Fundamentals of Convective Heat Transfer
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Module – 04
Convective Heat Transfer in External Flows – II
Lecture – 13
Solution of example problems

Hello, everyone. So, today we will solve few example problems of Convective Heat Transfer in External Flows. So, we have already covered few lectures in last two modules related to convective heat transfer in external flows and let us apply those knowledge to solve some numerical problems.

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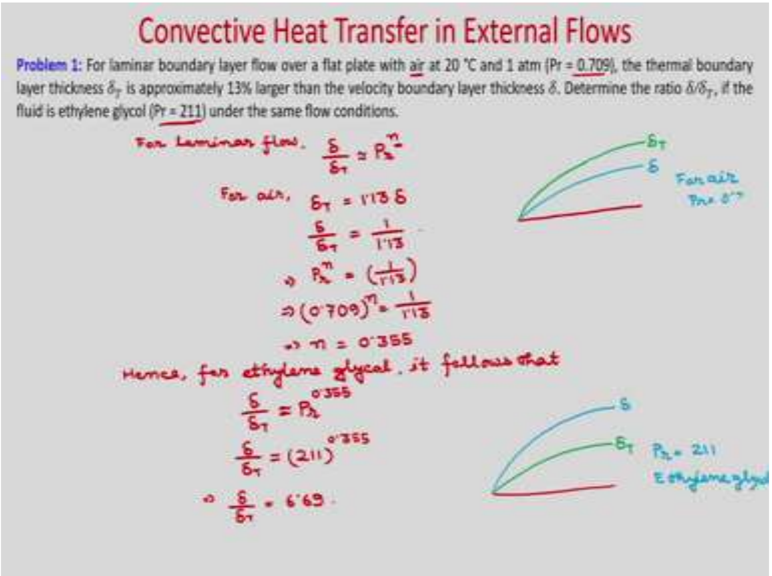

So, let us discuss the first problem;

<u>Problem 1</u>: For laminar boundary layer flow over a flat plate with air at 20^0C and 1 atm, Prandtl number = 0.709 the thermal boundary layer thickness δ_T is approximately 13 % larger than the velocity boundary layer thickness δ. Determine the ratio δ / δ_T if the fluid is ethylene glycol Prandtl number = 211 under the same flow conditions. So, you can see for a given condition δ / δ_T is given.

Now, we need to find the δ_T for ethylene glycol where Prandtl number is given. So, we know that for external flows if the flow is laminar then $\frac{\delta}{\delta_T} = \Pr^n$. So, this n for air we know it is 1/3, but for this particular case we need to determine from the given condition. So, we know that for laminar flow $\frac{\delta}{\delta_T} \approx \Pr^n$.

So, here you can see the first fluid is air and for air it is given that your δ_T is approximately 13 % larger than δ. So, you can write $\delta_T = 1.13\delta$ from the given condition. So, you can see that $\frac{\delta}{\delta_T} = \frac{1}{1.13}$. Now, you have the same flow conditions. So, under this now for air we have to find; what is the n?

So, you can see here $\frac{\delta}{\delta_T} = \frac{1}{1.13}$. So, if we put it here so, we will get $\Pr^n = \frac{1}{1.13}$. So, now, you can find Prandtl number is given as 0.709. So, you can see $(0.709)^n = \frac{1}{1.13}$. So, from here you find n; n = 0.355. So, for ethylene glycol it follows that $\frac{\delta}{\delta_T} \approx \Pr^n$; n = 0.355 because you have the same flow conditions, so n will be same.

So, now, Prandtl number is different. In this case Prandtl number is 211. So, if you put that. So, you will find $\frac{\delta}{\delta_T} = (211)^{.355}$ so hence you will get $\frac{\delta}{\delta_T} = 6.69$. So, you just notice here that for air $\frac{\delta}{\delta_T} < 1$. You can see here it will be less than 1, but in case of ethylene glycol $\frac{\delta}{\delta_T} > 1$.

So, if you plot the hydrodynamic boundary layer and thermal boundary layer, then you will see for air if it is your boundary layer hydrodynamic boundary layer then your thermal boundary layer will be higher than the hydrodynamic boundary layer. So, this is your δ, this is your δ_T, it is for air and if you consider ethylene glycol then if it is flat plate.

Then in this particular case this will be your thermal boundary layer and hydrodynamic boundary layer will be higher because Prandtl number >1. So, Prandtl number in this case it is 211; you have ethylene glycol and this is your Prandtl number 0.709.

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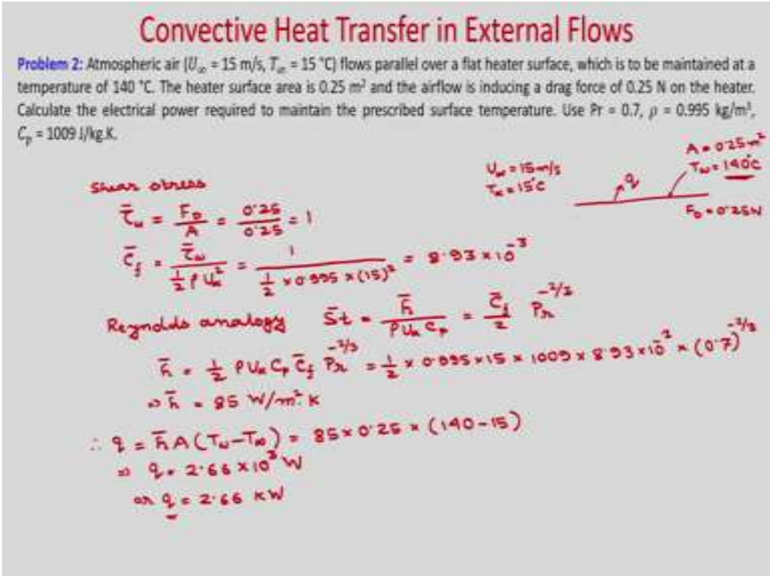

So, let us take the second problem

Problem 2: Atmospheric air U_∞ =15 m/s, T_∞ =15^{0}C flows parallel over a flat heater surface which is to be maintained at a temperature of 140^{0}C. The heated surface area is 0.25 m^2 and the air flow is inducing a drag force of 0.25N on the heater. Calculate the electrical power required to maintain the prescribed surface temperature. Use Prandtl number =0.7, ρ = 0.995 kg/m^3 and C_p = 1009 J/kgK.

So, in this particular case you can see your surface temperature is to be maintained at 140^{0}C. So, if you consider the flat plate so, the drag force acting on this flat plate is 0.25N, it is given and your surface temperature T_w = 140^{0}C.

And area of this plate is given as 0.25 m^2 and your U_∞ = 15 m/s which is your free stream velocity. Free stream temperature T_∞ = 15^{0}C and you need to find what is the electrical power required; q, to maintain the surface temperature as 140^{0}C.

So, we know the drag force acting on the plate. So, we will be able to calculate the shear stress acting on the surface. So, shear stress so, this is your average shear stress $\tau_w = \frac{F_D}{A} = \frac{0.25}{0.25} = 1$. So, if it is a average shear stress so, just I will denote with $\bar{\tau}_w$.

Similarly, the coefficient of drag, $\overline{C}_f = \frac{\bar{\tau}_w}{\frac{1}{2}\rho U_\infty^2}$. So, it is due to shear and it will be, $\frac{1}{\frac{1}{2} X\, 0.995 X (15)^2}$. So, if you calculate it you will get as $8.93 X 10^{-3}$.

So, now you know C_f, so, we will calculate the heat transfer coefficient using Reynolds analogy. So, Prandtl number you can see it is of the order of 1. So, you can use this Reynolds analogy to calculate the average heat transfer coefficient. So, you know that $\overline{St} = \frac{\bar{h}}{\rho U_\infty C_p}$ and these you can relate with $\frac{\overline{C_f}}{2} \Pr^{-2/3}$.

So, from here you can calculate the average heat transfer coefficient, $\bar{h} = \frac{1}{2}\rho U_\infty C_p \overline{C}_f \Pr^{-2/3}$. So, that will be $\frac{1}{2} X\, 0.995 X 15 X 1009 X 8.93 X 10^{-3} X (0.7)^{-2/3}$. So, if you calculate you will get the average heat transfer coefficient, $\bar{h} = 85 W / m^2 K$.

So, now, we have calculated average heat transfer coefficient, you know the total heat transfer area. So, you will be able to calculate the heat transfer rate. So, that is your using Newton’s law of cooling you can write as $\bar{h}A(T_w - T_\infty)$. So, $q = \bar{h}A(T_w - T_\infty)$, where it will be $85 X\, 0.25 X (140 - 15)$. So, if you calculate it so, you will get $q = 2.66 X 10^3 W$ or $q = 2.66 kW$.

So, this is the electrical power required to maintain the surface temperature at 140^0C.

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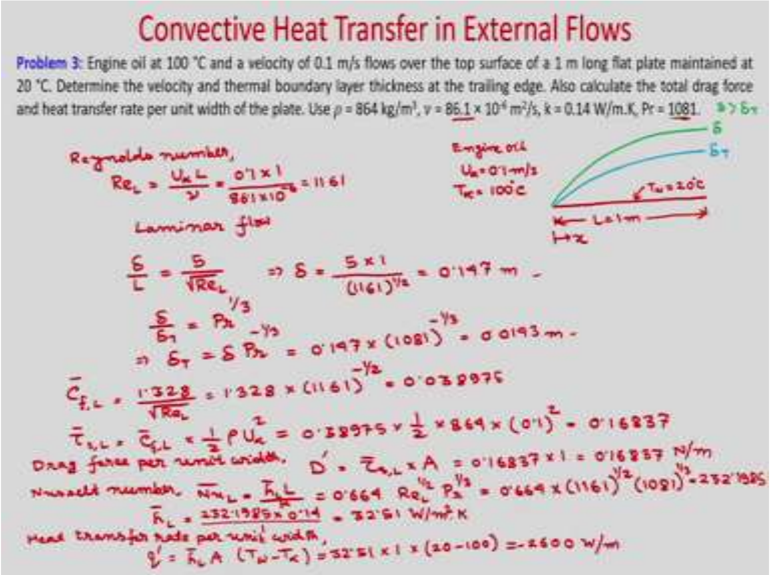

So, let us discuss the 3rd problem.

<u>Problem 3</u>; Engine oil at 100^0C and a velocity of 0.1 m/s flows over a top surface of a 1m long flat plate maintained at 20^0C. Determine the velocity and temperature boundary layer thickness at the trailing edge also calculate the total drag force and heat transfer rate per unit width of the plate; the properties are given here.

So, you have one flat plate; so, where U_∞ your free stream velocity is 0.1 m/s, T_∞=100^0C. So, this engine oil is flowing over this flat plate. Now, the length of the plate is given L is 1 m, x is measured from this leading edge, the T_w =20^0C.

Now, we need to determine the thermal boundary layer thickness δ_T and your hydrodynamic boundary layer thickness δ. So, we can see Prandtl number =1081, so, it is >> 1; obviously, $\delta > \delta_T$. So, first let us check whether flow is laminar or not. So, we will calculate the Reynolds number from the given data.

So, Reynolds number, Re based on the length L is $\mathrm{Re}_L = \frac{U_\infty L}{\nu}$ which is $\frac{0.1X1}{86.1X10^{-6}}$. So, if you calculate it you will get as 1161. So, we can see it is a laminar flow. So, now, you know for flow over flat plate $\frac{\delta}{L} = \frac{5}{\sqrt{\mathrm{Re}_L}}$.

So, from this you can calculate the hydrodynamic boundary layer thickness $\delta = \frac{5X1}{(1161)^{1/2}}$.

So, if you calculate this you will get 0.147m and we know $\frac{\delta}{\delta_T} = \Pr^{1/3}$. So, $\delta_T = \delta \Pr^{-1/3}$.

So, $0.147 \times (1081)^{-1/3}$. If you calculate you will get 0.0143m.

So, δ and δ_T now we are found, now we need to find the total drag force and heat transfer rate per unit width of the plate. So, now, $\overline{C}_{f,L} = \frac{1.328}{\sqrt{\mathrm{Re}_L}}$. So, this is the expression we have already derived for flow over flat plate. So, we can see it will be $1.328 \times (1161)^{-1/2}$. So, this is your Reynolds number to the power -1/2 . So, you will get 0.038975.

So, now, you will be able to calculate the shear stress. So, shear stress will be just $\overline{\tau}_{s.L} = \overline{C}_{f,L} \times \frac{1}{2}\rho U_\infty^2$. So, you will get as $0.38975 \times \frac{1}{2} \times 864 \times (0.1)^2$. So, if you calculate it you will get it as 0.16837.

So, now drag force per unit width you will be able to calculate as drag force per unit width. So, D unit width means perpendicular to this board so, in that direction. So, that is per unit width we are calculating and we are just prime we are telling per unit width as. So, it will be the $D' = \overline{\tau}_{s.L} \times A$. So, what will be your area? So, per unit width you are calculating. So, it is $0.16837 \times A$.

So, area is length into width per unit width we are calculating, so, into 1. So, we are considering only top surface of the plate because it is written that flows over the top surface ok. So, it will be $0.16837 \times A$; area we are considering on the top side so, it is $\mathrm{L} \times 1$. So, it is 1. So, you will get as 0.16837 N/m.

Now, we need to calculate the total heat transfer rate. So, Nusselt number you know relation for the Nusselt number average Nusselt number you can write as $\overline{Nu}_L = \frac{\overline{h}_L L}{K} = 0.664 \mathrm{Re}_L^{1/2} \Pr^{1/3}$. So, this relation we are using because you know that Prandtl number >> 1, 1081.

So, it will be $Re_L^{1/2} Pr^{1/3}$. So, from here you just calculate the average heat transfer coefficient. So, it will be, $0.664 \times (1161)^{1/2} (1081)^{1/3} = 232.1985$. So, heat transfer coefficient you will be able to calculate as $\frac{232.1985 \times 0.14}{L}$; L is 1 ok. So, this will give as 32.51 W/m^2 K.

So, now, heat transfer rate per unit width you will be able to calculate heat transfer rate per unit width. So, q' I am writing because per unit width we are writing. So, it will be $q' = \bar{h}_L A(T_w - T_\infty)$. So, it will be $32.51 \times$ A. So, we are considering only top surface one side of the plate we are considering and so, your area will be only 1 as we are considering per unit width in to $T_w - T_\infty$.

So, it will be $32.51 \times 1 \times (20 - 100)$. So, if you calculate it you will get approximately as 2600 W/m. So, what is the significance of this negative sign because we are calculating q from the surface to the fluid because we have $T_w - T_\infty$, but q is coming as negative; that means, heat transfer is taking place from the fluid to the wall fluid to the surface.

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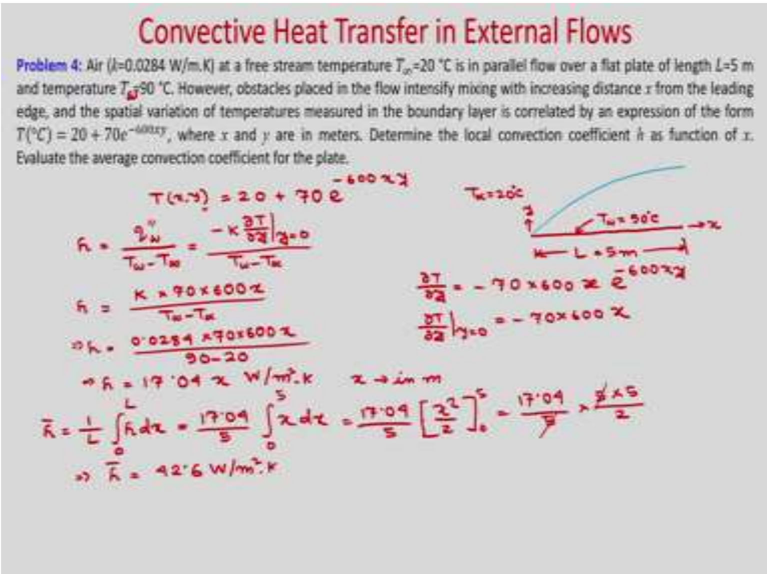

Now, let us discuss about the next problem.

<u>Problem 4</u>: Air thermal conductivity as 0.0284 W/m^2 K at a free stream temperature T_∞=20^{0}C is in parallel flow over a flat plate of length 5m and temperature T_w at 90^{0}C.

However, obstacles placed in the flow to intensify mixing with increasing distance x from the leading edge, and the spatial variation of temperatures measured in the boundary layer is correlated by the expression of the form $T(^0C) = 20 + 70e^{-600xy}$, where x and y are in meters. Determine the local convection coefficient h as a function of x and evaluate the average heat transfer coefficient for the plate.

So, in this particular case the temperature variation with the coordinate x and y is given. So, from there you will be able to calculate the temperature gradient at the wall. Once you know the temperature gradient at the wall you will be able to calculate; what is the heat flux from the surface and then you will be able to calculate the local heat transfer coefficient. Once you know the local heat transfer coefficient you will be able to calculate the average heat transfer coefficient for the plate.

So, $T(x, y) = 20 + 70e^{-600xy}$, where x and y are in meter. So, if you consider a flat plate L is 5m, your free stream temperature T_∞ =20^0C, your surface temperature T_w =90^0C and y is measured perpendicular to the plate and x along the plate, and your boundary layer thickness will be like this.

Now, you first calculate the local heat transfer coefficient. So, $h = \dfrac{q_w^{"}}{T_w - T_\infty}$ this is your local heat transfer coefficient, $\dfrac{-K \dfrac{\partial T}{\partial y}\Big|_{y=0}}{T_w - T_\infty}$. So, the temperature is given here. So, $\dfrac{\partial T}{\partial y} = -70 \times 600xe^{-600xy}$.

So, now, you see at $\dfrac{\partial T}{\partial y}\Big|_{y=0}$. So, at y = 0, what will be your $\dfrac{\partial T}{\partial y} = -70 \times 600x$. You will be able to calculate the local heat transfer coefficient h as, $h = \dfrac{K \times 70 \times 600x}{T_w - T_\infty}$. So, h will be $\dfrac{0.0284 \times 70 \times 600x}{90 - 20}$. So, hence you will be able to calculate h = 17.04x W/m^{2}K, x in meter.

Now, you know the local heat transfer coefficient now if we integrate over the flat plate and divided by the length of the plate will give you the average heat transfer coefficient. So, average heat transfer coefficient $\bar{h} = \frac{1}{L}\int_0^L h dx$. So, we can see $\bar{h} = \frac{17.04}{5}\int_0^5 x dx$.

So, it will be $\frac{17.04}{5}[\frac{x^2}{2}]_0^5$. So, it will be $\frac{17.04}{5} X \frac{5X5}{2}$; so these 5, 5 will get cancel. So, you can calculate the average heat transfer coefficient $\bar{h} = 42.6 W/m^2K$.

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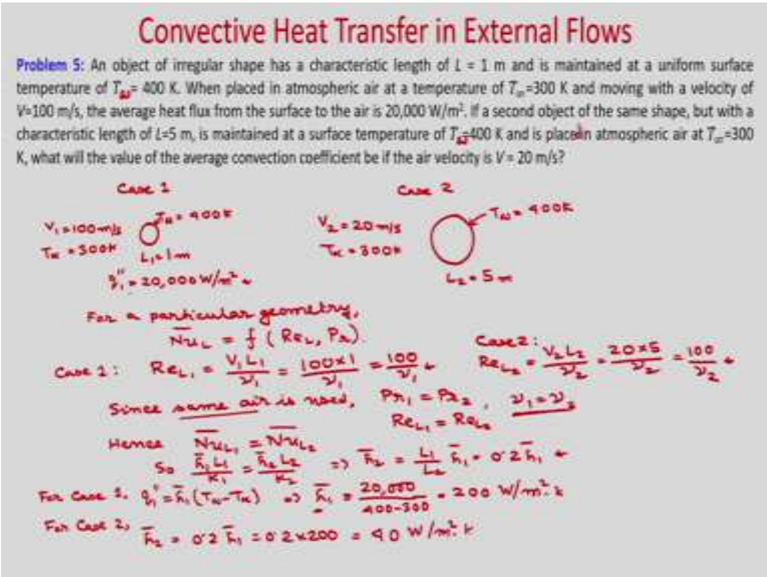

Now, let us take the next problem.

<u>Problem 5:</u> An object of irregular shape has a characteristic length of 1m and is maintained at a uniform surface temperature 400 K. When placed in atmospheric air at a temperature of T_∞ =300 Kelvin and moving with a velocity of 100 m/s, the average heat flux from the surface to the air is 20000 W/m^2.

If a second object of the same shape, but with a characteristic length of 5m, is maintained at a surface temperature 400 K and is placed in atmospheric air at T_∞ =300 K, what will be the value of average convection coefficient be if the air velocity is 20 m/s?

So, you have two irregular shape. So, if you see if those are geometrically similar then you will be able to equate the non-dimensional number Reynolds number and Nusselt number. So, let us take this irregular shape let us say case 1. So, you have let us say one shape like this whose characteristic length L_1 is 1 m, T_w =400 K, your v_1=100 m/s, T_∞=300 K and $q_1^" = 20000 W/m^2$.

Now, if you consider case 2 you have second object of the same shape. So, it is of the same shape, but characteristic length is different. So, in this case your characteristic length is 5m, wall temperature is 400 K, your velocity is 20 m/s and T_∞ =300 K. So, for a particular geometry you know that the Nusselt number is function of Reynolds number and Prandtl number. So, average Nusselt number is function of Reynolds number and Prandtl number.

So, for case 1 what is the Reynolds number? It will be $\mathrm{Re}_{L_1} = \frac{V_1 L_1}{\nu_1} = \frac{100 \times 1}{\nu_1} = \frac{100}{\nu_1}$. So, for case 2, what is the Reynolds number? Reynolds number based on the characteristic length L_2 it will be $\mathrm{Re}_{L_2} = \frac{V_2 L_2}{\nu_2}$. So, $\frac{20 \times 5}{\nu_2} = \frac{100}{\nu_2}$.

So, now, you can see thus since same air is used so, you will have $\mathrm{Pr}_1 = \mathrm{Pr}_2$. So, you will have $\nu_1 = \nu_2$. So, you can see that $\mathrm{Pr}_1 = \mathrm{Pr}_2$ because same air is used.

Similarly, your for the same air your kinematic viscosity will be same. So, you can write $\mathrm{Re}_{L_1} = \mathrm{Re}_{L_2}$ from this expression. So, now, as Nusselt number is function of Reynolds number and Prandtl number, but Prandtl number and Reynolds number are same for both the cases. So, Nusselt number also will be same.

So, hence $\overline{Nu}_{L_1} = \overline{Nu}_{L_2}$. So, you can write $\frac{\bar{h}_1 L_1}{K_1} = \frac{\bar{h}_2 L_2}{K_2}$. So, from here you can see you can write $\bar{h}_2 = \frac{L_1}{L_2} \bar{h}_1$ and $\frac{L_1}{L_2} = \frac{1}{5}$. So, $0.2\bar{h}_1$.

So, for case 1 the heat flux $q_1^"$ is given. So, you can write $q_1^" = \bar{h}_1 (T_w - T_\infty)$. So, you can write $\bar{h}_1 = \frac{q_1^"}{(T_w - T_\infty)} = \frac{20000}{400 - 300}$. So, it will be just 200 W/m^{2}K. So, once you know $\bar{h}_1$.

So, now you will be able to calculate the $\bar{h}_2$. So, for case 2, now you will be able to calculate your 0.2 $\bar{h}_1$. So, 0.2 $\times$200 so, it will be 40 W/m^{2}K.

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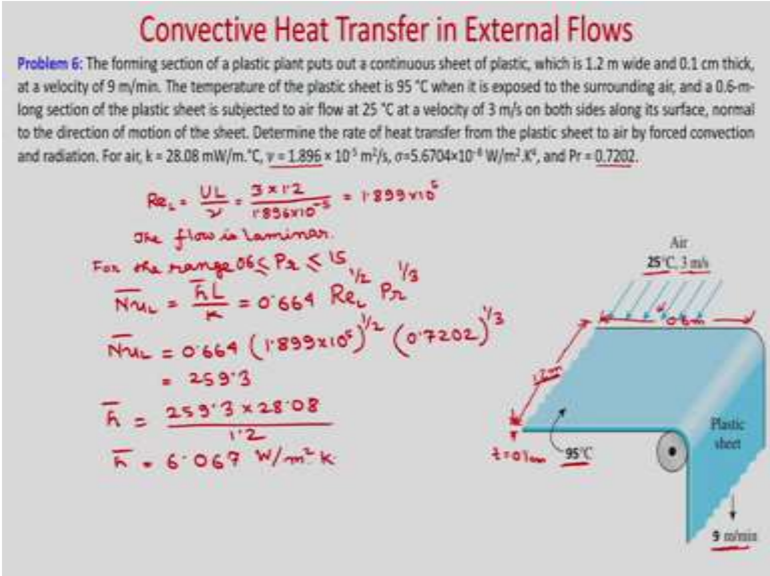

Let us consider now problem 6.

Problem 6: The forming section of a plastic plant puts out a continuous sheet of plastic, which is 1.2m wide and 0.1cm thick at a velocity of 9 m/min. The temperature of the plastic sheet is 95^0C where it is exposed to a surrounding air and a 0.6m long section of the plastic sheet is subjected to air flow at 25^0C at a velocity of 3m/s on both sides along its surface, normal to the direction of motion of the sheet.

Determine the rate of heat transfer from the plastic sheet to air by forced convection and radiation. For air, thermal conductivity, kinematic viscosity, Stefan Boltzmann constant and Prandtl number are given. So, you can see this is the continuous sheet of plastic so, the surface is maintained at 95^0C and air is flowing perpendicular to this direction of this sheet.

So, this is your 3m/s and air velocity and air velocity is 3m/s and temperature is 25^0C. And, the plastic sheet is having this velocity 9 m/min and you can see the thickness of this plastic sheet is 0.1cm then 1.2 m wide. So, this is your 1.2 m wide and you it is having 0.6 m long section. So, let us say that this is your 0.6m long section.

So, now, first let us calculate the Reynolds number and see whether the flow is laminar or turbulent. So, Reynolds number based on the width because flow is taking place in this direction. So, in this direction you can see if you consider this sheet as a flat plate then length of the flat plate will be 1.2 m. So, $\mathrm{Re}_L = \frac{UL}{\nu}$, where ν is the air velocity.

So, this is $\frac{3X1.2}{1.896X10^{-5}}$; so if you calculate the Reynolds number it will come as $1.899X10^5$ and you can see the flow is laminar because you know that if it is less than $5X10^5$ then the flow is laminar.

So, now, we can use the Nusselt number relation for the range of $0.6 \le \mathrm{Pr} \le 15$ and you know here Prandtl number is 0.72. So, obviously, we can use $\overline{Nu_L} = \frac{\bar{h}L}{K} = 0.664\,\mathrm{Re}_L^{1/2}\,\mathrm{Pr}^{1/3}$. So, from here you will be able to calculate the heat transfer coefficient.

So, if you calculate the Nusselt number, $\overline{Nu_L} = 0.664(1.899X10^5)^{1/2}(0.7202)^{1/3}$. So, if you calculate it you will get as 259.3. So, $\bar{h} = \frac{259.3X\,28.08}{1.2}$. So, h bar will be 6.067 W/m^{2}K.

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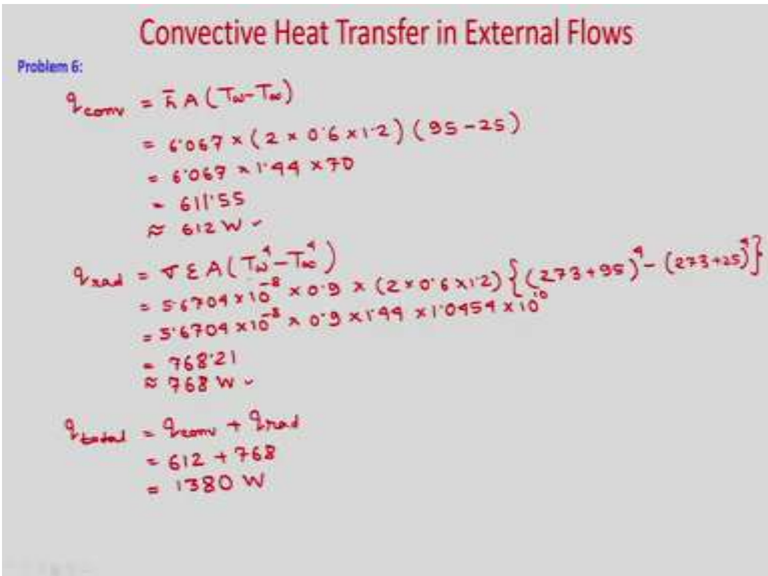

Now, let us calculate the heat transfer rate due to convection heat transfer rate due to convection you know it is $q_{conv} = \bar{h}A(T_w - T_\infty)$. So, what will be your area? So, $\bar{h}$ already we have calculated it is $6.067 \times (2 \times 0.6 \times 1.2)(95 - 25)$.

So, if you calculate it will come $6.067\times\ 1.4\ 4\ \times 70$. So, it will come as 611.55. So, you can write it as 612W. Now, let us calculate the heat transfer due to now let us calculate the heat transfer rate due to radiation. So, you know you can, $q_{rad} = \sigma\varepsilon A(T_w^4 - T_\infty^4)$. So, in this case this T you need to take in Kelvin because in radiation we know that the temperature you have to use in Kelvin.

So, $5.6704\times10^{-8}\times0.9\times(2\times0.6\times1.2)\{(273+95)^4-(273+25)^4\}$. So, if you calculate it will come as $5.6704\times10^{-8}\times0.9\times1.44\times1.0454\times10^{10}$. So, it will come around 768.21. So, let us take as 768W.

So, you see this is heat transfer rate due to convection; this is heat transfer rate due to radiation. So, the total heat transfer rate will be $q_{total} = q_{conv} + q_{rad}$. So, this is your 612+768. So, it will be 1380W.

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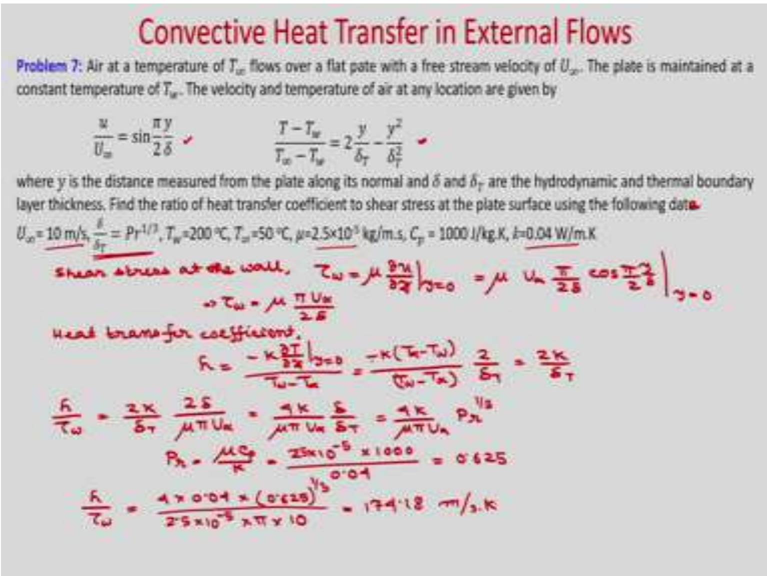

So, now let us discuss about the last problem:

Problem 7: Air at a temperature of T_∞ flows over a flat plate with a stream velocity of U_∞. The plate is maintained at a constant temperature of T_w. The velocity and temperature of air at any location are given by this is $\frac{u}{U_\infty}$.

And, this is your $\frac{T - T_w}{T_\infty - T_w}$, where y is the distance measured from the plate along its normal and δ and δ_T are the hydrodynamic and thermal boundary layer thickness. Find the ratio of heat transfer coefficient to shear stress at the plate surface using the following data.

So, you can see in this particular case your velocity distribution and temperature distribution are given. So, now, we have to calculate the ratio of heat transfer coefficient to the shear stress. So, as you know velocity distribution you will be able to calculate the shear stress and temperature distribution you know. So, you calculate the heat transfer coefficient and make the ratio.

Shear stress at the wall $\tau_w = \mu \frac{\partial u}{\partial y}|_{y=0}$. So, this is your $\frac{u}{U_\infty}$. So, this if you see $\mu \frac{\partial u}{\partial y}$. So, it will be $\mu U_\infty \frac{\pi}{2\delta} \cos \frac{\pi}{2} \frac{y}{\delta}|_{y=0}$. So, if you calculate this τ_w you will get as $\mu \frac{\pi U_\infty}{2\delta}$.

Now, heat transfer coefficient you can calculate as $h = \frac{-K \frac{\partial T}{\partial y}|_{y=0}}{T_w - T_\infty}$. So, this is the temperature distribution if you calculate $\frac{\partial u}{\partial y}|_{y=0}$, then you will get $\frac{-K(T_\infty - T_w)}{(T_w - T_\infty)}$ and at y=0. If you put then after taking the derivative with respect to y you will get $\frac{2}{\delta_T}$ and you see this is T_∞ - T_w if you write in terms of T_w -T_∞ then it will become plus and it will get cancel. So, you will get $\frac{2K}{\delta_T}$.

So, now, we need to calculate the ratio of $\frac{h}{\tau_w}$. So, $\frac{h}{\tau_w} = \frac{2K}{\delta_T} \frac{2\delta}{\mu \pi U_\infty}$. So, you will be able to see this as $\frac{4K}{\mu \pi U_\infty} \frac{\delta}{\delta_T}$ and $\frac{\delta}{\delta_T}$ you see it is given as $\Pr^{1/3}$. So, you can write as $\frac{4K}{\mu \pi U_\infty} \Pr^{1/3}$.

So, for this particular case Prandtl number you can calculate $\Pr = \frac{\mu C_p}{K}$. So, $\frac{2.5 \times 10^{-5} \times 1000}{0.04}$. So, if you see this if you calculate you will get 0.625. So, $\frac{h}{\tau_w} = \frac{4 \times 0.04 \times (0.625)^{1/3}}{2.5 \times 10^{-5} \times \pi \times 10}$.

So, if you calculate it you will get as 174.18 m/sK. So, now, we have found the ratio of heat transfer coefficient to the shear stress at the wall.

Thank you.