Fundamentals of Convective Heat Transfer
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Module – 05
Convection in Internal Flows – I
Lecture – 14
Hydrodynamic and thermal regions

Hello, everyone. So, in today's lecture we will start Convective Heat Transfer in Internal Flows. So, we will consider internal flows through channels such as ducts, pipes, and parallel plates. You can find the application in heat exchangers.

While considering these internal flows first we will assume the laminar flows. In general for this internal flows through a pipe or circular tube the Reynolds number based on average velocity and the diameter if it is less than 2300, then we consider it as laminar flows.

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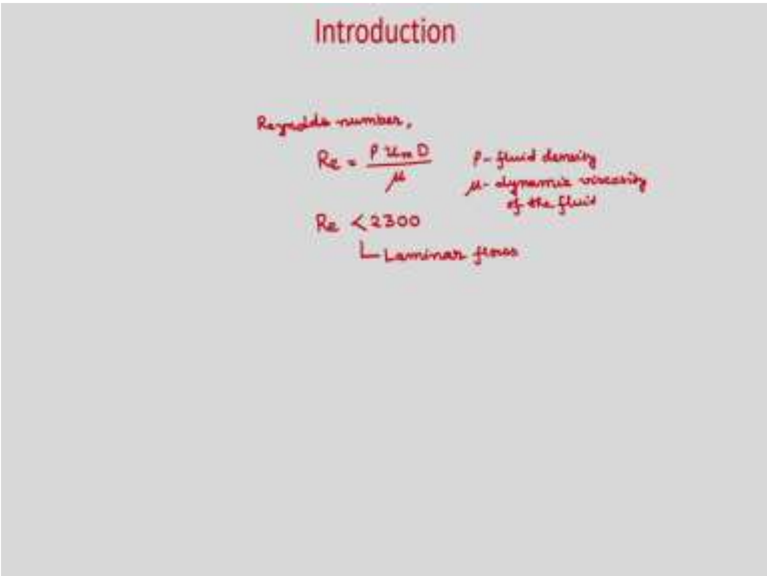

So, Reynolds number you know that it is a ratio of inertia force and the viscous force. So, these Reynolds number we consider as $\text{Re} = \frac{\rho u_m D}{\mu}$. So, ρ is the fluid density and μ is the dynamic viscosity of the fluid. So, if Reynolds number < 2300 then we consider it as laminar flows through pipe.

So, in this study for internal flows we will consider two different types of boundary conditions: one is uniform wall temperature where we will assume the temperature to be constant on the walls and secondly, we will consider uniform wall heat flux where we will consider heat flux to be constant on the walls. If heat flux is constant on the walls then wall temperature will vary in axial direction.

Similarly, if we consider uniform wall temperature boundary condition, then we will see that your heat flux will vary along the axial direction. In addition, we will study also the entry region and a developing region and fully developed region. So, you know that in entry region the boundary layer thickness will grow along x and it will merge at the central centre region.

When the entrance region extends to the from the inlet to the section where these boundary layer thickness merges at the central line and the fully developed region starts from there.

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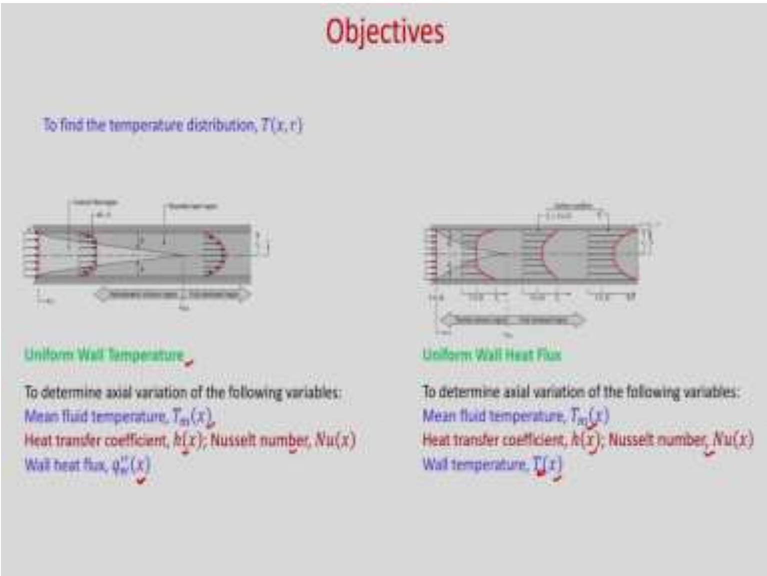

So, what is the objective to study these convective heat transfer in internal flows? Obviously, the first thing is to find the temperature distribution inside the domain. So, once you know the temperature distribution inside the domain then based on the thermal boundary condition on the wall we will have these objectives.

So, you can see that if you have uniform wall temperature, then our objective is to determine the axial variation of mean fluid temperature $T_m(x)$ which is the which is also known as bulk mean temperature, then heat transfer coefficient h_x your heat transfer coefficient will vary along the axial direction and hence we will calculate the Nusselt number. And, as it is uniform wall temperature we want to find the wall heat flux. So, this is the wall heat flux which is function of x.

Similarly, when we will consider the uniform wall heat flux, then we want to determine the axial variation of mean fluid temperature, axial variation of heat transfer coefficient and hence we will calculate the Nusselt number and also we want to calculate the axial variation of wall temperature. So, this is your T_w. So, wall temperature T_w as I told that for uniform wall heat flux condition, your wall temperature will vary along the axial direction.

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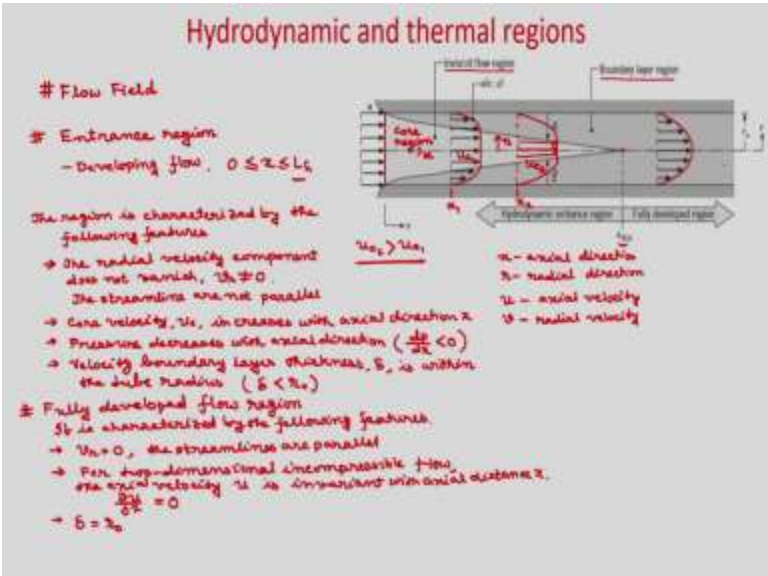

So, now let us discuss more about the entrance region and fully developed region. So, as we discussed in general we will consider at inlet you have uniform velocity u_i and uniform temperature T_i, where i denotes the inlet. Now, in the entrance region as we discuss that your velocity boundary layer will grow and it will merge in the central region. So, up to this point from the entrance it is known entrance region.

First let us consider the flow field only. So, you can see that you have a uniform velocity inlet at x = 0, in general x we will consider the axial direction. So, in the central region, so, this is your x. So, this is your central line. So, that is the x direction and from central region now we will consider r which is your radial direction and we will consider u as axial velocity and v as radial velocity in two-dimension case or axis symmetric case.

So, now we can see that when velocity will enter in the duct. So, your velocity boundary layer will start growing. So, then it will merge at the central line. So, up to this point it is known as entrance region or hydrodynamic entrance region as we are considering only the flow field. So, obviously, you can see this is your velocity boundary layer thickness δ and this δ is gradually increases and it merges at the central line and this region is known as entrance region.

So, you can see how the velocity profile will look like. So, obviously, at the inside the boundary layer so, velocity will gradually increase from the wall and it will go to the core region. So, this is known as core region and this is your viscous region and obviously, this is inviscid flow region. Which is your core region, inviscid flow region? And this is your boundary layer region which is your viscous region and at core region you can see you will have uniform velocity.

And, if you consider two different sections say let us say this is your x_1 and if you consider another section x_2 then you can see that your core velocity will increase. So, if this is your core velocity so, this is if your core velocity is constant if it c at section 1 and if it is core velocity u_c at section 2 then $u_{c_2} > u_{c_1}$ in the developing region or entrance region.

So, why it is so? So, you can see that your velocity boundary layer is gradually increasing. So, to have the same mass flow rate at section 1 and section 2 you should have more velocity at the core region at section 2. So, for that reason your core velocity at section 2 is higher than the core velocity at section 1.

So, in this case these developing region this length we will consider as L_h. So, first is your entrance region. So, this is your developing flow because you can see your boundary layer gradually increases and it is $0 \le x \le L_h$. So, whatever here it is mentioned x fully developed h. So, that length we are telling that it is your entrance length which is your L_h.

So, obviously, now the region this entrance region is characterized by the following features ok. So, first thing is that the. So, you can see that in the developing region your v velocity which is your radial direction velocity is not 0. So, obviously, for that reason your streamline will not be parallel in the developing region. So, the radial velocity component radial velocity component does not vanish so; that means, your $v_r \neq 0$ and hence the stream lines are not parallel.

The second observation already we discussed that core velocity u_c increases with axial direction; with axial direction and you can see that obviously, pressure decreases with axial direction pressure decreases with axial direction.

So, your $\frac{\partial u}{\partial x} < 0$ and most importantly your velocity boundary layer thickness; boundary layer thickness your δ is within the tube radius within the tube radius. So, δ will be less than r_0 in the developing region.

So, now, once you can see that this boundary layer reaches to the central line and merges then there will be no further growth of this hydrodynamic boundary layer. So, if there is no growth then obviously, your boundary layer thickness will be r_0 in the fully developed region.

And, you can see that as there will be no development of these hydrodynamic boundary layer your axial velocity u will not vary with x. So, that means, axial velocity distribution will remain constant in the fully developed region and hence $\frac{\partial u}{\partial x} = 0$ and this is the condition for fully developed flow.

So, now, we are discussing about the fully developed flow region. So, it is characterized by the following features. So, first thing is that $v_r = 0$. So, radial velocity will be 0 hence the stream lines are parallel stream lines are parallel.

Then, for two-dimensional incompressible flow the axial velocity u is invariant with axial distance x ok. Hence you can write $\frac{\partial u}{\partial x} = 0$. So, you can see in this region in fully developed region so, velocity becomes parabolic and as your boundary layer thickness merges here, so, your δ in fully developed region will be r_0. So, in fully developed region your δ always will be r_0.

And, at different section if you see in the axial direction these velocity profile will not change. Hence your $\frac{\partial u}{\partial x} = 0$ in fully developed region.

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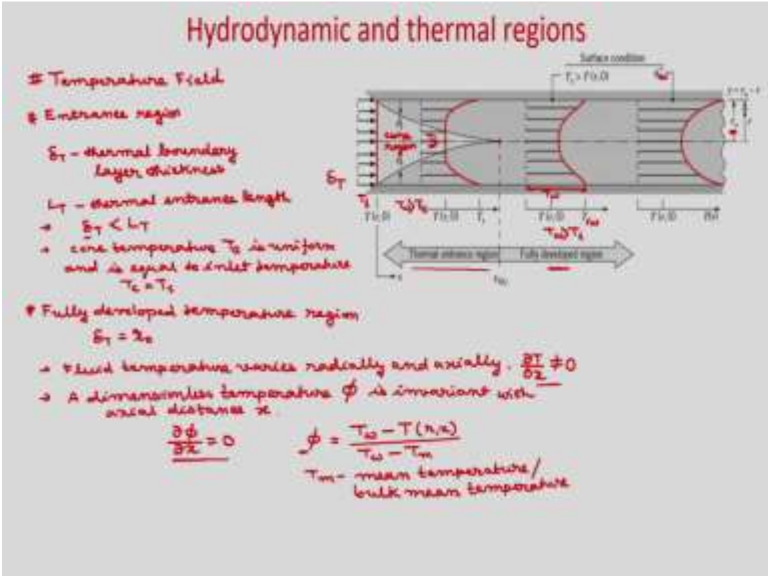

So, now let us consider the temperature field. So, similarly when your fluid enters at the inlet you have a uniform temperature T_i. Now, when it will come into contact with the wall which may have uniform wall temperature or uniform wall heat flux boundary condition, your thermal boundary layer thickness will grow and merge at central region and up to that region it is known as entrance region and it is known as the distance from the x = 0, it is known as thermal entrance length and after that you can see that thermal boundary layer thickness will not grow further.

So, δ_T which is your thermal boundary layer thickness will be equal to r_0 in the fully developed region. However, in this case your temperature distribution or temperature field will keep on changing with the axial direction in the fully developed region.

Unlike in your hydrodynamic case, where u does not vary in the axial direction in this case as always there will be heat transfer from the wall to the fluid your temperature profile will keep on changing. So, you cannot write in this case for two-dimensional incompressible flow $\frac{\partial T}{\partial x} = 0$, it is not true.

So, in this case entrance region, so, what is happening? You have uniform wall temperature T_i at inlet. So, gradually your thermal boundary layer thickness δ_T generally we consider thermal boundary layer thickness δ_T will start growing and it will merge at the central region, and these distance is known as thermal entrance length and this is your thermal entrance region.

After that this region is known as fully developed region. And, in fully developed region now your temperature will keep on changing in the axial direction. So, in the entrance region; so, δ_T is your thermal boundary layer thickness and when you have L_T if it is your thermal entrance length, then in the entrance region $\delta_T < L_T$. So, this is one condition and also you have core temperature.

Now, you can see this is your core region; it is your core region. So, the heat transfer is taking place from the surface to the fluid if $T_w > T_i$, then obviously, from the wall to the fluid there will be heat transfer and inside this thermal boundary layer thickness there will be variation of temperature. But, at the core region there will be the fluid will remain at temperature T_i. So, this is actually at temperature T_i.

So, there is no effect of the wall temperature in the core region in the thermal entrance region and if you consider two different sections then you will see that your T_i will remain same at two different cross section because your T_i will remain same in the core region. So, core temperature T_c is uniform and is equal to inlet temperature.

So, your T_c will be T_i in the core region and temperature or thermal entrance length or thermal boundary layer thickness will be less than L_T and if you consider a fully developed region so, obviously, now in this case your δ_T always will be r_0 because there will be no further growth of this boundary layer thickness.

So, if your r_0 is the tube radius then δ_T will be always r_0 and in this case your fluid temperature varies radially and axially. And, in this case $\frac{\partial T}{\partial x} \neq 0$ you can see it will keep on changing.

And, here we will define later one dimensionless temperature phi is invariant with axial distance x. And, in this case you can write $\frac{\partial \phi}{\partial x} = 0$ and $\phi = \frac{T_w - T(r,x)}{T_w - T_m}$.

So, it is the ratio of $\frac{T_w - T(r,x)}{T_w - T_m}$ where T m is the mean temperature or sometime it is known as bulk mean temperature or bulk temperature. So, in this case you need to calculate at that cross section what is the bulk mean temperature, we will learn how to calculate this balk mean temperature later.

So, these balk mean temperature we can see that the difference $T_w - T(r,x)$ varies similar way as $T_w - T_m$, so that your $\frac{\partial \phi}{\partial x} = 0$. And, this is the condition is valid for both the boundary conditions uniform wall temperature and uniform wall heat flux. And, in this case you can see that if you have uniform wall temperature T_w; so, this is your T_w.

So, these T_w if it is greater than your T than the T_i, then gradually your heat transfer will take place from the wall to the core fluid and from the surface to the fluid and you can see that your temperature distribution will look like this. Because this is the temperature T_w and there is a temperature and in the core region as $T_i < T_w$. So, it will have less temperature than the T_w.

And, if you have uniform wall heat flux which is $q_w^{"}$, then your temperature distribution may look like this. So, this is the temperature distribution in fully developed region and please remember that in this particular case $\frac{\partial T}{\partial x} \neq 0$ here $\frac{\partial \phi}{\partial x} = 0$ where Φ is the non-dimensional temperature or dimensionless temperature as defined $\phi = \frac{T_w - T(r,x)}{T_w - T_m}$.

So, as we are telling $T_w - T(r,x)$ varies in similar way as $T_w - T_m$, hence your Φ is invariant with axial direction and you can write $\frac{\partial \phi}{\partial x} = 0$ in this particular case.

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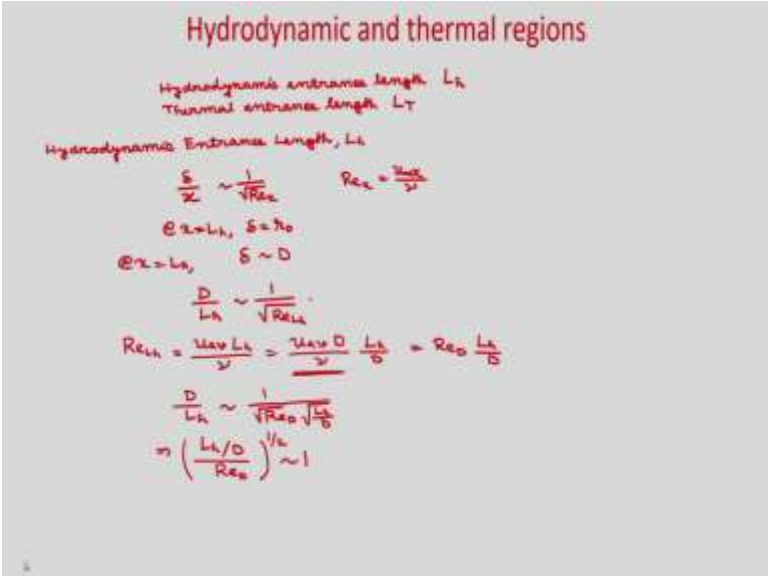

So, now let us see how to find this hydrodynamic entrance length and thermal entrance length. Already we discussed that hydrodynamic entrance length we have denoted as L_h and thermal entrance length we have denoted as L_T. So, first we will use the scale analysis and we will see what is the order of these hydrodynamic and thermal entrance length.

So, we know from the external flows that so, first let us consider hydrodynamic entrance length; hydrodynamic entrance length. So, L_h and we know from the external flows that $\frac{\delta}{x} \sim \frac{1}{\sqrt{\mathrm{Re}_x}}$ where R e we have defined $\mathrm{Re}_x = \frac{u_{avg} x}{\nu}$.

So, now, at the point where these velocity boundary layer merges then x at $x = L_h$ you have your $\delta = r_0$. Or so, you can write that the order of δ is at $x = L_h$ order of δ we can right diameter of the tube.

So, now we are doing the scale analysis order of magnitude we are seeing. So, the thermal so, the hydrodynamic boundary layer thickness you can write $\delta \sim D$. So, in that case now you can write $\frac{D}{L_h} \sim \frac{1}{\sqrt{\mathrm{Re}_{L_h}}}$. So, now, Re_{L_h} what we can write? It is just

$$\mathrm{Re}_{L_h} = \frac{u_{avg} L_h}{\nu}.$$

So, now we will convert it the Reynolds number based on the diameter. So, $\mathrm{Re}_{L_h} = \frac{u_{avg} D}{\nu} \frac{L_h}{D}$. So, you can see that you can write Re_D. So, this quantity is $\mathrm{Re}_{L_h} = \mathrm{Re}_D \frac{L_h}{D}$. So, now, if you substitute it here so, what you will get $\frac{D}{L_h} \sim \frac{1}{\sqrt{\mathrm{Re}_D}\sqrt{\frac{L_h}{D}}}$.

So, now you can write this $\left(\frac{L_h/D}{\mathrm{Re}_D}\right)^{1/2} \sim 1$. So, where L_h is the hydrodynamic entrance length, D is the tube diameter and Reynolds number now based on diameter we have defined. So, $\left(\frac{L_h/D}{\mathrm{Re}_D}\right)^{1/2} \sim 1$.

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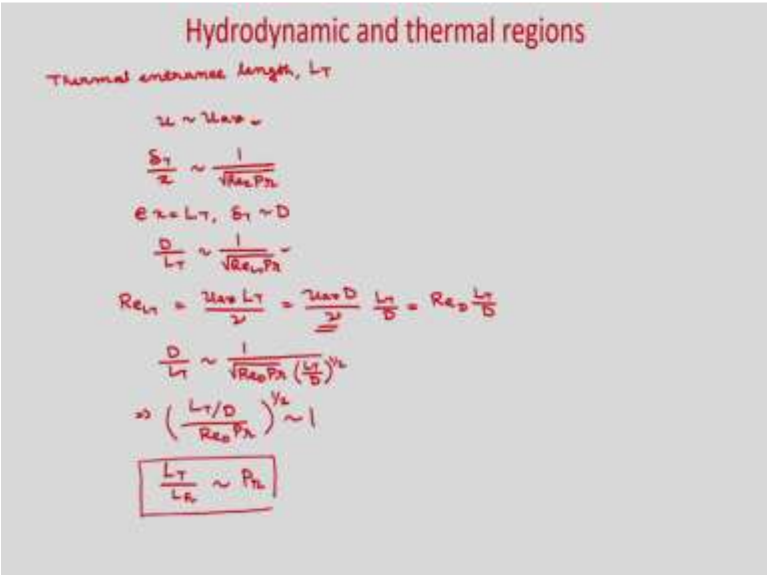

Similarly, now consider thermal entrance length thermal entrance length which we have denoted as L_T. So, in this particular case you can see that whether Prandtl number is > 1 or < 1 both cases the thermal boundary layer will merge at the core.

So, in this particular case we will consider that velocity u is order of u_∞ or u_{avg}. So, in this particular case the velocity will take the scale of u_{avg}, irrespective of the low Prandtl number fluids or high Prandtl number fluids because both will margin ultimately at the central region.

So, in this case now u will consider as u_{avg}. So, now, $\frac{\delta_T}{x} \sim \frac{1}{\sqrt{Re_x Pr}}$. So, that we have already shown for the external flows because it is a developing region. So, in the developing region you can see that it is kind of external flows and δ_T which is your thermal boundary layer thickness is increasing with x.

And, as we are considering the velocity scale as the average velocity irrespective of high Prandtl number fluids or low Prandtl number fluids. So, $\frac{\delta_T}{x} \sim \frac{1}{\sqrt{Re_x Pr}}$. Now, you can consider that x = L_T your $\delta_T \sim D$ because that will be your the thermal boundary layer thickness will be order of diameter. So, you can write $\frac{D}{L_T} \sim \frac{1}{\sqrt{Re_{L_T} Pr}}$.

So, now, you convert this $Re_{L_T} = \frac{u_{avg} L_T}{\nu}$. So, you can convert in terms of diameter. So, it will be $Re_{L_T} = \frac{u_{avg} D}{\nu} \frac{L_T}{D}$. So, this you can right now $Re_{L_T} = Re_D \frac{L_T}{D}$. So, if you put it in this equation then you will get $\frac{D}{L_T} \sim \frac{1}{\sqrt{Re_D Pr} (\frac{L_T}{D})^{1/2}}$.

So, this if you see then you can write $(\frac{L_T/D}{Re_D Pr})^{1/2} \sim 1$. So, for hydrodynamic case you have seen $(\frac{L_h/D}{Re_D})^{1/2} \sim 1$. And, in this case as you are considering the thermal flows, so, it will be $(\frac{L_T/D}{Re_D Pr})^{1/2} \sim 1$.

And, now you can see the ratio of this hydrodynamic entrance length and the thermal entrance length will be so, $\frac{L_T}{L_h}$. So, if you divide you can see $\frac{L_T}{L_h} \sim Pr$. So, using scale analysis we have found that $\left(\frac{L_h/D_h}{\text{Re}_{D_h}}\right)^{1/2} \sim 1$. So, now, after solving numerically or analytically you can find this entrance length.

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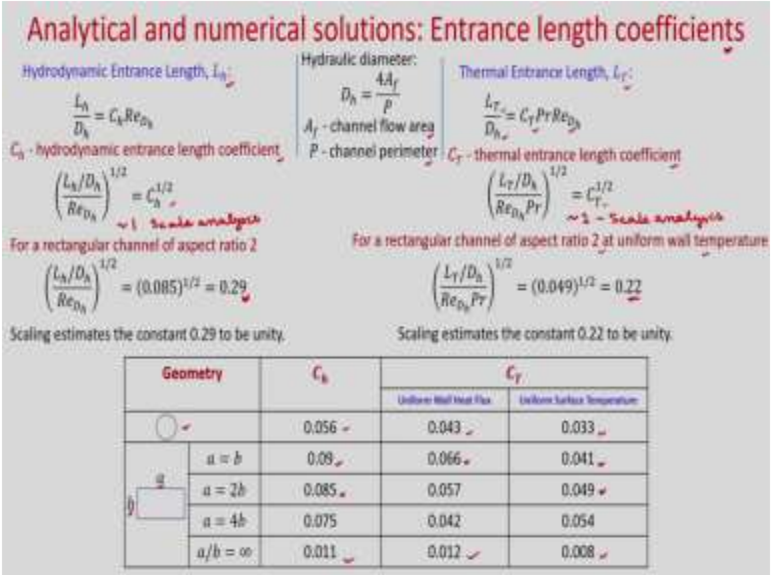

So, now we will define the entrance length coefficient you can see here entrance length coefficient. So, this is C_h is known as hydrodynamic entrance length coefficient and it is given as $\frac{L_h}{D_h} = C_h \text{Re}_{D_h}$.

And, you know that hydraulic diameter we define as $D_h = \frac{4A_f}{P}$, where A_f is your channel flow area and P is the channel perimeter and for circular tube you know that diameter of this pipe will be your hydraulic diameter.

Now, for thermal entrance length L_T we have seen that it is function of Prandtl number and Reynolds number. So, $\frac{L_T}{D_h} = C_T \Pr \text{Re}_{D_h}$, and this C_T is known as thermal entrance length coefficient.

So, you can see that these we can rewrite in this form $\left(\frac{L_T/D_h}{\mathrm{Re}_{D_h}}\right)^{1/2} = C_h^{1/2}$. And, this also you can write $\left(\frac{L_T/D_h}{\mathrm{Re}_{D_h}\,\mathrm{Pr}}\right)^{1/2} = C_T^{1/2}$. And, these using scale analysis we have shown that both are order of one.

So, after doing analytical or numerical solution this entrance length coefficients are found for different geometry. So, you see this table. So, if it is circular tube then your hydrodynamic entrance length co-efficient C_h = 0.056 and C_T which is your thermal entrance length coefficient it depends on the boundary conditions. So, for uniform wall heat flux C_T = 0.043 and for uniform surface temperature it is 0.033.

Now, if you consider a rectangular duct where sides are a b as shown in this figure. So, if a = b, then it is a square cross section channel; for that C_h=0.09 and C_T for uniform wall heat flux it is 0.066 and C_T for uniform surface temperature it is 0.041. And, similarly for a = 2b, a = 4b, and $\frac{a}{b} = \infty$; that means, it is almost infinite parallel plates, so, for that also C_h and C_T values are shown.

So, these are obtained from the analytical and or numerical solutions. So, now, let us see that what we derived using scale analysis so, how it matches with this analytical or numerical results. So, we have already shown that this is order of 1 using scale analysis we have already seen right. So, if you consider a rectangular channel of aspect ratio 2 then your C_h=0.085 and these to the other half is 0.29. So, you can see that scaling estimates the constant 0.29 to be unity.

Similarly, if you consider for a rectangular channel of aspect ratio 2 at a uniform wall temperature, then for this you can see it is 0.049 and $C_T^{1/2}$ it will give 0.22. So, we have seen that using scale analysis it is order of 1, $\left(\frac{L_T/D_h}{\mathrm{Re}_{D_h}\,\mathrm{Pr}}\right)^{1/2} \sim 1$. So, obviously, you can see that scaling estimates the constant 0.22 to be unity.

So, in today's lecture we have first discussed about the objectives to study the convection in internal flows, then we have discussed about the hydrodynamic entrance length and fully developed region. We have seen that in fully developed region the velocity does not vary in the axial direction, so, $\frac{\partial u}{\partial x} = 0$.

And, when we considered the thermal or temperature field then we have seen that in the core region your temperature will remain same as the inlet temperature in the thermal entrance region, but in a fully developed region your temperature will change gradually in the axial direction. So, $\frac{\partial T}{\partial x} \neq 0$ in this particular case.

So, we defined one dimensionless temperature $\phi = \frac{T_w - T(r,x)}{T_w - T_m}$. We discussed that $T_w - T(r,x)$ varies in similar way as $T_w - T_m$, where T_m is the mean temperature. Hence these non-dimensional temperature Φ does not vary in the axial direction. So, that means, your $\frac{\partial \phi}{\partial x} = 0$.

Then we used the scale analysis and we have seen the hydrodynamic entrance length $\left(\frac{L_h/D}{\mathrm{Re}}\right)^{1/2} \sim 1$, where Reynolds number is defined based on the diameter. And, when we considered the thermal entrance length in that particular case your $\left(\frac{L_T/D}{\mathrm{Re}_D \,\mathrm{Pr}}\right)^{1/2} \sim 1$. And, we have also seen the ratio of $\frac{L_T}{L_h} \sim \mathrm{Pr}$.

So, in next class, we will see that how we can find these thermal and hydrodynamic entrance length using analytical and numerical approach.

Thank you.