Fundamentals of Convective Heat Transfer

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Module – 05
Convection in Internal Flows - I
Lecture – 17
Velocity profile in fully-developed channel flows

Hello everyone. So, in today's lecture, we will find the fully developed velocity profile in channel flows. So, we will consider three different cases. One is flow between two parallel plates. Then we will consider flow between two parallel plates where one plate is moving with respect to the other. And finally, we will consider flow inside pipe. As you know that to solve the temperature or to solve the energy equation, you need to know the velocity profile.

So, if you have a fully developed condition under which if you want to find the velocity profile, so in today's lecture we will find this fully developed velocity profile for different situations.

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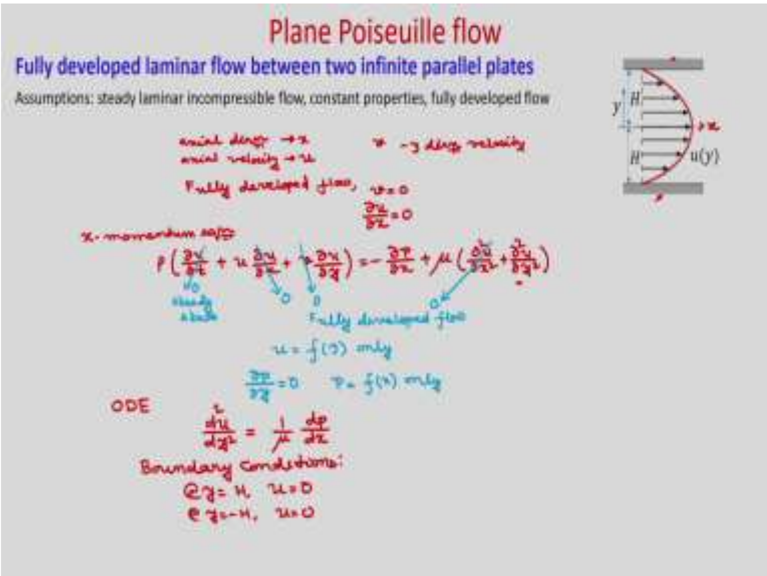

First we will consider plane Poiseuille flow which is known as fully developed laminar flow between two infinite parallel plates you can consider two infinite parallel plates. So, in third direction, as it is infinite you can consider as a two-dimensional flow. And you can have the assumptions of steady, laminar and incompressible flow. Then you can have the assumptions of constant properties, where density and viscosity remain constant, and we are anyway considering the fully developed flow.

You have already seen that in fully developed flow the velocity v in y direction, it is 0 or first let us consider that you have axial direction as x. So, this is your x, and axial velocity as u. And in y direction you have v velocity, and this is your y direction velocity.

So, from fully developed velocity profile, so for fully developed flow condition you can write v = 0 everywhere; and axial velocity is constant in the axial direction, so $\frac{\partial u}{\partial x} = 0$. So, under this assumptions now let us write the x momentum equation.

So, you can see your x momentum equation is $\rho(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}) = -\frac{\partial P}{\partial x} + \mu(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2})$. So, this is your x momentum equation.

So, as we have assumed that it is a steady state flow, then obviously, $\frac{\partial u}{\partial t} = 0$. So, this is your 0 as it is steady state, $\frac{\partial u}{\partial x} = 0$ fully developed condition; v = 0 fully developed condition; and $\frac{\partial^2 u}{\partial x^2}$ =0 as fully developed condition, so fully developed flow. So, we can see that as $\frac{\partial u}{\partial x} = 0$, then u is function of y only.

Similarly, if you write the y momentum equation, then you will find that only $\frac{\partial P}{\partial y} = 0$. So, all terms will get cancelled – the temporal term, convection term, the diffusion term, so all these terms will get cancelled and you will have just all these terms will be 0, and you will have $\frac{\partial P}{\partial y} = 0$. So, you will have from y momentum equation you will get $\frac{\partial P}{\partial y} = 0$, so that means, p is a function of x only.

So, under this situation this x momentum governing equation you can write as, so u is function of y only, so you can write $\frac{d^2u}{dy^2} = \frac{1}{\mu}\frac{dP}{dx}$, so because p is function of x only. So, this is your now you got ordinary differential equation. So, we started with the partial differential equation, but invoking the assumptions and putting some terms 0, you got the ordinary differential equation. Now, with proper boundary condition, you will be able to integrate this ordinary differential equation.

What are the boundary conditions? So, you can see that we have taken axis in the central line as x and y is measured from the central line from here so y. And these two parallel plates are separated by a distance 2 h. So, the boundary conditions you can write. So, at y=h, your u =0 as well as at y = - h ,u = 0, because you have no slip boundary condition on upper plate and the bottom plate.

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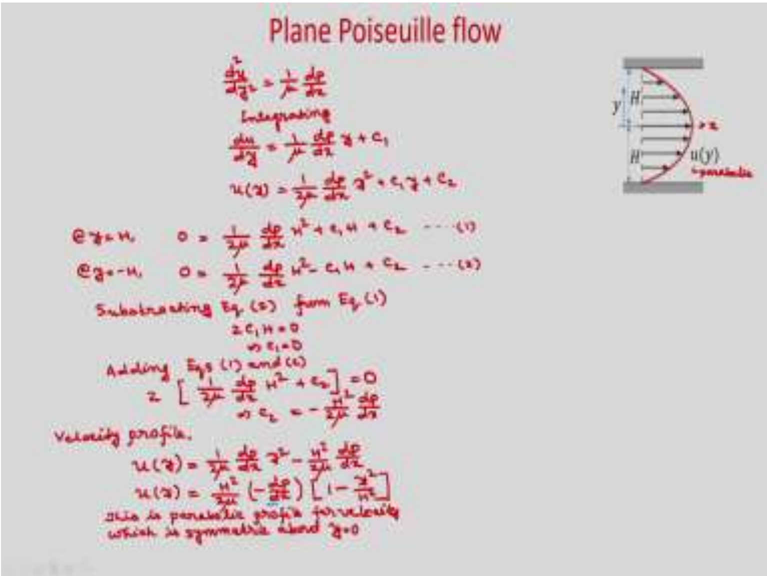

So, now let us integrate this equation. So, you have $\frac{d^2u}{dy^2} = \frac{1}{\mu}\frac{dP}{dx}$. So, if you integrate it, you will get $\frac{du}{dy} = \frac{1}{\mu}\frac{dP}{dx}y + C_1$. And again if you integrate, then you will get $u(y) = \frac{1}{2\mu}\frac{dP}{dx}y^2 + C_1 y + C_2$. So, C_1,C_2 are integration constant.

Now, you invoke the boundary conditions, you have two boundary conditions because there is as you have two boundary conditions you will be able to find the two constant C_1 and C_2. So, y =H , u = 0. So, you will get $0 = \frac{1}{2\mu}\frac{dP}{dx}H^2 + C_1H + C_2$ and similarly at y = -H, you have $0 = \frac{1}{2\mu}\frac{dP}{dx}H^2 - C_1H + C_2$.

So, if you say that this is equation number 1 and this is equation number 2, then if you subtract equation 2 from 1. So, subtracting equation 2 from equation 1 ; so what you will get? So, if you subtract, you will get $2C_1H = 0$. That means, $C_1 = 0$ and if you add these two equation adding equations 1 and 2, what you will get?

So, this term will get cancelled, this $C_1H - C_1H$. So, this will be 0. So, you will get $2[\frac{1}{2\mu}\frac{dP}{dx}H^2 + C_2] = 0$, so that means, you will get $C_2 = -\frac{H^2}{2\mu}\frac{dP}{dx}$.

So, if you put these values C_1, C_2, so you will get the final velocity profile. So, velocity profile you will get, $u(y) = \frac{1}{2\mu}\frac{dP}{dx}y^2 - \frac{H^2}{2\mu}\frac{dP}{dx}$. So, this you can write, $u(y) = \frac{H^2}{2\mu}(-\frac{dP}{dx})[1 - \frac{y^2}{H^2}]$.

So, you can see that it is a parabolic profile. So, this is parabolic profile for velocity which is symmetric about y = 0. So, you can see the velocity profile here. So, this is your x direction. So, it is actually symmetric about y = 0. So, you will get at y = 0 if you put, then velocity will be maximum, because there you will get maximum velocity which is your central line velocity and the profile will be parabolic. So, u y is parabolic.

You can see here that we have written this $\frac{dP}{dx}$ as -$\frac{dP}{dx}$. So, you can see that flow will takes place from high pressure region to low pressure region. So, in the positive x-direction or along the axial direction, your pressure will decrease. Hence -$\frac{dP}{dx}$ will be a positive quantity that is why we have written as-$\frac{dP}{dx}$. So, now, we are interested to find what is the mean velocity or average velocity at any cross section.

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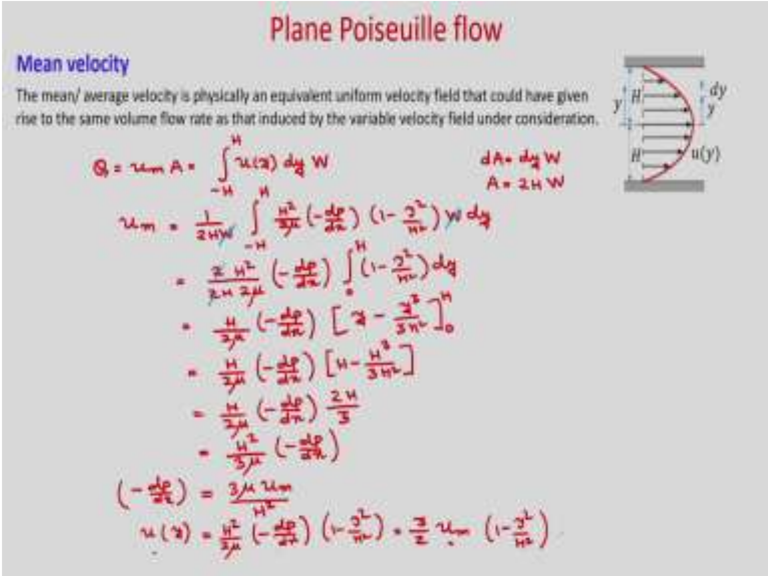

So, you can see that mean or average velocity is physically an equivalent uniform velocity field that could have given rise to the same volume flow rate as that induced by the variable velocity field under consideration. So, if you calculate the volume flow rate Q, so you can write $Q = u_m A$, where u_m is the mean velocity.

And that if you see now if you take a small elemental flow area of distance d at a distance y from the central line of distance dy. If you consider an elemental flow area at a distance y from the central line of distance dy, then your this flow area will be just dy into the width of the channel, width means perpendicular to this board whatever distance you have.

So, that if you consider as width, then you can write dA= Wdy. And total flow area will be, so this is your twice h is the distance between two parallel plates. So, it will be $A = 2HW$.

So, now, if you consider this elemental area, so what is flow is happening so that you can integrate from $\int_{-H}^{H} u(y)dyW$, so that is the elemental area dA= Wdy, so that we have written. So, now, your mean velocity will be $u_m = \frac{1}{2HW}\int_{-H}^{H}\frac{H^2}{2\mu}(-\frac{dP}{dx})(1-\frac{y^2}{H^2})Wdy$ s.

So, now, you will get, $u_m = \frac{2H^2}{2H2\mu}(-\frac{dP}{dx})\int_0^H (1-\frac{y^2}{H^2})dy$.

So, you can see that we have written this, $u_m = \frac{H}{2\mu}(-\frac{dP}{dx})[y-\frac{y^3}{3H^2}]_0^H$.

So, if you put H, then we are going to get $u_m = \frac{H}{2\mu}(-\frac{dP}{dx})[H-\frac{H^3}{3H^2}]$. So, this you can see that it will be 2 by 3. So, $u_m = \frac{H}{2\mu}(-\frac{dP}{dx})\frac{2H}{3}$. So, hence you can write $u_m = \frac{H^2}{3\mu}(-\frac{dP}{dx})$. So, now, you can see that you can represent the constant pressure gradient minus dp by dx in terms of the mean velocity.

So, you can write minus $(-\frac{dP}{dx}) = \frac{3\mu u_m}{H^2}$. So, we can see here that your right hand side, you have μ which is your fluid property, dynamic viscosity that is positive quantity, u_m which is your velocity mean velocity that is also constant and positive, and H which is your the distance from the central line so that is also constant. So, obviously, right hand side is constant, that means, $-\frac{dP}{dx}$ is constant.

So, now if you find the velocity profile, so you can write in terms of mean velocity, $u(y) = \frac{H^2}{2\mu}(-\frac{dP}{dx})(1-\frac{y^2}{H^2})$.

So, now, this $-\frac{dP}{dx}$ if you put it here, you are going to get, $u(y) = \frac{3}{2}u_m(1-\frac{y^2}{H^2})$. So, your velocity profile you can write in terms of mean velocity as, $u(y) = \frac{3}{2}u_m(1-\frac{y^2}{H^2})$. And if you find the maximum velocity, then you have to find the velocity at central line which is y =0.

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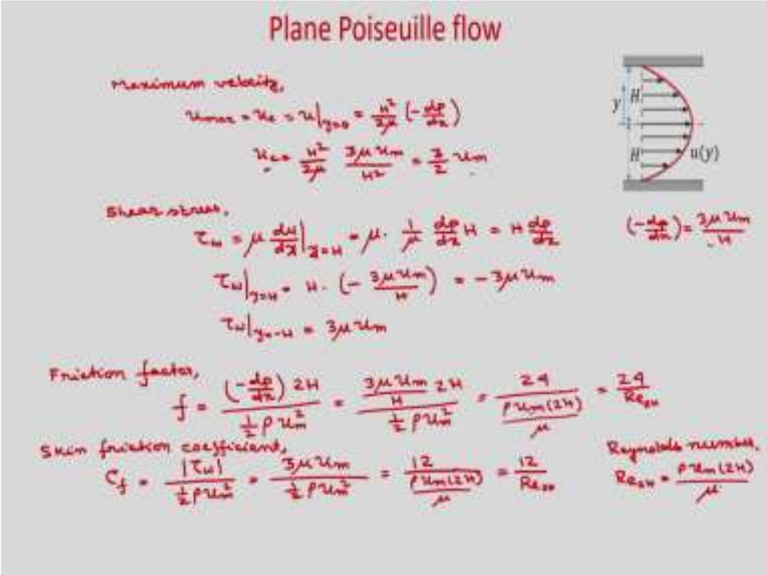

So, you will get maximum velocity. So, maximum velocity will occur at central line, where u at y =0. So, if you put it, you are going to get $u_{max} = u_c = u\big|_{y=0} = \frac{H^2}{2\mu}(-\frac{dP}{dx})$. And that if you write $-\frac{dP}{dx}$ in terms of $\frac{3\mu u_m}{H^2}$, then you are going to get , $u_c = \frac{H^2}{2\mu}\frac{3\mu u_m}{H^2}$.

So, it will be just $\frac{3}{2}u_m$. So, you can see that in case of flow fully developed flow inside two infinite parallel plates, your maximum velocity is 1.5 times the average velocity, because here you can see that u_c, which is your maximum velocity is 1.5 times the mean velocity for fully developed flow. Now, if you find what is the shear stress acting on the wall, then you can write shear stress, so that you can write $\tau_w = \mu\frac{du}{dy}$.

So, at any wall you can calculate. So, y = H let us calculate. So, you can write $\tau_w = \mu\frac{du}{dy}\big|_{y=H}$. So, what is $\frac{du}{dy}$? So, we can see this easily you will get $\tau_w = \mu\frac{1}{\mu}\frac{dP}{dx}H = H\frac{dP}{dx}$, and $(-\frac{dP}{dx}) = \frac{3\mu u_m}{H}$. So, if you put it, then, $\tau_w\big|_{y=H} = H(-\frac{3\mu u_m}{H}) = -3\mu u_m$.

So, similarly if you calculate, $\tau_w \big|_{y=-H} = 3\mu u_m$. And you can see that your shear stress inside the fluid, it will vary linearly, now we will define the friction factor. So, this is the non-dimensional representation of the pressure gradient. So, if you calculate non-dimensional pressure population, then that is known as the friction factor.

So, friction factor you can calculate, so this is $f = \dfrac{(-\dfrac{dP}{dx})2H}{\dfrac{1}{2}\rho u_m^2}$. So, this if you find, so what you are going to get? So, $-\dfrac{dP}{dx}$ is this one. So, you can write $f = \dfrac{\dfrac{3\mu u_m}{H}2H}{\dfrac{1}{2}\rho u_m^2}$. So, if you rearrange you will get this as, $\dfrac{24}{\dfrac{\rho u_m(2H)}{\mu}}$.

So, you can see this is your Reynolds number based on the mean velocity and the channel height, then you can write $\dfrac{24}{\text{Re}_{2H}}$. So, friction factor we have represented in the terms of the non-dimensional number Reynolds number based on the channel height. So, friction factor $f = \dfrac{24}{\text{Re}_{2H}}$.

If you calculate the skin friction coefficient, so this is dimensionless representation of the wall shear stress, so skin friction coefficient. So, it is represented as C_f . So, this is $C_f = \dfrac{|\tau_w|}{\dfrac{1}{2}\rho u_m^2} = \dfrac{3\mu u_m}{\dfrac{1}{2}\rho u_m^2}$.

So, you can rearrange it, and you will get $\dfrac{12}{\dfrac{\rho u_m(2H)}{\mu}}$, and you can write $\dfrac{12}{\text{Re}_{2H}}$. So, for the fully developed flow inside two parallel plates, we have calculated the shear stress on the wall.

And, hence we have calculated the skin friction factor which is your dimensionless representation of the wall shear stress as $\frac{12}{\text{Re}_{2H}}$ where Re_{2H} is the Reynolds number based on the channel height.

So, this is your Reynolds number. So, this is your based on 2H. So, Reynolds number we have defined based on mean velocity u_m, and the channel height 2H.

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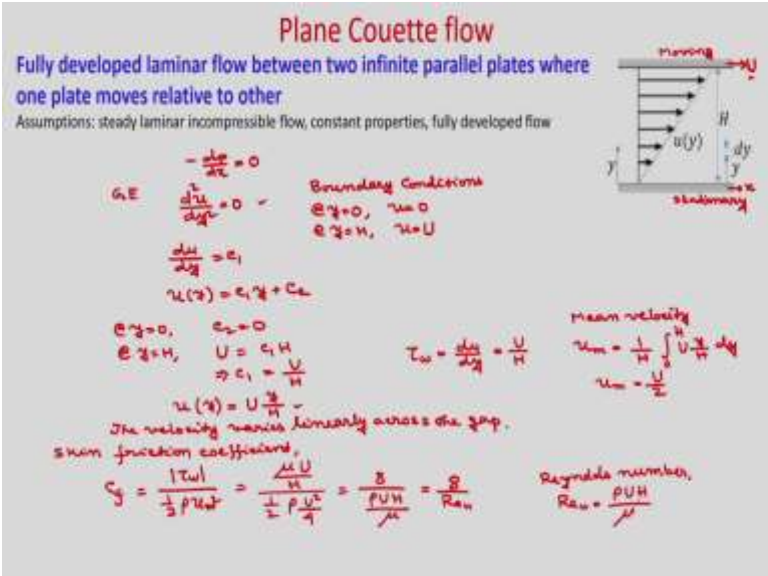

Next we will consider plane Couette flow. This is the flow between two infinite parallel plates where one plate is moving with respect to the other. So, you can see that these are the two fixed plates, and this upper plate is moving with velocity u in the axial direction ok. So, some constant velocity u it is moving with respect to the bottom wall.

So, this is your x-direction, and y is measured from the bottom wall, and bottom wall is stationary. So, bottom wall is stationary and upper wall is moving with constant velocity u, and y is measured from the bottom wall and the distance between two parallel plates is H.

So, in this scenario now you can see that the flow is taking place. So, here you can see that the flow is taking place due to the shear. So, the upper plate is moving. So, due to the shear there will be velocity inside the channel. So, this is known shear driven flow where your pressure gradient is absent in this particular case, so that is why we have told at as plane Couette flow. So, where you have the external pressure gradient as well along with the shear driven flow, then it will be Couette flow.

But here we are considering plane Couette flow where we are assuming that pressure gradient is 0. So, in this particular case, now we are considering $-\frac{dP}{dx}=0$ and fully developed condition. So, obviously, your governing equation whatever we have derived in earlier case, you can write $\frac{d^2u}{dy^2}=0$, because we have considered fully developed flow. So, your y direction velocity v =0 as well as your $\frac{\partial u}{\partial x}=0$, because there will be no change in the axial velocity in the axial direction. So, your governing equation is $\frac{\partial^2 u}{\partial y^2}=0$.

And what are the boundary conditions? Boundary conditions are at y = 0 bottom wall, u=0 and upper wall at y =H, it is moving with velocity U. So, you integrate this equation. So, you will get $\frac{du}{dy}=C_1$, and $u(y)=C_1 y+C_2$. So, C_1 and C_2 are constants. So, you can see that you will get a linear velocity profile.

So, now, invoke the boundary conditions at y = 0, u = 0, so u = 0, so we will get C_2 = 0; and at y = H, u = 0, so you will U = u. So, you will get $U=C_1 H$; C_2 = 0, so obviously $C_1=\frac{U}{H}$. So, hence you will get the velocity profile $u(y)=U\frac{y}{H}$. And you can see that y = 0, it is 0 velocity at y = H it is U, and it is linearly varying, it is linearly varying because from the velocity profile you can see that it is a velocity varies linearly across the gap.

And if you calculate the shear stress on wall, so you can see that $\frac{du}{dy}$ will be a constant, because here you can see $\tau_w=\frac{du}{dy}$ and $\frac{du}{dy}$ is constant C_1, so that is $\frac{U}{H}$. So, in both the plates, you can find that your wall shear stress $\tau_w=\frac{U}{H}$. So, along inside the flow also the shear stress will be constant. So, now, if you calculate the skin friction coefficient on what will be your mean velocity?

So, if you calculate the mean velocity, mean velocity, so it is $u_m = \frac{1}{H}\int_0^H U\frac{y}{H}dy$. And what is u? $u(y) = U\frac{y}{H}$ dy, so it will be $\frac{y^2}{2}$, hence $u_m = \frac{U}{2}$.

If you calculate the skin friction coefficient, so you will find for this particular case $C_f = \frac{|\tau_w|}{\frac{1}{2}\rho u_m^2}$. So, $\frac{\frac{\mu U}{H}}{\frac{1}{2}\rho\frac{u^2}{4}}$. So, if you rearrange, you will get $\frac{8}{\frac{\rho UH}{\mu}}$, so you will get $\frac{8}{\mathrm{Re}_H}$. So, we are writing this Reynolds number based on your; based on your upper velocity, upper plate velocity u, so that will be $\mathrm{Re}_H = \frac{\rho UH}{\mu}$.

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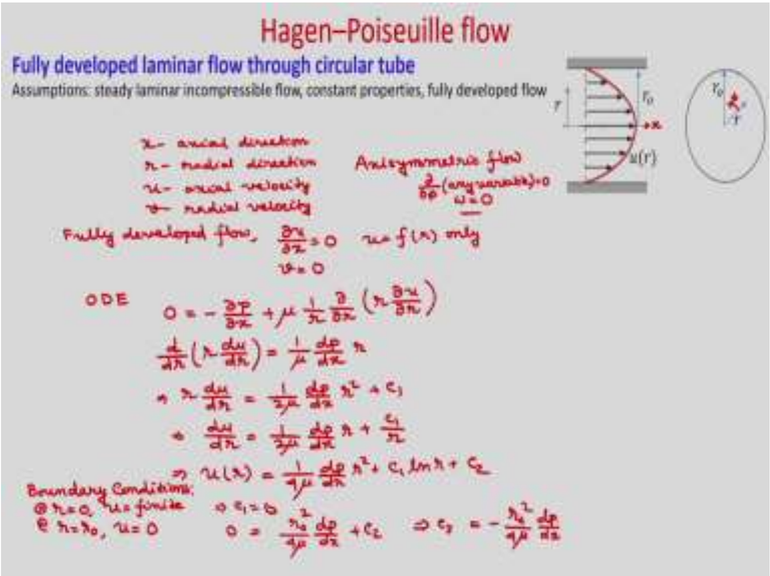

So, now, we will consider fully developed flow inside pipe. So, this flow is known as Hagen-Poiseuille flow. So, you can see this is your axial direction x, and radial direction is r.

And we are considering axisymmetric flow that means that in circumferential direction in phi direction there will be no variation of any variable, so that is your axisymmetric. So, in case of axisymmetric your u will be function of r only; and as it is fully developed condition then the axial velocity u will be constant or u is function of r only; it is not function of x. So, $\frac{\partial u}{\partial x} = 0$.

So, if you consider that. So, you can see that it is a pipe, and these are your pipe wall, and the pipe radius is r_0. And this is your central line, and r is measured from the center. So, if you write the fully developed condition, so in this particular case x is your axial direction ok, r is your radial direction, u is your axial velocity, and v is your radial velocity ok.

So, as it is a fully developed flow, so $\frac{\partial u}{\partial x} = 0$, because your in axial direction there will be no change in the velocity profile. So, $\frac{\partial u}{\partial x} = 0$. So, u is function of r only. And radial velocity everywhere v = 0. And one most important assumptions that we have taken it is axisymmetric flow.

So, if you considered that your circumferential direction if you miss that, so let us say this is your Φ or θ let us say Φ, then $\frac{\partial}{\partial \phi}(any\,\mathrm{var}\,iable) = 0$, and the velocity in Φdirection also 0. So, if you say that velocity w which is in Φ direction, so it will be this it will be 0 in axisymmetric flow.

So, if you consider the momentum equation, then you can see that you are invoking all these assumptions, you will get this ordinary differential equation which is your governing equation. You will get $0 = -\frac{\partial P}{\partial x} + \mu \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial u}{\partial r})$.

Here also you will find that p is function of x only, and you can write $\frac{\partial P}{\partial x} = \frac{dP}{dx}$, and u is function of r only. So, you can write $\frac{d}{dr}(r\frac{du}{dr}) = \frac{1}{\mu}\frac{dP}{dx}r$. So, now, if you integrate it, so you will get $r\frac{du}{dr}=\frac{1}{2\mu}\frac{dP}{dx}r^2+C_1$. If you divide by r, then $\frac{du}{dr}=\frac{1}{2\mu}\frac{dP}{dx}r+\frac{C_1}{r}$. And again if you integrate, then you will get the velocity profile as $u(r)=\frac{1}{4\mu}\frac{dP}{dx}r^2+C_1\ln r+C_2$.

So, now you invoke the boundary condition, what are the boundary conditions? At r = 0, you have u finite; and at r = r_0 which is your wall you have velocity 0, u = 0. So, if you write boundary conditions, boundary conditions, so at r = 0, u is finite;. So, you can see that if u is finite, then C_1 must be 0. So, C_1 is 0. And r = r_0 ,u = 0 because that is your wall, so it will be r_0.

So, you will get $0=\frac{r_0^2}{4\mu}\frac{dP}{dx}+C_2$. So, $C_2=-\frac{r_0^2}{4\mu}\frac{dP}{dx}$.

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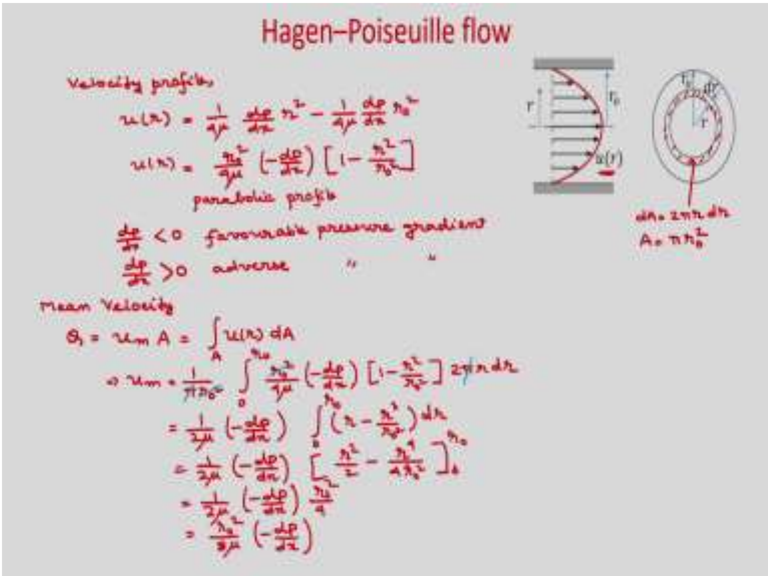

Hence you can write the velocity profile as so velocity profile you can write now, u is function of r only. And if you put the constant, so you can see that $C_2=-\frac{r_0^2}{4\mu}\frac{dP}{dx}$. So, you can write as $u(r)=\frac{1}{4\mu}\frac{dP}{dx}r^2-\frac{1}{4\mu}\frac{dP}{dx}r_0^2$.

So, hence you will get $u(r)=\frac{r_0^2}{4\mu}(-\frac{dP}{dx})[1-\frac{r^2}{r_0^2}]$. So, you can see that this is also your parabolic profile.

So, you can see you will get a parabolic velocity profile, where r = 0 you will get the maximum velocity; and at walls obviously it is 0. So, r = 0, r = r_0, it is 0. So, you will get a velocity profile which is u is function of r, so and you will get a parabolic profile.

And we are writing u as $-\frac{dP}{dx}$ in term, we are writing u(r) in terms of $-\frac{dP}{dx}$ because $-\frac{dP}{dx}$ is positive quantity because in axial direction you have $-\frac{dP}{dx}$ is positive, because your pressure gradient is decreasing in the axial direction. So, generally if $\frac{dP}{dx}$< 0, so this is known as favourable pressure gradient; and if $\frac{dP}{dx}$>0, then it is known as adverse pressure gradient, because you have pressure gradient as positive and your flow reversal may take place.

So, generally $\frac{dP}{dx}$<0, then it is known as favourable pressure gradient. And if $\frac{dP}{dx}$>0, then it is a adverse pressure gradient. Now, you want to calculate the mean velocity in Hagen-Poiseuille flow. So, you can calculate the mean velocity as, so $Q=u_m A=\int_A u(r)dA$. So, in this particular case, you can see that you have a circular cross section.

So, at a distance r you take one elemental flow area. So, this is your elemental flow area of distance dr. So, what will be your dA in this particular case? So, this is the elemental flow area. So, $dA=2\pi r dr$. So, this is the area. And total area is $A=\pi r_0^2$.

So, if you put it here, so you will get as $u_m=\frac{1}{\pi r_0^2}\int_0^{r_0}\frac{r_0^2}{4\mu}(-\frac{dP}{dx})[1-\frac{r^2}{r_0^2}]2\pi r dr$.

So, this π, π you cancel. And you can write it as, so these other r_0^2 also you can cancel because this is constant. So, you can write as $\mathrm{u}_m = \frac{1}{2\mu}(-\frac{dP}{dx})[r - \frac{r^3}{r_0^2}]\mathrm{dr}$.

So, if you integrate it, so you will get $\mathrm{u}_m = \frac{1}{2\mu}(-\frac{dP}{dx})[\frac{r^2}{2} - \frac{r^4}{4r_0^2}]_0^{r_0}$. So, if you put the value, so it will be $\mathrm{u}_m = \frac{1}{2\mu}(-\frac{dP}{dx})\frac{r_0^2}{4}$. So, hence you will get $\mathrm{u}_m = \frac{r_0^2}{8\mu}(-\frac{dP}{dx})$. So, now, you can express $-\frac{dP}{dx}$ in terms of mean velocity.

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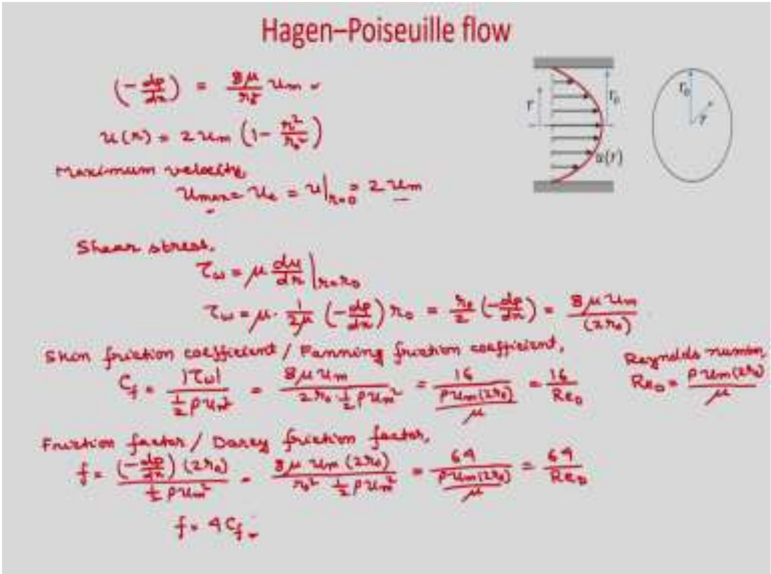

So, $-\frac{dP}{dx}$ you can express in terms of mean velocity, so that will be $-\frac{dP}{dx} = \frac{8\mu}{r_0^2}\mathrm{u}_m$. So, now, you can express this velocity profile in terms of the mean velocity. So, you can see that, $\mathrm{u(r)} = 2\,\mathrm{u}_m(1 - \frac{r^2}{r_0^2})$.

Similarly, now if you find the maximum velocity which will occur at the central line where r = 0, then you can write maximum velocity, $\mathrm{u}_{\max} = u_c = u|_{r=0} = 2u_m$. So, in this particular case, you can see that when you consider the flow inside pipe, then your maximum velocity will be twice of the average velocity or mean velocity.

So, this your u_{max}, you can see it is twice into the mean velocity. And when we consider flow between two parallel plates u_{max} was 1.5 times the mean velocity u_m. So, you should remember here.

So, when you consider pipe flow, your maximum velocity is twice the mean velocity; and when you consider flow between parallel plates, then your maximum velocity will be 1.5 times the average velocity. So, now, we want to calculate the shear stress at the wall, that means, at r = r_0 we want to calculate the shear stress. So, $\tau_w = \mu \frac{du}{dr}|_{r=r_0}$.

So, shear stress you can calculate $\tau_w = \mu \frac{du}{dr}|_{r=r_0}$. So, you can see $\tau_w = \mu \frac{1}{2\mu}(-\frac{dP}{dx})r_0$. So, it will be just $\frac{r_0}{2}(-\frac{dP}{dx})$. And $-\frac{dP}{dx} = \frac{8\mu}{r_0^2}\mathrm{u}_m$. So, you will you will see that if you put it, you will get $\tau_w = \frac{8\mu u_m}{(2r_0)}$.

So, now, if you want to calculate the skin friction coefficient, what is skin friction coefficient? It is the dimensionless shear stress, or sometime it is known as fanning friction coefficient. So, you can write $C_f = \frac{|\tau_w|}{\frac{1}{2}\rho u_m^2}$. So, you can see that $C_f = \frac{8\mu u_m}{2r_0 \frac{1}{2}\rho u_m^2}$.

So, if you rearrange it, you can write it as $\frac{16}{\frac{\rho u_m(2r_0)}{\mu}}$. So, you can write $\frac{16}{\mathrm{Re}_D}$. D is the diameter of the pipe, so that is $2r_0$. So, a Reynolds number we have defined here, Reynolds number we have defined here based on the diameter and mean velocity. So, $\mathrm{Re}_D = \frac{\rho u_m(2r_0)}{\mu}$.

Similarly, if you to want to calculate the friction factor which is your non-dimensional pressure gradient, so that you can write it as friction factor for this particular case it is known as Darcy friction factor also. So, you can see that $f = \frac{(-\frac{dP}{dx})(2r_0)}{\frac{1}{2}\rho u_m^2}$. So, $f = \frac{\frac{8\mu u_m}{r_0^2}(2r_0)}{\frac{1}{2}\rho u_m^2}$.

So, if you rearrange it, so you will get $\frac{64}{\frac{\rho u_m (2r_0)}{\mu}}$. So, you will get $\frac{64}{\mathrm{Re}_D}$. So, your friction factor is $\frac{64}{\mathrm{Re}_D}$. So, if you see that your friction factor is 4 times your skin friction coefficient. Friction factor, so this is your 16×4 and $C_f = \frac{16}{\mathrm{Re}_D}$. So, f=4c$_f$.

So, in today's lecture, we have found the fully developed velocity profile for three different cases. First case, we considered as flow between two infinite parallel plates, where your v velocity is 0, and $\frac{\partial u}{\partial x} = 0$ which is your axial velocity does not change in the x direction. So, with that condition, we found the velocity profile and we have found that it is parabolic in nature. And then we can calculated the mean velocity, and then we calculated the skin friction coefficient as well as the friction factor.

Next we considered the plane-Couette flow where we consider the shear driven flow between two parallel plates where bottom plate is stationary and upper plate is moving with velocity u in the positive x direction. And we have seen that velocity varies linearly from bottom plate to top plate.

Next we considered the Hagen-Poiseuille flow; this is the fully developed flow inside pipe. Here we found the velocity profile u as function of r, and that is also parabolic in nature. In this particular case also, we found the mean velocity, then skin friction coefficient and friction factor. And we have found that friction factor is 4 times the skin friction coefficient.

In Hagen-Poiseuille flow, we have seen that your maximum velocity is 2 times the average velocity; and in case of plane-Poiseuille flow your maximum velocity is 1.5

times the average velocity. In fully developed case, we have found the velocity profile in different channels, and we have found the mean velocity. These we will use in the next module when we will find that temperature distribution inside this channel flow.

Thank you.