Fundamentals of Convective Heat Transfer
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Module – 07
Convection in Internal Flows – III
Lecture – 23
Hydrodynamically developed and thermally developing flow through circular pipe with uniform wall temperature

Hello, everyone. Today, we will consider Hydrodynamically developed and thermally developing fluid flow through circular pipe with uniform wall temperature. So, this is also known as Graetz problem.

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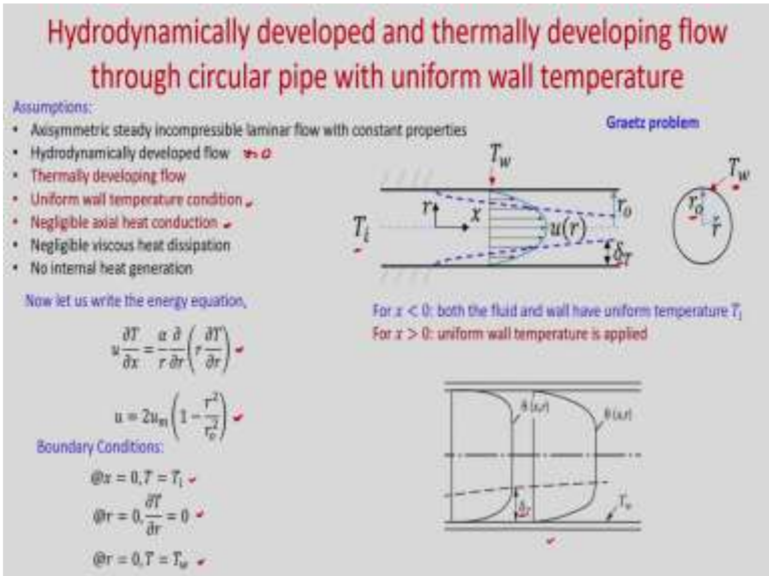

So, you can see that we are considering hydrodynamically developed and thermally developing; that means, you can see that your thermal boundary layer is growing. This is the thermal boundary layer thickness δ_T and in developing region we need to find what is the temperature distribution.

You can see that your temperature at inlet is T_i and up to x = 0 from the inlet it is adiabatic. So, the wall temperature and free temperature will remain at temperature T_i, but from x = 0 uniform wall temperature is applied at the wall. So, you can see at the wall T_w is applied. So, there will be formation of thermal boundary layer. In this region we are considering fully developed hydrodynamically developed flow; however, you have thermally developing temperature profile.

So, here you can see the radius of the tube is r_0 and r is measured from the central line. So, these are the assumptions already we have discussed only one important assumptions we are taking that it is uniform wall temperature condition and you are neglecting axial heat conduction.

So, with that boundary conditions, you can see that your temperature the energy equation will be $u\frac{\partial T}{\partial x} = \frac{\alpha}{r}\frac{\partial}{\partial r}(r\frac{\partial T}{\partial r})$ because we have neglected the axial heat conduction and as it is hydrodynamically developed flow so, v is 0.

So, your one convection term related to $v\frac{\partial T}{\partial y} = 0$ and fully developed velocity profile is this one which is parabolic and boundary conditions you can see that x = 0, you have temperature $T = T_i$ at r = 0. So, it is axisymmetric flow. So, $\frac{\partial T}{\partial r}$=0 and at r = 0 we have imposed uniform wall temperature, so, T will be T_w.

In this figure you can see along the axial direction if you go, so there will be change in the temperature profile. The temperature variation will occur only inside the thermal boundary layer and in the core region temperature will remain at temperature T_i. However, when $x \rightarrow\infty$; that means, thermally developed flow, then your axial or central line temperature will vary with x.

Now, today we will find the temperature distribution which is valid in both developing region, as well as developed region as well as we will find the Nusselt number.

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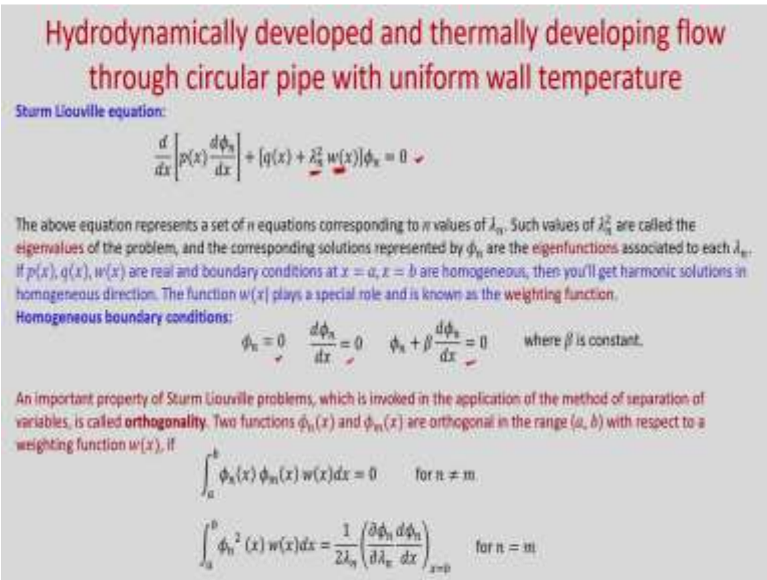

Again let us discuss about the Sturm Liouville equation in last class we have already discussed in details. So, this is the second order ordinary differential equation $\frac{d}{dx}\left[p(x)\frac{d\phi_n}{dx}\right]+\left[q(x)+\lambda_n^2 w(x)\right]\phi_n = 0$, where λ_n^2 is the eigen values and ϕ_n is the solution of this ordinary differential equation and these are the eigen functions associated with each λ_n and w(x) is the weighting function.

So, it is very important in this particular case when will apply the orthogonality condition it is required. In this analysis also we will use method of separation of variables. So, when can we use the separation of variables method? When the governing equation is linear and homogenous, and in one direction you have two homogenous boundary conditions. And, in homogenous direction it should give the characteristic value problem or harmonic solution in homogenous direction.

So, we can see what is homogenous direction? If the value of that variable is 0 or the gradient of that variable is 0 or combination of this two is 0. So, you can see homogenous boundary condition ϕ_n =0 or $\frac{d\phi_n}{dx}$ =0 or combination of this two where β is the constant is 0. So, in this form if you get the boundary condition then these are the homogenous boundary conditions.

So, we can use separation of variables method if in the homogenous direction if you get characteristic value problem. Now, you see the boundary condition in today's problem we have seen that in r direction it is we can make as homogenous using convenient non-dimensional quantity.

So, one important property of this Sturm Liouville problem is orthogonality. So, we can see the two function ϕ_n and ϕ_m are orthogonal in the range a, b with respect to weighting function w(x) if $\int_a^b \phi_n(x)\phi_m(x)w(x)dx = 0$ for n ≠m. And today, we will use another important property where n = m you will get $\int_a^b \phi_n^2(x)w(x)dx = \frac{1}{2\lambda_n}\left(\frac{\partial \phi_n}{\partial \lambda_n}\frac{d\phi_n}{dx}\right)_{x=b}$.

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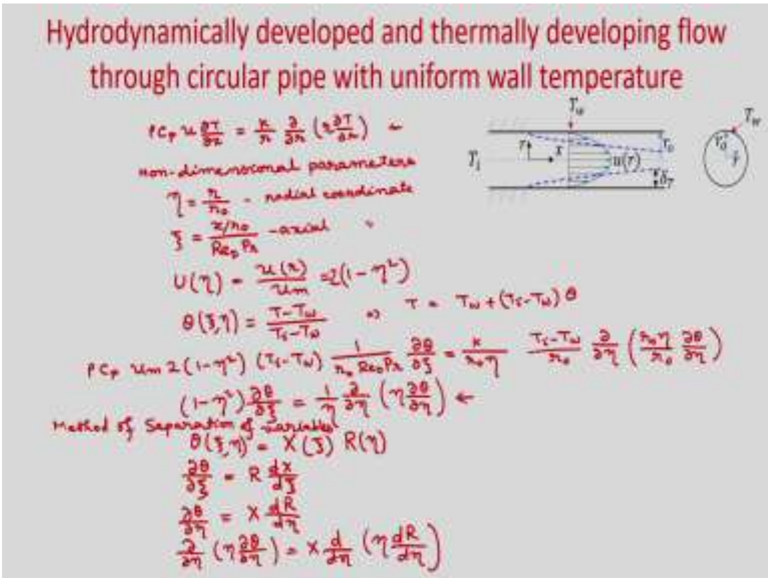

So, first let us define some suitable non dimensional quantity, so that we get in r direction both boundary conditions as homogenous. So, if you see we have written our energy equation as $\rho C_p u\frac{\partial T}{\partial x} = \frac{K}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$ now we are introducing the non-dimensional parameters non-dimensional parameters.

So, $\eta = \frac{r}{r_0}$. So, this is the radial coordinate non-dimensional radial coordinate and we will introduce $\zeta = \frac{x/r_0}{\mathrm{Re}_D \Pr}$. So, this is your axial coordinate. So, we can write the velocity $U(\eta) = \frac{u(r)}{u_m} = 2\left(1-\eta^2\right)$ and we are defining the non-dimensional temperature $\theta(\xi,\eta) = \frac{T - T_w}{T_i - T_w}$.

Now, you can see using these non-dimensional parameters in r direction at r = r_0 and $T=T_w$, so, θ will become 0 and at r = 0 , $\frac{d\theta}{d\eta}$ will become 0. So, you will get homogenous direction in r direction or η direction. So, if you put all these in these energy equation what you are going to get? $\rho C_p u_m 2\left(1-\eta^2\right)\left(T_i - T_w\right)\frac{1}{r_0 \mathrm{Re}_D \Pr}\frac{\partial\theta}{\partial\xi} = \frac{K}{r_0\eta}\frac{\left(T_i - T_w\right)}{r_0}\frac{\partial}{\partial\eta}\left(\frac{r_0\eta}{r_0}\frac{\partial\theta}{\partial\eta}\right)$. This we have carried out in last class as well . So, just after rearrangement you can write as $\left(1-\eta^2\right)\frac{\partial\theta}{\partial\xi} = \frac{1}{\eta}\frac{\partial}{\partial\eta}\left(\eta\frac{\partial\theta}{\partial\eta}\right)$. So, you carry out this algebra. So, you put the values of Re_D and Pr and you cancel out some parameters, then you will get finally, this equation.

Now, you see this equation is homogenous as well as linear and in r direction, you have two homogenous boundary condition. So, we will be able to use method of separations of variable now we will use separation of variables method where we will find the solution of this equation as theta which is product of two solution x and r, where X is function of ξ only and R is function of η only.

So, we can write θ method of separation of variable . So, $\theta(\xi,\eta) = X(\xi)R(\eta)$, so, now, you can see $\frac{\partial\theta}{\partial\xi} = R\frac{dX}{d\xi}$ you can write $\frac{\partial\theta}{\partial\eta} = X\frac{dR}{d\eta}$ and $\frac{\partial}{\partial\eta}\left(\eta\frac{\partial\theta}{\partial\eta}\right) = X\frac{d}{d\eta}\left(\eta\frac{dR}{d\eta}\right)$. So, all these you put in these equation and write.

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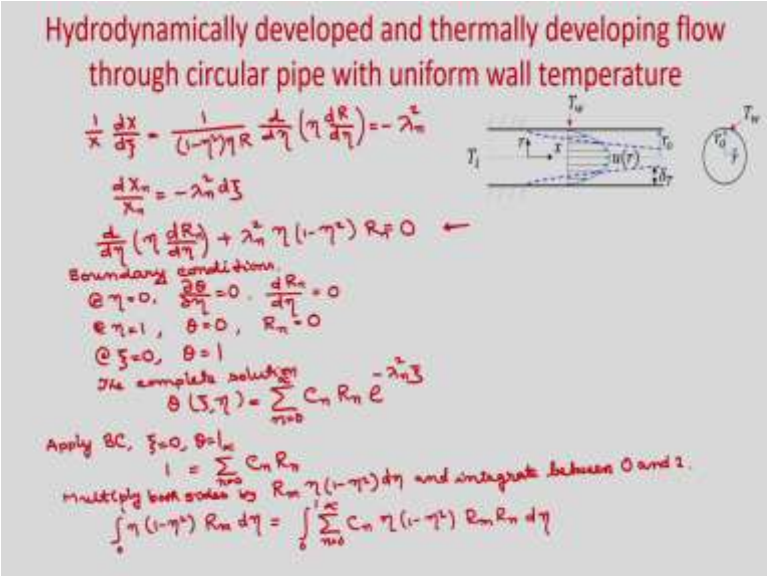

Finally, after rearrangement you can write $\frac{1}{X}\frac{dX}{d\xi} = \frac{1}{\left(1-\eta^2\right)\eta R}\frac{d}{d\eta}\left(\eta\frac{dR}{d\eta}\right)$. So, you can see left hand side is function of ξ only and right hand side function of η only. So, this will be equal to some constant and the sign of that constant you have to chose such a way that in r direction which is your homogenous direction you will get characteristic value problem; that means, it is solution will give you harmonic solution.

And, if you can constitute this equation such a way, that in r direction you will get the equation similar to Sturm Liouville equation. So, you will get solution as characteristic value problem. So, we will chose here is equal to $-\lambda_n^2$ because what different values of lambda you will get different solution X and R here λ_n^2 is your eigen values.

So, you can see you can write $\frac{dX_n}{X_n} = -\lambda_n^2 d\xi$. So, the solution you can see this will be exponential right, some constant into $e^{-\lambda_n^2\xi}$. And, the other equation you will get $\frac{d}{d\eta}\left(\eta\frac{dR_n}{d\eta}\right) + \lambda_n^2\eta\left(1-\eta^2\right)R_n = 0$.

Now, you compare this equation with the Sturm Liouville equation. So, you can see p =η , q = 0 and weighting function is $\eta\left(1-\eta^2\right)$. So, now, and also in η direction you have two boundary conditions as homogenous. So, boundary conditions if you write so, at η=0 $\frac{\partial \theta}{\partial \eta}$=0.

So, if $\frac{\partial \theta}{\partial \eta}$=0, so, obviously, $\frac{dR}{d\eta}$=0 and at η=1, θ=0; that means, R_n should be 0 at η= 1 and at ξ=0, θ=1 . So, now, you can write the solution right, but you can see that for different values of λ_n you will get a different solution X_n and R_n and as the governing equation is linear, you can super impose all the solutions right.

So, you can sum all the solution for different λ_n and it is possible as it is linear equation. So, we will write the final solution θ as the complete solution $\theta(\xi,\eta)=\sum_{n=0}^{\infty} C_n R_n e^{-\lambda_n^2 \xi}$.

So, now, we need to find what is the value of this constant C_n. So, we will apply the boundary conditions and R_n. What is R_n? R_n is the solution of this second order differential equation and you need to use some numerical technique to find the eigenfunctions of this second order differential equation and you can find the solution R_n.

So, this equation you need to use some numerical technique to find the eigen functions R_n and once R_n is known then you can write the complete solution as $C_n R_n e^{-\lambda_n^2 \xi}$. So, this is very important because these equation you need to solve and it is eigenfunction are R_n and in the complete solution this is coming as R_n.

Now, let us apply the boundary condition at ξ=0 , θ=1 . So, if you put it here you will get $1=\sum_{n=0}^{\infty} C_n R_n$. So, what we will do now? So, we will use the orthogonality condition to find this constant C_n.

So, we will multiply both side with R_n into the weighting function; in this case it is $\int_0^1 \eta\left(1-\eta^2\right) R_m d\eta = \int_0^1 \sum_{n=0}^{\infty} C_n \eta\left(1-\eta^2\right) R_m R_n d\eta$.

So, now, you use the orthogonality condition. In the right hand side you can see the summation n =0 to ∞. So, in this particular case you can see if you apply the orthogonality condition for this Sturm Liouville equation, then all the terms will become 0 except n = m . So, for n = to m the integral all will be 0.

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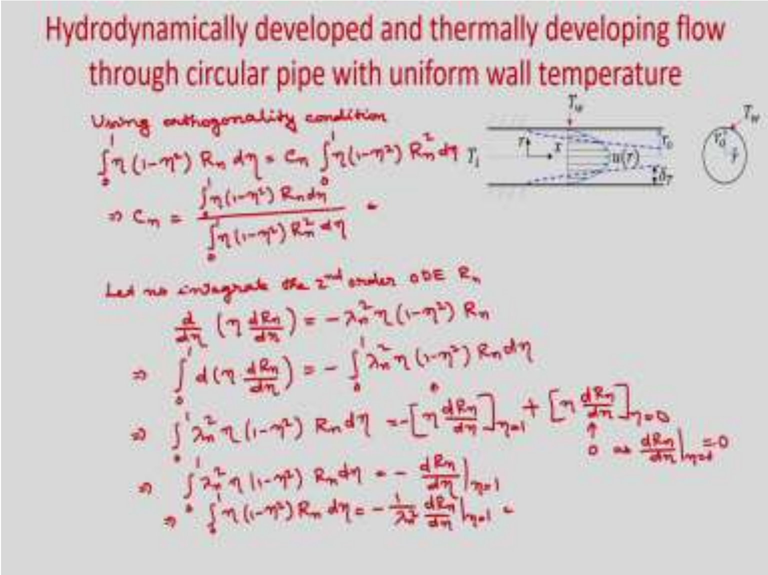

So, if you keep only n = m, then using orthogonality condition so, you can write $\int_0^1 \eta\left(1-\eta^2\right) R_n d\eta = \int_0^1 \sum_{n=0}^{\infty} C_n \eta\left(1-\eta^2\right) R_n^2 d\eta$.

So, now, you can write from here what is the value of constant. $C_n = \dfrac{\int_0^1 \eta\left(1-\eta^2\right) R_n d\eta}{\int_0^1 \eta\left(1-\eta^2\right) R_n^2 d\eta}$.

So, now we will use the second order differential equation of R_n and we will integrate it then we will find. So, first let us integrate the second order ODE of R_n. So, you can see our equation is $\dfrac{d}{d\eta}\left(\eta \dfrac{dR_n}{d\eta}\right) = -\lambda_n^2 \eta\left(1-\eta^2\right) R_n$.

So, now, if you integrate it, so, you will get $\int_0^1 d\left(\eta \dfrac{dR_n}{d\eta}\right) = -\int_0^1 \lambda_n^2 \eta\left(1-\eta^2\right) R_n d\eta$. So, you can see this integral then you can write $\int_0^1 \lambda_n^2 \eta\left(1-\eta^2\right) R_n d\eta = -\left[\eta \dfrac{dR_n}{d\eta}\right]_{\eta=1} + \left[\eta \dfrac{dR_n}{d\eta}\right]_{\eta=0}$.

So, now, we apply the boundary condition at η=0, $\frac{dR_n}{d\eta}$ =0. So, the last term will become 0. So, this is your 0 as $\frac{dR_n}{d\eta}|_{\eta=0}$. So, you will get only $\int_0^1 \lambda_n^2 \eta\left(1-\eta^2\right) R_n d\eta = -\frac{dR_n}{d\eta}|_{\eta=1}$. So, you can see in this constant C_n, so, in the numerator you can replace. So, now, this λ_n^2 you can take it outside. So, you can write $\int_0^1 \eta\left(1-\eta^2\right) R_n d\eta = -\frac{1}{\lambda_n^2}\frac{dR_n}{d\eta}|_{\eta=1}$. So, these value this integral value you can put it in the numerator because this is the integral.

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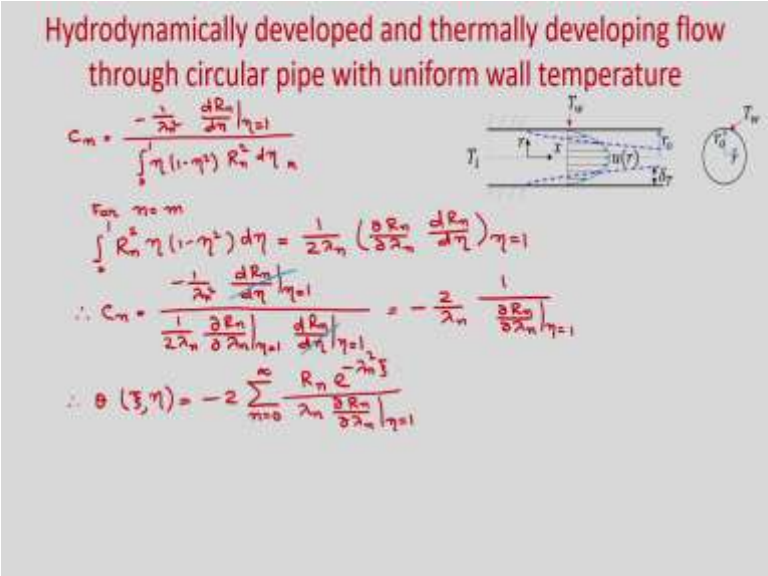

So, $C_n = \frac{-\frac{1}{\lambda_n^2}\frac{dR_n}{d\eta}|_{\eta=1}}{\int_0^1 \eta\left(1-\eta^2\right) R_n^2 d\eta}$. Now, apply the other important properties of orthogonality at n =m . So, already we have written for the Sturm Liouville equation. So, if you write it for n =m . So, you can write $\int_0^1 R_n^2 \eta\left(1-\eta^2\right) d\eta = \frac{1}{2\lambda_n}\left(\frac{\partial R_n}{\partial \lambda_n}\frac{dR_n}{d\eta}\right)_{\eta=1}$.

So, if you put in the denominator . So, this you see this is the left hand side this integral.

So, you can write the constant $C_n = \dfrac{-\dfrac{1}{\lambda_n^2}\dfrac{dR_n}{d\eta}\Big|_{\eta=1}}{\dfrac{1}{2\lambda_n}\dfrac{\partial R_n}{\partial \lambda_n}\Big|_{\eta=1}\dfrac{dR_n}{d\eta}\Big|_{\eta=1}}$. So, this and this we will get cancel. So, you will get $-\dfrac{2}{\lambda_n}\dfrac{1}{\dfrac{\partial R_n}{\partial \lambda_n}\Big|_{\eta=1}}$.

So, your temperature distribution, so, you can write C_n you know. So, $\theta(\xi,\eta) = -2\sum_{n=0}^{\infty}\dfrac{R_n e^{-\lambda_n^2\xi}}{\lambda_n \dfrac{\partial R_n}{\partial \lambda_n}\Big|_{\eta=1}}$.

So, if you can find the eigen function of R_n and its derivative with respect to the eigenvalues if you can get at η=1, then you will be able to find this temperature profile θ.

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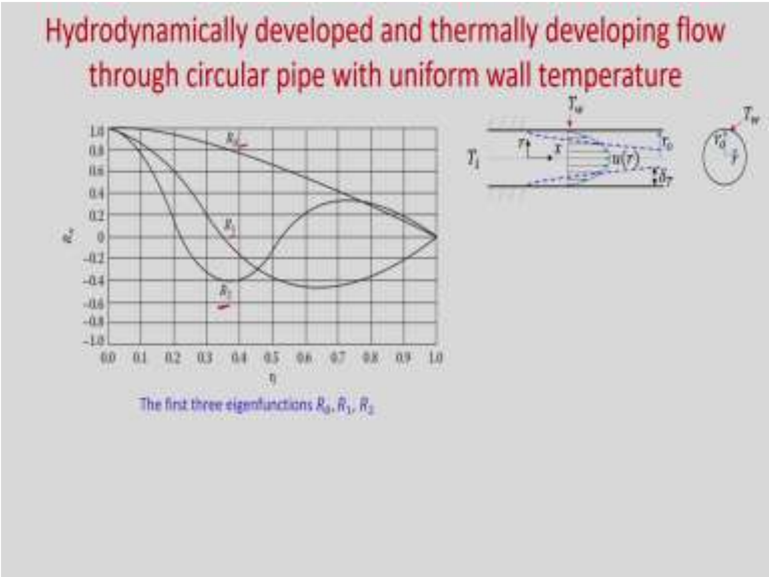

So, you can see the first three eigen functions R_0, R_1 and R_2 how it varies; first three eigen functions with η. So, the boundary condition you know . So, at η=1 R = 0 . So, you can see that here you are getting at η=1 and R = 0 and this is the R_0 variation, this is the R_1 variation and this is the R_2 variation . So, these are the eigen functions; first three eigen functions R_0, R_1 and R_2.

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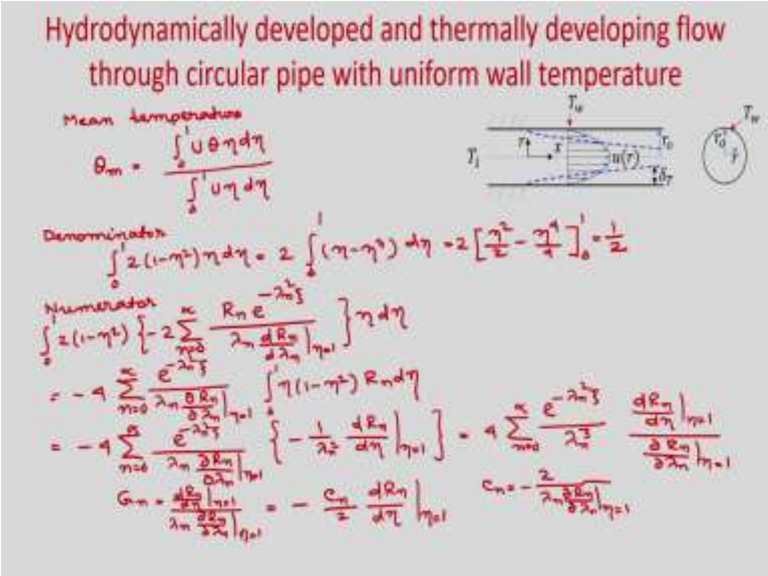

Now, we are interested to find the Nusselt number. So, to find the Nusselt number we have to find the difference we need to find the temperature difference between mean temperature and the wall temperature. So, for that first we need to find what is the mean temperature. So, you can see that bulk mean temperature mean temperature $\theta_m = \frac{\int_0^1 U\theta\eta d\eta}{\int_0^1 U\eta d\eta}$.

So, now if you write the denominator you integrate first . So, it will be $\int_0^1 2\left(1-\eta^2\right)\eta d\eta = 2\int_0^1\left(\eta-\eta^3\right)d\eta$ and it will be just $2\left[\frac{\eta^2}{2}-\frac{\eta^4}{4}\right]_0^1$. So, if you put the limits you will get $\frac{1}{2}$.

And, the numerator if you find the integration; so, $\int_0^1 2\left(1-\eta^2\right)\left\{-2\sum_{n=0}^{\infty}\frac{R_n e^{-\lambda_n^2\xi}}{\lambda_n \frac{dR_n}{d\lambda_n}\Big|_{\eta=1}}\right\}\eta d\eta$ k.

So, if you see if you find this integral, so, these you can see it is not function of η. So, whatever is function of η that you put inside the integral. So, you can write $-4\sum_{n=0}^{\infty}\frac{e^{-\lambda_n^2\xi}}{\lambda_n \frac{dR_n}{d\lambda_n}\Big|_{\eta=1}}\int_0^1 \eta\left(1-\eta^2\right)R_n d\eta$.

So, you can see that this integral already we have found right. So, this you can write as $-4\sum_{n=0}^{\infty}\frac{e^{-\lambda_n^2\xi}}{\lambda_n \frac{dR_n}{d\lambda_n}\Big|_{\eta=1}}\left\{-\frac{1}{\lambda_n^2}\frac{dR_n}{d\lambda_n}\Big|_{\eta=1}\right\}$.

So, now if you see, so, you can rearrange it as $4\sum_{n=0}^{\infty}\frac{e^{-\lambda_n^2\xi}}{\lambda_n^3}\frac{\frac{dR_n}{d\eta}\Big|_{\eta=1}}{\frac{\partial R_n}{\partial\lambda_n}\Big|_{\eta=1}}$. So, now, you can find the θ_m .

So, what we will do? We will now define another constant, $G_n=\frac{\frac{dR_n}{d\eta}\Big|_{\eta=1}}{\lambda_n \frac{\partial R_n}{\partial\lambda_n}\Big|_{\eta=1}}=-\frac{C_n}{2}\frac{dR_n}{d\eta}\Big|_{\eta=1}$. So, you can see the C_n. This is the, $C_n=-\frac{2}{\lambda_n \frac{\partial R_n}{\partial\lambda_n}\Big|_{\eta=1}}$.

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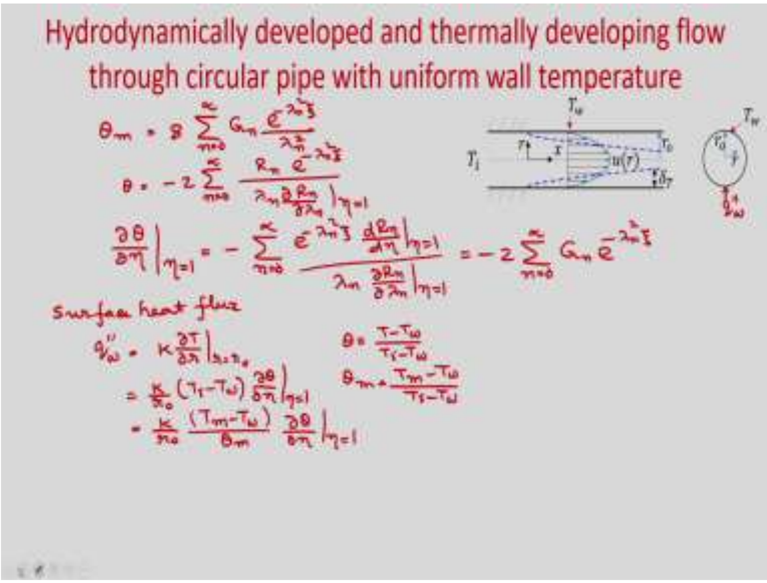

So, you will get the final expression for mean temperature $\theta_m = 8\sum_{n=0}^{\infty} G_n \frac{e^{-\lambda_n^2 \xi}}{\lambda_n^2}$. So, now, to calculate the Nusselt number we need the temperature gradient at η= 1; that means, at the wall. So, we need the temperature gradient at η=1.

So, we know theta which is $\theta = -2\sum_{n=0}^{\infty} \frac{R_n e^{-\lambda_n^2 \xi}}{\lambda_n \frac{\partial R_n}{\partial \lambda_n}\big|_{\eta=1}}$.

So, now $\frac{\partial \theta}{\partial \eta}\big|_{\eta=1} = -\sum_{n=0}^{\infty} \frac{e^{-\lambda_n^2 \xi} \frac{dR_n}{d\eta}\big|_{\eta=1}}{\lambda_n \frac{\partial R_n}{\partial \lambda_n}\big|_{\eta=1}}$.

So, this if you write in terms of G, then you can write $-2\sum_{n=0}^{\infty} G_n e^{-\lambda_n^2 \xi}$. We are now in a position to find the Nusselt number. So, first you find what is the surface heat flux. So, surface heat flux $q_w^{"} = K \frac{\partial T}{\partial r}\big|_{r=r_0}$. So, what we are telling that q we are taking in the negative direction of r. So, it will become positive.

So, you can see that $\theta = \frac{T - T_w}{T_i - T_w}$ and $\theta_m = \frac{T_m - T_w}{T_i - T_w}$. So, this you can see that it will be $q_w^{"} = \frac{K}{r_0}\left(T_i - T_w\right)\frac{\partial \theta}{\partial \eta}\big|_{\eta=1}$. And, in terms of mean temperature now if you write, so, it will be $q_w^{"} = \frac{K}{r_0}\frac{\left(T_m - T_w\right)}{\theta_m}\frac{\partial \theta}{\partial \eta}\big|_{\eta=1}$.

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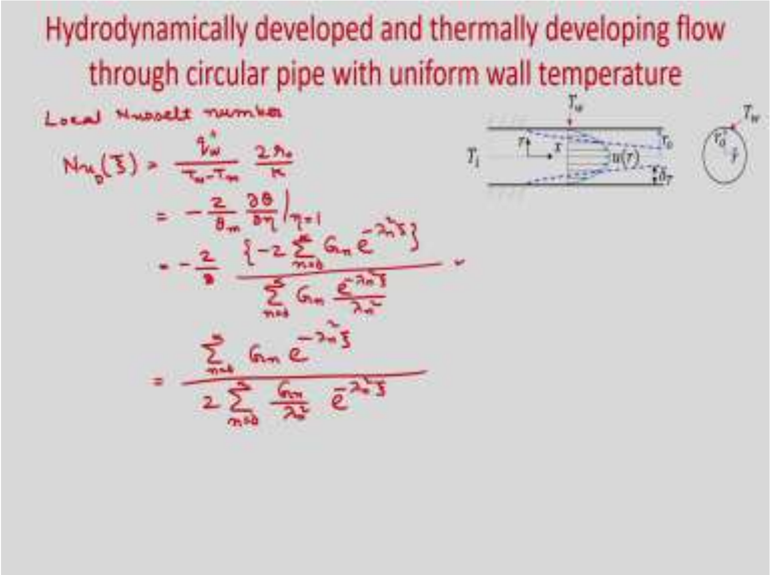

So, now you find the local Nusselt number. So, Nusselt number will be just $Nu_D = \frac{q_w^{"}}{T_w - T_m}\frac{2r_0}{K}$. So, you can see $-\frac{2}{\theta_m}\frac{\partial\theta}{\partial\eta}|_{\eta=1}$.

So, now, $\frac{\partial\theta}{\partial\eta}|_{\eta=1}$ already we have found. So, if you put it. So, you will get, $-\frac{2}{8}\frac{\left\{-2\sum_{n=0}^{\infty}G_n e^{-\lambda_n^2\xi}\right\}}{\sum_{n=0}^{\infty}G_n\frac{e^{-\lambda_n^2\xi}}{\lambda_n^2}}$.

So, this you can see that $\frac{\partial\theta}{\partial\eta}$ already we have found. So, if you put the θ_m expression and $\frac{\partial\theta}{\partial\eta}$ then you will get this expression. Finally, you will get this as $\frac{\sum_{n=0}^{\infty}G_n e^{-\lambda_n^2\xi}}{2\sum_{n=0}^{\infty}\frac{G_n}{\lambda_n^2}e^{-\lambda_n^2\xi}}$.

So, now, let us find the average Nusselt number. So, we will use the energy balance whatever we have derived earlier.

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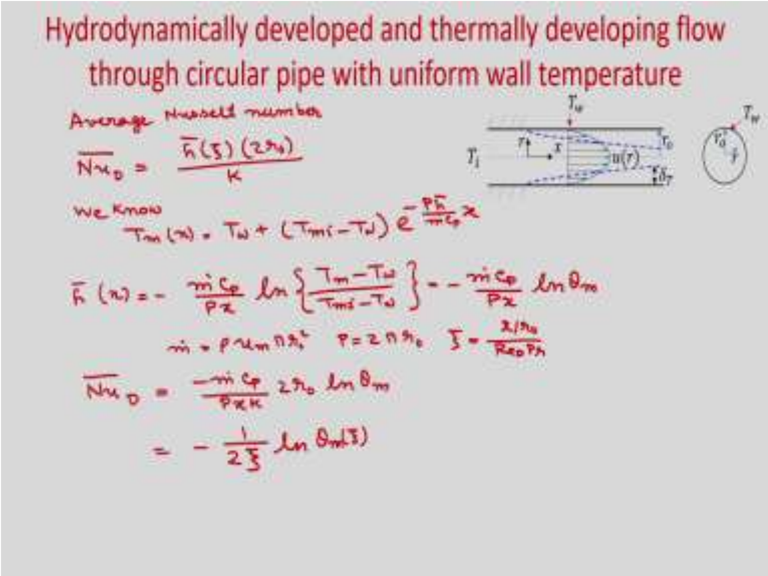

So, average Nusselt number if you calculate . So, you can write $\overline{Nu_D} = \frac{\bar{h}(\xi)(2r_0)}{K}$. So, it will be and from the energy balance we know $T_m(x) = T_w + (T_{mi} - T_w)e^{-\frac{P\bar{h}}{\dot{m}C_p}x}$.

So, from here you can find $\bar{h}(x) = -\frac{\dot{m}C_p}{Px}\ln\left\{\frac{T_m - T_w}{T_{mi} - T_w}\right\}$. So, it will be $-\frac{\dot{m}C_p}{Px}\ln\theta_m$. So, now, $\dot{m} = \rho u_m \pi r_0^2$ and $P = 2\pi r_0$ and $\xi = \frac{x/r_0}{\mathrm{Re}_D \Pr}$.

So, if you rearrange it so, and put it here this Nusselt number you are going to get $\overline{Nu_D} = -\frac{\dot{m}C_p}{Px}2r_0 \ln\theta_m$ after rearrangement you will get as $-\frac{1}{2\xi}\ln\theta_m(\xi)$. So, this is the average Nusselt number.

So, now, you can see so, to find this local Nusselt number average Nusselt number and the temperature distribution you need to find the eigenvalues, eigen function and those constants C_n and G_n. Once you find those numerically, then you will be able to find the temperature distribution and the Nusselt numbers.

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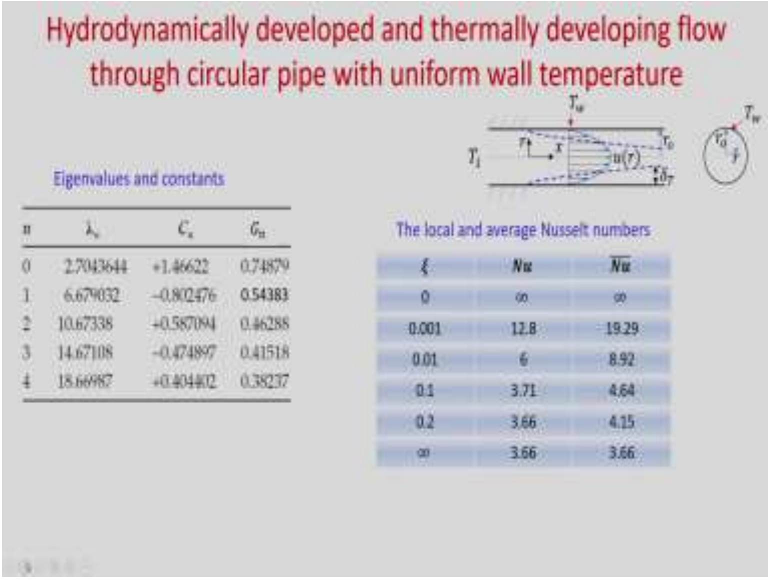

So, you can see for first five eigenvalues and constants from n = 0 to 4 we have written. So, this is the value of λ_n this is the constant C_n and Gn; G_n already we have written in terms of C_n. So, if C_n is known then G_n you will be able to evaluate. And, if you see the local and average Nusselt number so, as x varies. So, for different ξ 0 to ∞, I have written few terms.

Nusselt number, so, at $\xi=0$ it will be almost ∞ and average Nusselt number will be almost ∞ it will be very high value and as it $x \to \infty$, then it will be fully developed profile. And, if it is a hydrodynamically and thermally fully developed flow you should get the Nusselt number as 3.66 and that will be same as average Nusselt number 3.66.

So, you can see so, whatever expression we have derived for the temperature profile and the Nusselt number it is valid for developing region as well as developed region. So, you can see that if we put at $\xi \to \infty$ then we are getting back the Nusselt number which is constant and we have found earlier so, that is 3.66. So, you can see from this table.

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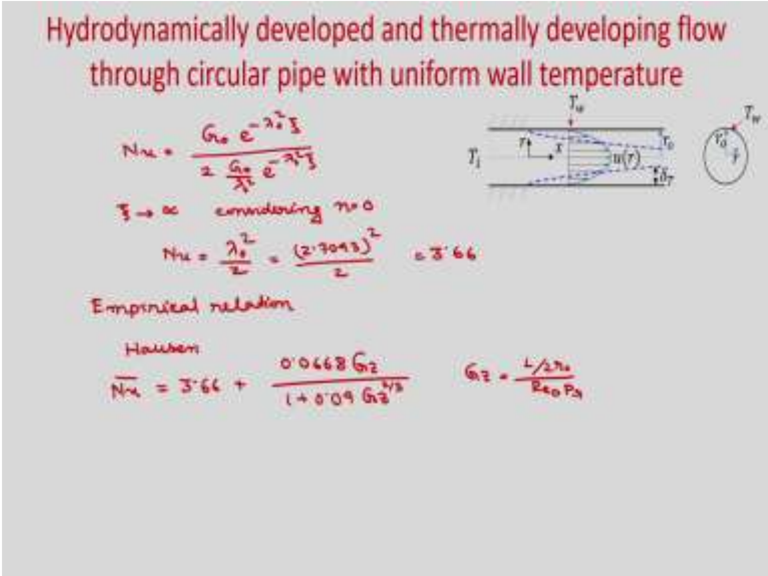

So, now, you see the limiting case. Limiting case at ξ→∞. So, $Nu = \dfrac{G_0 e^{-\lambda_0^2 \xi}}{2\dfrac{G_0}{\lambda_0^2} e^{-\lambda_0^2 \xi}}$. So, now, ξ→∞ considering the first value only for n = 0 neglecting the other terms with n >0, because all those terms are very small because exponentially decaying.

You can see that it will exponentially decaying, so, for n > 0 this will give very small value. So, you can see that $Nu = \dfrac{\lambda_0^2}{2}$ and λ_0^2 from this table if you see this is 2.7043. So, it will be $\dfrac{(2.7043)^2}{2}$; so, it will be 3.66 .

And, another empirical relations given by Hausen empirical relation proposed by Hausen. So, this is, $\overline{Nu} = 3.66 + \dfrac{0.0668Gz}{1+0.04Gz^{2/3}}$, where $Gz = \dfrac{L/2r_0}{\mathrm{Re}_D \,\mathrm{Pr}}$.

So, you can see the Nusselt number approaches constant value of 3.66 when the tube is sufficiently long because this term will become 0.

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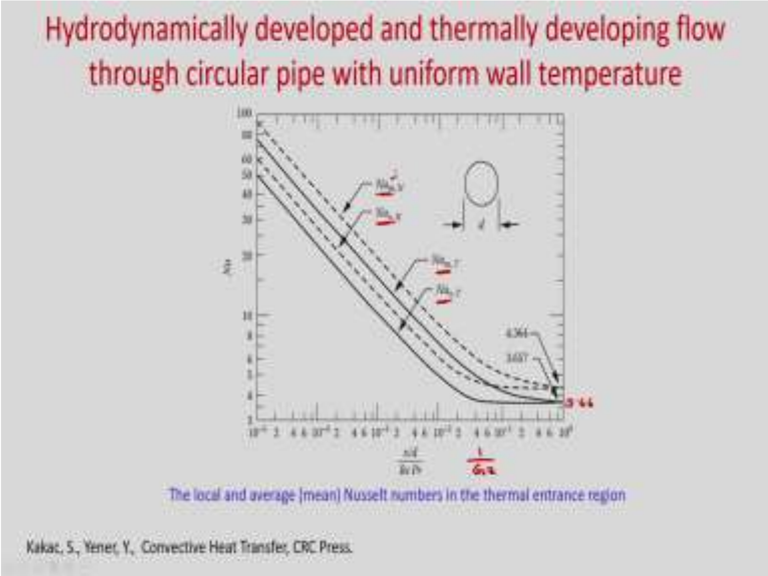

So, we can see the temperature profile. So, Nusselt number is plotted with $\frac{x/d}{\mathrm{Re}_D \Pr}$ which is $\frac{1}{Gz}$. So, for constant temperature, so, this is the average Nusselt number and this is the local Nusselt number. So, we can see local Nu_{NT} is very high value and it is exponentially decaying and it is becoming constant and it is you see it is almost 3.66 and it is for uniform wall heat flux boundary condition. So, local Nusselt number and the average Nusselt number.

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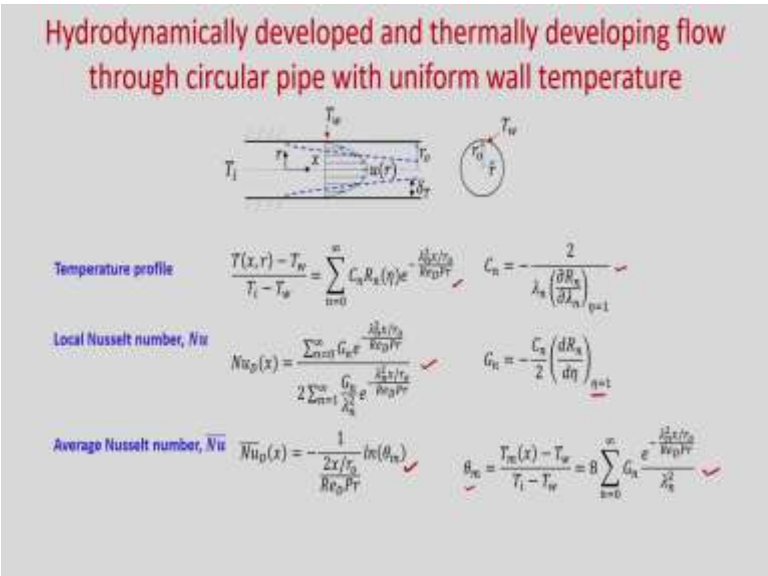

Finally, we got this temperature profile where C_n is given by this expression and the local Nusselt number we have derived like this, where $G_n = -\frac{C_n}{2}\left(\frac{dR_n}{d\eta}\right)_{\eta=1}$ and also we have derived the average Nusselt number. So, this is the expression where $\theta_m = \frac{T_m(x) - T_w}{T_i - T_w}$ with this expression.

So, today we considered hydrodynamically developed and thermally developing fluid flow through circular tube with uniform wall temperature. So, we used separation of variables method with suitable non-dimensional parameters; we defined the $\theta = \frac{T - T_w}{T_i - T_w}$. So, from there we found the eigen functions R_n from the second order differential equation and along x it varies exponentially.

So, with these two solutions product of these two solutions we found the temperature profile θ. Once we know the θ then we can find the Nusselt number and Nusselt number we have defined based on the temperature difference at wall and the mean temperature. And, Nusselt number based on diameter first we have found local Nusselt number, then we have found the average Nusselt number.

And, this is the general expression for Nusselt number when we put $x \to \infty$ then obviously, it becomes hydrodynamically and thermally fully developed flow. So, we should get back the constant Nusselt number and we derived earlier it as 3.66. And, today we have shown that if you put $x \to \infty$ you are getting back the Nusselt number as 3.66.

Thank you.