Fundamentals of Convective Heat Transfer
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Module – 09
Natural Convection - II
Lecture – 30
Natural convection over inclined plate and mixed convection

Hello everyone, so we have already derived the Nusselt number expression for Natural convection over a vertical flat plate using integral solution as well as similarity solution, but you will find the natural convection in other configurations as well.

Say if you have an inclined plates or horizontal plate, and in horizontal plate if top side is 1 or bottom side is 1, then you will get different types of flow and Nusselt number expression will be different. Even if you consider horizontal circular cylinder, then also you will get different Nusselt number expression. So, today we will discuss about these configurations, and later we will just discuss about mixed convection flow.

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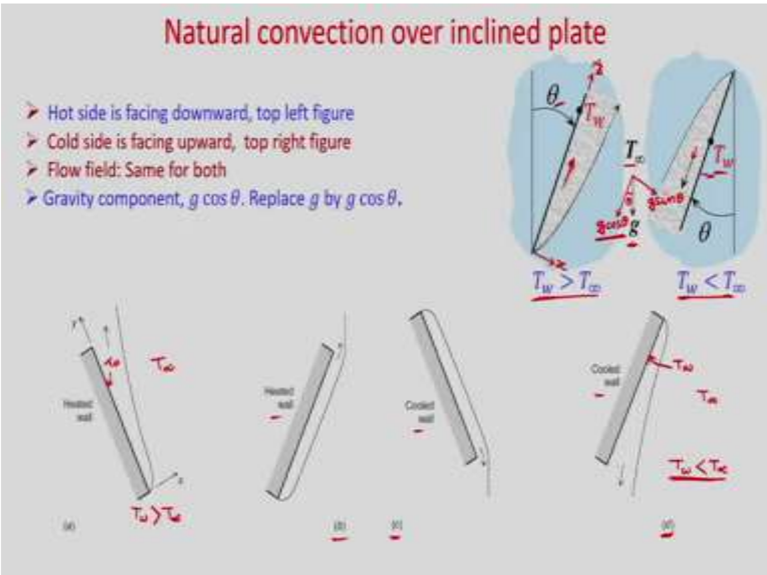

So, you can see in this case, say if it is an inclined plate where θ is measured from the vertical line and T_w is the wall temperature and ambient temperature is T_∞, and gravity is acting in negative y-direction. And if $T_w > T_\infty$, that means, your wall is maintained at higher temperature than the ambient temperature, then obviously, you will get this upward flow. And if your wall temperature $T_w < T_\infty$, then obviously you can see you will get downward flow.

So, you can see it depends hot side is facing downward. So, you can see here in this case top side is facing downward and $T_w > T_\infty$, and you are getting flow like this. And in this case, you can see your cold side is facing upward. So, this is the cold plate because $T_w < T_\infty$.

So, cold side is facing upward. So, here $T_w < T_\infty$, and flow is happening in downward direction. And here flow is happening in upward direction due to natural convection.

But if you see these two cases are same, because you will get the similar boundary layer and similar expression for the Nusselt number. So, you can see now if gravity is acting in negative y-direction, then in the flow direction, so you need to take gravity accordingly. So, when you are defining the buoyancy term in place of g, you just put g cosθ.

So, you will get two components. So, you can see this is your θ, and this will get g cosθ, and in this direction g sinθ. So, you can see in this particular case if y is along the plate, then in y momentum equation you will get g cosθ. So, you can see in place of g, you just write g cosθ in the buoyancy term.

Now, you can see here four different cases. In this case, the first case you can see heated wall tilted upward, so heated wall tilted upward. And the case d, you can see in this case cooled wall tilted downward.

So, when the natural convection is happening for this particular case, so here in this case $T_w > T_\infty$, and this is your T_w wall temperature and ambient temperature is T_∞. And similarly this is your T_w, and ambient temperature is T_∞, and $T_w < T_\infty$.

So, you can see already we have discussed here both the cases are same. You can see the effect of the angle θ is to thicken the tail end of the buoyancy layer and to give the wall jet a tendency to separate from the wall. So, you can see as it is going up, so you are at the tail end boundary layer is thicken. And in this case where $T_w < T_\infty$ also it is similar observation you can see.

The next one you can see here heated wall tilted in downward direction and cooled wall is tilted in upward direction . So, it is the opposite effect you will get. In this case, you can see the wall jet is pinched as it flows over the trailing edge, because in this case as it is going up you can see it is the jets is pinched as it flows over the trailing edge .

So, this is the different cases you can see when natural convection is taking place over a flat plate, but depending on the wall condition, you will get different flow physics . So, in this case b and c are similar.

So, although using the similarity solution and integral solution, we could find the Nusselt number expression for natural convection over a vertical flat plate, but you can also write some empirical relations. So, in the literature, different researchers performed experiments; and based on that they are proposed some relations which are known as empirical relations.

For these particular cases whatever we discussed, the similar expression you can use whatever we had already discussed from the similarity solution and integral solution just replacing g with the g cosθ. So, in the buoyancy term or the non-dimensional number whatever you are considering Grashof number or Rayleigh number. So, there you replace g with g cosθ then you will get the Nusselt number as same for this cases.

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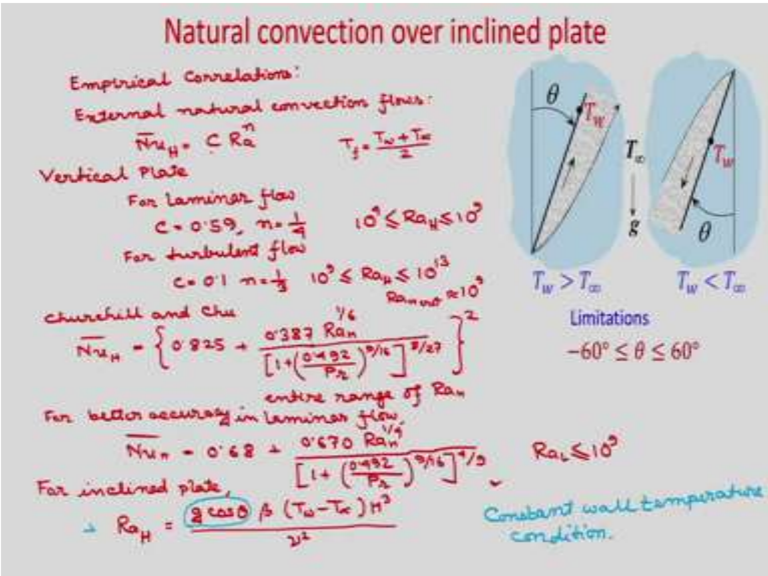

So, first let us write empirical correlations for external natural convection flows. So, Nusselt number, average Nusselt number based on some characteristic length, you can write as $\overline{Nu}_H = CRa^n$.

So, Ra^n, so where the properties are evaluated at film temperature you know that is $T_f = \frac{T_w + T_\infty}{2}$. So, it is the average temperature. So, that is known as film temperature. So, at that temperature, you need to find the properties and for different types of flows you will get different constant C and this power n.

So, for vertical plate, for laminar flow, you will get C = 0.59 and n = 1 / 4 in the range of $10^4 \leq \mathrm{Ra_H} \leq 10^9$, or in this case let us write H because we have used H.

So, generally in natural convection the flow becomes turbulent if $Ra_H \geq 10^9$. So, for turbulent flows you can write C =0.1, and n =1 / 3 in the range of $10^9 \leq Ra_H \leq 10^{13}$.

So, generally critical Reynolds number for turbulent flows, it is around 10^9. I hope that you will get the turbulent flows. And Churchill and Chu, they proposed for any range of Rayleigh number average, so it is, $\overline{Nu}_H = \left\{ 0.825 + \frac{0.387 Ra_H^{1/6}}{\left[1 + \left(\frac{0.492}{\mathrm{Pr}} \right)^{9/16} \right]^{8/27}} \right\}^2$.

So, these actually works good in reasonably good in the all range of Rayleigh number. So, it is entire range of Rayleigh number. But for better accuracy, in laminar flow, you can use this relation. $\overline{Nu}_H = 0.68 + \frac{0.670 Ra_H^{1/4}}{\left[1 + \left(\frac{0.492}{\mathrm{Pr}} \right)^{9/16} \right]^{4/9}}$. So, in the laminar flow range, you will get good accuracy if you use this correlation .

So, you can see these are for vertical plate. And as we are discussing inclined plate, so for inclined plate just you replace g = g cosθ. So, where you have Rayleigh number you just for inclined plate Rayleigh number will be $Ra_H = \frac{g \cos\theta \beta \left(T_w - T_\infty \right) H^3}{\nu^2}$.

So, you can see for inclined plate we have put g cosθ in place of g. And you remember in this case it is for all these relations we have used for constant wall temperature case. So, all these relations, you can use for inclined plate just using these Rayleigh number expression.

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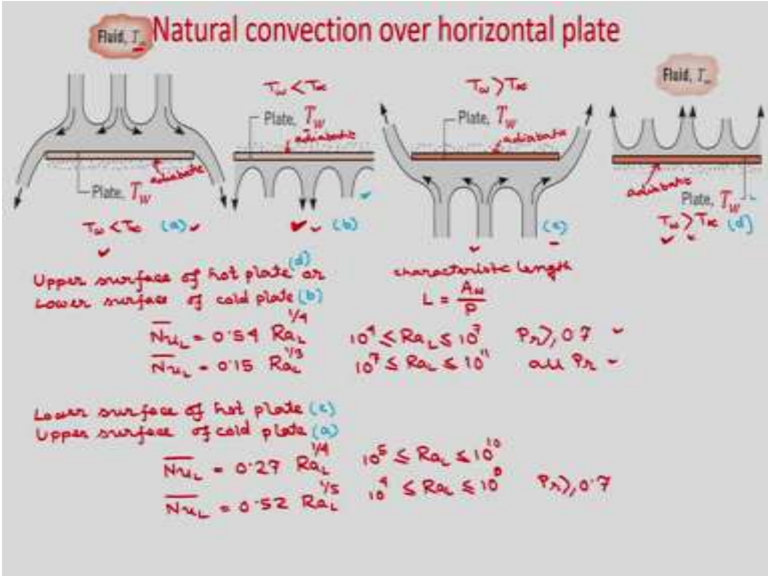

Now, if you consider horizontal plate, and in the horizontal plate if top side is warm or bottom side is warm, so depending on the thermal conditions, you will get different flow physics and you will get different Nusselt number expression. So, you can see here in this case say this is the horizontal plate and bottom side is adiabatic.

So, it is adiabatic. And topside is your T_w, and $T_w < T_\infty$. So, how, what is the flow physics you will get? See you can see as $T_w < T_\infty$ the fluid whatever is coming, so it will get cooled and these high density fluid will just go down. So, this way you will get the flow.

And now if topside is adiabatic, so there is no heat transfer across the top wall. And now bottom side temperature is T_w, and $T_w < T_\infty$, so that means it is cooled wall. So, in this particular case, obviously, the fluid coming into contact with the wall, so the density will become higher and it will go down. So, this way you will get that the high density fluid will be drained down.

Next case you can see it is also adiabatic top wall, but now $T_w > T_\infty$. If $T_w > T_\infty$ the fluid which coming into contact with the wall obviously the density will be lighter and it will go up.

So, this way you can see it is going up. And for this particular case, now this side is adiabatic and top side is now warm warmer than the ambient, so $T_w > T_\infty$. So, obviously, lighter fluid will go up, so this way. So, you will get this type of flow physics.

So, now, we will write the correlations for these cases. You can see this case and this case are almost similar. So, these are similar flow physics you are getting. And this case and this case you are getting similar flow physics. So, we can write the correlations as same for these two cases, and also for these two cases. So, you can see upper surface of hot plate or lower surface of cold plate.

So, if I say that this is the case a, this is the case b, this is the case c and this is the case d. Then upper surface of hot plates, that means, this is the case, so this is your case d . And lower surface of cold plate so; that means, this is lower surface of cold plate, so this is the case b .

So, for these two cases, you can write the correlation Nusselt number based on some characteristic length L. I am telling characteristic length L because $L = \frac{A_w}{P}$. So, this is the characteristic length, and it will be calculated as the heat transfer area – A_w divided by the perimeter , $L = \frac{A_w}{P}$.

So, for these you will write $\overline{Nu}_L = 0.54 Ra_L^{1/4}$ in the range of $10^4 \le Ra_L \le 10^7$, and Pr ≥ 0.7.

And for higher Rayleigh number, you can write Nusselt number. So, these are average $\overline{Nu}_L = 0.15 Ra_L^{1/3}, 10^7 \le Ra_L \le 10^{11}$ for all Prandtl number. So, you can see the first expression for laminar flow and it is for turbulent flow.

And now other two cases, you see this is the case a where upper surface of cold plate, and this is the bottom surface of the hot plate. So, the other cases you can write lower surface of hot plate. So, this is the case c. And you have upper surface of cold plate. So, this is the case.

So, you can write the case this is as c, and this is the case a. So, for these cases, this is the empirical relation you can use. $\overline{Nu}_L = 0.27 Ra_L^{1/4}$ in the range of $10^5 \le Ra_L \le 10^{10}$.

And also another expression is there $\overline{Nu_L} = 0.52 Ra_L^{1/5}$. This is also correlation. This is valid in the range of $10^4 \le Ra_L \le 10^9$, and $\Pr \ge 0.7$.

So, these are the correlations you can use, but here characteristic length you have to calculate as heat transfer area divided by the perimeter; it is not the just length of the plate. So, this way you can calculate. So, obviously, for different surfaces if it is a circular disc if it is a circular disc also you can use this relations using the characteristic length, $L = \dfrac{A_w}{P}$.

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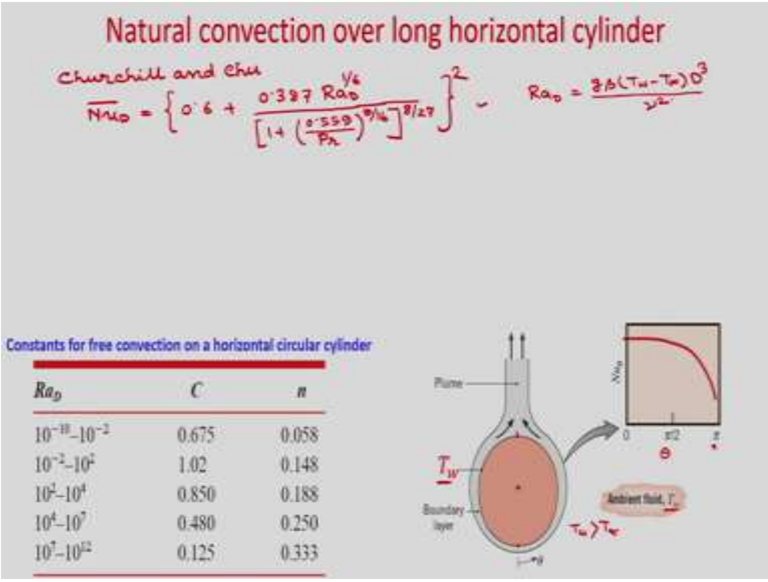

Now, let us consider flow over a horizontal circular cylinder. So, you can see this is the horizontal circular cylinder, where wall temperature is T_w, and ambient temperature is T_∞. In this case, $T_w > T_\infty$. So, obviously, you can see boundary layer will be developed over this cylinder surface. And due to buoyancy effect, it will go up. And this lighter fluid will go up.

So, if you plot the Nusselt number along θ from here, so it is symmetric about this vertical line. So, along θ, if you see the Nusselt number, obviously, at θ = 0. That means, at this point you will get maximum Nusselt number.

Why, because you have low boundary layer or lowest boundary layer thickness at this point, and temperature gradient will be highest, and obviously, you will get maximum Nusselt number at θ =0. This is θ. So, θ = 0 we will get maximum Nusselt number.

As you go along this surface , so your Nusselt number will decrease. So, you can see, so obviously, minimum Nusselt number will get at this point at θ =π . So, the correlations proposed by Churchill and Chu, for this long horizontal cylinder case as Nusselt number based on the diameter D.

It is $\overline{Nu}_D = \left\{ 0.6 + \dfrac{0.387 Ra_H^{1/6}}{\left[1+\left(\dfrac{0.559}{\Pr}\right)^{9/16}\right]^{8/27}} \right\}^2$. So, this is the empirical relation for natural convection over long horizontal cylinder, and Rayleigh number is based on diameter. So, it is $Ra_D = \dfrac{g\beta\left(T_w - T_\infty\right)D^3}{\nu^2}$.

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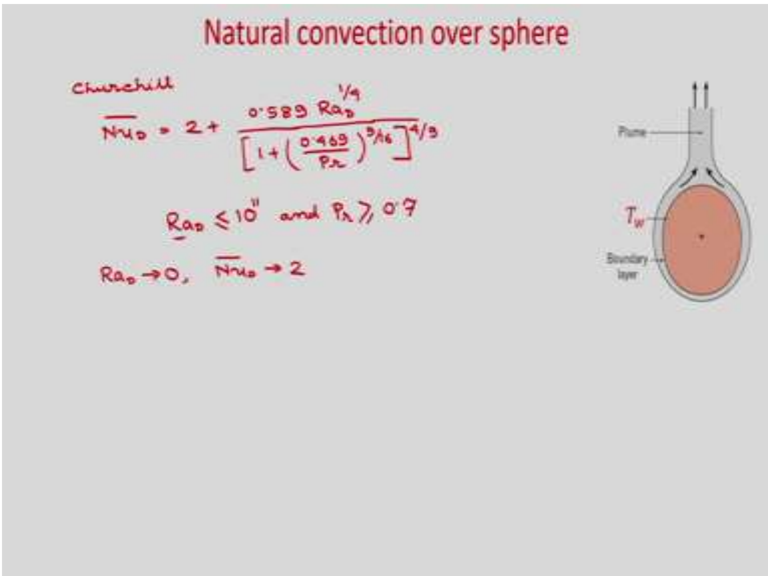

Now, if you consider a sphere and natural convection over a hot sphere, then we can write the correlation proposed by Churchill. So, this is also correlation. So, Churchill recommended this Nusselt number correlation as $\overline{Nu}_D = 2 + \frac{0.589 Ra_D^{1/4}}{\left[1+\left(\frac{0.469}{\Pr}\right)^{9/16}\right]^{4/9}}$.

And it is valid in the range of $Ra_D \le 10^{11}$, and $\Pr \ge 0.7$. So, you can see this is the sphere and diameter of the sphere is D, and based on that Rayleigh number is defined.

Now, in this expression if you put $Ra_D \to 0$, then what will happen? So, if you put, $Ra_D \to 0$, then $\overline{Nu}_D \to 2$. So, what does it mean? It means that heat transfer by conduction between a spherical surface and stationary infinite medium. So, only conduction will be taking place as $Ra_D \to 0$.

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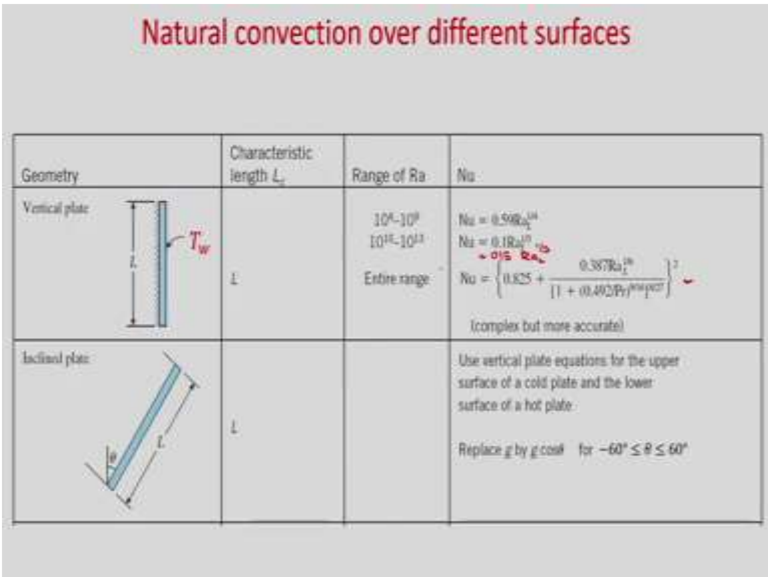

Now, let us summarize whatever we have discuss about the empirical relations, based on the experiments natural convection over different surfaces. So, vertical plate, you can see if you see characteristic length L, so in the range of 10^4 to 10^9, $Nu = 0.59 Ra_L^{1/4}$.

So, L is the characteristic length. And if it is high Rayleigh number in the range of turbulent flows, then it will be 0.1 or 0.15 also you can write 0.15, $Nu = 0.15 Ra_L^{1/3}$.

And for entire range of Rayleigh number, so this is the Churchill and Chu proposed. So, this is the expression you can use. If it is inclined plate and θ is measured from the vertical line, and just replace g by g cos θ, in the relations whatever we discussed for this vertical plate. And you can see we have a restriction that this θ should be between -60^0 to 60^0. So, in that range, you will get a good match.

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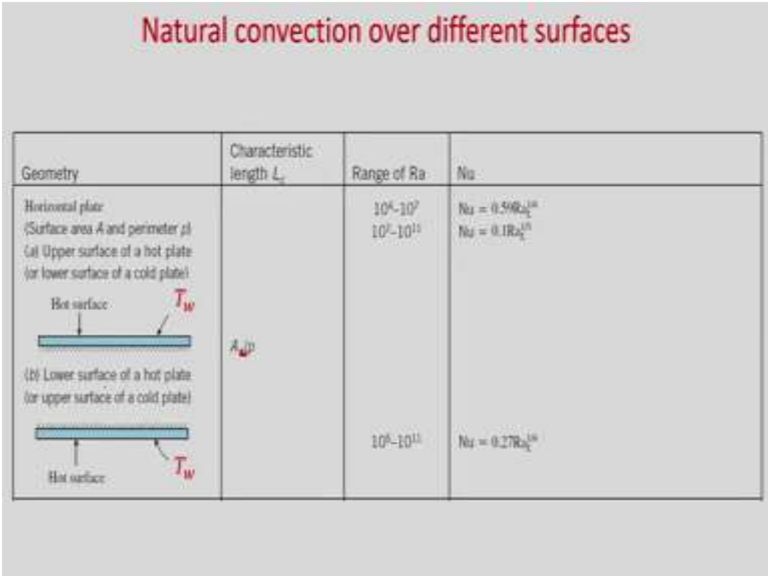
Natural convection over different surfaces

Geometry	Characteristic length L_c	Range of Ra	Nu
Horizontal plate (Surface area A and perimeter p) (a) Upper surface of a hot plate (or lower surface of a cold plate)	A_w/p	10^4–10^7	$Nu = 0.59Ra_L^{1/4}$
		10^7–10^{11}	$Nu = 0.1Ra_L^{1/3}$
(b) Lower surface of a hot plate (or upper surface of a cold plate)		10^5–10^{11}	$Nu = 0.27Ra_L^{1/4}$

So, for horizontal plate you can see that you can calculate the characteristic length as $L = \frac{A_w}{P}$. And when you have a hot surface on the top upper surface of a hot plate or lower surface of a cold plate, so you can use these Nusselt number expression for these range of Rayleigh number. And similarly lower surface of a hot plate or upper surface of a cold plate, you can use this relation.

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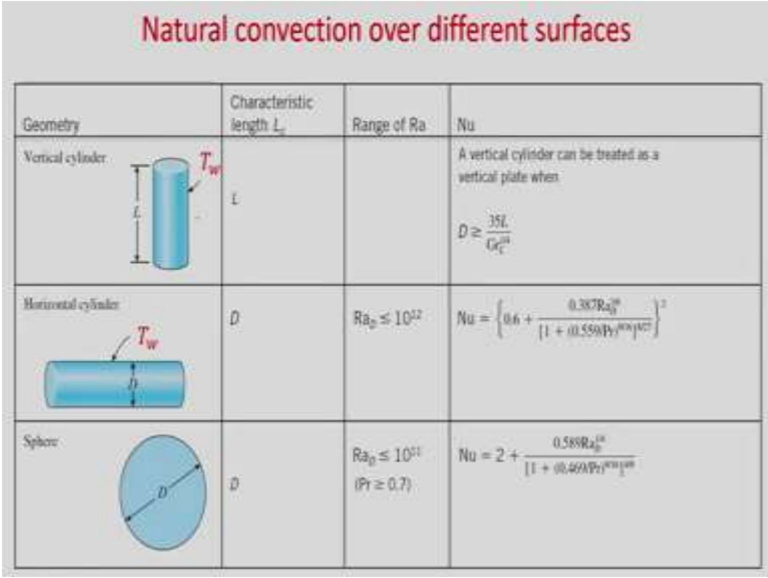

Natural convection over different surfaces

Geometry	Characteristic length L_c	Range of Ra	Nu
Vertical cylinder	L		A vertical cylinder can be treated as a vertical plate when $D \geq \frac{35L}{Gr_L^{1/4}}$
Horizontal cylinder	D	$Ra_D \leq 10^{12}$	$Nu = \left\{0.6 + \frac{0.387 Ra_D^{1/6}}{[1 + (0.559/Pr)^{9/16}]^{8/27}}\right\}^2$
Sphere	D	$Ra_D \leq 10^{11}$ $(Pr \geq 0.7)$	$Nu = 2 + \frac{0.589 Ra_D^{1/4}}{[1 + (0.469/Pr)^{9/16}]^{4/9}}$

If it is a vertical cylinder, then you can write a vertical cylinder can be treated as a vertical plate when, $D \geq \frac{35L}{Gr_L^{1/4}}$. So, the same relations for vertical plate whatever we have discussed you can use if this relation is satisfied.

For a horizontal cylinder, $Ra_D \leq 10^{12}$, you can use this Churchill and Chu correlation. And for a sphere also we have discussed for $Ra_D \leq 10^{11}$ and $Pr \geq 0.7$, you can use this Nusselt number relation.

So, now, we will discuss about the mixed convection which is also known as combined forced and natural convections. So, in this particular case, we have already discussed that both the forces forced and buoyancy are important and both are significant. So, we have shown earlier that Richardson number actually determines whether flow is natural convection, or forced convection, or mixed convection.

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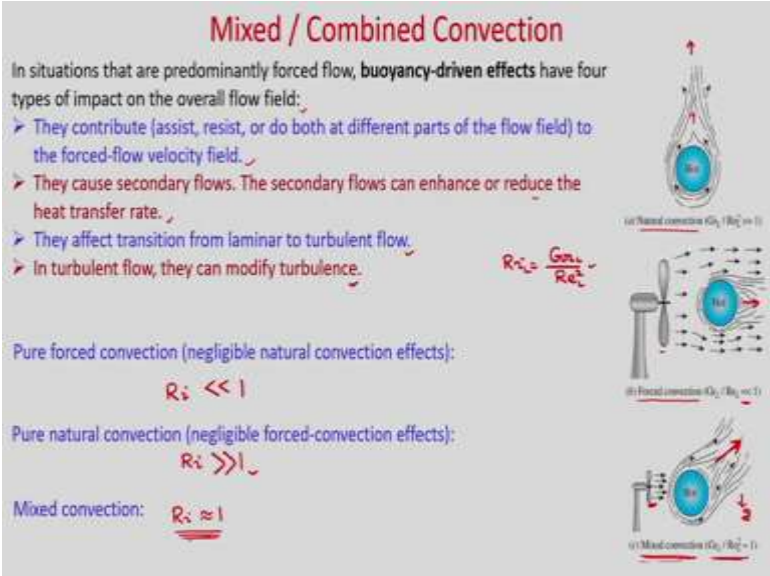

So, if you see that if Richardson number, we defined obviously as Grashof number by Reynolds number square right. So, if you some characteristic length if we define, then the $Ri_L = \dfrac{Gr_L}{\mathrm{Re}_L^2}$. So, this is the Richardson number. So, based on the Richardson number , we will determine whether flow is natural convection, or forced convection, or mixed convection.

So, there are many applications of mixed convection although if fluid velocity is less, then this forced convection and natural convection both will be significant. And in the application of electronics cooling or few heat exchangers, you will see that application of mixed convection.

In situations that are predominantly forced flow, buoyancy driven effects have four types of impact on the overall flow field. So, we can see they contribute either assist, resist, or do both at different parts of the flow field to the forced flow velocity field. They cause secondary flows.

The secondary flows can enhance or reduce the heat transfer rate . So, in presence of buoyancy effect whether it is assisting or resisting, based on that you will get the heat transfer enhancement or heat transfer reduction.

They affect transition from laminar to turbulent flows. And in turbulent flow, they can modify turbulence. So, we know that for pure forced convection, where you can neglect the natural convection effect. So, for this particular case, you can see that your Ri << 1, then you will get pure forced convection.

So, we can see example. So, you have a hot sphere and flow is taking place in the horizontal direction forced flow. So, it is a forced convection. So, where you have Ri<<1. For pure natural convection where you can neglect the forced convection effect, so their Ri>> 1.

So, this is the case you can see in the absence of forced convection, you will get only the natural convection. And due to buoyancy effect the flow will go up and flume will go up in this direction.

So, here actually with negligible forced convection effects, you can write or you can consider that it is a natural convection and the condition is Ri>> 1. But if Ri=1 or Ri ~ 1, then obviously, you will get the effect of both natural and forced convection, so that is known as mixed convection or combined convection.

So, you can see in this case there is a forced flow. And also you have buoyancy effect, and due to that the there will be effect of natural convection as well as forced convection here.

And it is known as mixed convection where Ri ~ 1. And you can see in this case forced convection flow will become only horizontal direction. In natural convection as you have absence of forced convection, flow will occur at the vertical direction due to the buoyancy effect. But when you have a mixed convection, so the both effect if you consider then you can see the flume will go up with an inclination. So, in this direction, it is going.

So, in this for a forced convection, you can see flow is occurring in this direction; in natural convection it is going in this direction. But for a mixed convection as it is having the combined effect of forced convection and natural convection, flow is occurring with an inclination. So, now, let us discuss whether this effect of buoyancy will assist or resist the flow.

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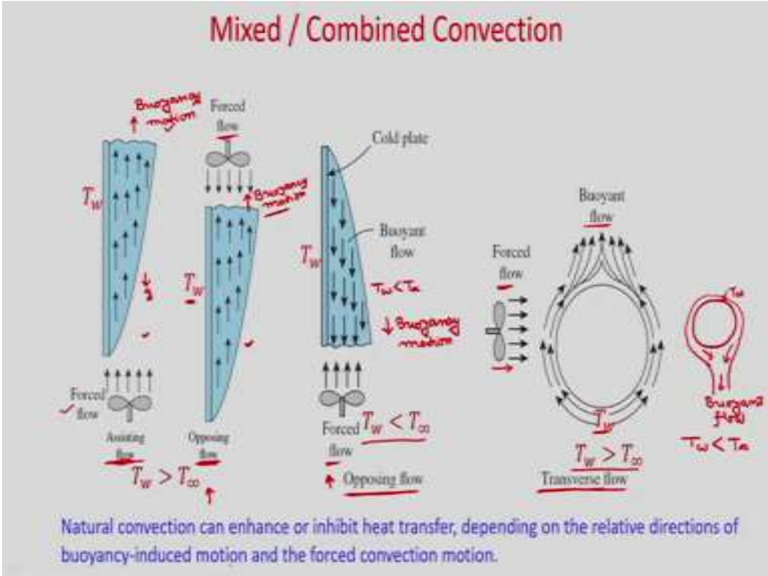

So, you can see here this particular case, you have a hot plate. And from the bottom, you are giving a forced flow. You have a fan; and from the fan this flow is going up. And buoyancy g is acting in this direction, and obviously, you can see buoyancy effect will be in this direction. Buoyancy emotion will be in this direction.

So, obviously, you can see that the forced flow and the buoyancy motion are in same direction, so that means, it is kind of assisting flow. So, this buoyancy effect is also assisting the forced flow. So, this is known as assisting flow.

And consider this case. So, you have a hot plate, and obviously, the buoyancy motion in this direction. So, in this particular case, flow is coming in opposite direction of the buoyancy motion. So, it is trying to resist the flow. So, this forced flow is trying to resist the flow, and that is why it is known as opposing flow.

So, you can see in the assisting flow, buoyancy motion and the forced flow are in same direction. But in opposing flow your forced flow and buoyancy motion are in opposite direction. In forced flow and assisting flow, the buoyancy motion and the forced flow are in same direction. So, this is known as assisting flow, and this is known as opposing flow where buoyancy motion and forced flow are in opposite direction.

If you consider now cold plate, so this is your cold plate where $T_w < T_\infty$, so obviously, your buoyancy motion will be in this directions buoyancy motion. Why? Because the fluid which will come into contact with the cold plate, its density will be higher and it will go down, and the flow will occur in the downward direction.

But you have a forced flow. So, now, you can see the buoyancy motion and forced flow are in opposite direction. So, obviously, it is also known as opposing flow. So, these two cases you see. In this case, $T_w > T_\infty$. So, flow is natural flow is occurring in upward direction, but forced flow is trying to resist it . Here forced flow and buoyancy motion are in opposite direction.

If you consider the if you consider these case also in this case $T_w < T_\infty$. And you are due to natural convection this buoyant flow is occurring in the downward direction. And in this case also forced flow and buoyancy motion are in opposite direction, and this is also opposing flow.

So, depending on the thermal condition and the direction of forced flow and the buoyant flow, you will define whether it is assisting flow or opposing flow. Assisting flow, sometime it is known as also aiding flow. And now if you see that forced flow is occurring in perpendicular direction of the buoyancy flow, then it is known as transverse flow.

So, this particular case, you see this is let us say you have a sphere at temperature T w. And $T_w > T_\infty$, obviously, your buoyant flow will occur in upward vertical direction.

So, in this case, it is due to natural convection this will happen, but you have forced flow in perpendicular direction of the buoyant flow. So, this is kind of cross flow. So, this is known as transverse flow. This is known as transverse flow because buoyant flow and the forced flow are occurring in perpendicular direction.

And if $T_w < T_\infty$, this is also transverse flow because you can see if it is your sphere and T_w is the wall temperature, and if $T_w < T_\infty$, then obviously you can see that your flow will occur buoyant flow will occur in the downward direction.

So, in this direction, the flow will occur. And you have this is your buoyant flow. And perpendicular to this, you can see you have a forced flow. So, this is also transverse flow. So, you can see natural convection can enhance or inhibit heat transfer depending on the relative directions of buoyancy induced motion and the forced convection motion.

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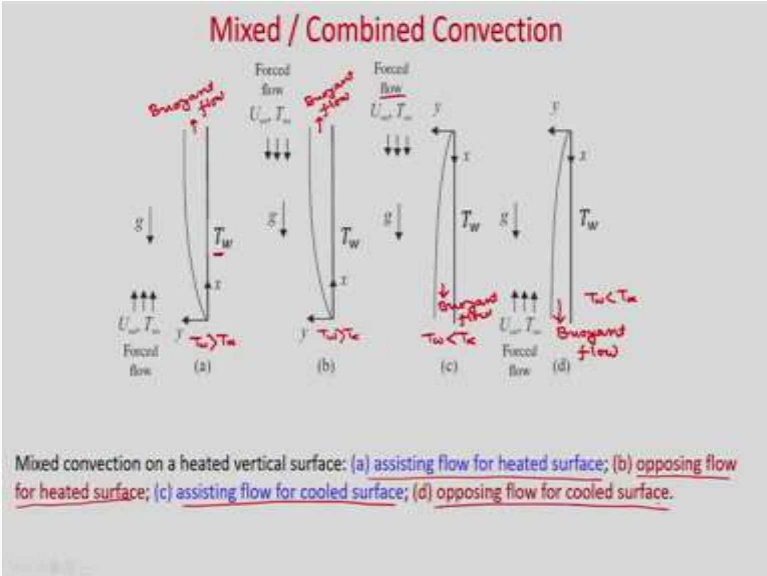

So, if you consider only flow over vertical plate, you can define the assisting or opposing flow for different thermal conditions. So, you can see in this case if it is heated surface, so $T_w > T_\infty$ in obviously, if it is forced flow direction in upward direction, then it is a assisting flow for heated surface.

In this case, here also $T_w > T_\infty$, and forced flow is just opposite to the buoyant motion, because buoyant motion will in these direction. These are buoyant motion or buoyant flow ok. So, you can see this is resisting the flow. So, this is known as opposing flow of heated surface.

Now, if you consider cooled surface where $T_w < T_\infty$ and forced flow is happening from top to bottom, then in these case buoyant flow in downward direction, and you will get assisting flow.

Assisting flow for cooled surface. Now, case d, if you see, here also it is a cooled surface and forced flow is occurring from bottom to upward direction, and buoyant flow is happening in these direction. So, these are in opposite direction. So, this is also opposing flow for cooled surface.

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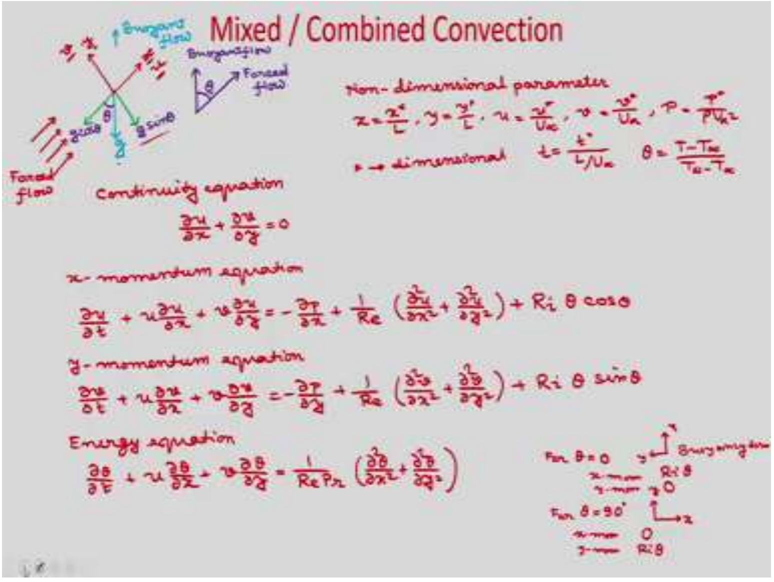

Now, let us write down the governing equations for this mixed convection. So, in this particular case, you can see your pressure gradient will also be there as well as there will be a buoyancy term. And this buoyancy term you can derive you can see from any book, and I am just writing the continuity equation momentum equation and the energy equation.

So, let us say that you have a forced flow in these direction. This is your forced flow. And if you define the axis x in the direction of forced flow, and perpendicular to this it is y. So, we are defining the coordinate system, in this way in the direction of forced flow x-direction and perpendicular to the direction of forced flow is y-direction.

So, now you can see obviously, your gravity will act in downward direction. So, this is your gravity. It is acting in downward direction. So, buoyant flow will be always in the upward direction. So, this is your buoyant flow. So, we will write the governing equations in x-direction and y-direction considering the buoyancy term.

So, now let us take two components of this gravity in x-direction and y-direction, and accordingly you can add it in the momentum equations. So, you can see in the if you take two components, this is one component in the negative x-direction you will get, and another component you will get in the negative y. So, and if you define the angle, so this will be your θ, so defining the θ,. So, you can see θ were defining, this is your buoyant flow; and this is your forced flow and this is the angle θ .

So, you can see this is your forced flow and this is your buoyant flow. So, this angle is we are defining as θ. So, you can have two components. So, one is g cosθ in the negative x-direction; and in negative y-direction, you have g sin θ. So, you see the definition of θ. And accordingly we are taking the component of g in negative x-direction and negative y-direction. So, and x-direction is due x-direction, we are taking in the direction of forced flow.

So, if you see that we have already written the non-dimensional equations using some non-dimensional parameters, and the governing equations in non-dimensional form we are going to write. So, you will see your continuity equation, you will get $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$. So, we are taking the velocity u in the x-direction, and velocity v in y-direction. So, this you have to consider.

So, x momentum equation if you see, it will be $\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+Ri\theta\cos\theta$, which is your non-dimensional temperature. And this velocity is u, v also we have written in non-dimensional form, and you have cos θ , because in x-direction you can see g cosθ right -g cos θ, so that you have written Ri θ cos θ.

And y momentum equation, so you will can see it will be, $\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+Ri\theta\sin\theta$. So, you see gravity; in negative y-direction, it is g sin θ.

So, if you consider the buoyancy term. So, sin θ will be there and energy equation you will get, $\frac{\partial \theta}{\partial t}+u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{\mathrm{RePr}}\left(\frac{\partial^2 \theta}{\partial x^2}+\frac{\partial^2 \theta}{\partial y^2}\right)$.

So, whatever non-dimensional number we are getting, it depends on that how you have chosen the non-dimensional parameters. For getting this non-dimensional equations, we have used the following non-dimensional parameters.

So, non-dimensional parameters we have used, non-dimensional parameter we have used, so $x = \frac{x^*}{L}$ now in this particular case I am just writing that star quantities dimensional. Because we have written the governing equations in non-dimensional form where u b we have given non-dimensional velocity so star quantities are dimensional.

So, and L is the characteristic length; $y = \frac{y^*}{L}$. You do not get confused because earlier we defined $x = \frac{x^*}{L}$ where x was the dimensional quantity here we are writing x_* is the dimensional quantity.

Just we have written the governing equations in non star form $u^* = \frac{u}{U_\infty}$; U_∞ is the characteristic velocity ; $v^* = \frac{v}{U_\infty}$, then $P^* = \frac{P}{\rho U_\infty^2}$, no these we have to write star do not get confused, and $t = \frac{t^*}{L/U_\infty}$. So, and $\theta = \frac{T - T_\infty}{T_w - T_\infty}$.

So, using these non-dimensional parameters, you can write the non-dimensional form of the governing equations as written here. So, if θ becomes 0, then what will happen? So, you will get sin θ as 0 and cos θ as 1.

And if θ is 0, then obviously you can see for θ = 0, you will get x and y like this; and you can see in the buoyant force and buoyant flow and forced flow will be in the same direction. And your x momentum equation will have Ri θ; and y momentum it will be 0 because sin θ is 0 and for $\theta = 90^0$.

So, what will happen? You will get x in this direction and y in this direction. So, in this case, x momentum equation, if it is 90^0 ,so x-momentum equation you will get buoyancy term, this is your buoyancy term buoyancy term. So, buoyancy term, we will get as 0, because $\cos 90^0 = 0$.

And in y momentum equation, you can see sin θ will be 1, so you will get desertion number into θ. So, you can use these equations for solving the mixed convection problem.

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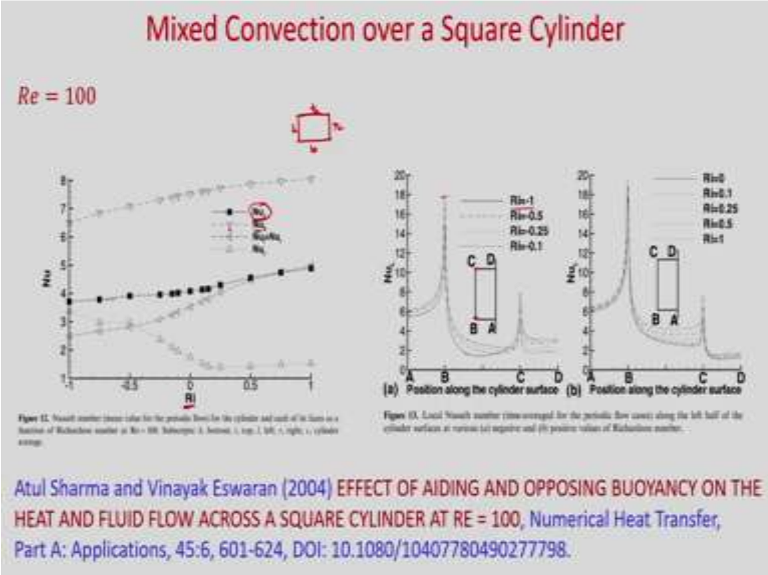

So, we will not go into details about finding the Nusselt number using some analytical approach, but we will show some results of mixed convection flow which are actually shown using numerical simulations.

So, first problem, you can see mixed convection over a square cylinder. So, you have a square cylinder maintained at temperature T_w and this square cylinder having the non-dimensional dimension as 1. And you have the vertical flow v_∞ and a main temperature is T_∞.

And these are the boundary conditions are used. And for Reynolds number 100 and for different Richardson number, this flow is calculated. So, it is taking this is taken from this paper Atul Sharma and Vinayak Eswaran Effect Of Aiding And Opposing Buoyancy On The Heat And Fluid Flow Across A Square Cylinder at Reynolds number 100 published in Numerical Heat Transfer, Part A, Applications.

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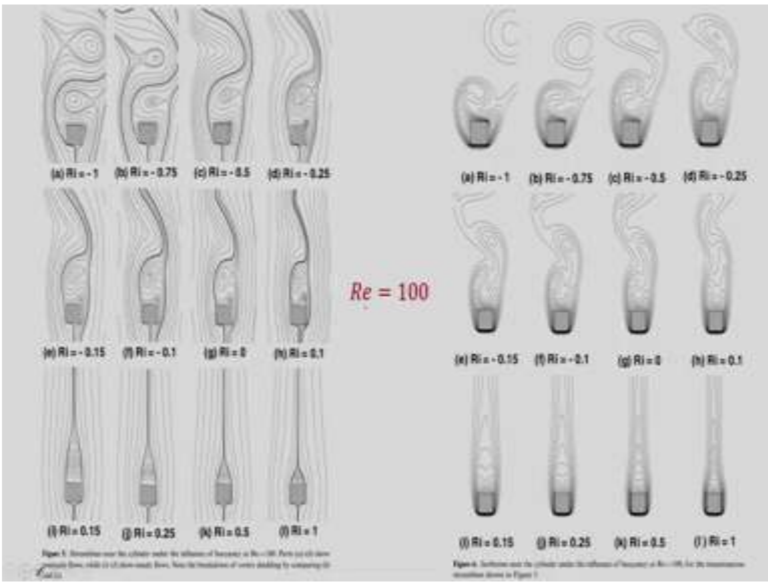

So, you can see that for Reynolds number 100, and Prandtl number is 0.7. You can see the flow physics. These are the streamlines near the cylinder under the influence of buoyancy. So, you can see these cases are from Richardson number 0.15 to 1, you can see you have a steady flow.

There is no setting of vertices behind these cylinder, so these are steady flow. But if you see in these ranges of Rayleigh number where it is 0.1, 0, and in the negative direction -0.1 to -1, you can see you have vortex shedding behind the cylinder and this is unsteady flow.

And if you see the isotherms in right side figures , so in the isotherms also, you can see that in this range Richardson number 0.15 to 1, you have steady flow. And you can see the clustering of the isotherms near to this section, obviously, you will get maximum Nusselt number here . So, and here also you can see the isotherms. So, these are also instantaneous isotherms for at shown at different Richardson number.

Now, if you see the Nusselt number, so this is average Nusselt number is shown with Richardson number. Nu_C means it is a combined Nusselt number means if you consider a square cylinder, so you have a bottom, you have a top, this is your left, and this is your right.

So, for these you can see different surfaces, these are the average Nusselt number. So, you can see Nusselt number bottom, this is the symbol. So, this is the Nusselt number variation with Richardson number. So, obviously, with in this case you can see increase with Richardson number your for the bottom wall, this Nusselt number is increasing.

And this is the average Nusselt number for the whole wall considering these bottom top left and right. So, this is the Nu_C. So, this is the combined for all the combining all the surfaces. So, this is the average Nusselt number. And with increase of Richardson number, it is increasing.

And this is the local Nusselt number along these ABCD, half of the domain it is shown because it is the time average Nusselt number is shown over the surfaces ABCD. And this is the local Nusselt number and we can see for Ri = 1, this is the plot solid line.

So, how the variation is happening? So, at A point, so you have a Nusselt number. And it is at this corner point, as you have clustering of the temperature, you will get a high Nusselt number, then again it will decrease along the C. And at C there will be another peak, because you have a corner and again it will decrease.

And it is from for different Richardson number 0 to 1, it is also shown. So, you have seen for either assisting or resisting buoyancy flow the flow over a square cylinder case.

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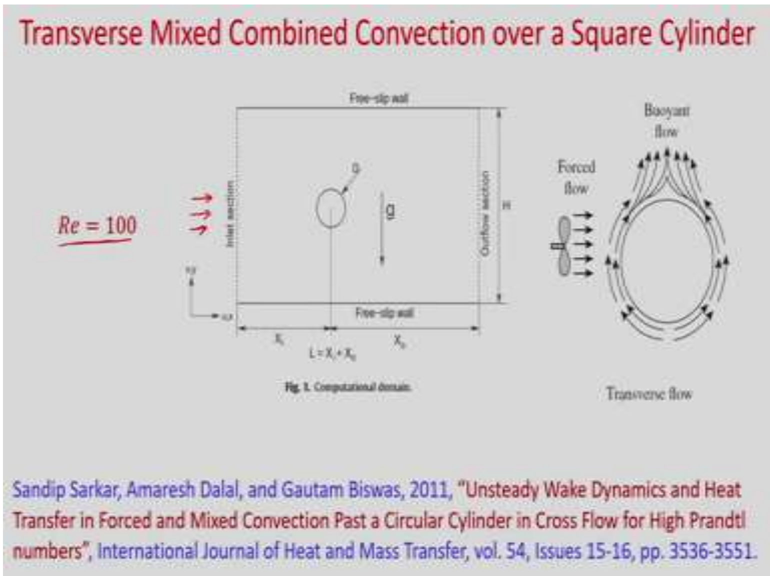

Now, if you have a transverse flow , so we are considering a transverse flow over a circular cylinder. So, this is taken from Sandip Sarkar, Amaresh Dalal and Gautam Biswas, Unsteady Wake Dynamics and Heat Transfer in a Forced and Mixed Convection Past a Circular Cylinder in Cross Flow for High Prandtl number.

So, this is a cross flow is happening. So, flow is happening in this direction. And you can see depending on the thermal boundary conditions, your buoyancy flow will act in perpendicular to this forced flow direction. And for Reynolds number 100, it is calculated. So, this is kind of transverse flow, or cross flow.

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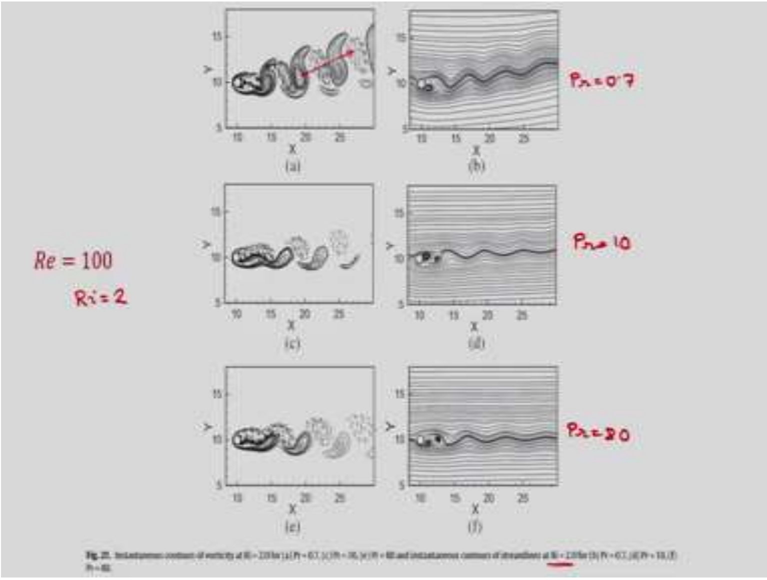

Now, you can see instantaneous contours of verticity, this left side figures; and in right side figure instantaneous contours of streamlines at Ri = 2. So, this is Ri=2, Reynolds number 100, and for different Prandtl number . So, Prandtl number is 0.7, this is your Pr=10, and this is for Pr = 80. So, you can see the effect of buoyancy .

So, when this vertices are shaded behind the cylinder , it is not going exactly in the horizontal direction, it is going in some inclined direction. You can see in this direction it is going. And these are the streamlines from streamlines also you can see similar thing. So, this is the transverse flow and obviously, with increase of Prandtl number, you can see the effect. So, this inclination is decreasing. And for higher Prandtl number, you can see it is going almost in the horizontal direction.

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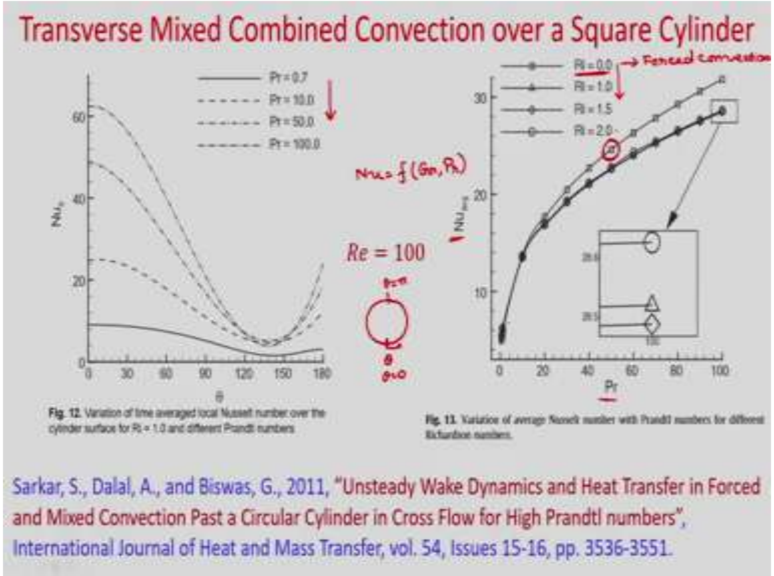

Now, if you see the Nusselt number, so this is the variation of time average local Nusselt number over the cylinder surface. So, you have a cylinder surface. And this from here this is your θ, and θ is measured from here to this. So, θ = 0 here, and θ =π here .

So, obviously, you can see for this particular case there is no corner. So, you will get maximum Nusselt number at this point. This is so you can see at θ = 0 you are getting maximum Nusselt number at any Prandtl number and it is decreasing. And obviously, again near to this π, it is increasing due to the effect of this buoyancy.

Now, you can see as you are increasing the Prandtl number, as you are increasing the Prandtl number, you can see your Nusselt number also increasing. So, you can see that Nusselt number is function of Grashof number and Prandtl number. And obviously, Prandtl number to the power something. So, as Prandtl number increases, Nusselt number also increases. So, you can see the average Nusselt number with Prandtl number for different Richardson number.

So, you can see; obviously, with increase of Prandtl number, your average Nusselt number is increasing. And at a particular Prandtl number, you can see at Richardson number 0. What is what does it mean? Richardson number 0, Richardson number 0 means your buoyancy effect is 0, that means, it is a forced convection. So, Richardson number 0 means it is a forced convection; natural convection is absent.

Here at a particular panel number if you see, you are getting maximum Nusselt number at a particular panel number you can see that you are getting a maximum Nusselt number at Richardson number 0. As you are increasing the Richardson number in this case of transverse flow, your Nusselt number is decreasing.

So, you can see that you also have a reduction in the heat transfer with increase of Richardson number, with increase of Richardson number, although your buoyancy effect is increasing with increase of Richardson number, but your Nusselt number is decreasing. So, in some flow, you will get reduction in some flow also we will get increase in the heat transfer.

So, in today's class, we first wrote some empirical relations for vertical flat plate cases and also inclined plates. If you replace the g with the g cos θ for the expression of vertical flat plate cases Nusselt number, you will can use those in the range of $60^0 \geq \theta \geq -60^0$.

Also we have discuss about correlations for flow over horizontal cylinder, flow over sphere. Then we introduced with the mixed convection. In mixed convection, we discussed about the assisting flow, opposing flow and the transverse flow.

Then we have written the governing equations in non-dimensional form for the case of mixed convection. And two different cases mixed convection flow over a square cylinder case we have discussed about the flow physics as well as the Nusselt number.

And later case also we have shown one example of transverse flow where you have mixed convection, but forced flow and the buoyant flow are perpendicular in direction. And for transverse flow over a circular cylinder, we have discussed about the flow physics as well as the Nusselt number.

Thank you.