Fundamentals of Convective Heat Transfer
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Module - 06
Convection in Internal Flows – II
Lecture - 18
Thermally fully developed laminar slug flow with uniform wall heat flux condition

So, today we will consider Thermally Fully developed laminar slug flow with uniform wall heat flux condition. So, today we will consider two different types of channel; first we will consider flow through parallel plates channel and next, we will consider flow through circular pipe. As you know that it is thermally fully developed flow; so obviously, the non-dimensional temperature, we have defined as $\frac{T_w - T}{T_w - T_m}$. So, it will not vary in axial direction and we will consider uniform wall heat flux and you know that in this particular case, $\frac{\partial T}{\partial x}$ will be constant.

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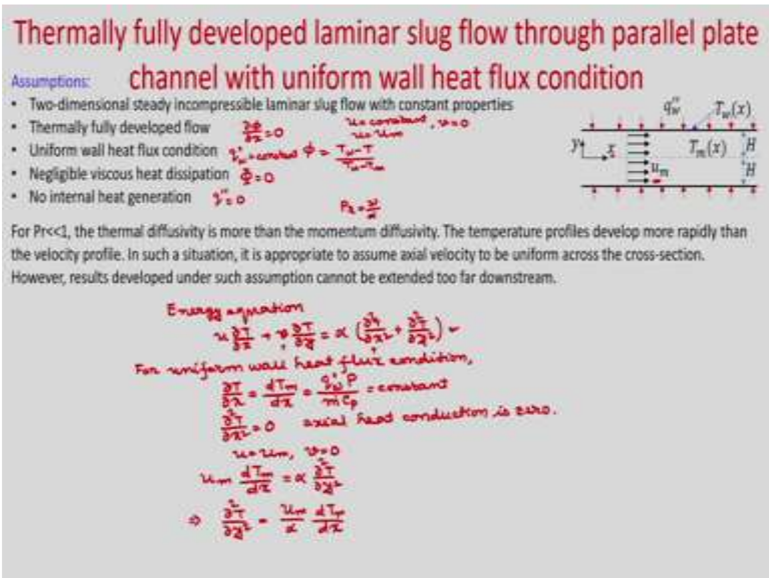

So, you can see that for Prandtl number << 1, where Prandtl number is defined as $\Pr = \frac{\nu}{\alpha}$. $\Pr = \frac{\nu}{\alpha}$, where ν is the momentum diffusivity; α is the thermal diffusivity. So, for Prandtl number << 1, the thermal diffusivity is more than the momentum diffusivity. The temperature profiles develop more rapidly than the velocity profile near to the inlet.

In such a situation, it is appropriate to assume axial velocity to be uniform across the cross section. However, results developed under such assumption cannot be extended too far downstream. And, these axial velocity when we considered as uniform that is known as slug flow.

So, the assumptions for today's class, we will consider two-dimensional steady incompressible laminar slug flow with constant properties. So, slug flow means u is constant, u is constant and in this particular case, we will take u =u_m. So, you can see this is the channel with two infinite parallel plates; x is the axial direction, y is measured from the center line, $q_w^{"}$ is the constant heat flux applied to both the walls. These two plates are separated by distance 2H and you have uniform velocity because it is slug flow. Obviously, if it is uniform, then your v velocity will be also 0.

So, v is also 0; v is the velocity in y direction. Thermally fully developed flow, so $\frac{\partial \phi}{\partial x} = 0$, where Φ is the non-dimensional temperature. $\frac{T_w - T}{T_w - T_m}$ uniform wall heat flux condition. So, $q_w^{"}$ is constant. Negligible viscous heat dissipation.

So, Φ = 0 and no internal heat generation. So, $q^{'''} = 0$. So, for this particular case, let us write the energy equation in general for steady two-dimensional situation. So, energy equation, $u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2})$, neglecting the viscous dissipation and heat generation.

So, now for uniform wall heat flux boundary condition, you know the $\frac{\partial T}{\partial x}$ right. We have already derived. So, for uniform wall heat flux, $\frac{\partial T}{\partial x} = \frac{dT_m}{dx} = \frac{q_w^{"} P}{\dot{m} C_P}$ and you can see that $q_w^{"}$ is constant, P is the perimeter.

If you have a constant cross-sectional channel, then P is also constant, $\dot{m}$ is constant, C_p is constant. So, this will be constant. Hence, $\frac{\partial^2 T}{\partial x^2}$ =0; because, $\frac{\partial T}{\partial x}$ is constant; so, $\frac{\partial^2 T}{\partial x^2}$ =0. So, for this particular case, you can see that your axial heat conduction is 0.

So, for slug flow, you can put u = u_m and b =0. So, u is equal to u_m and v = 0. So, all these if put it in this energy equation, what you will get? So, you will get u_m. So, $\frac{\partial T}{\partial x} = \frac{dT_m}{dx}$; then, v = 0. So, this term will get 0, $\frac{\partial^2 T}{\partial x^2}$ =0. So, you will write $\alpha \frac{\partial^2 T}{\partial y^2}$.

So, you can write it as, $\alpha \frac{\partial^2 T}{\partial y^2} = \frac{u_m}{\alpha} \frac{dT_m}{dx}$.

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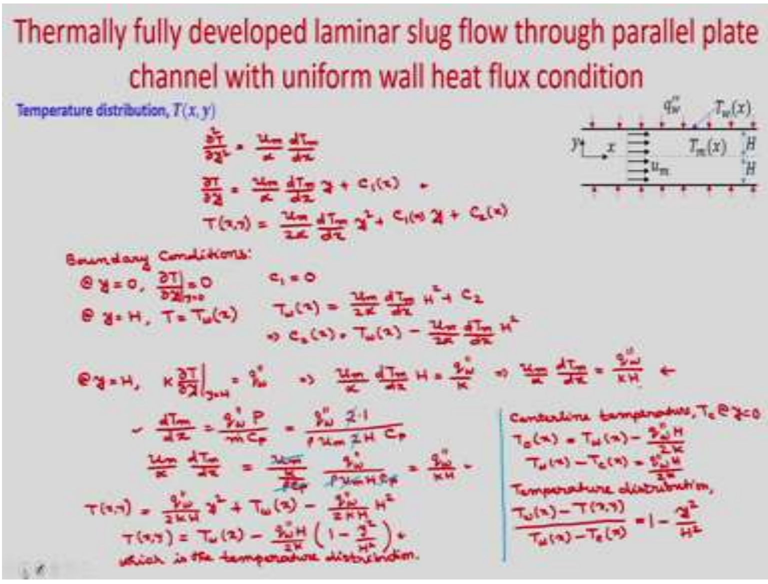

So, we have the equation, $\alpha \frac{\partial^2 T}{\partial y^2} = \frac{u_m}{\alpha} \frac{dT_m}{dx}$. So, this is constant. So, what you can do? Now, you can integrate it. So, you can write $\frac{\partial T}{\partial y} = \frac{u_m}{\alpha} \frac{dT_m}{dx} y + C_1(x)$; because,

$$\mathrm{T(x, y)} = \frac{u_m}{2\alpha} \frac{dT_m}{dx} y^2 + C_1(x) y + C_2(x).$$

So, now let us discuss about the boundary conditions. So, at central line, we have a finite temperature and you can see in this particular case about the center line, you have geometrical symmetry, because both walls are at distance H from the center line. At the same time, we have thermal boundary condition which is symmetric because in both walls, we have uniform wall heat flux boundary condition. So, you can see that it is geometrically and thermally symmetric.

So, at y = 0 at the center line, you will have either maximum or minimum temperature. Hence, you can write $\frac{\partial T}{\partial y}|_{y=0} = 0$; which is your center line will be 0. So, one boundary condition is; so, one boundary condition is at y = 0. You have $\frac{\partial T}{\partial y}|_{y=0} = 0$. And, another boundary condition at y = H, you can have temperature $T = T_w(x)$.

So, we are assuming that you have wall temperature T_w which varies in axial direction and that is $T = T_w$ at y = H. So, now if you put at y = 0, $\frac{\partial T}{\partial y} = 0$, then you can see from this equation, this equation you can see that C_1 will be 0 and at y = H, if you put $T = T_w$, then you can see that it will be $T_w(x) = \frac{u_m}{2\alpha}\frac{dT_m}{dx}H^2 + C_2$. So, hence your, $C_2(x) = T_w(x) - \frac{u_m}{2\alpha}\frac{dT_m}{dx}H^2$.

So, now let us simplify this $\frac{u_m}{2\alpha}\frac{dT_m}{dx}H^2$. So, let us apply the boundary condition at $r = r_0$, you have a uniform wall heat flux $q_w^{"}$=constant. So, obviously, you can write at y = H,. Now, $K\frac{\partial T}{\partial y}|_{y=H}$. So, it is y =0 will be $q_w^{"}$. So, you can see your y is in the positive upward direction, right. So, but $q_w^{"}$ is the negative y direction, what we considered in this diagram.

So, obviously, you are in negative y direction, the heat conduction will be, $K\frac{\partial T}{\partial y}|_{y=H} = q_w^{"}$. So, from here, now we can see $\frac{\partial T}{\partial y}$ from this equation $C_1 = 0$. So, $\frac{\partial T}{\partial y}$ if you put, then you will get K; K you just divide in the right hand side, so you will get $\frac{u_m}{\alpha}\frac{dT_m}{dx}H = \frac{q_w^{"}}{K}$. And hence, you can write $\frac{u_m}{\alpha}\frac{dT_m}{dx} = \frac{q_w^{"}}{KH}$. This we can also similar way, we can derive a straight forward from this expression of $\frac{dT_m}{dx}$, because you know that $\frac{dT_m}{dx}$ for this uniform heat flux boundary condition is $\frac{dT_m}{dx} = \frac{q_w^{"} P}{\dot{m} C_P}$. So, this is constant already you have written. So, now you tell me what is P perimeter?

So, this perimeter is the heaters per area on the wall. So, per unit width if you considered, then it will be 2 or if you w is the width if you considered; then, on the upper wall, it is w; bottom wall, it is w, then 2 w or per unit width if you consider, then 2 into 1. And, what is the flow area? Flow area is the distance between 2 parallel plates is 2 H. So, 2 H into w or per unit width if you consider, then it will be 2 H.

So, you can write $\frac{q_w^{"} 2.1}{\rho u_m 2HC_P}$. So, you can see that you can write $\frac{u_m}{\alpha}\frac{dT_m}{dx} = \frac{u_m}{\frac{K}{\rho C_p}} \frac{q_w^{"}}{\rho u_m H C_P}$ and similarly, from directly this expression also same, we got $\frac{q_w^{"}}{KH}$.

Now, you write the final temperature distribution putting the value of C_2. So, if you write the temperature distribution. So, C_1=0, C_2 is this, so will be, $T(x,y) = \frac{q_w^{"}}{2KH} y^2 + \mathrm{T}_w(\mathrm{x}) - \frac{q_w^{"}}{2KH} H^2$.

So, now you can see, so if you rearrange it. So, we will rearrange it as, $T(x,y) = \mathrm{T}_w(\mathrm{x}) - \frac{q_w^{"} H}{2K}(1 - \frac{y^2}{H^2})$.

So, this is the temperature distribution. This temperature distribution also we can write in terms of the centerline temperature. So, what will be centerline temperature? At y = 0, you will get the centerline temperature. So, if it is so, so you will get centerline temperature T_c at y = 0, right. So, y = 0, you will get the centerline temperature.

So, if you put y = 0 in this equation, you can find the center line temperature, $T_c(x) = \mathrm{T}_w(\mathrm{x}) - \frac{q_w^{"}H}{2K}$. So, you can see that if you can write $\mathrm{T}_w(\mathrm{x}) - T_c(x) = \frac{q_w^{"}H}{2K}$. And, if you put it here and you can write the temperature distribution in terms of the central line temperature.

So, it will be $\frac{T_w(x) - T(x,y)}{T_w(x) - T_c(x)} = 1 - \frac{y^2}{H^2}$. So, to calculate the heat transfer coefficient, we need to find the mean temperature. Because, we will define the Nusselt number or the heat transfer coefficient based on the difference between wall temperature and the mean temperature.

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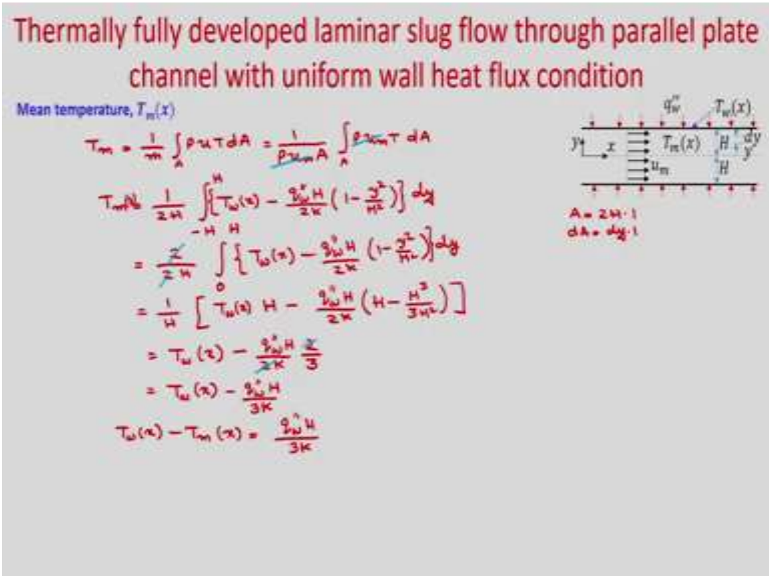

So, let us calculate the mean temperature now. You know that mean temperature, you can calculate as $T_m = \frac{1}{\dot{m}} \int_A \rho u T dA$. So, in this particular case, as you consider the slug flow. So, u_m is constant. So, you can write $\frac{1}{\rho u_m A} \int_A \rho u_m T dA$.

So, what will be the flow area in this particular case? So, particular case per unit width your flow area is 2 H. And, what is your d A? d A = d y. So, d y into 1; per unit width dA=dy. So, dy into 1 and this is also into 1. So, per unit width we are defining, so this ρu_m will cancel out.

So, you can write $T_m(x) = \frac{1}{2H}\int_{-H}^{H}\{T_w(x) - \frac{q_w^{"}H}{2K}(1-\frac{y^2}{H^2})\}dy$. So, now this integral minus H to H will write as 0 to H. So, 2 you can take it outside; so, 2 H.

So, it will be $T_m(x) = \frac{2}{2H}\int_{0}^{H}\{T_w(x) - \frac{q_w^{"}H}{2K}(1-\frac{y^2}{H^2})\}dy$. So, you can write now $T_m(x) = \frac{1}{H}[T_w(x)H - \frac{q_w^{"}H}{2K}(H - \frac{H^3}{3H^2})]$.

So, now if you multiply width 1 /H. So, it will be $T_m(x) = T_w(x) - \frac{q_w^{"}H}{2K}\frac{2}{3}$. So, you will get finally, T_m as T_w (x). So, the difference between the wall temperature and the mean temperature you can write as $T_w(x) - T_m(x) = \frac{q_w^{"}H}{3K}$.

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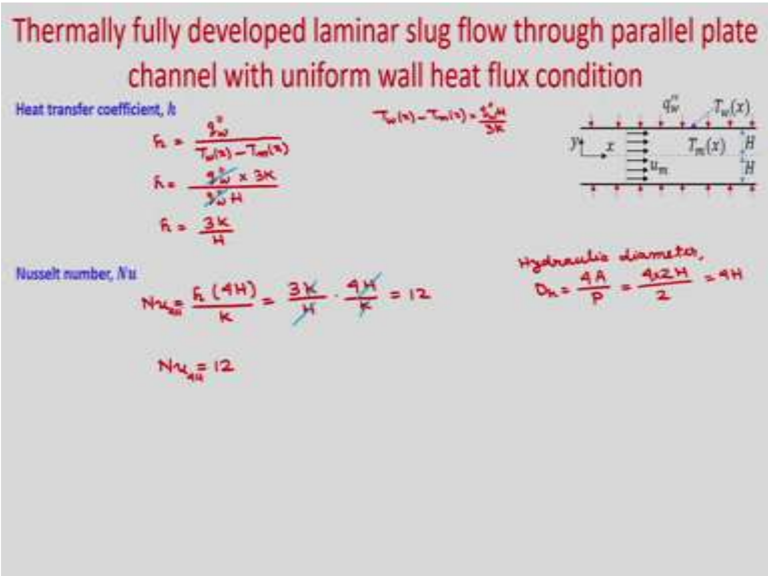

So, now we can calculate the heat transfer coefficient. So, what is your heat transfer coefficient? Heat transfer coefficient is $h = \frac{q_w^{"}}{T_w(x) - T_m(x)}$ that we got that equating the Fourier's law and the Newton's law of cooling.

So, you can write $h = \frac{q_w^{"} X 3K}{q_w^{"} H}$. So, you can write H in terms of thermal conductivity of the fluid and the distance from the centerline to the wall as, $h = \frac{3K}{H}$.

So, now you will be able to calculate the Nusselt number. Nusselt number you can calculate based on the hydraulic diameter. So, what is the hydraulic diameter of this particular case? So, you have a two parallel plates separated by distance 2 H. So, your hydraulic diameter in this particular case, so hydraulic diameter will be $D_h = \frac{4A}{P}$.

So, what will be that $\frac{4 \times 2H}{2}$. so, it will be 4H. So, we will define the Nusselt number based on the hydraulic diameter that is 4H and the difference between the mean temperature; difference between the wall temperature and mean temperature.

So, Nusselt number now will defined as, $Nu_{4H} = \frac{h(4H)}{K}$. So, hence you can see that it will be 12. So, Nusselt number for these slug flow and thermally fully developed flow inside the parallel plates channel is 12. So, it is based on 4 H, hydraulic diameter based on 4 H,. Next let us consider thermally fully developed laminar slug flow through circular pipe with uniform wall heat flux condition.

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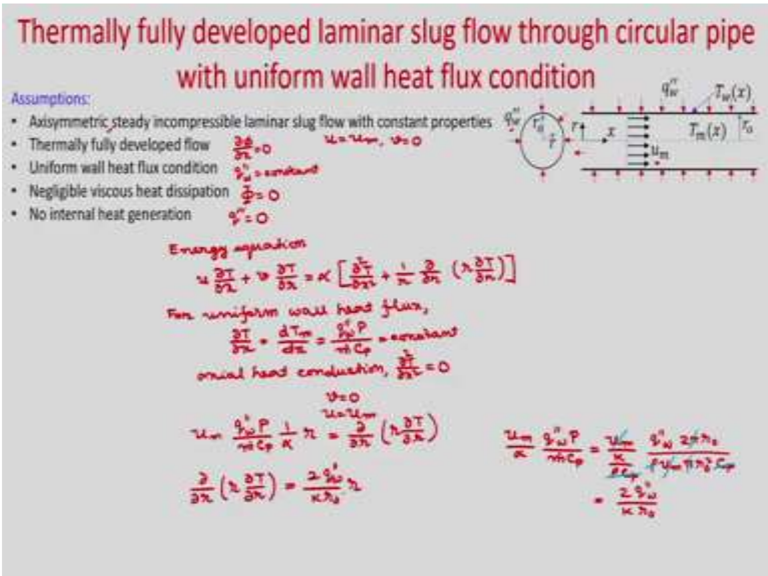

So, the assumptions will take that it is axisymmetric. Axisymmetric means circumferential variation is 0 and velocity is also 0. So, what is axisymmetric? Axisymmetric means the circumferential variation of any quantity is 0 and in that direction, velocity is also 0. And, you can see in this particular case geometrically and thermally because you have uniform wall heat flux boundary condition; so, it is axisymmetric.

So, you can write for axisymmetric steady incompressible laminar slug flow with constant properties so, u = u_m and v = 0. So, you can see x is the axial direction, r is the radial direction and it is measured from the center line; at r =r_0, your heat flux $q_w^{''}$ is constant and you have a slug flow.

So, your velocity profile is constant and that is equal to u_m. So, thermally fully developed flow so, $\frac{\partial \phi}{\partial x} = 0$. Uniform wall heat flux condition, so $q_w^{''}$ = constant. Negligible viscous heat dissipation, so Φ=0 and no internal heat generation, $q^{'''} = 0$.

So, now, let us write the energy equation in cylindrical coordinate for axisymmetric and steady incompressible and laminar flow. So, energy equation you can write in cylindrical coordinate as, $u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial r} = \alpha[\frac{\partial^2 T}{\partial x^2} + \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial T}{\partial r})]$.

So, now for constant heat flux boundary condition, we can write $\frac{\partial T}{\partial x}$=constant and hence, $\frac{\partial^2 T}{\partial x^2}$ =0. So, for uniform wall heat flux $\frac{\partial T}{\partial x} = \frac{dT_m}{dx} = \frac{q_w^{''} P}{\dot{m} C_P}$ = constant. Hence, your axial heat conduction $\frac{\partial^2 T}{\partial x^2}$ =0. And for slug flow, you can see v = 0. So, if you put these in these energy equation. So, you will get u = u_m. So, you can write, $u_m \frac{q_w^{''} P}{\dot{m} C_P}\frac{1}{\alpha} r = \frac{\partial}{\partial r}(r\frac{\partial T}{\partial r})$. So, now in left hand side, you put the values of P perimeter,

$\dot{m}$, mass flow rate and α as $\frac{K}{\rho C_p}$. So, if you put it and simplify it what you will get? So, you can see. So, you can write $\frac{u_m}{\frac{K}{\rho C_p}} \frac{q_w'' 2\pi r_0}{\rho u_m \pi r_0^2 C_P}$.

So, you will get finally this $u_m \frac{q_w'' P}{\dot{m} C_P} = \frac{2q_w''}{K r_0}$. So, if you put it here, so you will get $\frac{\partial}{\partial r}(r \frac{\partial T}{\partial r}) = \frac{2q_w''}{K r_0} r$.

So, now this equation, you can integrate twice and you can find the temperature distribution for thermally fully developed laminar slug flow through circular pipe with applying two boundary conditions at r = 0, you have T is finite and at $r = r_0$, you have wall temperature T_w right. So, that is why we have started with the simple problem considering the slug flow, because your right hand side is becoming constant. So, it is easy to integrate and find the temperature distribution and the heat transfer coefficient.

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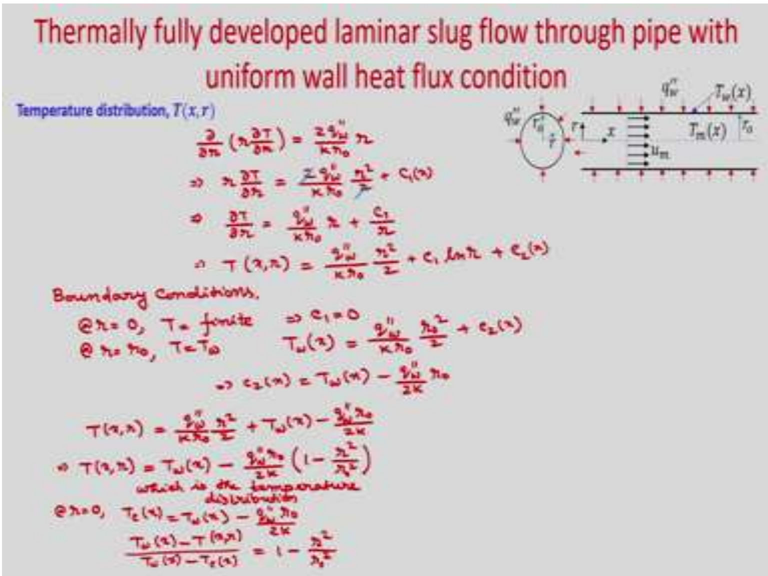

So, now let us find the temperature distribution. So, you have $\frac{\partial}{\partial r}(r\frac{\partial T}{\partial r})=\frac{2q_w^{"}}{Kr_0}r$. So, if you integrate it first, then you will get $r\frac{\partial T}{\partial r}=\frac{2q_w^{"}}{Kr_0}\frac{r^2}{2}+C_1(x)$. Then, if you divide by r, then you will get so these 2, 2 will get cancelled. So, you will write $\frac{\partial T}{\partial r}=\frac{q_w^{"}}{Kr_0}r+\frac{C_1}{r}$.

Now, if we integrate again; so, you will get, $T(x,r)=\frac{q_w^{"}}{Kr_0}\frac{r^2}{2}+C_1\ln r+C_2(x)$. So, what are the boundary conditions? So, one boundary condition is that at r = 0, T is finite. And, another boundary condition, you can write at $r = r_0$, you have a wall temperature T_w.

So, boundary conditions at r = 0, your T is finite. And also, you can see that geometrically and thermal it is symmetric. So, $\frac{\partial T}{\partial r}$=0. So, you can see if r = 0 if you put, so obviously, $C_1 = 0$. So, because this is your 0, so T is finite. So, this cannot be infinite. So, C_1 must be 0. So, C_1=0 and $r = r_0$, you have wall temperature T_w.

So, if you put it that $T_w(x)=\frac{q_w^{"}}{Kr_0}\frac{r_0^2}{2}+C_2(x)$. So, you can see $C_2(x)=T_w(x)-\frac{q_w^{"}}{2K}r_0$. So, now, these constants if you put in the temperature profile and find the temperature distribution. So, you can write $T(x,r)=\frac{q_w^{"}}{Kr_0}\frac{r^2}{2}+T_w(x)-\frac{q_w^{"}r_0}{2K}$. So, these you can write as $T(x,r)=T_w(x)-\frac{q_w^{"}r_0}{2K}(1-\frac{r^2}{r_0^2})$. So, which is your temperature distribution.

Now, if we write in terms of the centerline temperature, then you put at r = 0, T =T_c. So, at r = 0, your central line temperature will be $T_c(x)=T_w(x)-\frac{q_w^{"}r_0}{2K}$. So, if you put it here. So, if you rearrange, you are going to get this temperature profile $\frac{T_w(x)-T(x,r)}{T_w(x)-T_c(x)}=1-\frac{r^2}{r_0^2}$.

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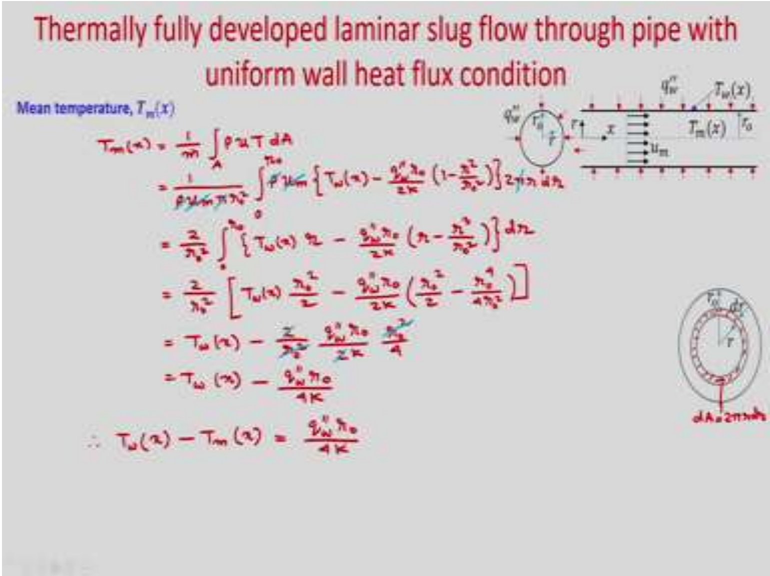

So, now next we need to find the mean temperature to find the heat transfer coefficient. So, you can write $T_m(x) = \frac{1}{\dot{m}} \int_A \rho u T dA$. So, now, what is $\dot{m}$? $\dot{m} = \frac{1}{\rho u_m}$. And, what is the area? Flow area is πr_0^2 and this you can write,

$$T_m(x) = \frac{1}{\rho u_m \pi r_0^2} \int_0^{r_0} \rho u_m \{T_w(x) - \frac{q_w^{"} r_0}{2K}(1 - \frac{r^2}{r_0^2})\} 2\pi r dr \, .$$

So, that you can write here. So, now, u_m u_m, ρ, ρ, this π, π will get cancelled.

So, and now you integrate it from 0 to r_0. So, what you will get? So, you can write $T_m(x) = \frac{2}{r_0^2} \int_0^{r_0} \{T_w(x) \mathrm{r} - \frac{q_w^{"} r_0}{2K}(r - \frac{r^3}{r_0^2})\} dr$.

So, now you integrate it. So, twice you can integrate it $T_m(x) = \frac{2}{r_0^2} [T_w(x) \frac{r_0^2}{2} - \frac{q_w^{"} r_0}{2K}(\frac{r_0^2}{2} - \frac{r_0^4}{4 r_0^2})]$.

So, if you put the limit, it will be $T_m(x) = T_w(x) - \frac{2}{r_0^2} \frac{q_w^{"} r_0}{2K} \frac{r_0^2}{4}$.You can write this as $T_m(x) = T_w(x) - \frac{q_w^{"} r_0}{4K}$. So, now, you can write the temperature difference $T_w(x) - T_m(x) = \frac{q_w^{"} r_0}{4K}$. So, now, you find the heat transfer coefficient. Heat transfer coefficient, you can write as $h = \frac{q_w^{"}}{T_w(x) - T_m(x)}$.

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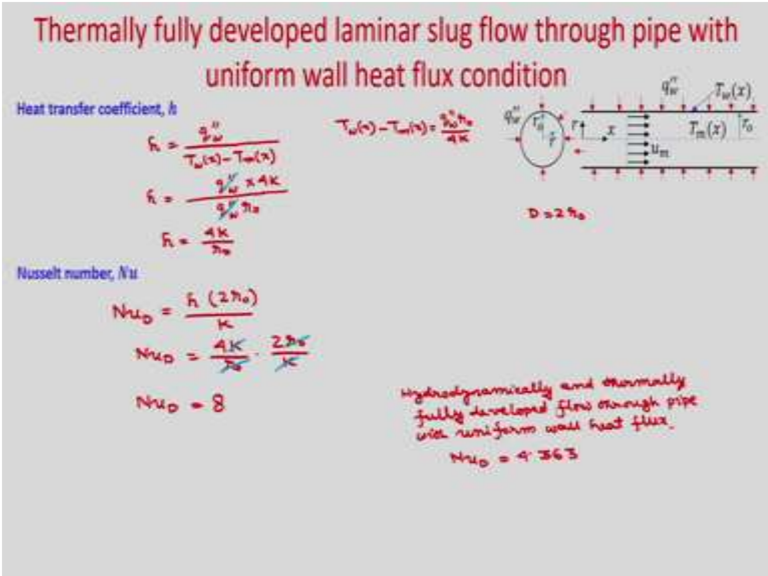

So, heat transfer coefficient, so $h = \frac{q_w^{"}}{T_w(x) - T_m(x)}$. So, in the last slide, we have found $T_w(x) - T_m(x) = \frac{q_w^{"} r_0}{4K}$. So, this h will be then, $h = \frac{q_w^{"} X\, 4K}{q_w^{"} r_0}$.

So, this you cancel, then you will get $h = \frac{4K}{r_0}$. Now, we will find the Nusselt number. Based on the hydraulic diameter and for a pipe, hydraulic diameter is just diameter of the pipe. So, it is 2 r_0. So, we will find the Nusselt number based on 2 r_0. So, Nusselt number based on diameter.

So, diameter is just 2 r_0. This is your hydraulic diameter. So, Nusselt number based on diameter, you can write $Nu_D = \frac{h(2r_0)}{K}$. So, Nusselt number based on diameter;

$$Nu_D = \frac{4K}{r_0} \frac{2r_0}{K}.$$

So, you will find Nusselt number based on diameter and the difference between the wall temperature and mean temperature will be 8. Later, we will calculate this thermally fully developed and hydrodynamically fully developed condition with uniform wall heat flux, for this condition for pipe flow.

And Nusselt number, you will find it as, so hydrodynamically and thermally fully developed; thermally fully developed flow through pipe with uniform wall heat flux this will calculate. So, we will calculate later. So, you can calculate Nusselt number based on diameter, it will be 4.363, you will get Nusselt number as 4.363. So, obviously, you can see that as you have uniform velocity obviously, near to the wall you have more velocity. Hence, you will get more heat transfer and that you can see for Nusselt number as 8. And also, you notice that your Nusselt number is constant for this fully developed flow.

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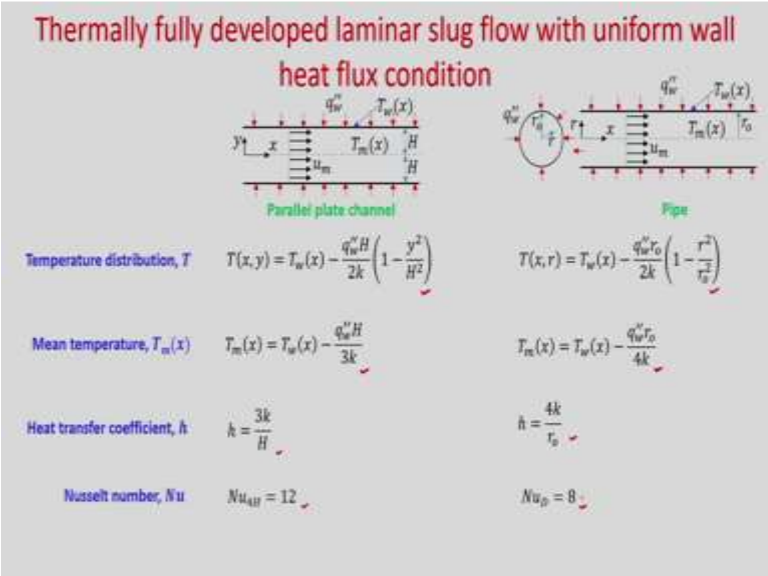

So, let me summarize. So, today, we have considered thermally fully developed laminar slug flow in two different types of channels; one is parallel plate channel and one is circular pipe. And, we considered uniform wall heat flux boundary condition. For both the cases, we calculated the temperature distribution first.

You can see, this is the temperature distribution for parallel plate channel. And for pipe, this is the temperature distribution here T is function of x and r. And in this particular case, T is function of x and y. Then, we calculated the mean temperature in terms of the wall temperature because wall temperature is also varying.

So, you can see for parallel plate channel, this is the mean temperature variation and for pipe flow, this is the mean temperature variation. Once we calculated the mean temperature, then we can calculate the difference between wall temperature and mean temperature.

And, from there we calculate the heat transfer coefficient and this is the heat transfer coefficient for parallel plate channel $\frac{3K}{H}$ and for pipe flow, it is $\frac{4K}{r_0}$. Then, we calculated the Nusselt number based on the hydraulic diameter and the temperature difference between the wall temperature and the mean temperature. And, we have found for this parallel plate channel Nusselt number based on hydraulic diameter as 12 and for pipe flow based on the diameter of the pipe as 8.

And, we have also discussed that if you consider the hydrodynamically fully developed flow then obviously, velocity will vary from u = 0 at the wall to r = 0 maximum velocity. And hence, you will get less heat transfer in fully developed flow. Here, we consider slug flow as your velocity is uniform and constant. Hence, we got higher heat transfer for this pipe flow. So, that is 8, we have shown and in the case of fully developed hydrodynamically fully developed flow it is 4.36.

Thank you.