Computational Continuum Mechanics
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Lecture - 38
Finite Element Formulation of Ductile Fracture in Coupled-Thermo-Elastoplastic Dynamic Contact Problems

So, welcome back. The topic of Finite Element Formulation of Ductile Fracture in Coupled Thermo Elastoplastic Dynamic Contact Problem was being discussed. And, we had already discussed what is meant by damage.

So, today we are going to discuss the mathematical modelling part of the ductile fracture in coupled thermo elastoplastic dynamic contact problem ok. So, we will derive the constitutive incremental constitutive relation between stress and strain, taking into account damage the thermal strain and the effect of plasticity.

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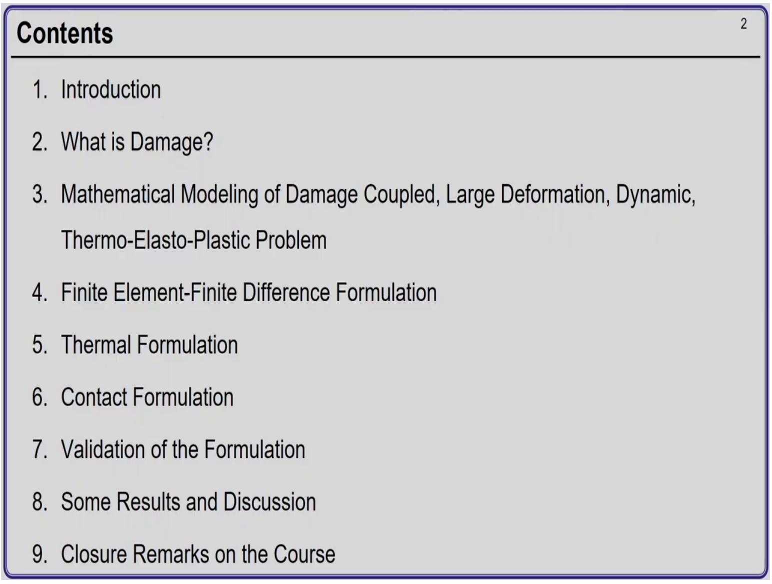

So, this is the topic that today we are going to cover ok.

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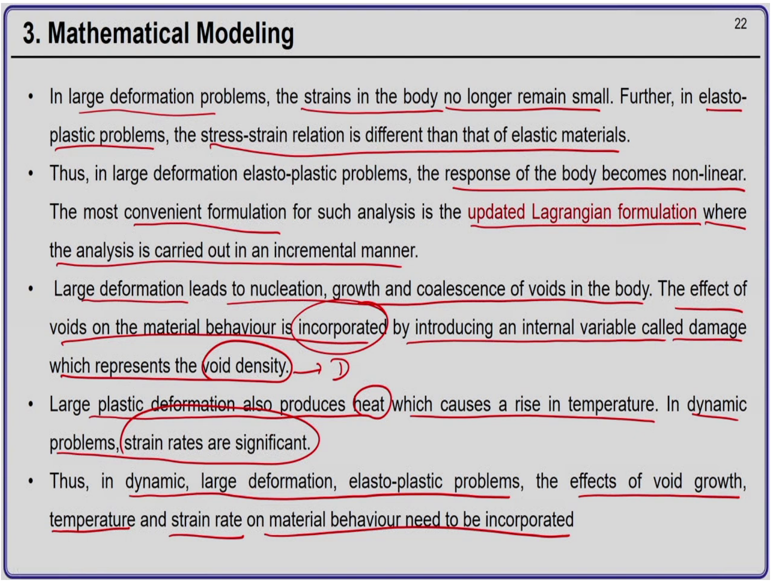

So, in large deformation problems the strains in the body no longer remain small ok. So, also in elastoplastic contact problems or elastoplastic problem, the stress strain relation is different from that of the elastic material ok.

So, the response of the body then becomes non-linear for the case of elastoplastic contact problem. And to deal with such problems to the most convenient find a formulation ok, is the updated Lagrangian formulation ok. Where the analysis is carried out in a incremental manner ok.

So, large deformation also leads to nucleation growth and collisions of voids in the body. So, that was discussed in previous slides and the effect of voids on the material behaviour ok, will be incorporated by introducing an internal variable called the damage which will represent the void density. So, our damage will be represented by symbol D ok.

And large plastic deformation also produces heat, which causes the rise in temperature and in dynamic problems the effect of strain rates is also very significant ok. Thus in dynamic large deformation elasto plastic problem the effect of void growth, temperature and strain rate on the material behaviour needs to be incorporated. And this is what we are going to do next ok.

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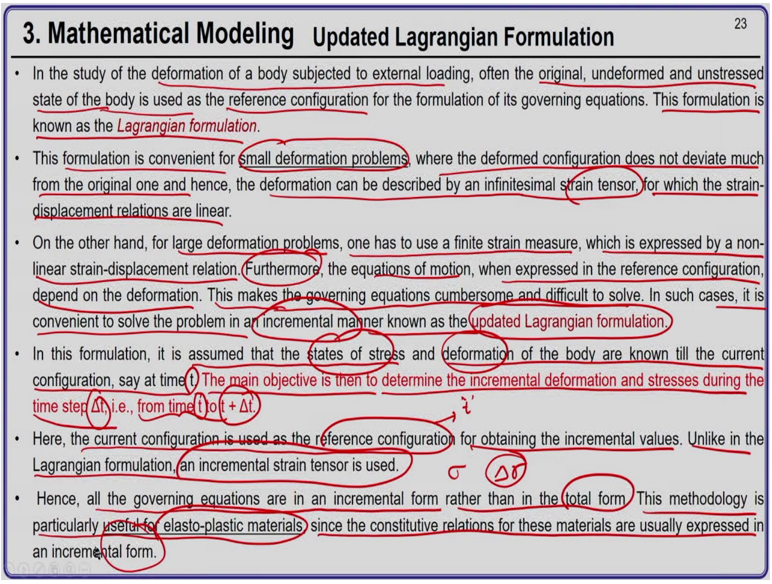

So, in the study of deformation of a body subjected to external loading, we often use the original undeformed or the unstressed state of the body ok, as the reference configuration ok. Now, this kind of formulation is known as the Lagrangian formulation. So, this we had already discussed.

However, this formulation is only convenient for small deformation problems, where the deformed configuration does not deviate much from the original one. And hence the deformation can be considered by an infinitesimal strain tensor, for which the strain displacement relation are linear. On the other hand for large deformation problems, one has to use a finite strain measure, which can be expressed by a non-linear strain displacement relation ok.

Furthermore the equation of motion when expressed in the reference configuration, depend on the deformation ok. So, if we use Lagrangian formulation this will make the governing equation cumbersome and difficult to solve. In such cases it is convenient to solve the problem, in an incremental manner ok.

And this kind of formulation is known as the updated Lagrangian formulation ok. So, in updated Lagrangian formulation, it is assume that the states of stress and deformation of the body are known till the last configuration say time t ok. So, we know everything about the body till time t.

And the main objective is then to determine the incremental deformation and stresses during this time step delta t, that is we want to find out what will be the configuration of the body at time t plus delta t given the configuration at time t ok.

So, hence the current configuration is used as the reference configuration ok. So, the configuration at time t, where we know the states of stress and deformation is used as the reference configuration ok, rather than the configuration at time t equal to 0 for obtaining the incremental values. So, unlike in the Lagrangian formulation an incremental strain tensor will now be used ok. So, hence all the governing equations are in an incremental form rather than in the total form ok.

So, which means that rather than writing say sigma ok, we will write delta sigma which delta represent the increment in the stress ok. So, all such quantities in the governing equation will be written in this kind of incremental form ok. So, this methodology is particularly useful for elastoplastic problems. Since the constitutive relations for these materials are usually expressed in the incremental form ok.

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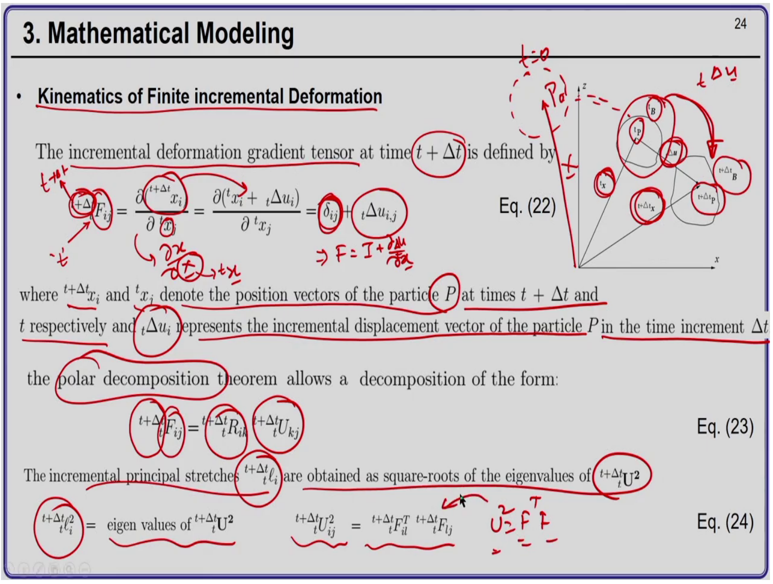

Now, coming to the kinematics of finite incremental deformation so, you consider you have a body at time t ok. So, we know the configuration at time t and consider a point P whose current coordinate is gained by t x.

And now over the time delta t, there is a displacement delta u and the body occupies the current position t plus delta t P ok. Therefore, the new position of the point P is t plus delta t P and the new position vector is given by t plus delta t x ok. Then, the incremental deformation gradient tensor at time t plus delta t will be written as t plus delta t t F ij ok.

So, here the lower subscript means we know everything at time t, and the upper subscript t plus delta t means we are going from time t to t plus delta t our objective is to determine the state of stress and displacement or the configuration at t plus delta t ok. And the deformation gradient tensor F is, then given by del by del x of t plus delta t x ok. So, this is just like our t x by del x ok.

Now, instead of the material initial position x, now we have because we are using updated Lagrangian formulation our reference configuration now is at time t therefore, the position is at time t x. We are not writing capital X, because we want to make it clear that it is for the previous configuration, converse configuration and not for the initial configuration ok.

Now, here t plus delta t x can be written as sum of t of x plus t of delta u and this is when we substitute. And then we open up the brackets we get delta i j plus t delta u i comma j so, the gradient of incremental displacement plus the Kronecker delta ok.

So, if you want to write in direct notation this will be F equal to I plus del u by del x and this is at time t ok delta nu ok. So, here t plus delta t x and t x denote the position vectors of particle P at time t plus delta t and t respectively. And this quantity t delta u represents the incremental displacement vector of a particle P in the time increment delta t ok.

So, delta u denotes the displacement of particle P from t to t plus delta t, remember the initial configuration may be at time t equal to 0 maybe somewhere here that may be P 0. And we started with t equal to 0 and we have reached t, where we know everything now we want to go from t to t plus delta t. So, this displacement t delta u is actually displacement from t to t plus delta t ok.

Now, using the polar decomposition theorem so, right polar decomposition theorem, states that F can be decomposed into an orthogonal tensor R and a symmetric tensor U, where U is also called the right stretch tensor ok. So, F can be written as R into U the only thing to notice now is there are subscripts and superscripts t and t plus delta t, which signify that we are trying to find quantities from t to t plus delta t ok.

Now, the incremental principle stretches given by t to t plus delta t l i are obtained from the square root of the eigenvalues of U ok. So, l square is nothing, but is the eigenvalue of U square ok. And then U square is given by F transpose F, this we know this is U square is F transpose F. So, this is nothing, but the initial notation ok.

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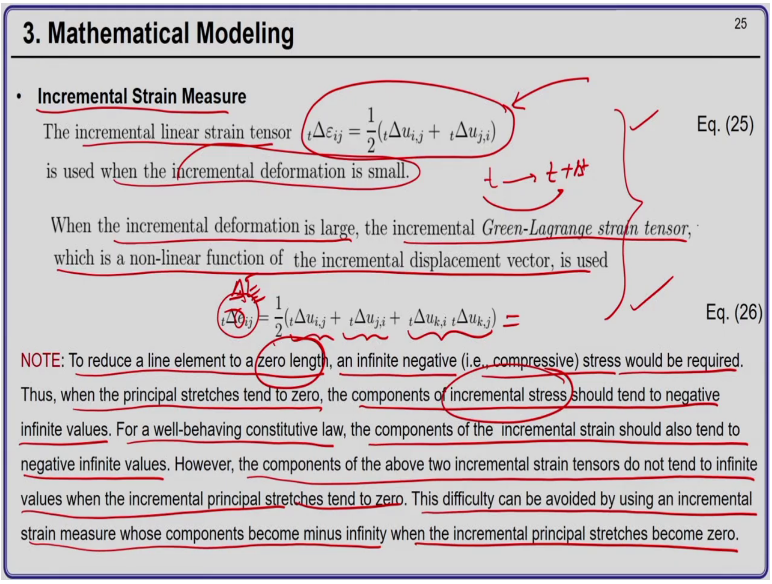

Then, once we have defined our incremental deformation gradient tensor, then we come to what is called the incremental strain measure ok. So, for small deformation; so, the incremental linear strain tensor is given by following formula ok. So, delta epsilon is equal to 1 by 2 delta u gradient of delta u plus the gradient of delta u transpose ok. So, now this incremental linear strain tensor is used, when the incremental deformation is small ok.

So, which means that going from t to t plus delta t, if in between the incremental deformation is small, you will can use the incremental linear strain tensor. However, we are not sure that from t to t plus delta t, then the deformation will be small it can be appreciable also. And in that case if the incremental deformation is large, the incremental Green-Lagrange strain tensor which is a non-linear function of incremental displacement vector is used ok.

So, remember we are using a different convention than the what we have established in our lectures. This is because this we have taken from the this follows the work of bathe the book from bathe so, we are using that convention here ok.

But, you can recognise this is nothing, but so this is nothing, but can be written as delta E, where E is your Green-Lagrangian strain tensor we are using small e ok. So, t delta e equal to 1 by 2 gradient of u plus the transpose of the gradient of u plus the product of gradient of u times gradient of u transpose ok.

Now, to one of the important things to note now is consider you have a line element ok. And now you want to reduce it to a zero length ok, you want to compress and reduce it to zero length. So, you will need an infinite amount of negative that is compressor stress ok. And thus when the principal stretches tend to zero which is in the previous slide, the components of the incremental stress should tend to be negative infinite values ok.

Because, principle stretch is nothing, but the ratio of the final length to initial length and now when you are compressing a line element to zero length the final length becomes zero. So, the principal stretch should become 0.

Now, to get that 0 value of the principal stretch the incremental stretch should be infinite only infinite stress can get you this 0 stretch. So, for a well behaving constitutive law the component of the incremental strain should also tend to negative infinite values ok. So, that your stress comes incremental stress comes out to be infinite.

However, the components of the above to incremental strain tensor do not tend to infinite values when the incremental principal stretches tend to zero ok. So, in one of our earlier lecture on kinematics, you have seen that for 1 D case we have derived the relation for linear strain and the Lagrange strain in terms of stretches in the 1 D direction. And if you plot that you will see that for 0 value of stretch these two strain tensor do not go to infinity ok.

So, hence you will not get a physically well behaving constitutive law you will not get negative infinite stress. So, this difficulty can be avoided by using an incremental strain measure whose component become minus infinity, when the incremental principal stretches become zero. So, if you use an incremental strain measure for which when the stretches go to 0, then the value of the strain incremental strain should go to infinite minus infinity and this can happen ok.

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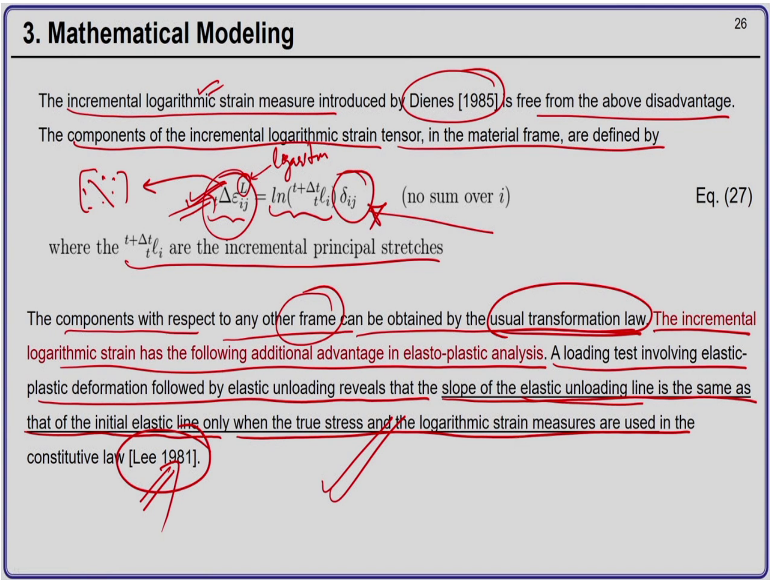

This can happen only when you use incremental logarithmic strain measure, which was introduced by Dienes in 1985. And this incremental logarithmic strain measure is free from the above disadvantage ok.

So, the components of the incremental logarithmic strain tensor in the material frame is defined by following equation. So, incremental logarithmic strain so, there is a capital L here that shows that is an incremental logarithmic strain it is logarithmic L for logarithmic ok.

So, this incremental logarithmic strain is nothing, but natural log of the of the stretches incremental principal stretches times the Kronecker delta ok, here this is the incremental principal stretch ok. Now, this is in the reference in the frame formed by the principal vectors.

So, the components of the logarithmic strain tensor in any other frame can be obtained by transformation rule ok. So, you can transform these components remember this is delta i j. So, this equation 27 will result in your diagonal tensor ok. Only your diagonal component will be nonzero and this is with respect to your principal vectors.

Now, you can use the principal vectors and transform it to another frame say your original coordinate frame of reference using transformation law that, we have already discussed in our discussion on tensors ok. So, incremental logarithmic strain has the following additional advantage in the elasto plastic analysis ok.

So, loading test involving elastic plastic deformation followed by elastic loading reveals that the slope of the elastic loading line is same as the that of the initial elastic line. Only when the two stress and logarithmic strain measures are used in the constitutive laws, and this was shown by Lee in 1981 ok.

So, apart from the advantage that the components of logarithmic strain tensor approach minus infinity as your value of stretches go to 0. It also has the additional advantage that the slope of the elastic unloading line will be same as the initial elastic line for elasto plastic case ok, and it was shown by lee ok. So, in our work which were showing here, we will be using the incremental logarithmic strain measure as our measure of strain ok.

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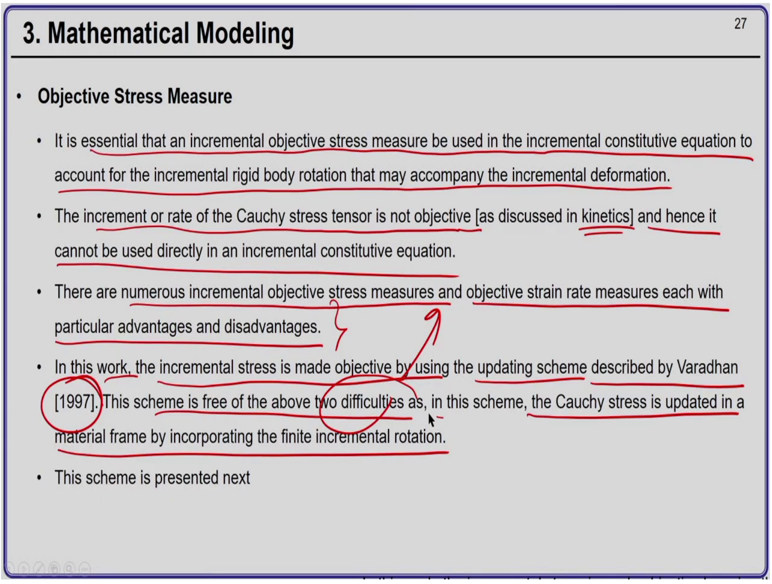

Now, comes the objective stress measures ok. So, it is essential that an incremental objective stress measure be used in an incremental constitutive equation to account for the incremental rigid body rotation that may accompany the incremental deformation. So, that we have discussed when we are discussing the objective stress measure that, if you have super impose rigid body motions apart from the deformation.

Then, the increment in the stress is only because of the deformation and not because of those rigid body rotation ok. So, the increment or the rate of Cauchy stress, we have already shown in our previous lecture is not objective this was already discussed. Hence, it cannot be used directly in an incremental constitutive equation. So, there are numerous incremental objective stress measures and objective strain rate measures, each which has particular advantages and disadvantages.

So, we have already shown that the Truesdell stress rate, Green Naghdi stress rate, this Jaumann stress rate there are many that we discuss and its own advantages and disadvantages ok. So, in our work what we do we do not compute the incremental stress, what we do is we rotate the stress in the material frame ok. And then, we compute the incremental stress add it there and revert it back we will discuss it later.

So, we use a rather than incremental stress computation procedure, we use what is called the updating scheme ok. So, the incremental stress in our work will be made objective by using an updating scheme, which was described by Varadhan in 1997. And this scheme is free from the two difficulties so, the above two difficulties which we measure.

So, the scheme is not having those shortcomings and the Cauchy stress is updated in a material frame by incorporating the finite incremental rotation. So, even if the rotation is large however it does not matter, we just update the Cauchy stress in the rotated material frame ok. So, we just describe this next.

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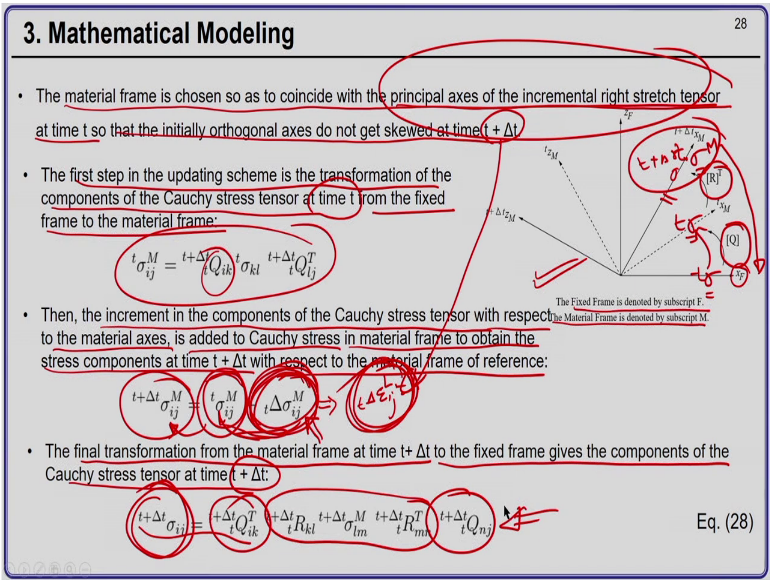

So, consider this particular figure, where the fixed frame is denoted by subscript F and the material flame is denoted by subscript M. So, fixed frame is what we use and the material frame is something which is attached to each gauss point of the material ok. So, now we are talking in terms of the finite element or even in the continuum setting for each point. In the material we have what is called the material frame?

And this material frame is like observer is sitting at that point and its observing the deformation. So, that observer will not be able to distinguish; that deformation not distinguish the rotation ok. So, he will just observe only the rotation ok. So, the material frame is chosen so, as to coincide with the principal axes of the incremental right stress tensor at time t ok.

So, that the initially orthogonal axes do not get skewed at time t plus delta t ok. Now, the first step in updating scheme ok, which we are using is the transformation of the components of the Cauchy stress at time t to the fixed frame at time t, which is in the fixed frame to the material frame ok. So, from fixed frame we have the stresses here, and we rotate those stresses from the fixed frame to the material frame at time t ok.

Next so, for this we use this particular formula and Q is the transformation matrix. Then, the increment in the components of the Cauchy stress tensor with respect to the material axis is added to the Cauchy stress in the material frame to obtain the stress component at time t plus delta t with respect to the material frame ok.

So, the Cauchy stress component in the material frame at time t plus delta t is nothing, but the Cauchy stress component, in the material frame at time t plus the incremental Cauchy stress in the material frame at time t ok. So, this is computed from the incremental logarithmic strain measure which was discussed in the previous slides ok.

So, this was already in the principal axis of the incremental right stretch tensor ok. So, these components of the logarithmic strain, were already in the principal axis of the incremental right stress tensor.

And now we have this stress also in the principle of principal axis of incremental right stretch tensor. So, we can add these two and then what we get will be the component of the Cauchy stress tensor with respect to material axis at time t plus delta t ok.

Next what we do? So, we will now get in this frame over here t plus delta t delta t sigma M ok. Now, once we have the Cauchy stress here we can finally, transform from material frame at time t plus delta t to the fixed frame giving the component of the Cauchy stress at time t plus delta t in the fixed frame ok. So, now I can go back to the fixed frame and from going back I use the rotation and I use the transformation ok.

So, I first use the rotation to go back at the material frame at time t ok. And then, I use the transformation matrix to go back to the fixed frame at time go back to the fixed frame at time t plus delta t and then we get the Cauchy stress at time t plus delta t ok. So, what we you can observe from our previous discussion on objective stress measure here, the incremental Cauchy stress is not being made objective by removing certain components.

We are directly using an objective form of we are directly using the incremental logarithmic strain, which is there in the rotated frame itself ok. So, the rotation has already been taken into account. So, we just multiply this by the material constitutive tensor to get this and then add to the stress in the rotated frame to get the new value.

So, we are not subtracting anything from the incremental stress, as we were doing in the previous objective stress measure ok. So, this will give you the objective stress ok. So, the effect of rotation will be taken out.

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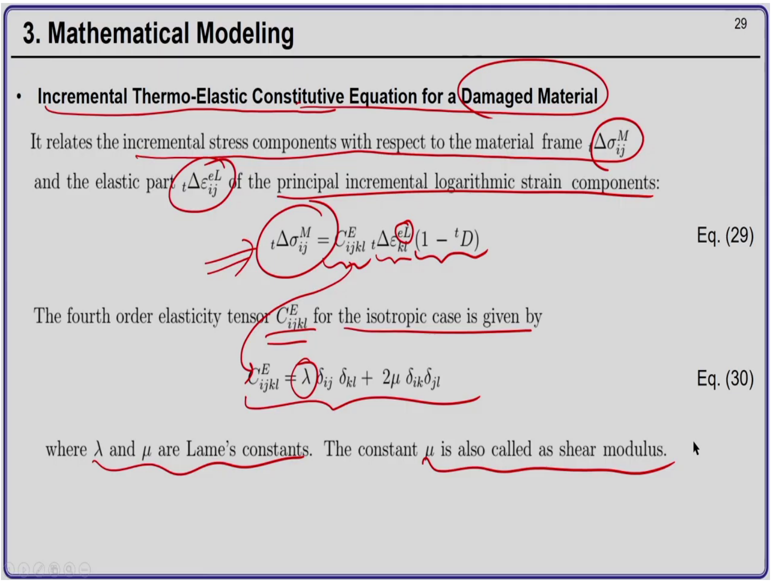

Now, we derive the incremental thermo elastic constitutive relation for a damaged material ok. So, this relation relates the incremental stress component with respect to the material frame t delta sigma M.

And the elastic part of the logarithmic strain tensor principal logarithmic strain component. So, the incremental stress in the material frame is equal to the material elasticity tensor C ok, times the elastic part of the principal logarithmic strain component delta epsilon e L ok.

So, this denotes the elastic part of the logarithmic strain times 1 minus D. Because, it’s a damage material from using the concept of effective stress this is what we get. Here the fourth order elasticity tensor for isotropic case is given by following formula ok. C ijkl is lambda delta ij delta k l plus 2 mu delta i k delta j l here, lambda and mu are lames constant and mu is also called the shear modulus ok.

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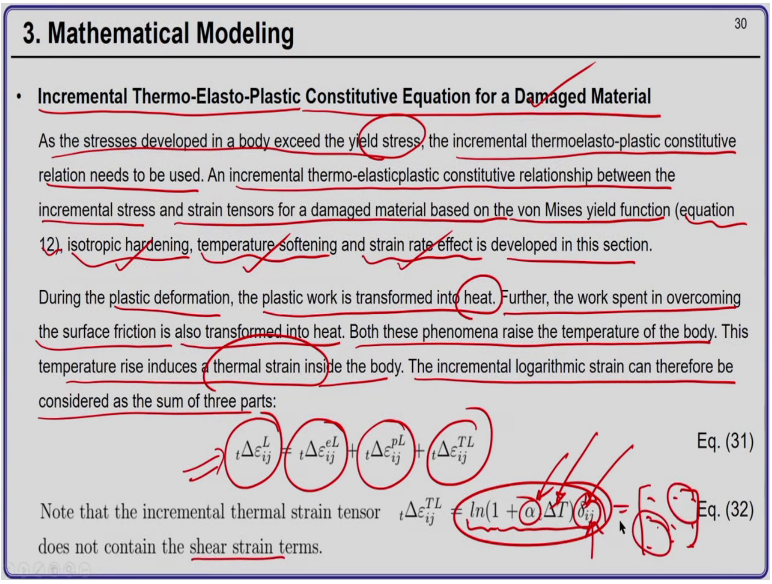

Now, once we have derived the incremental thermo elastic constitutive equation for damaged material, we derived the incremental thermo elasto plastic constitutive equation for a damaged material ok. So, as the stresses inside the body exceed the yield stress the incremental thermo elasto plastic constitutive relation needs to be used.

So, the previous relation that we derive was for elastic case. And now we have to derive the similar incremental equation for elasto plastic case ok. And this is needed because, our body may go into plastic state and the stress inside the body at certain points can exceed the yield stress.

So, an incremental thermo elastoplastic constitutive relation between the incremental stress and strain tensor for a damaged material based on the von Mises yield function which was given by equation 12 in the previous slide, isotropic hardening with temperature softening and strain rate effect will now be developed ok.

So, we will take into account isotropic hardening ok, thermal softening and strain rate effect and also we will take into account the damage ok. So, time temperature and damage all will be considered ok. Now, during plastic deformation the plastic work gets transformed into heat and this you would also have noted, whenever you bend a metal strip or something like this repeatedly like up and down up and down ok.

Then, the bent zone will if you touch you will see that it has got heated up and this is because, the plastic work is getting transformed into heat. So, furthermore the work spent in overcoming the surface friction is also transformed into heat. So, for a contact problem so, whenever there is a friction there will be frictional heat.

And both these phenomena raise the temperature of the body one in the bulk and the other in the surface contact surface. So, this temperature raise induces a thermal strain inside the body and the incremental logarithmic strain can therefore, be considered as the sum of three parts ok.

So, this is the incremental logarithmic strain, it is composed of three parts that is the incremental logarithmic strain corresponding to the elastic part, the incremental logarithmic strain corresponding to the plastic part. And the incremental thermal strain tensor, given by natural log of 1 plus alpha delta T into delta ij here, alpha is the your alpha is the constant material constant and delta is the Kronecker delta. And delta T is the temperature rise.

So, because there is a Kronecker delta this again will be a diagonal tensor ok. And you notice that there are no shear terms. So, all the off diagonal terms are 0, because here you have a Kronecker delta ok.

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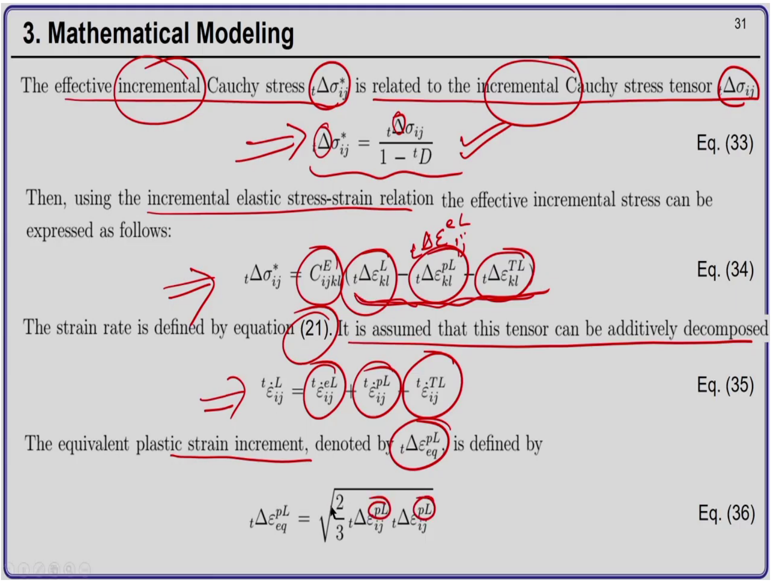

Then, the effective incremental Cauchy stress delta sigma star is related to the incremental Cauchy stress tensor delta sigma as following equation ok. So, this we directly take from our previous slide, where we did not have this incremental symbol. But, now we introduce that incremental symbol and we say that the relation which existed between effective Cauchy stress and in the and the Cauchy stress tensor is same ok, will be carried over to the effective incremental ok.

So, effective incremental Cauchy stress and the incremental Cauchy stress tensor ok. So, this is the equation. Therefore, now if I substitute the incremental elastic strain from equation 32 in our relation for the incremental elastic stress strain relation. So, I get the effective incremental Cauchy stress as C ijkl into this is nothing, but delta epsilon elastic ij ok. So, this from equation 32 is nothing, but this over here.

So, total strain minus the plastic part minus the thermal part ok. And the strain rate can be defined using equation number 21 and it is assumed that this tensor can be additively decomposed into the strain rate elastic strain rate.

The plastic strain rate and the thermal strain rate ok. And the equivalent plastic strain increment, which is denoted by this symbol over here, is given by root over 2 by 3 delta epsilon ij delta epsilon ij corresponding to the plastic part of the logarithmic strain tensor ok.

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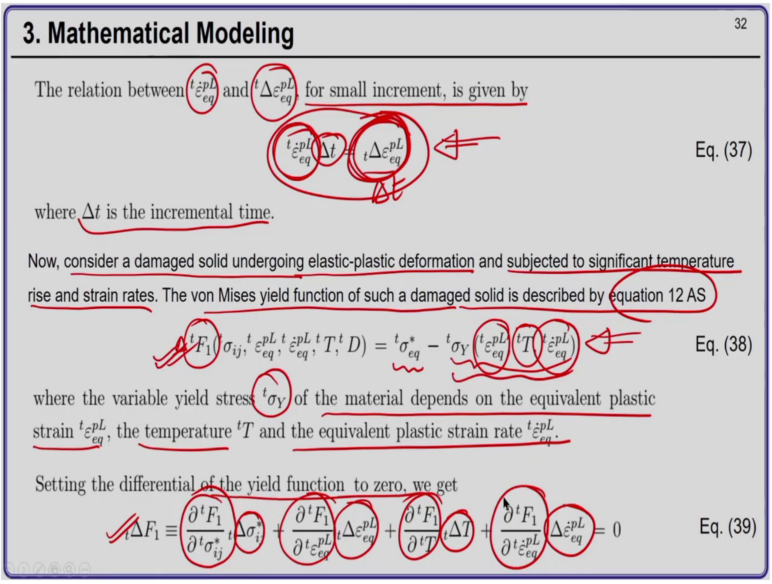

With this the relation between the equivalent plastic strain rate and the equivalent plastic strain, for small increment can be given by following formula so, the rate into the time step equal to the incremental value so, where delta t is the incremental time. So, in our computer implementation we will compute the increment in the plastic strain from this formula.

So, when we have to compute the plastic strain rate then that plastic strain rate will be nothing, but approximately equal to the incremental plastic strain divided by the time step delta t of that particular load step ok. Now, we consider a damaged solid undergoing elasto plastic deformation and subjected to significant temperature raise and strain rates ok. So, the von Mises yield function for a damaged material is given by equation 12 as following relation ok.

So, F 1 is nothing, but the equivalent effective stress minus the yield value. And this yield value of the material will be function of the plastic strain, the temperature and the strain rate. And this relation will have to be derived from the experiment most often, we use Johnson's-Cook formula ok, but right now we will leave it like this ok.

So, here the variable yield stress of the material depends on the equivalent plastic strain, the temperature and the equivalent plastic strain rate ok. Now, if we set the differential of this yield function to 0, we will get following relation ok. So, delta F 1 will become del F 1 by del sigma star into delta sigma star, del F 1 by del epsilon equivalent plastic into delta epsilon equivalent plastic plus del F 1 by del T into delta T plus del F 1 by del epsilon del of equivalent plastic strain rate into the incremental equivalent plastic strain rate ok.

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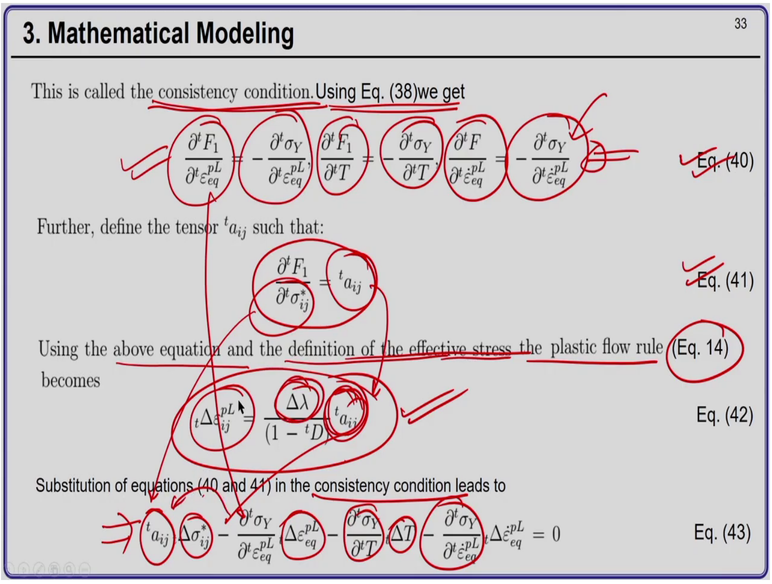

So, this condition is called the consistency condition ok, in plasticity this condition is called the consistency condition. Because, this forces that our material point always stays on the yield surface ok. Now, using equation number 38 you can write del F 1 by del epsilon equivalent plastic as minus del sigma Y by this del F 1 by delta T like this. And del F by del epsilon dot equivalent plastic as this ok.

Where, if you are given sigma Y you can explicitly in terms of these quantities, you can compute these relations in equation 40 ok. And furthermore I can define the derivative of the this potential F 1 with respect to e effective stress sigma as another tensor a given by t a ij ok.

So, using the above equation and defining and using the definition of effective stress the plastic flow rule, that is equation 14 that we had. We can show that the incremental plastic strain is given by delta lambda upon 1 minus D into this tensor a which is given by here ok.

So, if you substitute equation number 40 and equation number 41 in the consistency condition on the previous slide, you will get following relation ok. So, this was this here ok. And this was delta star and this was here into delta epsilon equivalent plastic into del sigma Y by delta t into delta T minus this ok. With this I can now substitute delta epsilon plastic ij ok.

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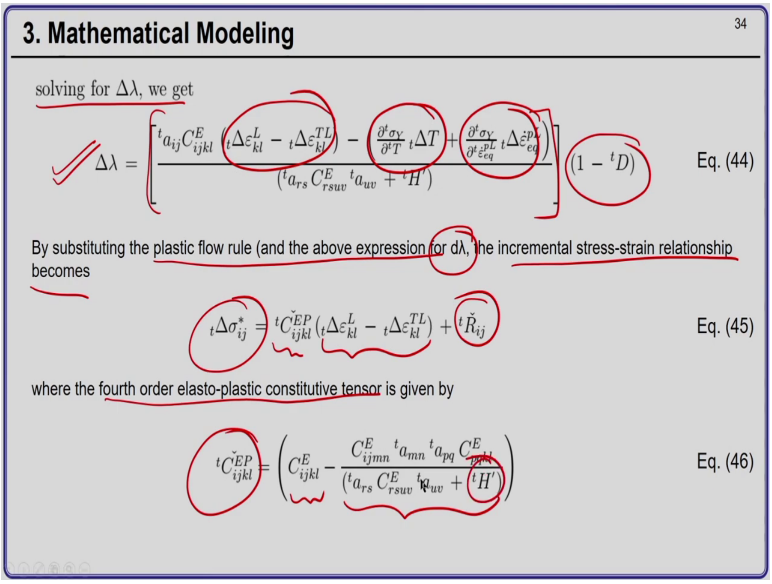

And solve ok. So, I can use this expression over here and I can put it here, and then I can compute for delta lambda ok. So, that step I am skip and I will directly write that solving for delta lambda, you can get delta lambda by this long expression ok.

So, here the effect of damage is into taken into account, the effect of strain rate is taken into account the effect of temperature is taken into account and; obviously, you have the effect of plasticity over here ok. So, by substituting the in the plastic flow rule the above expression for delta lambda, the incremental stress strain relationship will become like this ok.

So, the effective equivalent stress is given by C hat elastoplastic into the difference of the incremental logarithmic strain minus the incremental thermal strain plus a tensor R hat. Where, the fourth order elastoplastic constitutive tensor C hat becomes the fourth order elasticity tensor C minus the term which is here ok.

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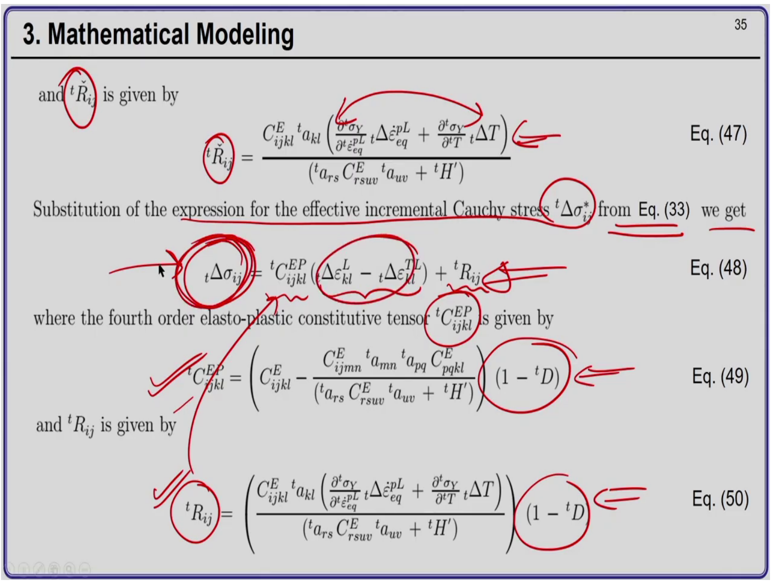

So, here this H dash will be computed from the yield value ok. And this we will see an R hat is what couples the plasticity with the temperature ok. So, R hat is given by following relation ok. So, substitution of the expression for the incremental Cauchy stress from equation number

33, we get that the incremental Cauchy stress, that is actually the stress incremental Cauchy stress is equal to C ijkl elastoplastic into the incremental the plastic strain plus R ok.

Where, fourth order elasto plastic constitutive tensor C EP is given by following relation. Now, you see the effect of damage also has come, remember this incremental stress is what is the true increment of the stress, delta sigma star was the equivalent stress in the virgin material, but our ours is not a virgin material.

So, in the damaged material this is the value of the incremental Cauchy stress. This stress is given by equation number 48 and equation number 49 and 50 gives you the expression for the tensor C and R respectively ok. So, once you have C and R you can substitute here and you can have the increment in the strain, you can directly compute the increment in the Cauchy stress ok.

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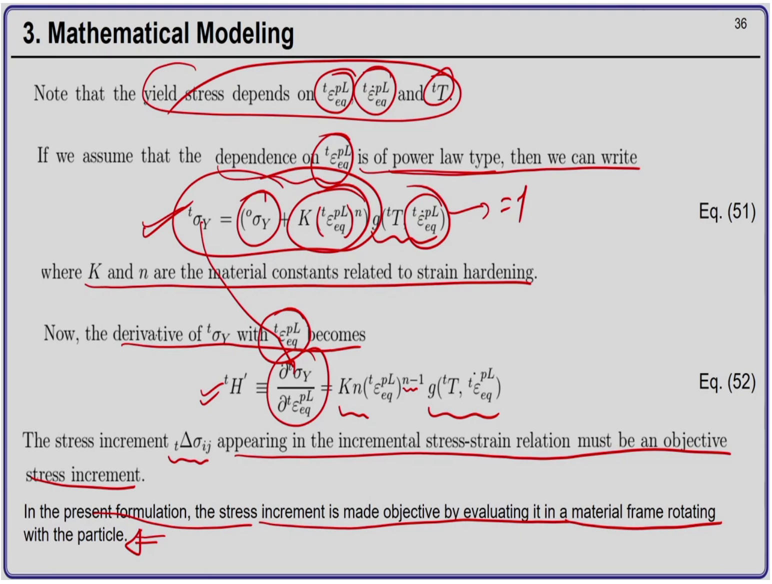

So, you notice that the yield stress depends on the plastic strain, equivalent plastic strain equivalent plastic strain rate in the temperature. And we, if we assume that the dependence of the equivalent plastic strain dependence of the sigma Y on the plastic strain equivalent plastic strain is of the power law type.

Then, I can write the yield stress at time t as the initial yield stress plus K times the epsilon equivalent plastic strain raised to power n into a function of temperature time temperature and the strain equivalent plastic strain rate ok. Here K and n are the material constants which are related to strain hardening.

Now, if I had a isothermal static case then that function will be equal to 1 and therefore, I get the usual power law type of isotropic hardening ok. Now, the derivative of sigma Y with equivalent plastic strain becomes t H dash equal to del sigma Y by del epsilon equivalent plastic equal to ok. So, you just substitute this here, you get K n into del epsilon equivalent plastic raised to power n minus 1 into function g ok.

Now, the incremental stress increment appearing in the incremental stress strain relation must be an objective stress increment ok. And in the present formulation as we discussed in the previous slide the stress increment is made objective, by evaluating it in the material frame rotating with the particle ok; while using the procedure that we discuss in our previous slide ok.

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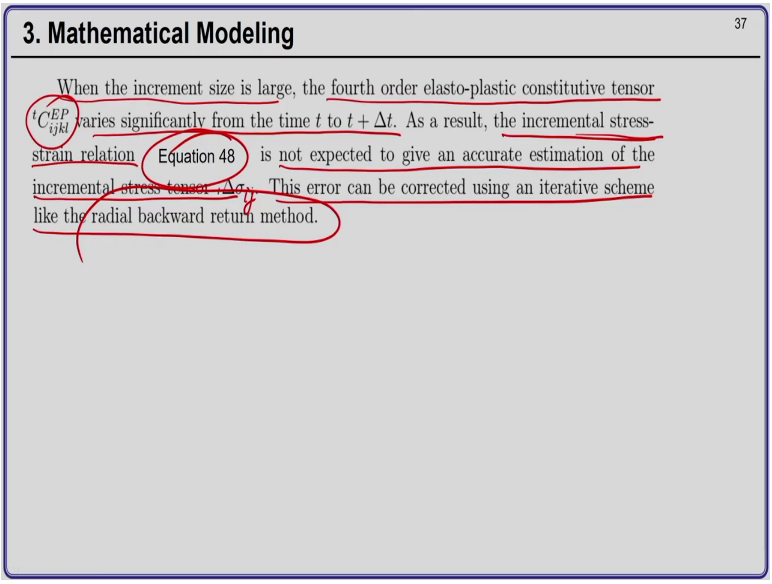

So, the final point before we move to finite element formulation is when the increment size is large, the fourth order constitutive elastoplastic tensor given by C EP varies significantly from t to t plus delta t. Which means; that the result that the incremental stress strain relation given by equation number 48 is not expected to give an accurate estimation of the incremental stress, given by t delta sigma ij.

So, this error can be corrected by using an iterative scheme for example, the radial backward return algorithm ok. So, this also we will discuss or we will just briefly touch upon in our finite element formulation ok. So, with this we end the mathematical modelling part, where we have derived the relation for the incremental Cauchy stress.

And in next we start the finite element formulation where we first derive the weak form, and then we will do the finite element discretization to get the Newton's second law in the discretized configuration and when we do the finite difference scheme to get the algebraic equations which will then be set for Newton Raphson iterative procedure ok.

So, with this I end today's lecture ok.

Thank you.