Fundamentals of Convective Heat Transfer

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Module - 05
Convection in Internal Flows - I
Lecture - 02
Energy balance in channel flow

Hello everyone. So, in today's lecture, we will consider different thermal boundary conditions and we will try to find the mean temperature variation as well as for different boundary conditions we will find either the variation of heat flux or variation of temperature on the walls.

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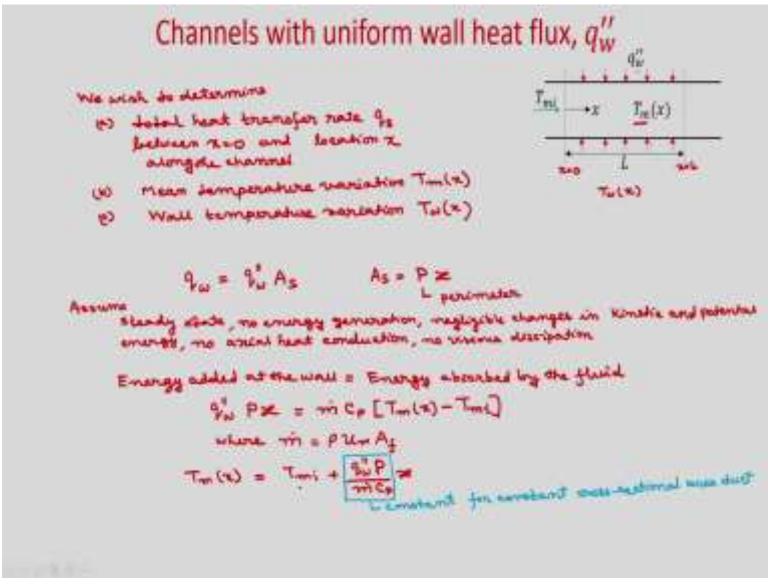

First, we will consider channels with uniform heat flux. So, you can see so, we have a channel where on the channel, we have uniform wall heat flux boundary condition so, q''_w we have given. Now, we consider that a section where x is measured from here where x = 0 and up to x = L.

So, this is the length. So, obviously, you can see at this section, if you considered the temperature mean temperature so, there will be inlet mean temperature will be T_{mi} and at any section x, you can find what is the mean temperature T_m. So, as we considered here uniform wall heat flux, then your temperature at the wall T_w will also vary in axial direction. So, T_w will be function of x.

So, here you can see as heat is added on the surface so, the fluid flowing in the channel, it will take the heat from the wall. So, here we wish to determine; we wish to determine first thing is that total heat transfer rate; total heat transfer rate q_s between x = 0 and location x along the channel.

Then, we want to find the mean temperature variation this mean temperature is also known as bulk mean temperature mean temperature variation so, which will be function of x and also in this case, we will see the wall temperature variation T_w which will be function of x.

In this particular thermal condition as heat flux is uniform, then we can calculate the total heat transfer rate from the wall to the fluid. So, the total heat transfer rate q_w will be just whatever heat flux you have given q_w'' into the surface area heat transfer area q_s. q_w'' So, into heat transfer area A_s. So, what is the A_s in this case? So, if P is the perimeter and x is the length, then your heat transfer area A_s will be P into x where P is the perimeter.

So, now from the energy balance, we will find what is the mean temperature variation. So, we are assuming that it is a steady state assume steady state, no energy generation, negligible changes in kinetic and potential energy, no axial heat conduction; no axial heat conduction and no viscous dissipation.

So, you can see that now with this assumptions, if you do the energy balance whatever heat is transferred from the wall to the fluid actually fluid gain the temperature from inlet while going from inlet to the distance x. So, energy added at the wall will be energy absorbed by the fluid. So, this is just simple energy balance.

So, now, what is energy added at the wall? That is nothing, but $q_w = q_w'' A_s$. So, $A_s = Px$ area is P into x where P is the perimeter and what is the energy absorbed by the fluid? So, at any section x, $q_w'' Px = \dot{m} C_p [T_m(x) - T_{mi}]$ because this is the temperature difference and where $\dot{m} = \rho u_m A_f$ so, flow area so, that will be for pipe it is πr_0^2 so, this will be flow area A_f.

And now, you can write, $T_m(x) = T_{mi} + \frac{q_s^{"}P}{\dot{m}C_p}x$. So, you can see that this is the expression for bulk mean temperature variation along the x where T_{mi} is the mean temperature at the inlet and q_s or you can write q_w here $q_w^{"}$ which is your heat flux constant.

P is the perimeter that is also constant for constant cross sectional channel, $\dot{m}$ is the mass flow rate and C_p is the specific heat and with x obviously, it will vary so, but you can see that $\dot{m}$. So, you can see that $\frac{q_w^{"}P}{\dot{m}C_p}$. So, this is your constant for this particular case for constant cross sectional; constant cross sectional area duct.

So, now, you can see the temperature is varying from inlet T_{mi} to the any distance x T_m. So, the fluid properties you need to determine at the average temperature. If it is T_{mi} and T_m at any location x, then at average temperature you determine the fluid properties while calculating the variation of this $T_m(x)$.

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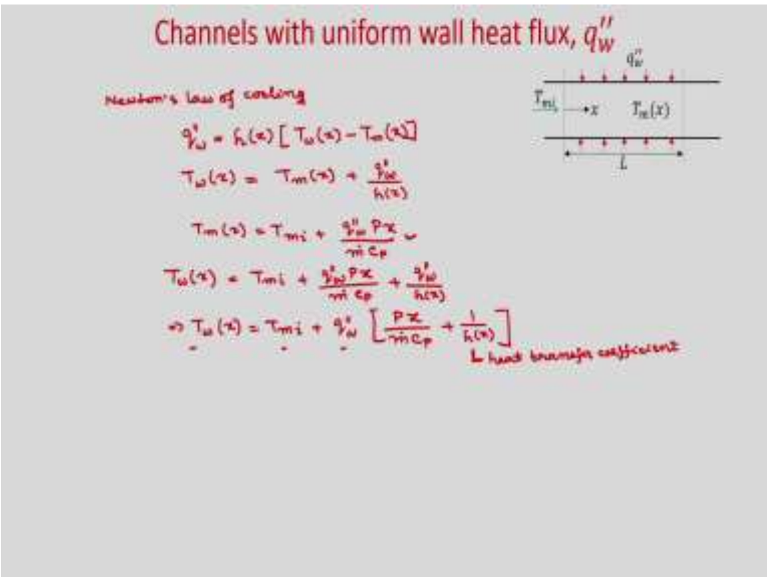

Now, we want to calculate the axial variation of wall temperature. So, for that we will do the we will use the Newton's law of cooling. So, from Newton's law of cooling what you can write? So, Newton's law of cooling what you can write?

So, $q_w^{"}$ what is your heat flux at the wall that you can write as, $q_w^{"} = h(x)[T_w(x) - T_m(x)]$.

In this case, when you consider the internal flows, you can see that you cannot take as T_∞ or T inlet as your reference temperature while calculating the heat transfer coefficient or the Nusselt number. Here, we will consider the bulk mean temperature at any cross section.

So, while writing the Newton's law of cooling, the wall heat flux we have written as the temperature difference as $T_w - T_m$. So, for internal flows we will consider $T_w - T_m$. So, you can write the, $T_w(x) = T_m(x) + \frac{q_w''}{h(x)}$ and T_{mx} already we know, $T_m(x) = T_{mi} + \frac{q_w'' P}{\dot{m} C_p} x$.

So, you can write in terms of T_{mi} now this T_w (x) so, if you substitute this, you can write $T_w(x) = T_{mi} + \frac{q_w'' P x}{\dot{m} C_p} + \frac{q_w''}{h(x)}$. So, you can rewrite it as, $T_w(x) = T_{mi} + q_w''[\frac{Px}{\dot{m} C_p} + \frac{1}{h(x)}]$.

Now, you can see that to calculate the wall temperature variation, you need to know the heat transfer coefficient because h is your heat transfer coefficient. So, you can see from this expression T_{mi} is your inlet mean temperature that will be known, this is your wall heat flux that will be known and it is constant, P is the perimeter, x is the axial direction so, at any location x, you can calculate the T_w, $\dot{m}$ and C_p are also known only unknown is h(x).

So, you can see that while deriving this, we did not take the assumptions whether it is laminar or turbulent or the region is developing or fully developed. So, these expression this T_m(x) and T_w(x) is the expression of T_w(x) and T_m(x) are valid for both laminar and turbulent flows as well as for entrance region as well as fully developed region.

Only thing is that while calculating the wall temperature, you need to calculate the h(x) and h(x) will depend whether it is developing region or fully developed region at the same time whether it is laminar or turbulent flows. So, this h(x) we need to find. So, depending on h(x) if you know h(x), then you can find T_w(x) for constant wall heat flux boundary condition.

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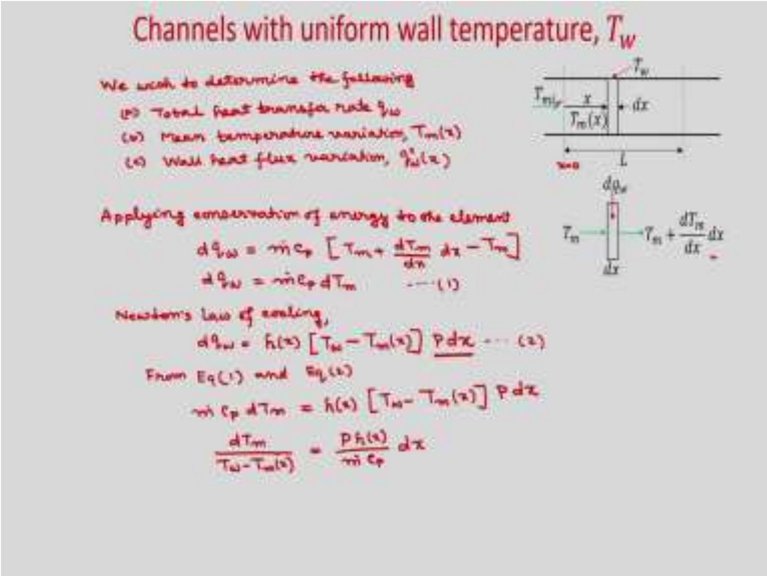

Now, let us consider channels with uniform wall temperature. So, you can see this is the channel at x=0, you have inlet mean temperature T_{mi} the walls are maintained at temperature T_w and T_w is constant.

So, in this case, you can see that heat flux is not constant. So, heat flux will vary in axial direction. So, for that reason, while solving this problem, we cannot consider the full length of the channel as we considered for the uniform heat flux case because in earlier case, the heat transfer rate q_w you could calculate using q''_w into the surface area A_s.

But here as q''_w is not constant for uniform wall temperature condition so obviously, we cannot consider the full length and do the energy balance for that we will consider a small elemental area in the flow region and we will do the energy balance.

In this particular case also, our objective is to find what is the axial variation of mean temperature, axial variation of the heat flux and what is the total heat transfer rate at the wall. So, we wish to determine the following: one is total heat transfer rate q_w, then mean temperature variation; mean temperature variation so, T_m as function of x and in this particular case as T_w is constant so, we want to calculate the wall heat flux variation q''_w as function of x.

So, now, in this region, you consider one small elemental volume at a distance x of distance dx. So, at this inlet of this elemental volume, you can see that we have the inlet mean temperature as T_m; obviously, at a distance dx using the Taylor series expansion what you can write?. So, at a distance dx using the Taylor series expansion you can write this.

And so, you have the wall temperature constant so, there will be the heat transfer from the wall to the fluid in this elemental volume so, that is your dq_w. So, whatever heat is added here the fluid which is passing through this dx distance that is actually that fluid absorbed this heat. So, you can do the energy balance as.

So, applying conservation of energy to the element what you can write? dq_w so, dq_w is the heat added to the fluid from the surface. So, what is heat absorbed that is your just $\dot{m} C_p \delta_T$. So, δ_T is nothing, but you can see it will be $T_m + \frac{dT_m}{dx} dx - T_m$. So, this is the temperature difference . So, you can write $dq_w = \dot{m} C_p dT_m$.

Now, you apply the Newton's law of cooling. So, Newton's law of cooling you can write dq_w so, that you can write the h which is function of x the temperature $[T_w - T_m(x)]A$, so, area is Pdx. So, this is the P, P is the perimeter and dx is the length. So, your heat transfer area will be Pdx.

So, now, this both you can equate. So, if it is equation number 1 and if it is equation number 2 so, from equation 1 and equation 2, you can write, $\dot{m} C_p dT_m = h(x)[T_w - T_m(x)]Pdx$. So, now, you rearrange it. So, $\frac{dT_m}{T_w - T_m(x)} = \frac{Ph(x)}{\dot{m} C_p} dx$.

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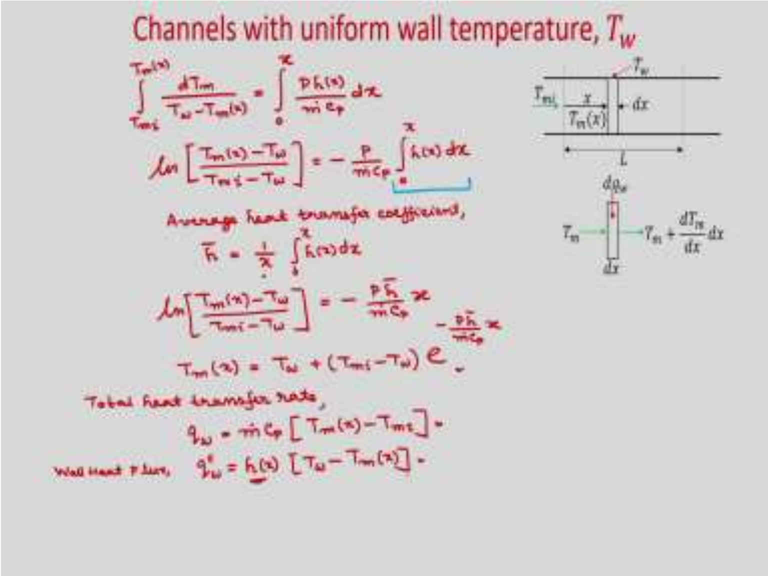

So, now, you integrate it from x =0 to any distance x so, if you do that. So, now, integrate this. So, from at x =0, you have T_{mi} and at any distance x, you have $\int_{T_{mi}}^{T_m(x)} \frac{dT_m}{T_w - T_m(x)} = \int_0^x \frac{Ph(x)}{\dot{m}C_p} dx$.

So, now, you can integrate and put the limits. So, this you can see it will be, $\ln[\frac{T_m(x) - T_w}{T_{mi} - T_w}] = -\frac{P}{\dot{m}C_p}\int_0^x h(x)dx$. So, if you know the variation of heat transfer coefficient in axial direction, then you will be able to integrate this.

So, we can find the average heat transfer coefficient as; average heat transfer coefficient ok. So, that will be $\bar{h} = \frac{1}{x}\int_0^x h(x)dx$. So, now, we will substitute this part as x $\bar{h}$. So, if you do so, you can write $\ln[\frac{T_m(x) - T_w}{T_{mi} - T_w}] = -\frac{P\bar{h}}{\dot{m}C_p} x$.

So, you can now find the axial variation of mean temperature as, $T_m(x) = T_w + (T_{mi} - T_w)e^{-\frac{P\bar{h}}{\dot{m}C_p}x}$. So, you can see your mean temperature varies exponentially right.

So, you can see that while deriving this expression, we did not assume whether flow is laminar or turbulent or whether it is entrance region or fully developed region. So, this is valid for all the conditions, but now you have to find what is the heat transfer coefficient. So, while finding the heat transfer coefficient, you need to have these assumption so, whether it is laminar or turbulent or it is entrance region or fully developed region.

So, now let us calculate the total heat transfer rate from the wall. So, we can write total heat transfer rate. So, total heat transparent rate q_w so, what is that we can write $q_w = \dot{m} C_p [T_m(x) - T_{mi}]$. So, you can write and $q''_w = h(x)[T_w - T_m(x)]$.

So, you can see that once you calculate the h(x) so, T_w is known so, you can calculate the mean temperature. Once you know the mean temperature, then you can calculate the total heat transfer rate as well as the heat flux. So, this is your wall heat flux. So, because h(x) you need to find once h(x) is known and T_{mx} is known from this expression, then you will be able to calculate what is the wall heat flux q''_w.

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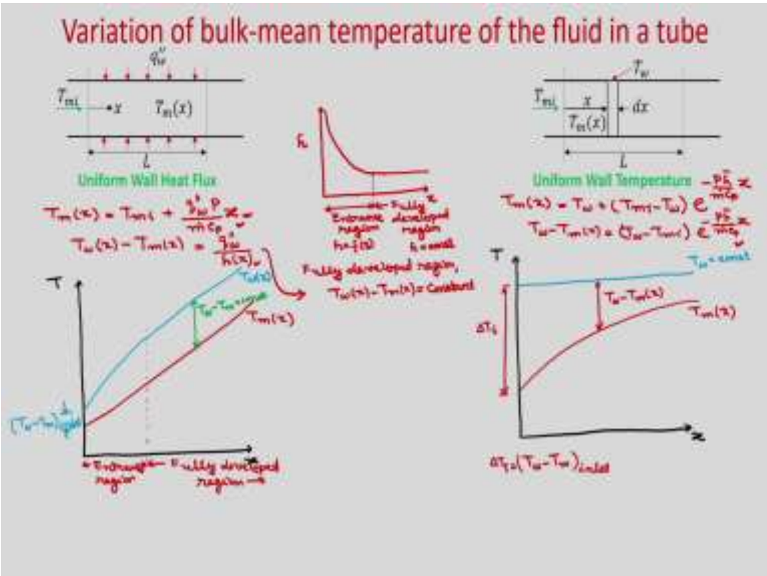

So, now for both the thermal conditions, we have found what is the axial variation of mean temperature as well as the total heat transfer rate. Now, let us see the variation of this bulk mean temperature of the fluid in the tube. So, you can see for uniform wall heat flux so, what is the variation of T_m?

T_m which is function of x we have written as $T_m(x) = T_{mi} + \frac{q_w'' P}{\dot{m} C_p} x$. So, how it is varying?

You can see your T_m is varying linearly because $\frac{q_w'' P}{\dot{m} C_p}$ this quantity is constant. So, T_m is varying linearly with x.

When we consider uniform wall heat flux, the T_m variation we have seen, $T_m(x) = T_w + (T_{mi} - T_w) e^{-\frac{P\bar{h}}{\dot{m} C_p} x}$. So, in this case, you can see your mean temperature varies exponentially.

So, now, if we plot. So, this is your temperature axis and this is your axial direction and in this case also this is your temperature axis and this is your axial direction. So, your this is your x and this is your T.

So, now, you can see that in this case, you have uniform wall heat flux right. So, uniform wall heat flux so, you can see that also we have found. So, here you can see that, $T_w(x) - T_m(x) = \frac{q_w''}{h(x)}$

So, now, you can see that in this particular case, h(x) is unknown obviously, your heat transfer coefficient will vary in axial direction, but we will show later that in fully developed region fully developed means both hydrodynamically and thermally, in that region h is not function of x, h is constant.

So, if you plot h with axial direction, then you will see that when the fluid is entering in the channel so, it will have very high heat transfer coefficient in the developing region it will gradually decrease, then once it will become fully developed both hydrodynamically and thermally, then your heat transfer coefficient will become constant and we will show it later that h is constant for a fully developed flow.

So, we plot this h as a function of x. So, in the developing region, it will decrease, then after that it will become constant. So, you can see if this is your entrance region or developing region, then it will gradually decrease and in fully developed region; fully developed region, you can see h is no longer function of x it is constant. So, in fully developed range, h is constant and in this region, h is function of x.

So, you can see that $T_w - T_m = \frac{q_w^{"}}{h(x)}$. So, in fully developed region, h(x) is constant hence, $T_w - T_m$ will be constant.

So, in fully developed region; fully developed region; fully developed region $T_w - T_m$ for uniform wall heat flux boundary condition will become constant. This is also true that fully developed region h is constant for uniform wall temperature. So, h is constant for both the thermal condition uniform wall temperature and uniform wall heat flux for both hydrodynamically and thermally fully developed region.

So, now, you can see here uniform wall heat flux. So, your T_m will vary linearly. You can see from this expression so, you draw this. So, this is linear variation. So, this is your T_m which is function of x it is varying linearly and you can see that $T_w - T_m$ will be constant in the fully developed region.

So, if you see that this is your fully developed region and this is your entrance region, then in this region fully developed region $T_w - T_m$ will be constant. So, you can see that your T_w also will vary linearly. So, it will also vary linearly. So, this is your T_w (x) and you can see that this difference in fully developed region, it is constant $T_w - T_m$ is constant.

However, in your entrance region, there will be variation. So, it may vary like this and this is your the temperature difference $T_w - T_m$ at inlet. So, this is the temperature variation. So, you can see that this T_w will vary linearly in the fully developed region and so, that $T_w - T_m$ will remain constant.

Now, you plot for uniform wall heat flux. So, for uniform wall heat flux so, T_w is constant right. So, you can draw this T_w it is constant. So, this is your T_w is constant and your T_m will vary exponentially so, if you plot it so, there will be variation like this. So, this is your T_m which will vary exponentially with x. So, this is $\Delta T_i = (T_w - T_m)_{inlet}$.

So, you can see that there will be always decrease in the temperature $T_w - T_m$. T_m is function of x, but T_w is constant and that you can see from here that it will how it will vary. So, you can see $T_w - T_m(x) = (T_w - T_{mi}) e^{-\frac{P\bar{h}}{\dot{m}C_p}x}$.

So, in today's lecture, we considered two different types of thermal conditions one is uniform wall heat flux and uniform wall temperature and we have just done the energy balance and we have tried to find what is the variation of mean temperature and the total heat transfer rate from the wall in both the cases.

However, in uniform wall temperature case, your heat flux at the wall also will vary so, we have found what is the variation of wall heat flux in axial direction and for uniform wall heat flux, your wall temperature will vary axially and we have found what is the variation of T_w with x.

Then, we have plotted this temperature; mean temperature and the wall temperature and also in this expression of mean temperature, we have seen that your heat transfer coefficient arises.

So, depending on whether the flow is laminar or turbulent or whether it is developing region or fully developed region, you need to find the heat transfer coefficient and also we have told that in fully developed region for both the thermal condition, your h is constant so, for a uniform wall heat flux condition $T_w - T_m$ will be constant. However, in uniform wall temperature boundary condition, $T_w - T_m$ will also vary exponentially.

We have plotted this mean temperature and wall temperature for both the thermal conditions and we have shown that in fully developed region, the temperature difference between wall and the bulk mean temperature will remain constant as your heat transfer coefficient remains constant in fully developed region for both the thermal conditions.

Thank you.