Fundamentals of Convective Heat Transfer
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Module – 07
Convection in Internal Flows – III
Lecture – 25
Solution of example problems

Hello everyone. So, today we will solve few example problems on Convection in Internal Flows.

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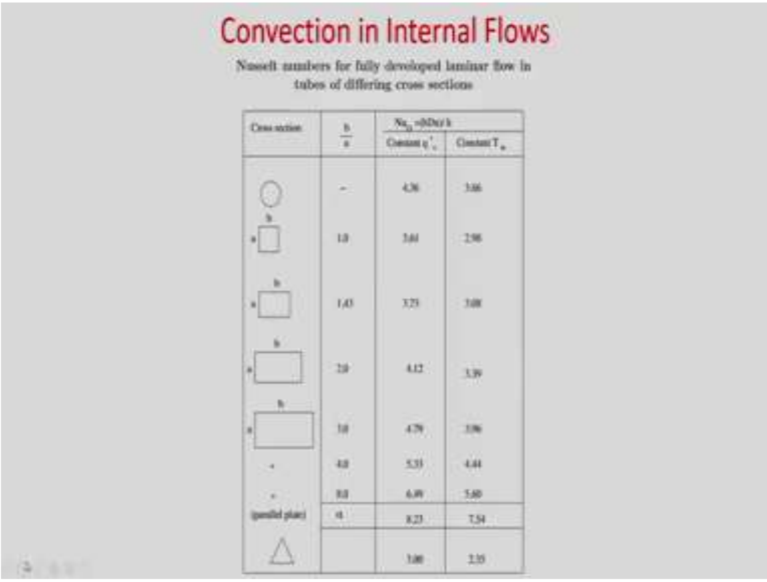

So, first let us summarize that the Nusselt number for thermally and hydro dynamically fully developed flow, you can see for different cross sectional pipe. So, this is your Nusselt numbers for fully developed laminar flow in tubes of differing cross sections. So, you can see here for circular cross section, we have already derived Nusselt number for this particular case.

If you see if it is a constant wall heat flux, then it is 4.36 and if it is constant wall temperature, then 3.66. These we have not derived, but you can see that for a square cross sectional duct. The Nusselt number for constant wall heat flux, it is 3.61 and for constant wall temperature it is 2.98.

Similarly, you see for a rectangular cross sectional channel where $\frac{b}{a}=1.43$. So, b is here a. So, for this particular case for constant wall heat flux boundary condition the Nusselt number is 3.73 and for constant wall temperature 3.08. If you change the $\frac{b}{a}$ ratio, you can see the Nusselt number for two different wall conditions.

Now, if it is a parallel plate so; that means, $\frac{b}{a}$ is infinity. So, in this particular case already we have derived the Nusselt number for constant wall heat flux, it is 8.23 and for constant wall temperature it is 7.54. And, if it is a triangular cross sectional channel then Nusselt number is 3 for constant wall heat flux and 2.35 for constant wall temperature. And you see that the Nusselt number, we have defined based on the hydraulic diameter.

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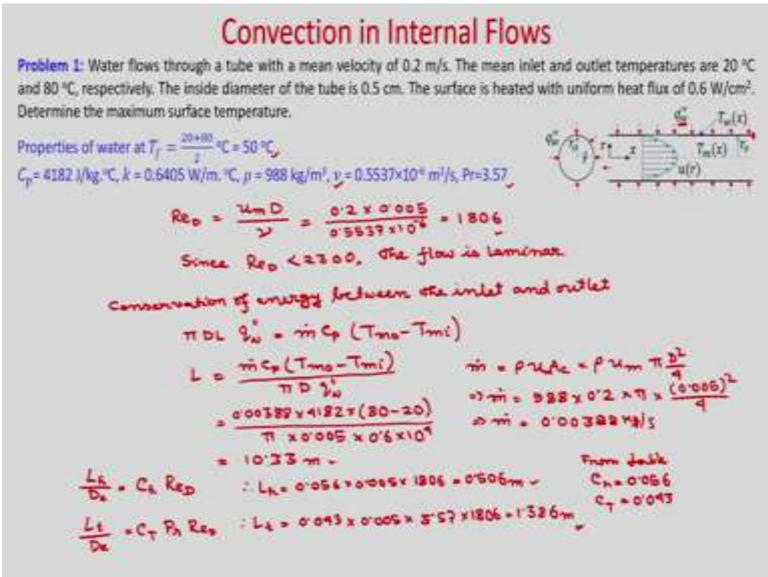

So, first let us take this problem; Water flows through a tube with a mean velocity of 0.2 m/s. The mean inlet and outlet temperatures are 20^0C and 80 ^{0}C respectively. The inside diameter of the tube is 0.5 cm, the surface is heated with uniform heat flux of 0.6 W/cm^2. Determine the maximum surface temperature. So, you can see it is uniform wall heat flux boundary condition.

So, your wall temperature will vary in axial direction and where it is expected to be maximum at the outlet right; so T_w will be maximum. Now, first for this problem first we have to find what is the Reynolds number. If the Reynolds number is in laminar zone and if it is a fully developed flow for both thermally and hydro dynamically then we know. What is the Nusselt number? So, for this particular case you know that the Nusselt number is 4.364.

So, first let us calculate the Reynolds number and the developing length or entrance length. So, properties of water at film temperature 50^0C. So, it is the average temperature. So, C_p ,k , ρ, ν and Prandtl number are given. So, you calculate first from here the Reynolds number.

So, this is your $Re = \frac{u_m D}{\nu}$. So, u_m is given because it is 0.2 m/s. So, it is $\frac{0.2 \times 0.005}{0.5537 \times 10^{-6}}$. So, if you calculate it will come as 1806 and as Reynolds number < 2300 the flow is laminar.

Now, we need to find the length of the tube that is also unknown and we have to find whether the exit is in developing zone or not. So, we have to find the developing length first so, that if it is a fully developed flow, then we can use Nusselt number = 4.364. How do you calculate the length of the pipe?

So, you use this conservation of energy between the inlet and outlet you can see. So, if $q_w^{''}$ is at the wall the heat flux is supplied. So, total heat transfer is your $\pi D L q_w^{''} = \dot{m} C_p \left(T_{mo} - T_{mi}\right)$.

So, length you can calculate as $L = \frac{\dot{m} C_p \left(T_{mo} - T_{mi}\right)}{\pi D q_w^{''}}$. You can see here all the parameters are known except $\dot{m}$. So, $\dot{m}$ you have to calculate. So, the mass flow rate $\dot{m}$ you can calculate as $\dot{m} = \rho u_m A_c$. So, $\dot{m} = \rho u_m \pi \frac{D^2}{4}$.

So, $\dot{m} = 988 \times 0.2 \times \pi \times \frac{(0.005)^2}{4}$. So, $\dot{m} = 0.00388 kg/s$. Now, you put it here then you find the length. So, it will be $L = \frac{0.00388 \times 4182 \times (80-20)}{\pi \times 0.005 \times 0.6 \times 10^4}$. So, length of the pipe is 10.33 m; so this length we have calculated.

Now, let us calculate the entrance length. For laminar flow in a tube the hydrodynamic and thermal lengths are given by $\frac{L_h}{D_e} = C_h \operatorname{Re}_D$. And for thermal developing length it is $\frac{L_T}{D_e} = C_T \Pr \operatorname{Re}_D$.

So, you know that from table that; C_h this coefficient is 0.056 and C_T is 0.043 for the circular cross sectional pipe. So, if you put it here and Reynolds number is 1806 Prandtl number is 3.57; let us calculate the L_h. So, $L_h = 0.056 \times 0.005 \times 1806$. So, it will be 0.056m and $L_T = 0.043 \times 0.005 \times 3.57 \times 1806$. So, thermal entrance length is 1.386 m.

So, you can see the length of the pipe is 10.33 m. And we want to calculate the maximum surface temperature which will occur at the exit. So, at L = 10.33 m and hydrodynamic entrance length and thermal entrance length are much less than the length of the tube. So, here it will be fully developed. So, you can see it is a laminar flow and it is a fully developed flow.

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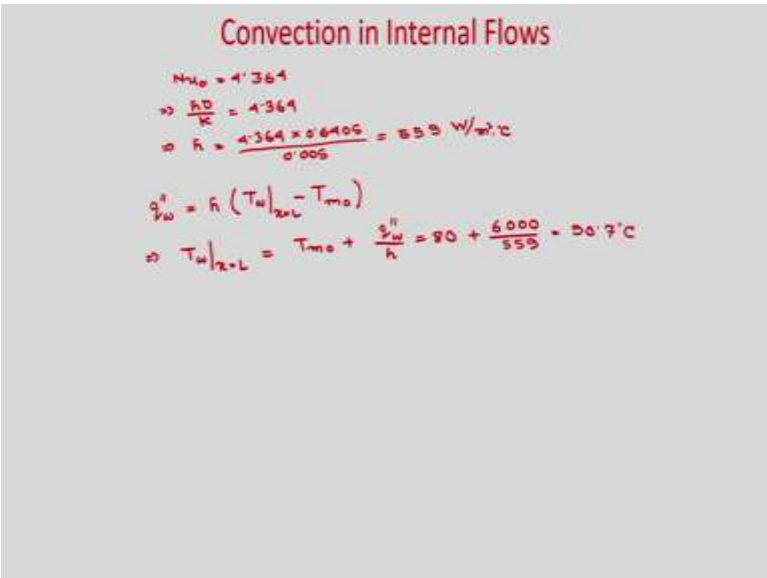

So, Nusselt number for this particular case will be 4.364 .

So, now let us calculate the heat transfer coefficient from here. So, you can see your $\frac{hD}{K}$ is 4.364. So, your; it is average heat transfer coefficient and local heat transfer coefficient as same because, it is a constant value so it is 4.364 thermal conductivity. So, it is $\frac{4.364 \times 0.6405}{0.005}$. So, your heat transfer coefficient is 559 W/m^2.^{0}C.

So, now we have calculate the heat transfer coefficient, but we need to calculate the maximum temperature. So, now, you do the heat flux at the outlet. So, $q_w^{"} = h\left(T_w\big|_{x=L} - T_{mo}\right)$.

So, from here $T_w\big|_{x=L} = T_{mo} + \frac{q_w^{"}}{h}$. So, $80 + \frac{6000}{559}$. So, if you calculate it you will get 90.7^{0}C. So, your maximum temperature at the outlet is 90.7 ^{0}C.

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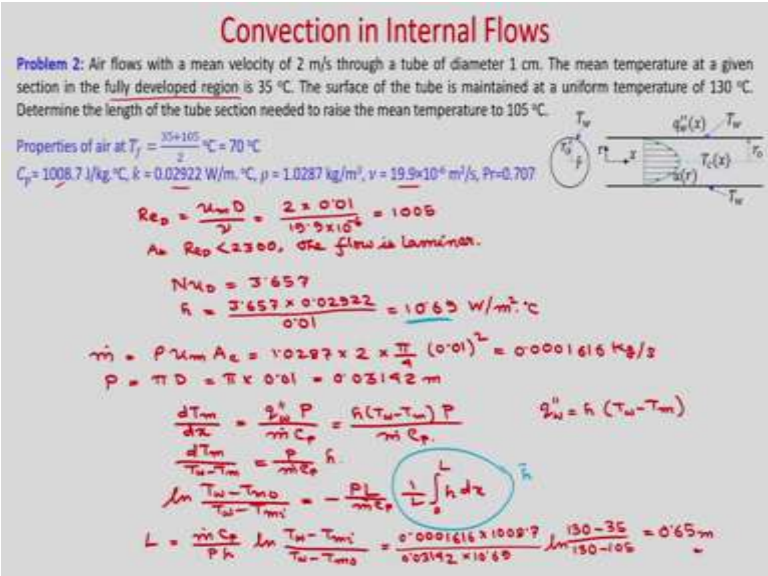

So, let us take the next problem; Air flows with a mean velocity of 2 m/s through a tube of diameter 1 cm. The mean temperature of a given section in the fully developed region is 35 ^{0}C.

The surface of the tube is maintained at a uniform temperature of 130 ^{0}C. Determine the length of the tube section needed to raise the mean temperature to 105 ^{0}C. So, we can see in this particular case it is a case for uniform wall temperature. And it is a fully developed region already it is told, your mean temperature at a given section it is given as 35 ^{0}C. Now, we have to calculate the length of the tube section, where the mean temperature is to be raised to 105 ^{0}C.

So, first we have to calculate so, first we have to calculate the Reynolds number and we have to check whether it is laminar zone or not. So, if it is a laminar flow and it is a fully developed already told in the problem. So, we can use the Nusselt number for a fully developed condition.

So, Reynolds number first let us calculate. So, $\mathrm{Re} = \frac{u_m D}{\nu}$. So, $\frac{2 \times 0.01}{19.9 \times 10^{-6}}$; this is equal to 1005.

So, as Reynolds number < 2300 the flow is laminar. So, the flow is laminar and it is a fully developed region already stated in the problem. So, we can use the Nusselt number based on diameter for this uniform wall temperature is 3.657.

We can calculate the h as $h = \frac{3.657 \times 0.02922}{0.01}$. So, it will be 10.69 W/m^2.^{0}C.

So, now let us calculate the mass flow rate. So, $\dot{m} = \rho u_m A_c$. So, $1.0287 \times 2 \times \pi \times \frac{(0.01)^2}{4}$. So, this if you calculate you will get 0.001616 kg /s. And the P = πD. So, it will be $\pi \times 0.01$=0.03142 m.

Now, from the energy balance analysis we have calculated the $\frac{dT_m}{dx} = \frac{q_w^{''} P}{\dot{m} C_p}$. So, this you know so now, $q_w^{''}$ you can write in terms of heat transfer coefficient so, if you write that.

So, what is $q_w^{''}$? $q_w^{''} = h(T_w - T_m)$. So, now if you see that, $\frac{dT_m}{T_w - T_m} = \frac{P}{\dot{m} C_p} h$.

So, if you integrate it from inlet to the outlet. So, you will get $\ln \frac{T_w - T_{mo}}{T_w - T_{mi}} = -\frac{PL}{\dot{m} C_p} \frac{1}{L} \int_0^L h dx$ So, what it is? If you remember this is nothing, but your $\bar{h}$ and in this case it is $\bar{h}$ =h. So, that we have calculated.

So, this value is known so, from here you will be able to calculate the length $L = \frac{\dot{m} C_p}{Ph} \ln \frac{T_w - T_{mi}}{T_w - T_{mo}}$; so this we have reverse because, this minus sign I have taken care here.

So, if you put all the values here. So, you can see it is, $\frac{0.0001616 \times 1008.7}{0.03142 \times 10.69} \ln \frac{130-35}{130-105}$. So, if you calculate this you will get 0.65 m. So, the length of the tube section needed to raise the mean temperature to 105 ^{0}C is 0.65 m.

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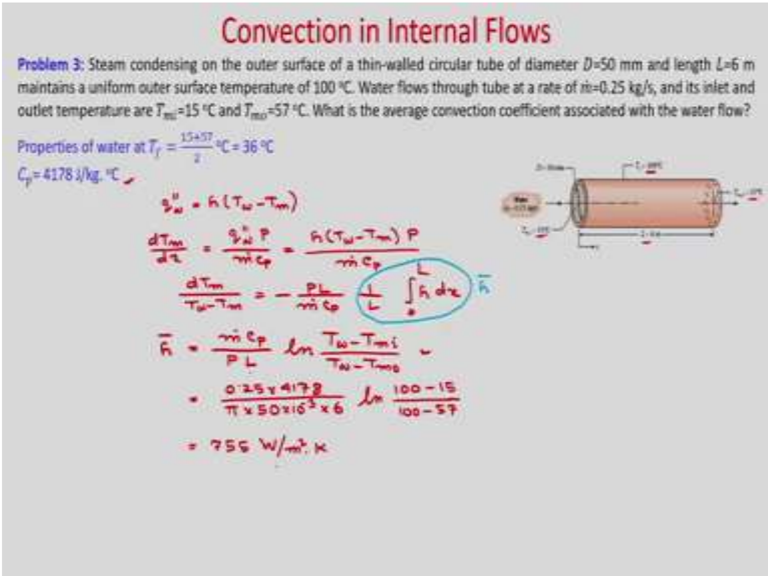

Let us discuss about the next problem; Steam condensing on the outer surface of a thin walled circular tube of diameter D = 50 mm and length L is 6 m, maintains a uniform outer surface temperature of 100 ^{0}C, water flows through tube at a rate of $\dot{m}$ = 0.25 kg /s. And its inlet and outlet temperatures are 15 ^{0}C and 57 ^{0}C respectively. What is the average convection coefficient associated with the water flow?

So, we can see this is the tube. So, inner surface is maintained at 100 ^{0}C and water is flowing through, it $\dot{m}$ = 0.25 kg /s. And inlet mean temperature is 15 ^{0}C outlet mean temperature is 57 ^{0}C , L is 6 m. So, now, we have to calculate the average heat transfer coefficient. So, properties of water at T_f = 36 ^{0}C is this one C_p we need for this calculation.

So, now if you see that we know heat flux $q_w^{"} = h(T_w - T_m)$. So, we have calculated just now $\frac{dT_m}{dx} = \frac{q_w^{"} P}{\dot{m} C_p}$. So, this is $\frac{h(T_w - T_m)P}{\dot{m} C_p}$. So, now, $\frac{dT_m}{T_w - T_m} = -\frac{PL}{\dot{m} C_p} \frac{1}{L}\int_0^L h dx$.

So, as earlier problem this represents your average heat transfer coefficient. So, from here you can see your $\bar{h}$, after integration you can write $\bar{h} = \frac{\dot{m} C_p}{PL} \ln \frac{T_w - T_{mi}}{T_w - T_{mo}}$. So, this we have already calculated in the last problem, there you calculate the L in this particular case we are calculating the $\bar{h}$. So, you put all the values.

So, $\dot{m}$ =0.25 kg /s. So, $\frac{0.25 \times 4178}{\pi \times 50 \times 10^{-3} \times 6} \ln \frac{100-15}{100-57}$. So, if you calculate it your $\bar{h}$ you will get 755 W/m^{2}K.

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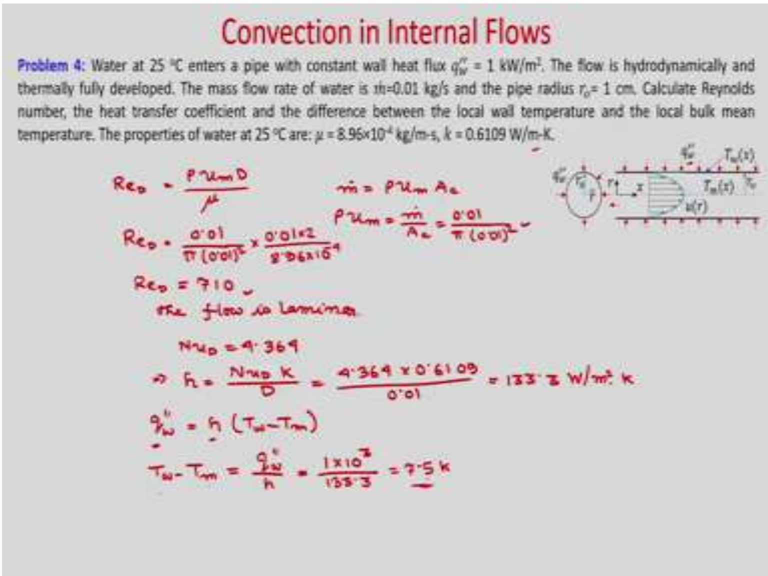

So, let us discuss about this problem; Water at 25 ^{0}C enters a pipe with constant wall heat flux $q_w^{"}$ =1 kW/m^2. The flow is hydro dynamically and thermally fully developed. The mass flow rate of water is $\dot{m}$ =0.01 kg /s and the pipe radius r_0 is 1cm calculate Reynolds number, the heat transfer coefficient. And the difference between the local wall temperature and the local bulk mean temperature.

The properties of water at 25 ^{0}C are given μ and k . So, this is a case of uniform wall heat flux boundary condition. So, for this particular case first you calculate the Reynolds number . So, it is water. So, rho you know so, your $\mathrm{Re} = \frac{\rho u_m D}{\mu}$. And mass flow rate is given $\dot{m}$.

So, you can calculate $\dot{m} = \rho u_m A_c$. So, what is ρu_m? $\rho u_m = \frac{\dot{m}}{A_c}$ and it is $\frac{0.01}{\pi(0.01)^2}$.

Now, Reynolds number you can calculate as $\mathrm{Re}_D = \frac{0.01}{\pi(0.01)^2} \times \frac{0.01 \times 2}{8.96 \times 10^{-4}}$. So, Reynolds number from here if you calculate you will get 710 . So, this is your; obviously, < 2300. So, the flow is laminar and in fully developed condition Nusselt number for constant wall temperature boundary condition as the flow is laminar and it is a fully developed flow.

So, the Nusselt number for uniform wall heat flux boundary condition is 4.36. So, from here you can see Nusselt number is 4.364. So, from here you can calculate $h = \frac{Nu_D K}{D}$. So, it will be $\frac{4.364 \times 0.6109}{0.01}$. So, if you calculate it you will get 133.3 W/m^2.K Now, what is the difference between the local wall temperature and the local bulk mean temperature?

So, we know $q_w'' = h(T_w - T_m)$ where q_w'' is known h we have found. So, you can calculate $T_w - T_m = \frac{q_w''}{h}$. So, q_w'' =1 kW/m^2. So, $\frac{1 \times 10^3}{133.3}$; so you will get 7.5 Kelvin. So, you can see the difference between the local wall temperature and local bulk mean temperature is 7.5 K.

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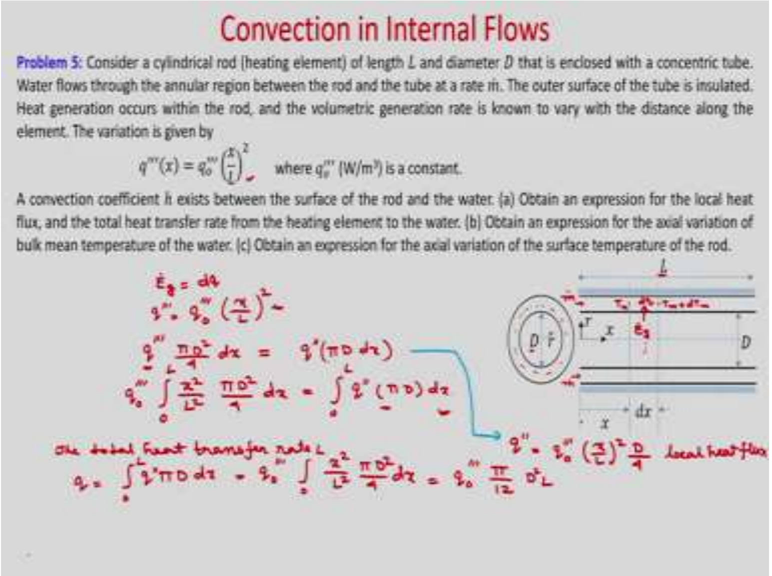

So, now we will consider this problem; Consider a cylindrical rod heating element of length L and diameter D that is enclosed with a concentric tube. Water flows through the annular region between the rod and the tube, at a rate m dot the outer surface of the tube is insulated heat generation occurs within the rod. And the volumetric generation rate is known to vary with a distance along the element. The variation is given by this relation.

So, q''' is the heat generation rate and per unit volume and q_0''' is a constant, a convection coefficient h exist between the surface of the rod and the water. So, obtain an expression for the local heat flux. And the total heat transfer rate from the heating element to the water, b) obtain an expression for the axial variation of the bulk mean temperature of the water.

And c) obtain an expression for the axial variation of surface temperature of the rod. So, you can see the inside tube. So, this diameter D is shown. So, for this is the solid rod and it is the heat generating rod heating element. And for the flow is occurring in the annulus.

So, you can see this is the space through which your flow is fluid is flowing. So obviously, you can see that if a at a distance x if you take a dx length. So, you can see in this volume whatever heat is generated so, a from the surface at a steady state whatever it is leaving the surface that is actually taken by the fluid. So, that energy balance we can do.

So, you can see it will be just $\dot{E}_g$ whatever is heat generated. So, that is the dq taken by the fluid. And if at the inlet the mean temperature is T_m. So, T_m + d T_m. So, there is a increase in the mean temperature d T_m and $\dot{m}$ is the mass flow rate. So, the energy balance you can see that $\dot{E}_g = dq$ from this elemental volume. So, $q''' = q_0'''\left(\frac{x}{L}\right)^2$.

So, you can see if you put it here $\dot{E}_g$. So, $\dot{E}_g$ is q''' into the volume. So, what is that? So, it will be $q''' = \frac{\pi D^2}{4}dx$. So, you can see the surface heat transfer surface is πDdx through which actually, this is going. And what is the heat is generated in the volume? So, this is the volume right.

So, this is the volume and D is the diameter. So, pi D square by 4 into dx. So, this is the volume of this heating element cylindrical rod. So, this volume is $\frac{\pi D^2}{4}dx$. So, into q''' will give you the total heat living the surface. And now the circumferential area through which your heat is transferred to the fluid that area is πDdx. So, that is the heat transfer area and the heat flux, if it is q double prime then $q''' = q''\left(\pi Ddx\right)$.

So, what we will do? Now, you put this expression here because, $q_0'''\int_0^L \frac{x^2}{L^2}\frac{\pi D^2}{4}dx = \int_0^L q''\left(\pi D\right)dx$. So, this is the expression so, from here you can see. .

What is q''? $q'' = q_0'''\left(\frac{x}{L}\right)^2\frac{D}{4}$. So, this is your local heat flux local heat flux. So, now, if you calculate the; total heat transfer rate. So, the total heat transfer rate. What is that? So, $q = \int_0^L q''\pi Ddx$.

So, now if you put this one only the left hand side then, you will get $q_0'''\int_0^L \frac{x^2}{L^2}\frac{\pi D^2}{4}dx$. So, if you carry out the integration you will get, $q_0'''\frac{\pi}{12}D^2L$.

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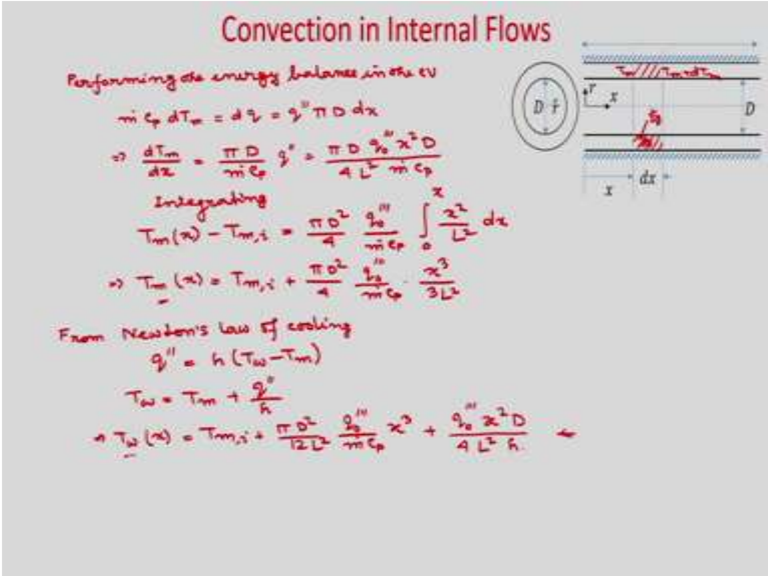

So, now you perform the energy balance in the control volume. So, performing the energy balance in the control volume. So, this is the control volume. So, whatever $\dot{E}_g$ is generated. So, dq is taken by the fluid. So, you can see you will get $\dot{m}C_p dT_m$ and dT_m is the change in the mean temperature.

So, $\dot{m}C_p dT_m = dq = q^{"}\pi D dx$. So, you can write $\frac{dT_m}{dx} = \frac{\pi D}{\dot{m}C_p} q^{"}$ and you can write $q^{"} = q_0^{'''}\left(\frac{x}{L}\right)^2 \frac{D}{4}$. So, if you write that expression. So, you will get $\frac{\pi D q_0^{'''} x^2 D}{4L^2 \dot{m}C_p}$.

Now, if you integrate it, the mean temperature $T_m(x) - T_{mi} = \frac{\pi D^2}{4} \frac{q_0^{'''}}{\dot{m}C_p} \int_0^x \frac{x^2}{L^2} dx$. So, your mean temperature variation you can see $T_m(x) = T_{mi} + \frac{\pi D^2}{4} \frac{q_0^{'''}}{\dot{m}C_p} \frac{x^3}{3L^2}$. So, you have found the mean temperature variation.

Now, you have to find the wall temperature variation along the axial direction. So, that if you do this. So, from Newton's law of cooling you can write $q_w^{"} = h(T_w - T_m)$ and T_m already we have found from this expression.

So, $T_w = T_m + \frac{q_w^{''}}{h}$. So, you can see you can write $T_w(x) = T_{mi} + \frac{\pi D^2}{12L^2} \frac{q_0^{'''}}{\dot{m} C_p} x^3 + \frac{q_0^{'''} x^2 D}{4L^2 h}$.

So, $q^{''}$ if you write. So, you can write as this expression. So, it will be $q^{''} = q_0^{'''} \left(\frac{x}{L}\right)^2 \frac{D}{4}$. So, you can see this is the expression of wall temperature variation along x.

So, today we solved total 5 example problems in most of the problem, you have seen that we have considered fully developed flow both hydro dynamically and thermally. But, before taking the Nusselt number for a fully developed condition first you cross check. What is the Reynolds number and the whether it is fully developed flow?

If Reynolds number < 2300, then the laminar the flow is laminar. And if you see the entrance length and this entrance length is less than the required length of the pipe, where you want to find the heat transfer coefficient or the heat flux. Then you can use the Nusselt number expression for thermally and hydro dynamically fully developed laminar flow condition.

So, you know that for uniform wall heat flux boundary condition, for flow through a pipe the Nusselt number is 4.364 and that we have used to calculate the heat transfer coefficient. In other case from the energy balance, you can calculate the heat transfer coefficient, but that heat transfer coefficient; obviously, it is a local heat transfer coefficient.

And after doing the integration you can calculate the average heat transfer coefficient. And these we have already studied in the initial lectures of these convection in internal flows. Also we have taken up one problem where inside the rod heat generation is taking place.

So, and heat generation per unit volume; we have considered and that we have equated with the heat flux at the wall, carried by the fluid. And at steady state you know whatever is heat is generated inside the heat generating element, that heat will be carried out by the fluid.

So, with that energy balance we have used and we have calculated the total heat transfer rate. And also we have done the energy balance in the fluid volume, where the mean temperature at inlet we have consider and mean temperature the outlet we have consider.

So, $\dot{m} C_p dT_m$ that is the fluid is carrying the energy equal to whatever the heat flux into the area is has come from the heat generating body. So, that we have equated and from that expression, we have calculated the variation of mean temperature along the axial direction and equating or using the Newton's law of cooling. We have calculated the variation of wall temperature in the axial direction.

So, if you have seen that in last three modules, we have solved several problems. Initially we considered thermally and hydro dynamically fully developed flow. Because, the calculation is easy the analysis is easy. And, also we started with the slug flow where you have a constant uniform velocity u m. And, then we considered fully developed condition, where the velocity profile is parabolic and also we considered thermally fully developed flow, where the non dimensional temperature of π does not change in the axial direction.

Later we considered fully hydro dynamically fully developed flow, but thermally developing flow. So, for this particular case also we considered two different types of boundary conditions constant wall heat flux and constant wall temperature boundary conditions.

In each cases we found the expression of local Nusselt number and the average Nusselt number. However, we have not studied where it is a thermally and hydro dynamically developing flow. So, it is thus analysis is more complicated, if you are interested you can refer some books convection heat transfer book and you can derive the Nusselt number for this particular case as a homework.

Thank you.