective Heat Transfer
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Module – 10
Numerical Solution of Navier-Stokes and Energy Equations
Lecture – 34
Solution of Navier-Stokes equations

Hello everyone, today we will consider unsteady three dimensional Navier-Stoke equations for laminar incompressible flows. And we will discretize this equation using finite difference method. In last class you have learnt how to discretize the first derivative and second derivative of any variable, we will use a famous technique Marker and Cell proposed by Harlow and Welch to discretize this Navier-Stoke equations using finite difference method.

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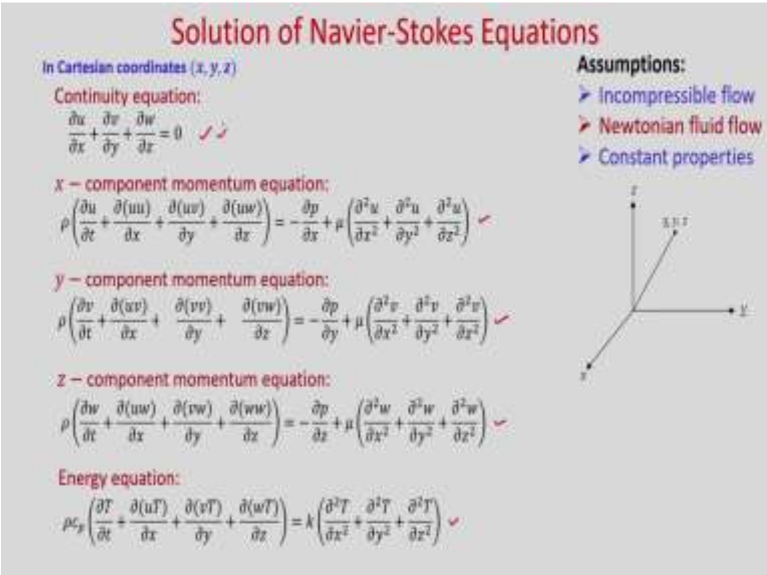

So, let us consider these equations in Cartesian coordinate. So, for incompressible Newtonian fluid flow with constant properties, this is the continuity equation. This is the x component of momentum equation, and this is y component of momentum equation, and this is the z component of momentum equation, and this is the energy equation. In today's class we will just discretize the Navier-Stoke equations and you can see how many variables are there? u, v, w, p,. And if you solve for energy then will be temperature.

So; obviously, you can see we have u, v, w for the Navier-Stoke equations and continuity equation. So, 4 equations and 4 unknowns u, v, w, and p. So, to find the pressure, there are no obvious equation we need to derive equation per to determine the pressure from the continuity equation.

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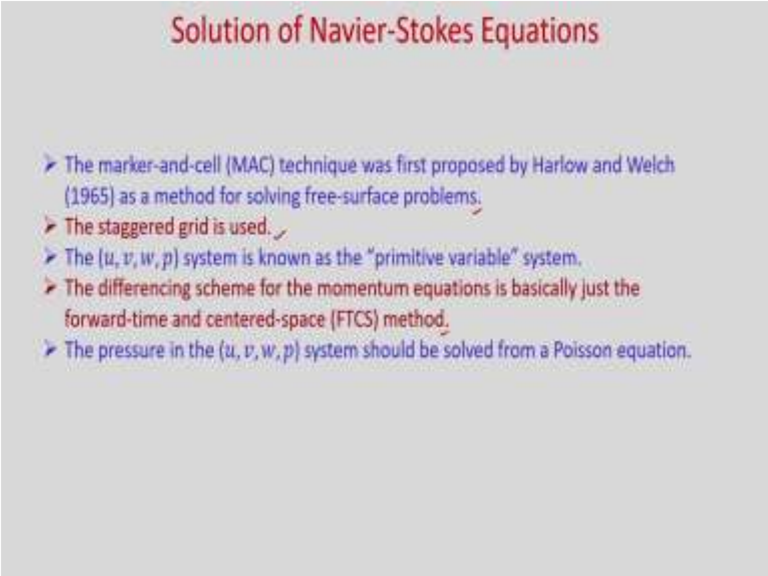

We will use the marker and cell technique first proposed by Harlow and Welch, for solving free-surface problems. And as we discussed in last class we will use staggered grid; you know that in staggered grid pressured and temperature will solve at cell center and velocities will solve in a staggered way, what is the advantage of using staggered grid? Because we will get a strong coupling between pressure and velocity.

The differencing scheme for the momentum equations is basically just the forward time and centered space FTCS method. And if you solve the system using u, v, w, p and temperature, then these system known as primitive variable approach and here we will solve equation for pressure separately.

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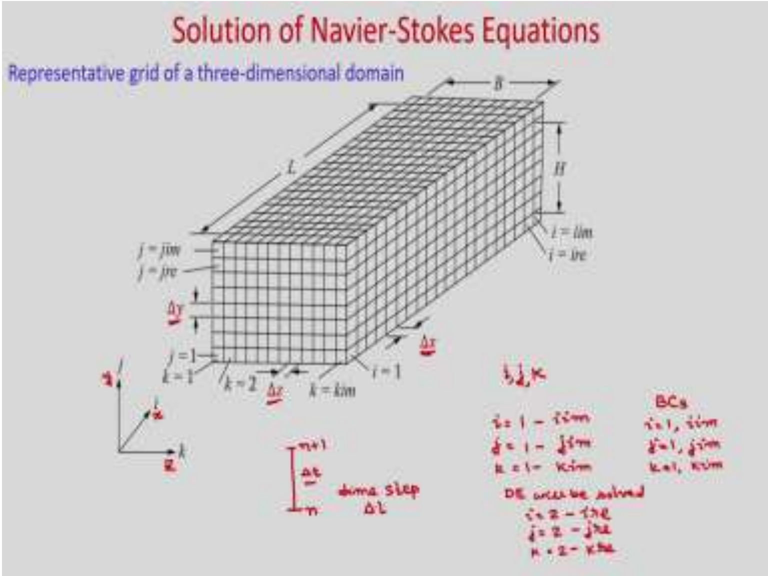

So, you know that first we need to discretize the domain into grids so, that we can solve the discretized equation in a discrete point. So, if you consider this three-dimensional domain. And you can see this is divided into grid x direction is in these direction, this is the y direction, and this is the z direction and the grid we are giving the index with i, j, and k. So, you can see this is your j = 1 and it is varying in this direction and it is j = j i m and previous cell is j r e.

Similarly, k = 1, this is k = 2 the last cell is k i m and here we will apply the boundary condition at the last cell and previous cell is your k r e. Similarly, in x direction this is the i = 1, i = 2 and this is the i = i r e and this is the i i m.

So, you can see your grid is varying i_1 to i i m, j_1 to j i m and k_1 to k i m. And boundary conditions will be applied at i = 1 and i i m and j = 1 and j i m, k = 1 and k i m. And the discretized equation we will solve in the interior domain; discretized equation will be solved and i = 2 to i r e, j = 2 to j r e, and k = 2 to k r e.

And you can see that in x direction we have uniform spacing of Δx, and in y direction we have uniform spacing with Δy, and in z direction we have uniform spacing Δz. So, grid size you can see this is Δx, Δ y and Δz. So, this is the discretization of this domain.

Now, we are considering unsteady equation. So, we need to march in time so, we will march from n to n + 1, where n is the previous time, and n + 1 is the current time or present time.

So, the variables at previous time n are already known from the previous solution and n+1 we need to determine. So, in MAC algorithm we use explicit method; that means, there will be only one unknown and all other terms will be known from the previous time n. When we will go in time marching so, we will go from n to n +1 and your time step will be Δt. So, here grid size is Δx, Δy, Δz and your time step is Δt.

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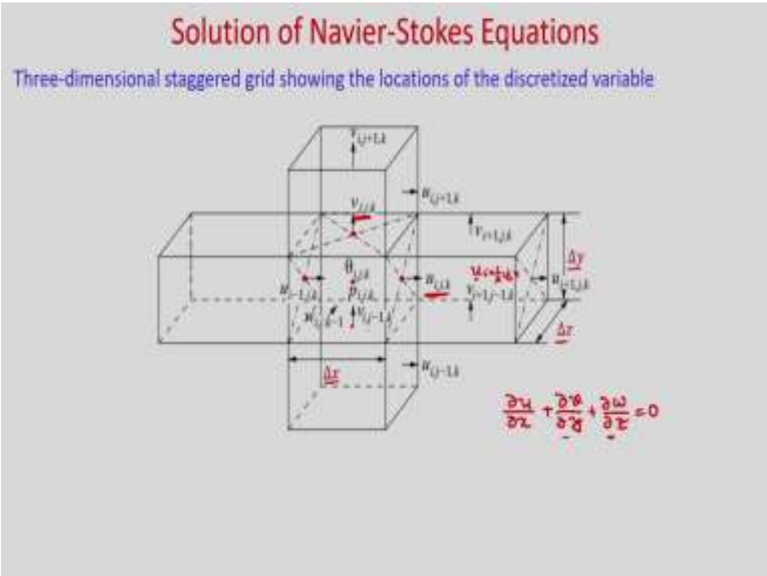

So, now let us see the cell, where we need to discretize the continuity equation as well as the energy equation. And the staggered grid where we need to solve the velocities. So, we can see this is the main cell, where at center you will solve for P and this cell is i j k index is i j k, and we need to solve for temperature, so, $t_{i,j,k}$ or $\theta_{i,j,k}$, where θ is non-dimensional temperature.

And velocities now you see we will solve in staggered way so, this is the point where we need to solve for the velocity $u_{i,j,k}$, and this is the staggered grid, where we need to solve for velocity $v_{i,j,k}$, and you can see in other direction we need to solve for $w_{i,j,k}$.

So, in that direction in inside you will solve for $w_{i,j,k}$ and this is the grid for $w_{i,j,k-1}$. And now if it is $u_{i,j,k}$ so; obviously, this will be your $u_{i-1,j,k}$ and if it is $v_{i,j,k}$ so, here you will get $v_{i,j-1,k}$. And in the other face the rear side you will solve for $w_{i,j,k}$ and this is the $w_{i,j,k-1}$.

So, we can see when we are solving for $u_{i,j,k}$. So, its neighbors are $u_{i-1,j,k}$ and $u_{i+1,j,k}$ and for $v_{i,j,k}$ this is the $v_{i,j+1,k}$ and this is $v_{i,j-1,k}$. And you can see now in this main cell we will satisfy the continuity equation. And when we satisfy the continuity equation so, velocities just difference will be just $\frac{\partial u}{\partial x}$ in x direction.

So, you can see your continuity equation is $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$. So, this equation we need to satisfy in this cell, where now $\frac{\partial u}{\partial x}$ you can see that it will be $\frac{u_{i,j,k}-u_{i-1,j,k}}{\Delta x}$, because that is the first derivative discretization.

Similar way you see this is $\frac{v_{i,j,k} - v_{i,j-1,k}}{\Delta y}$ so, you will get $\frac{\partial v}{\partial y}$. Similarly, you will can find $\frac{\partial w}{\partial z}$ and; obviously, you can see this is the grid size Δx this is your Δy and this is the Δz.

Here you notice that when you have $u_{i,j,k}$ and this is your $u_{i+1,j,k}$ and at the center. If you need to find the $u_{i,j,k}$ so, that will be $u_{i+\frac{1}{2},j,k}$ and this is unknown. So, this unknown will be just solved using the average velocity, using the neighbor points $u_{i,j,k}$ and $u_{i+1,j,k}$ and similarly for v and w.

So, we have now discussed about the staggered grid and how the variables are stored in the grid. Now, let us consider the x momentum equation and discretize term by term using finite difference method.

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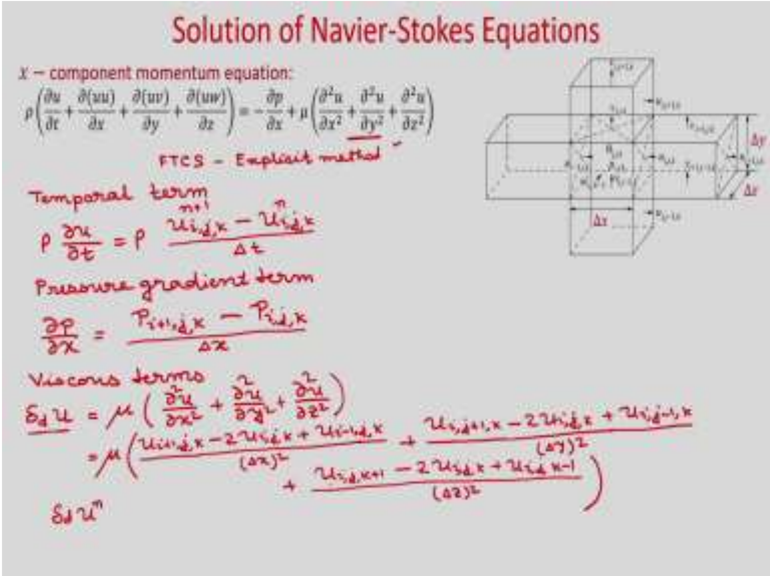

So, this is the x component momentum equation. So, first let us discretize temporal term so, we will use forward time central space and you know that it is explicit method. So, first let us discretize temporal term. So, what is temporal term? This is $\rho\frac{\partial u}{\partial t}$. So, now we will use forward time so, this is first order accurate scheme if we use then you will can write $u_{i,j,k}^{n+1}-u_{i,j,k}^{n}$. So, we are discretizing with respect to time. So, that is why we are writing $\rho\frac{\partial u}{\partial t}=\rho\frac{u_{i,j,k}^{n+1}-u_{i,j,k}^{n}}{\Delta t}$.

Next, let us discretize the pressure term so, pressure term if you discretize or pressure gradient term. So, that is $\frac{\partial p}{\partial x}=\frac{P_{i+1,j,k}-P_{i,j,k}}{\Delta x}$. Now, let us consider the viscous term in the right hand side. So, if you consider that term viscous terms so, this is the viscous term.

So, we can write this will denote as $\delta_d u$ so, when you will write the discretize equation we will use these notation. So, this is $\delta_d u=\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$. So, you can see this is the second derivative of u, with respect to x, y and z.

So, you know the central difference scheme. So, this is the second order accurate so, we can use the second order central difference approximation for this second derivative. So, you can write,

$$\delta_d u = \mu\left(\frac{u_{i+1,j,k} - 2u_{i,j,k} + u_{i-1,j,k}}{(\Delta x)^2} + \frac{u_{i,j+1,k} - 2u_{i,j,k} + u_{i,j-1,k}}{(\Delta y)^2} + \frac{u_{i,j,k+1} - 2u_{i,j,k} + u_{i,j,k-1}}{(\Delta z)^2}\right).$$

So, this is the discretized from of this equation, and when we will use the explicit scheme, we will write $\delta_d u^n$; that means, all these terms velocities will be from the nth time level. So, those will be known right.

Now, let us consider the convection terms. So, in the conversation terms you can see you have $\frac{\partial(uu)}{\partial x}$, similarly you have $\frac{\partial(uv)}{\partial y}$ and $\frac{\partial(uw)}{\partial z}$. So, here 2 velocities are coming. So, one velocity is convected velocity so, by the velocities u,v,w here u is convected right, because this is the x momentum equation so, your we are solving for the velocity u, but velocity u is convected by velocity u in x direction by velocity v in y direction and by velocity w in the z direction.

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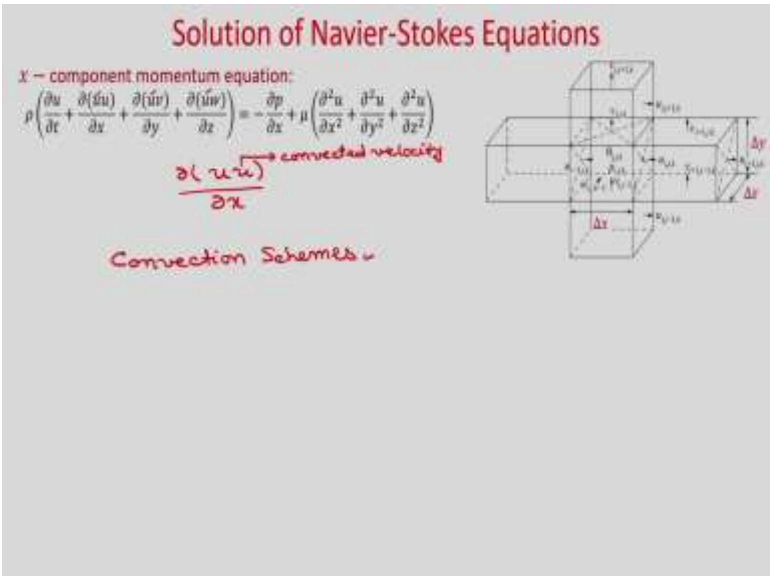

So, you can see this term, if you see $\frac{\partial(uu)}{\partial x}$ so, one is your convected velocity. So, here all these u are convected velocities and another u is just velocities, which actually convecting this variable convected velocity. So, if you use central difference then how you can use for this cell.

So, this is your $u_{i,j,k}$. So, this we will learn how to discretize this equation so, here we will use some convection schemes, because you have 2 velocities. So, one velocity we will calculate just at this phase centered, where it will be just average of the neighboring cells, but other velocity we will calculate using some convective schemes.

So, there are different convective schemes are available like first order accurate; first order upwind, second order upwind, third order upwind like quick even you have central difference, which is your second order accurate and you can have the combination of these within with some weighted function. So, here we will use convective scheme proposed by Hirt et. al.

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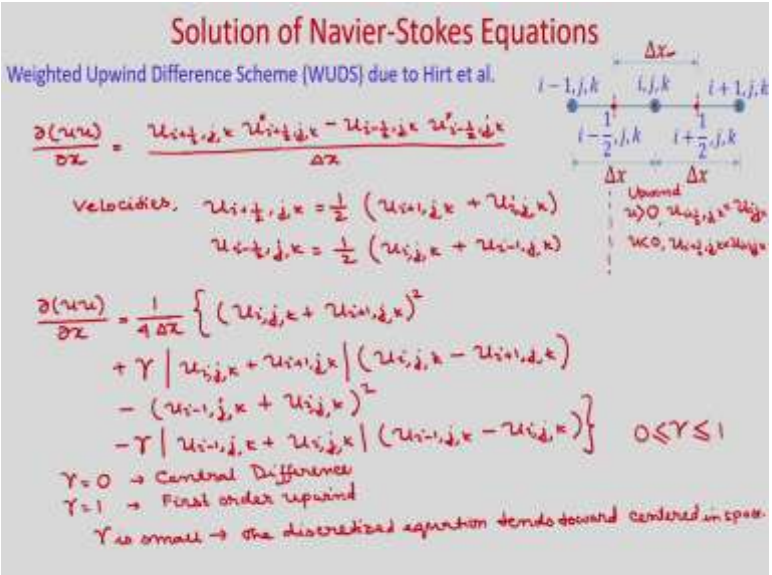

So, this is the scheme we will use weighted upwind difference scheme, which is known as WUDS due to Hirt et. al. So, you can see we have $\frac{\partial(uu)}{\partial x}$. So, what we will see so, this is to be discretized for the velocity here i,j,k. So, we will use $\frac{\partial(uu)}{\partial x}=\frac{u_{i+\frac{1}{2},j,k}u^{*}_{i+\frac{1}{2},j,k}-u_{i-\frac{1}{2},j,k}u^{*}_{i-\frac{1}{2},j,k}}{\Delta x}$.

So, here you see here at this point we are discretizing this u u term and we are discretizing so, you can see this is the midpoint between i, j, k and i +1, j k. So, this point we are considering and this point, which is your $i-\frac{1}{2}, j, k$. So, you can see so, the distance between these two points is Δx , but here we are velocity the velocities we are writing $u_{i+1,j,k}$ and another is stared quantity u*.

So, for the u * quantity we will use some convection scheme and for $u_{i+1,j,k}$ we will just take the average of velocities at i, j, k and i + 1 j, k. So, you can see in this case. So, these velocities $u_{i+\frac{1}{2},j,k}$ just we will use the average value, at this point taking the value from i, j, k and i +1 j, k.

So, you can write $u_{i+\frac{1}{2},j,k} = \frac{1}{2}\left(u_{i+1,j,k} + u_{i,j,k}\right)$. Similarly, $u_{i-\frac{1}{2},j,k} = \frac{1}{2}\left(u_{i,j,k} + u_{i-1,j,k}\right)$. So, you can see it is just average value at this point we have considered.

Now, this convected variable u*, we will use some convection scheme and we are going to use this weighted upwind difference scheme. So, you can write this equation like

$$\frac{\partial (uu)}{\partial x} = \frac{1}{4\Delta x}\left\{ \begin{matrix} \left(u_{i,j,k} + u_{i+1,j,k}\right)^2 + \gamma \left|u_{i,j,k} + u_{i+1,j,k}\right|\left(u_{i,j,k} - u_{i+1,j,k}\right) - \left(u_{i-1,j,k} + u_{i,j,k}\right)^2 \\ -\gamma \left|u_{i-1,j,k} + u_{i,j,k}\right|\left(u_{i-1,j,k} - u_{i,j,k}\right) \end{matrix} \right\}.$$

So, you see this factor gamma it varies between 0 and 1. So, if you put γ=0 what you are going to get. So, if you put γ=0 so, this term and this term we will get 0. So, you are going to get actually central difference, because if you use central difference you will get this value for this convected variable.

So, if you use that one then it will be 2 and another 2 is there from this value. So, it will be $\frac{1}{4\Delta x}$. So, this square minus this square so, these values. So, for γ=0 . so, 0≤γ≤1.

So, if γ=0 you will get central difference method, this is your convection scheme. And if you put γ= 1, so, γ=1 you will get upwind scheme, first order upwind you will get what is upwind scheme let us discuss.

So, if you see this value at $i+\frac{1}{2}, j, k$ you want to find, let us say u . So, now if at this point if velocity is greater than 0 u > 0, then you consider the value of $u_{i+\frac{1}{2},j,k}$ as $u_{i,j,k}$ and if u < 0 then you consider $u_{i+\frac{1}{2},j,k}$; that means, if it is less than 0.

So, velocity is coming from right to left. So, you take the value of u at this point as same as this point $u_{i+1,j,k}$; so, it will be $u_{i+1,j,k}$. So, this is known as upwind. So, we are taking the upwind point from the direction in which the velocities are coming.

So, you can see u > 0 so, $u_{i+\frac{1}{2},j,k}$ you are taking $u_{i,j,k}$ value and if it is u < 0 so, you are considering this value. So, similarly here if u > 0 these value you consider as these value $u_{i-1,j,k}$. And if it u < 0, then you consider u at this point as $u_{i,j,k}$.

So, this is known as upwind and if you put γ=1 you can see that you will get first order upwind and if γ is small then; obviously, you can see it will be towards central difference. So, the discretized equation tends to centered in space. Now, similarly you discretize the other convection terms, $\frac{\partial(uv)}{\partial y}$ and $\frac{\partial(uw)}{\partial z}$.

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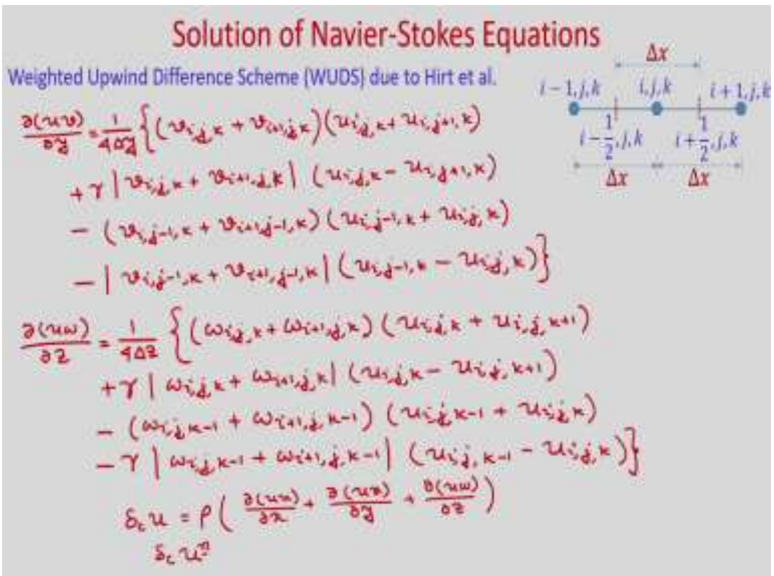

So, similarly you can write $\frac{\partial(uv)}{\partial y}=\frac{1}{4\Delta y}\begin{Bmatrix} \left(v_{i,j,k}+v_{i+1,j,k}\right)\left(u_{i,j,k}+u_{i,j+1,k}\right)+ \\ \gamma\left|v_{i,j,k}+v_{i+1,j,k}\right|\left(u_{i,j,k}-u_{i,j+1,k}\right) \\ -\left(v_{i,j-1,k}+v_{i+1,j-1,k}\right)\left(u_{i,j-1,k}+u_{i,j,k}\right) \\ -\gamma\left|v_{i,j-1,k}+v_{i+1,j-1,k}\right|\left(u_{i,j-1,k}-u_{i,j,k}\right) \end{Bmatrix}$. And this is the modulus you know that if these value is ≥ 0 then it will return this value, otherwise if it is < 0, then it will return the negative value.

Similarly, you can write $\frac{\partial(uw)}{\partial z}=\frac{1}{4\Delta z}\begin{Bmatrix} \left(w_{i,j,k}+w_{i+1,j,k}\right)\left(u_{i,j,k}+u_{i,j,k+1}\right)+ \\ \gamma\left|w_{i,j,k}+w_{i+1,j,k}\right|\left(u_{i,j,k}-u_{i,j,k+1}\right) \\ -\left(w_{i,j,k-1}+w_{i+1,j,k-1}\right)\left(u_{i,j,k-1}+u_{i,j,k}\right) \\ -\gamma\left|w_{i,j,k-1}+w_{i+1,j,k-1}\right|\left(u_{i,j,k-1}-u_{i,j,k}\right) \end{Bmatrix}$.

So, you can see that using weighted upwind difference scheme, we have discretized the convection terms, we can write considering all the convection terms as, $\delta_c u=\rho\left(\frac{\partial(uu)}{\partial x}+\frac{\partial(uv)}{\partial y}+\frac{\partial(uw)}{\partial z}\right)$.

And all these discretized equation we have already written so, that you can write and if you write $\delta_c u^n$; that means, we will consider u from the previous time level n, and in this discretized equation you can see whatever velocities are coming you consider from the known values at previous time level.

So, now we have already discretized the temporal term, convection terms, pressure gradient term, and the viscous terms. So, if you write down for the x component of momentum equations, this discretized equation then you can write like.

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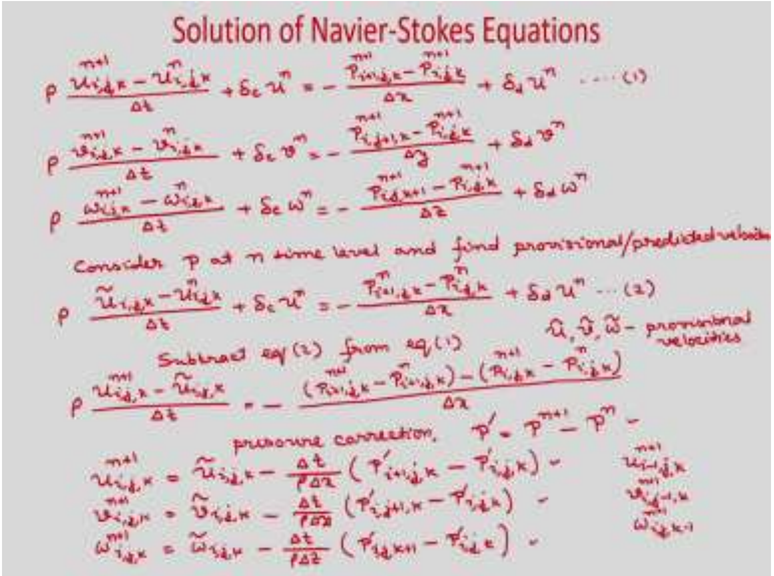

So, we are using forward time so, it is explicit method. So, this is the temporal term, now we have convection term, at time level n.

So, we have already discretized this term and we have $\rho \dfrac{u_{i,j,k}^{n+1} - u_{i,j,k}^{n}}{\Delta t} + \delta_c u^n = -\dfrac{P_{i+1,j,k}^{n+1} - P_{i,j,k}^{n+1}}{\Delta x} + \delta_d u^n$.

Similarly, you can write the discretized equation for y component and z component of momentum equation. So, we can write, $\rho \dfrac{v_{i,j,k}^{n+1} - v_{i,j,k}^{n}}{\Delta t} + \delta_c v^n = -\dfrac{P_{i,j+1,k}^{n+1} - P_{i,j,k}^{n+1}}{\Delta y} + \delta_d v^n$.

And for z component of momentum equation you can write, $\rho \dfrac{w_{i,j,k}^{n+1} - w_{i,j,k}^{n}}{\Delta t} + \delta_c w^n = -\dfrac{P_{i,j,k+1}^{n+1} - P_{i,j,k}^{n+1}}{\Delta z} + \delta_d w^n$. We need to solve these equations the pressure at current time level n +1 unknown, right.

So, you cannot solve this equation unless you know the value of pressure. So, what we will do we will assume the pressure, some value. So, with this gas pressure field; obviously, you will get the some provisional velocity. So, let us say that we are taking the pressure value from the previous time level n and whatever velocity we will get that is not the correct velocity field, but we will get some predicted or provisional velocity from the previous time level pressure values.

So, now consider p at n level n time level and find provisional or predicted velocity. So, for u momentum equation, if you write then you can write as ρ and whatever predicted velocity is there so, we will denote as, $\rho \frac{u_{i,j,k} - u_{i,j,k}^n}{\Delta t} + \delta_c u^n = -\frac{P_{i+1,j,k}^n - P_{i,j,k}^n}{\Delta x} + \delta_d u^n$.

So, let us say this is equation 1, and this is equation 2. Now, you subtract this equation 2 from equation 1. And you see that your $u, \tilde{v}, w$ if you write for other momentum equations so, these are provisional velocities .

So, if we subtract you see this, $\rho \frac{u_{i,j,k}^{n+1} - u_{i,j,k}}{\Delta t} = -\frac{\left(P_{i+1,j,k}^{n+1} - P_{i+1,j,k}^n\right) - \left(P_{i,j,k}^{n+1} - P_{i,j,k}^n\right)}{\Delta x}$.

So, finally, if you subtract equation 2 from the equation 1, you will get this expression. Now, let us tell that your corrected pressure so, the difference between the pressure at time level n plus 1 and time level n will denote as a pressure correction and this pressure correction you can denote as $P' = P^{n+1} - P^n$.

So, at i, j, k if you write then it will be just, $u_{i,j,k}^{n+1} = u_{i,j,k} - \frac{\Delta t}{\rho \Delta x}\left(P_{i+1,j,k}^{'} - P_{i,j,k}^{'}\right)$.

Now, similarly you can write this equation for velocity v and w, $v_{i,j,k}^{n+1} = \tilde{v}_{i,j,k} - \frac{\Delta t}{\rho \Delta y}\left(P_{i,j+1,k}^{'} - P_{i,j,k}^{'}\right)$. And similarly, $w_{i,j,k}^{n+1} = w_{i,j,k} - \frac{\Delta t}{\rho \Delta z}\left(P_{i,j,k+1}^{'} - P_{i,j,k}^{'}\right)$.

So, you can see that we have found the velocities at time level n +1 at point i, j, k in terms of the provisional velocity and the pressure correction term. So, you can see these $u_{i,j,k}$ you can write also in other grid point $u_{i-1,j,k}$ and $v_{i,j-1,k}$ and $w_{i,j-1,k}$.

Let us write the continuity equation and satisfy it in the main cell, and from there we will substitute these velocities and we will find the equation for pressure correction because that is unknown, right.

So, once we know the pressure correction value, then we can correct the pressure $P^{n+1} = P^n + P'$. So, we can correct the value of pressure.

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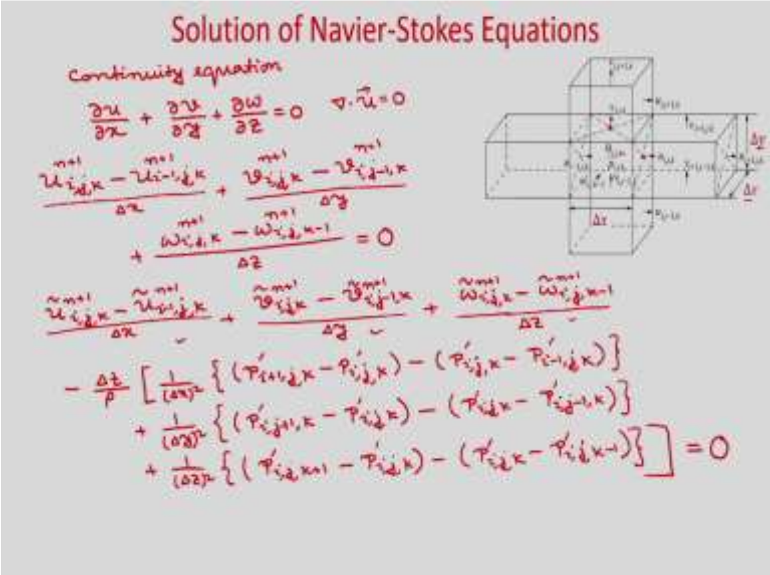

So, now you can see the continuity equation, we will satisfy in this main cell, here . We will satisfy the continuity equation in this main cell. So, when you will satisfy when you write, $\frac{\partial u}{\partial x} = \frac{u_{i,j,k}^{n+1} - u_{i-1,j,k}^{n+1}}{\Delta x}$.

Similarly, $\frac{\partial v}{\partial y} = \frac{v_{i,j,k}^{n+1} - v_{i,j-1,k}^{n+1}}{\Delta y}$. And in z direction now $\frac{\partial w}{\partial z} = \frac{w_{i,j,k}^{n+1} - w_{i,j,k-1}^{n+1}}{\Delta z}$.

So, you can see here u velocity we have found at this point; at this point v and in the rear surface here w and similarly you have $u_{i-1,j,k}$ and $u_{i+1,j,k}$ here $v_{i,j+1,k}$ and $v_{i,j-1,k}$ and similarly $w_{i,j,k-1}$. So, now, satisfy this continuity equation at this main cell. So, your continuity equation is $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial z} = 0$.

So, now, you write as, $\frac{u_{i,j,k}^{n+1} - u_{i-1,j,k}^{n+1}}{\Delta x} + \frac{v_{i,j,k}^{n+1} - v_{i,j-1,k}^{n+1}}{\Delta y} + \frac{w_{i,j,k}^{n+1} - w_{i,j,k-1}^{n+1}}{\Delta z} = 0$.

So, the value of u, v, w at n + 1, we have found from this relation you can see. So, the value of u, v, w at n + 1 you can write in terms of provisional velocity and the pressure correction. And similarly you can write for $u_{i-1,j,k}^{n+1}$, just you put here u_{i-1} and here, similarly you just change the pressure.

Similarly, $v_{i,j-1,k}^{n+1}$ you can find. And $w_{i,j,k-1}^{n+1}$. Similarly, you can write similar expression and if you; and if you substitute it here, $u_{i,j,k}$ and $u_{i-1,j,k}^{n+1}$, you are going to get.

So, in terms of provisional velocity now we are writing $\frac{\tilde{u}_{i,j,k}^{n+1}-\tilde{u}_{i-1,j,k}^{n+1}}{\Delta x}+\frac{\tilde{v}_{i,j,k}^{n+1}-\tilde{v}_{i,j-1,k}^{n+1}}{\Delta y}+\frac{\tilde{w}_{i,j,k}^{n+1}-\tilde{w}_{i,j,k-1}^{n+1}}{\Delta z}$ and you will have the pressure correction terms ok.

So, that will be $-\frac{\Delta t}{\rho}\begin{bmatrix}\frac{1}{(\Delta x)^2}\left\{\left(P'_{i+1,j,k}-P'_{i,j,k}\right)-\left(P'_{i,j,k}-P'_{i-1,j,k}\right)\right\}\\ +\frac{1}{(\Delta y)^2}\left\{\left(P'_{i,j+1,k}-P'_{i,j,k}\right)-\left(P'_{i,j,k}-P'_{i,j-1,k}\right)\right\}\\ +\frac{1}{(\Delta z)^2}\left\{\left(P'_{i,j,k+1}-P'_{i,j,k}\right)-\left(P'_{i,j,k}-P'_{i,j,k-1}\right)\right\}\end{bmatrix}$.

So, you can see the first three terms, this term this term and this term this is actually the continuity equation for the velocity for the provisional velocity, but; obviously, when you are starting the solution provisional velocity will not satisfy the continuity equation. But when the solution will converge then u^{n+1} will be your $\tilde{u}^{n+1}$, v^{n+1} will be $\tilde{v}^{n+1}$ and w^{n+1} will be $\tilde{w}^{n+1}$, because the provisional velocities will be same as the velocities at time level n +1 so, it will satisfy the continuity equation. So, you can write these three terms as a divergence form so, if you rewrite it.

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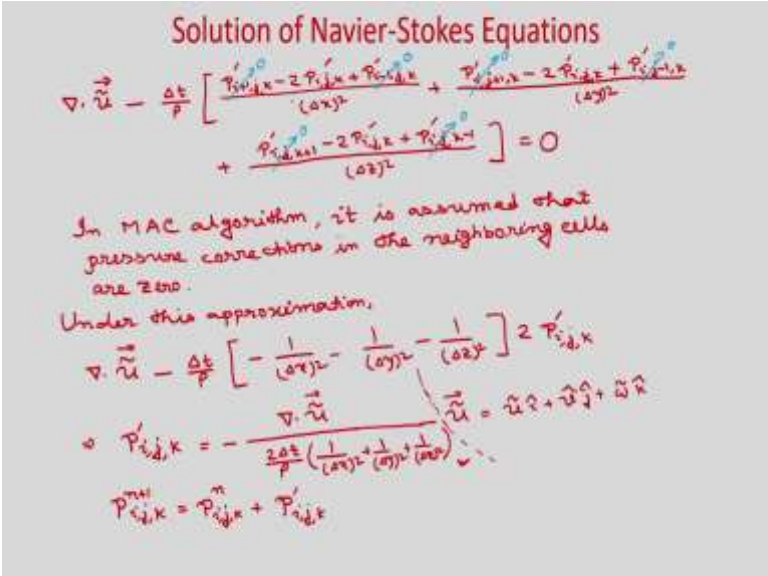

And you can write after rearranging

$$\nabla.\vec{u}-\frac{\Delta t}{\rho}\left[\frac{P'_{i+1,j,k}-2P'_{i,j,k}+P'_{i-1,j,k}}{(\Delta x)^2}+\frac{P'_{i,j+1,k}-2P'_{i,j,k}+P'_{i,j-1,k}}{\frac{1}{(\Delta y)^2}}+\frac{P'_{i,j,k+1}-2P'_{i,j,k}+P'_{i,j,k-1}}{(\Delta z)^2}\right]=0.$$

So, now in right hand side it will be 0 so, you can see here we will assume that pressure correction in the neighboring cells are 0. So, in MAC algorithm, it is assumed that the pressure correction in the neighboring cells are 0. So, under this approximation, so, you can see so, all these terms you can make it 0. So, this term is 0, this term because neighboring cell pressure correction we are just taking 0 for simple calculation and that is your MAC algorithm.

So, if you put 0 then you can write the equation for pressure. So, $\nabla.\vec{u}-\frac{\Delta t}{\rho}\left[-\frac{1}{(\Delta x)^2}-\frac{1}{(\Delta y)^2}-\frac{1}{(\Delta z)^2}\right]2P'_{i,j,k}$.

So, you can see now this is the equation for pressure. So, you will get the pressure value from this equation so, you can write $P'_{i,j,k}$. So, you can find it so, $P'_{i,j,k} = -\dfrac{\nabla.\vec{u}}{\dfrac{2\Delta t}{\rho}\left(\dfrac{1}{(\Delta x)^2}+\dfrac{1}{(\Delta y)^2}+\dfrac{1}{(\Delta z)^2}\right)}$.

So, you can see that we have already discretize the equations for the momentum and substituting those in the continuity equation in the main cell, we have found the equation for pressure using MAC algorithm; so, in the MAC algorithm we are neglecting or assuming the neighboring cell value of the pressure correction as 0 and following that you can write the equation for pressure as this.

So, once you find p prime then; obviously, you can correct it. So, you can correct $P^{n+1}_{i,j,k} = P^{n}_{i,j,k} + P'_{i,j,k}$. So, this if you solve then; obviously, you will be able to find.

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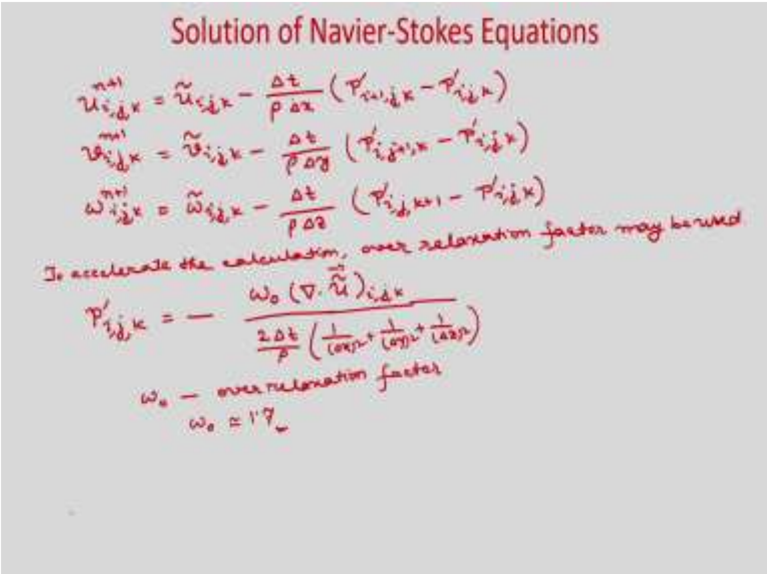

The values for the velocity $u^{n+1}_{i,j,k}$, once you know the pressure correction value then you can find from the provisional velocity $u^{n+1}_{i,j,k} = u_{i,j,k} - \dfrac{\Delta t}{\rho \Delta x}\left(P'_{i+1,j,k} - P'_{i,j,k}\right)$ then $v^{n+1}_{i,j,k} = \tilde{v}_{i,j,k} - \dfrac{\Delta t}{\rho \Delta y}\left(P'_{i,j+1,k} - P'_{i,j,k}\right)$ and $w^{n+1}_{i,j,k} = w_{i,j,k} - \dfrac{\Delta t}{\rho \Delta z}\left(P'_{i,j,k+1} - P'_{i,j,k}\right)$.

So, once you know the pressure correction values and from the known provisional velocities, you will be able to calculate the velocities at n + 1 level in the pressure equation to accelerate the calculation $P'_{i,j,k}$ some over relaxation factor is used.

So, you can write $P'_{i,j,k} = -\frac{\omega_0 \left(\nabla.\vec{u}\right)_{i,j,k}}{\frac{2\Delta t}{\rho}\left(\frac{1}{(\Delta x)^2}+\frac{1}{(\Delta y)^2}+\frac{1}{(\Delta z)^2}\right)}$. So, here this ω_0 is known as over relaxation factor.

So, to accelerate the calculation, over relaxation factor may be used and ω_0 value generally of the; obviously, it will be > 1. So, you can write of the order of 1.5 or 1.6 in that range.

So, in today's class we have used finite difference method to discretize the Navier-Stoke equations using a MAC algorithm, we have actually used staggered grid to discretize these unsteady Navier-Stoke equations. We have used finite difference method and written the difference equation for first derivative and second derivative using Taylor series expansion.

And we have used forward time and central space which is your explicit method after discretizing the each terms temporal term, pressure gradient term, viscous term and the convection term, we have written the governing equations, but pressure at n +1 level is unknown so, we have assume the pressure from the previous time level and we have solved for the provisional velocities first.

Then once you know the provisional velocities then you substitute it in the continuity equation in the main cell. Once you substitute it you can get the pressure Poisson equation, but if you neglect the pressure correction of neighboring cells. Then you will get the equation for pressure.

Once you can find the equation for pressure correction, then you can find the pressure at n + 1 time level after correcting the values from the previous time level p value and you can find the velocities at n + 1 time level from the provisional velocities, and the pressure correction terms. Sometime, this pressure correction equation you can use the over relaxation factor to accelerate the solution and these over relaxation factor generally or commonly, it is used as 1.5 to 1.8.

Thank you.