Fundamentals of Convective Heat Transfer

Prof. Amaresh Dalal
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Module - 05
Convection in Internal Flows - I
Lecture - 16
Determination of heat transfer coefficient

Hello everyone. So, in last lecture we have calculated the mean temperature for two different types of boundary conditions; uniform wall heat flux and uniform wall temperature. Today, we will discuss about the mean temperature first; then we will discuss about the dimensionless temperature in thermally fully developed region, then we will determine the heat transfer coefficient and Nusselt number.

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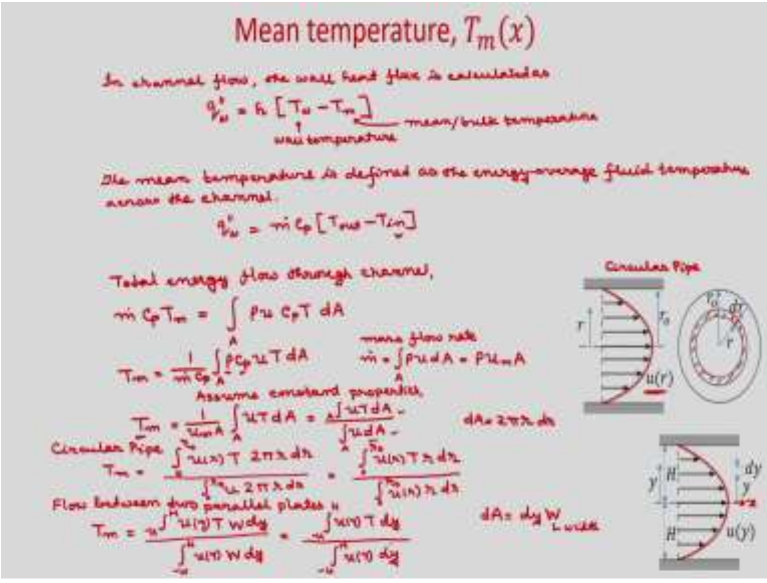

In external flow, we have seen that you can calculate the wall heat flux as $q_w^" = h[T_w - T_m]$ which is your Newton's law of cooling. Here, T_w is the wall temperature and T_∞ is the free stream temperature. But, internal flows you can see that, there is no such free stream temperature.

Hence, we need to find some constant temperature at different axial location. And, we will define the mean temperature or bulk temperature. So, you can see that in channel flow, the wall heat flux is calculated as $q_w^" = h[T_w - T_m]$. So, here you can see T_w is the wall temperature and T_m is the mean and sometime it is known as bulk temperature. And, depending on thermal boundary condition, T_w maybe constant or T_w may be function of x and T_m is always function of x.

Now, let us define the mean temperature. The mean temperature is defined as the energy average fluid temperature across the channel. You can see that, we can also calculate the wall heat flux as, $q_w^{"} = \dot{m} C_p [T_{out} - T_{in}]$. So, you can see that there will be variation of temperature at different axial location. So, at outlet and inlet you can have radial variation of temperature; so obviously, this will not be useful if you do not know a constant temperature at the outlet and inlet. So, now, this T_m will give you a proper area weighted average or a mean temperature at any location.

So, you can write that, total energy flow through channel. So, that is you can write $\dot{m} C_p T_m = \int \rho u C_p T dA$. So, if you see that, $T_m = \frac{1}{\dot{m} C_p} \int \rho u C_p T dA$.

Now, what is $\dot{m}$? $\dot{m}$ is your mass flow rate , mass flow rate. So, $\dot{m} = \int \rho u dA$; if you define a mean velocity at that cross section, then $\dot{m} = \rho u_m A$. So, flow cross sectional area, not the heat transfer area; it is a flow cross sectional area.

So, if you put it here and if you assume constant properties, assume constant properties, then this ρC_p you can take it outside and this $\dot{m}$ this ρ you can take it outside. So, you can write $T_m = \frac{1}{u_m A} \int uT dA$; because, ρ C_p you can cancel or you can write $\frac{\int_A uT dA}{\int_A u dA}$.

So, for any channel of different cross section maybe circular cross section or square cross section or flow between two parallel plates; you can calculate the mean temperature using this formula, where you can write $T_m = \frac{\int_A uT dA}{\int_A u dA}$. And, this is your bulk mean temperature T_m.

So, now if you consider a circular pipe; so this is your circular pipe. So, if you consider a fully developed region, then that will be your velocity distribution u r,. So, it is hydrodynamically fully developed ; hydrodynamically fully developed flow; then, your velocity will be only function of r, so u is functional of r.

And, if you see the flow cross sectional area, so that is circular, because it is a circular pipe. And, if you consider a small elemental area at a radial distance r of distance dr. So, this is the elemental flow area you are considering. So, this is your radial distance r and this thickness is dr; then, you can write for circular pipe.

For a fully developed profile u(r) you can write T_m as; so, $T_m = \frac{\int_0^{r_0} u(r) T 2\pi r dr}{\int_0^{r_0} u 2\pi r dr}$.

So, that is the elemental flow cross sectional area. So, this 2π you can cancel out in the denominator and numerator. So, you can write $\frac{\int_0^{r_0} u(r) T r dr}{\int_0^{r_0} u(r) r dr}$.

Similarly, if you consider flow between two parallel plates. So, in this particular case let us say two infinite parallel plates. So, you can have the central line as shown here. So, this is your axial direction x and this is your y. And, the parallel plates are separated by a distance 2 H. So, when you calculate the flow cross sectional area, so you can see that at a distance y, you take a small elemental area that is of distance dy.

So, in this particular case if you consider flow between two parallel plates; two parallel plates , then what will be your dA? So, dA in this particular case you can see that it will be dy , dy into third direction whatever width you have; so, into w we can write or per unit width also you can calculate. So, dA = dyW, where W is the width, so perpendicular to this board.

So, if you calculate the mean temperature in this particular case, then it will be you can see that your y is varying -H to H. So, u which is function of y, then your cross sectional area is dA = dyW; so, $T_m = \frac{\int_{-H}^{H} u(\text{y})TWdy}{\int_{-H}^{H} u(\text{y})Wdy}$. So, w is constant.

So, you can write now $T_m = \frac{\int_{-H}^{H} u(\text{y})Tdy}{\int_{-H}^{H} u(\text{y})dy}$. So, depending on the flow cross sectional area, you can consider the elemental flow area dA. And, accordingly you can calculate the mean temperature or it is also known as bulk temperature.

So, now let us discuss about the dimensionless temperature in thermally fully developed region. So, already we have introduced this dimensionless temperature Φ which is $\phi(r) = \frac{T_w - T}{T_w - T_m}$. So, in this particular case you can see that T_w-T varies in similar way as T_w-T_m; so that, in axial direction there is no variation of this dimensionless temperature phi in fully developed region. So, Φ is function of r only.

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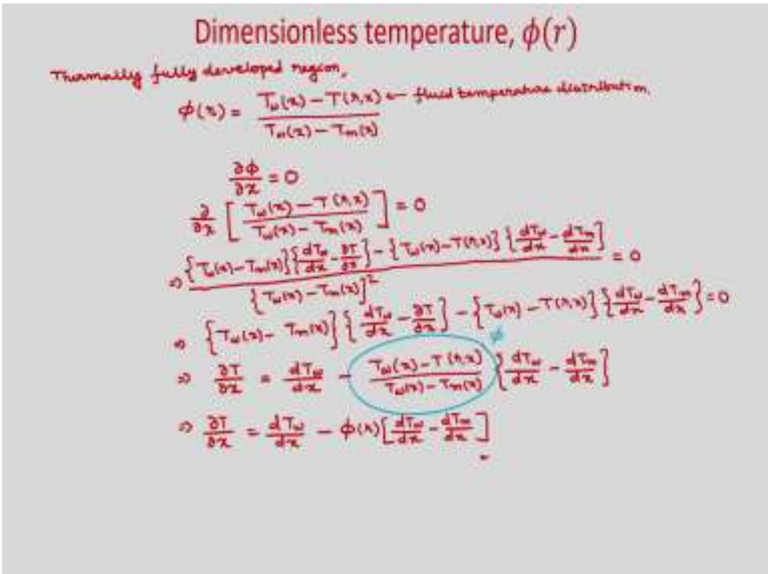

So, thermally fully developed region, we can introduce this dimensionless temperature $\phi(r) = \frac{T_w(x) - T(r,x)}{T_w(x) - T_m(x)}$. So, you know that T_w is the wall temperature, T_m is the mean temperature and T(r,x) is the fluid temperature distribution. So, this is your fluid temperature distribution. So, this dimensionless temperature expression is valid for both thermal condition constant wall temperature and constant wall heat flux.

So, now, for thermally fully developed condition $\frac{\partial \phi}{\partial x} = 0$; because, Φ is function of r only, so $\frac{\partial \phi}{\partial x} = 0$. So, if it is so, so you can see that $\frac{\partial}{\partial x}[\frac{T_w(x) - T(r,x)}{T_w(x) - T_m(x)}] = 0$. So, if you take the derivative; so, you can write,

$$\frac{\{T_w(x) - T_m(x)\}\{\frac{dT_w}{dx} - \frac{\partial T}{\partial x}\} - \{T_w(x) - T(r,x)\}\{\frac{dT_w}{dx} - \frac{dT_m}{dx}\}}{[T_w(x) - T_m(x)]^2} = 0.$$

$$\{T_w(x) - T_m(x)\}\{\frac{dT_w}{dx} - \frac{\partial T}{\partial x}\} - \{T_w(x) - T(r,x)\}\{\frac{dT_w}{dx} - \frac{dT_m}{dx}\} = 0$$

So, if you rearrange it, you can write $\frac{\partial T}{\partial x} = \frac{dT_w}{dx} - \frac{T_w(x) - T(r,x)}{T_w(x) - T_m(x)}\{\frac{dT_w}{dx} - \frac{dT_m}{dx}\}$. So, you can see that, this quantity is nothing but, Φ. So, you can write it now $\frac{\partial T}{\partial x} = \frac{dT_w}{dx} - \phi(r)[\frac{dT_w}{dx} - \frac{dT_m}{dx}]$.

So, this result will be used in analyzing thermally developed flow in channels. So, later we will use this relation, but now let us consider two different boundary conditions. So, now, let us consider two different boundary condition and see the simplification in finding $\frac{\partial T}{\partial x}$.

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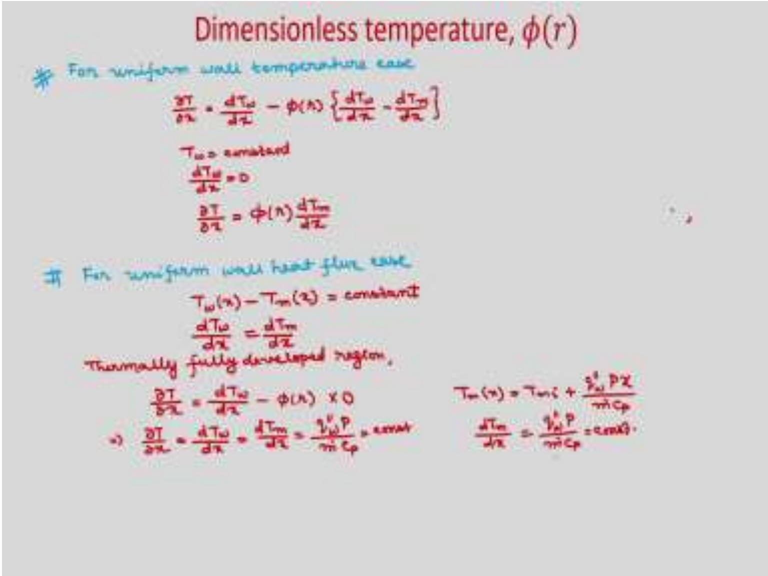

So, for uniform wall temperature case. So, if you consider this, then what you can write? So, our expression is $\frac{\partial T}{\partial x} = \frac{dT_w}{dx} - \phi(r)\{\frac{dT_w}{dx} - \frac{dT_m}{dx}\}$. So, in this particular boundary condition T_w = constant. So, $\frac{dT_w}{dx} = 0$. So, you put it in this expression. So, T_w = constant ; hence, $\frac{dT_w}{dx} = 0$. So, you can write $\frac{\partial T}{\partial x} = \phi(r)\frac{dT_m}{dx}$.

So, the axial variation of this temperature profile T, $\frac{\partial T}{\partial x}$ you can express in terms of $\phi(r)\frac{dT_m}{dx}$. Now, if you consider for uniform wall heat flux case, then these expression you can write as; so, for uniform wall heat flux case. So, for uniform wall heat flux case we know that, $T_w(x) - T_m(x)$ = constant. So, that we have shown.

So, this is your constant. So, if it is constant, then you can write $\frac{dT_w}{dx} = \frac{dT_m}{dx}$; because this is constant, so it will be 0. So, $\frac{dT_w}{dx} = \frac{dT_m}{dx}$. So, thermally fully developed region, you can write $\frac{\partial T}{\partial x} = \frac{dT_w}{dx} - \phi(r) X 0$, because you can see $\frac{dT_w}{dx} = \frac{dT_m}{dx}$. So, it will be 0.

So that means, you have $\frac{\partial T}{\partial x}=\frac{dT_w}{dx}=\frac{dT_m}{dx}$. And, already we know that you have $T_m(x)=T_{mi}+\frac{q_w^{''}Px}{\dot{m}C_p}$. So, $\frac{dT_m}{dx}=\frac{q_w^{''}P}{\dot{m}C_p}$. So, now, you can see this quantity is constant, because $\frac{q_w^{''}P}{\dot{m}C_p}$ all are constant. So, $\frac{dT_m}{dx}$ is constant. So, this you can write $\frac{q_w^{''}P}{\dot{m}C_p}$=constant. So, these are the simplification to find the $\frac{\partial T}{\partial x}$ for a thermally fully developed region for both the thermal conditions. So, now, to calculate the heat transfer coefficient, first we will use the scale analysis.

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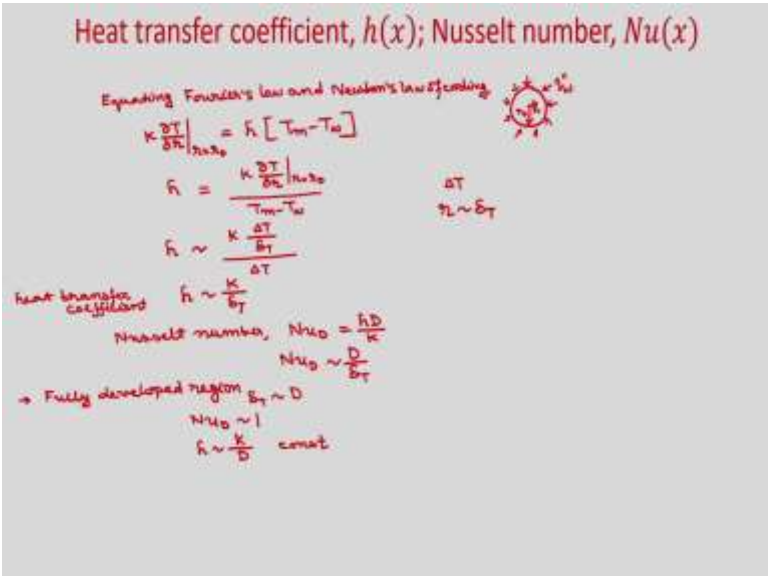

So, you can see that, if you have let us say a circular pipe of radius r_0 and this is your radial direction. And, if you take the q_w in inward direction. So, $q_w^{''}$; then, you can define whatever heat is conducted that will be conducted. So, from is equating the Fourier's and Newton's law, you can write equating Fourier's law and Newton's law of cooling. What you can write? You can write; so, $K\frac{\partial T}{\partial r}|_{r=r_0}$.

So, at r $=r_0$. So, it is plus, it is not minus, because you are taking in a negative r direction, because this $K\frac{\partial T}{\partial r}$ in general for Fourier's law heat conduction we write minus; but in this particular case you can see that radial direction is in this outward direction, but $q_w^{"}$ we are considering in the inward direction. So, it will be plus, $K\frac{\partial T}{\partial r}|_{r=r_0} = h[T_m - T_w]$.

So, you can see that $h = \frac{K\frac{\partial T}{\partial r}|_{r=r_0}}{T_m - T_w}$. So, as earlier we will use the scale analysis. So, the temperature difference we will take the scale of ΔT. So, we will take ΔT and the radius we will take order of the thermal boundary layer thickness δ_T. So, $h \sim \frac{K\frac{\Delta T}{\delta_T}}{\Delta T}$.

So, you can see that $h \sim \frac{K}{\delta_T}$. So, now, if you see the Nusselt number based on the diameter, then you can calculate Nusselt number. So, this is your heat transfer coefficient and Nusselt number now you can calculate as; so, $Nu_D = \frac{hD}{K}$. So, you can see $Nu_D \sim \frac{h}{K}$. So, it will be $Nu_D \sim \frac{D}{\delta_T}$.

So, now, if you consider entrance region and the fully developed region, then what will be your Nusselt number? So, you can see for fully developed region , we are talking about thermal fully developed region ; thermal fully developed region. So, in this case your δ_T will be order of diameter, because in a fully developed region your $\delta_T \sim D$. So, your $Nu_D \sim 1$. So, you can see in the fully developed region, Nusselt number is constant right. And, heat transfer coefficient you can see it is $h \sim \frac{K}{\delta_T}$.

So, in fully developed region, what will be your h? $h \sim \frac{K}{D}$; because your thermal boundary layer thickness merges at the center line. So, your thermal boundary layer thickness will be just δ_T as D; actual case the thermal boundary layer thickness will be order of r naught which is your radius of the pipe; but, as we are using the scale analysis,

$\delta_T \sim D$. So, you can write $h \sim \dfrac{K}{D}$. So, you see k is the for a constant properties k is constant, D is the diameter is constant. So, it will be also constant in a fully developed region.

So, using scale analysis we are showing that heat transfer coefficient and Nusselt number are constant in a thermally fully developed region. And, later when you will actually calculate the value of Nusselt number, it will be constant. Now, if you consider entrance region. So, in the entrance region your thermal boundary layer thickness will start growing. So, you can take $\dfrac{\delta_T}{x} \sim \dfrac{1}{\sqrt{\text{Re}_x \text{Pr}}}$.

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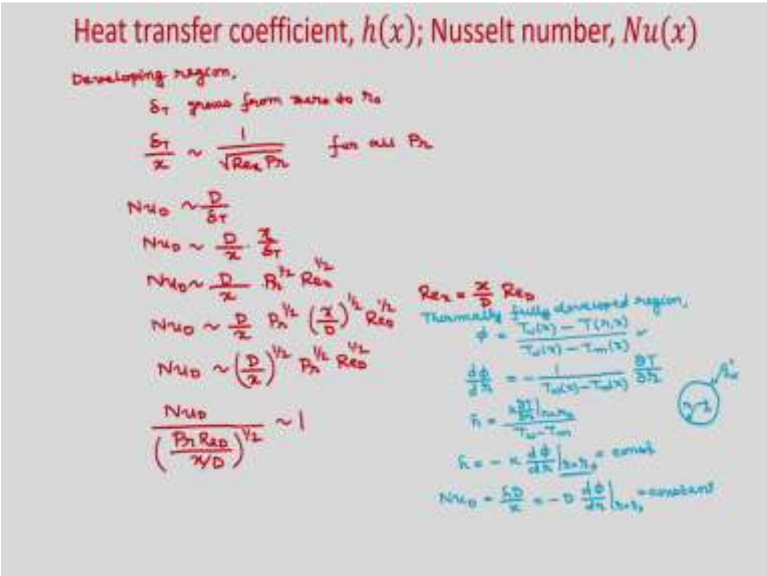

So, for developing region or entrance region; developing region or entrance region; so, what you can write? δ_T grows from 0 to r_0, right. So, your $\dfrac{\delta_T}{x} \sim \dfrac{1}{\sqrt{\text{Re}_x \text{Pr}}}$ that we have shown in external flows right, $\dfrac{\delta_T}{x} \sim \dfrac{1}{\sqrt{\text{Re}_x \text{Pr}}}$ for all Prandtl number range.

So, now, $Nu_D \sim \dfrac{D}{\delta_T}$. So, you can write $Nu_D \sim \dfrac{D}{x}\dfrac{x}{\delta_T}$. So, Nusselt number D will be; now, $Nu_D \sim \dfrac{D}{x} \text{Pr}^{1/2} \text{Re}_x^{1/2}$.

So, now this convert this Reynolds number based on x to Reynolds number based on diameter. So, $\mathrm{Re}_x = \frac{x}{D}\mathrm{Re}_D$,right. So, $Nu_D \sim \frac{D}{x}\mathrm{Pr}^{1/2}(\frac{x}{D})^{1/2}\mathrm{Re}_D^{1/2}$. So, $Nu_D \sim (\frac{D}{x})^{1/2}\mathrm{Pr}^{1/2}\mathrm{Re}_D^{1/2}$.

So, in $\frac{Nu_D}{(\frac{\mathrm{Pr}\,\mathrm{Re}_D}{x/D})^{1/2}} \sim 1$. So, you can see that your Nusselt number will be will depend on Prandtl number and Reynolds number and it is in developing region, the Nusselt number will be $\mathrm{Pr}^{1/2}\mathrm{Re}_D^{1/2}$ into some constant.

So, later we will consider one case, where we will consider a fully developed, hydrodynamically fully developed flow and thermally developing flow where we will calculate the Nusselt number. But, in the other two cases, we will consider both hydrodynamically and thermally fully developed region. So, it is easy to calculate the Nusselt number and you have we have shown now that Nusselt number will be constant value for both the thermal boundary conditions.

Now, if you write this heat transfer coefficient in terms of dimensionless temperature; then, $\phi = \frac{T_w(x) - T(r,x)}{T_w(x) - T_m(x)}$, where T_m is the mean temperature. So, if you take the derivative with respect to r. So, what we will get? $\frac{d\phi}{dr} = -\frac{1}{T_w(x) - T_m(x)}\frac{\partial T}{\partial r}$. So, now, let us calculate the heat transfer coefficient h.

So, you can see that h equating the Fourier's law and the Newton's law, you can write h equal to; so, equating the Fourier's law and the Newton's law of cooling, you can write $\mathrm{h} = \frac{K\frac{\partial T}{\partial r}|_{r=r_0}}{T_w - T_m}$. So, we are considering this as r, tube radius is r_0 and q_w we are considering this. So, now, you can see. So, $\frac{\partial T}{\partial r}$ if you put it here; so, what you will get? So, h is equal to; so, $h = -K\frac{d\phi}{dr}|_{r=r_0}$.

So, and you can see here k is thermal conductivity, right. So, for constant properties r is constant and $\frac{d\phi}{dr}|_{r=r_0}$, at r = r_0; so obviously, this will be also constant. So, for a fully developed region, so you can write that, this is equal to constant.

So, from the scale analysis we have already shown that, your h and Nusselt number will be constant for a thermally fully developed region. And, in this case also you can see k is the thermal conductivity that is constant and $\frac{d\phi}{dr}$, because Φ is function of r only, right. So, $\frac{d\phi}{dr}$ you are calculating at r = r_0; that means, at the tube surface.

So, obviously $\frac{d\phi}{dr}$ will also constant, so h will be constant. And, similarly Nusselt number you can write $Nu_D = \frac{hD}{K}$. So, from here you can see it will be $-D\frac{d\phi}{dr}|_{r=r_0}$. So, you can see here also this is constant. So, for thermally fully developed region you can calculate the $Nu_D = -D\frac{d\phi}{dr}|_{r=r_0}$, where Φ is the dimensionless temperature defined as this and it is true for both the thermal boundary conditions.

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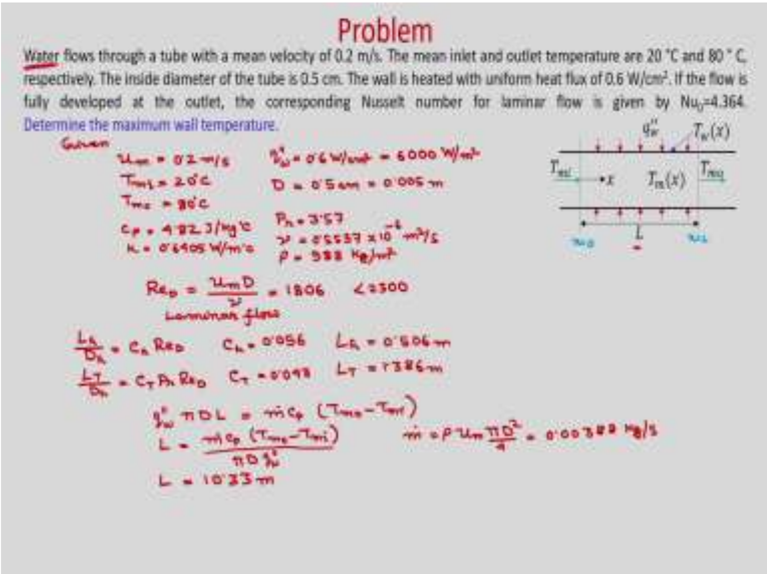

So, now let us solve one example problem; Water flows through a tube with a mean velocity of 0.2 m/s. The mean inlet and outlet temperature are 20^0C and 80^0C respectively. The inside diameter of the tube is 0.5 cm. The wall is heated with uniform heat flux of 0.6 W/cm^2. If the flow is fully developed at the outlet, the corresponding Nusselt number for laminar flow is given by Nu_D = 4.364. Determine the maximum wall temperature.

So, you can see that, first we have to consider whether this is laminar flow or turbulent flow. And, whether it is developing region or fully developed region. If it is fully developed region, then the Nusselt number is given for this particular constant wall heat flux condition. So, you can see schematically. So, this is your q_w right, constant wall heat flux is given and at x =0 you have T_{mi} and at x = L you have T_{mo}. And, in this case T_w is function of x and T_m is also function of x. And, you can see that your mean velocity is given as 0.2 m/s.

Your inlet mean temperature is given as 20^0C and outlet mean temperature is given as 80^0C, $q_w^{"}$ =0.6 W/cm^2. So, if you convert it to W/m^2. So, it will be 6000 W/m^2. And, diameter of the pipe is 0.5 cm, so it will be 0.005 m. And, the properties are also provided; so, at mean temperature. So, the properties are given as C_p = 4.82 J/kgK or J/kg^0C, thermal conductivity 0.6405 W/m^0C, Prandtl number is given as 3.57.

Kinematic viscosity of the fluid is given as 0.5537 $X10^{-6}$ m^2/s, and density is given as 988 kg/m^2. So, the fluid is a water, because it is already given water. So, now, you calculate the Reynolds number. So, Reynolds number you can calculate as $\mathrm{Re}_D = \frac{u_m D}{\nu}$. So, if you put all these values, you will get 1806. So, you can see that the Reynolds number < 2300, obviously this is the laminar flow. Now, you see, whether it is developing or fully developed region. So, we know that $\frac{L_h}{D_h} = C_h \mathrm{Re}_D$.

And, from here you can see that C_h for the circular pipe is given as 0.056 from the table we have already shown. So, L_h will be 0.506 m. And, $\frac{L_T}{D_h} = C_T \mathrm{Pr}\,\mathrm{Re}_D$. And, for this circular pipe your C_T =0.043, for the constant wall heat flux boundary condition; so, L_T will be 1.386 meter. So, now, let us calculate the tube length, because this is unknown, right.

So, these L we have to calculate from this equation. So, we can see that $q_w^{"}$ into the heat transfer area. So, that is your, $q_w^{"}\pi DL = \dot{m}C_p(T_{mo} - T_{mi})$. So, your $L = \frac{\dot{m}C_p(T_{mo} - T_{mi})}{\pi D q_w^{"}}$.

So, you can see that $\dot{m} = \rho u_m \frac{\pi D^2}{4}$, and all the properties are given and q_w also is given. So, T_{mo} and T_{mi} are given. So, you just calculate the length of the pipe. And, this length of pipe is you will get as; $\dot{m}$ if you calculate, it will be 0.00388 kg /s. So, L will be if you put it 10.33 m.

So, now, the tube length is 10.33 m and your entrance length, you can see L_h and L_T it is just 0.506 m hydrodynamic entrance length, and thermal entrance length is 1.386 m. So, it is very very small compared to the length of the pipe. And, you are asked to find the maximum wall temperature at the maximum wall temperature, and maximum wall temperature will occur at the outlet for this particular case.

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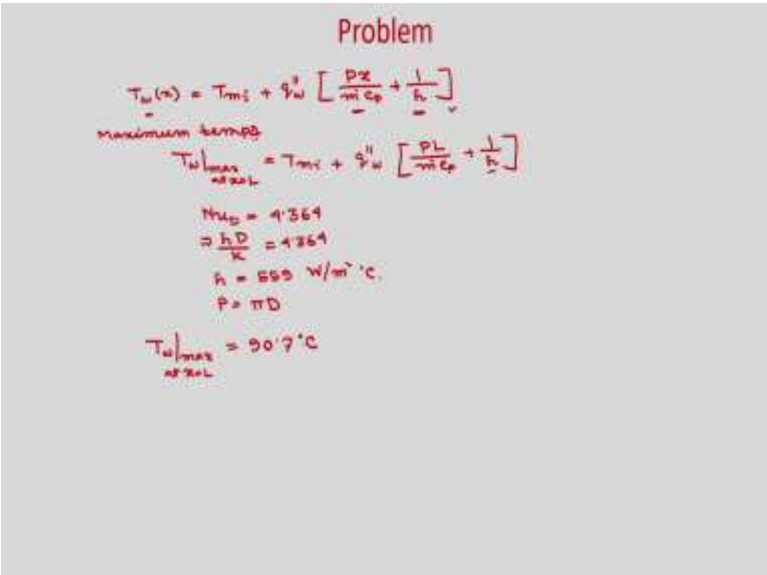

So, you can see your T_w (x) for this particular case you know that it is $T_w(x) = T_{mi} + q_w^{"}[\frac{Px}{\dot{m}C_p} + \frac{1}{h}]$. So, in this particular case as it is a fully developed flow, so h is constant. So, in this expression you can see that, if this term is constant; so as x increases, your T_w also will increase.

So, maximum temperature will occur, maximum temperature you will get, $T_w \big|_{\max @ x=L} = T_{mi} + q_w^{"}[\frac{PL}{\dot{m}C_p} + \frac{1}{h}]$. So this heat transfer coefficient for fully developed flow,

Nusselt number is given. So, that is your 4.364. So, this is nothing but, $\frac{hD}{K} = 4.364$. So, if you calculate the h you will get 559 W/m^2 ^{0}C.

So, now this T_w maximum you can now calculate h you know, P L $\dot{m}C_p$. What is P? P is nothing but, $P = \pi D$, $\dot{m}$ already you have calculated, $q_w^{"}$ you know, T_{mi} you know; you put all the values this $T_w \big|_{\max @ x=L}$ you will get as 90.7 ^{0}C.

So, in this particular case we have to calculate first, whether it is the flow is laminar or turbulent and where are you are calculating at outlet, whether it is developing region or fully developed region that we have calculated the entrance length for both thermal and hydrodynamic.

So, we have seen that it is very very small compared to the length of the pipes. So, at the outlet it will become anyway fully developed flow and fully developed flow Nusselt number, from Nusselt number you can calculate the heat transfer coefficient. So, in today's lecture, first we have calculated the mean temperature, ok. So, if you know the velocity profile and the temperature profile, then you can calculate the mean temperature at any cross section.

Then, we have discussed about the dimensionless temperature phi in a thermally fully developed region. And, we have simplified the $\frac{\partial T}{\partial x}$ for two thermal boundary conditions. Then, we have calculated the heat transfer coefficient and the Nusselt number using scale analysis. And, we have shown that in fully developed region h is constant and also Nusselt number constant.

However, in developing region as delta increases with x, so your h and Nusselt number both will increase in axial, both h and Nusselt number will vary in axial direction. Then, also we have calculated the heat transfer coefficient and a Nusselt number in terms of the dimensionless temperature getting $\frac{d\phi}{dr}$. And, here also we have shown for a fully developed for thermally fully developed region, the heat transfer coefficient and Nusselt number are constant.

Thank you.