Fundamentals of Convective Heat Transfer

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Module – 09
Natural Convection - II
Lecture – 32
Solution of example problems

Hello everyone. So, today we will solve few problems on Natural Convection Flows. Although, we will discuss about turbulent flows in post convection in detail in week 11, but today we will just show few correlations for turbulent natural convection flows.

So, generally if $Ra > 10^9$, then natural convection flows is to be said as turbulent flows. Whatever heat transfer relations we have derived in last 3 lectures, so those are valid for laminar flows, where $Ra > 10^9$.

Later, Bejan proposed that Rayleigh number is not the criteria for determining whether it is turbulent or laminar flows, it is the Grashof number. So, if $Gr > 10^9$, then it is turbulent natural convection flows, and few correlations will also write based on the experiments.

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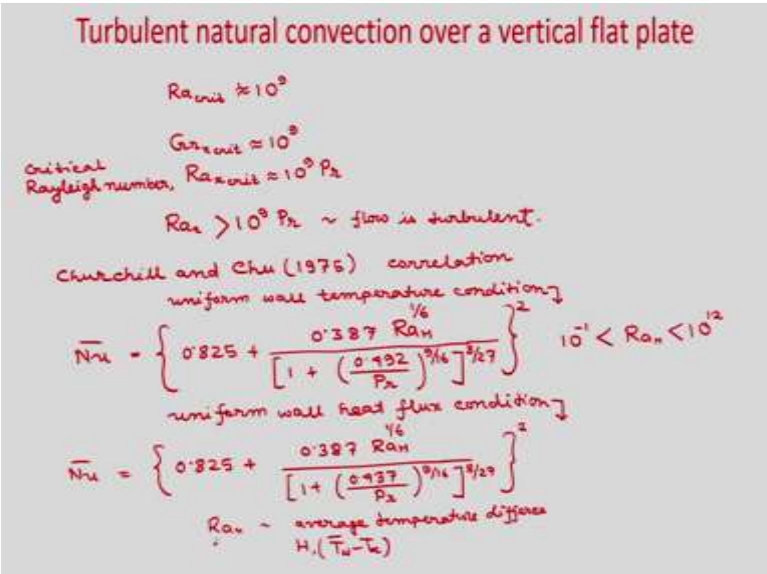

Turbulent natural convection over a vertical flat plate for these critical Rayleigh number, earlier it was proposed as based on your characteristic length was of the order of 10^9. But later Bejan proposed now it is the Grashof number, which actually determines this is the critical Grashof number which determines whether it is laminar or turbulent flow. So, $\mathrm{Gr}_{xcrit} = 10^9$.

So, as you know that Rayleigh number is the product of Grashof number and Prandtl number, so $Ra_{xcrit} = 10^9\,\mathrm{Pr}$.

So, depending on Prandtl number, so Rayleigh number will vary. So, this is the critical Rayleigh number. So, it will determine whether the flow is laminar or turbulent. If $Ra_x > 10^9 Pr$ then the flow is turbulent. So, obviously, it is natural convection flow.

So, Churchill and Chu in 1975, they proposed one correlation for turbulent flows and let us write that correlation first. So, Churchill and Chu relations in 1975, so they proposed for uniform wall temperature condition.

$$\overline{Nu} = \left\{ 0.825 + \frac{0.387 Ra_H^{1/6}}{\left[1 + \left(\frac{0.492}{\mathrm{Pr}}\right)^{9/16}\right]^{8/27}} \right\}^2$$

. So, this is for turbulent flows and this is derived based on the experimental value. So, this is known as correlation. So, Churchill and Chu correlation.

And for uniform wall heat flux boundary condition, so this 0.492 you can replace with 0.437. So, this is the same relation except the value of these for 0.492. So, it will be,

$$\overline{Nu} = \left\{ 0.825 + \frac{0.387 Ra_H^{1/6}}{\left[1 + \left(\frac{0.437}{\mathrm{Pr}}\right)^{9/16}\right]^{8/27}} \right\}^2$$

. And this relation is valid for $10^{-1} < Ra_H < 10^{12}$.

So, you can see this is also valid for $Ra_H < 10^9$. So, it will give reasonably good results. And you can see in case of uniform wall heat flux condition, this Rayleigh number will be based on your height of the plate and the temperature difference that will be average temperature difference because for uniform wall heat flux condition you know that temperature will vary along the plate.

So, obviously, your it will be based on H and your T_W -T_∞. So, this wall temperature you have to consider as average wall temperature. So, then it will be average temperature difference, based on that this Rayleigh number is defined. Even for laminar flow Gr < 10^9 , Churchill and Chu proposed another relation.

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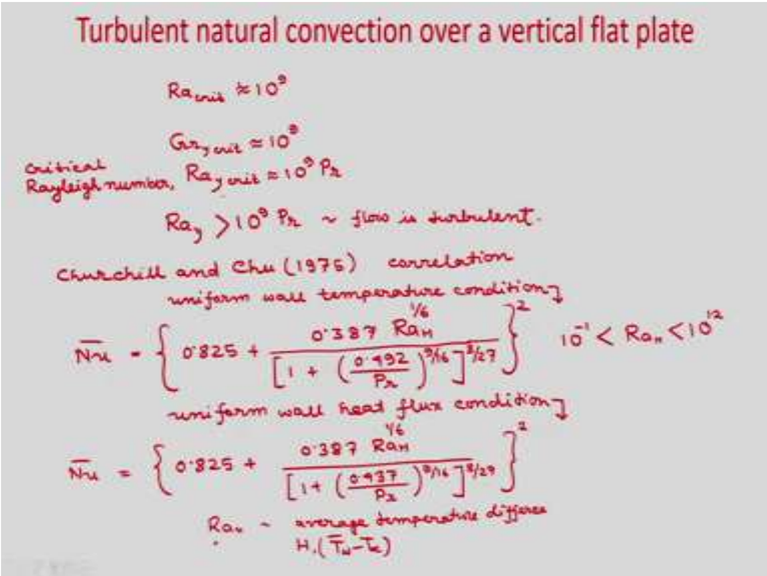

So, for laminar flow, where Gr < 10^9 another corelation was proposed by Churchill and Chu in 1975, and it is valid for both uniform surface temperature and uniform wall heat flux, and uniform wall heat flux conditions. So, that is given as,

$$\overline{Nu} = 0.68 + \frac{0.67 Ra_H^{1/4}}{\left[1 + \left(\frac{0.492}{\Pr}\right)^{9/16}\right]^{4/9}}.$$

And Eckert and Jackson also part from the integral analysis for turbulent natural convection flows, and they proposed this Nusselt number relations. So, let us write that. So, using integral relation or using integral analysis for turbulent natural convection; Eckert and Jackson proposed.

So, Eckert and Jackson derived the $Nu = 0.0295 \left[\frac{\Pr^7}{\left(1 + 0.494 \Pr^{2/3}\right)^6}\right]^{1/5} Gr_y^{2/5}$.

And $\overline{Nu}_H = 0.834 Nu\big|_{y=H}$. Because generally you have written in terms of y. So, this also you can write y, y because height of the plate, along that direction you have taken y, so this will be y.

So, while solving the problem first you have to determine the Rayleigh number and depending on the Rayleigh number you have to see whether it is laminar flow or turbulent natural convection flows. Then, you have to use suitable formula to calculate the average heat transfer coefficient or the heat transfer coefficient.

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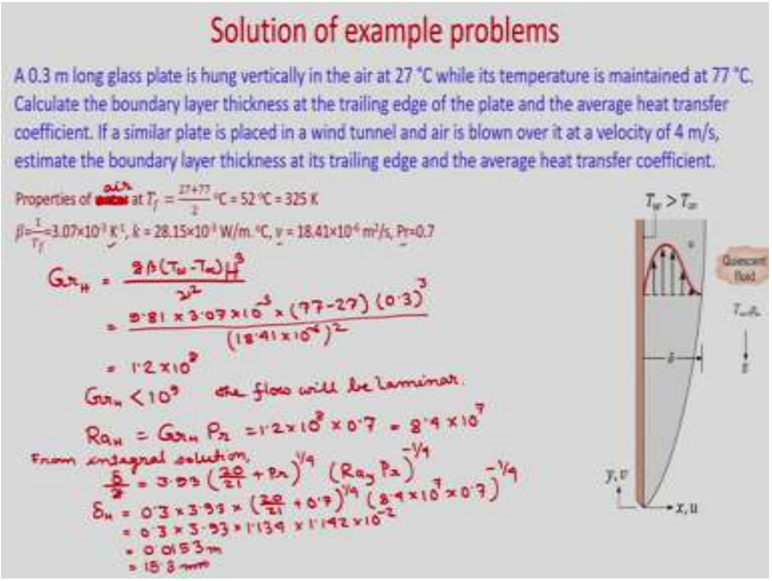

So, first let us solve this problem. If 0.3 m long glass plate is hung vertically in the air at 27^0C, while its temperature is maintained at 77^0C. Calculate the boundary layer thickness at the trailing edge of the plate and the average heat transfer coefficient.

If a similar plate is placed in a wind tunnel and air is blown over it at a velocity of 4 m/s estimate the boundary layer thickness at its trailing edge and the average heat transfer coefficient.

So, you can see first using natural convection you have to find the boundary layer thickness, and the heat transfer coefficient, and later case if it is a post convection flow, then you have to calculate the boundary layer thickness and the heat transfer coefficient.

So, in this case, first you have to know the properties. So, generally properties are calculated at the film temperature that means, average temperature and thermal expansion coefficient $\beta = \frac{1}{T_f}$, where T_f is the film temperature. And this film temperature you have to write in K.

So, this is air at T_f. So, $T_f = \frac{27+77}{2}$ = 52^0C, so 325 K. So, $\beta = \frac{1}{T_f}$ as this and the thermal conductivity, the kinematic viscosity, and Prandtl number are given.

So, first you find what is the Grashof number, so Grashof number you can calculate as, $Gr_H = \frac{g\beta\left(T_w - T_\infty\right)H^3}{\nu^2}$. So, you can see you can calculate $\frac{9.81 \times 3.07 \times 10^{-3} \times \left(77-27\right)\left(0.3\right)^3}{\left(18.41 \times 10^{-6}\right)^2}$. So, you will get it as 1.2×10^8 k. So, you can see obviously, $Gr_H < 10^9$, so the flow will be laminar.

And, once you know Grashof number you can calculate the Rayleigh number. So, it will be $Ra_H = Gr_H \Pr$. So, it is $1.2 \times 10^8 \times 0.7 = 8.4 \times 10^7$.

So, using integral solution we have derived the boundary layer thickness $\frac{\delta}{x}$, and also, we have found what is the Nusselt number and from there you can calculate the heat transfer coefficient. So, we will use those relations here. So, from integral solution we have found, $\frac{\delta}{y} = 3.93\left(\frac{20}{21} + \Pr\right)^{1/4}\left(Ra_y \Pr\right)^{-1/4}$.

So, in this case, at the trailing edge you have to calculate the boundary layer thickness. So, we will write y = H. So, you can write,

$\delta_H = 0.3 \times 3.93 \times \left(\frac{20}{21} + 0.7\right)^{1/4}\left(8.4 \times 10^7 \times 0.7\right)^{-1/4}$.So, this you can write, $0.3 \times 3.93 \times 1.134 \times 1.142 \times 10^{-2}$. So, finally, you will get 0.0153 m. So, if you write in terms of mm then, so 15.3 mm.

Now, you can write the average heat transfer coefficient. So, first let us write the expression for average Nusselt number and from there we will calculate the heat transfer coefficient.

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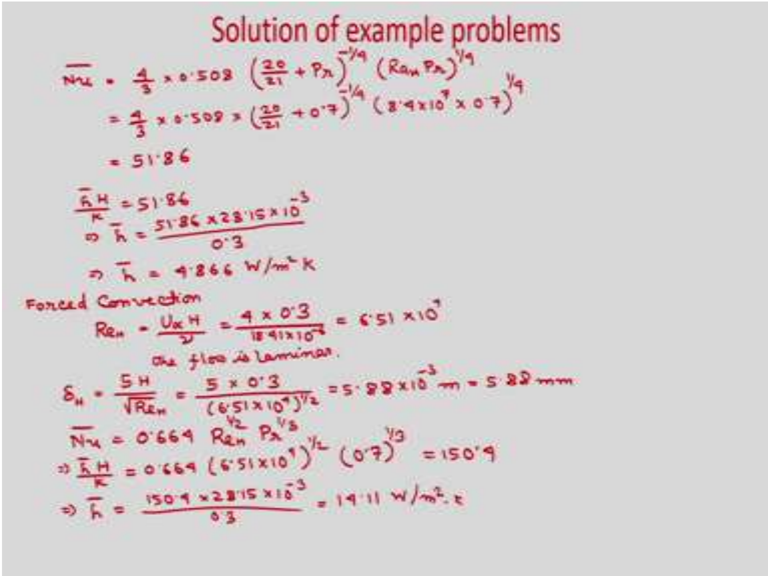

So, you know the relation from the integral solution as, $\overline{Nu} = \frac{4}{3} \times 0.508 \left(\frac{20}{21} + \Pr \right)^{-\frac{1}{4}} \left(Ra_H \Pr \right)^{\frac{1}{4}}$.

So, if you put these values, so $\frac{4}{3} \times 0.508 \times \left(\frac{20}{21} + 0.7 \right)^{-\frac{1}{4}} \left(8.4 \times 10^7 \times 0.7 \right)^{\frac{1}{4}}$. So, if we evaluate these values you will get finally, as 51.86. So, now, you can write, $\frac{\bar{h}H}{K} = 51.86$.

So, average heat transfer coefficient you can write as $\bar{h} = \frac{51.86 \times 28.15 \times 10^{-3}}{0.3}$, so you will get, $\bar{h} = 4.866 W/m^2 K$.

Now, you have to consider the post convection and first you calculate the Reynolds number, then check whether it is laminar or turbulent flows. You know that if $Re_H < 10^5$ then flow will be laminar for flow over flat plate case. And then, you use the suitable relation.

So, for forced convection $\mathrm{Re}_H = \frac{U_\infty H}{\nu}$. So, it will be $\frac{4 \times 0.3}{18.41 \times 10^{-6}}$. So, you will get 6.51×10^4, so the flow is laminar. And if you remember from the (Refer Time: 19:30) solution we calculate $\delta_H = \frac{5H}{\sqrt{\mathrm{Re}_H}}$.

So, you will be able to calculate $\frac{5 \times 0.3}{\left(6.51 \times 10^4\right)^{1/2}}$. So, this you will get as 5.88×10^{-3} m, so it will be 5.88 mm. So, you can see in case of post convection flow your boundary layer thickness is much much smaller than the natural convection boundary layer thickness.

So, now calculate the Nusselt number. So, $0.664 \mathrm{Re}_H^{1/2} \mathrm{Pr}^{1/3}$. So, you can see you can write $0.664 \times \left(6.51 \times 10^4\right)^{1/2} (0.7)^{1/3}$. So, if you calculate these you will get 150.4 and $\bar{h} = \frac{150.4 \times 28.15 \times 10^{-3}}{0.3}$. So, you will get as 14.11 W/m^{2}K

So, you can see here that in force convection flow obviously, your heat transfer is higher than the natural convection flow.

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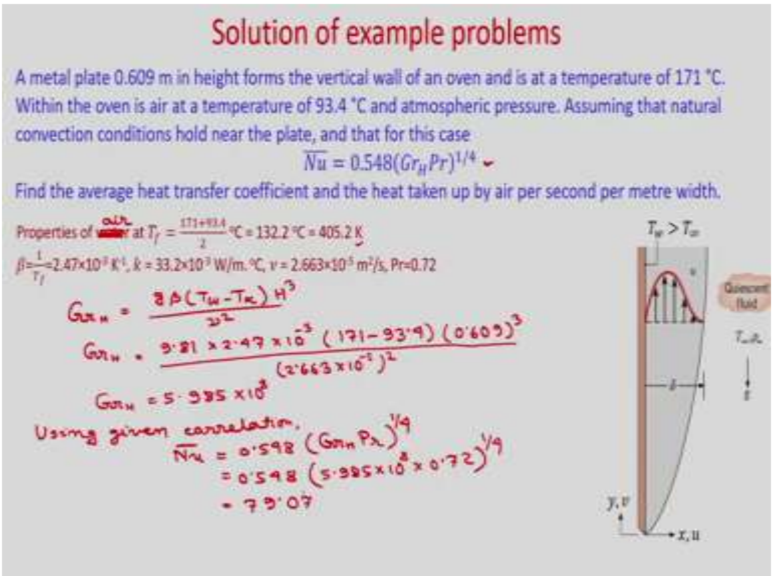

Now, let us consider the next problem. A metal plate 0.609 m in height forms the vertical wall open oven and is at a temperature of 171 ^{0}C. Within the oven is air at a temperature of 93.4 ^{0}C and atmospheric pressure. Assuming that natural convection conditions hold near the plate and that is for this case the average Nusselt number is given as this relation. Find the average heat transfer coefficient and the heat taken up by air per second per meter width.

So, the relation for the Nusselt number is given, so you have to use it. And properties for air, so it is not water, is evaluated at a film temperature 4.502 K and β thermal conductivity kinetic viscosity and Prandtl number are given.

So, first you calculate the Grashof number. Check whether flow is laminar or turbulent. So, you can see it is $Gr_H = \frac{g\beta(T_w - T_\infty)H^3}{\nu^2}$. So, Grashof number you can write as, $\frac{9.81 \times 2.47 \times 10^{-3} \times (171 - 93.4)(0.609)^3}{(2.663 \times 10^{-5})^2}$. So, if you evaluate it you will get Grashof number as 5.985×10^8. So, you can consider the flow is laminar however, the Nusselt number relation is given. So, you have to use it. So, using given correlation you can write $\overline{Nu} = 0.598(Gr_H \Pr)^{1/4}$. So, from here you can see you can calculate $\overline{Nu} = 0.548(5.985 \times 10^8 \times 0.72)^{1/4}$. So, Nusselt number you will get as 79.07.

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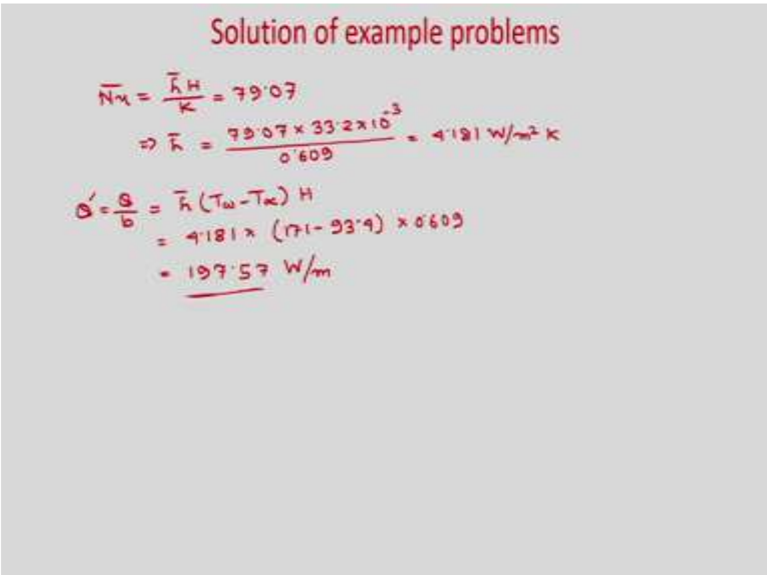

So, now Nusselt number you can write as $\frac{\bar{h}H}{K}$, so it will be 79.07. So, $\bar{h} = \frac{79.07 \times 33.2 \times 10^{-3}}{0.609}$. So, from here if you evaluate these you will get 4.181 W/m^{2}K.

So, average heat transfer coefficient we have calculated, now we are asked to calculate the heat transfer rate, per unit width of the plate. So, that you can use the Newton's law of cooling. So, we will write the heat transfer rate per unit width as $Q' = \frac{Q}{b}$, so per meter actually.

So, it will be $\bar{h}\left(T_w - T_\infty\right)H$. So, it is $4.181 \times \left(171 - 93.4\right) \times 0.609$. So, if you evaluate it you will get 197.57 W/m. So, this is the heat transfer rate.

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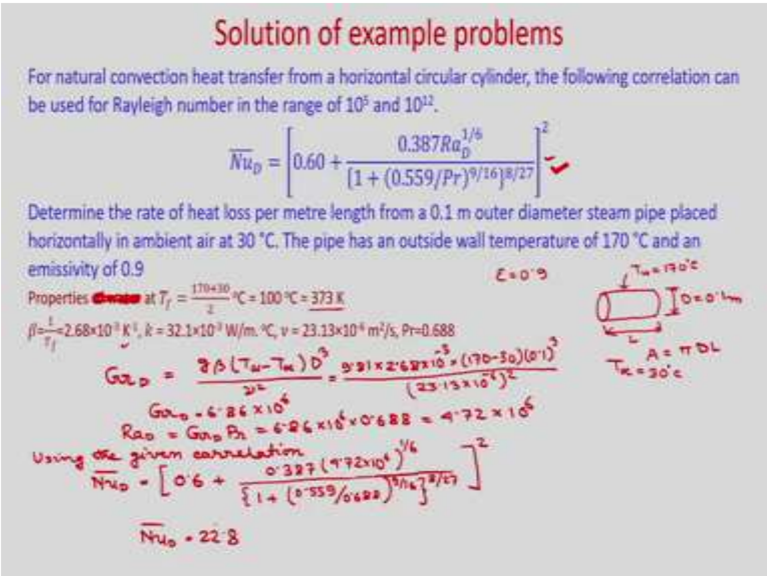

So, now let us discuss about the next problem. For natural convection heat transfer from a horizontal circular cylinder, the following correlation can be used for Rayleigh number in the range of 10^5 and 10^{12}. So, Nusselt number correlation is given for the natural convection heat transfer from a horizontal circular cylinder. So, horizontal circular cylinder you have. So, from here this is the correlation given.

Determine the rate of heat loss per meter length from a 0.1 m outer diameter. So, outer diameter is given as 0.1 m and if this is the length then A= π DL. And per unit length it will be π D.

So, determine the rate of heat loss per meter length from a 0.1 m outer diameter steam pipe placed horizontally in ambient air at 30 ^{0}C. So, T_∞ is 30 ^{0}C. The pipe has an outside wall temperature of 170 ^{0}C. So, T_w is given as 170 ^{0}C. And emissivity also given, so you have to find what is the radiative heat transfer rate.

So, again the properties are calculated at the film temperature. So, you can see the film temperature is 373 K and $\beta = \frac{1}{373}$. So, you can calculate the β and thermal conductivity, kinetic viscosity and Prandtl number are given. So, you can now calculate the Grashof number first.

So, Grashof number based on the diameter, $Gr_D = \frac{g\beta\left(T_w - T_\infty\right)D^3}{\nu^2}$. So, you can see $\frac{9.81\times 2.68\times 10^{-3}\times\left(170-30\right)\left(0.1\right)^3}{\left(23.13\times 10^{-6}\right)^2}$.

So, Grashof number you will get as 6.86×10^6 and $Ra_D = Gr_D \Pr$. So, it will be $6.86\times 10^6 \times 0.688$ and you will get as 4.72×10^6.

Now, using the given correlation you can calculate the Nusselt number and from there you can calculate the average heat transfer coefficient. Once you know the average heat transfer coefficient, then you will be able to calculate the heat transfer rate.

So, from this relation, you calculate the Nusselt number using the given correlation, $\overline{Nu_D} = \left[0.6 + \frac{0.387\left(4.72\times 10^6\right)^{1/6}}{\left\{1+\left(\frac{0.559}{0.688}\right)^{9/16}\right\}^{8/27}}\right]^2$. So, if you evaluate it you will get as 22.8.

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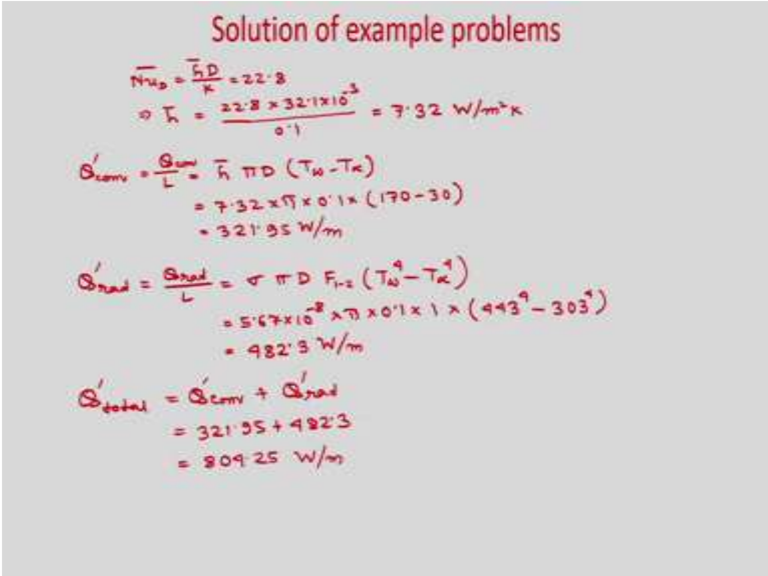

So, now you calculate the heat transfer coefficient. So, $\overline{Nu}_D = \frac{\bar{h}D}{K}$. This your average heat transfer coefficient 22.8. So, $\bar{h} = \frac{22.8 \times 32.1 \times 10^{-3}}{0.1}$. So, you can calculate these, $\bar{h} = 7.32 W/m^2 K$.

Now, you can calculate the heat transfer rate per unit length, because A= π DL. So, you will write the per unit length, so it will be π D. So, $Q'_{conv} = \frac{Q_{conv}}{L}$. So, it will be, $\bar{h}\pi D(T_w - T_\infty)$. So, this is a $7.32 \times \pi \times 0.1 \times (170-30)$, so you will get 321.95 W/m.

So, we have to calculate the heat transfer rate from the natural convection. Now, you have to calculate the heat transfer rate due to radiation. So, you can write $Q'_{rad} = \frac{Q_{rad}}{L}$. So, this is $\sigma \pi D F_{1-2}\left(T_w^4 - T_\infty^4\right)$. So, these temperature we have to write in K. So, $5.67 \times 10^{-8} \times \pi \times 0.1 \times 1 \times \left(443^4 - 303^4\right)$. So, you will get 482.3 W/m.

So, now, you have calculated heat transfer rate due to convection and radiation, so $Q'_{total} = Q'_{conv} + Q'_{rad}$. So, it will be 321.95 + 482.3, so you can write as 804.25 W/m.

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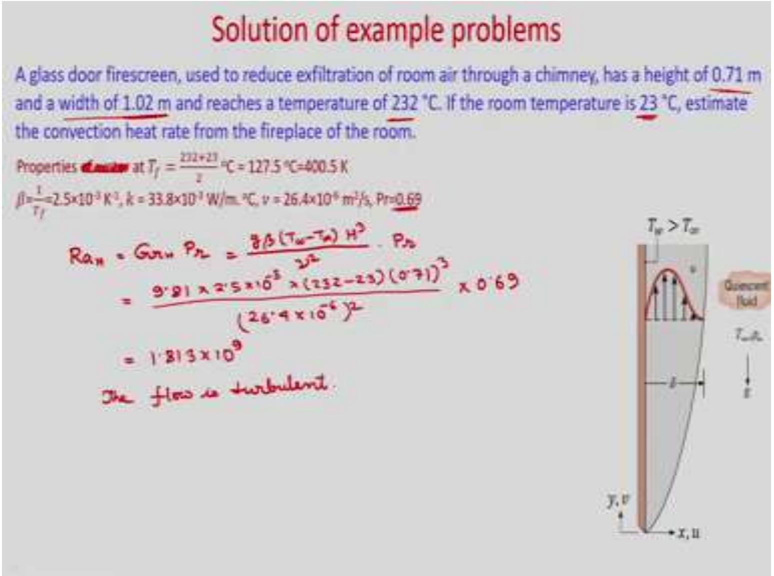

So, next let us consider one problem of turbulent flows. A glass door fire screen used to reduce exfiltration of room air through a chimney has a height of 0.71 m and width of 1.02 m and reaches a temperature of 232 ^{0}C. If the room temperature is 23 ^{0}C estimate the convection heat rate from the fires place of the room.

So, you can see your wall temperature will be 232 ^{0}C and ambient temperature is 3 ^{0}C. And height of the plate will be 0.71 m in this case width of the plate is given, 1.02 m. So, you can calculate the area as product of this height into width.

So, properties at film temperature you can calculate for the air, so it is 400.5 K, this is the film temperature. So, other properties are given here. So, the $Ra_H = Gr_H \Pr$.

So, $\frac{g\beta(T_w - T_\infty)H^3}{\nu^2}\Pr$. So, you can calculate Rayleigh number as $\frac{9.81\times 2.5\times 10^{-3}\times(232-23)\times(0.71)^3}{(26.4\times 10^{-6})^2}\times 0.69$. So, it is 1.813×10^9 and the flow is turbulent. So, you can use Churchill and Chu correlation for these turbulent flows.

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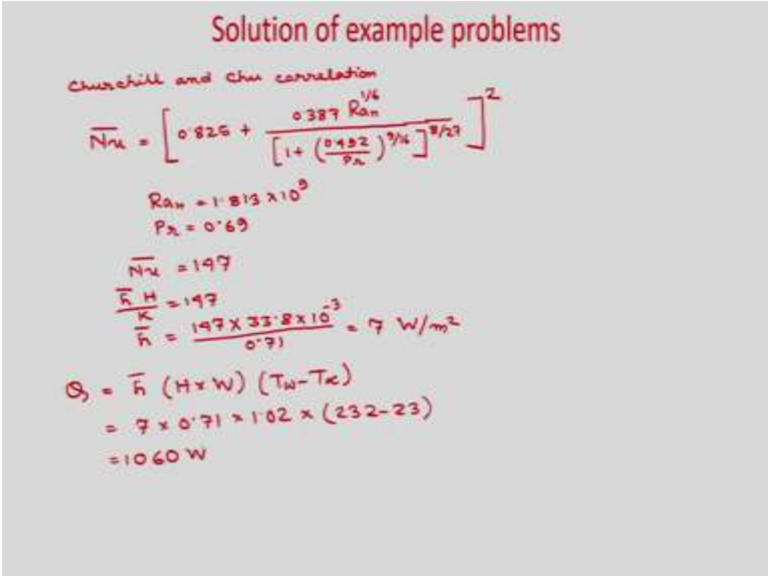

So, Churchill and Chu correlation is given as $\overline{Nu_D} = \left[0.825 + \frac{0.387 \left(Ra_H \right)^{1/6}}{\left\{ 1 + \left(\frac{0.429}{\Pr} \right)^{9/16} \right\}^{8/27}} \right]^2$.

So, you put the values of Rayleigh number and Prandtl number here. So, you will get, $Ra_H = 1.813 \times 10^9$ and Pr = 0.69. So, if you put this values and evaluate it you will get $\overline{Nu} = 147$.

So, from here now you calculate the average heat transfer coefficient and from there you will be able to calculate the heat transfer rate. So, we can write $\frac{\bar{h}H}{K}$ = 147. So, $\bar{h} = \frac{147 \times 33.8 \times 10^{-3}}{0.71}$. So, you will get as 7 W/m^2.

So, your heat transfer rate you can calculate as $\bar{h}A$. So, $Q = \bar{h}\left(H \times W \right)\left(T_w - T_\infty \right)$. So, $7 \times 0.71 \times 1.02 \times \left(232 - 23 \right)$. So, if you evaluate it you will get 1060 W.

So, in today's class first we discuss about the turbulent natural convection flows. And if the Gr > 10^9, then you can say that the flow is turbulent. So, critical $Gr_{crit} = 10^9$ to determine whether the flow is laminar or turbulent.

Then, we considered 4 problems. So, while solving the problem first you need to calculate the Grashof number and check whether the flow is laminar or turbulent, and suitably you use the relation for the Nusselt number. And one problem also we have solved using Churchill and Chu correlation for the turbulent flows. You can solve more problems from any textbook.

Thank you.