Fundamentals of Convective Heat Transfer
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Module – 06
Convection in Internal Flows -II
Lecture – 19
Hydrodynamically and thermally fully developed flow with uniform wall heat flux condition

Hello everyone. So, in last lecture we considered slug flow where axial velocity remained constant. Today, we will consider Hydrodynamically and thermally fully developed flow with constant or uniform wall heat flux boundary condition.

So, it is the velocity profile we have already derived for a fully developed condition earlier for two different channels; one is parallel plate channel and circular pipe. In today's lecture, we will consider these two types of channel; first we will consider flow through parallel plate channel, then we will consider circular pipe.

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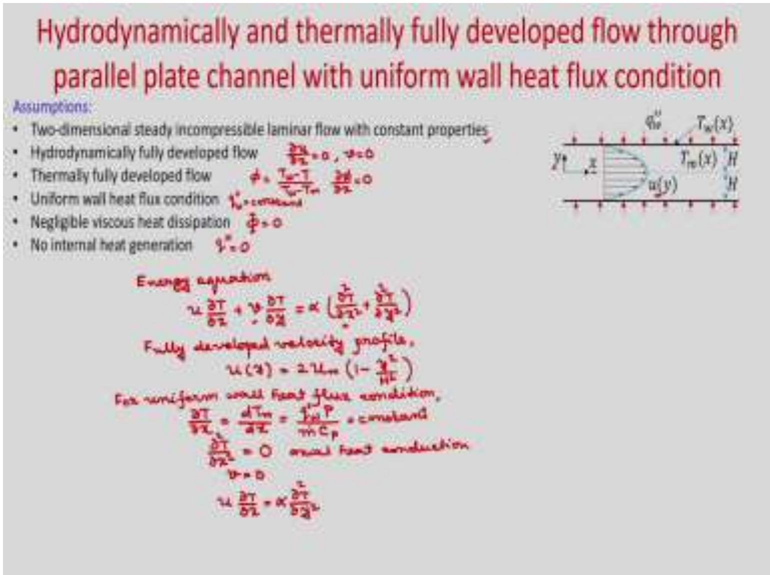

So, first let us consider the assumptions, we will consider two dimensional steady incompressible laminar flow with constant properties. Two dimensional we are considering; that means in the third direction it is in finite. So, there will be no change or no gradient in that direction; so obviously, we will consider for flow through parallel plates channel x as the axial direction and y we will measure from the center line.

You can see this is the two infinite parallel plates channel. We have the velocity profile which is fully developed and u is function of y only; x is the axial direction and y is measured from the central line and uniform wall heat flux is imposed on both walls. For this particular case you can see that, your wall temperature will vary in axial direction. So, T_w is function of x, and these two plates are separated by a distance two H. So, hydro dynamically fully developed flow. So, in this particular case, you know that $\frac{\partial u}{\partial x} = 0$ and v=0.

Thermally fully developed flow, so we have this non dimensional temperature which we defined as $\phi = \frac{T_w - T}{T_w - T_m}$, where T_w is the wall temperature and T_m is the mean temperature.

So, if it is a thermally fully developed flow, then we can write $\frac{\partial \phi}{\partial x} = 0$; because actual variation of this phi will be 0. Uniform wall heat flux condition, so q_w'' is constant, and we will have negligible viscous heat dissipation; that means phi will be 0 and no internal heat generation. So, $q''' = 0$.

Considering these assumptions, let us write the energy equation. So, what is your energy equation in for steady laminar flow in two dimensions? So, you can write energy equation. So, $u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2})$, neglecting the viscous heat dissipation and internal heat generation.

So, this is your energy equation, we will simplify invoking those assumptions now. First of all let us write the fully developed velocity profile. So, what is the fully developed velocity profile for this particular case? Fully developed velocity profile.

So, in this particular case, you know that $\mathrm{u(y)} = 2\,\mathrm{u}_m(1 - \frac{y^2}{H^2})$. Now, we know for constant heat flux boundary condition, $\frac{\partial T}{\partial x} = \frac{dT_m}{dx}$ and that is also constant.

So, for uniform wall heat flux condition, you know that $\frac{\partial T}{\partial x} = \frac{dT_m}{dx}$; we have already derived it in earlier classes. So, $\frac{dT_m}{dx} = \frac{q_w^{"} P}{\dot{m} C_P}$. So, where P is the perimeter, $\dot{m}$ is the mass flow rate.

And you can see, for this particular case $q_w^{"}$ constant; for a constant cross sectional channel P is constant, $\dot{m}$ is constant and C_p is specific heat that is also constant. So, this will be constant.

Now, if you take derivative with respect to x, then you can write $\frac{\partial^2 T}{\partial x^2}$. So, as it is constant it will be 0. So, we can see that in the diffusion term, you can actually write $\frac{\partial^2 T}{\partial x^2}$ =0. So, now, you invoke all the assumptions and write the energy equation.

So, this is your axial heat conduction is 0. So, axial heat conduction, for this particular case it will be 0 and fully developed flow. So, v = 0. So, you can see that v = 0 and this is 0; so your equation now you can write as $u\frac{\partial T}{\partial x} = \alpha \frac{\partial^2 T}{\partial y^2}$.

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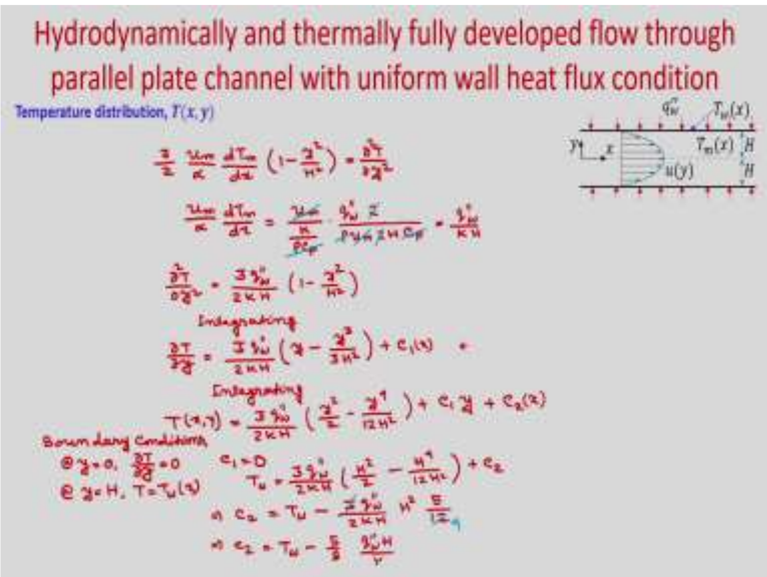

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Now, putting these values as $\frac{3}{2}\frac{u_m}{\alpha}\frac{dT_m}{dx}(1-\frac{y^2}{H^2})=\frac{\partial^2 T}{\partial y^2}$.

So, now let us find the term left hand side $\frac{u_m}{\alpha}\frac{dT_m}{dx}$. So, what is the value of this? So, u_m is the mean velocity, $\alpha=\frac{K}{\rho C_P};\frac{dT_m}{dx}=\frac{q_w^{"}P}{\dot{m}C_P}$, in this particular case P is your 2 X1 per unit width.

So, if you consider in third direction unit width, then it will be $\frac{u_m}{\frac{K}{\rho C_P}}\frac{q_w^{"}2}{\rho u_m 2HC_P}$. So, now if you simplify it, then ρ, ρ, C_p, C_p will get cancelled; u_m, u_m will get cancel; this 2, 2 will get cancel. So, you can write this as $\frac{q_w^{"}}{KH}$. So, now, you can write, $\frac{\partial^2 T}{\partial y^2}=\frac{3q_w^{"}}{2KH}(1-\frac{y^2}{H^2})$.

So, now this equation we will integrate twice and we will apply the boundary condition; then we will find the temperature distribution inside these parallel plates. So, integrating you will get, $\frac{\partial T}{\partial y}=\frac{3q_w^{"}}{2KH}(y-\frac{y^3}{3H^2})+\mathrm{C}_1(x)$. And again if you integrate, then you will get $\mathrm{T(x,y)}=\frac{3q_w^{"}}{2KH}(\frac{y^2}{2}-\frac{y^4}{12H^2})+\mathrm{C}_1 y+C_2(x)$. Now, we will apply two boundary conditions and find these constant C_1 and C_2. So, one boundary condition is that, at y = 0 which is your central line. So, your temperature gradient with respect to y, $\frac{\partial T}{\partial y}$=0, because the problem is thermally and geometrically symmetric.

So, the maximum or minimum temperature will occur at the central line. So, at y =0, we will put $\frac{\partial T}{\partial y}$=0. And another boundary condition we will take that, at the wall temperature as T_w which is function of x. So, at y = H, you have T_w

So, boundary conditions. So, at y = 0, we will put $\frac{\partial T}{\partial y}$=0. So, you can see from this equation; if you put y = 0, then this right hand side first term will become 0. So, $\frac{\partial T}{\partial y}$=0.

So, C_1 will be 0. And at y = H, we will put T as T_w which is function of x. So, if you put that, so you will get T_w; here I am not writing function of x, only Tw I am writing here. So, $T_w = \frac{3q_w^{''}}{2KH}(\frac{H^2}{2} - \frac{H^4}{12H^2}) + C_2$.

So, you can see that your, $C_2 = T_w - \frac{3q_w^{''}}{2KH} H^2 \frac{5}{12}$. So, if you rearrange it, so you will get $C_2 = T_w - \frac{5}{8}\frac{q_w^{''}H}{K}$.

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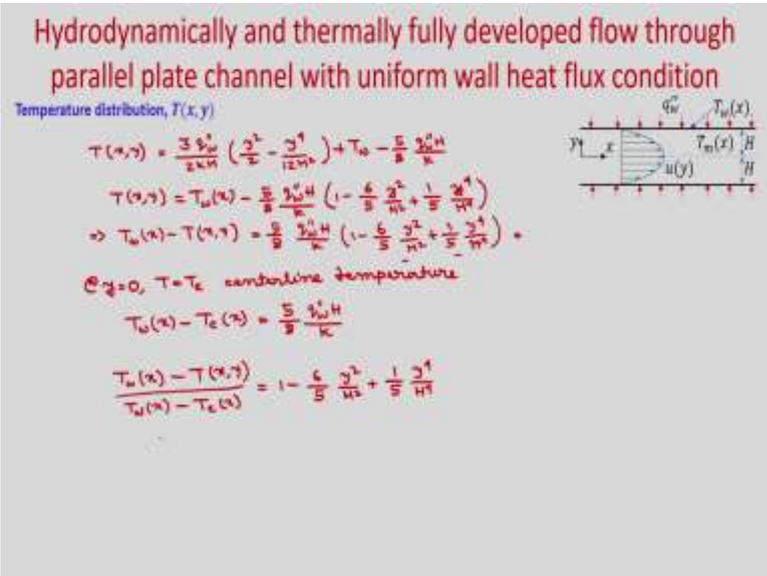

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So, now, let us put this constant in the temperature distribution and find the temperature distribution T(x, y). So, $T(x, y) = \frac{3q_w^{''}}{2KH}(\frac{y^2}{2} - \frac{y^4}{12H^2}) + T_w - \frac{5}{8}\frac{q_w^{''}H}{K}$.

So, if you rearrange it in this form. So, you can write, $T(x, y) = T_w(x) - \frac{5}{8}\frac{q_w^{''}H}{K}(1 - \frac{6}{5}\frac{y^2}{H^2} + \frac{1}{5}\frac{y^4}{H^4})$.

So, now I can also write this as $T_w(x) - T(x, y) = \frac{5}{8}\frac{q_w^{"}H}{K}(1 - \frac{6}{5}\frac{y^2}{H^2} + \frac{1}{5}\frac{y^4}{H^4})$. So, this is the temperature distribution along the y and also it is function of x and y; because T_w is function of x.

Now at the center y = 0. So, what will be the temperature, centerline temperature T_c? So, it will be either maximum or minimum temperature depending on what is the inlet temperature. So, let us write the centerline temperature as at y = 0, T = T_c which is your centerline temperature. So, you can write $T_w(x) - T_c(x) = \frac{5}{8}\frac{q_w^{"}H}{K}$.

Now, we can just divide this equation with this equation, then what you will get? $\frac{T_w(x) - T(x, y)}{T_w(x) - T_c(x)} = 1 - \frac{6}{5}\frac{y^2}{H^2} + \frac{1}{5}\frac{y^4}{H^4}$.

So, you can see that we have written this temperature distribution in non-dimensional form; because we are dividing with T_w - T_c, where T_c is the centerline temperature and right hand side you can see this is also non-dimensional. So, this is the temperature distribution.

Now, to find the heat transfer coefficient; we need to find, what is the mean temperature? Because we need to define the Nusselt number based on the difference between the wall temperature and the mean temperature. So, first let us find, what is the mean temperature?

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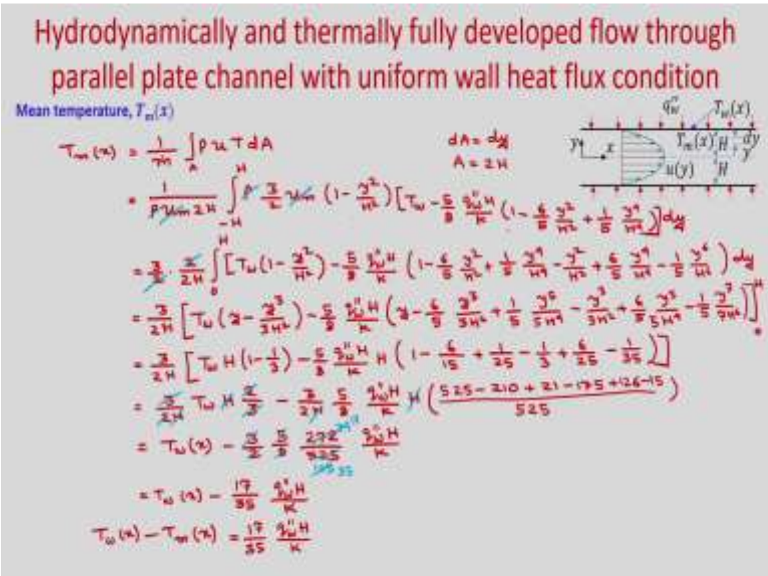

So, mean temperature you can find $T_m(x) = \frac{1}{\dot{m}} \int_A \rho u T dA$. So, if you consider from the centreline, one small elemental flow area of distance dy at a distance y from the centreline.

And in other direction if you take a unit width, then it will be dA = dy X 1and the total flow area will be 2 H X 1. So, this if you write, then you will get $\dot{m} = \rho u_m 2H$. Now, you integrate, $T_m(x) = \frac{1}{\rho u_m 2H} \int_{-H}^{H} \frac{3}{2} u_m \left(1 - \frac{y^2}{H^2}\right) \left[T_w - \frac{5}{8} \frac{q_w'' H}{K} \left(1 - \frac{6}{5}\frac{y^2}{H^2} + \frac{1}{5}\frac{y^4}{H^4}\right)\right] dy$.

So, dA = dy. So, you can see here, you can this u_m, u_m, ρ, ρ you can cancel and you can write this,

$$T_m(x) = \frac{3}{2}\frac{2}{2H}\int_0^H \left[T_w \left(1 - \frac{y^2}{H^2}\right) - \frac{5}{8}\frac{q_w'' H}{K}\left(1 - \frac{6}{5}\frac{y^2}{H^2} + \frac{1}{5}\frac{y^4}{H^4} - \frac{y^2}{H^2} + \frac{6}{5}\frac{y^4}{H^4} - \frac{1}{5}\frac{y^6}{H^6}\right)\right] dy .$$

So, now it will be easy to integrate. So, if you cancel it 2, 2; so you can write

$$T_m(x) = \frac{3}{2H}\left[T_w \left(y - \frac{y^3}{3H^2}\right) - \frac{5}{8}\frac{q_w'' H}{K}\left(y - \frac{6}{5}\frac{y^3}{3H^2} + \frac{1}{5}\frac{y^5}{5H^4} - \frac{y^3}{3H^2} + \frac{6}{5}\frac{y^5}{5H^4} - \frac{1}{5}\frac{y^7}{7H^6}\right)\right]_0^H .$$

So, after integration we have written this, but we need to put the limit from 0 to H. So, if you see y = 0 if you put all terms will become 0; only H you put, then what you will get $T_m(x) = \frac{3}{2H}\left[T_w H\left(1 - \frac{1}{3}\right) - \frac{5}{8}\frac{q_w'' H}{K}\left(1 - \frac{6}{15} + \frac{1}{25} - \frac{1}{3} + \frac{6}{25} - \frac{1}{35}\right)\right]$. So, now, if you see; so you can write, $T_m(x) = \frac{3}{2H} T_w H \frac{2}{3} - \frac{3}{2H}\frac{5}{8}\frac{q_w'' H}{K} H\left(\frac{525 - 210 + 21 - 175 + 126 - 15}{525}\right)$.

So, you can see this H, H will get cancel; 2, 2; 3, 3 here also H, H will get cancel. So, you can write it as $T_m(x) = T_w(x) - \frac{3}{2}\frac{5}{8}\frac{272}{525}\frac{q_w'' H}{K}$.

So, you can see, if you rearrange. So, you will get $T_m(x) = T_w(x) - \frac{17}{35}\frac{q_w'' H}{K}$.

So, we have now represented the mean temperature which is function of x in terms of the wall temperature and the heat flux. So, from here now you can actually find the temperature difference between the wall temperature and the mean temperature.

So, you can see, you can write $T_w(x) - T_m(x) = \frac{17}{35}\frac{q_w^{"}H}{K}$. So, now, we need to calculate the heat transfer coefficient.

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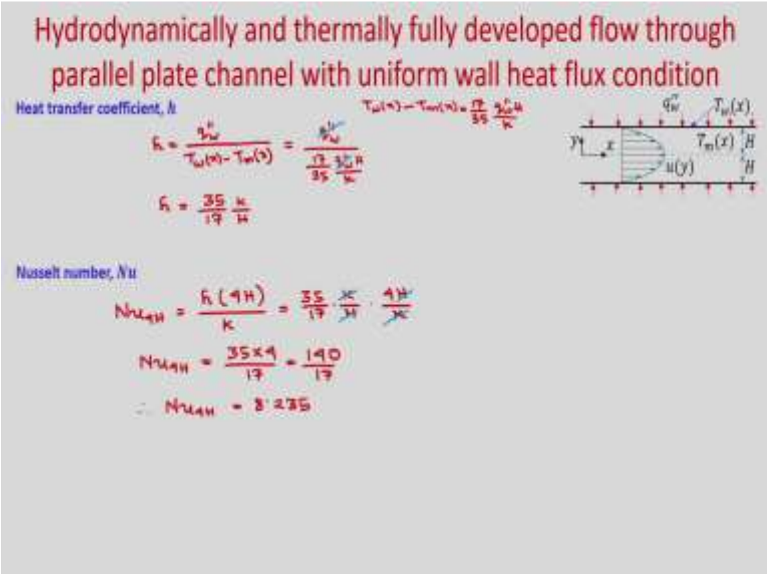

So, heat transfer coefficient will be. So, heat transfer coefficient is $h = \frac{q_w^{"}}{T_w(x) - T_m(x)}$. So, in earlier slide we have found what is $T_w(x) - T_m(x)$. So, that is $T_w(x) - T_m(x) = \frac{17}{35}\frac{q_w^{"}H}{K}$.

So, you can see if you put it here. So, we will get $h = \frac{q_w^{"}}{\frac{17}{35}\frac{q_w^{"}H}{K}}$.

So, you can write it as $h = \frac{35}{17}\frac{K}{H}$. So, now, Nusselt number; so Nusselt number we will calculate, Nusselt number we will calculate based on the hydraulic diameter and the difference between wall temperature and the mean temperature. So, what is the hydraulic diameter? Already you have calculated for this particular geometry, so that is your 4 H right, so 4 H. So, based on 4 H we will calculate the Nusselt number.

So, Nusselt number will be based on 4 H and the difference between wall temperature and the mean temperature. So, $Nu_{4H} = \frac{h(4H)}{K}$. So, $h = \frac{35}{17}\frac{K}{H}$ and thus $Nu_{4H} = \frac{35}{17}\frac{K}{H}\frac{4H}{K}$. So, you can see H, H will get cancel. So, you will get Nusselt number based on 4 H, $Nu_{4H} = \frac{35 \times 4}{17}$.

So, that will be $Nu_{4H} = \frac{140}{17}$. So, $Nu_{4H} = 8.235$. So, you can see the Nusselt number for hydrodynamically and thermally fully developed flow it is constant and independent of Reynolds number and Prandtl number.

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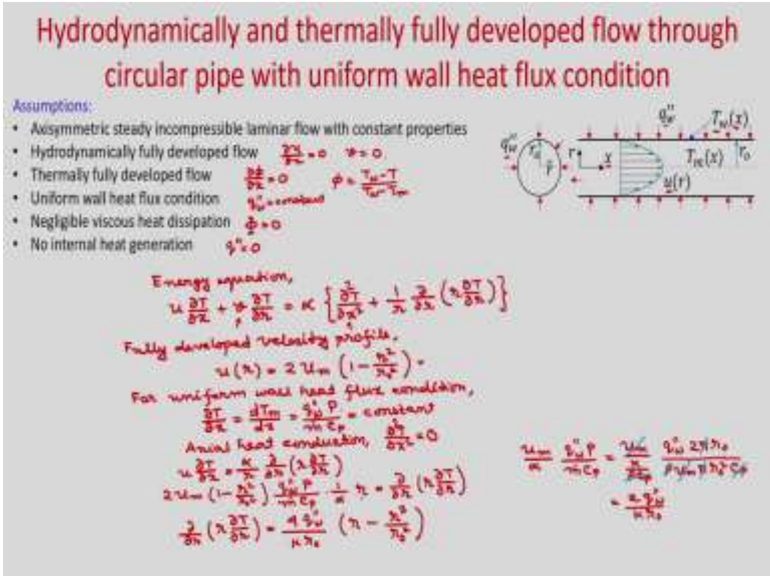

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Now, let us consider hydrodynamically and thermally fully developed flow through circular pipe with uniform wall heat flux boundary condition.

So, you can see this is your cross section of the circular pipe $q_w^{''}$ is constant and r_0 is the radius of this pipe; x is the axial direction, r is measured from the centreline, and we have uniform wall heat flux on the wall and velocity profile is parabolic u(r) and already we have derived it. So, T_w in this particular case also will be varying in axial direction. So, T_w will be function of x. So, you can see the assumptions in this particular case will take axisymmetric steady incompressible laminar flow with constant properties.

So, what is axisymmetric flow? So, you can see that in if circumferential direction velocity is zero and any gradient in that direction is zero; then it is axisymmetric flow. So, you can see in this particular case we have a circular pipe. So, we have a solid wall, and velocity obviously will be zero in circumferential direction and your thermal boundary condition is also uniform over the wall. So, it is geometrically and thermally axisymmetric.

So, we can have the assumption that your temperature will vary in r and x direction. Hydrodynamically fully developed flow, so hydrodynamically fully developed flow means, your $\frac{\partial u}{\partial x}=0$; and if v is the velocity in radial direction, then v will be also 0. Thermally fully developed flow, so $\frac{\partial \phi}{\partial x}=0$, where Φ is the non-dimensional temperature; already we have defined as, $\phi=\frac{T_w-T}{T_w-T_m}$.

Uniform wall heat flux condition, so for uniform wall heat flux condition $q_w^{"}$ will be constant. Negligible viscosity heat dissipation, so Φ will be 0; no internal heat generation, $q^{'''}$ =0. So, in cylindrical coordinate now let us write the energy equation.

So, energy equation you can write as assuming the axisymmetric flow, $u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left\{\frac{\partial^2 T}{\partial x^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)\right\}$.

So, now let us invoke all these assumptions in this energy equation and let us simplify it. First let us write the uniform velocity profile. So, first let us write the fully developed velocity profile. So, fully developed velocity profile for circular pipe, what is that? So, u is function of r only right in this particular case. So, $u(r)=2u_m\left(1-\frac{r^2}{r_0^2}\right)$.

And similarly for uniform wall heat flux boundary condition, you can write $\frac{\partial T}{\partial x}$ = constant. And hence $\frac{\partial^2 T}{\partial x^2}$ = 0; that means axial heat conduction will be zero in this particular case, or in this thermal boundary condition where $q_w^{"}$ = constant.

So, for constant or uniform for uniform wall heat flux boundary condition, you can write $\frac{\partial T}{\partial x}=\frac{dT_m}{dx}=\frac{q_w^{"}P}{\dot{m}C_P}$ which is constant; P is the constant, $\dot{m}$ is constant, C_p is constant, so that means this is equal to constant. Hence your axial heat conduction $\frac{\partial^2 T}{\partial x^2}$ = 0. So, and for hydrodynamically fully developed flow v = 0. So, you can see here v = 0 and $\frac{\partial^2 T}{\partial x^2}$ =0.

So, you can write invoking this condition the energy equation as $u\frac{\partial T}{\partial x}=\frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$. So, now, $\frac{\partial T}{\partial x}$ you can put this value. So, if you put this value; so what you will get?

So, you can write u is the velocity profile, you put this one. So, it will be $2u_m\left(1-\frac{r^2}{r_0^2}\right)\frac{q_w^{"}P}{\dot{m}C_P}\frac{1}{\alpha}r=\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$.

So, now in the left hand side let us simplify it. So, you can see, you have $\frac{u_m}{\alpha}\frac{q_w^{"}P}{\dot{m}C_P}=\frac{u_m}{\frac{K}{\rho C_P}}\frac{q_w^{"}2\pi r_0}{\rho u_m \pi r_0^2 C_P}$.

So, you can see if you simplify it. So, this ρ C_p , ρ C_P will get cancel, u_m, π π and one r_0. So, you can write this as $\frac{2q_w^{"}}{Kr_0}$. So, this equation now we can write. So, first let us write, $\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)=\frac{4q_w^{"}}{Kr_0}\left(r-\frac{r^3}{r_0^2}\right)$.

So, this equation now if we integrate twice, then you will be able to find the temperature distribution. So, now, this differential equation we will integrate twice and find the temperature distribution inside the pipe.

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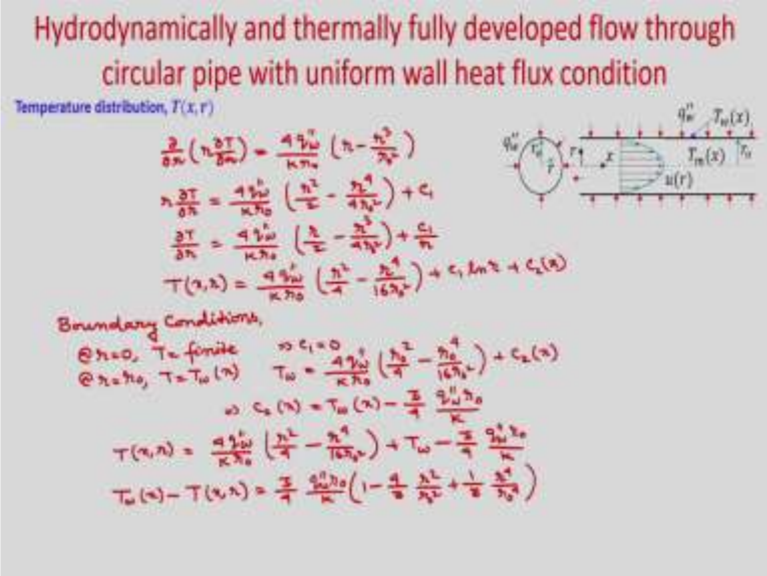

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So, what is your equation finally we got? So, that is your $\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)=\frac{4q_w^{"}}{Kr_0}\left(r-\frac{r^3}{r_0^2}\right)$.

So, you integrate twice. So, first time if you integrate, then you will get $r\frac{\partial T}{\partial r}=\frac{4q_w^{"}}{Kr_0}\left(\frac{r^2}{2}-\frac{r^4}{4r_0^2}\right)+C_1$. And you divide by r both side, then you will get, $\frac{\partial T}{\partial r}=\frac{4q_w^{"}}{Kr_0}\left(\frac{r}{2}-\frac{r^3}{4r_0^2}\right)+\frac{C_1}{r}$. So, now, another time if you integrate; so you will get, $T(x,r)=\frac{4q_w^{"}}{Kr_0}\left(\frac{r^2}{4}-\frac{r^4}{16r_0^2}\right)+C_1\ln r+C_2(x)$.

So, this is the temperature distribution we got with the two integration constant C_1 and C_2. So, we need two boundary conditions; one boundary condition is that at centerline r=0 you have finite temperature, and at r = r_0 you have wall temperature T_w. So, boundary conditions if you put, boundary conditions. So, at r = 0 ,T is finite, right.

So, if you see in this equation if you put r = 0. So, to have left hand side T finite, C_1 must be 0. So, C_1 = 0, and at r = r_0, T = T_w (x). If you put it that, so, $T_w=\frac{4q_w^{"}}{Kr_0}\left(\frac{r_0^2}{4}-\frac{r_0^4}{16r_0^2}\right)+C_2(x)$.

So, you can find, $C_2(x) = T_w(x) - \frac{3}{4}\frac{q_w^{"} r_0}{K}$.

So, final temperature distribution now we can write, $T(x,r) = \frac{4q_w^{"}}{Kr_0}\left(\frac{r^2}{4} - \frac{r^4}{16r_0^2}\right) + T_w(x) - \frac{3}{4}\frac{q_w^{"} r_0}{K}$.

So, if you rearrange it, you can write it as $T_w(x) - T(x,r) = \frac{3}{4}\frac{q_w^{"} r_0}{K}\left(1 - \frac{4}{3}\frac{r^2}{r_0^2} + \frac{4}{3}\frac{r^4}{r_0^4}\right)$. Now, we will find the centerline temperature putting the r = 0.

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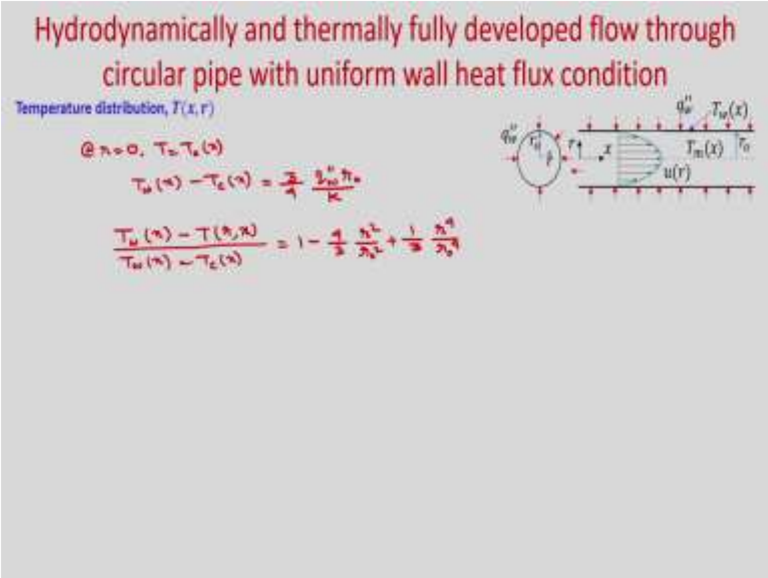

So, at r = 0, T = T_c which is function of x. So, if you write it. So, you will get $T_w(x) - T(x,r) = \frac{3}{4}\frac{q_w^{"} r_0}{K}$. So, this is your centerline temperature distribution. Now, you can also write the final temperature distribution $\frac{T_w(x) - T(x,r)}{T_w(x) - T_c(x)} = 1 - \frac{4}{3}\frac{r^2}{r_0^2} + \frac{4}{3}\frac{r^4}{r_0^4}$.

So, this is your temperature distribution. So, next we will find the mean temperature to find the Nusselt number.

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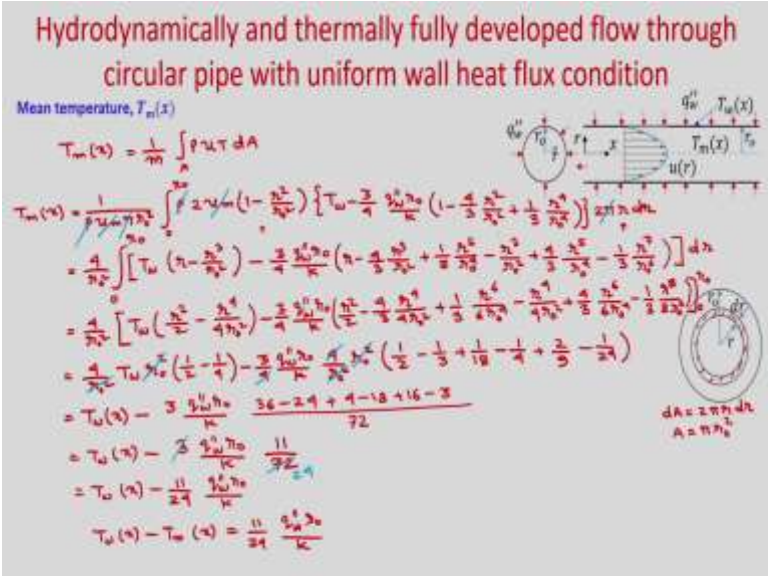

So, now let us calculate the mean temperature. So, that you can calculate $T_m(x)=\frac{1}{\dot{m}}\int_A \rho u T dA$. So, if you consider this elemental flow area d A at a distance r of distance dr. So, dA will be your 2πrdr. And what will be the flow area? It will be πr_0^2.

So, if you put it here. So, you will get, you see you will get $T_m(x)=\frac{1}{\rho u_m \pi r_0^2}\int_0^{r_0}\rho 2u_m\left(1-\frac{r^2}{r_0^2}\right)\left\{T_w-\frac{3}{4}\frac{q_w'' r_0}{K}\left(1-\frac{4}{3}\frac{r^2}{r_0^2}+\frac{4}{3}\frac{r^4}{r_0^4}\right)\right\}2\pi r dr$.

So, these u_m, u_m is constant ρ is constant, π π will get cancel. So, you can write it as, $T_m(x)=\frac{4}{r_0^2}\int_0^{r_0}\left[T_w\left(r-\frac{r^3}{r_0^2}\right)-\frac{3}{4}\frac{q_w'' r_0}{K}\left(r-\frac{4}{3}\frac{r^3}{r_0^2}+\frac{1}{3}\frac{r^5}{r_0^4}-\frac{r^3}{r_0^2}+\frac{4}{3}\frac{r^5}{r_0^4}-\frac{1}{3}\frac{r^7}{r_0^6}\right)\right]dr$. So, now, you integrate it. So, it will be,

$$T_m(x)=\frac{4}{r_0^2}\left[T_w\left(\frac{r^2}{2}-\frac{r^4}{4r_0^2}\right)-\frac{3}{4}\frac{q_w'' r_0}{K}\left(\frac{r^2}{2}-\frac{4}{3}\frac{r^4}{4r_0^2}+\frac{1}{3}\frac{r^6}{6r_0^4}-\frac{r^4}{4r_0^2}+\frac{4}{3}\frac{r^6}{6r_0^4}-\frac{1}{3}\frac{r^8}{8r_0^6}\right)\right]_0^{r_0}.$$

So, if we put r =0, all terms will become 0. So, you put just upper limit r = r_0 and rearrange it. So, what you will get,

$T_m(x) = \frac{4}{r_0^2} T_w r_0^2 \left(\frac{1}{2} - \frac{1}{4}\right) - \frac{3}{4}\frac{q_w^{''} r_0}{K}\frac{4}{r_0^2} r_0^2 \left(\frac{1}{2} - \frac{1}{3} + \frac{1}{18} - \frac{1}{4} + \frac{2}{9} - \frac{1}{24}\right)$. So, you simplify it. So, these r_0^2, r_0^2 will cancel. $\mathrm{T}_m(x) = \mathrm{T}_w(x) - 3\frac{q_w^{''} r_0}{K}\frac{36-24+4-18+16-3}{72}$.

So, we will get $\mathrm{T}_m(x) = \mathrm{T}_w(x) - 3\frac{q_w^{''} r_0}{K}\frac{11}{72}$. So, you can see this will be $\mathrm{T}_m(x) = \mathrm{T}_w(x) - \frac{11}{24}\frac{q_w^{''} r_0}{K}$.

So, you can write the temperature difference $\mathrm{T}_w(x) - \mathrm{T}_m(x) = \frac{11}{24}\frac{q_w^{''} r_0}{K}$. So, now, you are in a position to calculate the heat transfer coefficient right; because you can calculate q double prime w minus the temperature difference between wall temperature and mean temperature.

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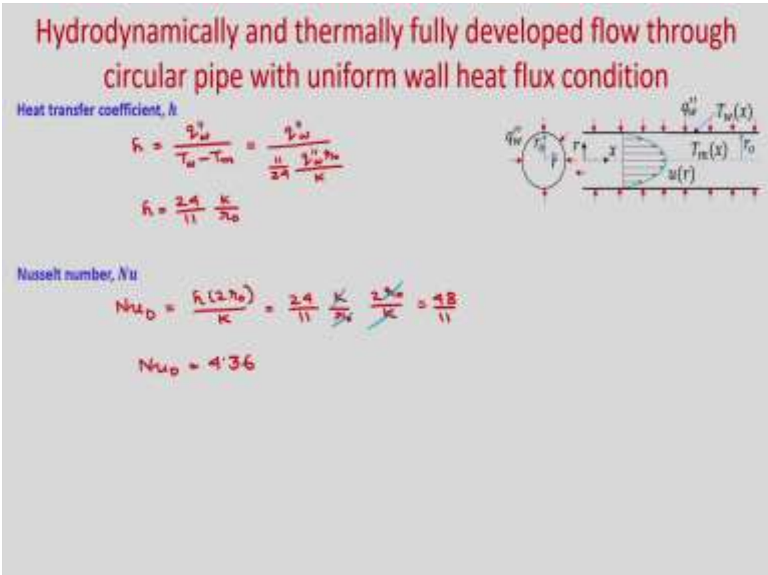

So, we will calculate the heat transfer coefficient, $h = \frac{q_w^{''}}{T_w - T_m}$. And that you can write $h = \frac{q_w^{''}}{\frac{11}{24}\frac{q_w^{''} r_0}{K}}$.

So, $h = \frac{24}{11}\frac{K}{r_0}$. Hence, now, Nusselt number you can calculate based on the hydraulic diameter and in this case hydraulic diameter is D.

So, it will be $Nu_D = \frac{h(2r_0)}{K}$. So, if you put the value of h, it will be $\frac{24}{11}\frac{K}{r_0}\frac{2r_0}{K}$. So, it will be 24 $\times$2. So, it will be $\frac{48}{11}$.

So, Nusselt number based on the diameter it is 4.36. So, you can see in this also particular case, your Nusselt number is independent of Reynolds number and Prandtl number and it is constant. So, thermally and hydrodynamically fully developed flow in a circular pipe, your Nusselt number is 4.36 and which is constant.

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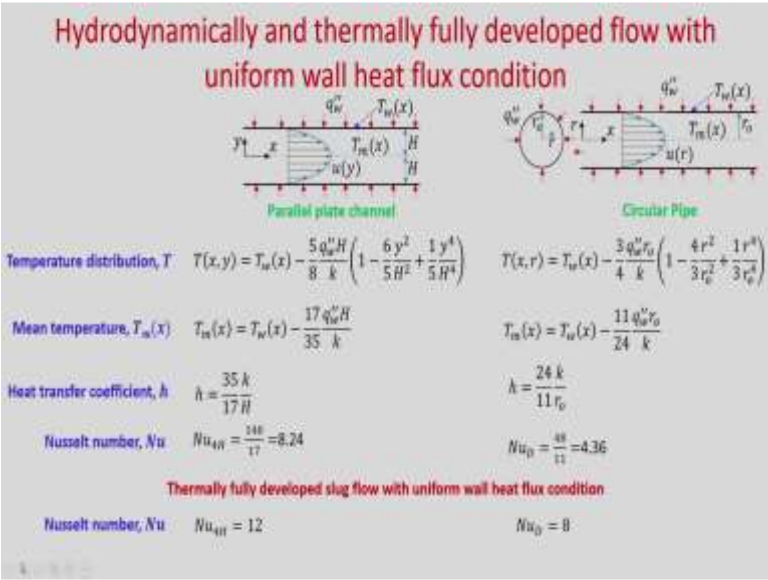

So, let us summarize what we have done in today's class. Today we considered two different types of duct; one is parallel plate channel and second is circular pipe.

We considered hydrodynamically and thermally fully developed flow with uniform wall heat flux boundary condition. In both the cases, first we have found the temperature distribution; you can see that we know the fully developed velocity profile put it putting it in the energy equation, we and integrating the equation we got the temperature profile.

And you can see here, the temperature distribution for parallel plates this one and for the circular pipe is this one. Now, we have represented this temperature which is function of x and r in terms of the wall temperature which is function of x.

Then we found the mean temperature right in both the cases. So, you can see for the parallel plate channel, your mean temperature is $T_m(x) = T_w(x) - \frac{17}{35}\frac{q_w'' H}{K}$; and whereas in circular pipe, it is $\mathrm{T}_m(x) = \mathrm{T}_w(x) - \frac{11}{24}\frac{q_w'' r_0}{K}$.

So, now, we know the temperature difference between wall temperature and the mean temperature. Hence you can calculate the heat transfer coefficient and you can see that for parallel plate channel, heat transfer coefficient is $h = \frac{35}{17}\frac{K}{H}$; in case of circular pipe $h = \frac{24}{11}\frac{K}{r_0}$. Then from heat transfer coefficient we calculated the Nusselt number. And we have seen in both the cases, Nusselt number is constant and independent of Reynolds number and Prandtl number.

So, when you consider thermally and hydrodynamically fully developed flow for uniform wall heat flux boundary condition, Nusselt number for parallel plate channel we calculated as 8.24 and for circular pipe we calculated as 4.36.

In last lecture we calculated the Nusselt number for thermally fully developed slug flow with uniform wall heat flux boundary condition and we found the Nusselt number for parallel plate channel as 12 and for circular pipe as 8 8. So, you can see that when you consider the fully developed velocity profile, then it is 8.24; but when you consider slug flow, where you have the everywhere you have the same velocity u m.

So, in that case you get higher Nusselt number in both the cases; that means when you have fully developed velocity profile, that means your velocity is decreasing towards the wall, so obviously you are getting less heat transfer compared to the slug flow. Because in slug flow, you have the velocity is same near to the wall as near to the centerline. So, velocity is higher, so you are getting higher heat transfer rate. So, that can be found from this expression Nusselt number 12 and Nusselt number 8 for circular pipe.