Fundamentals of Convective Heat Transfer
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Module - 06
Convection in Internal Flows - II
Lecture - 20
Hydrodynamically and thermally fully developed flow through parallel plate channel with uniform wall temperature

Hello everyone, today we will consider fully developed laminar flow through parallel plate channel with uniform wall temperature. So, you have seen that in earlier classes we considered different cases with uniform wall heat flux boundary condition. In that case, if you remember we have seen that temperature gradient with respect to x; $\frac{\partial T}{\partial x}$ is constant, as you have constant heat flux boundary condition.

Hence, the second derivate of temperature with respect to x, $\frac{\partial^2 T}{\partial x^2} = 0$; that means, axial heat conduction was 0, for the case with uniform wall heat flux boundary condition. But today, we are considering uniform wall temperature boundary condition, hence your axial heat conduction will not be 0.

However, in this particular case we will make a special assumption that your axial heat conduction is very very small compared to your radial heat transfer. So, we can neglect $\frac{\partial^2 T}{\partial x^2}$, as $\frac{\partial^2 T}{\partial x^2} << \frac{\partial^2 T}{\partial y^2}$.

So, when we will consider uniform wall temperature boundary condition, we will neglect the axial heat conduction. So, it is the major assumption we are considering in today's class.

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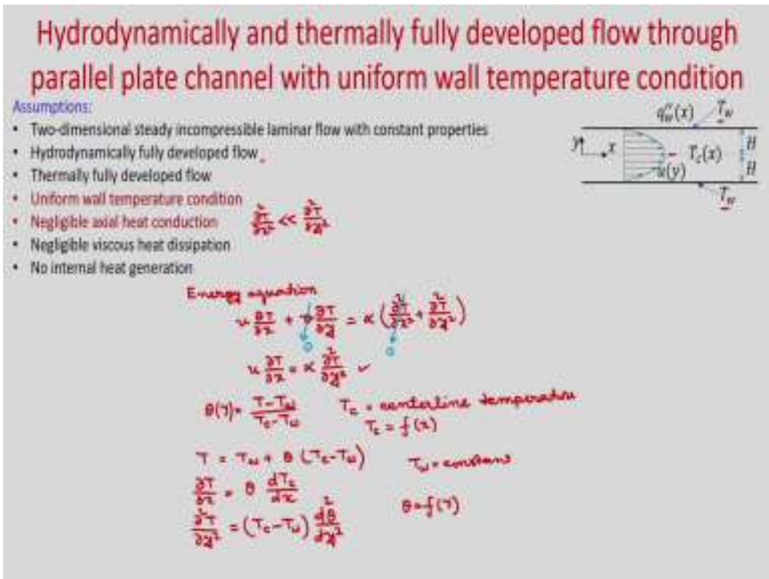

So, you can see in this figure; so, we have parallel plate channel and we have uniform wall temperature boundary condition, x is the axial direction, y is measured from the centre of the channel and these two parallel plates are separated by a distance 2H.

So, we are considering thermally fully developed and hydrodynamically fully developed condition. So, the assumptions are two-dimensional steady incompressible laminar flow; with constant properties, hydrodynamically fully developed flow; so that means, b will be 0 and thermally fully developed flow, uniform wall temperature condition we are considering.

And this is the important assumptions we are making that negligible axial heat conduction; that means, your $\frac{\partial^2 T}{\partial x^2} << \frac{\partial^2 T}{\partial y^2}$; so, you can neglect the axial heat conduction. And also we are assuming that negligible discuss it dissipation and no internal heat generation.

So, now with these assumptions we will start with the energy equation and we will invoke these assumptions and make it simplified. So, energy equation is $u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$; so this is your energy equation.

Now, we will invoke the assumptions like fully developed condition. So, v = 0 and your axial heat conduction is very very small; so this is also 0. So, you can write the simplified energy equation as, $u\frac{\partial T}{\partial x}=\alpha\frac{\partial^2 T}{\partial y^2}$.

So, now we will consider one non dimensional temperature and we will define with respect to the difference between the temperature of centre line and the wall temperature. So, we will consider one non dimensional temperature $\theta(y)=\frac{T-T_w}{T_c-T_w}$; So, T is your center line temperature here, so T_c obviously, is function of x.

But, if you see the $\frac{T-T_w}{T_c-T_w}$ is no longer function of x, it is only function of y. Like, we defined the non dimensional temperature π with respect to the mean temperature; so similarly here we are defining another non dimensional temperature θ with respect to T_c-T_w.

So, here θ will be function of y only, it will not vary along the axial direction. We are taking this non dimensional temperature so that our calculation will be easier and it will be easy to calculate the Nusselt number. So, now from here you can see that $T=T_w+\theta\left(T_c-T_w\right)$. So, $\frac{\partial T}{\partial x}=\theta\frac{dT_c}{dx}$.

So, here T_W is constant right and hence you can also take the derivative of T with respect to y; $\frac{\partial^2 T}{\partial y^2}$, so you can write as; so now T_C is function of x only and θ is function of y only. So, you can write $\frac{\partial^2 T}{\partial y^2}=\left(T_c-T_w\right)\frac{d^2\theta}{dy^2}$ because θ is function of y and we are taking the second derivative of T with respect to y; so it is $\frac{\partial^2 T}{\partial y^2}=\left(T_c-T_w\right)\frac{d^2\theta}{dy^2}$. So, now you put these values in the energy equation.

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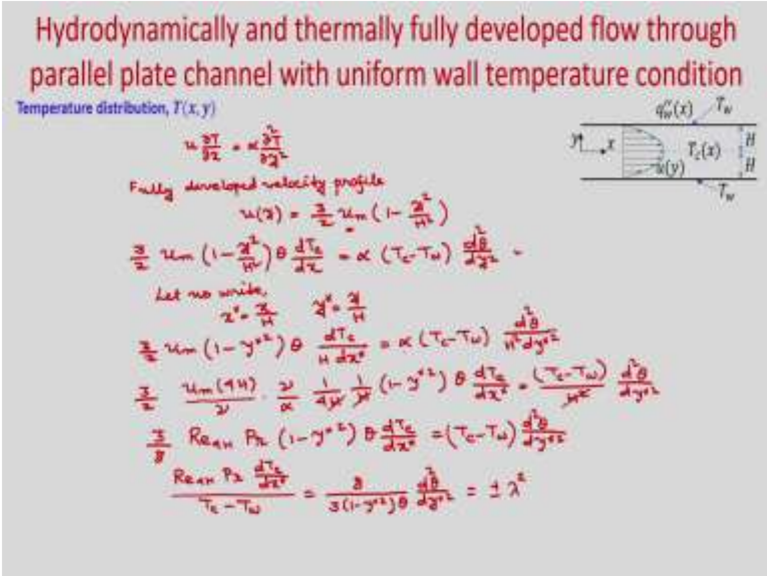

So, what you will get? Our energy equation is $u\dfrac{\partial T}{\partial x}=\alpha\dfrac{\partial^2 T}{\partial y^2}$. Now, if you put these values; you will get and what is your fully developed velocity profile? Fully developed velocity profile for flow through parallel plate channel, it is $u(y)=\dfrac{3}{2}u_m\left(1-\dfrac{y^2}{H^2}\right)$.

For this configuration this is the mean velocity, fully developed velocity profile. So, for this configuration; this is the fully developed velocity profile and u_m is the mean velocity. So, if you put all these things in this energy equation, you will get $\dfrac{3}{2}u_m\left(1-\dfrac{y^2}{H^2}\right)\theta\dfrac{dT_c}{dx}=\alpha\left(T_c-T_w\right)\dfrac{d^2\theta}{dy^2}$.

So, now we will also non dimensionalize the x coordinate and y coordinates. So, let us write $x^*=\dfrac{x}{H}$ and $y^*=\dfrac{y}{H}$. So, if you put it in this equation, so what you are going to get? $\dfrac{3}{2}u_m\left(1-y^{*2}\right)\theta\dfrac{dT_c}{Hdx^*}=\alpha\left(T_c-T_w\right)\dfrac{d^2\theta}{Hdy^{*2}}$.

So, if you rearrange it; so you can write as $\dfrac{3}{2}\dfrac{u_m(4H)}{\nu}\dfrac{\nu}{\alpha}\dfrac{1}{4H}\dfrac{1}{H}\left(1-y^{*2}\right)\theta\dfrac{dT_c}{dx^*}=\dfrac{\left(T_c-T_w\right)}{H^2}\dfrac{d^2\theta}{dy^{*2}}$.

So, now you see; so you can write $\frac{3}{8}\text{Re}_{4H}\,\text{Pr}\left(1-y^{*2}\right)\theta\frac{dT_c}{dx^*}=\left(T_c-T_w\right)\frac{d^2\theta}{dy^{*2}}$.

So, now we will separate the variables; so we will put the, which are function of x in the left hand side and which terms are function of y, we will take in the right hand side. So, if you do that then we can write as $\frac{\text{Re}_{4H}\,\text{Pr}\frac{dT_c}{dx^*}}{T_c-T_w}=\frac{8}{3\left(1-y^{*2}\right)\theta}\frac{d^2\theta}{dy^{*2}}$.

Now, you see the left hand side; so left hand side your T_c is function of x only, T_w is constant, Reynolds number is constant, Prandtl number is constant and also $\frac{dT_c}{dx^*}$ is function of x only.

So, in the left hand side is function of x only; now if you consider the right hand side, so if you see θ ; θ is function of y only and also $\frac{d^2\theta}{dy^{*2}}$ is function of y only, so right hand side is function of y only. So, we have separated the variables; left hand side is function of x only, right hand side function of y only; so this should be equal to some constant.

So, now what constant we will take, $\pm\lambda^2$. Now, whether it will take plus or minus? So, that we can see that we will use the rules of separation of variables method, before that let us write the boundary conditions.

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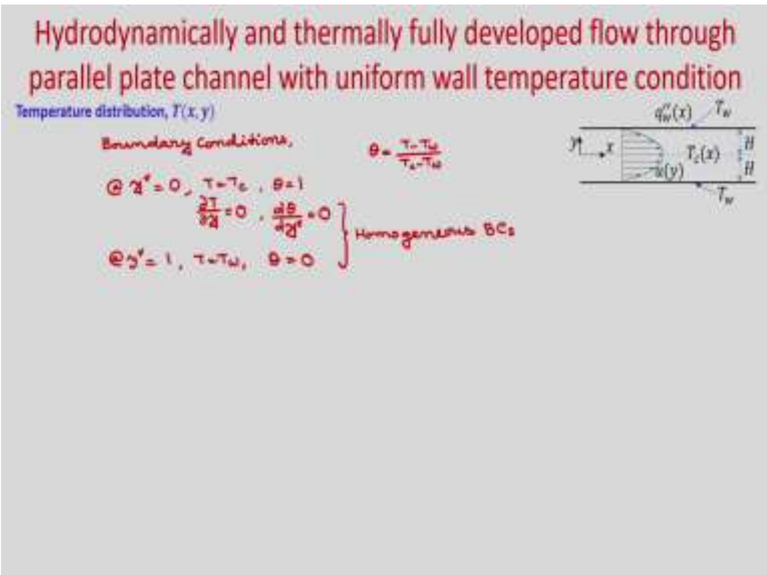

So, what are the boundary conditions? Boundary conditions are; you see, at y* = 0; that means, y is measured from the centreline; so y* = 0 means at the centerline. So, θ we have defined as $\theta(y) = \dfrac{T - T_w}{T_c - T_w}$. So, at y* = 0; obviously, θ = 1 because T = T_c . So, let us write y* = 0, T = T_c; hence your θ = 1.

Again, you see that this problem is geometrically symmetric; as well as thermally symmetric because both the walls are maintained at constant wall temperature T_w and from the centerline both the plates are separated at a distance H; so, it is geometrically symmetric and thermally symmetric. So, maximum or minimum temperature will occur at the centerline; that means you can write $\dfrac{d\theta}{dy^*} = 0$.

So, in this case; you can write $\dfrac{\partial T}{\partial y} = 0$ at y* = 0 or you can write in terms of θ ; $\dfrac{d\theta}{dy^*} = 0$. And at wall which is y* = 1; so obviously, T = T_w; so θ = 0. Now, you see the boundary conditions in y direction.

So, at y* = 0, at the centreline; your $\dfrac{d\theta}{dy^*} = 0$ and at y* = 1, on the wall your θ = 0; that means, you have homogeneous boundary conditions. So, both are homogeneous boundary conditions; so; that means, y is your homogeneous direction.

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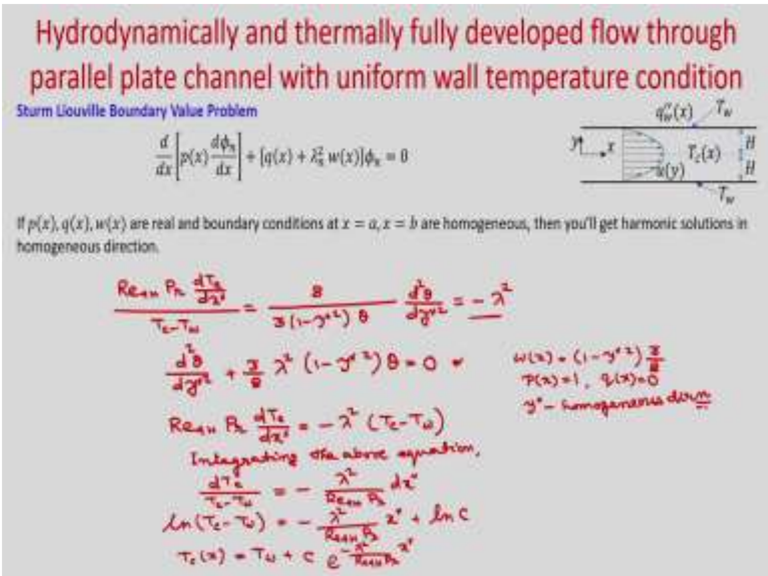

So, now let us consider this Sturm Liouville boundary value problem. So, what it is; Sturm Liouville boundary value problem? So, this is your second order differential equation.

So, in this case if p(x), q(x) and w(x) are real and boundary conditions at x =a and x = b are homogeneous, then you will get harmonic solutions in homogeneous directions. And in that direction, so you have to choose the value of λ square such a way that in the homogeneous direction, you get a harmonic solutions.

So, now let us look back the equations; so our equation, if you see we have written $\frac{\mathrm{Re}_{4H} \Pr \frac{dT_c}{dx^*}}{T_c - T_w} = \frac{8}{3\left(1 - y^{*2}\right)\theta} \frac{d^2\theta}{dy^{*2}}$; now, whether we will choose $\pm\lambda^2$?

So, you have seen that the boundary conditions in the y direction are homogeneous; so that is the homogeneous direction and we have to get the λ square such a way that in y direction, we get a Sturm Liouville problem.

So that means, if you choose this equal to - λ^2; then what equation you will get? You will get, $\frac{d^2\theta}{dy^{*2}} + \frac{3}{8}\lambda^2\left(1 - y^{*2}\right)\theta = 0$. And another equation, you will get, $\mathrm{Re}_{4H} \Pr \frac{dT_c}{dx^*} = -\lambda^2\left(T_c - T_w\right)$.

Now, you see the; this equation, so now you can see. So, if we compare with this Sturm Liouville boundary value problem, you can see you have the waiting function, $w(x) = \left(1 - y^{*2}\right)\frac{3}{8}$.

And p(x)=1 and q(x) = 0 and y star is homogeneous direction. So, as y* is homogeneous direction and p, q, w are real; so you can have a periodic solution or harmonic solutions in y star direction. So, we have chosen minus λ^2 to get the Sturm Liouville boundary value problem in y* direction .

So, now for this equation if you see; so integrate, so what you will get? So, you can see; so you can write $\frac{dT_c}{T_c - T_w} = -\frac{\lambda^2}{\mathrm{Re}_{4H} \Pr} dx^*$.

So, if you see, so you will get $\ln\left(T_c - T_w\right) = -\dfrac{\lambda^2}{\mathrm{Re}_{4H}\,\mathrm{Pr}} x^* + \ln\mathrm{C}$ and you can write,

$\mathrm{T}_c(x) = T_w + Ce^{-\frac{\lambda^2}{\mathrm{Re}_{4H}\,\mathrm{Pr}} x^*}$.

So, now we have to see the first equation; so now, we have to find the solution of this equation. Now, we want to find a series solution of this equation, now whether we can use the series solution for this particular second order differential equation, let us see.

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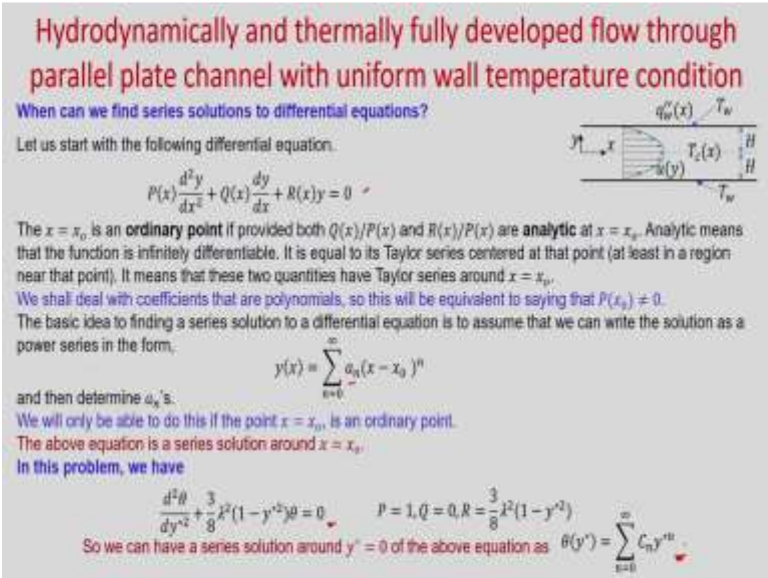

So, when can we find series solution to differential equations? So, you let us consider this differential equation, second order differential equation; $P(x)\dfrac{d^2y}{dx^2} + Q(x)\dfrac{dy}{dx} + R(x)y = 0$.

Now, we are just checking or when we can use the series solution, let us see. The x = x_0 is an ordinary point, if provided both $\dfrac{Q(x)}{P(x)}$ and $\dfrac{R(x)}{P(x)}$ are analytic at x = 0. Analytic means that the function is infinitely differentiable, it is equal to its Taylor series centred at that point; at least in a region near that point. It means that these two quantities have Taylor series around x =x_0.

We shall deal with coefficients that are polynomials; so this will be equivalent to saying that $P(x_0) \neq 0$. So, the basic idea to finding a series solution to a differential equation is to assume that we can write the solution as a power series in the form; $y(x)=\sum_{n=0}^{\infty} a_n (x-x_0)^n$.

We will only be able to do this, if the point x = x_0 is an ordinary point. So, the above equation is a series solution at x = x_0. So, now for our problem let us see; so if you compare our differential equation is this one. So, if you compare with this equation, you see P = 1, Q = 0 and $R=\frac{3}{8}\lambda^2\left(1-y^{*2}\right)$.

So, now you can see that $\frac{Q(x)}{P(x)}$ and $\frac{R(x)}{P(x)}$ are analytic. So, at x = 0 is an ordinary point; And we can have a series solution around y*=0 of the above equation, as $\theta(y^*)=\sum_{n=0}^{\infty} C_n y^{*n}$.

As y*=0 is an analytic point because you have seen that $\frac{Q(x)}{P(x)}$ and $\frac{R(x)}{P(x)}$ are analytic. So, we can find the solution of this equation as a series solution and this series solution, we will consider.

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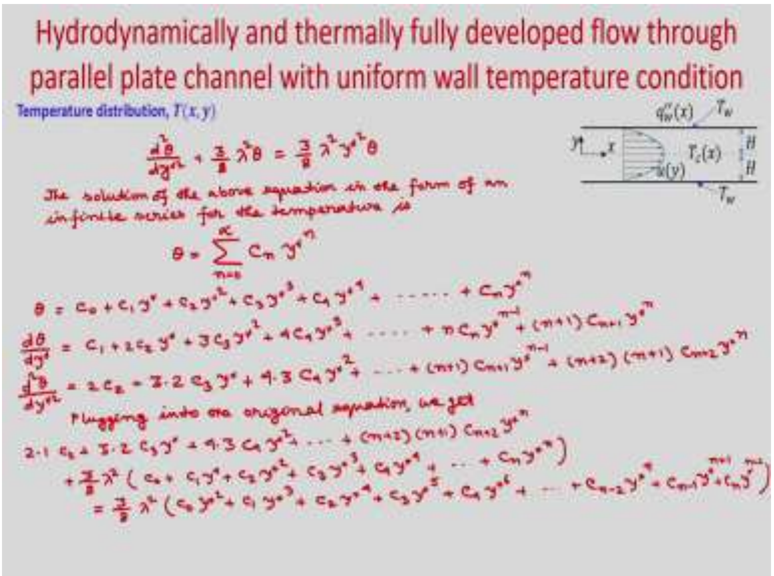

So, our differential equation is $\frac{d^2\theta}{dy*^2}+\frac{3}{8}\lambda^2\theta=\frac{3}{8}\lambda^2 y*^2\,\theta$. So, now the solution of the above equation in the form of an infinite series for the temperature is, $\theta=\sum_{n=0}^{\infty}C_n y*^n$.

So, now let us take the derivative of θ; so θ if we expand,

$$\theta=C_0+C_1y*+C_2\,y*^2+C_3\,y*^3+C_4\,y*^4+\ldots\ldots\ldots\ldots\ldots\ldots\ldots+C_n\,y*^n\,.$$

Now, if you take the derivative; if you take the derivative with respect to y*, then $\frac{d\theta}{dy*}=C_1+2C_2\,y*+3C_3\,y*^2+4C_4\,y*^3+\ldots\ldots\ldots\ldots\ldots\ldots\ldots+nC_n\,y*^{n-1}+(n+1)C_{n+1}\,y*^n$.

The next term if you write, so it will be

$$\frac{d^2\theta}{dy*^2}=2C_2+3.2C_3\,y*+4.3C_4\,y*^2+\ldots\ldots\ldots\ldots\ldots+(n+1)C_{n+1}\,y*^{n-1}+(n+2)(n+1)C_{n+2}\,y*^n\,.$$

Now, you plug into the original differential equation. So, if you do that; plugging into the original equation ok, we get so, it is

$$2.1C_2+3.2C_3\,y*+4.3C_4\,y*^2+\ldots\ldots+(n+2)(n+1)C_{n+2}\,y*^n+\frac{3}{8}\lambda^2\left(C_0+C_1y*+C_2\,y*^2+C_3\,y*^3+C_4\,y*^4+\ldots\ldots+C_n\,y*^n\right)$$

So, this is your $\frac{d^2\theta}{dy*^2}$. And in the right hand side now,

$$\frac{3}{8}\lambda^2\left(C_0y*^2+C_1y*^3+C_2\,y*^4+C_3\,y*^5+C_4\,y*^6+\ldots\ldots\ldots\ldots+C_{n-2}\,y*^n+C_{n-1}\,y*^{n+1}+C_n\,y*^{n+2}\right)$$

so, this is the right hand side. Now, what we will do? We will equate the equal power of y*.

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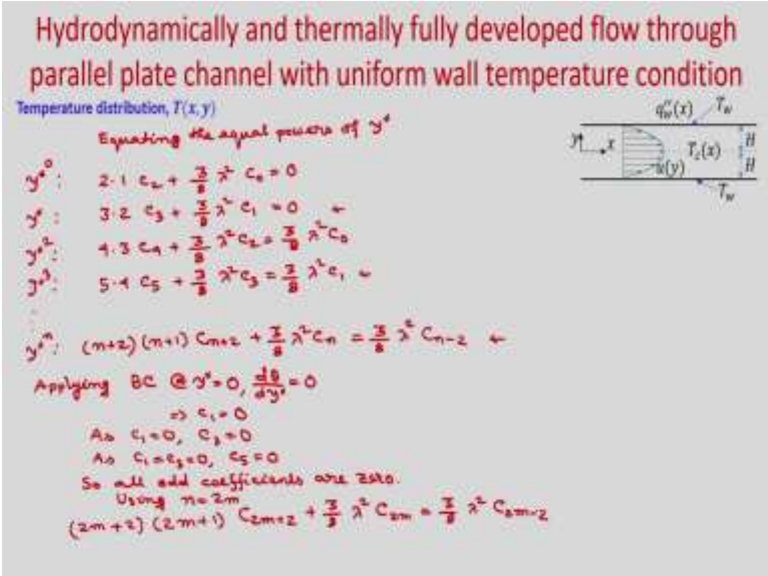

So, equating the equal powers of y*; so if you see y^{*0}, if you see its coefficient; so you can see here, so this is your $y^{*0}: 2.1\mathrm{C}_2 + \frac{3}{8}\lambda^2 C_0$ and then this one and right hand side, there is no term; so, it is equal to 0. So, you can write $2.1\mathrm{C}_2 + \frac{3}{8}\lambda^2 C_0 = 0$.

so if you see $y^{*}: 3.2\mathrm{C}_3 + \frac{3}{8}\lambda^2 C_1 = 0$;

Then, $y^{*2}: 4.3\mathrm{C}_4 + \frac{3}{8}\lambda^2 C_2$. Then, right hand side $\frac{3}{8}\lambda^2 C_0$.

So, if you write the $y^{*2}: 4.3\mathrm{C}_4 + \frac{3}{8}\lambda^2 C_2 = \frac{3}{8}\lambda^2 C_0$. So, similarly you find the other powers ok the coefficient of other powers and equate it.

So, just I am writing here; $y^{*3}: 5.4\mathrm{C}_5 + \frac{3}{8}\lambda^2 C_3 = \frac{3}{8}\lambda^2 C_1$. And if you see $y^{*n}: (n+2)(n+1)\mathrm{C}_{n+2} + \frac{3}{8}\lambda^2 C_n = \frac{3}{8}\lambda^2 C_{n-2}$.

So, now let us apply the boundary conditions. So, applying boundary condition at y* = 0; $\frac{d\theta}{dy^*} = 0$. So, if $\frac{d\theta}{dy^*} = 0$; you see from this equation, this equation you see; so, if at y* =

0, $\frac{d\theta}{dy^*} = 0$; that means, C_1 will be 0 because rest all other terms will become 0; so C_1 will be 0.

So, that means, C_1 is 0; now, you see here if C_1 is 0, then from this equation; you see from this equation, if C_1 is 0, C_3 will become 0. Then, if C_1, C_3 is 0; from here you can see C_1, C_3 are 0; so C_5 will be 0. So, you can see all the odd coefficient C will be 0 so; that means, you can write as $C_1 = 0$, $C_3 = 0$. As $C_1 = C_3 = 0$; $C_5 = 0$; so all odd coefficients are 0.

So, what we can write, we can write using n = 2 m, from here you can see; from this equation you can write $(2m+2)(2m+1)C_{2m+2} + \frac{3}{8}\lambda^2 C_{2m} = \frac{3}{8}\lambda^2 C_{2m-2}$.

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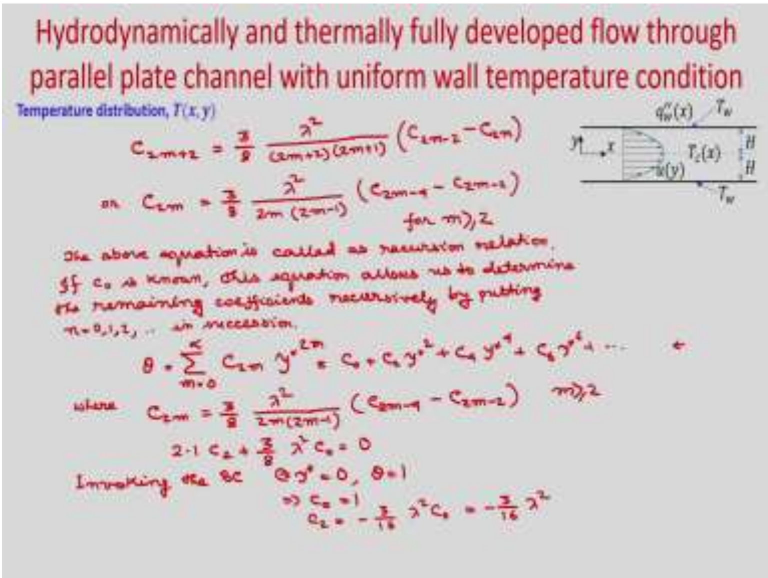

If you divide it; so you can write $C_{2m+2} = \frac{3}{8}\frac{\lambda^2}{(2m+2)(2m+1)}(C_{2m-2} - C_{2m})$ or we can write $C_{2m} = \frac{3}{8}\frac{\lambda^2}{2m(2m-1)}(C_{2m-4} - C_{2m-2})$.

So, we can see this equation is the recursion relation; so it is for m ≥2. So, the above equation is called as recursion relation, if C_0 is known this equation allows us to determine the remaining coefficient recursively by putting n = 0, 1, 2,…..in succession.

So, now let us write the temperature distribution; so your temperature distribution θ , now you can write because you know that odd coefficients are 0, so, $\theta = \sum_{m=0}^{\infty} C_{2m} y^{*2m} = C_0 + C_2 y^{*2} + C_4 y^{*4} + C_6 y^{*6} + \ldots.. $.

So that means, you can write $C_{2m} = \frac{3}{8}\frac{\lambda^2}{2m(2m-1)}\left(C_{2m-4} - C_{2m-2}\right)$, for m ≥2. And the first equation if you see here, so this is also valid; $2.1C_2 + \frac{3}{8}\lambda^2 C_0 = 0$; so this also we have. So, we also have $2.1C_2 + \frac{3}{8}\lambda^2 C_0 = 0$.

So, if you see; if C_0 is known, C_2 will be known and if C_2 is known then other terms will be known because m; for m ≥, you can use this recursive relation. So, now invoke the other boundary condition; at y*= 0, you have θ = 1.

If y*= 0, θ = 1; so, you can see from this equation; $\theta = C_0 + C_2 y^{*2}$. So, these are; all will become 0, at y*= 0 and θ = 1; so C_0 = 1.

So, if C_0 = 1; then you can write the value of C_2, from this equation. $C_2 = -\frac{3}{16}\lambda^2 C_0$, C_0 = 1; so it will be $C_2 = -\frac{3}{16}\lambda^2$. Now, C_0, C_2 are known. Now, you put this recursive relation and find C_4 and C_6.

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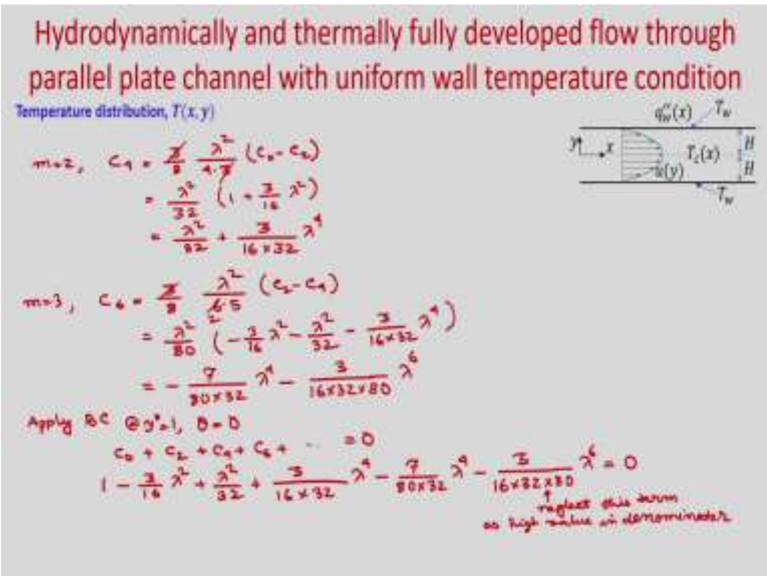

So, you can write C_4, so if you put m = 2; $C_2 = \frac{3}{8}\frac{\lambda^2}{4.3}(C_0 - C_2)$; C_2 and C_0 you know, so you can write λ^2; so these 3, 3; you can cancel. So, it is $C_4 = \frac{\lambda^2}{32}\left(1 + \frac{3}{16}\lambda^2\right)$.

So, it will be $C_4 = \frac{\lambda^2}{32} + \frac{3}{16\times 32}\lambda^4$. Now, you put m = 3; so, you can find $C_6 = \frac{3}{8}\frac{\lambda^2}{6.5}(C_2 - C_4)$.

Now, C_4 already we have found here; so, it will be 2; so it will be $\frac{3}{16\times 32}\lambda^4$.

So, if you rearrange it, you will get $C_6 = -\frac{7}{80\times 32}\lambda^4 - \frac{3}{16\times 32\times 80}\lambda^6$. So, now let us apply the another boundary condition at y* = 1, θ =0. Apply boundary condition at y*=1, θ = 0.

So, if you see θ expression; so, θ expression is this and y*=1 if you put, this will become all 1. So, it is just $C_0 + C_2 + C_4 + C_6 + ... = 0$.

Now, we will consider only the first three coefficients, other terms we will neglect. So, you can write, $1 - \frac{3}{16}\lambda^2 + \frac{\lambda^2}{32} + \frac{3}{16\times 32}\lambda^4 - \frac{7}{80\times 32}\lambda^4 - \frac{3}{16\times 32\times 80}\lambda^6 = 0$.

So, you can see the denominator value $16\times 32\times 80$. So, it is a very high value; so if you find, so this will become very small value; so, you neglect this term as you have high value in denominator.

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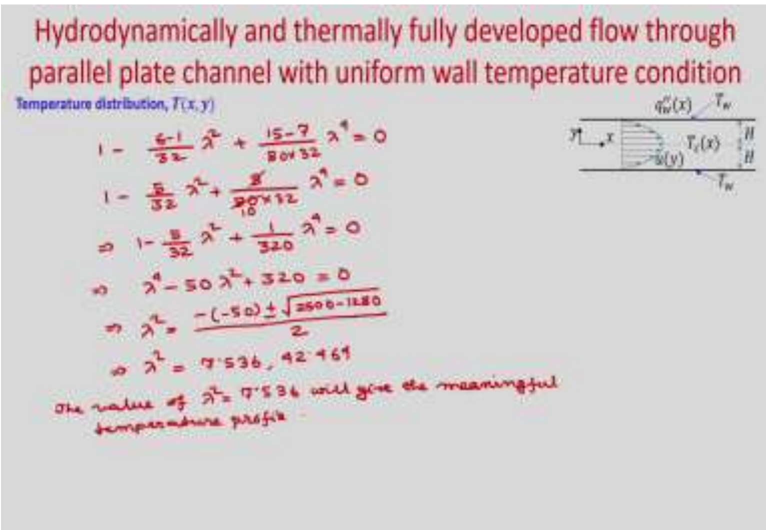

So, if you neglect it; so rest you just rearrange. So, what you will get? $1-\frac{6-1}{32}\lambda^2+\frac{15-7}{80\times 32}\lambda^4=0$. So, $1-\frac{5}{32}\lambda^2+\frac{8}{80\times 32}\lambda^4=0$. So, it will be just, $1-\frac{5}{32}\lambda^2+\frac{1}{320}\lambda^4=0$.

So, you will get a quadratic equation; $\lambda^4-50\lambda^2+320=0$. So, that means $\lambda^2=\frac{-(-50)\pm\sqrt{2500-1280}}{2}$. So that means, we will have $\lambda^2=7.536$; one value and another value will be 42.464.

So, you will see that if you take $\lambda^2=7.536$, it will give physically correct temperature profile, but if you consider $\lambda^2=42.464$, then it will not give a physically correct temperature value. So, you can neglect that; so you just consider the value of $\lambda^2=7.536$. So, the value of $\lambda^2=7.536$ will give the meaningful temperature profile.

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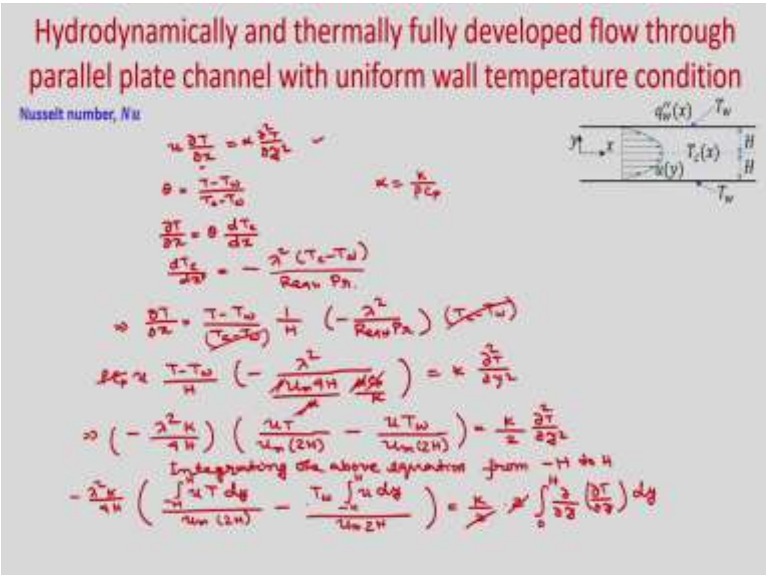

So, now we know the value of λ^2; so if we put in the equation, in the series solution; then you will get the temperature profile. So, in the series solution, if you see that you have found the value of C_0; then C_2, C_4 and C_6; so if you put this, so you will get the temperature profile.

Now, next target is to find the Nusselt number; so we will start from the governing equations. So, it is $u\frac{\partial T}{\partial x}=\alpha\frac{\partial^2 T}{\partial y^2}$ and if you remember, θ we have defined as $\theta=\frac{T-T_w}{T_c-T_w}$. And we have seen that $\frac{\partial T}{\partial x}=\theta\frac{dT_c}{dx}$ and $\frac{dT_c}{dx*}=-\frac{\lambda^2\left(T_c-T_w\right)}{\mathrm{Re}_{4H}\,\mathrm{Pr}}$.

Hence, $\frac{\partial T}{\partial x}=\frac{T-T_w}{T_c-T_w}\frac{1}{H}\left(-\frac{\lambda^2}{\mathrm{Re}_{4H}\,\mathrm{Pr}}\right)\left(T_c-T_w\right)$, so this is your $\frac{\partial T}{\partial x}$. So, if we put this value in the governing equation and $\alpha=\frac{K}{\rho C_P}$.

So, now, $\frac{\partial T}{\partial x}$; you put it here and $\alpha=\frac{K}{\rho C_P}$. So, you can see; it will be,

$$\rho C_P u\frac{T-T_w}{H}\left(-\frac{\lambda^2}{\frac{\rho C_P u_m}{\mu}\frac{\mu C_P}{K}}\right)=K\frac{\partial^2 T}{\partial y^2}.$$

So, if you see; so ρ C_p, ρ C_p; you can cancel, then μ, μ will get cancel and now if you write, it will be minus $\left(-\frac{\lambda^2 K}{4H}\right)\left(\frac{uT}{u_m(2H)}-\frac{uT_w}{u_m(2H)}\right)=\frac{K}{2}\frac{\partial^2 T}{\partial y^2}$.

So, now, integrating the above equation from -H to H. So, you see minus $-\frac{\lambda^2 K}{4H}\left(\frac{\int_{-H}^{H}uTdy}{u_m(2H)}-\frac{T_w\int_{-H}^{H}udy}{u_m(2H)}\right)=\frac{K}{2}2\int_0^H\frac{\partial}{\partial y}\left(\frac{\partial T}{\partial y}\right)dy$. So, this right hand side time you see the integration. So, this $\frac{\partial T}{\partial y}$y at H; at H, so that will give you the wall heat flux and $\frac{\partial T}{\partial y}$; at y = 0, it is centerline temperature gradient and that is equal to 0 because from boundary condition, you have seen.

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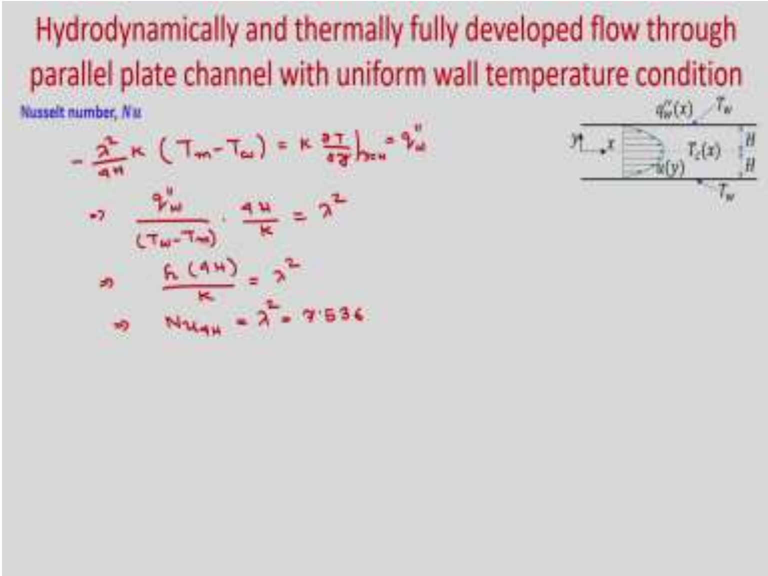

So, you can write $-\frac{\lambda^2 K}{4H}$. Now, you see this term $\frac{\int_{-H}^{H} uTdy}{u_m(2H)}$, so that means, it is your mean temperature right, mean temperature definition; so this is the mean temperature definition. And if you see, this is $\frac{\int_{-H}^{H} udy}{u_m(2H)}$, so obviously, this will get cancel; so you will get $T_m - T_w$.

So, you will get $T_m - T_w$; so we have written the definition of mean; bulk mean temperature and because we need to define a Nusselt number based on $T_m - T_w$. So, it will be $K\frac{\partial T}{\partial y}|_{y=H}$ and it is nothing, but $K\frac{\partial T}{\partial y}|_{y=H} = q''_w$.

So, now you can see $\frac{q''_w}{T_m - T_w}\frac{4H}{K} = \lambda^2$. So, you can see it is $\frac{h(4H)}{K} = \lambda^2$. So that means, so Nusselt number based on hydraulic diameter 4 H $Nu_{4H} = \lambda^2$ and λ^2 value, we have found as 7.536.

So, we have seen that the Nusselt number based on the hydraulic diameter, for this problem the hydraulic diameter is $Nu_{4H} = \lambda^2$ and λ^2; we have already found; so that is your 7.536.

So, let us summarise; so today we considered the fully developed laminar flow through parallel plate channels with constant wall temperature. We considered the non dimensional temperature as $\frac{T - T_w}{T_c - T_w}$ where T_c is the centerline temperature for easy calculation.

From there, we have found the separation of variables we have we; we use the separation of variable method and we have written the Sturm Liouville problem because the boundary conditions are homogeneous in y star direction.

And then, we have found the solution of that governing equation using series solution. And from there we have found the values of λ^2 and then we found the Nusselt number, starting from the governing equation and Nusselt number; we have defined with respect to $T_m - T_w$ where T_m is the mean temperature.

And you can see that for this particular case, the Nusselt number is also constant; independence of Reynolds number and the Prandtl number.

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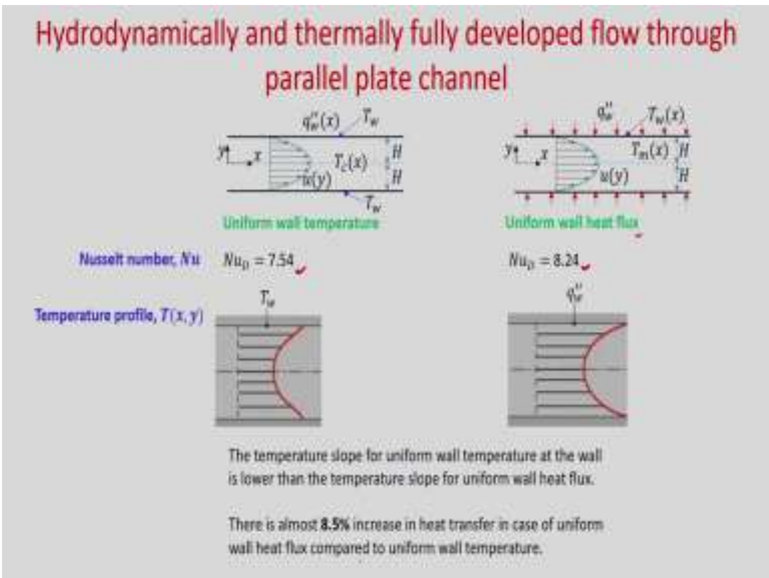

So, you can see that for uniform wall temperature case where you have that T_w as constant, Nusselt number is 7.54. In earlier lectures, we have found with uniform wall heat flux; for this flow through parallel plate channel, we have found Nusselt number as 8.24. And if you see the temperature profile, for this uniform wall temperature, you can see the slope at the wall is lower than the temperature slope of uniform wall heat flux.

So, if you see the temperature profile here; this slope is higher than this temperature profile, hence your Nusselt number is higher in case of uniform wall heat flux; compared to uniform wall temperature. And there is almost 8.5 % increase in heat transfer, in case of uniform wall heat flux compared to uniform wall temperature.

Thank you.