Fundamentals of Convective Heat Transfer
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Module - 06
Convection in Internal Flows - II
Lecture - 21
Hydrodynamically and thermally fully developed flow through circular pipe with uniform wall temperature

Hello everyone, today we will consider fully developed laminar flow through circular pipe with uniform wall temperature. For this flow through circular pipe, already we have found the temperature distribution and the Nusselt number with uniform wall heat flux boundary condition. When we will consider uniform wall temperature boundary condition, one important assumptions we have to take that axial heat conduction is negligible compared to the radial heat conduction.

Earlier case when we considered uniform wall heat plus boundary condition, your $\frac{\partial T}{\partial x}$ was constant as your heat flux $q_w^{"}$ was constant. So, hence the second derivative of T with respect to x, $\frac{\partial^2 T}{\partial x^2}$ becomes automatically 0 for the uniform wall heat flux boundary condition. However, in this particular case when we consider uniform wall temperature boundary condition, we need to assume that your axial heat conduction is negligible compared to the radial heat conduction.

Another assumptions we will take; that it is axisymmetric. What does it mean? It means that in circumferential direction there is no change of any property, that means, $\frac{\partial}{\partial \theta}$ of any quantity is 0. If geometry is symmetric and thermal boundary condition is symmetric, in this particular case we are considering uniform wall temperature boundary condition, hence you have a symmetric thermal boundary condition as well as it is wall circumferential direction there is no change of any quantity. Hence axisymmetric assumption is valid for this particular case.

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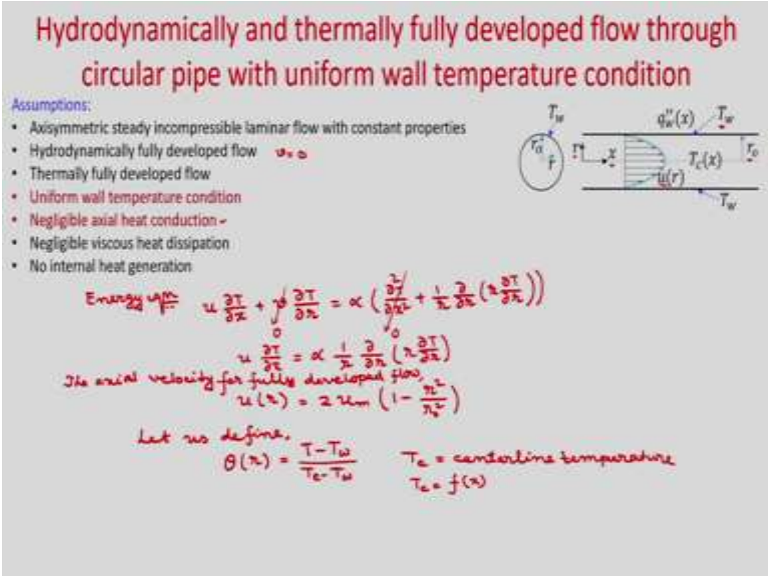

So, let us consider fully developed flow inside circular pipe. So, you can see x is the axial direction, r is the radial direction. The circular pipe is having radius r_0, wall is maintained at temperature T_w. So, you can see this is the circular pipe we have considered. So, these are the assumptions axisymmetric steady incompressible laminar flow with constant properties, hydrodynamically fully developed flow.

So, the we have considered u is function of r only because it is a fully developed flow, and we can write in terms of mean velocity. Thermally fully developed flow, that means, $\frac{d\phi}{dr}=0$ and we have considered uniform wall temperature condition. And we are neglecting the axial heat conduction. As well as we have assumed that negligible viscous dissipation, and no internal heat generation.

So, first let us write the governing equation. So, after invoking all these condition, you will be able to write the governing equation as; $u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)\right)$. So, this is your energy equation in general.

So, as it is a fully developed flow hydrodynamically fully developed flow, so v = 0 so. this is 0. And we are neglecting the axial heat conduction, so this is also 0. So, you will get $u\frac{\partial T}{\partial x}=\alpha\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$. And you know the fully developed you know that fully developed velocity profile is, $u(r) = 2u_m\left(1 - \frac{r^2}{r_0^2}\right)$. So, this is your the axial velocity for fully developed flow.

Now, we will define on non-dimensional temperature $\theta = \frac{T - T_w}{T_c - T_w}$ where T_c is your centerline temperature. In this case also, θ will not vary in the axial direction. So, θ is function of r only. So, let us define θ which is function of r only as $\theta(r) = \frac{T - T_w}{T_c - T_w}$, where T_c is your centerline temperature, and T_c is function of x only.

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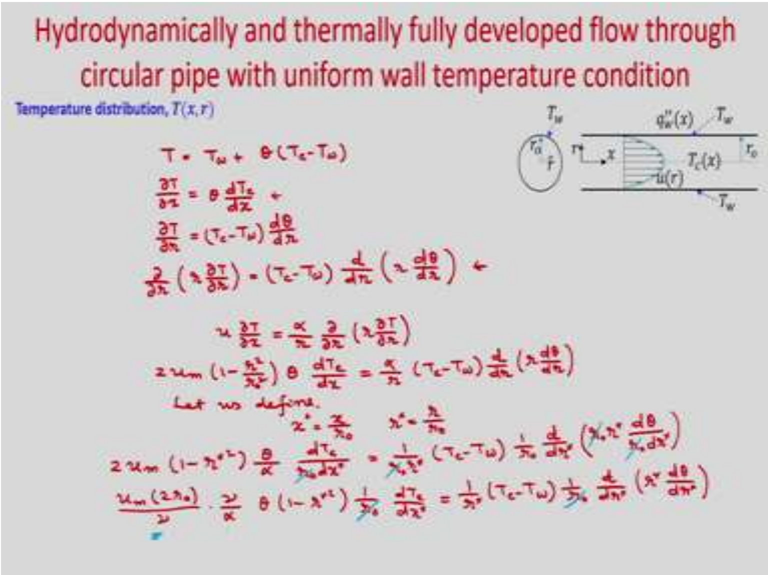

So, if you now take the derivatives of temperature with respect to x and r, then we can write, so $T = T_w + \theta(T_c - T_w)$. So, you can see T_w is constant. So, θ is function of r only, so you can write $\frac{\partial T}{\partial x} = \theta \frac{dT_c}{dx}$. Similarly, you can write $\frac{\partial T}{\partial r} = (T_c - T_w)\frac{d\theta}{dr}$. And if you write $\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = (T_c - T_w)\frac{d}{dr}\left(r\frac{d\theta}{dr}\right)$.

So, now all these you put in the energy equation. So, you have the energy equation as $u\frac{\partial T}{\partial x} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$. So, u is the axial velocity for fully developed flows. So, now, you put all the values in this energy equation. So, u is the axial velocity for fully developed flow and it can be written as, $2u_m\left(1-\frac{r^2}{r_0^2}\right)\theta\frac{dT_c}{dx}=\frac{\alpha}{r}(T_c-T_w)\frac{d}{dr}\left(r\frac{d\theta}{dr}\right)$.

So, now let us define; non-dimensional x coordinate as $x^*=\frac{x}{r_0}$, and radial non-dimensional radial coordinate $r^*=\frac{r}{r_0}$. So, if you put it here, then you will get twice $2u_m\left(1-r^{*2}\right)\frac{\theta}{\alpha}\frac{dT_c}{r_0dx^*}=\frac{1}{r_0r^*}(T_c-T_w)\frac{1}{r_0}\frac{d}{dr^*}\left(r_0r^*\frac{d\theta}{r_0dr^*}\right)$.

So, if you see this r_0, this r_0, you can cancel, then one r_0 here you can cancel; now, you simplify it. So, you will write,

$$\frac{u_m(2r_0)}{\nu}\frac{\nu}{\alpha}\theta\left(1-r^{*2}\right)\frac{1}{r_0}\frac{dT_c}{dx^*}=\frac{1}{r^*}(T_c-T_w)\frac{1}{r_0}\frac{d}{dr^*}\left(r^*\frac{d\theta}{dr^*}\right).$$

Here you can cancel this r_0, this r_0 and this r_0. So, you see what is this? So, this is your Reynolds number based on diameter 2 r_0, and $\frac{\nu}{\alpha}$ is your Prandtl number.

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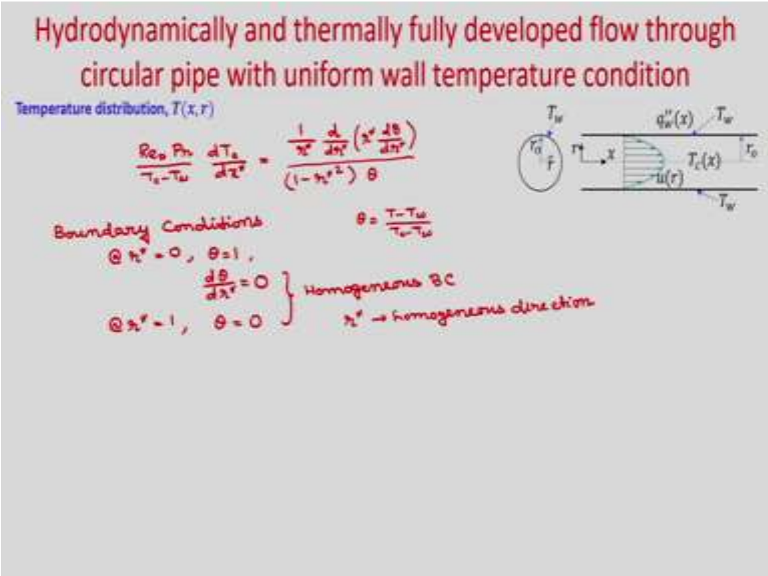

So, you can write, $\frac{\mathrm{Re}_D \Pr}{T_c - T_w}\frac{dT_c}{dx^*} = \frac{\frac{1}{r^*}\frac{d}{dr^*}\left(r^*\frac{d\theta}{dr^*}\right)}{\left(1-r^{*2}\right)\theta}$. So, after rearranging, you can write in this form. Now, you see the left hand side Reynolds number and Prandtl number are constant, T_c is the centerline temperature which is function of x only, T_w is constant, and $\frac{dT_c}{dx}$ is also function of x, so left hand side is function of x only.

Now, right hand side if you see here all terms are function of r* only. So, now, we have separated the variables. Left hand side is function of x only; right hand side is function of r* only. So, you can see that left hand side is function of x or x*, and right hand side is function of r or r* only. So, we have separated the variables now, these equal to some constant.

So, how we will choose the constant? First let us see the boundary conditions which is the homogeneous direction first let us see. Then accordingly we will choose the sign of this constant such that in homogeneous direction we get the harmonic solution, so that is the rule of using separation of variables method.

So, what are the boundary condition, first let us see. So, you can see that at r *= 0, that means, centerline temperature. How we have defined the θ? θ we have defined as $\theta = \frac{T - T_w}{T_c - T_w}$. So, at r *= 0 that means that the central line, obviously, T = T_c. So, if we put T_c, θ = 1.

Now, at the same time you can see that the problem is axisymmetric, and it is it is geometrically and thermally symmetric. So, at the center, you will have either maximum of minimum temperature. So, we can write that $\frac{d\theta}{dr^*}$ =0. So, although θ = 1, but another you can write at r *= 0, $\frac{d\theta}{dr^*}$ =0. And at the wall, what is the boundary condition at r *= 1, so you see T = T_c right, so θ will be 0.

Now, you see at r *= 0, you have $\frac{d\theta}{dr^*}$ =0, you have $\frac{d\theta}{dr^*}$ =0, and r *= 1 you have θ =0, so that means, these are homogeneous boundary conditions. What is homogeneous boundary condition? If the value of that variable is 0 or its gradient is 0 or combination of these two is 0, so that is known as homogeneous boundary condition.

So, you have in r direction both the boundary conditions are homogeneous; so. it is a homogeneous direction. So, r* is the homogeneous direction. So, you should choose the value of constant, you should choose the sign of the constant such a way that in homogeneous direction you get the harmonic solution. So, this is your homogeneous boundary condition. And r * or r is the homogeneous direction homogeneous direction.

So, now how we will determine that you will have the harmonic solution in the homogeneous direction? So, for that we will use the Sturm Liouville boundary value problem.

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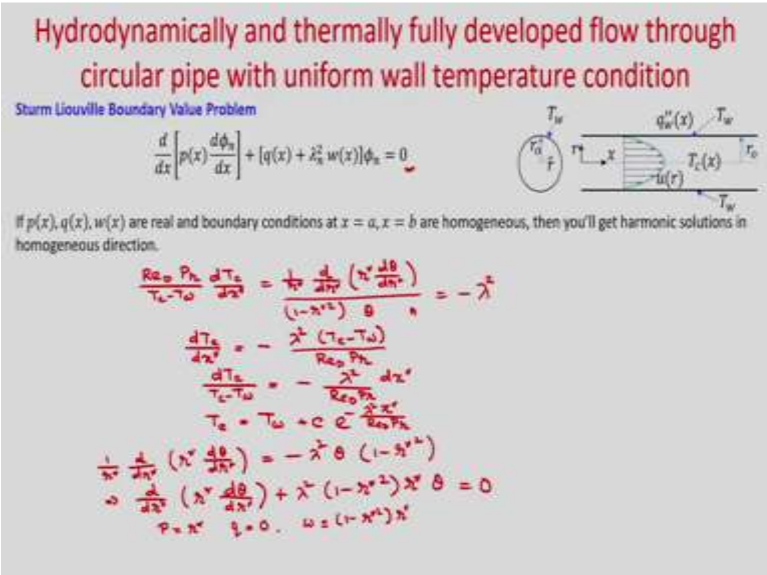

So, you see Sturm Liouville boundary value problem; it is actually given by this second order ordinary differential equation. So, you can see $\frac{d}{dx}\left[p(x)\frac{d\phi_n}{dx}\right]+\left[q(x)+\lambda_n^2 w(x)\right]\phi_n=0$, where w(x) is the weighting function. If p(x), q(x), w(x) are real, and boundary conditions at x = a, and x = b are homogeneous, then you will get harmonic solutions in homogeneous direction.

So, now let us see that if we choose the sign of the constant as $-\lambda^2$, then what will happen. So, let us say that this $\frac{\mathrm{Re}_D \Pr}{T_c - T_w}\frac{dT_c}{dx^*} = \frac{\frac{1}{r^*}\frac{d}{dr^*}\left(r^*\frac{d\theta}{dr^*}\right)}{\left(1-r^{*2}\right)\theta}$ = some constant, and the sign of that constant we are taking $-\lambda^2$. λ^2 we are taking for convenience, it is any constant. So, we are taking the sign of this constant as minus, so that in r* direction we get the harmonic solution.

Now, if you write the equation, so you will get the, if you write first one, so you will get $\frac{dT_c}{dx^*} = -\frac{\lambda^2 (T_c - T_w)}{\mathrm{Re}_D \Pr}$. And you can write $\frac{dT_c}{T_c - T_w} = -\frac{\lambda^2}{\mathrm{Re}_D \Pr} dx^*$. Now, if you integrate it, so you will get $T_c = T_w + Ce^{-\frac{\lambda^2 x^*}{\mathrm{Re}_D \Pr}}$. So, this is the variation of centerline temperature.

So, now if you write for this one, so you will get $\frac{1}{r^*}\frac{d}{dr^*}\left(r^*\frac{d\theta}{dr^*}\right) = -\lambda^2\theta\left(1-r^{*2}\right)$. So, if you rearrange, you well get $\frac{d}{dr^*}\left(r^*\frac{d\theta}{dr^*}\right) + \lambda^2\left(1-r^{*2}\right)r^*\theta = 0$.

If you compare this equation with the Sturm Liouville boundary value problem, then p=r*, q = 0 and the weighting function w $=\left(1-r^{*2}\right)r^*$. So, now, you see p, q, r are real and boundary condition in the r* direction both the boundary conditions are homogeneous. So, r * is the homogeneous direction. Hence the solution of this second order differential equation will give harmonic solution.

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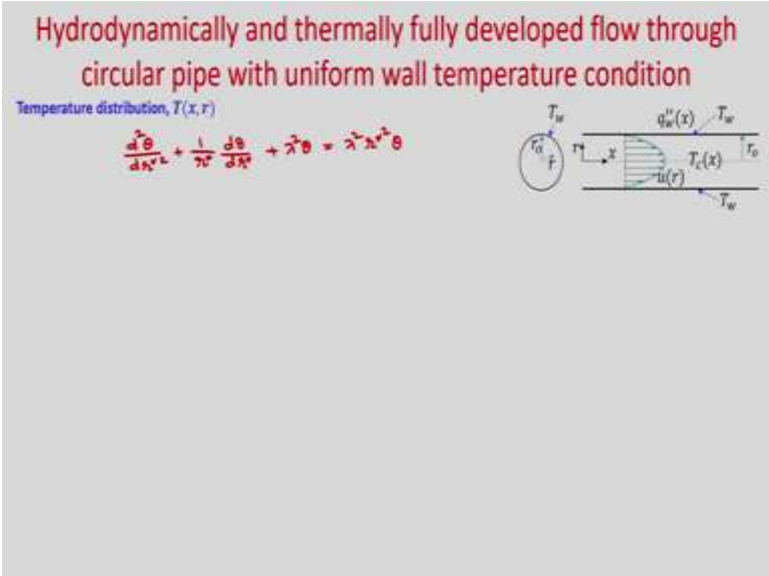

So, now if you write it, so you will get after rearranging $\frac{d^2\theta}{dr^{*2}} + \frac{1}{r^*}\frac{d\theta}{dr^*} + \lambda^2\theta = \lambda^2 r^{*2}\,\theta$.

So, now we want to seek the solution of this differential equation as a series solution and in last class we have already discussed that whether we can have the series solution of these ordinary differential equation or not for that let us revisit it again.

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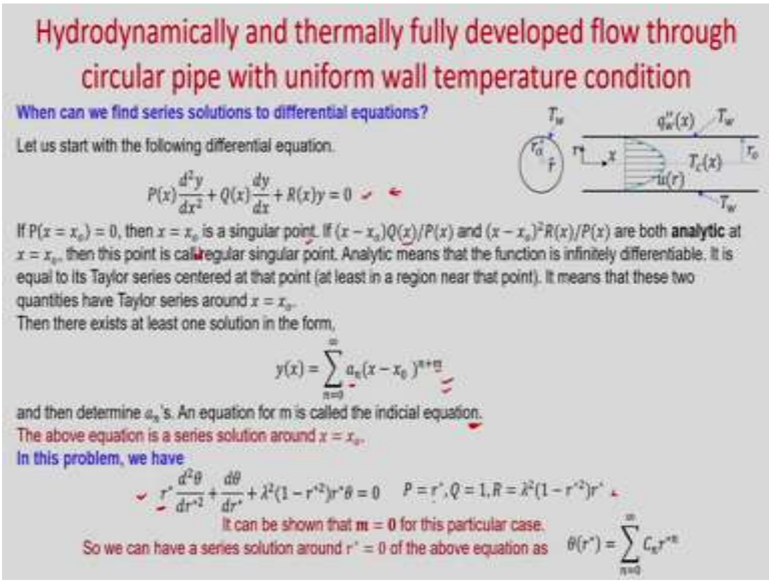

So, before going to the solution, let us first see when can we find series solution to differential equations. So, this is the second order ordinary differential equation. In this case, we will see that P at x $=x_0$, if it is 0, then x $=x_0$ is a singular point. So, if $\left(x-x_0\right)Q(x)/P(x)$, and $\left(x-x_0\right)^2 R(x)/P(x)$ are both analytic at x = x_0, then this point is called regular singular point.

Analytic means that; the function is infinitely differentiable, so that we already discussed in last class. Now, if we have this, then we can have the solution in the from $y(x)=\sum_{n=0}^{\infty} a_n\left(x-x_0\right)^{n+m}$.

So, here then we have to determine the coefficient a_n. And this a_n, an equation for m is called the indicial equation, so that we also need to find. So, the above equation is just series solution at around x $=x_0$.

So, for our present problem, you can see that our governing equation whatever we have derived this is the second order differential equation where P = r *, Q = 1, and $R=\lambda^2\left(1-r*^2\right)r*$. So, comparing with this equation just we have found it.

So, if you see that this condition, so for this present case $r*\frac{Q}{P}$, and $r*^2\frac{R}{P}$ are both analytic. And at r = 0 it is singular point. So, you can see that we can write the solution in this form, but we have not written +m, because it can be shown that m = 0 for this particular case.

So, this derivation will be easier because we have already assumed that m = 0, it can be shown actually. So, for this particular case, this is m = 0. So, we can have a series solution around r * = 0 of the above equation is this one. So, now you can see that for this differential equation, your $\frac{Q}{P}$, and $\frac{R}{P}$ are analytic. So, hence you can have the series solution.

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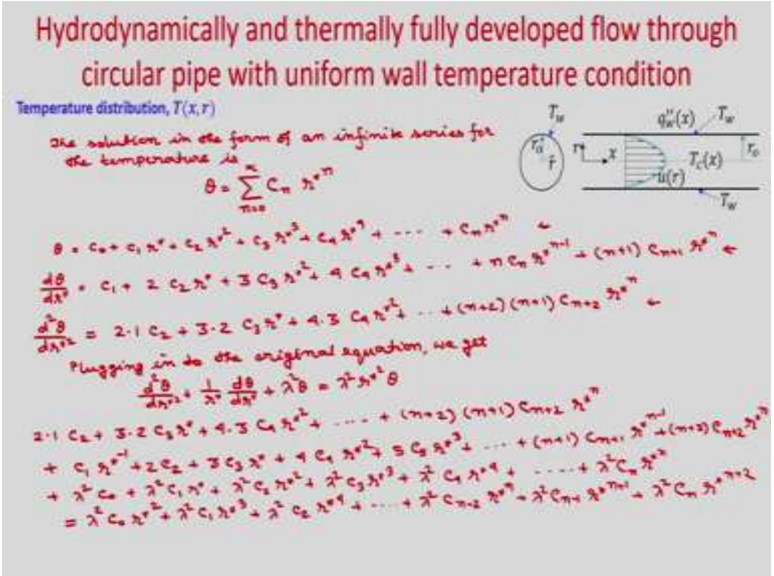

So, if you write the series solution about x about r* = 0, then we can write the solution in the form of an infinite series for the temperature is, so you can write, $\theta = \sum_{n=0}^{\infty} C_n r^{*n}$.

So, first we will find the derivative of θ with respect to r*, then we will put back to the ordinary differential equation, and we will try to find the coefficient equating the similar power. So, we will write $\theta = C_0 + C_1 r^* + C_2 r^{*2} + C_3 r^{*3} + C_4 r^{*4} + ... + C_n r^{*n}$.

So, now you write the derivative of θ with respect to r* as $\frac{d\theta}{dr^*} = C_1 + 2C_2 r^* + 3C_3 r^{*2} + 4C_4 r^{*3} + ... + nC_n r^{*n-1} + (n+1)C_{n+1} r^{*n}$. Then let us find the second derivative of θ. So, you can see that it is $\frac{d^2\theta}{dr^{*2}} = 2.1C_2 + 3.2C_3 r^* + 4.3C_4 r^{*2} + ... + (n+2)(\mathrm{n}+1)C_{n+2} r^{*n}$.

So, now you plug all these into the ordinary differential equation. So, what is our ordinary differential equation? So, plugging into the original equation we get. So, we have the equation $\frac{d^2\theta}{dr^{*2}} + \frac{1}{r^*}\frac{d\theta}{dr^*} + \lambda^2\theta = \lambda^2 r^{*2}\,\theta$. So, this is your equation.

Now, if you put it, so you will get,

$$2.1C_2 + 3.2C_3 r^* + 4.3C_4 r^{*2} + ... + (n+2)(\mathrm{n}+1)C_{n+2} r^{*n}$$

$$+C_1 r^{*-1} + 2C_2 + 3C_3 r^* + 4C_4 r^{*2} + 5C_4 r^{*3} ..... + (n+1)C_{n+1} r^{*n-1} + (n+2)C_{n+2} r^{*n}$$

$$+\lambda^2 C_0 + \lambda^2 C_1 r^* + \lambda^2 C_2 r^{*2} + \lambda^2 C_3 r^{*3} + \lambda^2 C_4 r^{*4} + ... + \lambda^2 C_n r^{*n}$$

So, left hand side all we have written, now you write the right hand side. So, right hand side is $\lambda^2 C_0 r^{*2} + \lambda^2 C_1 r^{*3} + \lambda^2 C_2 r^{*4} + ... + \lambda^2 C_{n-2} r^{*n} + \lambda^2 C_{n-1} r^{*n+1} + \lambda^2 C_n r^{*n+2}$. So, now let us equate the power of r*. So, now, let us equate the equal power of r* and find the coefficient.

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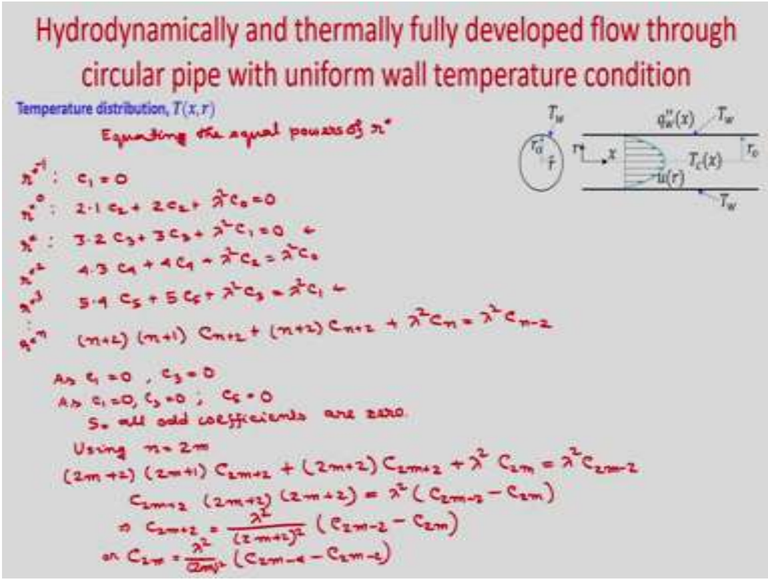

So, equating the equal power powers of r*; so first we will find r^{*-1} that means, $\frac{1}{r^*}$. So, for that what are the if you see left hand side in first line there is no term, the second it is there C_1, and third there is no term and right hand side there is no term; so only C_1 will become 0, so here C_1= 0.

Now, equate the equal powers of r^{*0} that means, in the left hand side you see; so there is no r *, so it is $r^{*0}: 2.1C_2 + 2C_2 + \lambda^2 C_0 = 0$. Now, you equate the power of r *; $r^*: 3.2C_3 + 3C_3 + \lambda^2 C_1 = 0$.

Similarly, if you write for r^{*2}; so you can write $r^{*2}: 4.3C_4 + 4C_4 + \lambda^2 C_2 = \lambda^2 C_0$. So, we have written this one.

Similarly, $r^{*3}: 5.4C_5 + 5C_5 + \lambda^2 C_3 = \lambda^2 C_1$.

And $r^{*n}: (n+2)(\mathrm{n}+1)C_{n+2} + (n+2)C_{n+2} + \lambda^2 C_n = \lambda^2 C_{n-2}$.

So, now you see the first term C_1= 0 right; if C_1= 0, then from here you can see if C_1= 0, then C_3 will become 0 right. And if C_1,C_3= 0, then from this equation you can see C_5= 0. So, you can see all the odd coefficients are 0; C_1, C_3, C_5, C_7, all odd coefficients will become 0 as C_1=0. So, you can see that as C_1=0 from there you can see C_2,C_3=0, then as C_1=0, C_3=0; then from this equation C_5= 0. So, you can see so all odd coefficients are 0.

So, we can replace n = 2 m , because there will be no odd coefficient only the even coefficients will be there. So, you can write using n = 2 m, we can write twice $(2m+2)(2m+1)C_{m+2} + (2m+2)C_{m+2} + \lambda^2 C_{2m} = \lambda^2 C_{2m-2}$.

So, so now if you rearrange it, so you see $C_{m+2}(2m+2)(2m+2) = \lambda^2\left(C_{2m-2} - C_{2m}\right)$.

So, you can write $C_{m+2} = \dfrac{\lambda^2}{(2m+2)^2}\left(C_{2m-2} - C_{2m}\right)$; or you can write $C_{2m} = \dfrac{\lambda^2}{(2m)^2}\left(C_{2m-4} - C_{2m-2}\right)$.

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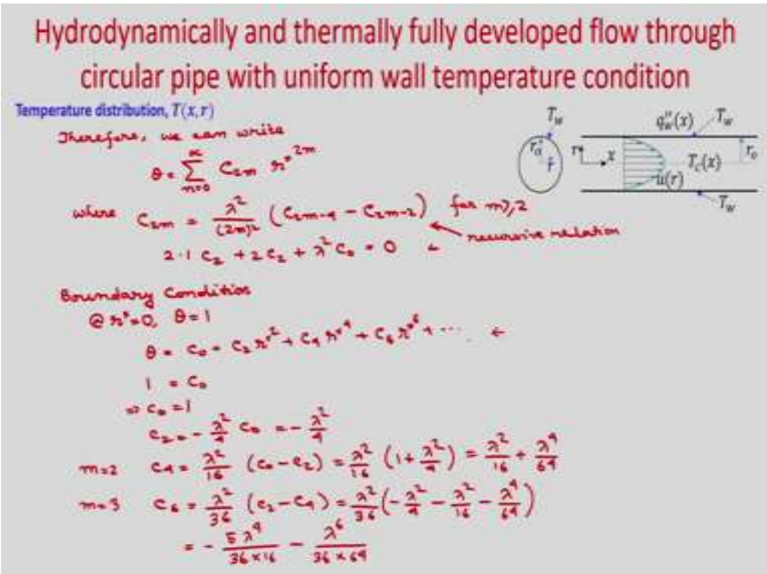

So, now as all odd coefficients are 0, now we can write the solution of θ as $\theta = \sum_{n=0}^{\infty} C_{2m} r^{*2m}$, where we have already found; what is C_{2m},

$C_{2m} = \frac{\lambda^2}{(2m)^2}\left(C_{2m-4} - C_{2m-2}\right)$ and this is valid for m ≥ 2. And we have another expression this one , so this we can write $2.1C_2 + 2C_2 + \lambda^2 C_0 = 0$.

So, now let us find the value of the coefficients C_2, C_4, C_6, in terms of C_0. And if we apply the boundary condition, then we will be able to find what is the value of C_0. So, first let us write the boundary condition at r * = 0; θ = 1. So, you can write θ = 1 in the left hand side and right hand side; so you can write first let us expand it, then it will be easier $\theta = C_0 + C_2 r^{*2} + C_4 r^{*4} + C_6 r^{*6}$ ....

So, if you put θ = 1 and r * = 0, so first term will remain and other terms will become 0 that means, $C_0 = 1$. So, now to find the other coefficient C_2, C_4, C_6, in terms of C_0. And you can see that this term this expression $C_{2m} = \frac{\lambda^2}{(2m)^2}\left(C_{2m-4} - C_{2m-2}\right)$, it is a recursive relation.

So, if you find C_0, then from there you can find C_2; and if you know C_0 and C_2, then you can find C_4, and C_6, C_8 you can find accordingly. So, from this expression what is the value of C_2? So, you can see $C_2 = -\frac{\lambda^2}{4} C_0 = -\frac{\lambda^2}{4}$.

Then m = 2 if you put in the recursive relation, so this is your recursive relation; so if you put m = 2, then you will get $C_4 = \frac{\lambda^2}{16}(C_0 - C_2)$. Now, C_0 C_2 you know; so if we put the value, $\frac{\lambda^2}{16}(1 + \frac{\lambda^2}{4}) = \frac{\lambda^2}{16} + \frac{\lambda^2}{64}$.

Similarly, if you put m = 3, then $C_6 = \frac{\lambda^2}{36}(C_2 - C_4) = \frac{\lambda^2}{36}(-\frac{\lambda^2}{4} - \frac{\lambda^2}{16} - \frac{\lambda^2}{64})$.

So, if you rearrange it, so you will get $-\frac{5\lambda^4}{36\times16}+\frac{\lambda^6}{36\times64}$. So, if you can find the other constant, then you will be able to find; if you find the other coefficients, then you will be able to find the temperature profile θ, because θ you can write using this expression. So, C_0, C_2, C_4, C_6 already we have found; another coefficients if you find, then you will be able to find the temperature distribution.

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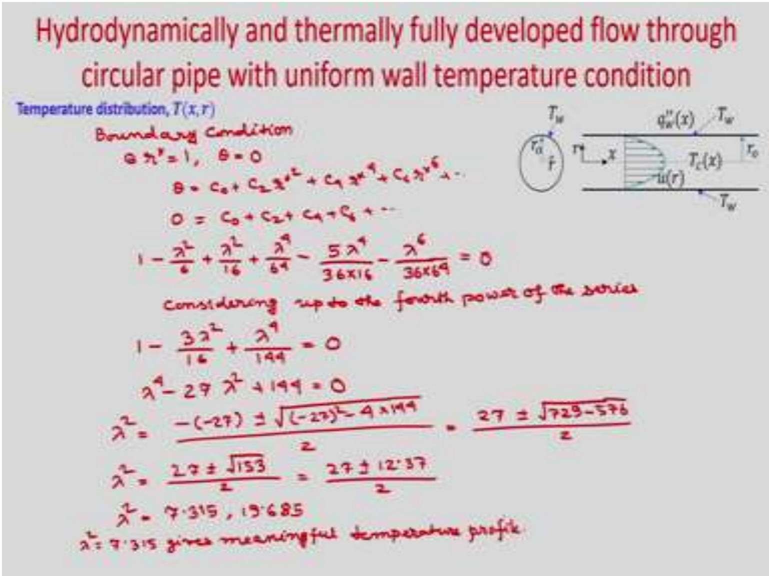

Now, let us find the value of λ^2. Now, apply another boundary condition at r * = 1 means at the wall, θ =0. So, now you know the expression $\theta = C_0 + C_2 r^{*2} + C_4 r^{*4} + C_6 r^{*6}$......... So, at r * = 1, so θ = 0; so that means $0 = C_0 + C_2 + C_4 + C_6 + ...$

Now, we know the value of C_0, C_2, C_4, C_6 already we have found let us put it in this expression. Then you can write C_0, C_2, C_4, C_6 in the left hand side, so we can write $1-\frac{\lambda^2}{6}+\frac{\lambda^2}{16}+\frac{\lambda^2}{64}-\frac{5\lambda^4}{36\times16}-\frac{\lambda^6}{36\times64}=0$. So, we have considered up to the fourth power of series.

You can see that after that if you see this term $\frac{\lambda^6}{36\times64}$, so the denominator is very high value. So, it will contribute very less here, so and this other term. So, neglecting all the other terms and considering up to the fourth power of series you can write as, $1-\frac{3\lambda^2}{16}+\frac{\lambda^4}{144}=0$. so if you rearrange it, you will get $\lambda^4-27\lambda^2+144=0$.

And λ^2 if you find, so it will be $\lambda^2=\frac{-(-27)\pm\sqrt{(-27)^2-4\times 144}}{2}$. So, it will be $\frac{27\pm\sqrt{729-576}}{2}$ and you will get $\frac{27\pm\sqrt{153}}{2}$; so you will get $\frac{27\pm 12.37}{2}$, so you will get the value of $\lambda^2=7.315, 19.685$.

So, the first value $\lambda^2=7.315$ will give you the physically correct temperature profile, the other one will not give; so you consider the value of $\lambda^2=7.315$. So, $\lambda^2=7.315$ gives meaningful temperature profile, so we are considering only this value .

So, if you now you know the λ^2, you know the values of C_0, C_2, C_4, C_6 and so forth. If you put it in the expression of θ, then you will be find you will be able to find the temperature profile.

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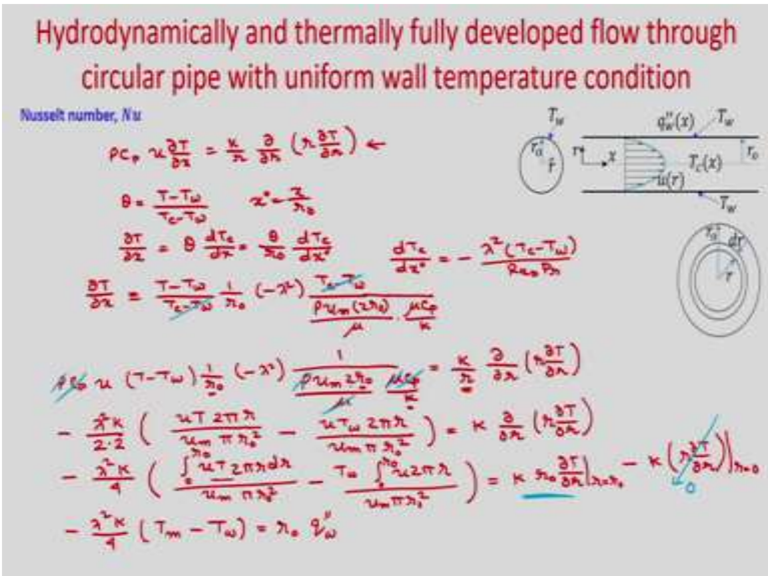

Now, let us find what is the Nusselt number? So, to calculate the Nusselt number we will start from the again governing equation, so it is $\rho C_P u \frac{\partial T}{\partial x} = \frac{K}{r}\frac{\partial}{\partial r}(r\frac{\partial T}{\partial r})$. So, we know $\theta = \frac{T - T_w}{T_c - T_w}$, and also $x^* = \frac{x}{r_0}$, you know $\frac{\partial T}{\partial x} = \theta \frac{dT_c}{dx} = \frac{\theta}{r_0}\frac{dT_c}{dx^*}$.

If you see we have found $\frac{dT_c}{dx^*} = -\frac{\lambda^2 (T_c - T_w)}{\mathrm{Re}_D \Pr}$. So, if you put it here, so you will get $\frac{\partial T}{\partial x} = \frac{T - T_w}{T_c - T_w}\frac{1}{r_0}(-\lambda^2)\frac{T_c - T_w}{\frac{\rho u_m (2r_0)}{\mu}\frac{\mu C_P}{K}}$.

So, you can see this you can cancel $T_c - T_w$, $T_c - T_w$. And you can write in the governing equation if you put it, then you will get, $\rho C_P u (T - T_w)\frac{1}{r_0}(-\lambda^2)\frac{1}{\frac{\rho u_m (2r_0)}{\mu}\frac{\mu C_P}{K}} = \frac{K}{r}\frac{\partial}{\partial r}(r\frac{\partial T}{\partial r})$.

So, you see what are the terms you can cancel, ρC_P, ρC_P; this μ, you can cancel. So, now, if you see you can rearrange it as $-\frac{\lambda^2 K}{2.2}\left(\frac{uT 2\pi r}{u_m \pi r_0^2} - \frac{uT_w 2\pi r}{u_m \pi r_0^2}\right) = K\frac{\partial}{\partial r}(r\frac{\partial T}{\partial r})$.

So, we have done some rearrangement; now we will integrate both side $-\frac{\lambda^2 K}{4}\left(\frac{\int_0^{r_0} uT 2\pi r dr}{u_m \pi r_0^2} - \frac{T_w \int_0^{r_0} u 2\pi r dr}{u_m \pi r_0^2}\right) = K r_0 \frac{\partial T}{\partial r}\Big|_{r=r_0} - K(r\frac{\partial T}{\partial r})\Big|_{r=0}$.

What is this term? So, this is nothing but the heat flux at the wall right, because $q_w'' = K\frac{\partial T}{\partial r}\Big|_{r=r_0}$ at the wall. So, if you now see this term, what does it mean u T X da you see at a distance r; if you take a small step dr, so whatever the area that is $2\pi r dr$ and that is uTda. So, integral $\frac{\int_0^{r_0} uTda}{u_m \pi r_0^2}$ what is that that is nothing but the definition of mean temperature, so we can write this term as the mean temperature.

Second term $\int_0^{r_0} u 2\pi r dr$, so that is the $u_m \pi r_0^2$ right, so that means this will get cancel. So, essentially you will get $-\frac{\lambda^2 K}{4}(T_m - T_w) = r_0 q_w^{"}$.

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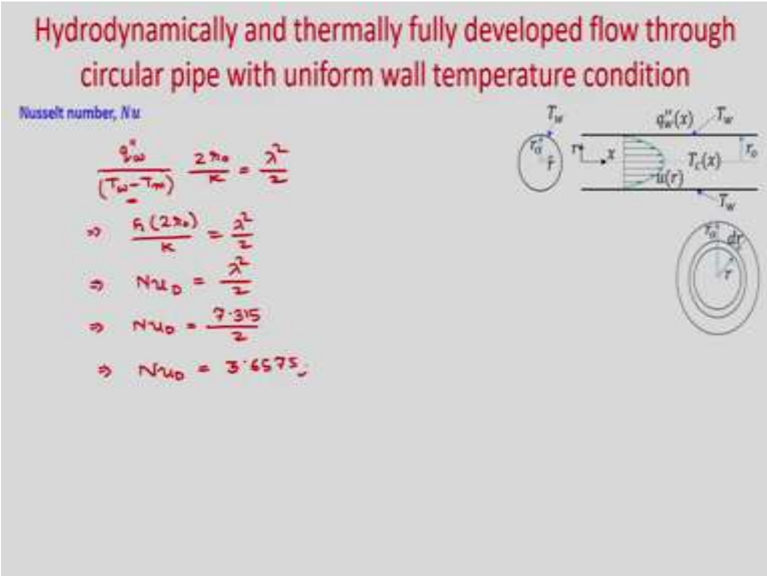

So, now if you rearrange it, so you can see $\frac{q_w^{"}}{(T_m - T_w)} \frac{2r_0}{K} = \frac{\lambda^2}{2}$ that means, so what is this; this is a local heat transfer coefficient that is $\frac{h(2r_0)}{K} = \frac{\lambda^2}{2}$. What is $\frac{h(2r_0)}{K}$ nothing but local Nusselt number, so $Nu_D = \frac{\lambda^2}{2}$. And we know the value of λ^2 as 7.315 , so it will be, $Nu_D = \frac{7.315}{2}$; so your Nusselt number is 3.6575.

So, you can see for this particular case when we consider fully developed laminar flow through circular pipe, the Nusselt number is independent of Reynolds number and Prandtl number, because it is a constant value. When we consider uniform wall temperature, the value of $Nu_D = 3.6575$.

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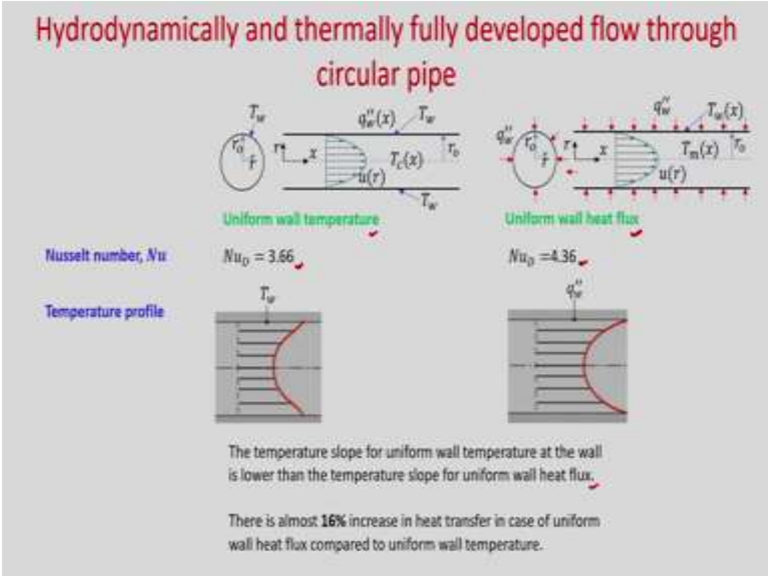

Now, let us summarise what we have done today. So, today we considered fully developed laminar flows through circular pipe with uniform wall temperature boundary condition. In this particular case, one important assumptions we have made that is axial heat conduction is negligible compared to the radial heat conduction, and also we have assumed that it is a axisymmetric flow.

Then we have started from the energy equation, then we have separated the variables in terms of x and r and we have compared the homogeneous direction whatever governing equation you are getting, ordinary differential equation with the second order differential equation which is your Sturm-Liouville boundary value problem. And we have shown that that is your in radial direction which is your homogeneous direction, you will get the harmonic solution.

Then we have checked whether we can use the power series solution or not, and considering that we have taken the solution as a power series and we have found the coefficient C_0, C_2, C_4, C_6; and from there we have found the value of lambda square. And then we have found the Nusselt number from the starting from the governing equation, and we have found Nusselt number for this particular case is independent of Reynolds number and Prandtl number and it is constant value.

Now, if you see earlier, we have already found the Nusselt number for this particular case flow through a circular pipe with uniform wall heat flux as 4.36. And when you consider today, uniform wall temperature for this flow through circular pipe the Nusselt number is 3.66. And if you see the temperature profile for uniform wall temperature, if you see the gradient at the wall , the temperature gradient at the wall, and if you see the uniform wall plus case, the temperature gradient at the wall obviously you can see it is much higher than the uniform wall temperature gradient.

So, the temperature slope for uniform wall temperature at the wall is lower than the temperature slope for uniform wall heat flux. And if you see the Nusselt number, there is almost 16 % increase in heat transfer in case of uniform wall heat flux compared to the uniform wall temperature, and it is due to the higher gradient at the wall of this temperature, higher gradient of the temperature at the wall for the case of uniform wall heat flux.

Thank you.