Fundamentals of Convective Heat Transfer
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Module – 10
Numerical Solution of Navier–Stokes and Energy Equations
Lecture – 33
Basics of finite difference method

Hello, everyone. So, we have already solved analytical solution for external flows and for internal flows, for fully developed condition we could get some analytical solution, but if you have developing internal flows then obviously, it is very difficult to study because we cannot have the analytical solution, but we can solve the governing equations numerically.

So, using computational fluid dynamics technique we can solve the Navier–Stokes equations as well as the energy equation with viscous dissipation or with heat generation term in the energy equation. So, in this module we will study the solutions of Navier–Stokes equations and energy equations using numerical technique. There are three different types of discretization method – finite difference method, finite volume method and finite element method. In this course we will use finite difference method to discretize the Navier–Stokes equations and energy equation.

So, in today's class we will study the basics of finite difference method. So, before starting the basics of finite difference method let us know the classification of partial differential equations because ultimately we are going to discretize the partial differential equations and before using some numerical skills it is very much needed to know the classification of these PDEs.

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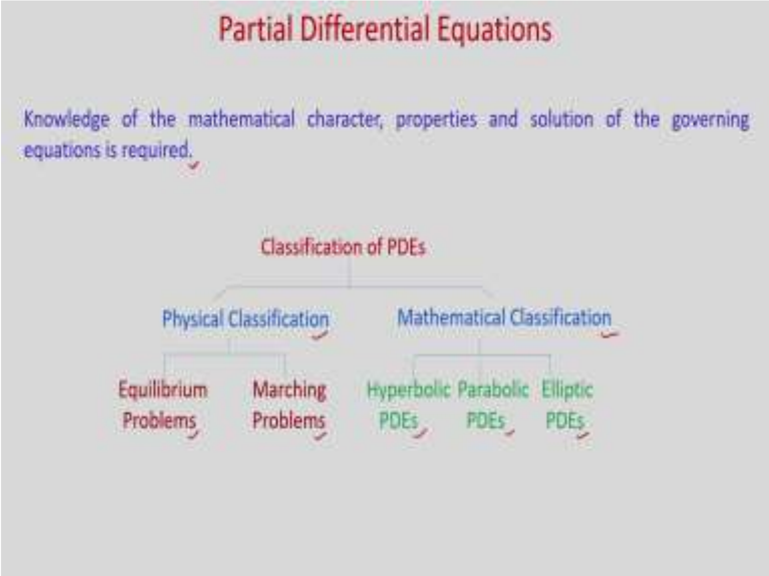

So, knowledge of mathematical character properties and solution of the governing equations is required. So, you can see we can classify the PDEs as physical classification and mathematical classification. In physical classification we can have equilibrium problems and marching problems; and, in mathematical classification we have hyperbolic partial differential equations, parabolic partial differential equations and elliptic partial differential equations.

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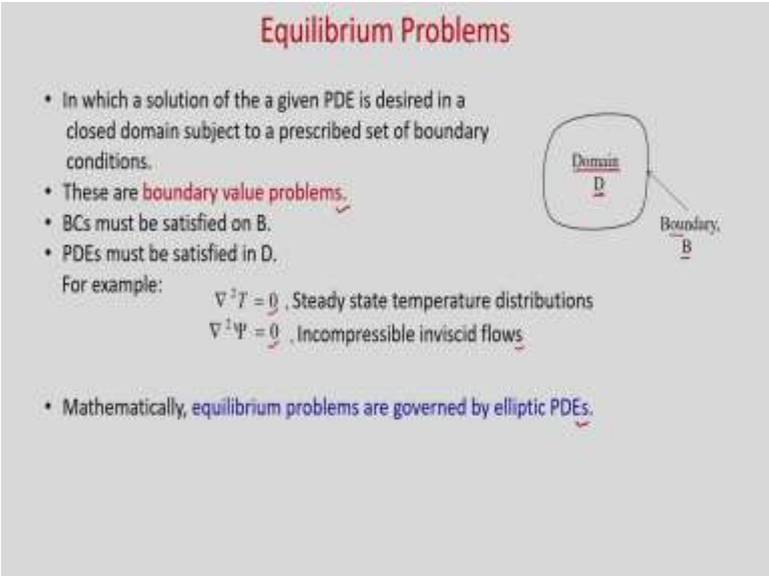

So, first let us discuss about the equilibrium problems. Let us say that we have a domain D and this is the boundary B. So, obviously, whatever partial differential equation you need to solve you will solve inside this domain and apply the boundary condition at the boundary. So, these problems are known as boundary value problem like you have steady state temperature distribution.

So, the equation is heat conduction equation $\nabla^2 T = 0$ or incompressible inviscid flows $\nabla^2 \psi = 0$. So, these you can see that these equations you can solve in the domain and with proper boundary condition you can apply in the boundary. So, mathematically equilibrium problems are governed by elliptic partial differential equations.

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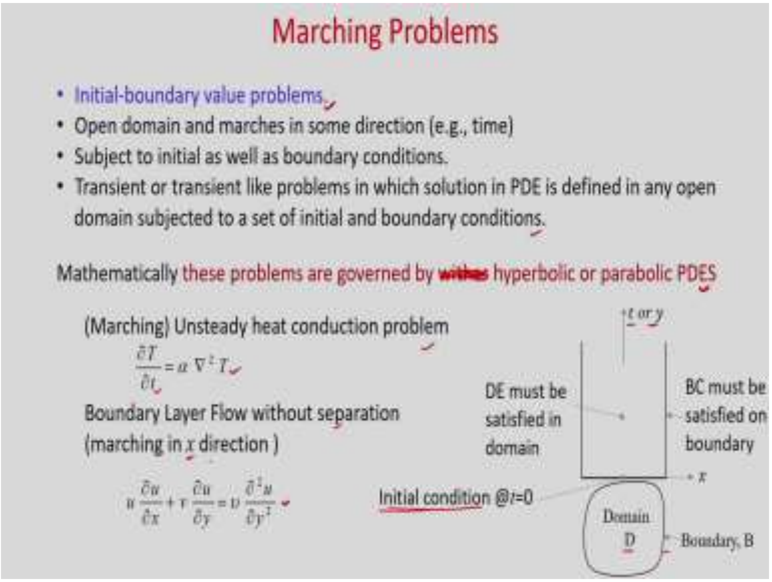

Marching problems: so, marching problems are also known as initial boundary value problems. In marching problem so, you have domain where you need to solve the partial differential equations and at the boundary you will apply the boundary conditions, but it will march in some direction maybe in time or maybe in space.

So, if it is a unsteady problem obviously, it will march in time and at time T = 0 you need to specify the value of that variable inside the domain. So, you can see here. So, this is the domain. So, governing equation at each time level will solve in this domain and also you will apply the boundary condition at boundary B and you can see at t = 0, you will apply the initial condition and you will march in time or in some direction.

So, differential equation must be satisfied in the domain and boundary condition must be satisfied on boundary. So, these are tangent or tangent like problems in which the solutions in PDE is defined in any open domain subjected to a set of initial and boundary conditions. So, mathematically these problems are governed by hyperbolic or parabolic partial differential equations.

So, you can have the example like unsteady heat conduction equation. So, $\frac{\partial T}{\partial t} = \alpha \nabla^2 T$. So, you can see that in this case you will march in time, but if you have a boundary layer flow without separation then you can have the boundary layer equation. So, this equation is also parabolic and it you will march in the direction x because at x = 0 you will specify the free stream velocity infinity and in the x direction you will march.

Now, let us discuss about the mathematical classification. First let us consider a second order partial differential equation.

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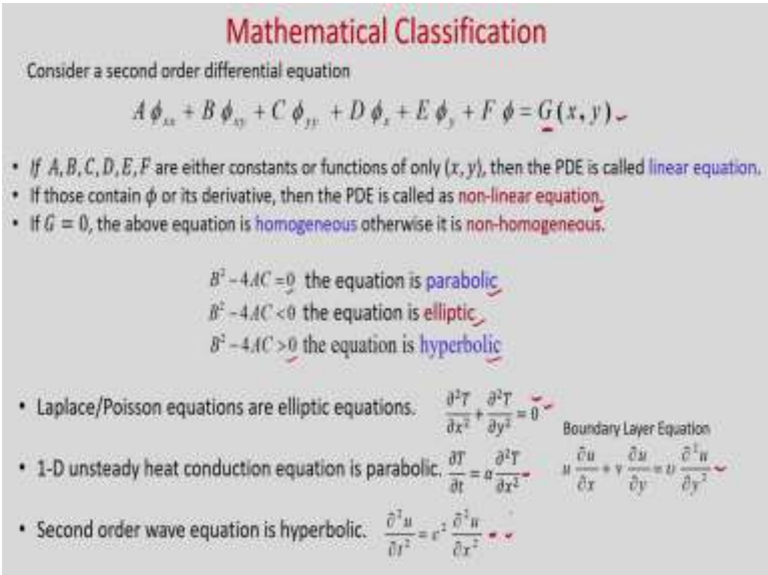

So, consider this equation $A\phi_{xx} + B\phi_{xy} + C\phi_{yy} + D\phi_x + E\phi_y + F\phi = G(x, y)$. So, you can see these coefficients A, B, C, D, E, F if these are constants or function of only x, y, then this partial differential equation is called as linear equation. If these coefficients contain Φ or its derivative, then the partial differential equation is called as non-linear equation and if G = 0, then this equation is homogeneous otherwise it is non-homogeneous.

So, for this equation if you find the discriminant so, if $B^2 - 4AC = 0$, then mathematically this equation is known as parabolic equation; $B^2 - 4AC < 0$, the equation is known as elliptic equation and $B^2 - 4AC > 0$, then the equation is hyperbolic. So, you can have some example.

So, you can have this Laplace equation, heat conduction equation. So, if you find $B^2 - 4AC < 0$, then it is elliptic. 1-D unsteady heat conduction equation, this is if you find you will get $B^2 - 4AC = 0$. So, it is parabolic; this is equation also is parabolic. And, second order wave equation is hyperbolic. So, this equation if you find $B^2 - 4AC > 0$ and this equation is hyperbolic in nature.

So, the detail of mathematical and physical classification you can study from any CFD book. Now, let us introduce the finite difference method. So, in finite difference method we considered the partial differential equation and its derivative we expressed in terms of its values using Taylor series expansion. First let us consider any function f which is a function of x.

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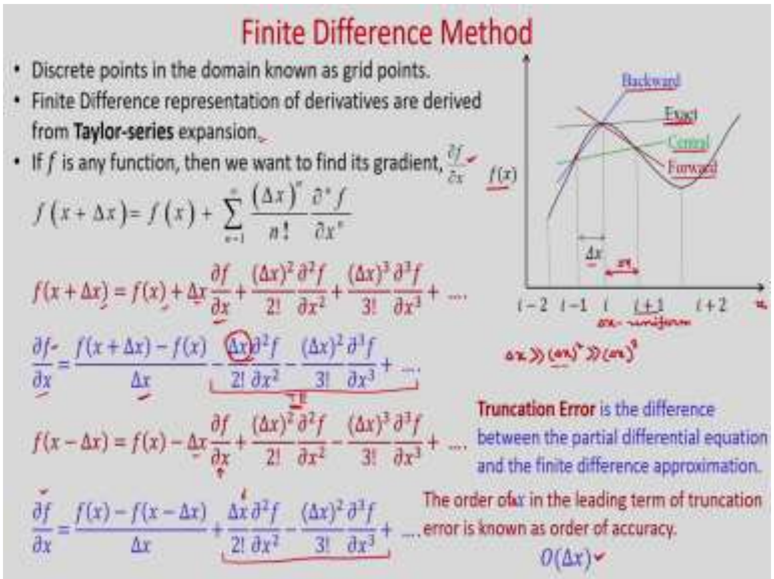

So, you can see this is the function f(x) and x we are plotting in x-direction. If you take this variation of f with x and at this point let us say this is the discrete point i and this is i+1 with a distance Δx and in left side if you take another discrete point i - 1. So, this distance is also Δx. Let us say Δx is uniform. So, everywhere Δx is same.

So, now if you want to find the derivative at this point , the derivative of f at this point what you will do? Just you will draw one tangent. So, that is the exact determination of this derivative. So, this tangent will give you the $\frac{\partial f}{\partial x}$. Now, if you use two points to find it is derivative, let us say we are using one forward point forward point means i + 1, at point i we want to find the derivative. So, this point and this point we are using as you are using forward point so, it is a forward difference.

So, if you join this line then you will get a forward difference approximation; if you use one backward point so, we want to find the $\frac{\partial f}{\partial x}$ at this point and we are using another point at i -1 which is your backward point. So, if you join these two points, then you will get backward approximation and if you use one forward and one backward point and if you join then, you will get the central approximation to find the derivative.

So, finite difference representation of derivatives are derived from Taylor series expansion and from this graph you can see that central approximation is more closer to the exact because you see in backward and forward it is more deviated, but central is almost parallel to the exact. So, that means, it gives determination of this $\frac{\partial f}{\partial x}$ close to the exact solution.

Now, let us consider the Taylor series and expand it. So, if, $f\left(x+\Delta x\right)=f\left(x\right)+\sum_{n=1}^{\infty}\frac{\left(\Delta x\right)^{n}}{n!}\frac{\partial^{n}f}{\partial x^{n}}$. So, if you take one forward point and if you go in forward direction Δx, then you can write,

$$f\left(x+\Delta x\right)=f\left(x\right)+\Delta x\frac{\partial f}{\partial x}+\frac{\left(\Delta x\right)^{2}}{2!}\frac{\partial^{2}f}{\partial x^{2}}+\frac{\left(\Delta x\right)^{3}}{3!}\frac{\partial^{3}f}{\partial x^{3}}+.......$$

So, now if you want to find the first gradient this derivative, $\frac{\partial f}{\partial x}=\frac{f\left(x+\Delta x\right)-f\left(x\right)}{\Delta x}-\frac{\Delta x}{2!}\frac{\partial^{2}f}{\partial x^{2}}-\frac{\left(\Delta x\right)^{2}}{3!}\frac{\partial^{3}f}{\partial x^{3}}+.......$

So, you can see that these derivative we have expressed in terms of the discrete points using a forward point at i + 1 that is $f\left(x+\Delta x\right)$. So, the approximation using finite difference method of this derivative is $\frac{f\left(x+\Delta x\right)-f\left(x\right)}{\Delta x}$ and these higher order terms you can neglect. So, these terms are known as truncation error.

So, if you see if Δx is very small then Δ x > $\left(\Delta x\right)^2$ and $\left(\Delta x\right)^3 << \left(\Delta x\right)^2$. So, anyway you can neglect the higher order terms and this is whatever you are actually neglecting that is your truncation error. So, what is the definition of truncation error? You can see from this expression the truncation error is the difference between the partial differential equation and the finite difference approximation.

If you see the order of x in the leading term of truncation error and that is known as order of accuracy. So, if you can see in this truncation error the leading term is this one and here the order of x is Δx, so its order of accuracy is Δx and it is first order accurate approximation.

Similarly, now if you use one backward point and if you use $f\left(x-\Delta x\right)$, then you can write from the Taylor series expansion as, $f\left(x-\Delta x\right)=f\left(x\right)-\Delta x\frac{\partial f}{\partial x}+\frac{\left(\Delta x\right)^2}{2!}\frac{\partial^2 f}{\partial x^2}-\frac{\left(\Delta x\right)^3}{3!}\frac{\partial^3 f}{\partial x^3}+.......$

So, now if we want to approximate this gradient $\frac{\partial f}{\partial x}$ using one backward point then you can write $\frac{\partial f}{\partial x}=\frac{f\left(x-\Delta x\right)-f\left(x\right)}{\Delta x}+\frac{\Delta x}{2!}\frac{\partial^2 f}{\partial x^2}-\frac{\left(\Delta x\right)^2}{3!}\frac{\partial^3 f}{\partial x^3}+......$. So, you can see the finite difference approximation of this $\frac{\partial f}{\partial x}$ is $\frac{f\left(x-\Delta x\right)-f\left(x\right)}{\Delta x}$.

So, you have represented these $\frac{\partial f}{\partial x}$ using two discrete points f(x) and $f\left(x-\Delta x\right)$ and this is your truncation error and in the leading order term the order of Δ x. So, this is order of Δx. So, order of Δx is 1. So, its order of accuracy is order of Δx.

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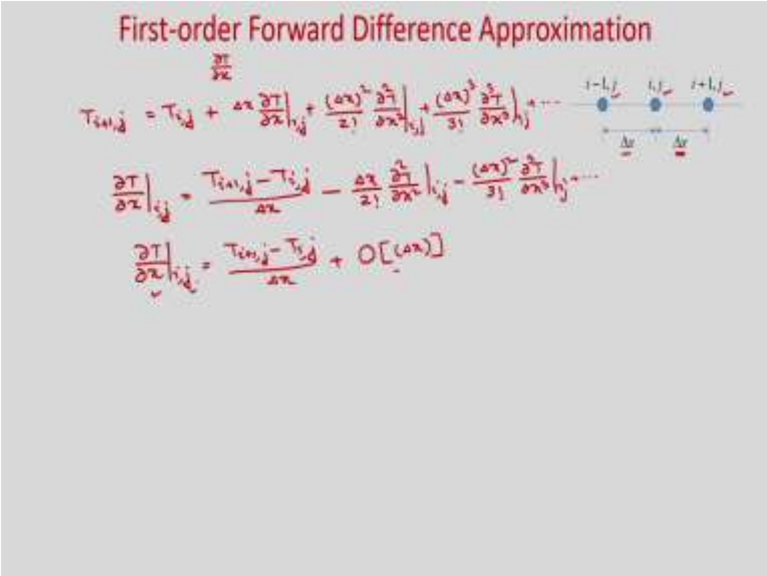

Now, let us consider the derivative of t with respect to x and the discrete points are i, j; i+1, j and i -1, j. So, let us consider these points i, j; i + 1, j and i - 1, j and you have a uniform grid. So, the step size is Δx and we went to find the derivative $\frac{\partial T}{\partial x}$, where T is the temperature.

So, you can use the Taylor series expansion. So, you can write $T_{i+1,j} = T_{i,j} + \Delta x \frac{\partial T}{\partial x}\Big|_{i,j} + \frac{(\Delta x)^2}{2!}\frac{\partial^2 T}{\partial x^2}\Big|_{i,j} + \frac{(\Delta x)^3}{3!}\frac{\partial^3 T}{\partial x^3}\Big|_{i,j} + ...$

So, if you want to find the derivative $\frac{\partial T}{\partial x}\Big|_{i,j}$ so, you can rearrange and write,

$$\frac{\partial T}{\partial x}\Big|_{i,j} = \frac{T_{i+1,j} - T_{i,j}}{\Delta x} - \frac{\Delta x}{2!}\frac{\partial^2 T}{\partial x^2}\Big|_{i,j} + \frac{(\Delta x)^2}{3!}\frac{\partial^3 T}{\partial x^3}\Big|_{i,j} + ...$$

So, you can see your $\frac{\partial T}{\partial x}$ so, you can represent using forward point $T_{i+1,j}$ and $T_{i,j}$. So, what will be the order of accuracy? So, if you neglect this truncation error then order of accuracy is Δx. So, it is a first order accurate discretization and it is known as first order forward difference approximation because we are using one forward point and order of accuracy is first order.

And, you can see the distance between $T_{i+1,j}$ and $T_{i,j}$ is Δ x; so, $\frac{T_{i+1,j} - T_{i,j}}{\Delta x}$. So, it is the finite difference approximation of the first gradient $\frac{\partial T}{\partial x}\Big|_{i,j}$.

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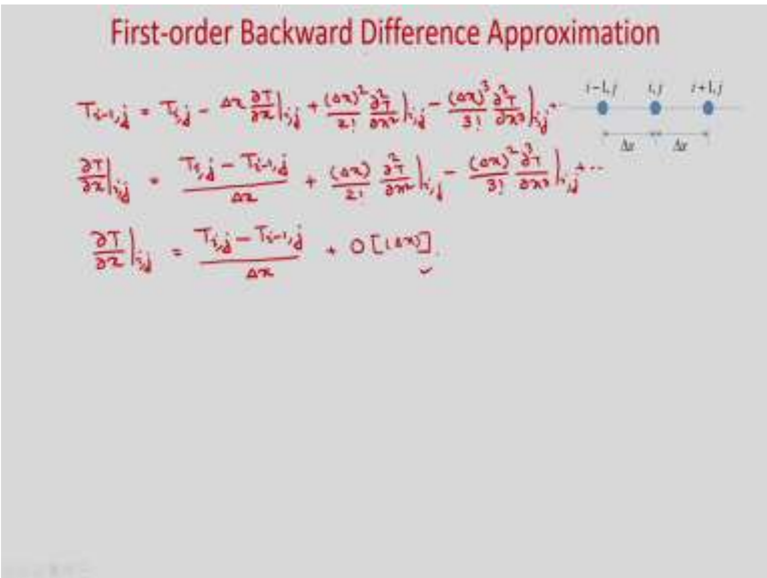

Now, similarly if you consider one backward point i - 1, j then you can find the derivative of T with respect to x as. So, you use Taylor series expansion,

$$T_{i-1,j} = T_{i,j} - \Delta x \frac{\partial T}{\partial x}\Big|_{i,j} + \frac{(\Delta x)^2}{2!}\frac{\partial^2 T}{\partial x^2}\Big|_{i,j} - \frac{(\Delta x)^3}{3!}\frac{\partial^3 T}{\partial x^3}\Big|_{i,j} + \ldots\ldots.$$

So, now if you want to represent this first gradient then you can rearrange it and you can write, $\frac{\partial T}{\partial x}\Big|_{i,j} = \frac{T_{i,j} - T_{i-1,j}}{\Delta x} + \frac{(\Delta x)}{2!}\frac{\partial^2 T}{\partial x^2}\Big|_{i,j} - \frac{(\Delta x)^2}{3!}\frac{\partial^3 T}{\partial x^3}\Big|_{i,j} + \ldots\ldots.$

So, you can write $\frac{\partial T}{\partial x}\Big|_{i,j} = \frac{T_{i,j} - T_{i-1,j}}{\Delta x}$. So, you can see this is the finite difference approximation of this first gradient using one backward point and order of accuracy is Δx so, this is known as first order backward difference approximation. Now, use one backward and one forward points and find the approximation of this first derivative $\frac{\partial T}{\partial x}$ and the second derivative $\frac{\partial^2 T}{\partial x^2}$.

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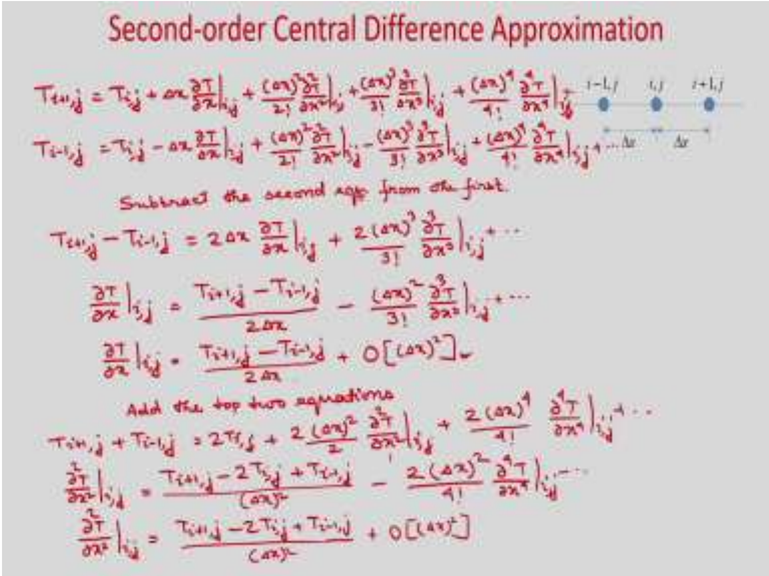

So, using Taylor series expansion you can write $T_{i+1,j} = T_{i,j} + \Delta x \frac{\partial T}{\partial x}\Big|_{i,j} + \frac{(\Delta x)^2}{2!}\frac{\partial^2 T}{\partial x^2}\Big|_{i,j} + \frac{(\Delta x)^3}{3!}\frac{\partial^3 T}{\partial x^3}\Big|_{i,j} + \frac{(\Delta x)^4}{4!}\frac{\partial^4 T}{\partial x^4}\Big|_{i,j} + ...$ and other term and $T_{i-1,j} = T_{i,j} - \Delta x \frac{\partial T}{\partial x}\Big|_{i,j} + \frac{(\Delta x)^2}{2!}\frac{\partial^2 T}{\partial x^2}\Big|_{i,j} - \frac{(\Delta x)^3}{3!}\frac{\partial^3 T}{\partial x^3}\Big|_{i,j} + \frac{(\Delta x)^4}{4!}\frac{\partial^4 T}{\partial x^4}\Big|_{i,j} + ...$. So, we are writing the Taylor series expansion. So, you can have the higher order terms. So, what you do? So, you first subtract the second one from the first equation. So, we want to find the $\frac{\partial T}{\partial x}$. So, if you subtract the second equation from the first.

So, if you subtract what you will get? You can see you will get, $T_{i+1,j} - T_{i-1,j} = 2\Delta x \frac{\partial T}{\partial x}\Big|_{i,j} + 2\frac{(\Delta x)^3}{3!}\frac{\partial^3 T}{\partial x^3}\Big|_{i,j} + ...$

So, now if you write the approximation of first derivative $\frac{\partial T}{\partial x}$. So, you can see it will be $\frac{\partial T}{\partial x}\Big|_{i,j} = \frac{T_{i+1,j} - T_{i-1,j}}{2\Delta x} - \frac{(\Delta x)^2}{3!}\frac{\partial^3 T}{\partial x^3}\Big|_{i,j} + ...$. So, you can write $\frac{\partial T}{\partial x}\Big|_{i,j} = \frac{T_{i+1,j} - T_{i-1,j}}{2\Delta x} + O\left[(\Delta x)^2\right]$. So, it is a second order accurate.

So, it is second order central difference approximation because we are using one forward and one backward point and we are finding the derivative at i, j and you can see these two points are separated by a distance 2 Δx. So, $\frac{\partial T}{\partial x}\Big|_{i,j} = \frac{T_{i+1,j} - T_{i-1,j}}{2\Delta x} + O\left[\left(\Delta x\right)^2\right]$.

Now, you add these two equations so, and find the second derivative of T. So, if you add. So, if you add it you will get $T_{i+1,j} + T_{i-1,j} = 2T_{i,j} + 2\frac{\left(\Delta x\right)^2}{2!}\frac{\partial^2 T}{\partial x^2}\Big|_{i,j} + 2\frac{\left(\Delta x\right)^4}{4!}\frac{\partial^4 T}{\partial x^4}\Big|_{i,j} + ...$

So, now you can see you can represent this second derivative using three points, $\frac{\partial^2 T}{\partial x^2}\Big|_{i,j} = \frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{\left(\Delta x\right)^2} - 2\frac{\left(\Delta x\right)^4}{4!}\frac{\partial^4 T}{\partial x^4}\Big|_{i,j} + ...$

So, you see the finite difference approximation of the second derivative about point i, j as $\frac{\partial^2 T}{\partial x^2}\Big|_{i,j} = \frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{\left(\Delta x\right)^2} + O\left[\left(\Delta x\right)^2\right]$. It is second order. So, generally whatever equations we will consider we will have first derivative and second derivative. You can see that in Navier–Stokes equations, as well as in the energy equations you will get the first derivative of temperature and the second derivative of this temperature.

So, we have derived $\frac{\partial^2 T}{\partial x^2}$ so similar way you can write $\frac{\partial^2 T}{\partial y^2}\Big|_{i,j} = \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{\left(\Delta y\right)^2} + O\left[\left(\Delta y\right)^2\right]$ because in the y-direction you are taking the derivative.

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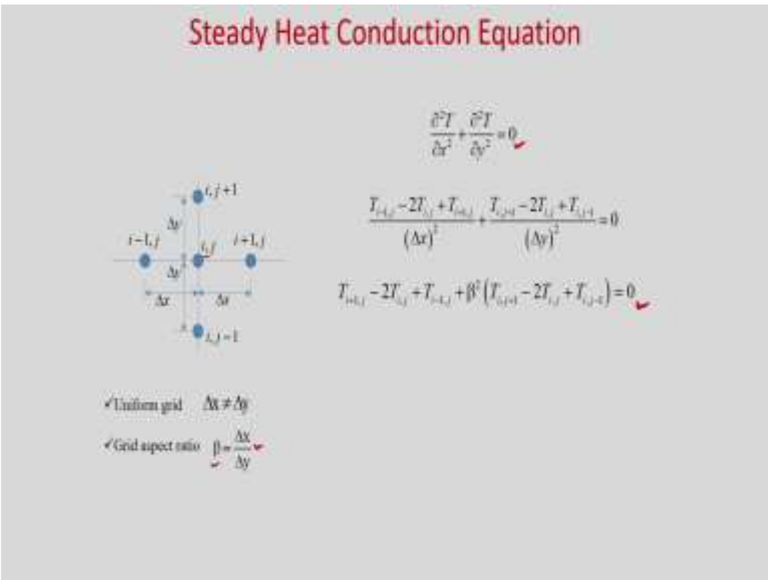

Now, we have learned how to use the Taylor series expansion to find the first derivative and second derivative. Now, let us consider steady state heat conduction equation and discretize using finite difference method. So, you can see we have this equation 2-dimensional steady state heat conduction equation these are the discrete points. So, this is known as grid.

So, we will discretize this equation about this point i, j. So, we have i +1, j; i -1, j these are the index and i, j + 1 and i, j -1 we have a uniform grid Δx in the x-direction and in y-direction Δy. So, Δx may not be same as Δy and grid aspect ratio we are defining $\beta = \frac{\Delta x}{\Delta y}$.

So, if you use the second order central difference approximation then you can write $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$ and $\frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{\left(\Delta x\right)^2} + \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{\left(\Delta y\right)^2} = 0$. If you rearrange it you can write, $T_{i+1,j} - 2T_{i,j} + T_{i-1,j} + \beta^2 \left(T_{i,j+1} - 2T_{i,j} + T_{i,j-1}\right) = 0$.

So, these equation is your discretized equation. So, this is your algebraic equation. So, that we have discretized about this point i, j. So, this is the equation we have written for this i, j. So, you can write similar equation for each point. So, you will get a system of algebraic equations. So, once you get the system of algebraic equation at each point, then you will you need to solve using some iterative method.

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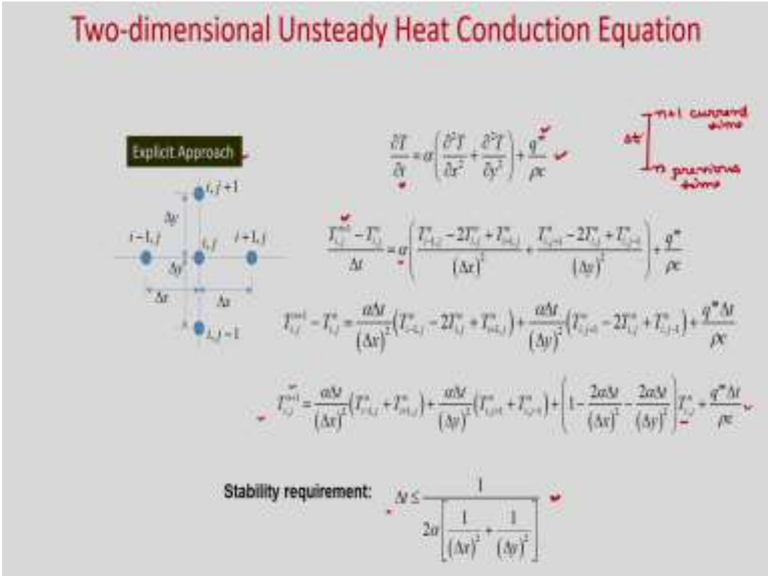

Now, let us consider two-dimensional unsteady heat conduction equation. So, this is the unsteady heat conduction equation with your heat generation term and we will use explicit approach; that means, there will be one unknown. So, we will use forward time discretization.

So, now, we need to march in the direction of time. So, we will march in time from n to n + 1. So, n is your previous time and this is your current time or present time. So, we are just marching in time from n to n +1.

So, these derivative now we are using forward time. So, $\dfrac{T_{i,j}^{n+1}-T_{i,j}^{n}}{\Delta t}=\alpha\left(\dfrac{T_{i-1,j}^{n}-2T_{i,j}^{n}+T_{i+1,j}^{n}}{\left(\Delta x\right)^2}+\dfrac{T_{i,j+1}^{n}-2T_{i,j}^{n}+T_{i,j-1}^{n}}{\left(\Delta y\right)^2}\right)+\dfrac{q'''}{\rho c}$.

So, now you can write you can see here you have one unknown term n + 1 and all are at time level n. So, you are you can express,

$$T_{i,j}^{n+1}=\frac{\alpha\Delta t}{\left(\Delta x\right)^2}\left(T_{i-1,j}^{n}+T_{i+1,j}^{n}\right)+\frac{\alpha\Delta t}{\left(\Delta y\right)^2}\left(T_{i,j+1}^{n}+T_{i,j-1}^{n}\right)+\left(1-\frac{2\alpha\Delta t}{\left(\Delta x\right)^2}-\frac{2\alpha\Delta t}{\left(\Delta y\right)^2}\right)T_{i,j}^{n}+\frac{q'''\Delta t}{\rho c}.$$

So, this is your equation and it is you see you have only one unknown and all are known from the previous time level and this is known as explicit approach, but limitation of using the time is you will get from the stability criteria and here ΔT you have to choose from this stability requirement. So, $\Delta t \le \dfrac{1}{2\alpha\left[\dfrac{1}{(\Delta x)^2}+\dfrac{1}{(\Delta y)^2}\right]}$.

So, obviously, you can see if you refine the grid then Δx Δy will be smaller and accordingly, you have to decrease the time step ΔT and when you are going from n to n+1 so, this is the time step ΔT.

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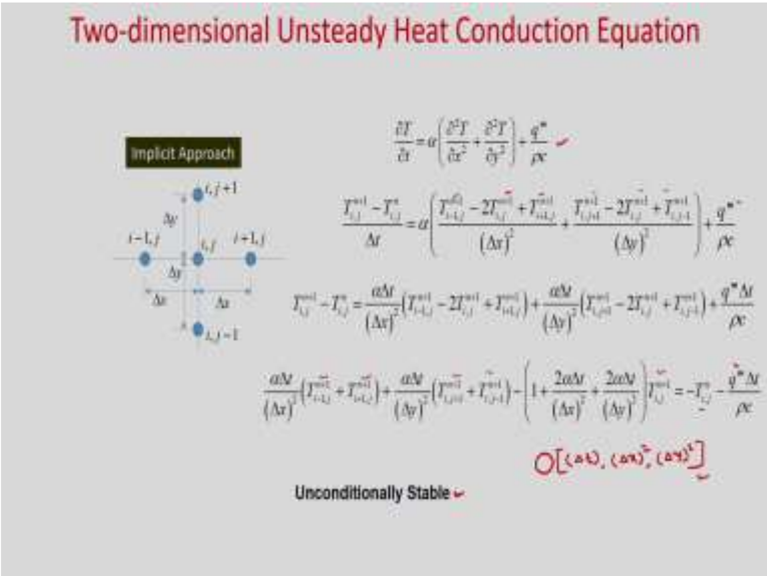

Now, we will use backward time and in that you will get more than one unknown and you will get a system of linear algebraic equations and this method is known as implicit method. So, the same equation we are considering two-dimensional unsteady heat conduction equation.

Now, if you discretize using first order this is your $\dfrac{T_{i,j}^{n+1}-T_{i,j}^{n}}{\Delta t}=\alpha\left(\dfrac{T_{i-1,j}^{n+1}-2T_{i,j}^{n+1}+T_{i+1,j}^{n+1}}{(\Delta x)^2}+\dfrac{T_{i,j+1}^{n+1}-2T_{i,j}^{n+1}+T_{i,j-1}^{n+1}}{(\Delta y)^2}\right)+\dfrac{q'''}{\rho c}$

So, if you rearrange it so, you will get in the left hand side we have written all unknown terms you see n + 1 and in right hand side $T_{i,j}^{n}$ is known and the source term is also known from the known heat generation per unit volume term.

So, this you can see this you if you write the system of equations you will get a printer diagonal matrix. So, that you need to solve using some method and the advantage of this implicit method is that it is unconditionally stable. If you do the 1-dimension stability analysis you will get it is unconditionally stable.

So, what is the order of accuracy of this discretization? So, the order of accuracy for this explicit method and implicit method is order of ΔT because it is first order accurate in time and second order accurate in space, $(\Delta x)^2$ and $(\Delta y)^2$. Because you can see this is order of $(\Delta x)^2$, this is the order of $(\Delta y)^2$ and this is the order of ΔT. So, this is the order of accuracy of this method.

Using the finite difference discretization method we have converted the partial differential equation to system of algebraic equations. So, that, you need to solve at each discrete point.

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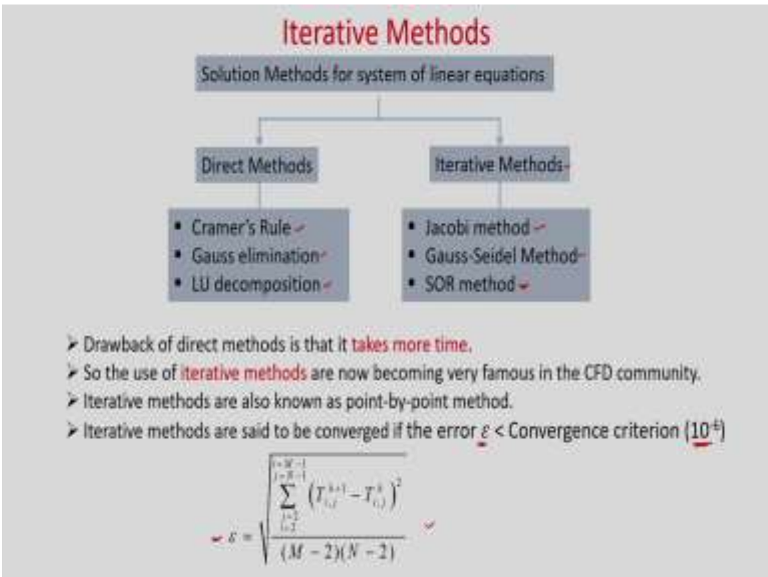

So, there are many iterative methods are there. So, one is direct methods are very time consuming. So, direct methods like Cramer's rule, Gauss elimination or LU decomposition. So, generally these are not used in CFD. In iterative methods you have Jacobi method, Gauss-Seidel method and successive over election method. So, these are mostly point by point method.

And, also nowadays you can use some advance iterative method like conjugate gradient or by conjugate gradient method, and using these iterative methods you need to solve those algebraic equations. So, you can see iterative methods are said to be converged if the error epsilon is less than the convergence criteria.

So, you have to set the convergence criteria let us say 10^{-6} or 10^{-5} and this ε you need to calculate from the value of the discrete points. So, at each point if you have, $\varepsilon = \sqrt{\dfrac{\sum_{\substack{i=2 \\ j=2}}^{\substack{i=M-1 \\ j=N-1}} \left(T_{i,j}^{k+1} - T_{i,j}^{k}\right)^2}{(M-2)(N-2)}}$. Then, you find the ε if it decreases with time and when it will go below the given convergence criteria let us say 10^{-6}, then your iteration will stop.

Now, we will discuss about the variable storage, say when you are solving Navier–Stokes equations and the energy equation how many variables you have? You have velocity u, v, w in 3-dimensions and temperature and you have pressure p because pressure is appearing in momentum equations.

So, obviously, you have total five unknowns; three velocities, pressure and temperature. So, where will you store or where will you solve these variables? So, depending on the storage variable storage we can have staggered grid and collocated grid.

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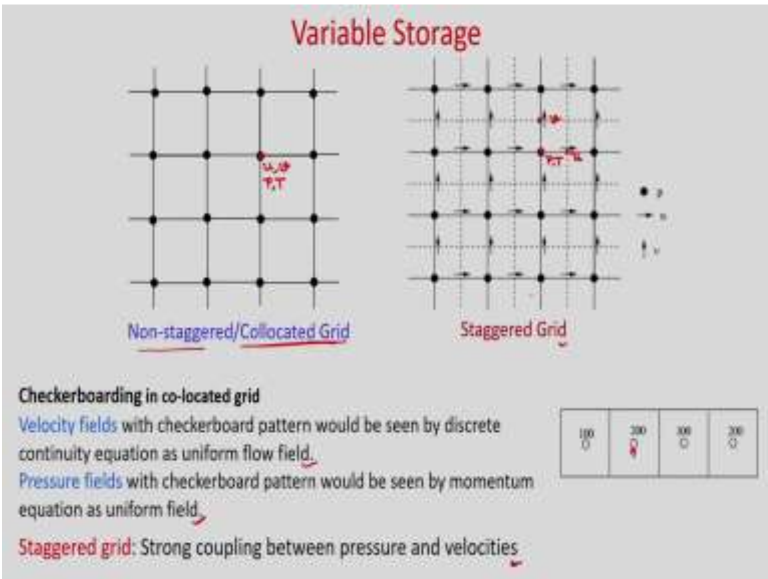

So, first let us discuss about the collocated grid or non-staggered grid. So, you can see you can divide the domain into discrete points. So, that is known as grid and these are the grid points and in each grid point if you solve all the variables in two-dimension you solve u, v, pressure, temperature or any other species like mass fraction.

So, if you solve at same grid point then this is known as collocated grid. So, all the variables you are solving at the same point. But, it is having some disadvantage. So, that is known as pressure velocity decoupling. So, when you are solving the Navier–Stokes equation and energy equation, you will get pressure velocity decoupling; that means, pressure and velocity will not talk each other.

So, if you see here the let us say this is the pressure distribution. So, you have discrete points and at discrete points you have the pressure distribution 100, 200, 100 and 200. So, for some physical situation it may arise that your pressure is varying like this. Now, at this point if you want to find the pressure gradient using collocated grid for the Navier–Stoke equation, then you will find $\frac{\partial p}{\partial x}$.

So, $\frac{\partial p}{\partial x}$ is the if you central difference you can see $\frac{100-100}{2\Delta x}$. So, it will become 0. So, you can see pressure gradient is 0, but pressure gradient is the driving force for the velocity, but here pressure gradient is becoming 0. So, that is known as pressure velocity decoupling.

So, you can see for a checkerboard kind of pressure distribution or velocity distribution in collocated grid velocity fields with checkerboard pattern would be seen by discrete continuity equation as uniform flow field and pressure fields with checkerboard pattern would be seen by momentum equation as uniform field because it seems like a uniform field so that you do not have the pressure gradient. But, you have pressure gradient, but it is varying.

So, similarly continuity equation you see if we apply $\frac{\partial u}{\partial x}$ here it will become 0. So, it will field that it is a you have a uniform flow field, but you have a velocity variation. So, that is the disadvantage of collocated grid, but you have some way to overcome this problem and you can use momentum interpolation like Eddie and Joe proposed the moment of interpolation. So, that, you can use to overcome these velocity pressure decoupling problem.

Another grid is staggered grid. So, in the staggered grid the velocities are solved in staggered grid point ok. So, you can see here. So, this is the grid. So, if you see this is the grid point. In staggered grid, pressure and temperature or any other scalar like mass fraction these are solved at this main grid point, but velocities are solved in a staggered point. So, here velocity u is solved and here velocity v is solved.

So, this is the velocity u and this is the velocity v and at these point pressure is solved and any other scalar like temperature are solved. So, you can see velocities are solved in staggered way that is why it is not it is known as staggered grid and these pressure velocity decoupling is avoided using these staggered grid. And, that is why in staggered grid you will get strong coupling between pressure and velocities.

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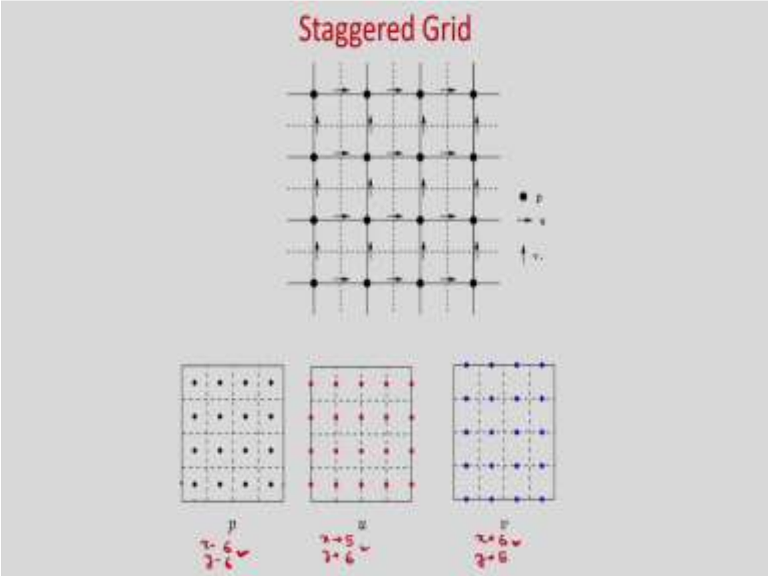

So, you can see in the staggered grid you have pressure at these black dot velocity u in this red dot and v velocity you will have in these blue dots. But, here you can see you have some problem in book keeping. So, for pressure you can see you have interior points 1, 2, 3, 4 and if you consider the boundary points then here you have one boundary point, here another boundary points. So, you have five.

But, when you are considering u so, in x-direction how many points are there including the boundary points? 1, 2, 3, 4, 5 and in y direction you if you consider the boundary points so, 1, 2, 3, 4, 5, 6. If you consider v in x-direction if you consider the boundary point 1, 2, 3, 4, 5, 6, but in y-direction 1, 2, 3, 4, 5 and for pressure in x-direction 1, 2, 3, 4, 5, 6 and in y-direction 1, 2, 3, 4, 5, 6including the boundary points.

So, you can see you have total number of points for solving the velocities u v are different in x and y direction and it is also different than the p. So, this book keeping is the difficult in staggered grid, but you need to be careful while solving these velocities and pressure and accordingly, you need to show for the interior grid points.

So, that you need to be very careful while solving the equations using staggered grid. But, the main advantage of staggered grid that you can avoid this pressure velocity decoupling and obviously, you will get a strong coupling between pressure and velocity.

So, in today's class we have introduced the partial differential equation and we have discussed about different classification of partial differential equation; physical classification and mathematical classification. In mathematical classification, we have elliptic equations, parabolic equations and hyperbolic equations and in physical classification you have equilibrium problems and marching problems.

So, in equilibrium problems are mostly governed by elliptic equations and marching problems are generally governed by parabolic and the hyperbolic equations. Then we have introduced the finite difference approximation using the Taylor series expansion we have used a forward point and backward point and we have written the expression for the first derivative and also using central difference we have written the approximation of the first derivative and second derivative.

So, you can see the forward difference approximation and backward difference approximation the order of accuracy is 1. So, that is the order of Δx, but when you use central difference method then order of accuracy is 2 because order of $(\Delta x)^2$. Then, we discretize the partial differential equation of steady state heat conduction equation and unsteady heat conduction equation.

So, after discretizing you have seen that you will get system of algebraic equations and you need to use suitable iterative method to solve the system of algebraic equations, because at each grid point if you write that equation then you will get a system of algebraic equations.

And, for explicit method you will have only one unknown, but Δ T has the restriction due to the stability criteria, but in most of the time in unsteadies two-dimensional heat conduction equation, we have seen that this implicit method is unconditionally stable.

Then we have discussed about the variable storage, depending on that we have classified the grid as staggered grid and collocated grid. In collocated grid all the variables are stored at the same point, but it is having the problem of velocity and pressure decoupling. To avoid that problem you can use staggered grid where pressure and any scalar like temperature and species is stored at the main grid point and velocities u, v are stored in a staggered manner. So, in the staggered grid you will get a strong coupling between pressure and velocity.

Thank you.